Defining Customer Export Limits in PV-Rich Low Voltage Networks

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***Abstract*—The growing adoption of residential photovoltaic (PV) systems around the world is presenting distribution network operators (DNOs) with technical challenges, particularly on low voltage (LV) networks. The need to mitigate these issues with simple yet effective measures in countries with high PV penetrations is likely to drive the adoption of limits on the very exports that affect this infrastructure. Defining the most adequate limit, however, requires understanding the tradeoffs between the technical benefits and the effects on PV owners. This paper proposes two methodologies: an optimal power flow (OPF) based technique to define the export limit that solves technical problems with minimal curtailment, and a Monte Carlo based analysis to investigate the spectrum of such tradeoffs considering different PV penetrations and export limits. A real U.K. residential LV network with 180 customers is analyzed using realistic 1-min resolution daily load and PV generation profiles across seasons. Results demonstrate that, for DNOs, the OPF-based approach is effective in determining the most technically adequate export limit. However, for policy makers, the spectrum of tradeoffs provided by the Monte Carlo approach can help defining export limits that reduce curtailment at the expense of partially mitigating technical issues.**

***Index Terms*—Low voltage networks, optimal power flow, photovoltaics, PV management.**

I. Introduction

THE need to reduce CO_2 emissions combined with attractive incentive mechanisms such as feed-in tariffs (FIT) or net metering schemes have encouraged in the last few years the growth of the global photovoltaic (PV) generation capacity, which is now expected to exceed 500 GW by 2020 [1]. A significant share of this capacity, however, corresponds to residential PV systems, which are connected to low voltage (LV) networks not traditionally designed to host large volumes of generation. This is presenting distribution network operators (DNOs) with significant challenges to tackle technical issues, such as overvoltages and congestion [2], [3], caused by the aggregated exports from houses with PV systems during periods of high generation and low demand.

Different solutions to mitigate problems on residential LV networks with PV systems have been recently proposed and trialed, including the use of new devices (e.g., LV on-load tap changers [4], [5], storage [6], [7]) and topological arrangements (e.g., ring operation [8]). Nonetheless, their adoption by DNOs depends not only on their cost-effectiveness (as opposed to traditional reinforcements) but also on their practicality. A simple yet effective alternative that can be implemented by DNOs, albeit affecting PV owners, is to impose a fixed, network-wide limit on the very exports that affect the LV network. For instance, in Germany, where more than one million PV systems are connected to LV networks [9], customers with installed capacities smaller than 30 kWp are not allowed to export (i.e., generation minus demand) more than 70% of their installed capacity [10]. In Australia and Hawaii some DNOs are also imposing export limits to approve the connection of new PV systems [11], [12]. Depending on the country or region, the export limit could either be mandated by the DNO or the regulator. The definition of this limit, however, has been done considering mainly one side, either the effects on PV owners or the impacts on the LV network.

Most of the studies in the literature assess the implications of different export limits for PV owners (i.e., curtailment) considering only a single (typical) installed capacity and a small number of simplified demand and generation profiles, without quantifying the effects on the LV network [13]–[15]. A few studies investigate, among other solutions, the benefits brought by export limits to LV networks [16]–[18]. Different limits were considered in [16], [17], focusing particularly on voltage issues and adopting real LV networks. Although the effects on PV curtailment were quantified, no methodology per se was proposed to identify the most suitable export limit. An optimization-based methodology was presented in [18] to define the export limit capable of mitigating voltage issues in a small real LV feeder. The corresponding formulation, however, neglects thermal constraints which could be an issue in other circumstances.

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All the aforementioned studies do not account for the uncertainties related to PV installed capacities and location as well as demand and generation behavior, which is necessary for a more comprehensive assessment. More importantly, these studies have not considered the trade-offs between the effects on PV owners and the impacts on LV networks when determining the export limit; an aspect of particular interest for policy makers and/or DNOs. In this context, this work proposes two methodologies: an AC Optimal Power Flow (OPF)-based technique to define the export limit that solves technical problems with minimal curtailment for a given PV penetration, and a Monte Carlo-based analysis to investigate the spectrum of such trade-offs considering different PV penetrations and export limits.

The proposed methodologies consider the uncertainties related to demand and PV generation such as location, size and behavior [19], as well as the unbalanced (three-phase) nature of LV networks. Voltage and asset utilization are adopted as the key metrics to assess the technical performance of export limits in the Monte Carlo-based analysis but also as constraints in the AC OPF-based technique to determine the most adequate export limits. From the perspective of PV owners, the effects of export limits are quantified considering the resulting energy curtailment. Furthermore, a new index is proposed to understand the extent to which curtailment makes PV systems less profitable under different incentives mechanisms such as net metering and FIT. Both methodologies are applied to a real UK residential LV network with 180 single-phase customers considering realistic time-series high-resolution profiles for demand and PV generation.

This paper is organized as follows. Section II describes the concept of customer export limits. The proposed methodologies are provided in Section III. Their corresponding application on the real UK LV network and results are presented in Section IV. Finally, Section V provides a discussion and the conclusions are drawn in Section VI.

II. Limiting Customer Exports

The concept of the export limit is described here for a single customer whose demand and generation in instant t and phase ϕ are denoted by $P_{t,\phi}^{D}$ and $P_{t,\phi}^{G}$, respectively.

The operation under export limits consists of imposing a constant power limit, P_{ϕ}^{Limit}, on the corresponding net generation $P_{t,\phi}^{N}$, calculated using (1), that can be injected into the LV network; exceeding this limit, shown in (2), triggers the curtailment of PV generation. For simplicity, as this would be applied to different PV installations, this power limit is usually expressed as a percentage, γ, of the PV rated capacity in phase ϕ, P_{ϕ}^{Rated}, as denoted by (3). Because of this proportionality, the larger the PV sizes across the same network, the lower the export limit (γ) so as to ensure that the individual power injections do not violate technical constraints.

$$P_{t,\phi}^{N} = P_{t,\phi}^{G} - P_{t,\phi}^{D} \tag{1}$$

$$P_{t,\phi}^{N} \leq P_{\phi}^{Limit} \tag{2}$$

$$P_{\phi}^{Limit} = \gamma \cdot P_{\phi}^{Rated} \tag{3}$$

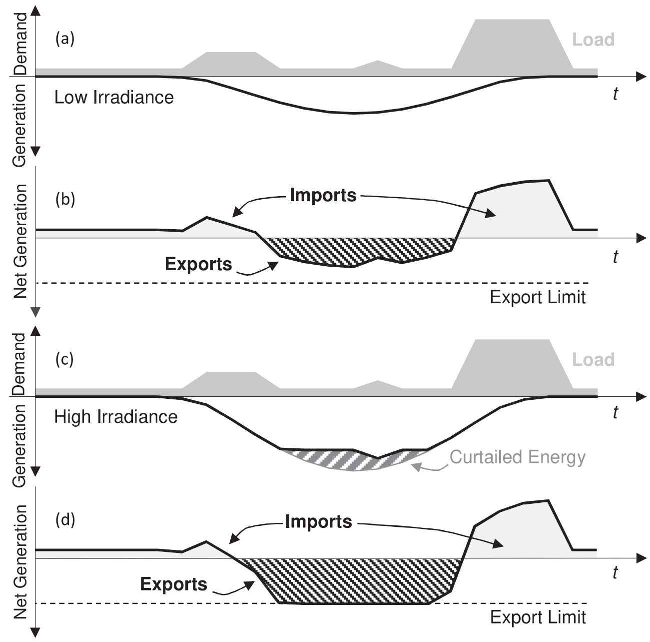

Fig. 1. (a) Demand and generation for a low irradiance scenario. (b) Net generation for the low irradiance scenario. (c) Demand and generation for a high irradiance scenario. (d) Net generation for the high irradiance scenario.

This mechanism can be visualized in Fig. 1. With low irradiance, Fig. 1 (a) and Fig. 1 (b), the resulting net generation is less than the export limit (dashed line) and, therefore, no curtailment action is needed by the PV system. However, with high irradiance, Fig. 1 (c), PV generation needs to be reduced to comply with the export limit. To achieve this, the PV inverter would be required to continuously check the demand, $P_{t,\phi}^{D}$, and the PV power produced by the panels (before the inverter), $P_{t,\phi}^{P}$, so as to regulate the PV generation (after the inverter), $P_{t,\phi}^{G}$, according to (4) and shown in Fig. 1 (d).

$$P_{t,\phi}^{G} = \begin{cases} P_{t,\phi}^{P}, & \text{if } P_{t,\phi}^{P} - P_{t,\phi}^{D} \leq \gamma \cdot P_{\phi}^{Rated} \\ \gamma \cdot P_{\phi}^{Rated} + P_{t,\phi}^{D}, & \text{if } P_{t,\phi}^{P} - P_{t,\phi}^{D} > \gamma \cdot P_{\phi}^{Rated} \end{cases} \tag{4}$$

The curtailed energy, Fig. 1 (c), important to understand the effects on PV owners from adopting different export limits, can be calculated using (5), in which Δt is the time interval.

$$E_{curt} = \sum_{t} \sum_{\phi} P_{t,\phi}^{P} \Delta t - \sum_{t} \sum_{\phi} P_{t,\phi}^{G} \Delta t \tag{5}$$

The value of γ is intended to be the same for all the PV systems in the network. Since all the PV systems are likely to contribute to the aggregated reverse power flows and, consequently, to the resulting voltage rise and congestion issues, then, having a pro-rata approach to limit their injections can be considered socially fair, particularly for residential customers. Furthermore, as a policy, using the same export limit across the network (or an entire region) is more practical to define and enforce.

III. Methodology

This section presents two different methodologies to assist decision makers (DNOs, regulators, etc.) defining the most suitable PV export limits depending on their objectives and considering both the effects on PV owners and the technical benefits to LV networks. First, the AC OPF-based technique able to define the export limit that solves technical problems with minimal curtailment is formulated; appropriate if the objective of the decision maker is to only meet the technical constraints of the LV networks. Then, the framework of the Monte Carlo-based trade-off assessment is described; which is suitable for a decision maker willing to find the spectrum of technical benefits to LV networks and the corresponding effects on PV owners resulting from curtailment. Finally, the adopted performance metrics are presented.

A. OPF-Based Export Limit

The multi-period AC OPF formulation proposed in [20] is adapted here to find, for a given PV penetration level, the most suitable export limit that minimizes curtailment whilst keeping voltages and power flows within the statutory and thermal limits, respectively. Furthermore, an expansion of the method is also proposed so that the export limit is determined considering multiple scenarios for each penetration level.

1) Three-Phase OPF Formulation: The OPF determines the most suitable export power limit, γ^{opt}, same for all the houses with PV systems (set H_{PV}, included in the set of houses, H) and for each time step (set T). The objective function minimizes the curtailed energy, given in (5), in terms of the curtailed power for each PV system, as given in (6). To adequately cater for the unbalanced nature of the LV networks, the AC OPF previously developed in [20] is extended to three phases including their corresponding couplings.

$$minimize \sum_{t\in T}\sum_{h\in H_{PV}}\sum_{\phi}\left(P^{P}_{h,t,\phi}-P^{G}_{h,t,\phi}\right)\Delta t \tag{6}$$

This objective is subject to the AC OPF constraints formulated as an extension of the branch flow equations (i.e., power balances and voltage drops) for three-phase networks. Moreover, additional constraints are also included to ensure that voltage and thermal limits are satisfied.

The active and reactive power balance is formulated in terms of the active/reactive power flow arriving node n through line/transformer mn, $P_{mn,\phi,t}$ and $Q_{mn,\phi,t}$ for phase ϕ, with mn in the set $L\cup T_r$ (lines and transformers), as shown in (7) and (8). $S^{loss}_{mn,\phi,t}$ is the complex power losses in that element; the active/reactive powers imported/exported from the grid are represented by $P_{x,\phi,t}$ and $Q_{x,\phi,t}$ (x representing the external grid and belonging to set X); the active/reactive powers demanded by a house for phase ϕ are $P^{D}_{h,t,\phi}/Q^{D}_{h,t,\phi}$, with h in the set H; the net generation of a house corresponds to $P^{N}_{h,t,\phi}$ (for h in H).

$$\sum_{mn\in L}\left(P_{mn,\phi,t}+\mathrm{Re}\left\{S^{loss}_{mn,\phi,t}\right\}\right)-\sum_{km\in L}P_{km,\phi,t} = \sum_{x\in X\mid \beta_x=m}P_{x,\phi,t}+\sum_{h\in H\mid\beta_h=m}P^{N}_{h,t,\phi},\forall m,\phi,t \tag{7}$$

$$\sum_{mn\in L}\left(Q_{mn,\phi,t}+\mathrm{Im}\left\{S^{loss}_{mn,\phi,t}\right\}\right)-\sum_{km\in L}Q_{km,\phi,t} = \sum_{x\in X\mid \beta_x=m}Q_{x,\phi,t}-\sum_{h\in H\mid\beta_h=m}Q^{D}_{h,t,\phi},\forall m,\phi,t \tag{8}$$

where β_u maps the location of each element to its corresponding bus ($u\subset\{H,X,L\}$). For simplicity, the power balance equations consider wye-connections for the loads. Delta connections could also be modeled (see Appendix-C).

The voltage drop in line mn is defined by (9) in terms of the square of the voltage, $V^{sqr}_{m,\phi,t}$, the power flows through the line, the nominal voltage at node m, V^{nom}_m, and the equivalent impedance $Z'_{mn,\phi,\psi}$ between phases ϕ and ψ, equals to $Z_{mn,\phi\psi}\angle\theta_{\phi\psi}$, in which $\theta_{\phi\psi}$ is the corresponding phase angle difference:

$$V^{sqr}_{m,\phi,t}-V^{sqr}_{n,\phi,t}=\sum_{\psi}\left|Z'_{mn,\phi,\psi}\right|^2\frac{(P_{mn,\psi,t})^2+(Q_{mn,\psi,t})^2}{(V^{nom}_m)^2}+ 2\sum_{\psi}\left(\mathrm{Re}\left\{Z'_{mn,\phi,\psi}\right\}P_{mn,\psi,t}+\mathrm{Im}\left\{Z'_{mn,\phi,\psi}\right\}Q_{mn,\psi,t}\right), \forall mn,\phi,t \tag{9}$$

The OPF constraints should also keep the voltages within the statutory limits, V^-/V^+, by limiting $V^{sqr}_{m,\phi,t}$ as defined by (10). In addition, current/power flows should satisfy the thermal limits of lines and transformers, $\bar{I}_{mn}$ and $\bar{S}_{mn}$, as shown in (11) and (12), respectively.

$$\left(V^{-}_m\right)^2\le V^{sqr}_{m,\phi,t}\le\left(V^{+}_m\right)^2,\forall m,\phi,t \tag{10}$$

$$(P_{mn,\phi,t})^2+(Q_{mn,\phi,t})^2\le V^{sqr}_{m,\phi,t}\left(\bar{I}_{mn}\right)^2, \forall mn\in L,\phi,t \tag{11}$$

$$\left(\sum_{\phi}P_{mn,\phi,t}\right)^2+\left(\sum_{\phi}Q_{mn,\phi,t}\right)^2\le\left(\bar{S}_{mn}\right)^2,\forall mn\in T_r,t \tag{12}$$

In addition, the net power of a house with a PV unit is defined considering the PV power generation $P^{G}_{h,t}$ and the demand $P^{D}_{h,t}$:

$$P^{N}_{h,t,\phi}=P^{G}_{h,t,\phi}-P^{D}_{h,t,\phi}\ \forall h\in H_{PV},t,\phi \tag{13}$$

As defined in Section II, if $P^{P}_{h,t}-P^{D}_{h,t}$ is below or equal the corresponding power limit (P^{Limit}_h), then curtailment is zero and $P^{N}_{h,t}$ is equal to $P^{P}_{h,t}-P^{D}_{h,t}$. On the other hand, if $P^{P}_{h,t}-P^{D}_{h,t}>P^{Limit}_h$, then $P^{N}_{h,t}$ should be equal to the corresponding power limit. These conditions are represented by (14).

$$P^{N}_{h,t,\phi}=\min\left\{P^{P}_{h,t,\phi}-P^{D}_{h,t,\phi},\ \gamma^{opt}\cdot P^{Rated}_{h,\phi}\right\},\ \forall h\in H,t,\phi \tag{14}$$

where $P_{h,\phi}^{Rated}$ is the rating of PV system in house h, phase ϕ and the export limit, γ^{opt}, is within zero and one. Note that $P_{h,\phi}^{Limit} = \gamma^{opt} \cdot P_{h,\phi}^{Rated}$. The minimum function used in (14) is implemented via binary variables. In addition, (9), (11), and (12) have been linearized and a two-stage approach is performed to improve the accuracy of the AC OPF based on the approaches used in [20]. Since the objective function and resulting constrains are linear and some of the variables are binary, the mathematical formulation is a Mixed-Integer Linear Programming (MILP) problem.

It is important to highlight that the proposed formulation is an approximation of the non-linear AC OPF and, therefore, the obtained results could not be called optimal. Nevertheless, the adopted linearizations provide a solution with low errors, as illustrated in the Appendix-A. It is also worth noting that, although convex and successive linear programming approaches [21] could be used, a linearized formulation brings a simpler representation combined with a reduced computational effort required to its solution, as verified by [22]. This is especially critical when problems involve a large number of binary variables (necessary here to model the export limit). It is worth highlighting that alternative linearization approaches such as successive linear programming, polyhedral approximation and McCormick relaxations can also be adopted to obtain a linear formulation from non-linear AC OPF [23]–[26]. Adopting more advanced solution processes to guarantee optimality of the AC OPF is not part of the scope of this work.

In general, the more complex modelling considerations (e.g., grounding, presence of other conductors, detailed PV system modelling, etc.) are needed, the more complex the AC OPF formulation. In those cases, the use of alternative approaches, such as the Monte Carlo-based method proposed in Section III-B, might prove more adequate.

2) Expansion to Multiple Scenarios: To cater for the uncertainties, the OPF-based methodology can be expanded to run a set of scenarios S (indexed by s). For each scenario, different locations and sizes of the PV systems as well as generation and demand profiles can be considered and an optimal export limit γ_s^{opt} obtained. In this case, the export limit, γ, will be the one that addresses the technical issues for a given α-th percentile, $\mathrm{P}\alpha$, as shown in (15):

$$\gamma = \mathrm{P}\alpha \left\{ \gamma_s^{opt} \,|\, s \in S \right\} \tag{15}$$

If the DNO desires to address the issues of 100% of the scenarios, α should be equal to 0, i.e., γ is the minimum among all the γ_s^{opt} values. If addressing the issues of 95% of the scenarios is enough, then $\alpha = 0.05$, and so on. This is aligned with DNO practices, as they are interested in identifying the most plausible technical violations (e.g., non-compliant customers or congested assets) rather than their extent; no matter how small a violation could be, it would need to be fixed to comply with local regulations and ensure the integrity of the network. Nonetheless, while technical constraints are critical to define γ, the curtailment minimization has already been implicitly considered in each scenario.

B. Monte Carlo-Based Trade-Off Assessment

In practice, the value of an export limit should be defined considering not only the mitigation of technical issues but also the acceptable level to which PV owners could be affected (defined based on local policies, economics, etc.). More specifically, for high PV penetration levels the optimal export limit needed to solve technical issues, as proposed in the previous section, can be very restrictive for PV owners–even if the corresponding curtailment has been minimized. Thus, to adequately define export limits it is important to assess the trade-offs between the technical benefits and the effects on PV owners. Here, the framework of a Monte Carlo-based methodology is presented to investigate the spectrum of such trade-offs considering different PV penetrations and export limits. The adopted performance metrics are also described.

1) Assessment Framework: To assess the trade-offs, this approach requires covering as much as possible the spectrum of combinations between PV penetration levels and export limit values (both from 0 to 100%). The assessment is carried out for each of the $N_{PL} \times N_{EL}$ combinations; where N_{PL} and N_{EL} are the number of PV penetration levels and the number of export limits to be evaluated, respectively. To cope with the uncertainties related to demand and PV generation (such as location, size, and behavior), a Monte Carlo analysis is carried out, i.e., N_{sim} scenarios are assessed for each combination of N_{PL} and N_{EL}.

The creation of the N_{sim} scenarios is done following the procedure proposed in [19]. This, essentially, considers the random allocation of PV systems to customers according to the penetration level and a given distribution of installed capacities (e.g., national or regional statistics), and the random allocation of realistic irradiance and load profiles (e.g., based on weather information and statistics of occupancy). A similar approach can be used to define the set of scenarios for the expanded OPF-based analysis proposed in Section III-A-2.

The principle of carrying out multiple scenarios for a single PV penetration level is similar to the methodology proposed in [19]. The key difference is that, here, multiple export limits (γ) are also evaluated to find the spectrum of trade-offs between the benefits of the export limits to mitigate the technical issues in the LV network and the corresponding effects on PV owners. For each scenario, a time-series, three-phase power flow is run and the corresponding performance metrics are calculated. The results can then be presented statistically (average, percentiles, and standard deviation) to assess the trade-offs.

2) Performance Metrics: The adopted performance metrics quantify the impacts of export limits on the LV network and PV owners. The three metrics used to quantify the technical performance of the LV network are presented below.

- *Transformer utilization level:* This metric is the hourly maximum apparent power divided by the transformer capacity. The power at each hour is calculated by averaging the corresponding intra-hour values.

- *Feeder utilization level:* Similar to the previous, this metric is the hourly maximum current divided by the ampacity at the head of the feeder.
- *Percentage of customers with voltage problems:* This metric takes the daily voltage profile calculated at each customer connection point and checks compliance with the corresponding standard (e.g., BS EN50160 [27]).

The effects on the PV owners are quantified using the two indices defined below.

- *Exported energy index (EEI):* As shown in (16), it is the percentage of exported energy when an export limit is applied relative to the energy that would otherwise be exported without curtailment (no limit, $\gamma = 1$).

$$EEI = \frac{\sum_{t \in T | P_t^N > 0} P_t^N \Delta t \Big|_{\gamma}}{\sum_{t \in T | P_t^N > 0} P_t^N \Delta t \Big|_{\gamma=1}} \cdot 100\% \qquad (16)$$

where P_t^G and P_t^D are as defined in Section II.

- *Net benefit index (NBI):* As defined in (17), it calculates the percentage change in the net benefit a customer will experience from the introduction of an export limit (relative to the case without a limit). It considers the benefits from reducing imports, B^{IR}, calculated by (18), and from exporting into the grid, B^X.

$$NBI = \left(\frac{B^{IR} + B^X \big|_{\gamma}}{B^{IR} + B^X \big|_{\gamma=1}} - 1 \right) \cdot 100\% \qquad (17)$$

$$B^{IR} = \left(\sum_{t \in T} P_t^D \Delta t - \sum_{t \in T | P_t^D > P_t^G} \left(P_t^D - P_t^G \right) \Delta t \right) \cdot \lambda^e \qquad (18)$$

Since the net benefits depend on how customers are compensated from exporting into the grid, the calculation of the NBI varies accordingly. Equations (19) and (20) are used for this purpose considering the cases of net metering and FIT, respectively.

$$B^X = \sum_{t \in T | P_t^N > 0} P_t^N \Delta t \cdot \lambda^e \qquad (19)$$

$$B^X = \sum_{t \in T | P_t^N > 0} P_t^N \Delta t \cdot \lambda^{exp} + \sum_{t \in T} P_t^G \Delta t \cdot \lambda^{gen} \qquad (20)$$

where λ^e, λ^{exp}, and λ^{gen} are the electricity price, export tariff, and generation tariff, respectively. The generation tariff only applies to particular incentive mechanisms, and should be null if not applicable.

While the NBI is a more accurate indication of the impacts on PV customers, it depends on electricity and FIT prices. The EEI, on the other hand, is more generic as only considers changes in exported energy and, therefore, is independent of specific conditions related to price and incentive mechanisms.

C. OPF-Based Method vs. Monte Carlo-Based Approach

Each method provides a different type of results. For a given PV penetration, the OPF-based is designed to provide a single export limit value that meets technical constraints. On the other hand, the Monte Carlo-based assessment is designed to explore the trade-offs between network impacts and effects on PV owners by investigating multiple export limits. Consequently, the OPF-based is meant to provide a direct answer for a given PV penetration and set of constraints while the Monte Carlo-based assessment allows decision makers to assess the spectrum of options considering their effects on networks and customers. For comparison purposes, a summary of the key characteristics of each method is presented in Table I.

TABLE I
KEY CHARACTERISTICS OF THE PROPOSED METHODS

Characteristic	OPF-based Export Limit	Monte Carlo-based Assessment
Data Requirements	Full three-phase network modelling, including demand and generation profiles.	Same.
Implementability	Requires scripting, a mathematical modelling tool and adequate solvers. DNOs are not necessarily familiar with using these platforms.	Requires scripting and a power flow analysis tool. DNOs are familiar with using these platforms.
Network Modelling	More complexity in adapting the OPF formulation.	Easier to model complex equipment and connection arrangements in power flow platforms.
Output	A single export limit value that meets technical constraints whilst minimizing PV curtailment. No further processing is required. Creates opportunities for expediting assessments.	Network impacts and effects on PV owners for each of the investigated export limits. Post processing and a decision maker are required.
Final Outcome	Direct use of the export limit.	An export limit is defined based on the decision maker's desired trade-off between technical impacts and their effects on PV owners.
Application	DNOs and/or policy makers willing to avoid network investment through the (least possible) curtailment of PV generation.	DNOs and/or policy makers trying to strike a balance between network investment (to address technical impacts) and an acceptable level of effects on PV owners (due to curtailment).

Given the characteristics of the proposed methods, the choice will depend on the context in which the decision maker is in. A DNO in country A is facing the rapid uptake of PV systems. A quick export limit is needed where the priority is to avoid reinforcements. In this case, the OPF approach could be adopted as a tool to expedite such assessments. On the other hand, a DNO in country B is required to enable a given PV penetration. Since it is preferable for this DNO to avoid customer complaints due to curtailment, there is willingness to invest in network reinforcements. The Monte Carlo approach could be used to decide on an export limit that strikes the right balance between effects on PV owners and impacts on networks (which in turn can be translated into cost).

IV. CASE STUDY

This section presents the application of the proposed methodologies on a real UK LV network. First, the data used to model the LV network as well as the corresponding demand and PV generation profiles is presented. The results are then discussed for both the OPF and Monte Carlo methodologies, highlight-

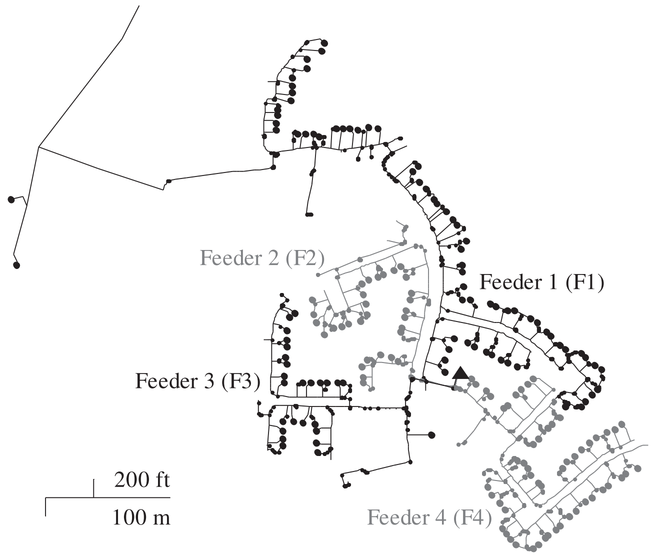

Fig. 2. LV network topology.

TABLE II
NUMBER OF CUSTOMERS AND LENGTH PER LV FEEDER

Feeder	F1	F2	F3	F4	Network
No. Customers	70	31	26	53	180
Total Length (m)	651	303	273	294	1521

ing the benefits to the network and effects on PV owners from adopting export limits.

A. LV Network and Profiles

A real LV network from the North West of England, provided by ENWL, is used in all the case studies presented [28]. The network supplies 180 single-phase residential customers through four feeders connected to an 800 kVA transformer. The transformation ratio is considered to be 11/0.416 kV so as to mimic typical busbar values in the UK. The topology is shown in Fig. 2, where the transformer is represented by a triangle, the customers by small dots, and the feeders are labelled accordingly. The number of customers per feeder and the length of each feeder are presented in Table II, with feeder F1 being the longest and the largest in terms of number of customers. This case study can be considered as a large test residential LV network as it embeds the IEEE European Low Voltage Test Feeder [29] (corresponding only to F4 in Fig. 2), expanding it to four feeders and 180 customers. For the Monte Carlo-based assessment, the network was fully modelled in OpenDSS [30]. For the OPF-based methodology, the formulation of the optimization problem was implemented in AIMMS using the well-established commercial solver CPLEX [31]. Other MILP solvers, commercial or not, can also be used.

The percentage of customers with voltage problems is quantified following the standard BS EN50160 [27], which states that a voltage rise problem occurs if the 10-min average voltage exceeds 10% (253 V line-to-neutral given that the nominal LV supply is 230 V). Residential demand and PV generation daily profiles with 1-min resolution were used in the Monte Carlo-based assessment. For the OPF-based methodology, 10-min profiles were adopted to reduce the corresponding computational burden whilst being aligned with the BS EN50160 requirements. The single-phase profiles were created using the CREST tool [32] and UK statistics of residential PV systems installed capacity and occupancy, presented in Table III and Table IV. The usage of realistic distributions is highly important, as it affects the definition of the export limit value. It is also worth mentioning that, in the UK, residential PV installations are, in general, limited to 16 A per phase; therefore, this case study considered PV systems of maximum 4 kWp [35]. Pools of 10,000 PV generation and 1,000 residential demand profiles (for weekdays and weekends) were created for each month of the year. Finally, the electricity, export and generation prices used are 0.151, 0.0503, and 0.0407 £/kWh, respectively. The generation and export tariff values correspond to those published by the UK regulator (Ofgem) for July 2017 to September 2017 [36]. The electricity price is the average price listed by three energy suppliers [37].

TABLE III
UK STATISTICS OF RESIDENTIAL PV SYSTEMS CAPACITY [33]

PV Size (kW)	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Probability (%)	1	1	8	13	14	14	12	37

TABLE IV
UK STATISTICS OF HOUSEHOLD OCCUPANCY [34]

Number of Residents	1	2	3	≥4
Probability (%)	29	35	16	20

B. OPF-Based Export Limit

The OPF-based methodology, expanded for multiple scenarios as presented in Section III-A-2, was applied to find the export limit for 10 PV penetration levels, from 10 to 100% in 10% steps. For each PV penetration level, the value of γ is calculated considering 100 scenarios. This number of scenarios is selected based on the findings in [19], where the authors demonstrate, also using UK LV networks, that the adoption of 100 scenarios is a sensible trade-off between accuracy and computational efforts. With each scenario, the following parameters are changed, based on the distributions provided in Table III and Table IV: demand profiles (to reflect different number of occupants), PV system locations and sizes, and PV generation profiles. To ensure an export limit that caters for most technical problems, these scenarios are comprised of summer weekdays (June; low demand and high irradiance) and are limited to the sunlight hours (7am to 7pm, i.e., 72 periods of 10-min each). Two values of α are investigated, 5 and 50%, which ensure export limits that cover 95 and 50% of the scenarios, respectively. The results are presented in Fig. 3 along with all the values of γ_s^{opt} for each penetration level (dots). For illustration purposes, the found export limit γ considering a single scenario is also presented. For instance, for a 70% PV penetration, this single scenario, has an aggregated load demand (180 customers) that varies from 20 to 90 kW and an aggregated generation that varies from 0 to 257 kW. The maximum aggre-

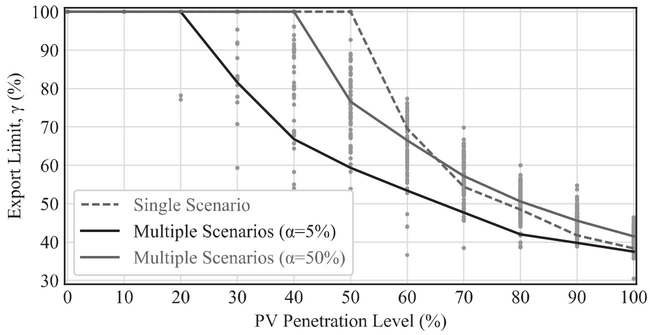

Fig. 3. Export limits (γ) and distributions of γ_s^{opt} for 100 scenarios per penetration level obtained using the OPF-based methodology.

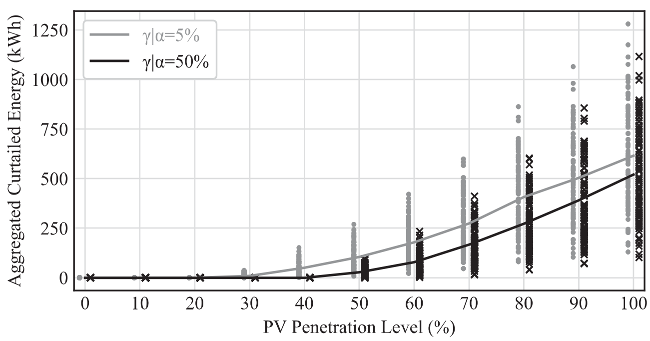

Fig. 4. Aggregated curtailed energy (individual scenarios and average) per penetration level using the OPF-based export limits (shown in Fig. 3).

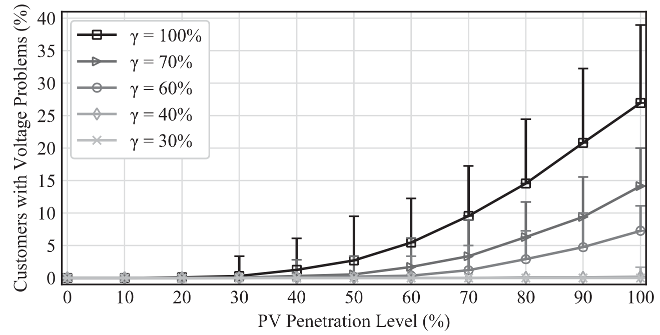

Fig. 5. Percentage of customers with voltage problems for different export limits (γ) and PV penetration levels.

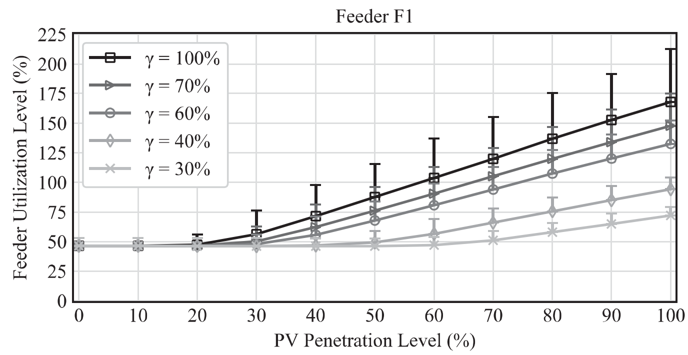

Fig. 6. Feeder F1 utilization level for different export limits (γ) and PV penetration levels.

gated net demand is 90 kW and the minimum is -224 kW (reverse power flow). The corresponding γ was found to be 54.4%.

The results with the multiple scenarios show that, as expected, the higher the PV penetration, the more reverse power flows and, thus, the lower the value of γ. Also, as the penetration increases, the dispersion of γ_s^{opt} tends to reduce. This suggests that an adequate value of γ could be found using only a single scenario for high PV penetration levels. Conversely, for low to medium PV penetration levels, the use of a single scenario can lead to a γ that is unlikely to cater for uncertainties. In fact, this can be seen when comparing the single and multiple-scenario curves in Fig. 3. Based on this single scenario, no export limit would be necessary ($\gamma = 100\%$) for 50% of penetration level; however, based on multiple scenarios, export limits of 59 and 77% would be needed for the same penetration for α equal to 5 and 50%, respectively.

As expected, the smaller the value of α the smaller the resulting export limit so as to ensure network constraints are satisfied in more scenarios. This, however, leads to higher volumes of curtailed energy despite the minimization done by the OPF. To illustrate this, the aggregated curtailed energy resulting from applying the export limits found for each PV penetration for α equal to 5 and 50% (shown in Fig. 3) to the net household profiles of each of the 100 scenarios is calculated using (4) and (5). The results are presented in Fig. 4 (dots for $\alpha = 5\%$ and crosses for $\alpha = 50\%$) including the corresponding averages. For 50% of PV penetration, the aggregated curtailed energy is, in average, 105 kWh using $\gamma = 59\%$ ($\alpha = 5\%$). This is almost four times higher than when using an export limit of 77% with $\alpha = 50\%$; which results in 27 kWh. For higher PV penetrations, however, this ratio becomes much smaller as the export limit values become closer for both α (see Fig. 3). For full PV penetration, an export limit of 38% ($\alpha = 5\%$) would result in an average aggregated curtailment of 616 kWh. With $\gamma = 41\%$ ($\alpha = 50\%$), this value is 520 kWh.

The results demonstrate that it is possible to define the export limit in such a way that voltage and thermal constraints are met for a pre-defined fraction of the investigated scenarios. Whilst this is useful to DNOs as they could potentially eliminate the impacts from PV installations, such a prescriptive approach leaves no room to explore the trade-offs (for customers and the DNO) from adopting different values. This is explored in the next sections.

C. Monte Carlo-Based Export Limits

To explore how progressive values of γ can provide different levels of mitigation for the LV Network, this section applies the methodology presented in Section III-B-1 to define the export limits based on a Monte Carlo assessment. To cover the entire spectrum of combinations between PV penetration levels and export limits, both were varied from 0 to 100% in 10% steps, resulting in 121 combinations ($N_{EL} = 11$ and $N_{PV} = 11$). Each combination is explored considering 100 scenarios, each representing summer weekdays (June), as discussed in Section IV-B.

The percentage of customers with voltage problems and the feeder utilization level are shown in Fig. 5 and Fig. 6, respectively, for selected values of γ; the lines correspond to the av-

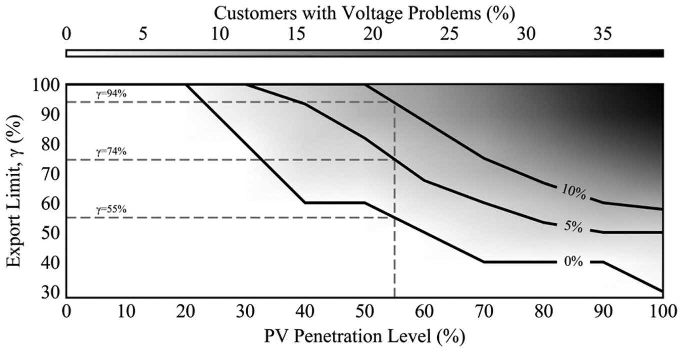

Fig. 7. Export limits (γ) required to keep the percentage of customers with voltage problems limited in 0, 5, or 10% for 95% of the scenarios.

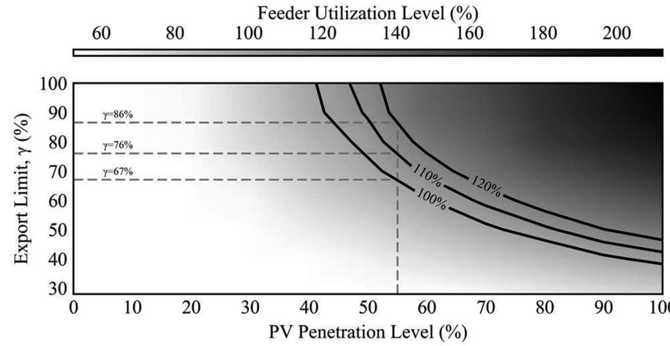

Fig. 8. Export limits (γ) required to keep the feeder utilization level limited in 100, 110, or 120% for 95% of the scenarios.

erages and the error bars represent the 95th percentile (i.e., covering 95% of the scenarios). For the feeder utilization level, results are presented for feeder F1, the most impacted by the aggregated exports from houses with PV systems. Since the transformer utilization level is less than 100% even for full PV penetration without export limit, the results are not shown.

Without export limits ($\gamma = 100\%$) and considering 95% of the scenarios, voltage issues start to appear for 30% of PV penetration, when the percentage of customers with voltage problems is 3.3%. For full PV penetration, this metric increases to 39% of customers. However, as expected, if an export limit is applied, the occurrence of problems is postponed. If γ is 70, 60, or 40%, voltage issues will be postponed to 40, 50, or 100% of penetration level. With these values of γ and full PV penetration, the percentage of customers with voltage problems is reduced to 20, 11, and 2%, respectively. With a γ of 30%, no problems occur for any penetration. Regarding thermal issues, without an export limit, problems start at 50% of PV penetration. With values of γ equal to 70, 60, or 40%, problems in Feeder 1 would start later on: 60, 70, or 100% of penetration level, respectively. As expected, the lower the export limit, the lower the level of technical problems. From Fig. 5 and Fig. 6, it can be concluded that for this LV network, a value between 30 and 40% would ensure that 95% of the scenarios (for any PV penetration level) would have neither voltage nor thermal issues; which is aligned with that found by the OPF considering multiple scenarios (38% for $\alpha = 5\%$).

The performance metrics quantified in Fig. 5 and Fig. 6 show how different values of γ reduce the impacts of PV installations. However, those graphs do not directly show the value of γ needed to meet a desired level of impacts (e.g., up to 5% of customers with voltage problems); which could assist DNOs in identifying a suitable export limit. For this purpose, the statistical results from Fig. 5 or Fig. 6 can be projected in the plane *penetration level vs. export limit* to produce heat maps for the corresponding LV network performance metric. Using interpolation, contour lines can then be calculated to represent desired levels for those metrics. To illustrate this, Fig. 7 shows three contour lines that relates, as a function of the penetration level, the value of γ required to keep the percentage of customers with voltage problems limited to 0, 5, or 10% for 95% of the scenarios. Similarly, Fig. 8 presents three contour lines to relate the export limit required to limit the feeder utilization level to 100, 110, and 120%. These figures allow also the identification of values of γ for penetration levels different from those accessed in Fig. 5 and Fig. 6. For instance, for 55% of PV penetration level (dashed line in Fig. 7), an export limit of 55% is required to avoid customers with voltage problems. Increasing γ to 74% results in up to 5% of customers with voltage problems and 110% of feeder utilization level, as shown in Fig. 8; if these metrics are further relaxed, γ equal to 86% results in up to 10% of customers with voltage problems and 120% of feeder utilization level.

The analyses with feeder utilization levels above 100% are intended to show the extent to which export limits could be increased if the network had larger thermal capabilities, not to recommend the overloaded operation of the network; although some DNOs might be able to tolerate such levels for certain periods. It is worth mentioning that the values of γ obtained using the OPF-based methodology (Fig. 3) can be higher than those obtained using the Monte Carlo approach using the same constraints, i.e., 0% of customers with voltage problems (Fig. 7) and up to 100% of feeder utilization (Fig. 8). This is because in the OPF γ changes continuously, while in the Monte Carlo this is carried out in 10% steps. For instance, to accommodate 70% of PV penetration, a γ of 47.7% is obtained by the OPF. For the Monte Carlo, a γ equal to 40% results in no issues but a 50% would lead to 2.8% of customers with voltage problems; therefore a γ of 40% is adopted by the Monte Carlo. If the assessment is done using smaller steps for the values of γ (e.g., 1%), it would lead to values more aligned with those found by the OPF, as discussed in the Appendix-B.

Although Fig. 7 and Fig. 8 are useful to define the required export limits to meet a desired level of impacts, they are still incomplete because they neglect the effects of export limits on PV owners. In the following subsection, those results will be combined with the corresponding effects of export limits on PV owners, allowing a complete trade-off assessment.

D. *Effects on PV Owners*

To assess the extent to which different export limits affect PV owners, the performance metrics EEI and NBI presented in Section III-B-2 are calculated using year-long profiles to capture changes due to seasonality and demand behavior. This ensures a fair quantification of the curtailment seen by PV own-

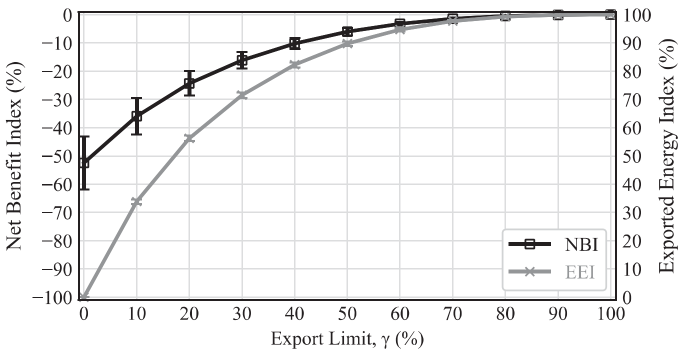

Fig. 9. EEI and NBI for different export limits (γ).

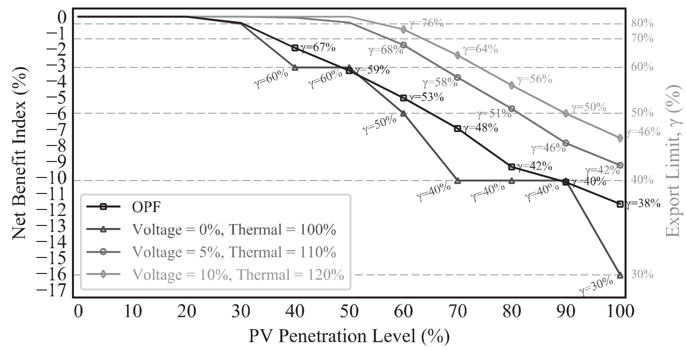

Fig. 10. Export limit trade-off chart for selected combinations of mitigation levels (voltage and thermal issues) and for the OPF-based methodology.

ers throughout the year and not only during critical periods such as summer (likely to have more curtailment).

The EEI and NBI are calculated using household energy flows only (i.e., independent from network interactions) considering 100 individual customers with continuous 365-day 1-min resolution demand and generation profiles (produced using the CREST tool [32]) to assess each of the 11 investigated export limits (from 0 to 100% in 10% steps). The results are shown in Fig. 9, in which the lines indicate the mean value and the error bars correspond to a standard deviation.

Both the EEI and NBI plots display a behavior that resembles the cumulative distribution function of a negative exponential distribution. This means that conservative export limits ($\gamma > 50\%$) result in minor impacts on PV owners. For instance, for $\gamma = 50\%$, PV owners could see a 10% reduction on exports ($EEI = 90\%$) and 6% reduction on their net benefit ($NBI = -6\%$). Conversely, when the export limit is reduced towards 0%, the EEI converges to 0% and the NBI to -52% (half of the benefits that otherwise could have been obtained). With this information is now possible to carry out a trade-off assessment to define the export limit.

E. Trade-Off Assessment

By combining the information from the EEI and NBI (Fig. 9), the contour lines for the technical problems (Fig. 7 and Fig. 8), and the priorities of the decision maker, an export limit can be defined by intersecting the corresponding values.

For instance, for a customer-centric decision maker, an export limit of 50% would be acceptable due to the small effect on PV owners. This value can then be used to identify the maximum PV penetration that does not exceed a desired level of technical impacts. For example, using Fig. 7 and Fig. 8 with $\gamma = 50\%$ and assuming no voltage or thermal issues are acceptable, the maximum PV penetration would be 60%. This value can increase if a certain level of technical impacts is acceptable. For instance, allowing 5% of customers with voltage problems (Fig. 7) and 120% of feeder utilization (Fig. 8) can push the maximum PV penetration to 90%.

To allow a more straightforward trade-off assessment, three export limits per penetration level obtained from the Monte Carlo analysis presented in Fig. 7 and Fig. 8 are combined with the NBI per export limit level shown in Fig. 9 to produce Fig. 10, the trade-off chart. For completeness, the export limits per penetration level obtained from the OPF analysis for $\alpha = 5\%$, shown in Fig. 3, are also included.

Fig. 10 facilitates defining the most suitable export limit for a given PV penetration considering the trade-off between effects on PV owners and LV networks. For instance, to accommodate 100% PV penetration without technical problems it would be required to impose a $\gamma = 30\%$ which leads to a 16% reduction in benefits for the PV owners. Alternatively, by accepting some technical impacts (which might require reinforcements), the export limit could be raised to 42% or 46% which in turn results in lesser effects on PV owners (9.4 and -7.7% of NBI, respectively). It should be noted that as the export limit found from the OPF analysis is optimized to minimize curtailment, the resulting reduction in NBI, for all PV penetrations, is less than when compared to the equivalent Monte Carlo case. For instance, for the OPF, 100% of PV penetration requires a $\gamma = 38\%$ which in turn leads to an 11.4% reduction of the benefits to customers.

V. Discussion

The proposed methodologies are generic and, therefore, can be applied to any LV network and incentive mechanism. The challenge, however, is the large number of LV networks that exist in a given DNO area. To address this, a relatively small set of representative LV networks [38] can be analyzed so as to define an area-wide export limit.

The quantification of net benefits is heavily dependent on electricity and FIT prices. In this case study, the obtained NBI values are expected to be higher (less impact on PV owners) with the gradual reduction of FIT prices and the increase of electricity prices over the coming years.

VI. Conclusions

To mitigate the technical issues from high PV penetrations in residential LV networks, PV export limits can be introduced. Defining the most adequate limit, however, requires understanding the trade-offs between the technical benefits and the effects on PV owners. To assist decision makers, this paper proposed two methodologies: an OPF-based formulation that minimizes energy curtailment and a Monte Carlo-based trade-off assessment.

TABLE V
LARGEST ERRORS FOR THE VOLTAGE CALCULATED BY THE LINEARIZED POWER FLOW (PU)

	Phase 1	Phase 2	Phase 3
Linearized Power Flow	1.0972	1.0499	1.0982
OpenDSS	1.0977	1.0497	1.0987
Error	-0.0005	0.0002	-0.0005

The results, based on a real UK LV network, showed that the OPF approach is an effective, direct tool to find an export limit when network constraints need to be met. However, the application of the Monte Carlo assessment allowed quantifying the technical benefits and the effects on PV owners for different export limits, making it possible to explore the spectrum of values through trade-off charts.

For instance, to encourage PV installations, a decision maker could use this to determine the extent of acceptable technical issues (a proxy for the need of further mitigating actions) from using export limits that do not heavily penalize customers. Conversely, the adoption of strict export limits could be used to influence PV owners to shift their demand to times of high PV generation (e.g., heaters, laundry machines, etc.) or to purchase energy storage systems to store excess generation; resulting in a more efficient use of PV generation.

APPENDIX

A. Accuracy of the Linearized Power Flow

A simple test using the real UK LV network has been carried out to compare the power flow results between the linearized formulation and OpenDSS. This test considers a scenario with 70% of PV penetration without curtailment. The largest voltage error per phase of the linearized power flow, shown in Table V, occur at a time period corresponding to 3:50 pm and is limited to 0.05%. Moreover, the largest error in the apparent power through the transformer is lower than 0.12% (256.66 kVA calculated by the linearized power in comparison to 256.36 kVA obtained by OpenDSS). These low errors indicate that, although it is not exact, the linearized power flow can be accurate enough for practical applications.

B. Analysis of Export Limit Steps

To demonstrate that for a specific condition (when network constraints are met) both methods are numerically very close to each other, thus validating the results, the same case study presented in Section IV, for a 70% PV penetration, is partially reassessed here for the Monte Carlo-based assessment considering steps of 1% (instead of 10%). Customers with voltage problems and the utilization level of feeder F1 are shown in Table VI for $40\% \leq \gamma \leq 50\%$. The values correspond to the 95th percentiles (i.e., covering 95% of the 100 scenarios assessed).

Considering voltage and thermal issues, a γ equal to 47% results in no issues. However, a 48% would lead to 1.1% of customers with voltage problems. This value ($\gamma = 47\%$) obtained by the Monte Carlo-based assessment using steps of 1% is much closer to the value obtained by the OPF-based methodology ($\gamma = 47.7\%$). Consequently, the smaller the steps are, the closer the results found by both methodologies will be. However, it is worth mentioning that, from a policy perspective, and to ensure a straightforward implementation, an adequate value is likely to be a simple one (e.g., a multiple of 10 or 5 such as 70%) instead of a very precise one (e.g., 73.5%).

TABLE VI
PERCENTAGE OF CUSTOMERS WITH VOLTAGE PROBLEMS AND FEEDER F1 UTILIZATION LEVEL FOR DIFFERENT EXPORT LIMITS (γ) AND 70% OF PV PENETRATION LEVEL

Export Limit γ (%)	*Customers with Voltage Problems (%)*	*Feeder F1 Utilization Level (%)*
50	2.78	97.01
49	2.25	95.11
48	1.14	93.20
47	0	91.30
46	0	89.39
45	0	87.47
44	0	85.56
43	0	83.64
42	0	81.72
41	0	79.79
40	0	77.86

C. Delta-Connected Loads Within the AC OPF

For delta-connected loads, active and reactive powers per phase in (7) and (8) can be calculated by (21) and (22) in terms of the active/reactive load between phases $\phi - \psi$ and estimated voltages ($V_{h,t,\psi}$ for phase ψ). This, however, requires special considerations (e.g., using balanced voltages and angles) to produce the corresponding linearization.

$$P^D_{h,t,\phi} = Re\left\{V_{h,t\phi} \angle \theta_\phi \sum_{\psi \neq \phi} \frac{P^D_{h,t,\phi\psi} + jQ^D_{h,t,\phi\psi}}{V_{h,t\phi}\angle\theta_\phi - V_{h,t,\psi}\angle\theta_\psi}\right\} \tag{21}$$

$$Q^D_{h,t,\phi} = Im\left\{V_{h,t\phi} \angle \theta_\phi \sum_{\psi \neq \phi} \frac{P^D_{h,t,\phi\psi} + jQ^D_{h,t,\phi\psi}}{V_{h,t\phi}\angle\theta_\phi - V_{h,t,\psi}\angle\theta_\psi}\right\} \tag{22}$$

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