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Generalized relativistic harmonic oscillator in minimal length quantum mechanics

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Abstract

We solve the generalized relativistic harmonic oscillator in $1 + 1$ dimensions in the presence of a minimal length. Using the momentum space representation, we explore all the possible signs of the potentials and discuss their bound-state solutions for fermions and antifermions. Furthermore, we also find an isolated solution from the Sturm–Liouville scheme. All cases already analyzed in the literature are obtained as particular cases.

Keywords: generalized uncertainty principle, dirac equation, relativistic oscillator

1. Introduction

The concept of a minimal measurable length scale, which is expected to be of the order of the Planck length, given by $l_p = 1.62 \times 10^{-33}$ cm, has emerged as a condition required for a consistent formulation of quantum theory of gravity [1]. The existence of such minimal length, which arise when quantum fluctuations of the gravitational field (at Planck scale) are taken into account, is a common feature among most of the theories of quantum gravity such as string theory [2], loop quantum gravity [3], quantum cosmology [4–6], noncommutative field theories [7–10], and black hole physics [11–16]. One of the interesting implications of introducing this minimal length is the modification of the standard commutation relation between position and momentum, which is transformed into a generalized relation that includes an additional quadratic term in momentum, namely, $[X, P] = \mathrm{i}\hbar\left(1 + \beta P^2\right)$, where $\beta = \beta_0 l_p/\hbar^2$, $\beta_0$is a dimensionless constant. This generalized commutation relation leads to the modification of the Heisenberg uncertainty principle to a generalized uncertainty principle (GUP). This generalization, on the other hand, would imply in the modification of the properties of the quantum system under consideration, namely the eigenfunctions and the eigenvalues. This is the main reason why recent works on so-called minimal length quantum mechanics (MLQM) have undergone a significant growth [17–47].

The Dirac equation in $3+1$ dimensions with a mixture of spherically symmetric scalar, vector and anomalous magnetic-like (tensor) interactions can be reduced to the $1+1$ Dirac equation with a mixture of scalar (V_s), time-component vector (V_t) and pseudoscalar (V_p) couplings when the fermion is limited to move in just one direction [48]. In this restricted motion the scalar and vector interactions preserve their Lorentz structures while the anomalous magnetic-like interaction becomes pseudoscalar. So, in the context of $1+1$ dimensions the potential composed by V_s, V_t and V_p is the most general combination of Lorentz structures because there are only four linearly independent 2×2 matrices. In a recent published work, Hassanabadi *et al* [33] solved the minimal length Dirac equation with harmonic oscillator potential (scalar and vector interactions) and the energies and solutions are showed in quite a simple and systematic manner. To the best of our knowledge, no one has reported on the solution of the Dirac equation with a generalized relativistic harmonic oscillator in $1+1$ dimensions in the presence of a minimal length, and we believe that this problem deserves to be explored.

In this context, the purpose of this work is to investigate the effects of GUP when brought into problem of relativistic fermions moving in $1+1$ dimensions when a vector, scalar and pseudoscalar potentials are applied. We consider the effect of GUP on the definition of momentum operator and then we obtain a generalized Dirac equation and solve exactly the corresponding eigenvalue problem for the case of a generalized relativistic harmonic oscillator. This problem is mapped into a Schrödinger-like equation embedded in a symmetric Pöschl–Teller potential. We explore all the possible signs of the potentials, thus paying attention to bound states of fermions and antifermions. Finally, we show that our results obtained for the energy spectrum and the eigenfunctions are a generalization to those obtained in [19] (Dirac oscillator) and [33] (mixed vector–scalar harmonic oscillator). Also, in the limit of the ordinary quantum mechanics ($\beta \to 0$) we are able to reproduce the case of generalized relativistic harmonic oscillator [49].

2. The generalized uncertainty principle

We consider the following one-dimensional deformed commutation relation

$$[X, P] = \mathrm{i}\hbar \left(1 + \beta P^2\right), \tag{1}$$

where $0 \leqslant \beta \leqslant 1$. The limits $\beta \to 0$ and $\beta \to 1$ correspond to the ordinary quantum mechanics (OQM) and the extreme quantum gravity (EQG), respectively. This deformed commutation relation leads to the following generalized uncertainty principle (GUP)

$$\Delta X \Delta P \geqslant \frac{\hbar}{2}\left[1 + \beta(\Delta P)^2\right]. \tag{2}$$

The peculiarity of (2) is that it implies the existence of a non-zero minimal uncertainty in position (minimal length). The minimization of (2) with respect to ΔP gives

$$(\Delta X)_{\min} = \hbar\sqrt{\beta}. \tag{3}$$

Following [17], we consider the following simple one-dimensional realization of the position and momentum operators obeying the relation (1):

$$X = \mathrm{i}\hbar \left(1 + \beta p^2\right) \frac{\partial}{\partial p}, \qquad P = p. \tag{4}$$

It is important to note that the scalar product in this case is not the usual one, but is defined as

$$\langle f \mid g\rangle = \int_{-\infty}^{+\infty} \frac{\mathrm{d}p}{(1+\beta p^2)} f^*(p)g(p). \tag{5}$$

3. The Dirac equation in 1 + 1dimensions

The $1+1$ dimensional time-independent Dirac equation for a fermion of rest mass m under the action of vector (V_t), scalar (V_s) and pseudoscalar (V_p) potentials can be written, in terms of the combinations $\Sigma = V_t + V_s$ and $\Delta = V_t - V_s$, as

$$H\psi = E\psi, \tag{6}$$

with

$$H = c\sigma_1 P + \sigma_3 mc^2 + \frac{I+\sigma_3}{2}\Sigma + \frac{I-\sigma_3}{2}\Delta + \sigma_2 V_p, \tag{7}$$

where E is the energy of the fermion and P is the momentum operator. The matrices σ_1, σ_2 and σ_3 denote the Pauli matrices, and I denotes the 2×2 unit matrix. The positive definite function $|\psi|^2 = \psi^\dagger\psi$, satisfying a continuity equation, is interpreted as a position probability density and its norm is a constant of motion. This interpretation is completely satisfactory for single-particle states [50].

3.1. Equations of motion and isolated solution

If we now write the spinor ψ in terms of its components $\psi^T = (f,\, g)$, the Dirac equation gives rise to two coupled first-order equations for the upper, f, and the lower, g, components of the spinor:

$$[cP - \mathrm{i}V_p(X)]\, g = \left[E - mc^2 - \Sigma(X)\right] f, \tag{8}$$

$$[cP + \mathrm{i}V_p(X)]\, f = \left[E + mc^2 - \Delta(X)\right] g. \tag{9}$$

In terms of f and g the spinor is normalized as

$$\int_{-\infty}^{+\infty} \frac{\mathrm{d}P}{1+\beta P^2}(|f|^2 + |g|^2) = 1, \tag{10}$$

so that f and g are square integrable functions.

For $\Delta = 0$ with $E \neq -mc^2$, the Dirac equation becomes

$$g = \frac{(cP + \mathrm{i}V_p)f}{E + mc^2}, \tag{11}$$

$$c^2P^2 f - \mathrm{i}c[V_p, P]f + \left[(E+mc^2)\Sigma + V_p^2\right] f = (E^2 - m^2c^4)f, \tag{12}$$

and for $\Sigma = 0$ with $E \neq mc^2$, the Dirac equation becomes

$$f = \frac{(cP - \mathrm{i}V_p)g}{E - mc^2}, \tag{13}$$

$$c^2P^2 g + \mathrm{i}c[V_p, P]g + \left[(E-mc^2)\Delta + V_p^2\right] g = (E^2 - m^2c^4)g. \tag{14}$$

Either for $\Delta = 0$ with $E \neq -mc^2$ or $\Sigma = 0$ with $E \neq mc^2$ the solution of the relativistic problem is mapped into a Sturm–Liouville problem in such a way that solution can be found by solving a Schrödinger-like problem.

The solutions for $\Delta = 0$ with $E = -mc^2$ and $\Sigma = 0$ with $E = mc^2$ (isolated solution) can be obtained directly from the original first-order equations (8) and (9). They are

$$(cP - \mathrm{i}V_p(X))g = \left[-2mc^2 - \Sigma(X)\right] f, \tag{15}$$

$$(cP + \mathrm{i}V_p(X))f = 0, \tag{16}$$

for $\Delta = 0$ with $E = -mc^2$, and

$$(cP - \mathrm{i}V_p(X))g = 0, \tag{17}$$

$$(cP + \mathrm{i}V_p(X))f = \left[2mc^2 - \Delta(X)\right] g, \tag{18}$$

for $\Sigma = 0$ with $E = mc^2$. It is worthwhile to note that this sort of isolated solution cannot describe scattering states and is subject to the normalization condition (10). Because f and g are normalizable functions, the possible isolated solution presupposes $V_p(X) \neq 0$.

3.2. The nonrelativistic limit

In the nonrelativistic limit (with small potential energies compared to mc^2 and $E \sim mc^2$), the equation (12) becomes

$$\frac{P^2}{2m}f + \left(\frac{V_p^2}{2mc^2} - \frac{\mathrm{i}\,[V_p, P]}{2mc} + \Sigma\right) f = \mathcal{E}f, \tag{19}$$

where $\mathcal{E} = E - mc^2$ is the nonrelativistic energy and f obeys a Schrödinger equation with a effective potential expressed in terms of the original potentials Σ and V_p. Note that, in this approximation, Σ preserves its original structures itself. However, V_p provides two terms proportional to $\mathrm{i}\,[V_p, P]$ and V_p^2, which do not have a nonrelativistic counterpart. Therefore, we can say that V_p is a potential intrinsically relativistic. Furthermore, the term V_p^2 in (19) allows us to infer that even a potential unbounded from below could be a confining potential.

4. The generalized relativistic harmonic oscillator

Let us consider

$$\Sigma = k_1X^2, \qquad \Delta = 0, \qquad V_p = k_2X. \tag{20}$$

Note that, by making the changes $\Sigma \to \Delta$, $m \to -m$, $V_p \to -V_p$, $f \to g$ and $g \to f$ in equation (8) (or equation (12)), we obtain equation (9) (or equation (14)). This symmetry also is present in the isolated solution and can be clearly seen from the two equation pairs (15)–(18). This symmetry provides a simple mechanism by which one can go from the results from the case $\Sigma = k_1X^2$, $\Delta = 0$, $V_p = k_2X$ to the case when $\Delta = k_1X^2$, $\Sigma = 0$, $V_p = k_2X$ by just changing the sign of m and of k_2 in the relevant expressions.

The configuration for the potentials (20) was chosen conveniently so that one obtains Schrödinger-like equations with the harmonic oscillator potential. We can note that, for the case $\Delta = 0$ ($V_t = V_s$) and a confining vector potential ($\sim X^2$), the scalar potential V_s is also a confining potential. In this case, the Σ potential, which appears in the Schrödinger-like equation for the component f (equation (12)) becomes proportional to X^2 (confining potential). For $\Sigma = 0$ ($V_t = -V_s$) and a confining vector potential ($\sim X^2$), the scalar potential is not a confining potential ($\sim -X^2$). In this case, the Δ potential, which appears in the Schrödinger-like equation for the component g (equation (14)) also becomes proportional to X^2 (confining potential). In both cases for a linear pseudoscalar potential, we always have a confining potential in the Schrödinger-like equation for f and g components. The linear pseudoscalar potential is associated with a well known system, the Dirac oscillator [51]. The Dirac oscillator is a natural model for studying properties of physical systems, it is an exactly solvable model; several research have been developed in the context of this theoretical framework in recent years [48].

4.1. Isolated solution

The isolated solution with $E = -mc^2$ is obtained from equations (15) and (16). Substituting (20) in equations (15) and (16) and using the operator relation (4), we obtain

$$\frac{\mathrm{d}g(p)}{\mathrm{d}p} + \frac{cp}{\hbar k_2\,(1+\beta p^2)} g(p) = Q(p)f(p), \tag{21}$$

and

$$\frac{\mathrm{d}f(p)}{\mathrm{d}p} - \frac{cp}{\hbar k_2\,(1+\beta p^2)} f(p) = 0, \tag{22}$$

respectively. It is important to mention that the expression for $Q(p)$ is irrelevant to our analysis. The general solutions for (21) and (22) are given by

$$f(p) = N_+ \left(1+\beta p^2\right)^{\frac{c}{2\beta\hbar k_2}}, \tag{23}$$

$$g(p) = \left(1+\beta p^2\right)^{-\frac{c}{2\beta\hbar k_2}} \left[N_+ I(p) + N_-\right], \tag{24}$$

where N_+ and N_- are normalization constants, and

$$I(p) = \int Q(p)\left(1+\beta p^2\right)^{\frac{c}{\beta\hbar k_2}} \mathrm{d}p. \tag{25}$$

Observing (23) and (24), we can conclude that it is impossible to have both nonzero components simultaneously as physically acceptable solution. A normalizable solution for $k_2 > 0$ and $\beta \neq 0$ is possible if $N_+ = 0$. Thus,

$$\psi(p) = N(1+\beta p^2)^{-\frac{c}{2\beta\hbar k_2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{26}$$

where

$$N = \sqrt{\frac{1}{\delta}}, \tag{27}$$

with

$$\delta = \int_{-\infty}^{+\infty} \frac{\mathrm{d}p}{(1+\beta p^2)^{\left(\frac{\beta\hbar k_2 + c}{\beta\hbar k_2}\right)}}. \tag{28}$$

4.2. Solution for $\Delta = 0$ and $E \neq -mc^2$

For $\Delta = 0$ and $E \neq -mc^2$, the equation (12) takes the form

$$\left(c^2 + \hbar c\beta k_2\right) P^2 f(P) + \kappa^2 X^2 f(P) = \varepsilon^2 f(P), \tag{29}$$

where

$$\kappa^2 = \left(E + mc^2\right) k_1 + k_2^2, \tag{30}$$

$$\varepsilon^2 = E^2 - m^2c^4 - \hbar c k_2, \tag{31}$$

and the equation (11) becomes

$$g(P) = \frac{cP + ik_2 X}{E + mc^2} f(P). \tag{32}$$

Using the operator relation (4), the equation (29) gets

$$\frac{\mathrm{d}^2 f(p)}{\mathrm{d}p^2} + \frac{2\beta p}{1+\beta p^2}\frac{\mathrm{d}f(p)}{\mathrm{d}p} + \frac{\eta^2}{(1+\beta p^2)^2}\left[\varepsilon^2 - \left(c^2 + \hbar c\beta k_2\right)p^2\right] f(p) = 0, \tag{33}$$

where $\eta^2 = \frac{1}{\hbar^2\kappa^2}$, and (32) becomes

$$g(p) = -\frac{\hbar k_2\left(1+\beta p^2\right)}{E + mc^2}\left[\frac{\mathrm{d}}{\mathrm{d}p} - \frac{cp}{\hbar k_2\left(1+\beta p^2\right)}\right] f(p). \tag{34}$$

Implementing a change of variable defined by

$$q = \frac{1}{\hbar\sqrt{\beta}\kappa}\arctan\left(\sqrt{\beta}p\right), \tag{35}$$

we can rewrite the equation (33) as

$$\frac{\mathrm{d}^2 f(q)}{\mathrm{d}q^2} - \frac{c^2 + \hbar c\beta k_2}{\beta}\tan^2\left(\hbar\sqrt{\beta}\kappa q\right) f(q) + \varepsilon^2 f(q) = 0. \tag{36}$$

where we recognize the effective potential as the exactly solvable symmetric Pöschl–Teller potential [52] with $k_2 > -\frac{c}{\hbar\beta}$. The corresponding effective eigenenergy is given by

$$\frac{\varepsilon^2}{\hbar^2\beta\kappa^2} = n\left(n + 2\lambda\right) + \lambda, \tag{37}$$

where n is a non-negative integer and

$$\lambda = \frac{1}{2} + \frac{1}{2\hbar\beta}\sqrt{\frac{\hbar^2\beta^2(E+mc^2)k_1 + (\hbar\beta k_2 + 2c)^2}{(E+mc^2)k_1 + k_2^2}}. \tag{38}$$

Now, (31), (37) and (38) tell us that

$$E^2 = \hbar^2\beta\left[(E+mc^2)k_1 + k_2^2\right]\left(n^2 + n + \frac{1}{2}\right)$$
$$+ \left(n + \frac{1}{2}\right)\hbar\frac{\sqrt{\hbar^2\beta^2(E+mc^2)k_1 + (\hbar\beta k_2 + 2c)^2}}{\left[(E+mc^2)k_1 + k_2^2\right]^{-1/2}} + m^2c^4 + \hbar c k_2. \tag{39}$$

The solutions of (39) determine the eigenvalues of our problem. This equation can be solved easily with a symbolic algebra program.

The solution for the differential equation (36) becomes

$$f(q) = A\left(\cos\left(\hbar\sqrt{\beta}\kappa q\right)\right)^{\lambda} C_n^{\lambda}\left(\sin\left(\hbar\sqrt{\beta}\kappa q\right)\right), \tag{40}$$

where A is a normalization constant and $C_n^{\lambda}(z)$ are the Gegenbauer polynomials [53]. The lower component obtained from (34) is given by

$$g(q) = -\frac{2iAc\left(\cos\left(\hbar\sqrt{\beta}\kappa q\right)\right)^{\lambda+1}}{\sqrt{\beta}\left(E+mc^2\right)} C_{n-1}^{\lambda+1}\left(\sin\left(\hbar\sqrt{\beta}\kappa q\right)\right). \tag{41}$$

Using the relation (35) the solutions (40) and (41) can be rewritten in function of the original variable p as

$$f(p) = A\left(1+\beta p^2\right)^{-\lambda/2} C_n^{\lambda}\left(\frac{p\sqrt{\beta}}{\sqrt{1+\beta p^2}}\right), \tag{42}$$

and

$$g(p) = -\frac{2iAc\left(1+\beta p^2\right)^{-\lambda-1}}{\sqrt{\beta}\left(E+mc^2\right)} C_{n-1}^{\lambda+1}\left(\frac{p\sqrt{\beta}}{\sqrt{1+\beta p^2}}\right), \tag{43}$$

respectively. The normalization condition (10) dictates that the normalization constant can be written as

$$A = \frac{2^{\lambda}\beta^{1/4}}{\sqrt{2\pi}}\left[\frac{\Gamma(2\lambda+n)}{n!(n+\lambda)\left(\Gamma(\lambda)\right)^2} + \frac{c^2}{\beta\left(E+mc^2\right)^2}\frac{\Gamma(2\lambda+n+1)}{(n-1)!(n+\lambda)\left(\Gamma(\lambda+1)\right)^2}\right]^{-1/2}. \tag{44}$$

5. Particular cases on minimal length quantum mechanics (MLQM)

5.1. Pure pseudoscalar linear potential (one-dimensional Dirac oscillator)

For $\beta \neq 0$ and $k_1 = 0$ the expression (39) reduces to

$$E^2 - m^2c^4 = \hbar^2\beta k_2^2\left(n^2+n+\frac{1}{2}\right) + \hbar\left(n+\frac{1}{2}\right)|\hbar\beta k_2 + 2c||k_2| + \hbar c k_2. \tag{45}$$

One can readily envisage that two different classes of solutions can be distinguished depending on the sign of k_2.

For $k_2 > 0$, we obtain

$$E = \pm\sqrt{m^2c^4 + 2\left(n+1\right)\hbar c k_2 + \beta\hbar^2 k_2^2\left(n+1\right)^2}. \tag{46}$$

For $-\frac{c}{\hbar\beta} < k_2 < 0$, we obtain

$$E = \pm\sqrt{m^2c^4 + 2n\hbar c|k_2| + \beta\hbar^2 k_2^2 n^2}. \tag{47}$$

This last result is exactly the equation (37) of [19]. Note that $n \geqslant 0$ for $k_2 > 0$ and $n \geqslant 1$ for $-\frac{c}{\hbar\beta} < k_2 < 0$, because for the lower component is proportional to a Gegenbauer polynomial of degree $n-1$. Our results (46) and (47), allow us to conclude that

$$E_{\pm} = \pm\sqrt{m^2c^4 + 2\,(n+1)\,\hbar c|k_2| + \beta\hbar^2 k_2^2\,(n+1)^2}, \tag{48}$$

with $n = 0, 1, \ldots$, and it is independent of the sign of k_2. Note that both particle (E_+) and antiparticle (E_-) energy levels are members of the spectrum.

Let us consider the limit of the ordinary quantum mechanics ($\beta \to 0$). In this limit (48) becomes

$$E_{\pm} = \pm\sqrt{m^2c^4 + 2\,(n+1)\,\hbar c|k_2|}, \tag{49}$$

which is in accordance with [49].

5.2. Mixed vector–scalar harmonic oscillator potentials

For $\beta \neq 0$ and $k_2 = 0$ the expression (39) reduces to

$$E^2 - m^2 = \left(n + \frac{1}{2}\right)\hbar\frac{\sqrt{\hbar^2\beta^2(E+mc^2)k_1 + 4c^2}}{\left[(E+mc^2)k_1\right]^{-1/2}} + \hbar^2\beta\left(n^2 + n + \frac{1}{2}\right)\left[(E+mc^2)k_1\right]. \tag{50}$$

This result is equivalent to equation (12) of [33]. The quantization condition expressed by (50) can be rewritten as

$$\begin{aligned}(E - mc^2)&\sqrt{(E+mc^2)\,\mathrm{sgn}(k_1)} \\ &= \mathrm{sgn}(k_1)\left[\frac{\left(n+\frac{1}{2}\right)\hbar\sqrt{|k_1|}}{\left[\hbar^2\beta^2(E+mc^2)\mathrm{sgn}(k_1)|k_1| + 4c^2\right]^{-1/2}}\right. \\ &\qquad \left. + \hbar^2\beta\left(n^2 + n + \frac{1}{2}\right)\sqrt{(E+mc^2)\,\mathrm{sgn}(k_1)}|k_1|\right].\end{aligned} \tag{51}$$

This result implies that when $k_1 > 0$ there are only bound states for fermions with $E > mc^2$. On the other hand, for $k_1 < 0$ there are only bound states for antifermions with $E < -mc^2$. Therefore, the positive (negative) energies for fermions (antifermions) never sink into the Dirac sea of negative (positive) energies. This fact means that there is no channel for spontaneous fermion–antifermion creation (Klein's paradox).

Now, let us consider the limit of the ordinary quantum mechanics ($\beta \to 0$). In this limit (51) becomes

$$(E - mc^2)\sqrt{(E+mc^2)\,\mathrm{sgn}(k_1)} = \mathrm{sgn}(k_1)\left(n + \frac{1}{2}\right)\hbar c\sqrt{4|k_1|}, \tag{52}$$

which is in accordance with [49].

5.3. Generalized relativistic harmonic oscillator (ordinary quantum mechanics)

Now, let us consider the limit of the ordinary quantum mechanics ($\beta \to 0$). In this limit (39) reduces

$$E^2 - m^2c^4 = (2n+1)\,\hbar c\sqrt{(E+mc^2)\,k_1 + k_2^2} + \hbar c k_2. \tag{53}$$

This result is exactly the equation (44) of [49].

6. Final remarks

In this paper, we have studied the problem of relativistic fermions moving in $1+1$ dimensions under the influence of mixture of a vector, scalar and pseudoscalar potentials (most general Lorentz structure) in the context of minimal length quantum mechanics. Using the momentum space representation and a convenient representation of the Dirac matrices, we solved the first order Dirac equation and found a isolated solution for $\Delta = 0$ with $E = -mc^2$ for the case of a generalized relativistic harmonic oscillator. It is important to highlight that those solutions only exist for $\beta \neq 0$ and are normalizable when $k_2 > 0$. Normalizable solutions for $\Sigma = 0$ with $E = mc^2$ can be easily obtained from the symmetries of the system, as mentioned above.

The expression for the energy spectrum and the corresponding eigenstates for $\Delta = 0$ and $E \neq -mc^2$ were obtained exactly from the second–order differential equation for the Dirac spinor components after being mapped into a Sturm–Liouville problem (Schrödinger-like) with a symmetric Pöschl–Teller potential. For the general case, the quantization condition can be solved easily with a symbolic algebra program and the Dirac spinor were expressed in terms of the Gegenbauer polynomials. We discussed in detail all the possible signs of the potentials and determined which values of k_1 and k_2 allow there to be a spectrum of both fermion and antifermion bounded solutions simultaneously or just one of these type of solutions. Furthermore, a remarkable feature of this problem is that we were able to reproduce well–known particular cases of relativistic harmonic oscillator in the presence of a minimal length, as for instance: the cases of harmonic oscillator potential (scalar and vector couplings) [33] and the so–called Dirac oscillator [19]. Also, the results obtained in this work are consistent in the limit of the ordinary quantum mechanics ($\beta \to 0$) with those found in [49] for the generalized relativistic harmonic oscillator.

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