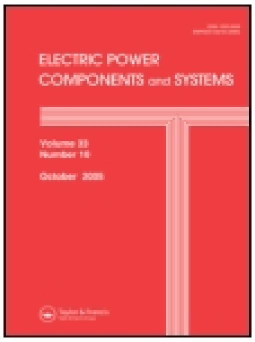

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Electric Power Components and Systems

ISSN: 1532-5008 (Print) 1532-5016 (Online) Journal homepage: https://www.tandfonline.com/loi/uemp20

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To cite this article: Carolina Goulart de Carvalho, Eduardo C. Marques Costa, Sérgio Kurokawa & José Pissolato (2016) Alternative Phase-domain Model for Multi-conductor Transmission Lines Using Two Modal Transformation Matrices, Electric Power Components and Systems, 44:3, 291-301, DOI: 10.1080/15325008.2015.1110733

To link to this article: https://doi.org/10.1080/15325008.2015.1110733

Electric Power Components and Systems, 44(3):291–301, 2016

ISSN: 1532-5008 print / 1532-5016 online
DOI: 10.1080/15325008.2015.1110733

Alternative Phase-domain Model for Multi-conductor Transmission Lines Using Two Modal Transformation Matrices

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Abstract—A new model for multi-conductor transmission lines is proposed based on an alternative modal decoupling technique. The line decoupling is accomplished using two modal transformation matrices: the real and constant Clarke's matrix and a reduced frequency-dependent matrix for decoupling the remaining mutual terms in the impedance and admittance matrices (quasi-modes). This procedure results in a simple matrix formulation in the frequency domain, which time-domain results can be obtained using inverse transforms. The proposed line model provides accurate results because the phases of a multi-conductor line are decoupled into exact propagation modes from a second modal transformation, using a reduced frequency-dependent matrix. Each propagation mode is modeled in the frequency domain as three independent two-port circuits. The proposed model is evaluated in the frequency and time domains based on results obtained from the line modeling using the exact modal transformation matrix of the line. The advantage of the proposed line model is the simplified modeling based on a constant and real matrix and a reduced matrix with dimension two per two instead a frequency-dependent transformation matrix with dimension three per three.

Keywords: power system modeling, transmission lines, electromagnetic transients, modal analysis

Received 23 June 2015; accepted 3 October 2015

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1. INTRODUCTION

Reliability is one of the main issues concerning power distribution, transmission, and generation. Several important issues are directly associated with the maintenance of a reliable power system, *e.g.*, the correct parametrization of the protection system, previous knowing of possible over-voltage transients, the insulation coordination, and the propagation characteristics of the line. These issues are dependent on the accuracy in which power transmission systems are represented and modeled. An accurate transmission line modeling (TLM) is essential for prevention of possible fault occurrences and for development of new analysis techniques for global parametrization of power systems.

Several line models are available in the technical literature on TLM that vary in accuracy depending on the electromagnetic transient to be simulated and the fault characteristics. The line models can be classified into two principal groups: the time-domain models using equivalent electric circuits and the frequency-domain models based on the direct representation of the line parameters by a two-port circuit. The two modeling methods present restrictions and advantages depending on the transmission system to be modeled and transient characteristics [1–3].

Time-domain line models are developed based on the approach by lumped R, L, and C elements. The frequency effect in the line impedance Z(ω) is included in the line model using fitting techniques, *i.e.*, the frequency-dependent line impedance Z(ω) is approached by a rational function [4]. Thus, the frequency dependence of the line parameters is modeled directly in the time domain. However, the fitting of the line parameters for a wide range of frequencies could result in inaccuracies in simulations of fast and impulsive transients [1, 5].

The frequency-domain models are developed directly from the frequency-dependent parameters of the line, representing accurately the distributed nature of the line impedance $Z(\omega)$. The transient currents and voltages are simulated in the frequency domain and the time-domain results are obtained using inverse transforms [2, 6]. Though frequency-domain models provide accurate results for a wide range of frequencies, these models have some difficulties in the inclusion of non-linear and time-variable elements during the simulations [1, 5].

The multi-phase representation is also an important issue directly related to the performance of transmission line models. An efficacious method, widely approached in technical literatures in TLM, is the line modal decoupling using transformation matrices. This modal technique consists in to decouple an n-phase transmission line into n independent propagation modes that can be modeled as n single-phase lines in the time domain using lumped circuits or in the frequency domain using two-port circuit [1, 4]. The modal decoupling is usually performed using a modal transformation matrix calculated as a function of the frequency-dependent parameters of the line [7]. However, depending on the line model, the frequency-dependent matrix can be substituted by an approximated matrix composed of constant and real terms [1]. Although the procedure with an approximated matrix is sometimes required, and represents a significant simplification in the line modeling, errors are introduced in the modeling and simulation process [8].

This article proposes an accurate frequency-dependent model for multi-conductor transmission lines directly from the frequency-dependent parameters. The original contribution of this research is the multi-phase representation using two transformation matrices, to eliminate the implicit errors in the modal decoupling, and the analytical formulation based on a three-phase two-port circuit. The exact modal decoupling is obtained using two matrices: the constant and real Clarke's matrix and a two-per-two frequency-dependent matrix to decouple the remaining mutual terms from the first modal decoupling using the Clarke's matrix. This procedure ensures the exact modal decoupling without the use of a three-dimensional frequency-dependent transformation matrix using the Newton–Raphson method [7].

2. MODAL DECOUPLING OF THREE-PHASE TRANSMISSION LINES

Modal decoupling techniques applied to multi-conductor transmission lines consists in to decouple an n-phase transmission line into n independent propagation modes, *i.e.*, an n-phase line is represented as n propagation modes completely decoupled from each other. Thus, the n propagation modes can be modeled as n single-phase lines without modeling the mutual terms of the impedance matrix $[Z]$ and admittance matrix $[Y]$. The impedance and admittance matrices of an n-phase transmission line are expressed as follows:

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix}, \tag{1}$$

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix}. \tag{2}$$

Matrices $[Z]$ and $[Y]$ are square matrices with dimension n. The terms in the main diagonal are self-parameters, whereas the remaining terms are the mutual parameters between phases. The self and mutual impedances in $[Z]$ are composed of frequency-dependent resistances and frequency-dependent inductances, generically expressed as

$$Z_{ij} = R_{ij}(\omega) + j\omega L_{ij}(\omega). \tag{3}$$

The impedance parameters are variable with the angular frequency ω because of the earth-return current through the soil and the skin effect in the wires at low frequencies [9]. If $i = j$, the term in $[Z]$ is in the main diagonal and is denoted as a self-impedance. If $i \neq j$, the impedance is an off-diagonal term and is denoted as a mutual impedance. An analogous terminology is applied for the admittance matrix $[Y]$.

The self and mutual admittance terms in [Y] are expressed by a constant conductance G and capacitance C, calculated from the geometrical and physical characteristics of the line:

$$Y_{ij} = G_{ij} + j\omega C_{ij}. \tag{4}$$

Usually, conductance G is neglected in TLM. Thus, self and mutual admittances are only composed of C parameters [9].

The explicit modeling of the mutual parameters in TLM represents a critical issue because there is no consensus on the physical frequency-dependent modeling of the longitudinal parameters between two phases [10]. A usual tool to overcome this problem is the use of modal decoupling techniques and modal transformation matrices.

For a three-phase transmission line ($n = 3$), the line decoupling into three propagation modes is performed using the following mode-phase relationships [8]:

$$[Z_M] = [T]^T [Z] [T], \tag{5}$$

$$[Y_M] = [T]^{-1} [Y] [T]^{-T}. \tag{6}$$

The *exact transformation matrix* $[T]$ is variable with the frequency, the rows and columns of which are calculated based on the eigenvalues and eigenvectors of the matrix product $[Y][Z]$. Matrices $[T]^{-1}$ and $[T]^T$ are the inverse matrix and the transposed matrix of the modal transformation matrix [7].

Since the modal transformation is performed using the exact transformation matrix, the propagation modes are ideally transposed and the modal matrices $[Z_M]$ and $[Y_M]$ are described as

$$[Z_M] = \begin{bmatrix} Z_\alpha & 0 & 0 \\ 0 & Z_\beta & 0 \\ 0 & 0 & Z_0 \end{bmatrix}, \quad [Y_M] = \begin{bmatrix} Y_\alpha & 0 & 0 \\ 0 & Y_\beta & 0 \\ 0 & 0 & Y_0 \end{bmatrix}. \tag{7}$$

In Eq. (7), all mutual terms are null because the propagation modes α, β, and 0 are completely decoupled from each other.

The same relationship between phase and modal domain can be extended to the currents and voltages of a three-phase transmission line. The currents and voltages can be calculated in the modal domain using the same exact transformation matrix in Eqs. (5) and (6):

$$[I_M] = [T]^{-1} [I], \tag{8}$$

$$[V_M] = [T]^T [V]. \tag{9}$$

Vectors $[I]$ and $[V]$ represent the currents and voltages, respectively, at phases 1, 2, and 3 of a three-phase transmission line. The vectors $[I_M]$ and $[V_M]$ are the currents and voltages, respectively, at the propagation modes α, β, and 0. The phase values are expressed in Eq. (10) whereas the modal vectors are described in Eq. (11):

$$[I]^T = [i_1\, i_2\, i_3], \quad [V]^T = [v_1\, v_2\, v_3], \tag{10}$$

$$[I_M]^T = [i_\alpha\, i_\beta\, i_0], \quad [V_M]^T = [v_\alpha\, v_\beta\, v_0], \tag{11}$$

However, there are several line models in the technical literature that use a constant and real matrix instead of the exact frequency-dependent transformation matrix [1, 11]. This approach is useful for line models developed directly in the time domain and represents a significant simplification in the modeling and simulation process. Thus, Eqs. (5) and (6) can be restructured as

$$[Z_M] = [T_{Ck}]^T [Z] [T_{Ck}], \tag{12}$$

$$[Y_M] = [T_{Ck}]^{-1} [Y] [T_{Ck}]^{-T}. \tag{13}$$

The modal transformation matrix $[T_{Ck}]$ is the *Clarke's matrix*:

$$[T_{Ck}] = \begin{bmatrix} \frac{2}{\sqrt{6}} & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}. \tag{14}$$

From Eqs. (12) and (13), the matrices $[Z_M]$ and $[Y_M]$ in Eq. (7) are not totally decoupled, resulting in remaining mutual terms between modes α and 0:

$$[Z_M] = \begin{bmatrix} Z_\alpha & 0 & Z_{\alpha 0} \\ 0 & Z_\beta & 0 \\ Z_{\alpha 0} & 0 & Z_0 \end{bmatrix}, \quad [Y_M] = \begin{bmatrix} Y_\alpha & 0 & Y_{\alpha 0} \\ 0 & Y_\beta & 0 \\ Y_{\alpha 0} & 0 & Y_0 \end{bmatrix}. \tag{15}$$

This means that α and 0 are not independent from each other and are denominated as quasi-modes [11]. Several time-domain line models have been developed using the Clarke's matrix and the remaining mutual terms from the quasi-modes in Eq. (15) are neglected. However, this approach leads to errors in time-domain simulations, especially those composed of a wide range of frequencies [8].

The line model proposed in this research is characterized by a first modal decoupling using the Clarke's matrix and a subsequent modal decoupling using a frequency-dependent transformation matrix for the quasi-modes α and 0 ($Z_{\alpha 0}$ and $Y_{\alpha 0}$). Thus, the exact decoupling of the line can be accomplished using the approach by the Clarke's matrix and a two-per-two transformation matrix instead of a frequency-dependent three-per-three transformation matrix, representing a simplification in the modeling and simulation process.

3. MULTI-CONDUCTOR LINE MODELING USING THE EXACT TRANSFORMATION MATRIX

The Z and Y parameters of the three independent propagation modes in Eq. (7) can be obtained from the modal relationships in Eqs. (5) and (6), using the exact transformation matrix $[T]$ and the iterative method of Newton–Raphson [7]. Since $[Z_M]$ and $[Y_M]$ are known, the propagation modes can be modeled as three independent two-port circuits in the frequency domain as follows:

$$[V_{MA}] = [A][V_{MB}] - [B][I_{MB}], \tag{16}$$

$$[I_{MA}] = [C]\,[V_{MB}] - [A][I_{MB}]. \tag{17}$$

The vectors with the modal currents and voltages in Eqs. (16) and (17) are expressed in Eq. (11). Subscripts A and B mean the sending and receiving ends of the line. Thus, $[I_{MA}]$ and $[I_{MB}]$ are vectors with currents at the sending and receiving ends, respectively, of the propagation modes α, β, and 0. Vectors $[V_{MA}]$ and $[V_{MB}]$ are the voltages at the sending and receiving ends, respectively, of the propagation modes α, β and 0. Matrices $[A]$, $[B]$, and $[C]$ are expressed as functions of the propagation function γ and the characteristic impedance Z_C of each propagation mode:

$$[A] = \begin{bmatrix} \cosh(\gamma_\alpha d) & 0 & 0 \\ 0 & \cosh\left(\gamma_\beta d\right) & 0 \\ 0 & 0 & \cosh(\gamma_0 d) \end{bmatrix}, \tag{18}$$

$$[B] = \begin{bmatrix} Z_{C\alpha}\sinh(\gamma_\alpha d) & 0 & 0 \\ 0 & Z_{C\beta}\sinh\left(\gamma_\beta d\right) & 0 \\ 0 & 0 & Z_{C0}\sinh(\gamma_0 d) \end{bmatrix}, \tag{19}$$

$$[C] = \begin{bmatrix} \frac{1}{Z_{C\alpha}}\sinh(\gamma_\alpha d) & 0 & 0 \\ 0 & \frac{1}{Z_{C\beta}}\sinh\left(\gamma_\beta d\right) & 0 \\ 0 & 0 & \frac{1}{Z_{C0}}\sinh(\gamma_0 d) \end{bmatrix}. \tag{20}$$

The term d is the line length. The propagation function γ and the characteristic impedance Z_C are given as functions of the Z and Y parameters in the modal domain [9].

From the two-port equations in Eqs. (16) and (17), the currents and voltages can be calculated at the terminals A and B of the propagation modes α, β, and 0. The phase values in Eq. (10), [I], and [V], can be calculated using the same phase-mode relationships in Eqs. (8) and (9). The sequence phase-mode-phase transformation and modal modeling using the two-port equations can be illustrated in Figure 1.

The modeling using the two-port representation, Eqs. (16) and (17), is possible because the propagation modes are ideally decoupled from the use of the exact transformation matrix. The proposed procedure can also be achieved using the approach by the Clarke's matrix and a reduced frequency-dependent matrix, as described in the following section.

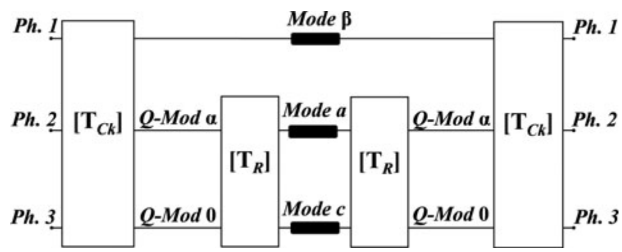

FIGURE 1. Phase-mode-phase transformation using the exact transformation matrix.

4. MULTI-CONDUCTOR LINE MODELING USING TWO MODAL MATRICES

As demonstrated in Eqs. (12)–(15), the modal decoupling using the Clarke's matrix results in one exact propagation mode and two quasi-modes. Thus, the three-phase line is not ideally decoupled because remaining mutual terms are observed in the modal matrices $[Z_M]$ and $[Y_M]$ in Eq. (15).

The quasi-modes α and 0 cannot be modeled as two independent two-port circuits because they are not totally decoupled from each other. Several well-established line models in the TLM literature use the Clarke's matrix as a modal transformation matrix and neglect the remaining mutual terms between the modes α and 0. Usually this approach leads to inaccuracies in the electromagnetic transient simulations [1, 8].

The alternative modeling proposed in this article is based on two sequential modal procedures. The first is achieved using the Clarke's matrix as modal transformation matrix, decoupling the line into an exact propagation mode β and two quasi-modes α and 0, such as described in Eq. (15). The following modal procedure is accomplished using a reduced frequency-dependent transformation matrix to decouple only the quasi-modes α and 0. The reduced transformation matrix has two-per-two dimension, differently of the exact matrix [T] that is a three-per-three matrix. The proposed modal decoupling can be described as in Figure 2.

After the first line decoupling using the Clarke's matrix, the quasi-modes α and 0 are not totally decoupled from each

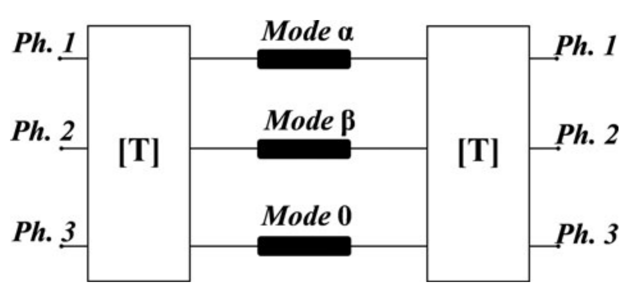

FIGURE 2. Phase-mode-phase transformation using Clarke's matrix and a reduced matrix.

other, as verified from the mutual terms in $[Z_M]$ and $[Y_M]$ in Eq. (15). Thus, the self and remaining mutual parameters of α and 0 can be rewritten as

$$\left[Z'\right]=\begin{bmatrix} Z_\alpha & Z_{\alpha 0} \\ Z_{\alpha 0} & Z_0 \end{bmatrix}, \quad \left[Y'\right]=\begin{bmatrix} Y_\alpha & Y_{\alpha 0} \\ Y_{\alpha 0} & Y_0 \end{bmatrix}. \tag{21}$$

The Z and Y parameters of the quasi-modes α and 0, as expressed in Eq. (21), are equivalent to a two-phase transmission line. Therefore, an exact transformation matrix can be obtained for $[Z']$ and $[Y']$ from the same procedure applied for a three-phase line, as previous described for a multi-conductor line modeling using the exact transformation matrix $[T]$. The modal transformation matrix $[T_R]$, for the two-phase line in Eq. (21), is composed of columns that are eigenvectors associated with the eigenvalues of the matrix product $[Y']\,[Z']$ [7]. The procedure for calculation of the reduced transformation matrix $[T_R]$ is described in details in Appendix A.

From the expressions in Eqs. (5) and (6), the modal transformations of $[Z']$ and $[Y']$ are expressed as

$$\left[Z'_M\right]=[T_R]^T\left[Z'\right][T_R], \tag{22}$$

$$\left[Y'_M\right]=[T_R]^{-1}\left[Y'\right][T_R]^{-T}. \tag{23}$$

As described in Figure 2, the quasi-modes α and 0 are decoupled into exact propagation modes denoted as a and c, such as expressed in the following impedance and admittance matrices, respectively:

$$\left[Z'_M\right]=\begin{bmatrix} Z_a & 0 \\ 0 & Z_c \end{bmatrix}, \quad \left[Y'_M\right]=\begin{bmatrix} Y_a & 0 \\ 0 & Y_c \end{bmatrix}. \tag{24}$$

From the modal parameters of the propagation mode β in Eq. (15) and from the modal parameters of the exact propagation modes a and b, the two-port equations in Eqs. (16) and (17) can be expressed for the proposed line model as

$$[V'_{MA}]=[A'][V'_{MB}]-[B'][I'_{MB}], \tag{25}$$

$$[I'_{MA}]=[C'][V'_{MB}]-[A'][I'_{MB}]. \tag{26}$$

The currents and voltages at the terminals A and B of the propagation modes a, β, and c can be generically expressed in the vector form as

$$[I'_M]^T=\left[i_a\ i_\beta\ i_c\right], \quad [V'_M]^T=\left[v_a\ v_\beta\ v_c\right]. \tag{27}$$

The matrices $[A']$, $[B']$, and $[C']$ are expressed based on Eqs. (18)–(20) as functions of the propagation function γ and the characteristic impedance Z_C of the propagation modes:

$$\left[A'\right]=\begin{bmatrix} \cosh(\gamma_a d) & 0 & 0 \\ 0 & \cosh(\gamma_\beta d) & 0 \\ 0 & 0 & \cosh(\gamma_c d) \end{bmatrix}, \tag{28}$$

$$\left[B'\right]=\begin{bmatrix} Z_{C\alpha}\sinh(\gamma_\alpha d) & 0 & 0 \\ 0 & Z_{C\beta}\sinh(\gamma_\beta d) & 0 \\ 0 & 0 & Z_{Cc}\sinh(\gamma_c d) \end{bmatrix}, \tag{29}$$

$$\left[C'\right]=\begin{bmatrix} \frac{1}{Z_{Ca}}\sinh(\gamma_a d) & 0 & 0 \\ 0 & \frac{1}{Z_{C\beta}}\sinh(\gamma_\beta d) & 0 \\ 0 & 0 & \frac{1}{Z_{Cc}}\sinh(\gamma_c d) \end{bmatrix}. \tag{30}$$

However, the new mode-phase relationship of currents and voltages, Eqs. (8) and (9), are expressed from a modified modal transformation matrix:

$$[I'_M]=[Q_V]^{-1}[I], \tag{31}$$

$$[V'_M]=[Q_I]^T[V]. \tag{32}$$

Matrices $[Q_V]$ and $[Q_I]$ are result of the successive modal transformations using the Clarke's matrix and the reduced matrix $[T_R]$, such as demonstrated in Figure 2. The modified transformation matrices $[Q_V]$ and $[Q_I]$ are expressed in Eqs. (33) and (34), respectively, and their algebraic development are described in Appendix B;

$$[Q_V]=\begin{bmatrix} \frac{2q_{11}}{\sqrt{6}}+\frac{q_{21}}{\sqrt{3}} & \frac{q_{21}}{\sqrt{3}}-\frac{q_{11}}{\sqrt{6}} & \frac{q_{21}}{\sqrt{3}}-\frac{q_{11}}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{2q_{12}}{\sqrt{6}}+\frac{q_{22}}{\sqrt{3}} & \frac{q_{22}}{\sqrt{3}}-\frac{q_{12}}{\sqrt{6}} & \frac{q_{22}}{\sqrt{3}}-\frac{q_{12}}{\sqrt{6}} \end{bmatrix}, \tag{33}$$

$$[Q_I]=\begin{bmatrix} \frac{2p_{11}}{\sqrt{6}}+\frac{p_{21}}{\sqrt{3}} & \frac{p_{21}}{\sqrt{3}}-\frac{p_{11}}{\sqrt{6}} & \frac{p_{21}}{\sqrt{3}}-\frac{p_{11}}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{2p_{12}}{\sqrt{6}}+\frac{p_{22}}{\sqrt{3}} & \frac{p_{22}}{\sqrt{3}}-\frac{p_{12}}{\sqrt{6}} & \frac{p_{22}}{\sqrt{3}}-\frac{p_{12}}{\sqrt{6}} \end{bmatrix}. \tag{34}$$

Terms q_{11}, q_{12}, $q_{21,}$ and q_{22} are elements of the reduced transformation matrix $[T_R]$, whereas p_{11}, p_{12}, $p_{21,}$ and p_{22} are the elements of $[T_R]^{-1}$.

Substituting in Eqs. (25) and (26) the current and voltage vectors $[I'_M]$ and $[V'_M]$, Eqs. (31) and (32), respectively, the following expressions are described:

$$[Q_I][V_A]=\left[A'\right][Q_I][V_B]-\left[B'\right][Q_V][I_B], \tag{35}$$

$$[Q_V][I_A]=\left[A'\right][Q_V][I_B]+\left[C'\right][Q_I][V_B]. \tag{36}$$

From some algebraic manipulations in Eqs. (35) and (36), the phase voltage and the current vectors at the terminal A of the line can be expressed as follows:

$$\begin{aligned}[V_A]&=[Q_I]^{-1}\left[A'\right][Q_I][V_B]-[Q_I]^{-1}\left[B'\right][Q_V][I_B]\\&=[\theta_1][V_B]-[\theta_2][I_B],\end{aligned} \tag{37}$$

$$\begin{aligned}[I_A]&=[Q_V]^{-1}\left[A'\right][Q_V][I_B]+[Q_V]^{-1}\left[C'\right][Q_I][V_B]\\&=[\theta_3][I_B]+[\theta_4][V_B].\end{aligned} \tag{38}$$

The matrices $[\theta_1]$, $[\theta_2]$, $[\theta_3]$, and $[\theta_4]$ are expressed as

$$[\theta_1]=[Q_I]^{-1}\left[A'\right][Q_I], \tag{39}$$

$$[\theta_2]=[Q_I]^{-1}\left[B'\right][Q_V], \tag{40}$$

$$[\theta_3] = [Q_V]^{-1} [A'] [Q_V], \quad (41)$$

$$[\theta_4] = [Q_V]^{-1} [C'] [Q_I]. \quad (42)$$

From Eqs. (37) and (38), the phase currents and voltages at the two terminals of a three-phase line are expressed as

$$\begin{bmatrix} V_A \\ I_A \end{bmatrix} = \begin{bmatrix} \theta_1 & \theta_2 \\ \theta_3 & \theta_4 \end{bmatrix} \begin{bmatrix} V_B \\ I_B \end{bmatrix}. \quad (43)$$

The voltages and currents at the terminals A and B of the line, at the three-phases, are obtained in the frequency domain from the expression in Eq. (43). The time-domain results are obtained using Laplace transform, based on the computational algorithm described in detail in [6].

5. VALIDATION OF THE PROPOSED LINE MODEL

The validation of the proposed model is given by comparison with the line modeling using the exact frequency-dependent transformation matrix (Section 3). Two input signals are considered: an ideal switching impulse and an atmospheric impulse. The switching impulse is represented by an ideal step function with 0 volts for the instant $t = 0$ and 1 p.u. for $t > 0$. The ideal step function is mostly composed of frequencies no greater than 100 Hz. The atmospheric impulse can be modeled by as a double exponential function with front-wave time of 1.2 μs and a tail time of 50 μs (time interval from the voltage peak of 1 p.u. to the voltage tail magnitude of 0.5 p.u.). The proposed atmospheric impulse signal is applied for standard high-voltage testing of various power devices and is established by the International Electrotechnical Commission (IEC) [12]. The atmospheric impulse covers a wide range of frequencies, enabling the validation of the proposed line model for frequencies up to 1 MHZ.

A conventional 440-kV transmission line is considered for modeling of the proposed model and the reference model using the exact transformation matrix, as illustrated in Figure 3.

The impedance and admittance parameters of the single-circuit line in Figure 3 were calculated following the geometrical and physical characteristics of the towers and conductors provided by a Brazilian company of the power transmission segment [1]. These parameters were obtained using the Carson and Bessel formulations that are well established in the technical literature on power systems [9]. The line parameters were calculated up to 1 MHz.

The proposed model is validated from frequency- and time-domain simulations considering two well-established testing procedures in TLM: open- and short-circuit tests [1, 3, 8]. Both line configurations are described in Figure 4.

In the two line circuits, the sending end of phase 1 is connected to a voltage source and a switch that represents the voltage function is applied to the line modeling. The receiving end of the line is open in Figure 4(a), whereas in Figure 4(b), the receiving ends of phases 1, 2, and 3 are in short and grounded. This two line circuits are standard procedures for analyses in

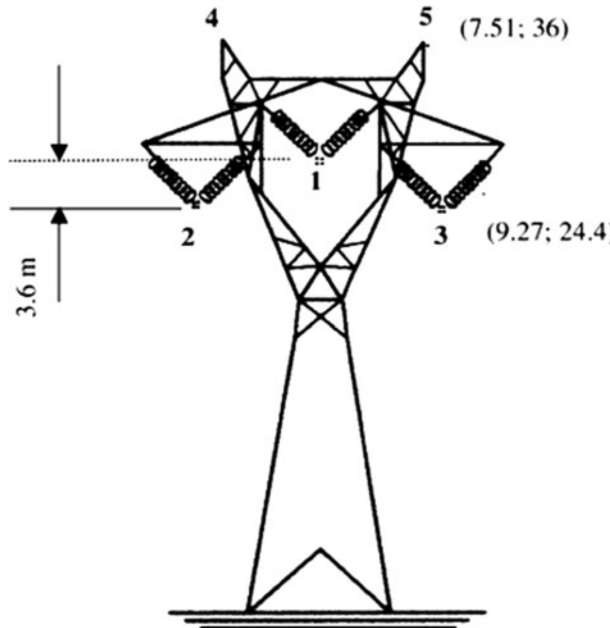

FIGURE 3. Conventional 440-kV three-phase transmission line.

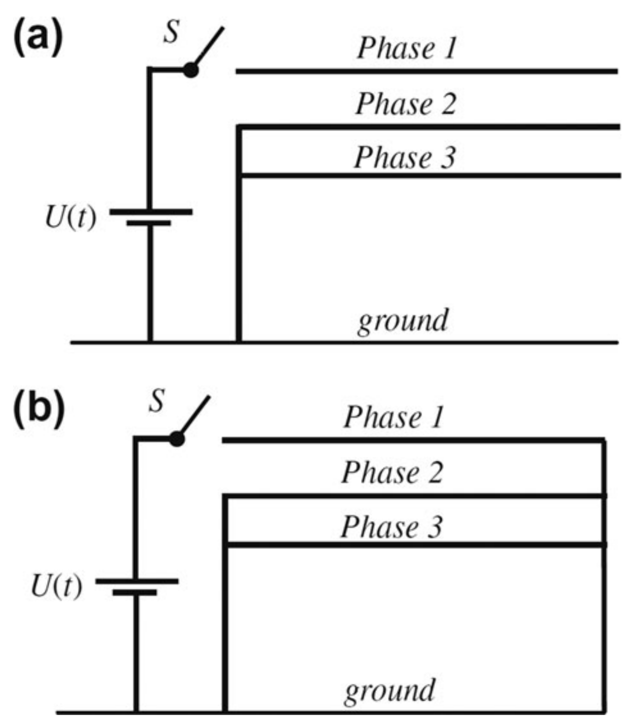

FIGURE 4. Test: (a) open circuit and (b) short circuit.

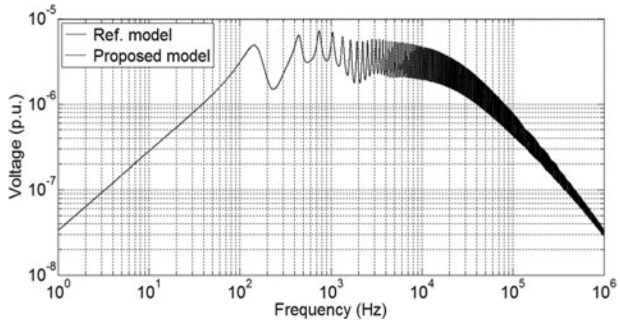

FIGURE 5. Transient voltage responses of the proposed line model (dotted curve) and reference model (solid curve) for an atmospheric impulse in the frequency domain.

TLM because the open and shorted receiving ends represent maximum voltage and current reflection coefficients, resulting in higher and more oscillatory electromagnetic transients along the transmission line [9].

5.1. Analyses in the Frequency Domain

Initially, the proposed line model is evaluated in the frequency domain up to 1 MHz considering an input signal represented by a 1.2/50-μs double exponential function with voltage peak of 1 p.u. (atmospheric impulse). The results obtained from the proposed and reference line models are compared based on the open and short-circuit tests in Figure 4.

The frequency-domain response of the two line models for an atmospheric impulse, considering the open-circuit test, are described in Figure 5.

The transient voltage in Figure 5 is at the receiving end of phase 1. The dotted and solid curves are referred to the proposed and reference line models, respectively. The two curves are overlapped and the relative errors for each frequency step are null.

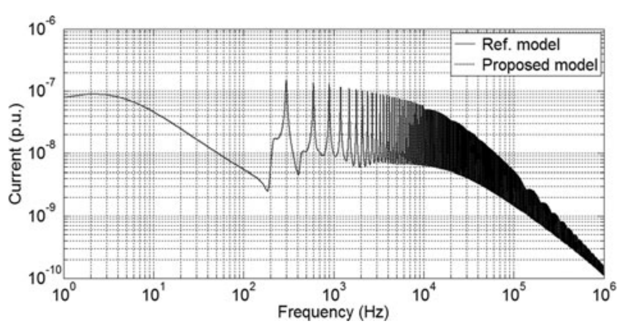

FIGURE 6. Transient current responses of the proposed line model (dotted curve) and reference model (solid curve) for an atmospheric impulse in the frequency domain.

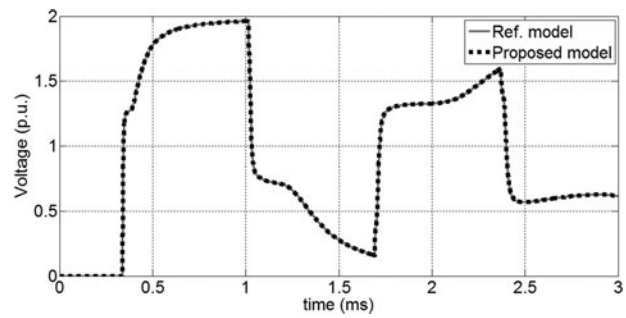

FIGURE 7. Transient voltage obtained from the proposed line model (dotted curve) and reference model (solid curve) for a switching impulse in the time domain.

The transient currents at the receiving end of phase 1 are simulated considering the short-circuit test, as described in Figure 6.

Results obtained from the proposed and reference models are exactly the same. The two curves in Figure 6 are overlapped and the relative error calculated based on the reference model are null, which validates the proposed model for the considered frequency range.

5.2. Analyses in the Time Domain

A proper evaluation of the proposed model in the time domain is accomplished based on the line open circuit in Figure 4. The switching and atmospheric impulses are considered as input voltage signals and the electromagnetic transients obtained from the proposed and reference models are compared.

An ideal switching impulse is applied at the sending end of the phase, as described in Figure 4(a) in the open-circuit test. The electromagnetic transient at the receiving end of phase 1, simulated by the two line models, is presented in Figure 7 as follows:

Such as in the frequency-domain analyses, results obtained using the proposed line model (dotted curve) and from the

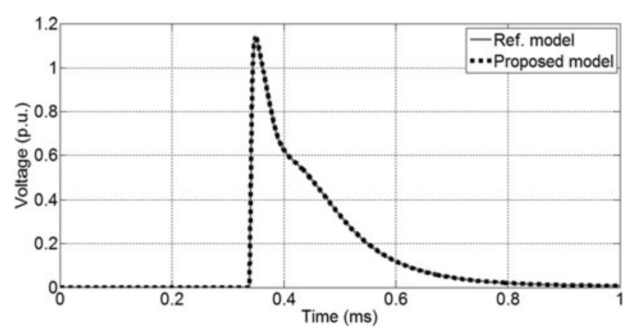

FIGURE 8. Transient voltage obtained from the proposed line model (dotted curve) and reference model (solid curve) for an atmospheric impulse in the time domain.

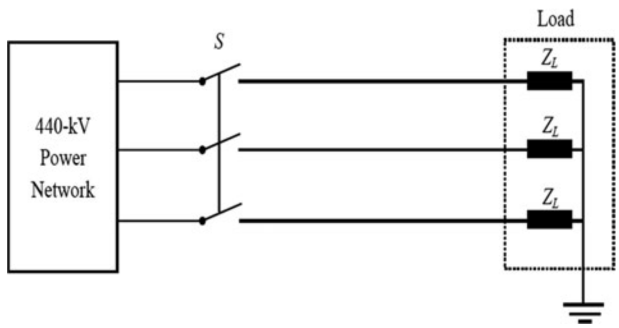

FIGURE 9. Switching simulation of a balanced load to a 440-kV power grid.

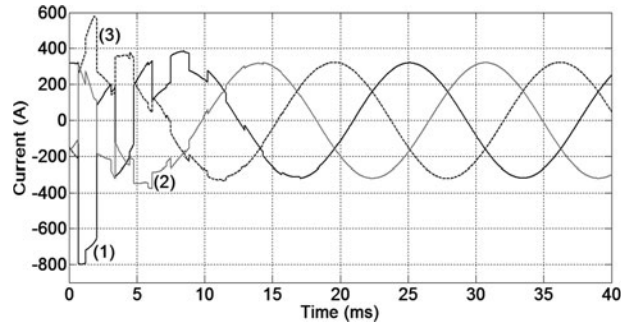

FIGURE 11. Currents at the receiving end of phases 1, 2, and 3.

reference model (solid curve) are exactly the same. This conclusion is possible because the relative errors between the two results were calculated for each time step and are null, *i.e.*, the results are exactly the same. Furthermore, the simulation time of the proposed line model is 43% faster than the reference model because only four frequency-dependent terms are calculated during simulation, which are associated to the transformation matrix $[T_R]$, whereas nine frequency-dependent terms are calculated for the exact transformation matrix $[T]$ in the reference line model.

As previously discussed, an ideal switching impulse is composed by low frequencies. In order to provide a time-domain validation for the entire frequency range considered for the line parameters calculation, an atmospheric impulse is applied at the sending end of phase 1, taking into account the same open-circuit test. The first wave reflections at the receiving end of phase 1, obtained from both line models, are described in details in Figure 8.

The voltage transient obtained from the proposed and reference models are practically overlapped. The relative error is zero for each time step, such as observed in the frequency domain. Thus, the proposed line model has the same accuracy as the reference model using the exact transformation matrix. The principal difference is that the proposed line model was developed based on the approach by the Clarke's matrix and using a two dimensional transformation matrix instead of the exact three-per-three matrix.

6. ELECTROMAGNETIC TRANSIENT SIMULATIONS USING THE PROPOSED LINE MODEL

The proposed line model is applied for switching simulation of a generic load and a three-phase voltage source. The transmission line is 200 km long and its sending end is switched to an infinite bus bar of 440 kV. The receiving end of the three-phase transmission line (Figure 3) is connected to a 500-MVA load with power factor of 0.97, which is modeled considering three identical impedances Z_L, as described in Figure 9.

The voltage profile at the load terminal, simulated using the proposed line model, is described in Figure 10.

The currents through the load terminal for the three phases are shown in Figure 11.

The current magnitudes assume inverse values to the respective voltages at each phase, as observed in Figures 10

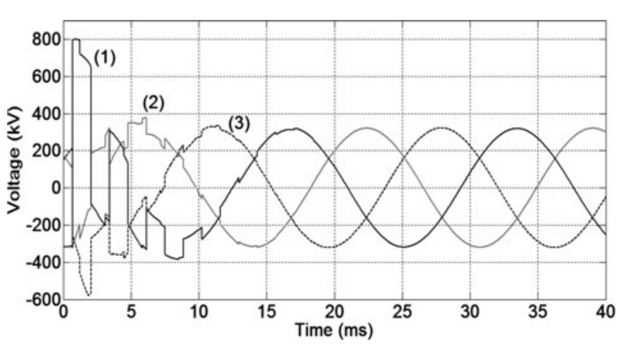

FIGURE 10. Voltages at the receiving end of phases 1, 2, and 3.

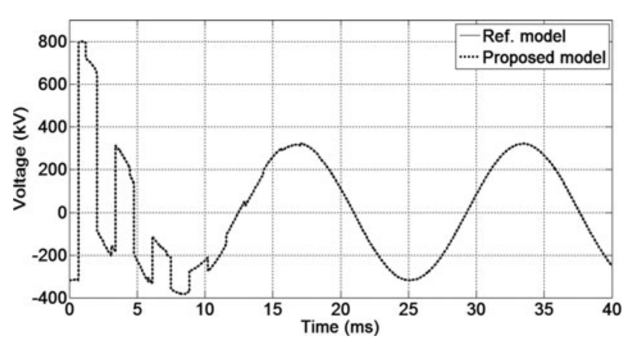

FIGURE 12. Voltages at the load terminal using the reference and proposed line models.

and 11. This behavior occurs because of the negative signal notation of the currents in Eqs. (25) and (26).

Although the proposed model was previously validated for a wide range of frequencies in Section 5, the voltage at the load terminal of phase 1, simulated using the proposed line model, is compared to the reference line model in Figure 12.

Similarly to previous results in Figures 7 and 8, the proposed and reference line models show similar results during the fast oscillatory transients, after 10 ms past the switching at the sending end of the line, and also during steady state up to 40 ms of simulation.

7. CONCLUSION

An alternative line model was proposed without the use of the exact transformation matrix in the modal decoupling. The premise of the proposed model is the use of the Clarke's matrix and a reduced frequency-dependent matrix, to decouple the remaining quasi-modes, instead of the frequency-dependent exact transformation matrix with dimension three per three. In fact, this replacement in the modal decoupling represents a simplification in the use of the exact line model because the Clarke's matrix is composed of real and constant terms and though the reduce matrix $[T_R]$ is also composed of frequency-dependent terms, it is a two-per-two matrix with less than five elements than the exact transformation matrix used in the reference line model. Although the proposed model uses two transformation matrices in the line decoupling, the use of the Newton-Raphson method for calculation of a three dimensional exact transformation matrix is still a more complex procedure in TLM. Furthermore, the proposed line model shows to be more than 40% faster than the reference model (simulation time), which means a significant improvement in terms of computational performance.

The proposed line model presented similar results, in the frequency and time domains, compared to the reference model using the exact transformation matrix. Both models represent the transmission line as a multi-phase two-port circuit, which is classified by various technical literatures as being an exact representation for a generic propagation guide. The transmission line representation by a three-phase two-port circuit is an original contribution of the proposed research.

The proposed line model has two principal advantages compared to the reference model by using the exact transformation matrix. The first is the simplification using a real and constant matrix and a reduced transformation matrix composed of only four frequency-dependent terms. The second advantage is the reduced computational cost, which leads to simulation times around 40% faster than the line model using the exact transformation matrix.

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APPENDIX A

As demonstrated in Eqs. (21)–(23), the reduced transformation matrix results in the product:

$$[Z'][Y'] = [S] = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}. \tag{A.1}$$

The eigenvalues λ_1 and λ_2 of $[S]$ can be analytically calculated as

$$\lambda_1 = \frac{1}{2}\left(s_{22} - s_{11} + \sqrt{(s_{11} - s_{22})^2 - 4(s_{11}p_{22} - s_{12}s_{21})}\right), \quad (A.2)$$

$$\lambda_2 = \frac{1}{2}\left(s_{22} - s_{11} - \sqrt{(s_{11} - s_{22})^2 - 4(s_{11}s_{22} - s_{12}s_{21})}\right). \quad (A.3)$$

Although the eigenvalues for a matrix with $n = 2$ are analytically expressed by Eqs. (A2) and (A3), the eigenvalues can be easily obtained by a single line command using MATLAB or any other similar software.

Calculating the eigenvalues, the reduced transformation matrix $[T_R]$ can be obtained:

$$[T_R] = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}. \quad (A.4)$$

The elements of $[T_R]$ are calculated as follows [7]:

$$q_{11} = -\frac{s_{12}}{\sqrt{s_{12}^2 + (s_{11} - \lambda_1)^2}}, \quad q_{12} = -\frac{s_{12}}{\sqrt{s_{12}^2 + (s_{11} - \lambda_2)^2}}, \quad (A.5)$$

$$q_{21} = -\frac{s_{11} - \lambda_1}{\sqrt{s_{12}^2 + (s_{11} - \lambda_1)^2}}, \quad q_{22} = -\frac{s_{11} - \lambda_2}{\sqrt{s_{12}^2 + (s_{11} - \lambda_2)^2}}. \quad (A.6)$$

APPENDIX B

Based on the optimized procedure for modal decoupling in TLM, provided in [8] and Eqs. (8) and (9), it can be reformulated using the Clarke's matrix as follows:

$$[I_M] = [T_{Ck}]^{-1}[I], \quad (B.1)$$

$$[V_M] = [T_{Ck}]^T[V]. \quad (B.2)$$

As described in Eq. (15), the quasi-modes α and 0 are not totally decoupled each other. From Eqs. (B1) and (B2), the currents and voltages of the quasi-modes α and 0 can be analytically expressed as

$$i_\alpha = \frac{2}{\sqrt{6}}i_1 - \frac{2}{\sqrt{6}}i_2 - \frac{2}{\sqrt{6}}i_3, \quad i_0 = \frac{1}{\sqrt{3}}(i_1 + i_2 + i_3), \quad (B.3)$$

$$v_\alpha = \frac{2}{\sqrt{6}}v_1 - \frac{2}{\sqrt{6}}v_2 - \frac{2}{\sqrt{6}}v_3, \quad v_0 = \frac{1}{\sqrt{3}}(v_1 + v_2 + v_3). \quad (B.4)$$

The second decoupling of the quasi-modes α and 0, using a reduced transformation matrix $[T_R]$, is described in Figure 2. The total modal decoupling of α and 0 into the exact propagation modes a and c are expressed as

$$\begin{bmatrix} i_a \\ i_c \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}^{-1} \begin{bmatrix} i_\alpha \\ i_0 \end{bmatrix}, \quad (B.5)$$

$$\begin{bmatrix} v_a \\ v_c \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}^{T} \begin{bmatrix} v_\alpha \\ v_0 \end{bmatrix}. \quad (B.6)$$

The transformation matrices in Eqs. (B5) and (B6) are

$$[T_R] = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}, \quad (B.7)$$

$$[T_R]^{-1} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}. \quad (B.8)$$

Substituting Eqs. (B3) and (B4) into Eqs. (B5) and (B6), respectively, the following analytical equations for the propagation modes a and c are expressed:

$$i_a = i_1\left(\frac{2p_{11}}{\sqrt{6}} + \frac{p_{21}}{\sqrt{3}}\right) + i_2\left(\frac{p_{21}}{\sqrt{3}} - \frac{p_{11}}{\sqrt{6}}\right) + i_3\left(\frac{p_{21}}{\sqrt{3}} - \frac{p_{11}}{\sqrt{6}}\right), \quad (B.9)$$

$$i_c = i_1\left(\frac{2p_{12}}{\sqrt{6}} + \frac{p_{22}}{\sqrt{3}}\right) + i_2\left(\frac{p_{22}}{\sqrt{3}} - \frac{p_{12}}{\sqrt{6}}\right) + i_3\left(\frac{p_{22}}{\sqrt{3}} - \frac{p_{12}}{\sqrt{6}}\right), \quad (B.10)$$

$$v_a = v_1\left(\frac{2q_{11}}{\sqrt{6}} + \frac{q_{21}}{\sqrt{3}}\right) + v_2\left(\frac{q_{21}}{\sqrt{3}} - \frac{q_{11}}{\sqrt{6}}\right) + v_3\left(\frac{q_{21}}{\sqrt{3}} - \frac{q_{11}}{\sqrt{6}}\right), \quad (B.11)$$

$$v_c = v_1\left(\frac{2q_{12}}{\sqrt{6}} + \frac{q_{22}}{\sqrt{3}}\right) + v_2\left(\frac{q_{22}}{\sqrt{3}} - \frac{q_{12}}{\sqrt{6}}\right) + v_3\left(\frac{q_{22}}{\sqrt{3}} - \frac{q_{12}}{\sqrt{6}}\right), \quad (B.12)$$

The propagation mode β was ideally decoupled in the first line decoupling using the Clarke's matrix, as described in Figure 2. The current and voltage in the mode β is analytically expressed as

$$i_\beta = i_2\frac{1}{\sqrt{2}} - i_3\frac{1}{\sqrt{2}}; \quad v_\beta = v_2\frac{1}{\sqrt{2}} - v_3\frac{1}{\sqrt{2}}. \quad (B.13)$$

Based on Eqs. (B9)–(B13), vectors $[I'_M]$ and $[V'_M]$ are expressed in Eq. (27) and in the mode-phase relationship in Eqs. (31) and (32). The transformation matrices $[Q_V]$ and $[Q_I]$ are also expressed in Eqs. (33) and (34), respectively.

BIOGRAPHIES

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