Ilha Solteira

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"JÚLIO DE MESQUITA FILHO"
FACULDADE DE ENGENHARIA
CAMPUS DE ILHA SOLTEIRA

LAURA MÓNICA ESCOBAR VARGAS

SPECIALIZED MODELS FOR THE LONG-TERM TRANSMISSION NETWORK EXPANSION PLANNING PROBLEM

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Ph.D. thesis presented to the Faculty of Engineering, Campus of Ilha Solteira – UNESP, as part of the requirements for obtaining the title of PhD in Electrical Engineering. Field of knowledge: Automation.

Prof. Dr. Rubén Augusto Romero Lázaro
Orientador

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AUTORA: LAURA MONICA ESCOBAR VARGAS
ORIENTADOR: RUBEN AUGUSTO ROMERO LAZARO

Aprovada como parte das exigências para obtenção do Título de Doutora em ENGENHARIA ELÉTRICA, área: Automação pela Comissão Examinadora:

Prof. Dr. RUBEN AUGUSTO ROMERO LAZARO
Departamento de Engenharia Elétrica / Faculdade de Engenharia de Ilha Solteira

Prof. Dr. JOHN FREDY FRANCO BAQUERO
Departamento de Engenharia Elétrica / Universidade Estadual Paulista Júlio de Mesquita Filho, Câmpus Experimental Rosana

Dr. JONATAS BOAS LEITE
Departamento de Engenharia Elétrica / Faculdade de Engenharia de Ilha Solteira

Prof. Dr. EDUARDO NOBUHIRO ASADA
Departamento de Engenharia Elétrica e de Computação / Escola de Engenharia de São Carlos - USP

Prof. Dr. LUIZ CARLOS PEREIRA DA SILVA
Departamento de Sistemas de Energia Elétrica / Universidade Estadual de Campinas - UNICAMP

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RESUMO

A análise de sistemas altamente complexos quando e analizado o problema de planejamento de expansão de redes de transmissão de longo prazo, é o foco principal deste trabalho. Os modelos e metodos propostos são aplicados ao problema de planejamento estático tradicional, que é um problema de otimização matemática classificado como NP-completo, não-linear inteiro misto. O qual envolve no investimento, variáveis operacionais contínuas e variáveis inteiras. O comportamento normal de cada sistema pode conter informação essencial para a criação de novos métodos, como os planos de corte baseados em cortes de diferença de ângulos para problemas de grande escala, o que é a base é o ponto de partida deste trabalho, derivando em desigualdades válidas é ciclos críticos. Os cortes angulares básicos reduzem o espaço de busca do problema e o tempo total de cálculo deste problema, enquanto ao método de inequações válidas que pode ser usado para fornecer limites inferiores sólidos no investimento ótimo do planejamento de transmissão, já que a diferença entre o modelo DC (modelo exato) e o modelo de transporte (modelo mais relaxado) são as restrições angulares. Os ciclos críticos têm sido desenvolvidos para melhorar alguns dos modelos tradicionais do problemas de planejamento da expansão da rede de transmissão de longo prazo. A razão por trás disso é a ausência da segunda lei de Kirchhoff, que completa a representação do sistema, mas aumenta a complexidade. Para resolver os problemas resultantes, este trabalho usa a linguagem de modelagem AMPL com o solver CPLEX. Assim, este trabalho apresenta um método novo e mais eficiente para reduzir o espaço de busca do problema, a fim de melhorar o processo de solução através de diferentes tipos de desigualdades válidas e a combinação de ciclos com modelos tradicionais. Por ultimo, uma heurística de baixo esforço é proposta para permitir que o modelo identifique as opções de investimento mais atraentes, que devem ser consideradas no processo de otimização que descartará as opções de investimentos pouco promissoras que complicarão o problema sem contribuir para a solução final. Os testes fazem uso de quatro sistemas clássicos da literatura especializada: o sistema sul-brasileiro, o sistema colombiano, e o sistema norte-nordeste brasileiro.

Palavras-chave: Planejamento da expansão de redes de transmissão. Cortes de diferença de ângulo . Ciclos críticos. Desigualdades válidas. Otimização. Modelo de transmissão de baixo esforço. Modelo de transporte disjuntivo melhorado. Modelo de transporte disjuntivo melhorado. Segunda lei de Kirchhoff.

ABSTRACT

The analysis of highly complex systems when solving the long-term transmission network expansion planning problem is the main focus of this work. The proposed improved models and methodology are applied to the traditional static planning problem, which is a mathematical optimization problem classified as NP-complete and mixed-integer nonlinear problem. It involves continuous operating variables and integer investment variables. The normal behavior of each system can be shown essential information to the creation of new methods, as the cutting-planes based in bus-angle difference cuts for large-scale problems which were the starting point of this work, deriving in valid inequalities and critic cycles. The angular cuts aim to reduce the search space of the problem and the total computation time of this NP-hard problem as for the valid inequalities method that can be used to provide strong lower bounds on the optimal investment of the transmission planning, since the difference between the DC model (exact model) and the transport model (more relaxed model) are the angular constraints. Critic cycles has been develop in order to improve some of the traditional long-term transmission network expansion planning problem models. The reason behind it is the absence of second Kirchhoff's law which completes the representation of the system, but increase the complexity. In order to solve the resulting problems, this work uses the modeling language AMPL with the solver CPLEX. In test systems with many variables and constraints, Hence, this work presents a new, more efficient method to reduce the search space of the problem in order to improve the solution process through different types of valid inequalities and the combination of cycles with traditional models. Lastly, a low effort heuristic is proposed to allow the model to identify the most attractive investment options, which should be considered in the optimization process which will discard the unpromising investment options that will complicate the problem without contributing to the final solution. The tests make use of four classic systems in the specialized literature: the Southern Brazilian system, the Colombian system, and the Brazilian north-northeastsystem.

Keywords: Transmission network expansion planning. Bus-angle difference cuts. Critic cycles. Valid inequalities. Optimization. Low effort transmission model. Improved disjunctive transportation model. Improved disjunctive transportation model. Second Kirchhoff law.

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1 INTRODUCTION

The objective of the transmission-network expansion planning problem (TEP) is to find the least costly investment options in new transmission devices in order to ensure proper power system operations in the future (GARVER, 1970). The transmission network is a complex and importantsystem, and it is one of the most critical engineering infrastructureof the modern era. It transmits the power generated at a variety of facilities and distributes it to end users, over long distances. Addressing this problem is importantbecause the transmission network belongs to the so-called heavy technologies, and it is both expensive and difficult to withdraw from them or relocate once the new elements are installed (DOMINGUEZ, 2017). Inadequate long-term planning can lead to low service quality, excessive oversizing, inefficient systems with high operating costs, and delays in the expansion of electricity markets. While new systems are growing in size and the demands imposed on them are increasing, infrastructure budgets are sharply decreasing. Hence, it is critical to derive solutions that maximize cost efficiency. For these reasons, it is necessary to devise new planning methodologies that can effectively deal with the associated combinatorial difficulties of the underlying TEP optimization models. Operation at higher voltage and higher power levels allows the use of economies of scale. This gives rise to the problem of planning the expansion of electric power transmission networks. The transmission capacity of the planned transmission network has significant impacts on the introductionof competition in the electricity sector. The need for planning also corresponds to the scope and complexity of the electricity system, including the different actors responsible for the evolution of the sector, both on the supply side and the demand side. Their decisions impacts all agents and influence the future of the systems. These need systematic processes of support and decision, especially regarding the prospects for the future.

The TEP considers that the following aspects have been previously established:

- The growth of demand in the bus bar of the system.
- The location and quantity of demand for new bus bars.
- The quantity and location of a new and existing generation.
- The additional capacity of existing generation plants that are being or will be re-powered.
- The location, cost and electrical characteristics of the new transmission corridors.

- The cost associated with additional circuits that can be connected in parallel with existing circuits in existing corridors.
- The location, size and cost of new substations.
- The type of conductors to use in the network.
- The voltage levels at which the new and existing corridors will operate.
- The voltage levels associated with the new substations.
- The new options to consider: HVDC links, Flexible AC Transmission Systems (FACTS), and Energy Storage Systems (ESS), among others.
- The base topology and the planning horizon

1.1 IMPORTANCE OF THE TRANSMISSION SYSTEM EXPANSION PLANNING

The planning and development of a power system infrastructure to supply the needs of a continuous growing network is of great importance at any point in time. However, it is becoming increasingly difficult, as, in recent years, there has been an increase of new technologies added to the system, new generation and transmission technologies, unexpected inflation in equipment or labor costs or change of national income, which can all mean that the system plans may go in another direction. Additionally, the increase on the general demand with the migration of people from the rural areas to the urban cities.

Although the transmission expansion planning of the network is carried out according to a logical and coherent structure, the accompanying uncertainty can lead to misdirection. Adequate planning of transmission expansion planning (TEP) not only puts the supply of electrical energy at risk, but it also highlights safety and security. For this reason, the the TEP must be periodically rethought, in order to periodically adjust to the changes that are presented either by the entry of new projects or by changes in the predictions of generation and demand. The main consequence of a bad planning process is an inefficient system operating for many years, once a transmission line or a substation has been built, as its removal or transfer is unfeasible (DUQUE; ESCOBAR AND GALLEGO, 2014).

In developing countries, power system planning has become more difficult, but it has also become even more important to provide the necessary information in order to enable decisions to be made today about circumstances many years in the future (SCHRAMM, 1990; WILBANKS, 1990). In almost all cases, planning must be done in the face of many uncertainties (SILVA; RIDER AND ROMERO, 2006; CHARLIN; RUDNICK; ARANEDA, 2015; WEN; HAN; LI; CHEN; YI; LU, 2015; CHOI; EL-KEIB; TRAN, 2005), and this misinformation can represent a big problem when analyzing these problems; one of the most common mistakes is a lack of awareness when it comes to the normal behavior of the system, which can give enough clues of exactly where to to find better techniques to solve these new networks faster and more effectively.

Since the 1970s, (GARVER, 1970), optimization concepts and techniques have been part of the transmission system expansion planning problem. The development of computers, the algorithms and the mathematical optimization methods allowed the application of mathematical optimization to the solution of this problem, and its implementation in computer systems. Today, the development of new mathematical models is crucial to obtain the optimal solution of great size systems with new investment options, that have not been solved yet. Once we have a mathematical model that represents the problem properly, it is necessary to choose the most appropriate method to solve it. We can use exact methods, when possible, or heuristic, hyperheuristic, metaheuristic or matheuristic techniques. The recent development of efficient commercial solvers based on exact optimization methods, such as the branch and cut method, has increased the interest of researchers in the development of more complex and more realistic mathematical models for the transmission expansion planning problem. If the commercial solvers can find the optimal solution in all cases, the planner does not need to develop specialized solution methods, which unfortunately, is not the case at present.

The transmission expansion planning problem can be represented as a linear or nonlinear problem, with integers and continuous variables. In the relaxed versions, the integer variables are eliminated and are replaced by continuous variables. According to the mathematical representation this problem may be classified as follows:

- Linear Programming (LP), where the term 'linear' indicates that all constraints, as well as the objective function, are linear, and the variables are continuous.

- Nonlinear Programming (NLP), which aims to work with problems involving nonlinear constraints or nonlinear objective functions, and the variables are continuous.

- Mixed Integer Linear Programming (MILP), which is a special type of LP, where some of the decision variables are integers.

- Mixed Integer Nonlinear Programming (MINLP), a special type of NLP where some of the decision variables are integers.

There are several well-known optimization techniques, according to each type of mathematical model. The LP problems can be solved using the SIMPLEX or the interior point method. For the NLP problems there is a wide variety of methods available, Newton's method, gradient method, quadratic programming, and successive linear programmingare some examples.

We can solve MILP problems using the algorithms: branch and bound, branch and cut, branch and price, branch and cut and price, Benders' decomposition, and Gomory's cutting planes, among others. But the MINLP problem are more complicated to solve, and the commercial solvers use algorithms based on branch and bound algorithms, sensitivity and barrier methods, and interior point methods that can not always provide an optimal global solution even when the system is small. In these cases it is necessary to use heuristics, hyperheuristics, metaheuristics, or matheuristics that do not guarantee the optimal solution, but do find sub optimal solutions of good quality. In recent years, the commercial solvers based on classical optimization techniques have become extremely efficient. Solvers that target LP and MILP problems such as CPLEX IBM (2017, last access october 2018), KNITRO, and GUROBI, have become extremely efficient compared to prior versions. Specialized solvers for MINLP problems are still in progress.

In its standard form, the DC model, TEP consists of linear and non-linear constraints that include continuous variables (voltage angles, power flows, etc) and integer variables (the number of lines to be added to the network). Accordingly, TEP is a non-convex, mixed-integer nonlinear programming problem. It is NP-complete, which makes its solution generally intractable (ROMERO; GARCIA; HAFFNER, 2002; LATORRE; CRUZ; AREIZA, 2003). This is exacerbated by the fact that in large-scale systems, the number of network components and associated restrictions can number in the hundreds or thousands. That is, the size and/or topology of the transmission network and the inclusion of discrete variables for representing possible transmission investments lead to a combinatorial explosion of potential solutions. Due to these complications, in general, TEP cannot be practically solved using standard optimization techniques. Different modeling techniques and algorithms have been proposed to expedite the solution times (HAGHIGHAT; ZENG, 2018; SILVA; FREIRE, HONÓRIO, 2016; CABRERA; ALCARAZ; GIL, 2018; CHOI; EL-KEIB; TRAN, 2005). The exact methods require larger calculation times when compared to those required by metaheuristic techniques such as Tabu Search (GALLEGO; MONTICELLI; ROMERO, 2000; GARCIA-MARTINEZ; ESPINOSA-JU AREZ; RICO-MELGOZA, 2015) and Genetic Algorithms (ROMERO; GALLEGO; MONTICELLI, 1998; JALIL ZADEH, 2009), among others. However, the latter techniques generally do not provide formal optimality guarantees. In small and medium-sized systems, the ideal solution can be found using methods such as branch-and-bound or branch-and-cut when a disjunctive integer linear model is utilized (BINATO; PEREIRA; GRANVILLE, 2001; SOUSA; ASADA, 2011; DINAVAHI, 2017). Such methods provide formal guarantees, but they are computationally demanding.

The process of continuous restructuringof the electricity sector produces the emergence of new agents and new requirements, that generate different competitive schemes with specific characteristics for each system, making it difficult to obtain general solutions to the different problems that appear in the market efficiency process (ZOLEZZI; RUDNICK, 2002). TEP is more complex to develop in restructuredelectrical systems because, in these systems, the new investment decisions are the result of market forces rather than centralized decision processes. The future expansion plan affects the electricity market because: it 1) reduces the operating cost of the system. (cost of future dispatches, cost of transmissionconstraints and cost of safety generation, among others); 2) reduces the investment cost; (3) reduces the technical losses, and 4) increases electrical safety. Network planning is a process that adopts the future system to the growth of demand for electricity, which is accompanied by the growth of generation. The new systems are becoming larger and the demands imposed are increasing, so it is necessary to develop new planning methodologies that can also adapt to the new technologies, involving the construction of equipment and available elements, and to the new mathematical optimization techniques. Currently, the world population is around 7300 million people with an energy consumption of approximately 350 million MWh and it is estimated that, by 2050, the world economy will grow almost fourfold and the world populationwill be 10000 million people with an energy consumption80% higher (OECD, 2012, Last access in january 2018). This growth will affect the increase of generation plants in the world and therefore, transmission systems will have to evolve to more efficient and reliable energy transportation systems. There are several solutions to improve the capacity of transportingenergy in a transmission system, such as identifying characteristics in the connections between nodes, which restricts the operation of other lines or may cause future problems in the operation of the transmission network.

An importantpart of analyzing these discrepancies is how they highlight possible defects in a network operation that, when not analyzed in depth, can go unnoticed, hidden among the thousands of data points analyzed and containing the information necessary to find small real behaviors that can lead to major problems. Combining all this new informationcan improve the performance of the model used and in the same way in the process of creating new constraints that help to use these characteristics to find a better solution or in defect the reduction of the solution space, which consequently reduces the computational effort in the search for the solution of the problem. In other words, it seeks to know what the appropriate behavior is in the system and, thus, ensure that the model can find an optimal and robust solution for the future network.

The new high-performance computers and the advances in techniques have produced changes in the planning of the power system, from the experimental design to the intelligent and low-cost design (DOMINGUEZ, 2012). Analyze in depth large transmission systems is highly importantto reduce the space of the solution, in this work is proposed the use of angular cuts in combinationwith exact methods based on branch and cut. Using this procedure, the idea is to find a linear inequality that induces close limitations on the facets of the convex hull of the MILP problem.

In the specialized literature (SILVA; RAHMANI; RIDER, 2014; RAHMANI; ROMERO, RIDER, 2013), there are several proposals for the reduction of variables and/or the reduction of the upper limits of the investment variables in each corridor. In this way, in addition to deciding which transmission corridors are those that are selected for analysis, the number of lines to be added is also limited. In Mendonça, (2016, static planning is solved by using a constructive heuristic method combined with an adaptive search procedure (GRASP to reduce the solution space. The method, which uses the largest flow through the lines based on Garver (1970) as a sensitivity index, executes in a first step a DC load flow with all the relaxed variables, and then build the line with the highest sensitivity index. In the next step, the DC load flow is resolved again, but this time with the new variables to add lines of all type and if the line that was selected by the sensitivity index is part of the base network. This is an iterative process that stops according to the value of the unmet demand. In the end, the candidate transmission corridors are reduced to those where the lines were built. In the final part, these corridors are the only ones where the lines can be added and the planning is solved with the metaheuristic method particle swarm optimization. In Mendonça, (2016, the solution space for portfolio-based static planning is also reduced, in the end, the technique that solves the problem is particle swarm optimization. The reduction of variables and/or the reduction of the upper limits of the investment variables in each corridor, in combination with the angle-cuts theory, use the more powerful features of both ideas to reduce computational effort and reduce solution space. In this work, we use the Brazilian Northeastern system as a test system. Rahmani (2013) finds better solutions for the Brazilian Northeastern system applying heuristics to establish the maximum number of circuits and applying specialized constraints that function as cuts in the space of solution. Duque, Escobar and Gallego (2014) finds the same solution as in the previous work, in a much shorter computational time, applying the concept of network division. In this work, we use a reduction of variables, reduction of the upper limits of the investment variables and bus-angle cuts with very good results.

1.2 MOTIVATION TO CREATE NEW VALID CONSTRAINTS BASED ON NORMAL SYSTEM BEHAVIOR

With everyday analysis, there are a vast number of different characteristics and behaviors that can lead to new methods that can improve the computational effort and in some cases, find optimal solutions. As can be found in the specialized literature data mining has been an important step in the research process for different kinds of high complexity problems, where a really good solution can be the starting point to identify what makes a good solution good (FLORIAN; KENNETH, 2017). Based on this analysis normal system behavior can show us which path to follow to find these starting points and active local solutions, from which the new optimal solutions can be found in the future.

Reducing the computation effort can be a dividing topic on the research field, since, for the majority of cases, companies will be more interested in the optimal solution, than in the time needed to find it, this underestimates the importance of this new methods and heuristics and the impact they have on finding better solutions in the future. Moreover, the continual restructuring process of the electricity sector produces different competitive schemes with specific characteristics for each system. Accordingly, network planning needs to adapt to the future growth of demand, which is accompanied by the growth of generation (ZOLEZZI; RUDNICK, 2002).

Due to the combinatorial explosion of TEP, it is not possible to find an optimal solution for large-scale systems using standard analysis, off-the-shelf algorithms. The computational difficulty of the problem is related directly to the size of the system to be analyzed. However, other factors increase computational difficulty, including the connectivity of the buses or how well the system is enmeshed. In TEP, only a small number of instances can be solved to optimality with mixed-integer linear programming methods. Usually, the complexity of the problem is directly related to the size of the system being analyzed. However, for TEP other factors also contribute to complexity, as previously explained, including the degree of connectivity of the nodes, the degree of the radial or meshed structure of the system, the number of isolated nodes, and the presence of the Second Kirchhoff's Law (KSL) constraints. In particular, the KSL becomes complicated when a set of connected circuits (new and existing) forms a loop.

This effect produced for the connection in parallel of circuits of low capacity-reactance product with circuits of high capacity-reactance product can provide insight into the system behavior which is a topic that can be exploited to find new paths to better solve the TEP. Since this is a complicating constraint when added to the traditional models the need to develop new methodologies which can replace the original constraints with a conjunct of new constraints based in cycles or add new constraints, or valid cuts, that reduce the solution space. An improved transportationmodel can be used too, to obtain the active power flow, which in some cases is not distributed equally in all the lines in parallel or some of them presents a power flow equal to its maximum capacity, when some other lines present a power flow much lower than its medium capacity, when we use the traditional transport model. All these ideas will be developed later on this work.

1.3 OBJECTIVES

1.3.1 General objective

Development of a methodology that solves the transmission expansion planning problem using the strongest cuts based in the bus-angle difference to reduce the solution space.

1.3.2 Specific objectives

- Present the state of the art for the transmission expansion planning problem.
- Present the models used for the transmission expansion planning problem.
- Develop new models for the transmission expansion planning problem.
- Incorporate the idea of less effort focused differently to find the maximum lines limits needed in the future.
- Develop the strongest cuts based in capacity-reactance constraints, bus-angle difference constraints, and paths of transmission constraints.
- Test the behavior of the methodology using instances of the specialized literature, specifically, the Brazilian Northeastern test system.

1.4 STATE OF THE ART FOR TRANSMISSION EXPANSION PLANNING

The TEP problem evolved both in the mathematical model used to represent the problem and in how to solve the problem. Garver (1970) was the first to propose a model based on the concept of load flow and was also the first to suggest the use of optimization techniques to solve the resulting problem. The exact methods of mathematical optimization were developed by combining linear programming and dynamic programming (KALTENBATCH; PERSON; GEHRIG, 1970; DUSONCHET; EL-ABIAD, 1973), with constructive heuristic methods based on sensitivity (MONTICELLI; SANTOS; PEREIRA, 1982). Several studies have arisen to develop models to solve this problem by using exact techniques based on heuristics and methods classified as intelligent systems. The relevant references for the specific problem to be solved in the doctorate were presented. This chapter presents approaches to the planning problem and a classification of the solution techniques used, based on heuristic procedures, classical optimization techniques (exact methods) and non-classical optimization techniques (metaheuristic techniques). The search methodology used is presented too. There are multiple research articles on the long-term transmission planning problem focused on different types of analysis and specific topics. The progressive search for articles closer to the problem to be solved can be organized in a list and enumeratedwith respect to relevance to the problem of this thesis.

1.4.1 Exact techniques

- Linear Programming. The traditional transmission planning problem solves the denominated: static planning. This problem assumes that all investments occur in the beginning of a single planning period. Garver (1970) solves a static transmission expansion planning problem using linear programming, in Hashimoto, Romero and Mantovani (2003), the analysis focuses on static planning and proposes to solve the problem using an efficient linear programmingalgorithm. In Romero and Monticelli (1994) the problem of static planning in several levels is solved using the transportmodel. Villasan, Garver and Salon (1985) solves the static transmission expansion planning problem using a hybrid linear model.

- Non-linear Programming. In Sanchez (2005) the traditional transmission planning problem is solved using the DC model and a nonlinear-programming technique. Rider, Garcia and Romero (2004) solves the short term static transmission expansion planning problem using an interior point method.

- Integer Linear Programming. In Choi (2005) a Method for Transmission System Expansion Planning Considering Probabilistic Reliability Criteria is solved using integer programming. In Choi, El-keibi and Tran (2005) the transmission expansion network is modeled as a fuzzy integer programming problem. Yu, Guo and Duan (2007) presents an approach for transmission network expansion planning in deregulated power system. The method adopts elasticity constraints and elasticity coefficients for transmission network expansion planning.

- Mixed-Integer Linear Programming. In Romero and Monticelli (1994) a hierarchical decomposition approach is presented for transmission network expansion planning. The implementation of the hierarchical decomposition approach utilizes three different levels of network modeling: transportation models, hybrid models, and linearized power flow models. Alguacil, Motto and Conejo (2003) presents a mixed-integer LP approach to the solution of the long-term transmission expansion planning problem. The problem is large-scale, mixed-integer, nonlinear and non-convex. The mixed-integer linear formulation considers losses. In Torre, Conejo and Contrerras (2008) a transmission expansion planning in electricity markets is presented. The paper presents a mixed-integer linear programmingformulation for the long-term transmission expansion planning problem in a competitive pool-based electricity market. In Wei (2006), a congestion-based model for the transmission expansion planning problem is presented. The paper presents a mixed-integer linear programming formulation for this problem. Roh, Shahidehpour and Fu (2007) shows a market-based coordination of transmission and generation capacity planning. The examples illustrate the coordinated planning of generation and transmission in restructured power systems. The constraints include linearized power flow equations and limits on circuit flows for all combinations of economic dispatch points, which capture hydrological variation.

Alizadeh, Jadid and Pozo (2011), Sauma and Contreras (2013) shows methodologies for new a generation location that minimizes the transmission expansion planning investment cost. Bustos (2018) presents a mixed-integer LP approach to the solution of the ESSs (Energy Storage Systems) as complements of renewable generation. Show these technical and economic options for the short-term operation and the long-term transmission expansion planning. Cabrera; Alcaraz; Gil, (2018) evaluates the effect on investment and the operational costs of including the operational constraints of generators (ramping), transmission losses, and (N-1) ecurity constraints on the TEP problem. The TEP is formulated as a mixed-integer

- linear Mixed-Integer programing.

Nonlinear Programming. In Oliviera, Binato and Pereira (2007) the planning problem is formulated as a large-scale mixed integer nonlinear optimization model and the objective function is to minimize the sum of investment costs and expected load-shedding costs.

- Stochastic Programming. In Serna, Duran and Camargo (1978), the transmission planning problem is solved using an algorithm that minimizes the investment cost and the load shedding. The lines and generators are connected and disconnected using probabilistic values and the other variables take random values. In Jirutitijaroen; Singh,(2008), the static transmission planning problem is solved by including the generation planning. The problem is solved using the Montecarlo simulation. In Akbar, Rahimikian and Kazemi (2011) uses a multistage fashion that combines the investment in lines and substations with energy storage systems (ESS). The randomcomponentis include in the load and the renewable generation. Alvarez Lopez, Ponnambalam and Quintana (2007) presents a generation and transmission expansion under risk using stochastic programming. Martín, Ramos and Alonso, (2005) solves a probabilistic midterm transmission planning problem in a liberalized market. Billint (2002) presents a composite system reliability evaluation incorporating an HVDC link and a static synchronous series compensator.
- Benders Decomposition. In Romero (1989,1993) the static transmission expansion planning with the investment problem separated from the operationproblem is presented. The two problems are solved in a sequence form using the Benders decomposition method. Binato, Pereira and Granville (2001) combines the Benders decomposition method with Gomory cuts in the static problem. Akbari, Rahimikian and Kazemi (2011) presents a multi-stage stochastic model for short-termtransmission expansion planning based on the Benders decomposition algorithm. Moreira, Street and Arroyo (2015) solves the transmission network expansion planning problem undergeneralized joint generation and transmission (n-K) security criteria. The resulting problem is solved by a primal-dual algorithm based on Benders decomposition, combined with a column-and-constraint generation procedure. Dilwal (2016) solves the transmission expansion planning problem using Benders decompositionand local branching. Adds to the process a set of new constraints to the local branching. Alizadeh-Mousavi and Zima-bockarjo Va (2016) presents an efficient Benders cuts for transmission expansion planning.
- Branch and Bound. Bahiense, Oliveira and Granville (2001) is presents the disjunctive linear model for the transmission expansion problem. The mixed-integer program is solved by a commercial branch and bound code. In Haffner, Garcia and Romero (2000) the problem is solved using a branch-and-bound technique and the transport model for

the network representation. In Rider, Garcia and Romero, (2008) the problem is solved by a branch-and-boundalgorithm. Choi, El-keibi and Tran (2005) solves an optimal strategy using a fuzzy set theory-based branch-and-bound method. Delgado, Pourakb Ari-kasmaei and Rider (2013) uses a modified branch-and-boundalgorithm to solve the transmission expansion planning problem. Gomez (2008) combines a GRASP and simulated annealing algorithm with a branch-and-boundmethod.

1.4.2 Heuristic techniques.

- Sensibility Index.

Monticelli, Santos and Pereira (1982) uses a heuristic based on the minimum effort criterion. This is a sensibility index used for the selection of the most attractive circuit to add in the investment process. The problem solved is the static problem. In Pereira and Pinto (1985) a minimum load shedding index and capacity of the load supply is used, in the static planning. In Sanchez (2005) the new circuits added uses an index based on the power flow of the new circuits. At the end of the process, with the added circuits an outage is simulated to verify whether they are necessary for the future operation or are redundant. In Cedeño and Arora (2009) a sensibility index is used, based on the congestion of the right of way in the static problem. Zeinaddini-Maymand (2011) uses a DC model for the network representation (non-linear). The integer variables are relaxing and the index is the number of circuits for addition. Escobar, Gallego and Romero (2011) presents several sensitivity indexes to obtain initial solutions of good quality for the long-term transmission expansion problem. Rider (2007), Manso (2014) uses a constructive heuristic algorithm based on the sensitivity index. Moghaddam, Monsef and Jafari (2011) uses a sensitivity index in a new heuristic method for transmission expansion planning.

- Sigmoid Functions.

In Oliveira (2005), the static transmissionexpansion planning problem is solved using an algorithm with an index based on the power flow and the load shedding. The method uses a sigmoid function with the selected circuits.

- Forward-Backward Strategy.

In Sei (2007), a two-step algorithm is used. In the first step, the method assumes that the total number of lines are built and, gradually, are eliminated one by one, the investment cost is then analyzed with the purpose of measuring the impact of each circuit.

The first step is finalized when the system is feasible for normal operational conditions. In the second step, the circuits are added to the network, considering (n-1) contingencies. The cost is verified. The algorithm stops when the system is feasible with the lower investment cost. The problem solved is the static problem and the substation expansions and the voltage level of the network are considered. In Goswami and Monalisa (2013), a similar algorithm is used with static planning and contingencies.

1.4.3 Intelligent systems.

- Genetic Algoritms (GA).

In Romero, Gallego and Monticelli (1998) Silva, Gil and Areiza (2000) the static transmission expansion planning problem is solved using a genetic algorithm. In Escobar, Gallego and Romero (2004) a multistage and coordinated planning is solved using a genetic algorithm. In Oliveira (2004), a genetic algorithm with modifications is used and the performance of the variants is evaluated. Domínguez, Escobar and Gallego (2014) solves the transmission network expansion problem, considering the selection of the wire size and the construction technology of the conductors for transmission lines for each new right of way. The optimization problem is solved using a specialized genetic algorithm that uses the logic of the Chu-Beasley Genetic Algorithm. In Gallego, Romero and Escobar (2000) the static planning of the Colombian transmission system is solved using genetic algorithms. Charlin, Rudnick and Araneda (2015) presents a dynamic expansion model that minimizes the maximum regret, for the transmission system expansion, and the problem is solved using a genetic algorithm.

- Simulated Annealing (SA).

In Romero, Gallego and Monticelli (1996), the static problem is solved using simulated annealing with good results in atest system from north-northwest Brazil. In Cortes-Carmona, Palma-Behnke and Moya (2009), a local search is incorporated for the adjustment of the parameters of the simulated annealing: the temperatureof the method.

- Tabu Search.

In Gallego, Monticelli and Romero (2000) and Escobar, Gallego and Toro (2009) the focus is on the static transmission expansion planning problem using tabu search algorithm for network synthesis. In Silva (2001) the static transmission planning problem is solved using a tabu search approach. Molina and Rudnick (2011) uses a tabu search in the transmission planning in a multiobjective problem.

- Greedy Randomized Adaptive Search Procedure (GRASP).

In Binato, Oliveira and Araujo (2001), a greedy randomized adaptive search procedure is presented for static transmission expansion planning. The method obtain optimal solutions with low computational effort. In Wu, Cheng and Xing (2008) the transmission network expansion planning problem is solved using GRASP, and studies the minimum load cutting problem existing in the process of transmission network expansion planning when the load is uncertain and expressed in interval number. Figueiredo, Silva and Poss (2012) presents transmission expansion planning with re-design, and solves the problem using a greedy randomized adaptive search procedure.

- Game Theory.

In Contreras and Wu (1999), a coalition formation is used in transmission expansion planning applied game theory. Zolezzi and Rudnick (2002) shows the transmission cost allocation by cooperative games and coalition formation. Styczynski (1999) presents transmission network planning using game theory.

- Variable Neighborhood Search.

In Taglialenha (2008), good solutions for the transmission expansion planning problem are obtained using a neighborhood search. The problem uses an evolutive algorithm with sets of alternatives for the investment problem.

- Path Relinking.

In Rahmani (2010), the traditional path relinking method is used to improve using an evolutive algorithm. In Escobar, Gallego and Toro (2010) the path relinking mechanism is used to improve the best solutions of the Braziliam Northeastern test system for the transmission planning problem. The modified relinking path obtains a new BKS (best known solution).

- Fuzzy Logic.

In Choi, El-Keib and Tran (2005), a fuzzy branch and bound-based transmission expansion planning system is presented for the highest satisfaction level of the decision maker. Sousa (2009) shows an application of fuzzy logic for the decision maker in the static transmission expansion planning problem.

- Particle Swarm Optimization (PSO).

Gomes and Saraiva (2015) presents static transmission expansion planning using an hybrid method that combines heuristic and metaheuristic techniques. First, the space of solution is reduced using a heuristic based in minimum effort and the transport model. In the second phase, the problem is solved using particle swarm optimization. Barreto, Torres and Castro (2013) shows a study of particle swarm optimization variations applied to static transmission expansion planning. This work presents different versions of particle swarm optimization. Torres and Castro (2012) shows an efficient parallel particle swarm optimization applied to the static transmission expansion planning problem. Mendonça, Junior and Marcato (2014) shows the static planning of the expansion of electrical energy transmission systems using particle swarm optimization.

- Ant Colony.

In Limsakul, Pothiya and Leeprechanon (2009) an application of ant colony optimization to static transmission network expansion planning is presented with a security constraint: (n-1) contingencies. This work uses the DC model and sensibility index in the process. Fuchs, Voller and Gjengedal (2011) presents a methodology for static transmission expansion planning based in ant colony and developed for the Nordic area and Great Britain. The authors use removable generation in their work and uses the DC model for network representation.

- Differential Evolution Algorithm (DEA) and Immune System.

In Sum-In (2009) a differential evolution algorithm is presented for static and multistage transmission expansion planning. Different versions of mutation mechanism of the differential evolution are presented and uses a parallel search. In Rezende, Silva and Honorio (2009) an artificial immune system and a differential evolution method are applied to multistage transmission expansion planning. Alhamrouni (2014a, 2014b) presents a differential evolution algorithm for transmission expansion planning based on the AC load flow model.

- Hybrid methods.

In Sisodia, Kumar and Wadhwani (2016) the transmission planning problem is solved using a hybrid method that combines a genetic and a Particle Swarm Optimization (PSO) method. The test shows that the methods are complementary. Souza (2009), a tabu search method combined with fuzzy techniques is used in the transmission planning problem. Gomez (2008) uses an GRASP algorithm with simulated annealing to initialize a branch-and-boundmethod. Faria Junior (2005) presents a new path relinking algorithm using two metaheuristics: variable neighborhood search and scatter search. Flores, Salonga and Nerves (2011) solves a multi-objective transmission expansion planning problem using an elitist non-dominated sorting genetic algorithm with fuzzy decision analysis. In Ugranli and Karatepe (2015) multiobjective transmission expansion planning considering minimization of curtailed wind energy is presented. Eghbal, Saha and Hasan (2011) solves the long term transmission planning using shuffled frog leaping, PSO and GA algorithms.

- Multiobjective

Garcia-Martinez, Espinosa-Ju Arez and Rico-Melgoza (2015) proposes an optimization algorithm to expand electrical networks based on the application of a mul-tiobjective tabu search technique. The proposed methodology searches the topology that minimizes the voltage sag/year number in the electrical system. In Escobar (2008) the static planning with multiple generation scenarios is solved using a multiobjective method based in a genetic algorithm. In Correa, Bolaños and Garcés (2015), a NSGA II algorithm is presented in a multiobjective problem with investment cost and contingencies. In others works, the transmissionnetwork expansion problem is combined with the gas expansion problem: Hu(2016) and is solved using an NSGA-II method. Ugranli and Karatepe (2015) solves a multiobjective transmissionexpansion planning problem considering the minimization of curtailed wind energy, and uses an NSGA-II method. Doagou-Mojarrad, Rastegar and Gharehpetian (2016) solves a probabilistic multiobjective HVDC/HVAC transmission expansion planning problem considering distant wind/solar farms.

1.4.4 Variants in the transmission expansion problem and in the network modeling.

An interesting computational tools that solves the transmission expansion planning problem for obtaining optimal or suboptimal solutions is shown in Proto (2009). In this work, a graphical interface can select one of a subset of metaheuristics to solve the problem. This tool was designed so that students interested in transmission planning can strengthen their knowledge.

The traditional transmission expansion planning problem uses a DC model in the network representation. There are works that have considered using models different from the traditional DC in the planning of transmission. Garver (1970), uses the transportmodel for the network. In Villasana A, Garver and Salon (1985) the network is represented through a hybrid model. Bahiense, Oliveira and Granville (2001) uses a mixed integer disjunctive model for transmission network expansion. The AC model is used in Rider (2007) and Rider, Garcia and Romero (2007) and Rider (2006) and Moghaddam, Monsef and Jafari (2011) and Manso (2014) and Akb Ari and Bina (2014) and Bent, Toole and Berscheid (2011) uses an optimal power flow model.Taylor and Hover (2011) and Alhamrouni (2014a) and Kim (2015) uses relaxed linear model sobtained from the ACl oad flow method.

With the introductionof electricity markets in deregulated systems, and bilateral contracts,newmodels appear where,forexample, the main objectiveisto maximize social welfareShrestha and Fonseka (2004) and Yu, Guo and Duan (2007) and Wei (2006). In Fang and Hill (2003) and Silvajunior (2005b), the economical spects in transmission planning should consider different operating scenarios. The planning process uses safety conditions and/or reliability concepts in Manso (2014) and Choi, Mount and Thomas (2007) and Choi (2005) and Verma, Panigrahi and Bijwe (2010) and Domínguez, Escobar and Gallego (2017). Alizadeh and Jadid (2011) introduce reliability conditions in a transmission-generation expansion problem. Kazerooni and Mutale (2010) introduces safety and environmental conditions in the problem. Lu, Dong and Saha (2006) uses electricity market constraints. Camac (2010) applied conditions of reliability and risk. In Baldick and Kahn (1993) and David and Wen (2001) and Xu, Dong and Wong (2003) uses competition conditions. The open access in systems with an electricity market is affected for the network constraints. Scott, Hogan and Pope (1997) and Fang and David (1999) and Metteb and Kurt (2007) and Papalexopoulos (1997) analyze the congestion in competitive environments. Lu, Dong and Saha (2005) and Shrestha and Fonseka (2006) considers congestion in the transmission expansion problem, and Eghbal, Saha and Hasan (2011) includes contingencies and congestion.

The transmission planning process can consider demand uncertainty and future generation Silva (2013) and Tor, Guven and Shahidehpour (2008), reliability analysis including uncertainty in demand and future generation Negrete (2010) and Charlin, Rudnick and Araneda (2015) oruncertaintyindemandanduncertaintyinthegeneration of renewable energy (YU; CHUNG; WONG, 2011; DOAGOU-MOJARRAD; RASTEGAR; GHAREHPETIAN, 2016; TANG, 2015). Escobar (2008) uses uncertainty in demand and uncertainty in the generation. Escobar, Escobar and Melchor (2014) uses extreme and feasible generation-demand scenarios in the planning process and obtains the limit investment transmission cost for open access. The time horizon used in the planning process can be represented by one stage Garver (1970) and Romero (1989) and Romero, Gallego and Monticelli (1998) several time intervals, or multistage planning Haffner (2000) and Rocha, Saraiva and Frias (2011) and Escobar, Gallego and Romero (2004) and Dominguez (2017) shows a transmission expansion planning with constraints and techniques for reducing the solution space in both traditional and multistage planning. Akbari, Rahimikian and Kazemi (2011) and Zhang (2012) and Silva (2011) and Maghouli (2011) and Silva, Rahmani and Rider (2014) and Vinasco (2014) and Goswami and Monalisa (2013) solves the multistage planning, too.

The transmission planning problem is solved in Maghouli (2009); Flores, Salonga and Nerves (2011) using a multiobjective methodology with investment costs, congestion costs and reliability analysis. In Maghouli (2009) the cost of the circuits, the cost of congestion and social welfare are considered with planning in a multistage problem.

Motamedi (2010) and Pereira and Institute (1987) includes transmission expansion and generation expansion. Xu, Dong and Wong (2006) shows a transmission expansion with uncertainty in the electricity market, reliability and safety criteria. Fan (2011) consider uncertainty in the electricity market. Khodaei, Shahidehpoura and Kamalinia (2010) presents a generation-transmission expansion and optimal dispatch, and Molina and Rudnick (2011) shows multiple objectives with a tabu search method, ordinal optimization and Pareto optimality.

Miasaki (2006) includes investments in substations and transmission lines, and consider FACTS devices in the network to reduce investment in transmission lines. Cebeci (2011) analyzes the location problem for new substations, in the expansion planning. Sepasian (2006) propose the location of new substations and the re-powering of existing ones. There are also works that consider expanding the transmission network based on possible terrorist attacks, such as Arroyo, Alguacil and Carrion (2010) and Carrión, Arroyo and Alguacil (2007) and Romero (2012), or based on spatial planning, that is, including the geographical environment Shu (2012). Kwon and Hedman (2015) and Domínguez, Escobar and Gallego (2014) shows a transmission expansion planning model considering conductor thermal dynamics and high temperature low sag conductors or HTLS conductors. Santos (2007) and Escobar Gallego and Toro (2009) presents a mathematical model and one methodology for the long-term transmission system expansion planning problem considering de-planning or the disconnection of existent circuits. Transmission planning also appears, considering rules for different zones based on the electricity exchanges Buijs and Belmans (2012). Doagou-Mojarrad, Rastegar and Gharehpetian (2016) presents a probabilistic approach with multi-objective HVDC/HVAC transmission expansion planning considering distant wind/solar farms. Domínguez, Escobar and Gallego (2017) shows a milp model for the static transmission expansion planning problem including HVAC/HVDC links, security constraints and power losses with a reduced search space. Gilles (1987) presents a optimum HVDC transmission expansion planning formulation, and Escobar, Escobar L., Romero and Gallego (2016) shows a long term transmission expansion planning considering generation-demand Scenarios combined with HVDC links. Lotfjou, Fu and Shahidehpour (2012) presents hybrid HVAC/HVDC transmission expansion planning. Torbaghan (2015) solves market-based transmission planning for an HVDC grid-case study of the North Sea.

In Cabrera, Alcaraz and Gil (2018) the transmission planning problem is solved considering an hourly demand curve. Seifi (2007) solves a multi-voltage approach to long-term network expansion planning. Obio and Mutale (2015) presents a comparative analysis of energy storage systems (ESS) and (N-1) network security in transmissionexpansion planning.

A complete review of the publications, models and approaches that have been used for the transmission expansion planning problem can be found in Latorre, Cruz and Areiza (2003) and Sum-im, (2006) and Lee (2006) and Molina and Rudnick (2010) and Hemmati, Hooshmand and Khodabakhs hian (2013)and Romero, Garcia and Haffner (2002) and Lumbreras and Ramos (2016) and Niharika, Verma and Mukherjee (2016) and Dedecca and Hakvoort (2016). Mccalley and Krishnan (2014) presents a survey of transmission technologies for planning long distance bulk transmission overlay.

1.5 ORGANIZATION

In Chapter 2, the basic models for the transmission expansion problem are described. In Chapter 3, the new improved models are explained. In Chapter 4, the theory for the valid inequality constraints and the cycle cuts are explained, as well as the low effort algorithm and its use in combination with the proposed methodology. In Chapter 5, the proposed strategy for reducing the search space of the TEP problem is used to solve the reduced disjunctive model of the transmission expansion planning problem finding the best known solution for the Brazilian Northeastern test system. As well as test and result with the cycles cuts, new specialized models and valid inequalities. In Chapter 6, conclusions and possible future work are presented. appendix A present the data of the test systems analyzed in this work. appendix B present the articles presented in conferences and papers on technical journals.

2 MODELS FOR TRANSMISSION EXPANSION PLANNING

2.1 INTRODUCTION

The mathematical modeling of any real-life problem consists of representing it in the most accurate way, defining all the possible variables of the problem, and finding all those interrelationships that make the constructed model work properly. Modeling is difficult to the extent that a better approximation to the problem is the main objective, which is related to a more efficient adjustment of the relations used and the addition of more variables and functions. This chapter presents the state-of-the-art models for the TEP problem. The investment cost of the transmission lines is considered to be an objective function and basic power flow equations are considered to be the constraints for the problem. In the TEP problem, the models are used to guide the process of determining the new investments required by the electric system in the future. The planning agencies, in different countries, use as reference the results obtained with these models to answer the following questions: *Where should the new elements be located? What elements should be installed? How many elements should be added? and what is the best time for them to be added?* In order to answer these questions, the appropriate mathematical models must be available to meet the operating, demand and generation constraints that appear in the course of time in order to have a minimum cost investment plan. According to the characteristics of the ideal problem, the transmission network should be represented by the AC load flow model, however, this model presents some peculiarities that make it difficult to use. In the TEP problem, the important aspect is the determination of the transmission paths that must follow the active power of the system. From this point of view, the DC load flow model (model of the MINLP type) has traditionally been considered the ideal model for long-term planning and is the most used in the specialized literature by the experts for being a good approximation of the AC model (NIHARIKA; VERMA; MUKHERJEE, 2016). Consequently, the DC model is used to exhaustively explore the solution space in search of the best alternatives for expansion, while the AC model is used to refine the search for the best solution in small solution sub-spaces.

The disjunctive linear model is the linear representation of the DC load flow model. The latter is considered the ideal model for the exhaustive exploration of investment options in the problem of transmission network expansion. The two models are equivalent from the mathematical point of view and therefore have the same optimal solution. The disjunctive

Linear model allows one to transform a MINLP problem in to a MILP problem, which is easier to solve than the original problem. A disadvantage is that it has a greater number of variables, and an advantage is that it is linear. There are exact techniques that solve this type of problems for systems of small and medium complexity. In this research, we use commercial CPLEX software that solves problems of the mixed-integer linear type (which corresponds to the nature of the linear disjunctive model) efficiently.

2.2 TRANSPORTATION MODEL

The transport model was the first systematic proposal of mathematical modeling used successfully in the problem of the planning of transmission systems. The model was proposed by Garver (1970), and it represented the beginning of a systematization of transmission planning problems, suggesting the use of different models for the problems of operation and planning (HAFFNER; GARCIA; ROMERO, 2000; LATORRE; CRUZ; AREIZA, 2003; SUM-IM, 2006; LEE, 2006; MOLINA; RUDNICK, 2010).

The transport model suggests that in a transmission network only the first Kirchhoff law must be fulfilled. The second Kirchhoff law is not considered. Through this model, one can find attractive topologies that serve as a starting point for more accurate models.

$$Min = v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} \tag{1}$$

$$s.a. \quad SP + g = d \qquad \forall_n \in B, \forall_{ij} \in \Omega \tag{2}$$

$$\left|P_{ij}\right| \leq (n_{ij} + n_{ij}^{o})\, \bar{P}_{ij} \qquad \forall_{ij} \in \Omega \tag{3}$$

$$0 \leq n_{ij} \leq \bar{n}_{ij} \qquad \forall_{ij} \in \Omega \tag{4}$$

$$0 \leq g_i \leq \bar{g}_i \qquad \forall_i \in B \tag{5}$$

$$n_{ij}, \ integer \qquad \forall_{ij} \in \Omega \tag{6}$$

On the transportationmodel, v is the investment due to additions of circuits in the system, c_{ij} is the cost of adding a circuit in branch $i-j$, n_{ij} is the numberof added circuits in branch ij, f is the vector of flows whose elements represent the total flow in the path $i-j$. S is the node-branch incidence matrix of the complete electrical system, g is the vector of nodal generations, d is the vector of nodal demands, n_{ij}^{o} is the number of existing circuits in the branch ij in the base or initial configuration, $\overline{f}_{ij}$ is the maximum flow allowed for a circuit in the path ij. $\overline{g}$ Is the maximum nodal generation vector, n_{ij} is the maximum number of circuits that can be added in the branch $i-j$ and Ω represents the set of existing transmission corridors in the base network and new transmission corridors.

2.3 TRADITIONAL DC MODEL

The DC model of transmission expansion planning (without power losses) is the most used model in transmission planning and numerous publications discuss this model. There are two different mathematical optimization models for the DC model: one is nonlinear, and the other is an equivalent linear disjunctive model. These models are discussed in the following sections. The DC load flow model is considered ideal to represent the transmission system in the TEP problem, and its performance has been tested again and again in different researches reported in the specialized literature (GALLEGO, 1997; RIDER; GARCIA; ROMERO, 2004; ROMERO; GALLEGO; MONTICELLI, 1998; ESCOBAR; ROMERO; GALLEGO, 2008; ESCOBAR; GALLEGO; ROMERO, 2004; ESCOBAR; GALLEGO; TORO, 2009; SILVA; FREIRE; HONÓRIO, 2016). The DC model considers the two laws of Kirchhoff. This model presents an optimization problem of the MINLP type, which assumes the following form.

$$Min = v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} \tag{7}$$

$$s.a.\ SP + g = d \qquad \forall_n \in B,\ \forall_{ij} \in \Omega \tag{8}$$

$$P_{ij}x_{ij} - (\theta_i - \theta_j)(n_{ij} + n_{ij}^o) = 0 \qquad \forall_{ij} \in \Omega \tag{9}$$

$$\left|\theta_i - \theta_j\right| \leq \bar{P}_{ij}x_{ij} \qquad \forall_{ij} \in \Omega \tag{10}$$

$$0 \leq g_i \leq \bar{g}_i \qquad \forall_i \in B \tag{11}$$

$$0 \leq n_{ij} \leq \bar{n}_{ij} \qquad \forall_{ij} \in \Omega \tag{12}$$

$$n_{ij},\ integer \qquad \forall_{ij} \in \Omega \tag{13}$$

$$P_{ij}, \theta_n, \quad \text{Unconstrained} \qquad \forall_{ij} \in \Omega \tag{14}$$

The DC model, defines an additional subset of variables and constraints, with respect to the transport model.

In the DC model, $c_i j$ represent the cost of adding a circuit in the branch ij, Ω represents the set of existing transmission corridors in the base network and new transmission corridors, Ω_i is the subset of load nodes. S is the node-branch incidence matrix of the electric system, f is the flow vector whose elements represent the total flow in the path $i-j$, g is the vector of nodal generations, d is the vector of nodal demands. θ is the vector of nodal angles, $\overline{P}_{ij}$ is the maximum flow allowed for a circuit in the path $i-j$, $\overline{g}_i$ is the maximum nodal generation vector, n_{ij} is the number of added circuits in the $i-j$ branch, $\overline{n}_{ij}$ is the maximum number of circuits that can be added in the $i-j$ branch, and n^o_{ij} is the number of existing circuits in the branch i-j in the base or initial configuration. The first two constraints of the model represent first and second Kirchhoff's laws, the remaining set of constraints are operational or investment constraints: power flow limits for the lines, generation limits and circuit addition limits. The problem is PNLEM, due to the presence of integer variables, such as the number of circuits added in each branch, and the existence of the constraints corresponding to KSL, which are non-linear.

In the DC model, the first constraints represent Kirchhoff's first law (KFL), The second constraints represent the Kirchhoff's second law (KSL) either for existing circuits or for the candidate circuits to be added to the transmission system, and the remaining set of constraints are operational limits or investment limits: active power flow limits for the circuits, active power generation limits and investment limits. The problem is MINLP, due to the presence of products between investment integer variables and bus-angle variables in the SKL.

2.4 HYBRID LINEAR MODEL

An alternative form to the previous model is the linear hybrid model. This model can be easier to solve than the one presented previously because it presents a linear relation between the variables. In this model new additions (n_{ij}) where circuits already exist and where they do not exist must satisfy only the KFL. The existing circuits (n^o_{ij}) must satisfy the KFL and KSL. This is equivalent to having two superimposed networks where the original network existing in the base configuration must comply with Kirchhoff's two laws and the newly added circuits must only comply with KFL. In the hybrid model, the power flows through circuits in the existing transmission circuits are represented separately of the flows of the candidate transmission circuits. The flow of existing transmission lines is represented by variable P^o_{ij} and for candidate transmission circuits by P_{ij} .

$$Min = v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} \tag{15}$$

$$s.a. \sum_{(l,i)\in\Omega} \left(P^0_{li} + P_{li}\right) - \sum_{(i,q)\in\Omega} \left(P^0_{iq} + P_{iq}\right) + g_i = d_i \qquad \forall_i \in B,\ \forall_{ij} \in \Omega \tag{16}$$

$$P^o_{ij} x_{ij} - (\theta_i - \theta_j) n^o_{ij} \ = 0 \qquad \forall_{ij} \in \Omega \tag{17}$$

$$-\bar{P}_{ij} n_{ij} \leq P_{ij} \leq \bar{P}_{ij} n_{ij} \qquad \forall_{ij} \in \Omega \tag{18}$$

$$-\bar{P}_{ij} n^o_{ij} \leq P^o_{ij} \leq \bar{P}_{ij} n^o_{ij} \qquad \forall_{ij} \in \Omega \tag{19}$$

$$0 \leq g_i \leq \bar{g}_i \qquad \forall_i \in B \tag{20}$$

$$P^0_{ij}, P_{ij}, \theta_i, g_i, \quad Unconstrained \tag{21}$$

The advantage of the hybrid model over the DC model is that it is much easier to solve whereas the solution may be unfeasible for the DC model. In the hybrid linear model, the constraint for the bus-angle difference is usually neglected. The hybrid model, defines an additional subset of variables and constraints, with respect to the transport model and the DC model.

2.5 LINEAR DISJUNCTIVE MODEL

The modeling considered to be an ideal representation of the TEP is the so-called DC model, which is a mixed integer non-linear programming problem (MINLP); however, the DC model has the possibility of being transformed into an equivalent problem whose modeling is a linear model. It is possible to transform a non-linear quadratic problem into a linear problem with the use of binary variables; this is achieved by using a transformation that allows the quadratic terms to be separated into linear relationships using disjunctive constraints. This process is obtained by incorporating a big M parameter into the problem, in which KSL can be represented in two parts for each possible addition in a network section. This transformation is achieved by representing each possible addition of transmission lines and/or substations by binary decision variables (1 to represent that the element is added and 0 to indicate otherwise). The inclusion of these binary variables implies a separation of the quadratic terms present in the DC model (products between θ_i and $n_{i,\,j}$), the big M parameter of very large value, so as to include the SKL associated with binary variables whose value is 1, or that does not affect otherwise the model. The most interesting aspect of the disjunctive linear model is that as a problem of linear programming with binary variables, it shares its optimal global solution with that of the DC model.

The disjunctive linear model has some advantages and disadvantages, the main advantage is that it converts a non-linear problem into a linear problem and its disadvantage is related to the increase in the dimension of the problem due to the introduction of a large number of binary variables. It depends on the size of the system and the maximum number of candidate lines in each corridor. Another disadvantage is the selection of the parameter M, since it represents a factor that can destabilize the solution of the disjunctive linear model.

$$Minv = \sum_{(i,j)\in\Omega} c_{ij} \sum_{k\in K} \omega_{ij,k} \tag{22}$$

$s.a.$

$$\sum_{(l,i)\in\Omega}\left(P^0_{li} + \sum_{k=1}^{K} P_{li,k}\right) - \sum_{(i,q)\in\Omega}\left(P^0_{iq} + \sum_{k=1}^{K} P_{iq,k}\right) + + g_i = d_i \qquad \forall_i \in B,\ \forall(i,j)\in\Omega \tag{23}$$

$$P^o_{ij}x_{ij} - (\theta_i - \theta_j)\, n^o_{ij} = 0 \qquad \forall_{ij}\in\Omega \tag{24}$$

$$-M(1-\omega_{ij,k}) \le P_{ij,k}x_{ij} - (\theta_i - \theta_j) \le M(1-\omega_{ij,k}); \qquad \forall_{ij}\in\Omega,\ k = 1\ldots K \tag{25}$$

$$-\bar{P}_{ij} n^o_{ij} \le P^o_{ij} \le \bar{P}_{ij} n^o_{ij} \qquad \forall_{ij}\in\Omega \tag{26}$$

$$\left|P_{ij,k}\right| \le \omega_{ij,k}\bar{P}_{ij} \qquad \forall_{ij}\in\Omega,\ k = 1\ldots K \tag{27}$$

$$0 \le g_i \le \bar{g}_i \qquad \forall_i \in B \tag{28}$$

$$\sum_{k\in K}\omega_{ij,k} \le \bar{n}_{ij} \qquad \forall_{ij}\in\Omega,\ k = 1\ldots K \tag{29}$$

$$\omega_{ij,k} \le \omega_{ij,k-1} \qquad \forall_{ij}\in\Omega,\ k = 2\ldots K \tag{30}$$

$$\omega_{ij,k} \in \{0,1\}, \qquad \forall_{ij}\in\Omega,\ k = 1\ldots K \tag{31}$$

$$P^0_{ij}, P_{ij,k}, \theta_i, g_i, \quad Unconstrained \tag{32}$$

Where M is a parameter with a very large value, included in the disjunctive variables; $\omega_{ij,k}$ is a binary variable that represents the addition of a circuit in position k of corridor ij, with value "1" in the case of addition and "0" in the opposite case. The disjunctive linear model, defines an additional subset of variables and constraints, with respect to the transport model and the DC model.

The following clarifications should be made about the model:

- The first constraints represent compactly all the constraints associated with FKL, when this applies to the n nodes of the system. All are linear equality constraints.

- The second constraints represent the SKL applied to the existing circuits in the ij corridor of the current network. In this expression, x_{ij} represents the reactance of one circuit of the path ij and it is assumed that all circuits have the same reactance and the same capacity.

- The third group of constraints represent the SKL for each candidate circuit that has been selected for addition. That is, if the element $\omega_{ij,k}$ = 1. If $\omega_{ij,k}$ = 0, the element is not selected for addition, and the SKL does not apply. These constraints are denominated disjunctive constraints.

2.6 REDUCED LINEAR DISJUNCTIVE MODEL

The reduced linear disjunctive model is a transformationof the linear disjunctive model (RAHMANI, 2013). The reduced linear disjunctive model is the best version of the specialized literature to solve the transmission expansion planning problem. As in the previous case, it is a MILP. In the reduced linear disjunctive model, the number of constraints and the number of variables are reduced with respect to the linear disjunctive model. In the reduced linear disjunctive model, each binary variable $\omega_{i\ j,k}$ activates 2^{k-1} circuits simultaneously. Consequently, the quantity of circuits added depends of the value of the binary variables in each problem. The number of circuits in the corridor ij can be calculated through the expression:

$$n_{ij} = 2^0\omega_{ij,1} + 2^1\omega_{ij,2} + 2^2\omega_{ij,3} + ... + 2^{K-1}\omega_{ij,K} \tag{33}$$

In the reduced linear disjunctive model, the number of binary variables needed to represent m investment options are reduced to $log_2(m+1)$.

$$Minv = \sum_{(i,j)\in\Omega} c_{ij} \sum_{k\in K} 2^{k-1}\omega_{ij,k} \tag{34}$$

$s.a.$

$$\sum_{(l,i)\in\Omega}\left(P^0_{li} + \sum_{k=1}^{K} P_{li,k}\right) - \sum_{(i,q)\in\Omega}\left(P^0_{iq} + \sum_{k=1}^{K} P_{iq,k}\right) +$$

$$+ g_i = d_i \qquad \forall_i \in B,\ \forall(i,j) \in \Omega \tag{35}$$

$$P^o_{ij}x_{ij} - (\theta_i - \theta_j)\, n^o_{ij} = 0 \qquad \forall_{ij} \in \Omega \tag{36}$$

$$\left|P_{ij,k}x_{ij} - 2^{k-1}(\theta_i - \theta_j)\right| \leq M(1 - \omega_{ij,k}); \qquad \forall_{ij} \in \Omega,\ k = 1 \ldots K \tag{37}$$

$$-\bar{P}_{ij}n^o_{ij} \leq P^o_{ij} \leq \bar{P}_{ij}n^o_{ij} \qquad \forall_{ij} \in \Omega \tag{38}$$

$$\left|P_{ij,k}\right| \leq 2^{k-1}\omega_{ij,k}\bar{P}_{ij} \qquad \forall_{ij} \in \Omega,\ k = 1 \ldots K \tag{39}$$

$$0 \leq g_i \leq \bar{g}_i \qquad \forall_i \in B \tag{40}$$

$$\sum_{k\in K} 2^{k-1}\omega_{ij,k} \leq \bar{n}_{ij} \qquad \forall_{ij} \in \Omega,\ k = 1 \ldots K \tag{41}$$

$$\omega_{ij,k} \in \{0,1\}, \qquad \forall_{ij} \in \Omega,\ k = 1 \ldots K \tag{42}$$

$$P^0_{ij}, P_{ij,k}, \theta_i, g_i \quad Unconstrained \tag{43}$$

This model uses the same nomenclature as the linear disjunctive model.

The equation that represents the bus-angle difference constraint or KSL, for every 2^{k-1} elements candidates to be added to the transmission system, becomes strict within the system of equations when the decision variable takes the value of 1. Otherwise, the big M parameter ensures that the particular constraint is irrelevant for the model. The bus-angle difference constraint is equivalent to the product of the maximum active power flow of *one* circuit (capacity in p.u.) and it's reactance, (in p.u.), and is denominated capacity-reactance constraint, in this work. It represents a stronger constraint in the power transmission process. These constraints produce more investment cost in the long-term planning (DC model) than either the planning without capacity-reactance constraints or the transport model.

3 NEW SPECIALIZED MODELS FOR TRANSMISSION EXPANSION PLANNING

3.1 INTRODUCTION

The problematic part of the disjunctive model is the KSL (Kirchhoff's second law), which causes the problem to increase in difficulty, this is the main characteristic of the normal behavior of TEP used in this work to create different methods to improve the computational effort and build the path to find better solutions in the future. Here, the objective is to find the best representation for KSL to replace it and have as a result a simpler model that can be solved in less time. The capacity-reactance nature present on this constraint has been commented on different works in the specialized literature in the past, as a well know behavior. Unfortunately, the most common way to solve these problems is based on adding more constraints to the model, in some cases based on other constraints such as KFL (Kirchhoff's first law). It is important to punctuate the difference between each system analyzed, and the amount of new information that can be used based on the diverse nature of each system that has been solving, as can be found in data mining the goal is to use this information to understand how this affects the solutions and find the correct structure or characteristics. Although this is all well-known information, this might prove beneficial to guide a heuristic search or a new methods development with knowledge about the problem, as a relatively simple example, we are giving the model a guide map of the problem.

In this work, we present two novel relaxed models and a novel model formulation is proposed to solve the TEP problem using the denominated: improved disjunctive transport model, and a formulation based in cycles.

The transport model can be presented using an alternative form: the disjunctive transport model. This model has more variables and constraints than the traditional transport model but can be useful in other contexts.

3.2 DISJUNCTIVE TRANSPORT MODEL

One of the most used models when solving the TEP problem is the transport model, it is also the simplest way to represent the transmission system and easy to solve, in this case the KSL is nor used, that been the main reason that even big size problems had a solution in a relatively short amount of time, in some cases is a relation of minutes versus days, as a total time needed to solve it. In this disjunctive version, we use binary variables for the investment options and disjunctive constraints. The formulation of this model is:

$$Min = \sum_{(i,j)\in\Omega} C_{ij} \sum_{k\in\Gamma} \omega_{ij,k} \tag{44}$$

$$s.a. \sum_{(p,i)\in\Omega_1} \left(P^0_{pi} + \sum_{k\in\Gamma} P_{pi,k} \right) - \sum_{(i,j)\in\Omega} \left(P^0_{ij} + \sum_{k\in\Gamma} P_{ij,k} \right) + g_i = d_i \tag{45}$$

$$\left|P^0_{ij}\right| \leq n^0_{ij} \bar{P}_{ij}, \qquad \forall\ i,j \in \Omega \tag{46}$$

$$\left|P_{ij,k}\right| \leq \omega_{ij,k} \bar{P}_{ij}, \qquad \forall\ i,j \in \Omega,\ k \in \Gamma \tag{47}$$

$$0 \leq g_i \leq \bar{g}_i \qquad \forall\ i \in B \tag{48}$$

$$\sum_{k\in\Gamma} \omega_{ij,k} \leq \bar{n}_{ij}, \qquad \forall\ (i,j) \in \Omega \tag{49}$$

$$\omega_{ij,k-1} \geq \omega_{ij,k}, \qquad \forall\ i,j \in \Omega,\ k \in \Gamma,\ k > 1 \tag{50}$$

$$P^0_{ij}, P_{ij,k}, \theta_i, g_i, \quad Unconstrained \tag{51}$$

This model uses the nomenclature presented before.

3.3 IMPROVED DISJUNCTIVE TRANSPORT MODEL

Taking as reference the disjunctive transport model, we implement a new set of constraints that will include the behavior of KSL for parallel circuits only, without using the SLK. This is reach by forcing the power flow to be the distributed equally in each parallel circuit. It can be obvious that circuits of equal reactance, connected in parallel, transport equal flow, however, in the traditional transport model the solutions present different flows in this case. This is because the traditional transport model does not include the KSL. The improved disjunctive transport model presented in this work is a relaxed version of the linear disjunctive model.

$$Min = \sum_{(i,j)\in\Omega} C_{ij} \sum_{k\in\Gamma} \omega_{ij,k} \quad (52)$$

$$s.a. \sum_{(p,i)\in\Omega_1} \left(P_{pi}^0 + \sum_{k\in\Gamma} P_{pi,k} \right) - \sum_{(i,j)\in\Omega} \left(P_{ij}^0 + \sum_{k\in\Gamma} P_{ij,k} \right) + g_i = d_i \quad (53)$$

$$\left| P_{ij}^o - n_{ij}^o P_{ij,1} \right| \leq M(1 - \omega_{ij,1}), \qquad \forall\, i,j \in \Omega, k \in \Gamma, n_{ij}^0 > 0 \quad (54)$$

$$\left| P_{ij,k} - P_{ij,k-1} \right| \leq M(1 - \omega_{ij,k}), \qquad \forall\, (i,j) \in \Omega, k \in \Gamma, k > 1 \quad (55)$$

$$\left| P_{ij}^0 \right| \leq n_{ij}^0 \bar{P}_{ij}, \qquad \forall\, i,j \in \Omega \quad (56)$$

$$\left| P_{ij,k} \right| \leq \omega_{ij,k} \bar{P}_{ij}, \qquad \forall\, i,j \in \Omega, k \in \Gamma \quad (57)$$

$$0 \leq g_i \leq \bar{g}_i \qquad \forall\, i \in B \quad (58)$$

$$\sum_{k\in\Gamma} \omega_{ij,k} \leq \bar{n}_{ij}, \qquad \forall\, (i,j) \in \Omega \quad (59)$$

$$\omega_{ij,k-1} \geq \omega_{ij,k}, \qquad \forall\, i,j \in \Omega, k \in \Gamma, k > 1 \quad (60)$$

$$P_{ij}^0, P_{ij,k}, \theta_i, g_i, \quad Unconstrained \quad (61)$$

In the improved disjunctive transport model, the equation (54) establishes a relationship between the active power flow of the existing circuits and the active power flow of the first candidate circuit of the corridor, for each transmission corridor:

$$-M(1 - \omega_{ij,k}) \leq P_{ij}^o - n_{ij}^o P_{ij,1} \leq M(1 - \omega_{ij,1}); \qquad \forall_{ij} \in \Omega,\ k \in \Gamma, n_{ij}^o \geq 1$$

This disjunctive constraint ensures that the power flows will be equally distributed among the existing circuits and the first investment option of the expansion circuits, connected in parallel, between the buses i and j. The equation (55) establishes a relationship between the active power flow of the candidate circuits in parallel, for each transmission corridor:

$$-M(1 - \omega_{ij,k}) \leq P_{ij,k} - P_{ij,k-1} \leq M(1 - \omega_{ij,k}) \qquad \forall_{ij} \in \Omega,\ k \in \Gamma, k > 1$$

This disjunctive constraint ensures that the power flows will be equally distributed among the new circuits connected in parallel, between the buses i and j.

This model produces the same optimal solution as the traditional transport model but presents active power flows with a behavior closer to that of the real-life system, and it will be used to guide the process of generation of angular cycles.

3.4 IMPROVED DISJUNCTIVE HYBRID MODEL

The disjunctive hybrid improved model is a simplified version of the linear disjunctive model and presents an optimal solution between the improved disjunctive transport model and the linear DC disjunctive model. This model only applies the SKL to the existing circuits. This makes it easier to solve than the linear DC disjunctive model. Similarly to the improved disjunctive transport model, this model establishes a relationship between the active power flow of the existing circuits and the active power flow of the candidate circuits, for each transmission corridor, and similarly ensures that the power flows will be equally distributed among the existing circuits and the new circuits, connected in parallel.

$$Min = \sum_{(i,j)\in\Omega} C_{ij} \sum_{k\in\Gamma} \omega_{ij,k} \tag{62}$$

$$s.a. \sum_{(p,i)\in\Omega_1} \left(P^0_{pi} + \sum_{k\in\Gamma} P_{pi,k} \right) - \sum_{(i,j)\in\Omega} \left(P^0_{ij} + \sum_{k\in\Gamma} P_{ij,k} \right) + g_i = d_i \tag{63}$$

$$P^o_{ij} x_{ij} - (\theta_i - \theta_j)\, n^o_{ij} = 0, \qquad \forall\, i,j \in \Omega \tag{64}$$

$$\left| P^o_{ij} - n^o_{ij} P_{ij,1} \right| \leq M(1 - \omega_{ij,1}), \qquad \forall\, i,j \in \Omega, k \in \Gamma, n^0_{ij} > 0 \tag{65}$$

$$\left| P_{ij,k} - P_{ij,k-1} \right| \leq M(1 - \omega_{ij,k}), \qquad \forall\, (i,j) \in \Omega, k \in \Gamma, k > 1 \tag{66}$$

$$\left| P^0_{ij} \right| \leq n^0_{ij} \bar{P}_{ij}, \qquad \forall\, i,j \in \Omega \tag{67}$$

$$\left| P_{ij,k} \right| \leq \omega_{ij,k} \bar{P}_{ij}, \qquad \forall\, i,j \in \Omega, k \in \Gamma \tag{68}$$

$$0 \leq g_i \leq \bar{g}_i \qquad \forall\, i \in B \tag{69}$$

$$\sum_{k\in\Gamma} \omega_{ij,k} \leq \bar{n}_{ij}, \qquad \forall\, (i,j) \in \Omega \tag{70}$$

$$\omega_{ij,k-1} \geq \omega_{ij,k}, \qquad \forall\, i,j \in \Omega, k \in \Gamma, k > 1 \tag{71}$$

$$P^0_{ij}, P_{ij,k}, \theta_i, g_i, \quad Unconstrained \tag{72}$$

The difference between the improved disjunctive transport model and the improved disjunctive hybrid model is the equation (64). This is the KSL of existing circuits of the corridors.

3.5 CYCLES FORMULATION

This formulation is achieved using the improved disjunctive transport model and a set of constraints associated with the critical cycles of the network that will replace the KSL of the

DC model. The new model has, as a result, the same optimal solution with less computational effort, in complex systems. First, the improved transportation model is solved. Then the existing corridors that have their power flows at maximum capacity are identified as well as the expansion corridors with new additions. With this set of corridors the new constraints will be created, using them as part of a cycle or a closed trajectory that must meet two conditions: It must include the identified corridor on the new cycle associating it with the corridors that are connected near it, and it must create a cycle with the corridors that have the smallest sum of capacity-reactance product (*critical cycle*) . The critical cycles found are added to the improved disjunctive transportation model, and the problem is solved again, repeating the previous process until no overloaded corridors or newly added candidate corridors appear in the solution of the improved transport model.

---

input : A Transmission Network of size $nb \times nl$
output : A set of critical cycles
Parameters: busbar data, circuit data, maximum number of critical cycles

solve the system using the disjunctive transport improved model;
while *there are circuits on their maximum capacity or expansion corridors with new additions* **do**
  Build and add to the transport improved model each of the critical cycles, by using each circuit on their maximum capacity or expansion corridors with new additions;
  if *all the critical cycles where build* **then**
    solve the system using the disjunctive transport improved model with the critical cycles added;
  else
    Build the next critical cycle;

If a circuit appears overloaded in different stages of the process, the following critical cycle that contains it is added. This procedure allows us to identify the most critical cycles, and find the optimal solution obtained with the DC model, without including the SKL. The most important part of this formulation is the cycles added to them.

To start the process we use the basic terminology of graph theory, which will guide us to the definition of the cycle. A non-oriented graph G is a pair (V, E), where V is a finite set and E is a family of pairs of elements of V. The elements of V are called nodes or vertices and the elements of E are called paths or corridors of G. Given a corridor connected between two vertices i and j, $\in V$, with $i \neq j$, we denote this corridor by (i, j). Therefore, for a corridor $e = (i, j) \in E$, i and j are called their endpoints or final vertices. On the same point, we say that the corridor e is incident to the vertex i and j. Similarly, we say that the vertex i is adjacent to the vertices j. It is important to note that as we assume a non-oriented graph, the adjacency relationship is symmetric. The degree of a vertex in a non-orientedgraph is the number of links incident on it, which we will denote as $deg(i_q)$. Summarizing, a path p of length k, which joins a vertex i to a vertex j, in a graph $G(V,E)$, is a sequence $\langle r_0, r_1, \ldots, r_k \rangle$ of vertex such that:

$$i = r_0 \, , \, j = r_k$$

with $(r_{m-1}, r_m) \in E$ for $m = 1, 2, ..., k$. a path is simple if all the vertices are different. In a non-orientedgraph, a path $\langle r_0, r_1, ..., r_k \rangle$ forms a cycle if $r_0 = r_k$ and $r_1, r_2, ..., r_k$ are different.

A graph $G' = (V', E')$ is a sub-graph of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. Given a set $V' \subseteq V$, the sub-graph of G induced by V' is the graph $G' = (V', E')$ where $E' = (i, j) \in E : i, j \in V'$

3.5.1 Base cycles

Let $G = (V, E)$ be a non-orientedgraph with m links and n vertices. A cycle of G is a sub-graph of G. The space vector generated by the incident vectors of cycles is called the space of cycles of G, which has the dimension:

$$m - n + \alpha(G)$$

Where m is the number of links in G, n is the number of nodes or vertex and $\alpha(G)$ is the number of related components of G. The maximal set of linearly independent cycles is called the base cycles.

It is importantto highlight that the links of G have an assigned weight or value. Therefore, base cycles where the sum of the weights of the cycles is minimum is called the minimum base cycle of G. In our case, the graph G' is not connected and through an iterative process, links and nodes can be added to the system, if this allows for lower cost solutions. Since the final system $G' = (V', E')$ obtained has a smaller dimension than G and is connected, we will denote the dimension of the final space of the cycles as:

$$N = m - n + 1$$

Where m is the number of links in the final solution of G', n is the number of nodes or vertices in the final solution G', and $\alpha(G') = 1$ because the resulting sub-graph, in the final solution, is always connected in TEP problem.

3.5.2 Solving methodology

In this part we present a methodology to solve the problem of TEP replacing the SKL by the concept of critical cycles.

A cycle is called a critical cycle if it meets one of the following two conditions:

- Presents circuits with power flow in its maximum capacity, form cycles with existing and/or candidate circuits, and the sum of the weights of the links are minimal.
- Presents additions of circuits in expansion corridors, form cycles with existing and/or candidate circuits, and the sum of the weights of the links are minimal.

Since the critical cycles replace the effect of KSL, in TEP, the weights on each line are associated with the bus-angle difference $(\theta_i - \theta_j)$ of the power system corridors. If $\mathscr{C}$ is a critical cycle of the system, then:

$$-M(R-z) \leq \sum_{(i,j)\in\Omega_a} \frac{P^0_{i,j}}{n^0_{i,j}} x_{i,j} + \sum_{(i,j)\in\Omega_b} P_{i,j} x_{i,j} \leq M(R-z) \tag{73}$$

Where $\Omega_a \in \mathscr{C}$, $\Omega_b \in \mathscr{C}$, and Ω_a is the subset of index-pair that represents the established corridors on denominated path p in the network G', Ω_b is the subset of the index-pair that represents the expansion corridors on the denominated path p in the network G', R is the number of the expansion corridors on the path p in the network G', and z is the sum of the binary variables associated with the first investment option of the expansion corridors included in the path:

$$z = \sum_{(i,j,1)\in\Omega_b} \omega_{i,j,1} \tag{74}$$

Where $\mathscr{C}$ is any oriented cycle and x_{ij} is the reactance on the way (i,j). Therefore, the mathematical model for the TEP problem that involves the SKL can be modified using the model given by equations (52) - (61) plus the minimum cycles given by equation (73).

Additionally, because these problems traditionally specify corridors from a lower index bus to a higher index bus, for the existing corridors we use:

$$\begin{array}{ll} P^0_{i,j} & if \;\; j > i \\ -P^0_{j,i} & if \;\; j < i \end{array}$$

And for the expansion corridors we use:

$$\begin{array}{ll} P_{i,j,k} & if \;\; j > i \\ -P_{j,i,k} & if \;\; j < i \end{array}$$

In all cases: $n^0_{i,j} = n^0_{j,i}$, $x_{i,j} = x_{j,i}$, $n_{i,j} = n_{j,i}$, $\omega_{i,j,k} = \omega_{j,i,k}$

3.5.3 Procedure for generation of the critical cycles

In a TEP problem, given the initial network of the system, the data of future generation, future demand and electrical characteristics of the investment options in lines and transformers, the problem will be solved using the improved disjunctive transport model, (which does not include the KSL). Then it is verified in the answer depending on whether the circuits appear in their maximum capacity, or there are added circuits in the expansion corridors. Next, the cycle that contains these circuits will be determined, and this will be a cycle with a minimum sum of weights. These cycles, called critical cycles, are added to the improved disjunctive transportationmodel and the process is repeated until there are not new circuits on the maximum limit or no more circuits are added.

As an example of application, in the South Brazilian test system one of the critical cycles assume the following structure:

$$\left(\frac{P^0_{12}}{n^0_{12}}\right) x_{12} + \left(\frac{P^0_{25}}{n^0_{25}}\right) x_{25} + \left(\frac{P^0_{58}}{n^0_{58}}\right) x_{58} - \left(\frac{P^0_{78}}{n^0_{78}}\right) x_{78} - \left(\frac{P^0_{17}}{n^0_{17}}\right) x_{17} = 0$$

That corresponds to the succession of vertices: $\langle 1,2,5,8,7,1\rangle$ and the sequence of corridors: $\{(1,2),(2,5),(5,8),(8,7),(7,1)\}$. The positive terms correspond to circuits that have the same orientation of the cycle and the negatives correspond to the links that have the opposite orientation of the cycle.

When the critical cycles include circuits in expansion corridors, the general form of equation (73) involves disjunctive constraints. For the South Brazilian test system, one of the critical cycles involving the new corridors correspond to the succession of the vertices: $\langle 5,6,46,19,18,13,8,5\rangle$ and the sequence of corridors:

$$\{(5,6),(6,46),(46,19),(19,18),(18,13),(13,8),(8,5)\}$$

This cycle assumes the following structure:

$$\left|\left(P_{5-6,1}\right)\cdot x_{5-6}+\left(P_{6-46,1}\right)\cdot x_{6-46}-\left(\frac{P^{0}_{19-46}}{n^{0}_{19-46}}\right)x_{19-46}-\left(\frac{P^{0}_{18-19}}{n^{0}_{18-19}}\right)x_{18-19}-\left(\frac{P^{0}_{13-18}}{n^{0}_{13-18}}\right)x_{13-18}-\left(\frac{P^{0}_{8-13}}{n^{0}_{8-13}}\right)x_{8-13}-\left(\frac{P^{0}_{5-8}}{n^{0}_{5-8}}\right)x_{5-8}\right|\leq M\left(2-\omega_{5-6,1}-\omega_{6-46,1}\right)$$

If $\omega_{5-6,1}=1$ and $\omega_{6-46,1}=1$, This constraint is equivalent to:

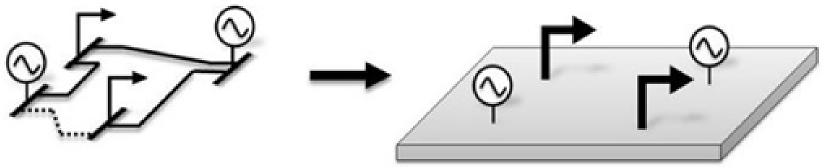
$$\left(P_{5-6,1}\right)x_{5-6}+\left(P_{6-46,1}\right)x_{6-46}-\left(\frac{P^{0}_{19-46}}{n^{0}_{19-46}}\right)x_{19-46}-\left(\frac{P^{0}_{18-19}}{n^{0}_{18-19}}\right)x_{18-19}+$$

$$-\left(\frac{P^{0}_{13-18}}{n^{0}_{13-18}}\right)x_{13-18}-\left(\frac{P^{0}_{8-13}}{n^{0}_{8-13}}\right)x_{8-13}-\left(\frac{P^{0}_{5-8}}{n^{0}_{5-8}}\right)x_{5-8}=0$$

3.6 LOW EFFORT METHOD

The principle of minimal effort is already widely known in the transmission expansion planning networks thanks to the constructive heuristic algorithm of minimum effort (MONTICELLI; SANTOS; PEREIRA, 1982). For the low effort method, the original system is modified by adding a large number of elements connected in parallel in all the transmission corridors, existing and candidates: $n_{i,j}^{0}=\bar{n}_{ij}$. This process is used to ensure the objective of the approximate behavior of the transmission network toward a plate of electrically conductive material that joins all the nodes of the system, as illustrated in Figure 1. This modified transmission network is called a low-effort network because it gives full freedom to the flow of active power in orderto find the route whose least resistance opposes its circulation, considering existing and expansion corridors.

Figure 1 – Low effort network.



Source: Created by the author.

So far an algorithm that allows us to define the maximum number of lines per corridor has not been discussed. In Rahmani, Romero and Rider (2013) a GRASP-CP with a sensitivity index is used to reduce the search space of the TEP problem, In large-scale systems, the number of binary decision variables is very large; as a result, the time needed for obtaining a solution grows exponentially, and the branch-and-bound used by the solver is unable to converge to the optimum solution or at least the high-quality solutions. Therefore, on Rahmani, Romero and Rider (2013), the search space of binary variables is reduced using GRASP-CP. However, it should be observed that the GRASP-CP may exclude the optimum point of the problem. In this work, once the low effort network of the original transmission network has been built for the DC model, it is resolved. The results of this model seek to identify the transmission corridors through which a significant amount of active power flows and to identify the direction they have.

3.6.1 Low effort to reduce the search space

Using the algorithm of low effort alone can only show the possible paths of flow on the system, unfortunatly, the paths themselves cannot make a good prediction of the behavior of TEP in the future. This is the main reason why the solution delivered by this algorithm cannot be used by itself to propose the maximum number of lines for future connections calculated by the model. To meet this objective, the low effort solution is used as a point of comparison with other relaxed solutions analyzed. In this work, we use the improved disjunctive transport the model and the disjunctive hybrid improved model, for the Brazilian north-northeast system. The added lines were compared to the answers found on Escobar, Gallego and Toro (2010) Rahmani, Romero and Rider (2013). The low effort will provide the point of comparison to determine which corridors are more important. With these results, we can reduce the investment options in transmission corridors that do not make important exchanges of active power. In the same way, it is possible to identify the transmission corridors that are not projected as being of great importance and that can be removed. For determining the maximum lines per corridor, the overlap of the results obtained with the relaxed models is used.

4 ANGULAR CUTS IN THE TRANSMISSION EXPANSION PLANNING PROBLEM

4.1 INTRODUCTION

In the transmission expansion planning problem, only a small number of instances can be solved to optimally with mixed-integer linear programming methods. Usually, the complexity of the problem is directly related to the size of the system being analyzed. However, for TEP, other factors also contribute to the complexity,including the degree of connectivity of the nodes, the degree of the radial or meshed structure of the system, the number of isolated nodes and the presence of the SKL constraints. In particular, KSL becomes complicated when a set of connected circuits (candidate circuits or existing circuits) forms a loop or a cycle. This effect is produced by the connection in parallel of circuits of low capacity-reactance product with circuits of high capacity-reactance product, and it can provide an insight on the system behavior, which is a topic that can be exploited to find new and better solutions for TEP.

The transmission planning problem has relaxed models that can be used to find backbones: solution parts that reoccur in their optimal solutions. In this work we uses the disjunctive continuous model, in which $\omega_{i,j,k}$ is not an integer variable, but a continuous variable the disjunctive transport improved model, which does not consider the KSL, and the disjunctive hybrid improved model, in which the KSL is only applied to the existing circuits. The bus-angle difference concept is used to generate cuts or valid inequalities for TEP. Another important idea is to identify the variables with the most impact on the problem and to use this information for the reduction of the space of solutions. From a practical standpoint, in realistic-sized instances with a high number of investment options per corridor, a low number of corridors present additions, next to the maximum number of additions and some expansion corridors are present in the investment options but they are not used in the solution of the problem. By combining the concepts of reduction of the space of solutions with cuts based on the bus-angle difference, we can reduce the computational effort for obtaining the optimal solution.

The primary aim of our experiments is to use cutting planes based on the bus-angle difference in the TEP for complex problems. Instances like these are well beyond the reach of current (exact) solution techniques because of the excessive computational effort involved. The angular cuts can be used to provide tighter lower bounds on the optimal investment of the transmission planning since the difference between the DC-based model (exact model) and the transportmodel (more relaxed model) is the bus-angle difference constraints (ESCOBAR; ROMERO, 2017). When the electrical model of the transmission system includes the bus-angle difference limits, some constraints of maximum angular distance or capacity-reactance constraints can be oversized with respect to the true limits, and it is possible to build cuts based on this information.

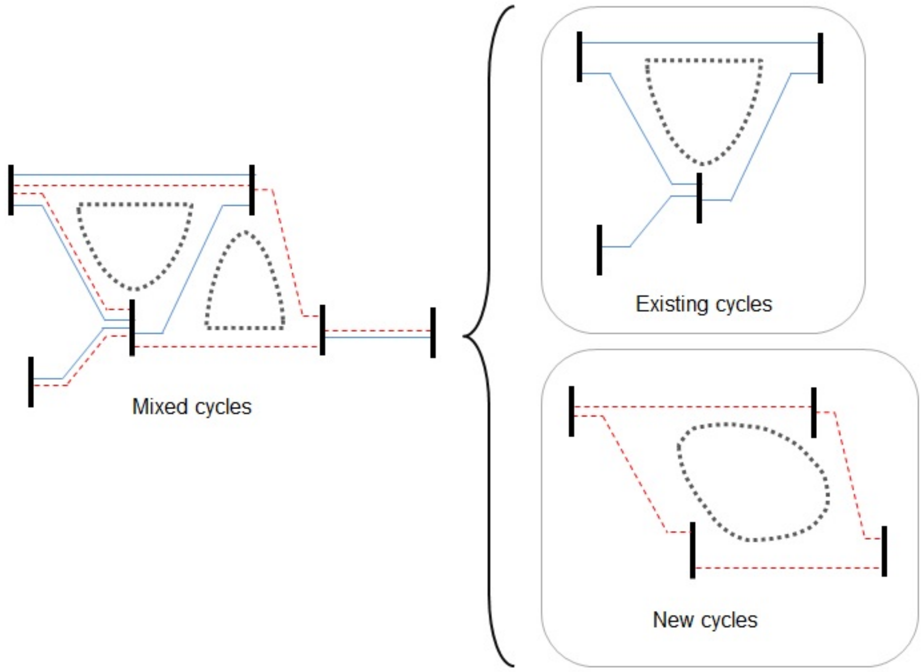
The methodology proposed in this work applied the following ideas:

a) If the bus-angle difference constraints or the products capacity-reactance limits of the circuits are relaxed in the transmission expansion planning problem, the power flows uses the shortest and low cost paths. b) The maximum bus-angle difference is a obligatory limit for the existing transmission circuits. This constraint can be unnecessary for circuits with low probability of selection in a first approximation.) If the cost of the investments is ignored, the problem with angle aperture constrains can be configured as a low effort transmission model. The relaxation of the cost will show attractive paths in the DC transmission model.

Is important know the structural characteristics of the problem and the characteristics of good solutions to solve realistic-sized instances, in a feasible time and to combine optimization methods with the knowledge of the problem too. It is necessary to use concepts and models that identify patterns and establish relationships between the variables and the constraints presents in the good solutions of the transmission planning problem. The known capacity-reactance constraints permit building angular cuts close to the facets of the convex hull. The combination of the relaxed solutions of the problem permit identified sub-spaces of good quality where an exact method can realize an exhaustive search. The primary aim of our work is to use the cutting-plane based in the bus-angle difference for large-scale problems.

All the mathematical models for the transmission expansion planning problem includes capacity-reactance constraints. In electric systems with high mesh level, the number of loops can be increases exponentially. In consequence it can be prohibitive to identify and verify each capacity-reactance constraint violated in the loops of the system. For large size systems, it is better to identify some potentials capacity-reactance constraints that can be violated in the exact model or if possible develop a method to identify angular cuts, or more restricted bus-angle constraints, and with this information generate stronger angular cuts close to the convex hull of the MILP problem.

Figure 2 – Cycles over current paths, new paths and mixed: current and new paths.



Source: Created by the author.

Figure 2 shows the three types of loops in a electric system. Each one produce one type of strong angular cut constraint.

4.1.1 Parallel connection between existing circuits with different capacity-reactance limits

The simplest case is when two existing circuits with different capacity-reactance products are connected in parallel, as in Figure 3. In this case, the capacity-reactance constraints for the circuits are:

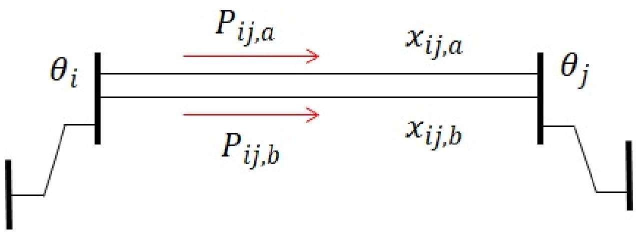
$$Limit\ for circuit\ a: \quad P_{ij,a}x_{ij,a} \leq (\theta_i - \theta_j)_{\max}$$

$$Limit\ for circuit\ b: \quad P_{ij,b}x_{ij,b} \leq (\theta_i - \theta_j)_{\max}$$

were:

$$(\theta_i - \theta_j)_{\max} = \min\left\{\bar{P}_{ij,a}x_{ij,a}\,,\ \bar{P}_{ij,b}x_{ij,b}\right\}$$

Figure 3 – Circuits with different capacity-reactance limits.



Source: Created by the author.

In this case, if the circuit b has lower capacity-reactance product that the circuit a. The normal bus-angle difference for circuit a is:

$$P_{ij,a}x_{ij,a} \leq \bar{P}_{ij,a}x_{ij,a}$$

And its strong angular cut is:

$$P_{ij,a}x_{ij,a} \leq \bar{P}_{ij,b}x_{ij,b}$$

In this case, circuit b limits the operation of the circuit a.

As a simple example, when analyzing the Brazilian north-northeasttest system, corridor 20 - 21 has two circuits of different capacity-reactance products in parallel:

Table 1 – Comparison between two corridors with different capacity - reactance

corridor	reactance	capacity	$xij * fij_{max}$
20 - 21	0.0715 p.u.	300 MW (3.0 p.u.)	0.0715*3.0 = 0.21450
20 - 21	0.1032 p.u.	170 MW (1.7 p.u.)	**0.1032*1.7 = 0.17544**

Source: Created by the author.

As can be seen in Table 1, the corridor with a lower capacity-reactance product is the last one. With this information an angular cut or valid inequality can be built.

$$(\theta_{20} - \theta_{21})_{\max} = \min\{0.21450\ ,\ 0.17544\} = 0.17544$$

Then this is a valid cut for this system:

$$|\theta_{20} - \theta_{21}| \leq 0.17544$$

The absolute value guarantees that the inequality continues to be valid for other operating conditions in which a change in the flow direction occurs. In this example, the second circuit limits the maximum power capacity of the first circuit, from $300MW$ to $245.3MW$. This represents a total functioning capacity for the first circuit of 81.7%. This condition is independentof the flow direction. For a specific operative condition, in which the flow direction is known, the cut can be written without using the absolute value. This allows one to reduce the number of added constraints. In the previous example, if the active power flows from node 20 to node 21, the added cut is:

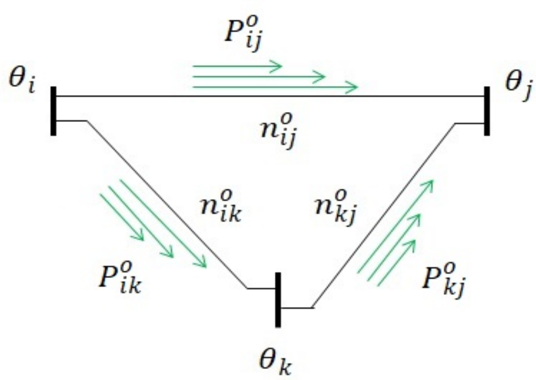
$$(\theta_{20} - \theta_{21}) \leq 0.17544$$

For two or more circuits in parallel, with a different capacity-reactance product, this cut is unnecessary, because it is well defined in the constraints in any version of the DC model. This cut is useful and improves the performance of the solution method for paths that represent the sequence of m connected corridors in the network, that share the bus-angle difference with other paths of corridors with the same starting/ending buses, but that have non-overlapping intermediate buses. The next section shows this characteristic.

4.1.2 Loop with different capacity-reactance limits in existing circuits

In this work, when three or more existing circuits form a loop or a cycle, a capacity-reactance cut is generated if the three proposed relaxed models are used: the improved transport model, improved hybrid model and low effort model, produce the same flow direction in each circuit. Figure 4 shows an example.

Figure 4 – Cycles over current paths, new paths and mixed: current and new paths.



Source: Created by the author.

In Figure 4, the capacity-reactance limit of the circuits ik and kj are added, and the result is compared with the capacity-reactance limit of the circuits ij. If the sum of the capacity-reactance limits of the circuits ik and kj are greater than the capacity-reactance limit of the circuit ij, a valid cut capacity-reactance or a valid bus-angle difference constraint added to the problem is:

$$(P^o_{ik} x_{ik} / n^o_{ik}) + (P^o_{kj} x_{kj} / n^o_{kj}) \leq \bar{P}_{ij} x_{ij}$$

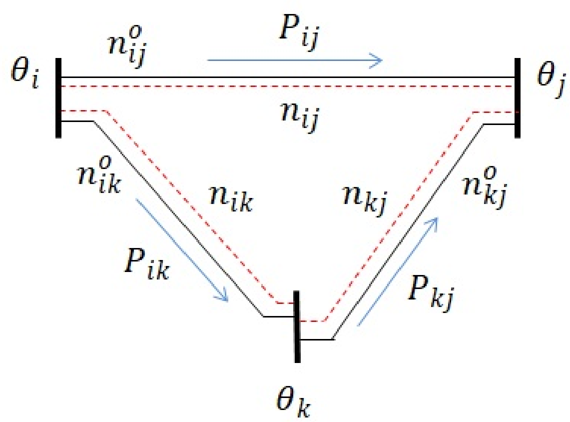
If the sum of the capacity-reactance limits of the circuits ik and kj is smaller than the capacity-reactance limit of the circuit ij, the cut capacity-reactance added to the problem is:

$$P^o_{ij} x_{ij} / n^o_{ij} \leq (\bar{P}_{ik} x_{ik}) + (\bar{P}_{kj} x_{kj})$$

In this constraint, the flows P^o_{ij} , P^o_{ik} and P^o_{kj} are the total active flow in the existing circuits and it is necessary divide it over the number of them in the corridor. If ij, ik, or kj are expansion corridors, the formulation is different.

4.1.3 Loop with different capacity-reactance limits in corridors with existing and new circuits

Figure 5 – Cycles between paths with investment options in parallel with existing circuits.



Source: Created by the author.

In this case we consider a system with n^o_{ij}, n^o_{ik} and n^o_{kj} existing circuits, and $w_{ij,p}$, $w_{ik,p}$ and $w_{kj,p}$, with $p = 1, 2, ..., P$ investment options, in each corridor. The capacity-reactance constraint of the circuits ik and kj is added and the result is compared with the capacity-reactance constraint of the circuit ij. If the sum of the capacity-reactance constraints of the circuit's ik and kj is greater than the capacity-reactance constraint of the circuit ij, the capacity-reactance cut added to the problem is:

$$\frac{P^o_{ik}}{n^o_{ik}} x_{ik} + \frac{P^o_{kj}}{n^o_{kj}} x_{kj} \leq \bar{P}_{ij} x_{ij}$$

In this case we use the active power flow of the existing circuits to obtain a linear valid inequality.

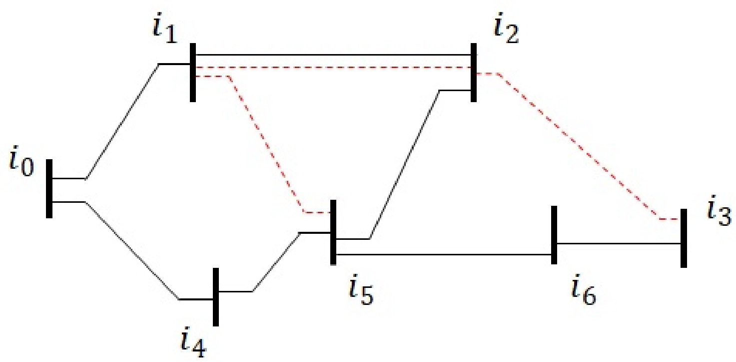
If the sum of the capacity-reactance constraint of the circuit's ik and kj are smaller than the capacity-reactance constraint of the circuit ij, the capacity-reactance cut added to the problem is:

$$\frac{P^o_{ij}}{n^o_{ij}} x_{ij} \leq \bar{P}_{ik} x_{ik} + \bar{P}_{kj} x_{kj}$$

4.2 PATH-BASED ANGULAR VALID INEQUALITIES DERIVATION CUTS

This section will describe the bus-angle valid inequalities derivation or capacity-reactance cuts.

Figure 6 – Network to illustrate the valid inequalities.



Source: Created by the author.

Figure 6 will be used to illustrate an application of each type of valid inequality or bus-angle difference cut.

We say then that (i, j) is an *established corridor* of the system G if $n^0_{i,\,j} > 0$; otherwise we say that (i, j) is an *expansion corridor* or a corridor of candidate circuits only. To introduce the bus-angle difference cut concept, we use $n^0_{i,\,j}$ and $P^0_{i,\,j}$ for the existing circuits. Use $n_{i,j} > 0$ and $P_{i,\,j}$ for the candidate circuits, $w_{i,\,j,k}$ for the investment options, where $1 \leq k \leq K$, is the number of candidate circuits in corridor $(i,\ j)$, and $P_{i,\ j,k}$ for the active power flow in the investment option $w_{i,\,j,k}$. All lines along the same corridor are assumed to have the same characteristics (i.e. reactance and capacity), including candidate circuits. Additionally, because these problems traditionally specify corridors from a lower index bus to a higher index bus, for the existing corridors we use:

$$\begin{array}{ll} P^0_{i_{q-1},i_q} & if\ \ i_q > i_{q-1} \\ -P^0_{i_q,i_{q-1}} & if\ \ i_q < i_{q-1} \end{array}$$

And for the expansion corridors we use:

$$\begin{array}{ll} P_{i_{q-1},i_q,k} & if\ \ i_q > i_{q-1} \\ -P_{i_q,i_{q-1},k} & if\ \ i_q < i_{q-1} \end{array}$$

In all cases: $n^0_{i_{q-1},i_q} = n^0_{i_q,i_{q-1}},\ x_{i_{q-1},i_q} = x_{i_q,i_{q-1}}\ ,\ n_{i_{q-1},i_q} = n_{i_q,i_{q-1}},\ \omega_{i_{q-1},i_q,k} = \omega_{i_q,i_{q-1},k}$

4.2.1 Single path over established corridors

Type 1 bus-angle valid inequality

Establish corridors can create cycles and paths in every power transmission system that are know before any analysis, since these elements are already presents on the network this makes the development of this type of cut much simpler.

If we let $\Omega^p_{i0,im} = \{(i_0, i_1), \ldots, (i_{m-1}, i_m)\}$ as a subset of an index-pair that represents the sequence of m connected corridors on denominated path p in the network G, starting with bus i_0 and ending with bus i_m, the sum of the bus-angle difference for consecutive node-pairs in $\Omega^p_{i0,im}$ can be written as:

$$(\theta_{i_0}-\theta_{i_m})^p = (\theta_{i_0}-\theta_{i_1})^p + (\theta_{i_1}-\theta_{i_2})^p + \ldots + (\theta_{i_{m-1}}-\theta_{i_m})^p$$

Where:

$$(\theta_{i_{q-1}}-\theta_{i_q}) = \frac{P^0_{i_{q-1},i_q}}{n^0_{i_{q-1},i_q}} x_{i_{q-1},i_q}; \qquad \forall\, q = 1,2,\ldots,m$$

If we defined π_0 as,

$$\pi_0 = \textstyle\sum_{q=1}^{m} x_{i_{q-1},i_q} \cdot \overline{P}_{i_{q-1},i_q} \tag{75}$$

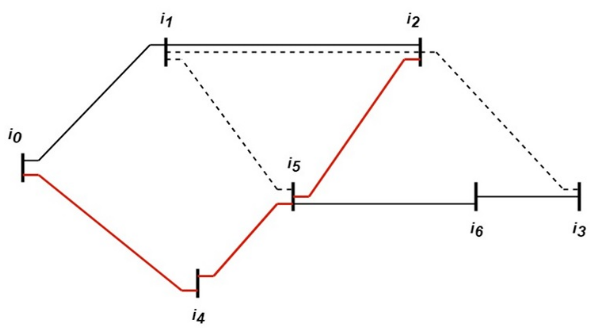
Then the following two-sided inequality is valid for TEP:

$$-\pi_0 \leq \sum_{q=1}^{m} \frac{P^0_{i_{q-1},i_q}}{n^0_{i_{q-1},i_q}} x_{i_{q-1},i_q} \leq \pi_0. \tag{76}$$

An alternative form of this bus-angle difference cut that extends this valid inequality to other paths with the same starting/ending buses but with non-overlapping intermediate buses, is:

$$-\pi_0 \leq (\theta_{i_0}-\theta_{i_m}) \leq \pi_0. \tag{77}$$

Figure 7 – Type 1 bus-angle valid inequality example.



Source: Created by the author.

As an example using Figure 7 the path $\Omega^1_{i0,i2} = \{(i_0,i_4),(i_4,i_5),(i_5,i_2)\}$ is a subset of an index-pair that represents the sequence of 3 connected corridors on denominated path 1 in the network, starting with bus i_0 and ending with bus i_2. The sum of the bus-angle difference for consecutive node-pairs in $\Omega^1_{i0,i2}$ can be written as:

$$(\theta_{i_0} - \theta_{i_2})^1 = (\theta_{i_0} - \theta_{i_4})^1 + (\theta_{i_4} - \theta_{i_5})^1 + (\theta_{i_5} - \theta_{i_2})^1$$

Which creates the example type 1 two-sided valid inequality:

$$\begin{aligned}
&-(\overline{P}_{i_0,i_4} x_{i_0,i_4} + \overline{P}_{i_4,i_5} x_{i_4,i_5} + \overline{P}_{i_2,i_5} x_{i_2,i_5}) \\
&\leq \frac{P^0_{i_0,i_4}}{n^0_{i_0,i_4}} x_{i_0,i_4} + \frac{P^0_{i_4,i_5}}{n^0_{i_4,i_5}} x_{i_4,i_5} - \frac{P^0_{i_2,i_5}}{n^0_{i_2,i_5}} x_{i_2,i_5} \\
&\leq \overline{P}_{i_0,i_4} x_{i_0,i_4} + \overline{P}_{i_4,i_5} x_{i_4,i_5} + \overline{P}_{i_2,i_5} x_{i_2,i_5}
\end{aligned}$$

An alternative form of this bus-angle difference cut that extends this valid inequality to other paths with the starting bus i_0 and ending bus i_2 but with non-overlapping intermediate buses, is:

$$\begin{aligned}
&-(\underline{P}_{i_0,i_4} x_{i_0,i_4} + \underline{P}_{i_4,i_5} x_{i_4,i_5} + \underline{P}_{i_2,i_5} x_{i_2,i_5}) \\
&\leq (\theta_{i_0} - \theta_{i_2}) \\
&\leq \underline{P}_{i_0,i_4} x_{i_0,i_4} + \underline{P}_{i_4,i_5} x_{i_4,i_5} + \underline{P}_{i_2,i_5} x_{i_2,i_5}
\end{aligned}$$

On the same note, in Figure 6 the path $\Omega^2_{i0,i2} = \{(i_0, i_1), (i_1, i_2)\}$ creates the type 1 two-sided valid inequality:

$$\begin{aligned}
&-(\overline{P}_{i_0,i_1} x_{i_0,i_1} + \overline{P}_{i_1,i_2} x_{i_1,i_2}) \\
&\leq \frac{P^0_{i_0,i_1}}{n^0_{i_0,i_1}} x_{i_0,i_1} + \frac{P^0_{i_1,i_2}}{n^0_{i_1,i_2}} x_{i_1,i_2} \\
&\leq \overline{P}_{i_0,i_1} x_{i_0,i_1} + \overline{P}_{i_1,i_2} x_{i_1,i_2}
\end{aligned}$$

The alternative form of this cut is:

$$\begin{aligned}
&-(\overline{P}_{i_0,i_1} x_{i_0,i_1} + \overline{P}_{i_1,i_2} x_{i_1,i_2}) \\
&\leq (\theta_{i_0} - \theta_{i_2}) \\
&\leq \overline{P}_{i_0,i_1} x_{i_0,i_1} + \overline{P}_{i_1,i_2} x_{i_1,i_2}
\end{aligned}$$

This has a similar approach as seen on the first part of this chapter, where the main idea is to constraint the angular aperture between nodes, in this case the cut used is the valid inequality associate with the problematic corridors or path with low capacity-reactance product ($\Omega^1_{i0,i2}$ or $\Omega^2_{i0,i2}$), as shown in the type 2 bus-angle valid inequality.

4.2.2 Parallel paths over established corridors

Type 2 bus-angle valid inequality

This valid inequality combines information of the paths that share their initial and final busbars.

If we let $\Omega^r_{i^r_0,i^r_m} = \{(i^r_0, i^r_1), \ldots, (i^r_{m-1}, i^r_m)\}$, with $r = 1,2,..,s$, as a subset of an index-pair that represents the sequence of m^r connected corridors on denominated path r in the network G, and exist s parallel paths starting with bus i^r_0 and ending with bus i^r_m, the sum of the bus-angle difference for consecutive node-pairs in $\Omega^r_{i^r_0,i^r_m}$ can be written as:

$$(\theta_{i^r_0} - \theta_{i^r_m}) = (\theta_{i^r_0} - \theta_{i^r_1}) + (\theta_{i^r_1} - \theta_{i^r_2}) + \ldots + (\theta_{i^r_{m-1}} - \theta_{i^r_m}) \;\; \forall\, r = 1,2,\ldots,s$$

Where:

$$(\theta_{i^r_{q-1}} - \theta_{i^r_q}) = \frac{P^0_{i^r_{q-1},i^r_q}}{n^0_{i^r_{q-1},i^r_q}} x_{i^r_{q-1},i^r_q}; \qquad \forall\, q = 1,2,\ldots,m^r;\; r = 1,2,..,s$$

If we defined π^r_0 as,

$$\pi^r_0 = \Sigma^m_{q=1} x_{i^r_{q-1},i^r_q} \cdot \overline{P}_{i^r_{q-1},i^r_q} \tag{78}$$

And,

$$\pi^t_0 = \min\{\pi^r_0\} \;\; \text{for} \;\; r = 1,\ldots,s. \tag{79}$$

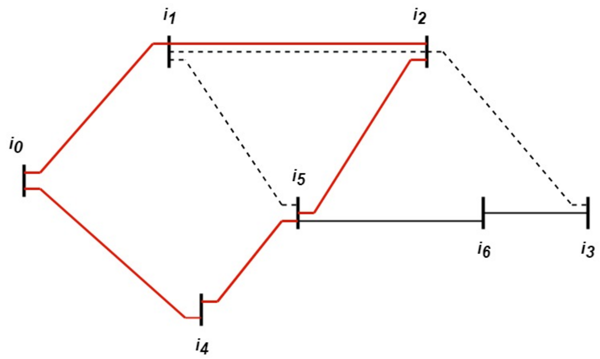
In this case π^t_0 is associate with the problematic corridors or path t with low capacity-reactance product. Then the following two-sided inequality is valid for TEP:

$$-\pi^t_0 \leq \sum_{q=1}^{m^r} \frac{P^0_{i^r_{q-1},i^r_q}}{n^0_{i^r_{q-1},i^r_q}} x_{i^r_{q-1},i^r_q} \leq \pi^t_0. \tag{80}$$

An alternative form of this bus-angle difference cut that extends this valid inequality to other paths with the same starting/ending buses but with non-overlapping intermediate buses, is:

$$-\pi_0^t \leq (\theta_{i_0} - \theta_{i_m}) \leq \pi_0^t. \tag{81}$$

Figure 8 – Type 2 bus-angle valid inequality example.



Source: Created by the author.

Continuing with the example of the previous subsection, in Figure 8, $\Omega^1_{i_0^1, i_2^1}$ creates an established parallel path with $\Omega^2_{i_0^2, i_2^2}$. Assuming that path $\Omega^2_{i_0^2, i_2^2}$ is the path with lower sum of capacity-reactance products, creates the example type 2 two-sided valid inequality:

$$\begin{aligned}
&-\bar{P}_{i_0,i_1} x_{i_0,i_1} - \bar{P}_{i_1,i_2} x_{i_1,i_2} \\
&\leq \frac{P^0_{i_0,i_4}}{n^0_{i_0,i_4}} x_{i_0,i_4} + \frac{P^0_{i_4,i_5}}{n^0_{i_4,i_5}} x_{i_4,i_5} - \frac{P^0_{i_2,i_5}}{n^0_{i_2,i_5}} x_{i_2,i_5} \\
&\leq \bar{P}_{i_0,i_1} x_{i_0,i_1} + \bar{P}_{i_1,i_2} x_{i_1,i_2}
\end{aligned}$$

The alternative form that extends this valid inequality to other paths with the same starting/ending buses but with non-overlapping intermediate buses, and with greater sum of capacity-reactance products, is:

$$\begin{aligned}
&-(\overline{P}_{i_0,i_1} x_{i_0,i_1} + \overline{P}_{i_1,i_2} x_{i_1,i_2}) \\
&\leq (\theta_{i_0} - \theta_{i_2}) \\
&\leq \overline{P}_{i_0,i_1} x_{i_0,i_1} + \overline{P}_{i_1,i_2} x_{i_1,i_2}
\end{aligned}$$

In this case $\Omega^2_{i_0^2,i_2^2}$ is associate with the problematic corridors or path with the lower sum of capacity-reactance products.

4.2.3 Single path over established and expansion corridors

Type 3 Valid inequality Cut

In this case $\Omega^p_{i0,im} = \{(i_0,i_1),\ldots,(i_{m-1},i_m)\}$ is a sequence of m connected corridors on denominated path p in the network G, starting with bus i_0 and ending with bus i_m, the sum of the bus-angle difference for consecutive node-pairs in $\Omega^p_{i0,im}$ can be written as:

$$(\theta_{i_0}-\theta_{i_m})^p = (\theta_{i_0}-\theta_{i_1})^p + (\theta_{i_1}-\theta_{i_2})^p + \ldots + (\theta_{i_{m-1}}-\theta_{i_m})^p$$

For established corridors we have:

$$(\theta_{i_{q-1}}-\theta_{i_q}) = \frac{P^0_{i_{q-1},i_q}}{n^0_{i_{q-1},i_q}} x_{i_{q-1},i_q}$$

And for expansion corridors with $\omega_{i_{q-1},i_q,1} = 1$:

$$(\theta_{i_{q-1}}-\theta_{i_q}) = P_{i_{q-1},i_q,1} x_{i_{q-1},i_q}$$

In this case we uses the traditional linear disjunctive model with *one* investment option for each $\omega_{i_{q-1},i_q,k}$ for $k=1,2,\ldots,K$, and consider the next constraint:

$$\omega_{i_{q-1},i_q,k} \leq \omega_{i_{q-1},i_q,k-1} \qquad \forall_{i_{q-1},i_q} \in \Omega,\ k = 2\ldots K$$

This constraint guarantees that the SKL is activated in the expansion corridors if the option $\omega_{i_{q-1},i_q,1}$=1. In the disjunctive form, are equivalent to:

$$-M(1-\omega_{i_{q-1},i_q,1}) \leq (\theta_{i_{q-1}}-\theta_{i_q}) - P_{i_{q-1},i_q,1} x_{i_{q-1},i_q} \leq M(1-\omega_{i_{q-1},i_q,1})$$

On the other side, we defined π_0 as,

$$\pi_0 = \textstyle\sum_{q=1}^{m} x_{i_{q-1},i_q} \cdot \overline{P}_{i_{q-1},i_q} \tag{82}$$

In this case, the following two-sided inequality is valid for TEP:

$$-M(R-z)-\pi_0$$
$$\leq \sum_{(i_{q-1},i_q)\in\Omega_a} \frac{P^0_{i_{q-1},i_q}}{n^0_{i_{q-1},i_q}} x_{i_{q-1},i_q} + \sum_{(i_{q-1},i_q)\in\Omega_b} P_{i_{q-1},i_q,1} x_{i_{q-1},i_q}$$
$$\leq \pi_0 + M(R-z)$$

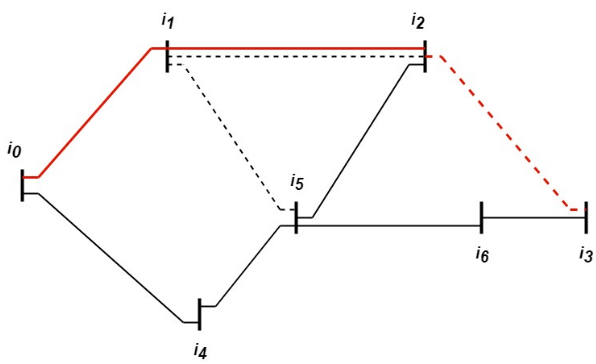
Where Ω_a is the subset of index-pair that represents the established corridors on denominated path p in the network G, Ω_b is the subset of index-pair that represents the expansion corridors on the denominated path p in the network G, R is the number of the expansion corridors of the path p in the network G, and z is the sum of the binary variables associate to the first investment option of the expansion corridors include of the path:

$$z = \sum_{(i_{q-1},i_q,1)\in\Omega_b} \omega_{i_{q-1},i_q,1}$$

The alternative formulation of this cut is:

$$-M(R-z)-\pi_0 \leq (\theta_{i_0}-\theta_{i_m}) \leq \pi_0 + M(R-z)$$

Figure 9 – Type 3 bus-angle valid inequality example.



Source: Created by the author.

In Figure 9, a new single path, $\Omega^3_{i_0,i_3} = \{(i_0,i_1),(i_1,i_2),(i_2,i_3)\}$ creates the next type 3 valid inequality:

$$-M(1-z) - \overline{P}_{i_0,i_1} x_{i_0,i_1} - \overline{P}_{i_1,i_2} x_{i_1,i_2} - \overline{P}_{i_2,i_3} x_{i_2,i_3}$$
$$\leq \frac{P^0_{i_0,i_1}}{n^0_{i_0,i_1}} x_{i_0,i_1} + \frac{P^0_{i_1,i_2}}{n^0_{i_1,i_2}} x_{i_1,i_2} + P_{i_2,i_3.1} x_{i_2,i_3}$$
$$\leq \overline{P}_{i_0,i_1} x_{i_0,i_1} + \overline{P}_{i_1,i_2} x_{i_1,i_2} + \overline{P}_{i_2,i_3} x_{i_2,i_3} + M(1-z)$$
$$z = \omega_{i_2,i_3,1}$$

In this cut, (i_0,i_1) and (i_1,i_2) are established corridors, and (i_2,i_3) is a expansion corridor.

The alternative formulation for this cut is:

$$\begin{aligned}
&-M(1-\omega_{i_2,i_3,1})-\overline{P}_{i_0,i_1}x_{i_0,i_1}-\overline{P}_{i_1,i_2}x_{i_1,i_2}-\overline{P}_{i_2,i_3}x_{i_2,i_3}\\
&\leq(\theta_{i_0}-\theta_{i_3})\\
&\leq\overline{P}_{i_0,i_1}x_{i_0,i_1}+\overline{P}_{i_1,i_2}x_{i_1,i_2}+\overline{P}_{i_2,i_3}x_{i_2,i_3}+M(1-\omega_{i_2,i_3,1})
\end{aligned}$$

4.2.4 Parallel paths over established and expansion corridors

Type 4 valid inequality

Let $\Omega^r_{i^r_0,i^r_m}=\{(i^r_0,i^r_1),\ldots,(i^r_{m-1},i^r_m)\}$, with $r=1,2,..,s$ alternative directed paths over established and expansion corridors in G, a subset of an index-pair that represents the sequence of m^r connected corridors on denominated path r in the network, and exist s parallel paths with the same starting/ending buses but with non-overlapping intermediate buses. The sum of the bus-angle difference for consecutive node-pairs in $\Omega^r_{i^r_0,i^r_m}$ can be written as:

$$(\theta_{i^r_0}-\theta_{i^r_m})=(\theta_{i^r_0}-\theta_{i^r_1})+(\theta_{i^r_1}-\theta_{i^r_2})+\ldots+(\theta_{i^r_{m-1}}-\theta_{i^r_m})\ \ \forall\, r=1,2,\ldots,s$$

For established corridors we have:

$$(\theta_{i^r_{q-1}}-\theta_{i^r_q})=\frac{P^0_{i^r_{q-1},i^r_q}}{n^0_{i^r_{q-1},i^r_q}}x_{i^r_{q-1},i^r_q};\qquad\forall\, r=1,2,..,s$$

And for expansion corridors with $\omega_{i^r_{q-1},i^r_q,1}=1$:

$$(\theta_{i^r_{q-1}}-\theta_{i^r_q})=P_{i^r_{q-1},i^r_q,1}x_{i^r_{q-1},i^r_q};\qquad\forall\, r=1,2,..,s$$

In this case we uses the traditional linear disjunctive model with *one* investment option for each $\omega_{i^r_{q-1},i^r_q,k}$ for $k=1,2,\ldots,K$.

If we defined π^r_0 as,

$$\pi^r_0=\Sigma^m_{q=1}x_{i^r_{q-1},i^r_q}\cdot\overline{P}_{i^r_{q-1},i^r_q}\tag{83}$$

And,

$$\pi_0^t = \min\{\pi_0^r\} \text{ for } r = 1, \ldots, s. \tag{84}$$

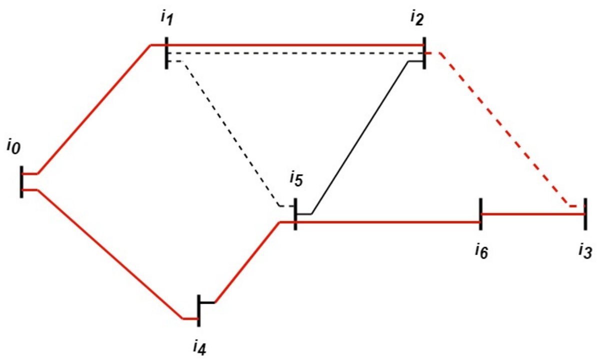
In this case π_0^t is associate with the problematic corridors or path t with the lower sum of capacity-reactance products. Then the following two-sided inequality is valid for TEP:

$$\begin{aligned} &-M(R-z) - \pi_0^t \\ &\leq \textstyle\sum_{(i_{q-1}^r, i_q^r) \in \Omega_a} \frac{P^0_{i_{q-1}^r, i_q^r}}{n^0_{i_{q-1}^r, i_q^r}} x_{i_{q-1}^r, i_q^r} + \sum_{(i_{q-1}^r, i_q^r) \in \Omega_b} P_{i_{q-1}^r, i_q^r, 1} x_{i_{q-1}^r, i_q^r} \\ &\leq \pi_0^t + M(R-z) \end{aligned}$$

The alternative form that extends this valid inequality to other paths with the same starting/ending buses but with non-overlapping intermediate buses, and with greater sum of capacity-reactance products, is:

$$-M(R-z) - \pi_0^t \leq (\theta_{i_0^r} - \theta_{i_m^r}) \leq \pi_0^t + M(R-z)$$

Figure 10 – Type 4 bus-angle valid inequality example.



Source: Created by the author.

In Figure 10, the new path $\Omega^4_{i_0,i_3} = \{(i_0,i_4),(i_4,i_5),(i_5,i_6),(i_6,i_3)\}$ is connected in parallel with the path $\Omega^3_{i_0,i_3} = \{(i_0,i_1),(i_1,i_2),(i_2,i_3)\}$. Assuming $\Omega^4_{i_0,i_3}$ is the parallel path with the lower sum of capacity-reactance products, creates the type 4 valid inequality:

$$\begin{aligned} &-M(1-z) - \overline{P}_{i_0,i_4} x_{i_0,i_4} - \overline{P}_{i_4,i_5} x_{i_4,i_5} - \overline{P}_{i_5,i_6} x_{i_5,i_6} - \overline{P}_{i_3,i_6} x_{i_3,i_6} \\ &\leq \frac{P^0_{i_0,i_1}}{n^0_{i_0,i_1}} x_{i_0,i_1} + \frac{P^0_{i_1,i_2}}{n^0_{i_1,i_2}} x_{i_1,i_2} + P_{i_2,i_3.1} x_{i_2,i_3} \\ &\leq \overline{P}_{i_0,i_4} x_{i_0,i_4} + \overline{P}_{i_4,i_5} x_{i_4,i_5} + \overline{P}_{i_5,i_6} x_{i_5,i_6} + \overline{P}_{i_3,i_6} x_{i_3,i_6} + M(1-z) \\ &z = \omega_{i_2,i_3,1} \end{aligned}$$

The alternative form that extends this valid inequality to other paths with the starting bus i_0 and ending bus i_3 but with non-overlapping intermediate buses, and with greater sum of capacity-reactance products, is:

$$
\begin{aligned}
&-M(1-z)-\overline{P}_{i_0,i_4}x_{i_0,i_4}-\overline{P}_{i_4,i_5}x_{i_4,i_5}-\overline{P}_{i_5,i_6}x_{i_5,i_6}-\overline{P}_{i_3,i_6}x_{i_3,i_6}\\
&\leq(\theta_{i_0}-\theta_{i_3})\\
&\leq\overline{P}_{i_0,i_4}x_{i_0,i_4}+\overline{P}_{i_4,i_5}x_{i_4,i_5}+\overline{P}_{i_5,i_6}x_{i_5,i_6}+\overline{P}_{i_3,i_6}x_{i_3,i_6}+M(1-z)\\
&z=\omega_{i_2,i_3,1}
\end{aligned}
$$

5 TESTS AND RESULTS

In this chapter, we carry out four types of studies, The first is related to the Improved Disjunctive Transport Model (IDTP) subsection (3.3), the second is related to the improved disjunctive hybrid model (IDHM) subsection (3.4), the third is related to the base cycles subsection (3.5) in combination with the improve disjunctive transport model subsection (3.3) and, finally, the fourth study relates to the reduced lineal disjunctive model (RLDM) subsection (2.6) in combination with the path-based angular valid inequalities derivation cuts subsection (4.2).

The purpose of studying the improved disjunctive transport model (IDTM) and the disjunctive hybrid improved model (IDHM) is to show how the inclusion of additional valid constraints improves the power flow results for these models. In this case, the new constraints keep to the traditional transport and hybrid model as simple as they can be and force the power flows to have the same distribution through all the corridors. This will allow the model to be solved with less computational effort and in less time.

All tests were executed using the laboratory computer and server with the following specifications:

- Dell | PowerEdge T430
- Operational system: Linux
- 2x computer processing units, Intel® Xeon® E5-2650 v4 2.2GHz, 30M Cache, 9.60GT/s QPI, Turbo, HT, 12Cores/24Threads (105W) Max Mem 2400MHz.
- Memory of 64GB.
- 2x Solid Discs of 480GB SATA Read Intensive MLC 6Gbps 2.5” hybrid carrier de 3.5”.

5.1 IMPROVED DISJUNCTIVE TRANSPORT MODEL

In this part, we show a comparative analysis of the transport model in two versions: the traditional disjunctive transport model and the improved disjunctive transport model. We use

In section (3.2), we present this model. This is the simplest way to represent the transmission system and it is easy to solve when the investment options are integers. In this case, the KSL is not used. When the transmission planning problem uses the traditional transport model, in the disjunctive version, it presents different active power flows for circuits in parallel with the same corridor. Table 2 shows the results of the four corridors when we solve the Colombian test system of 93 bus-bar and 155 corridors with this model. This table, presents two problematic aspects: One hand, there are different active power flows for parallel circuits in corridors $(43,88)$, $(14,31)$ and $(19,66)$. On the other hand, it shows circuits at its upper limit, although the corridor is below its capacity: circuits in corridors $(43,88)$, $(14,31)$ and $(19,66)$. This is caused by an inadequate distribution of power in the circuits of the same corridor. This characteristic induces the wrong results in the construction of base cycles. The traditional transport model in a disjunctive version presents an investment cost of $v = US\$315.35$.

Table 2 – Active power flows in traditional transport model.

i	j	n_{ij}^{o}	$\omega_{i,j,1}$	$\omega_{i,j,2}$	P_{ij}^{o}	$P_{i,j,1}$	$P_{i,j,2}$	$P_{ij}max$
52	88	0	1	0	0	106.5	0	300
43	88	0	1	1	0	-250.0	-156.5	250
14	31	2	1	0	500.0	136.2	0	250
19	66	1	1	1	350.0	350.0	189.9	350

Source: Created by the author.

The traditional transport model in a disjunctive version presents a investment cost of $v = US\$315.35$, and uses the next additions:

$n_{52-88} = 1$	$n_{43-88} = 2$	$n_{57-81} = 1$	$n_{14-31} = 1$
$n_{55-84} = 1$	$n_{55-62} = 1$	$n_{19-66} = 2$	$n_{68-86} = 1$

Source: Created by the author.

When we use the improved disjunctive transport model, the active power flows are the same. Table 3 shows the results of the four corridors from Table 2 when we solve the Colombian test system of 93 busbars and 155 corridors with the improved model. This table, presents the same active power flows for parallel circuits in the corridors $(43,88)$, $(14,31)$ and $(19,66)$. In this case, only the circuits of corridor $(14,31)$ are identified at its upper limit. This characteristic induces the correct results in the construction of base cycles. The improved disjunctive transport model presents the same investment cost of $v = US\$315.35$.

Table 3 – Active power flows in improved transport model.

i	j	n^o_{ij}	$\omega_{i,j,1}$	$\omega_{i,j,2}$	P^o_{ij}	$P_{i,j,1}$	$P_{i,j,2}$	$P_{ij}max$
52	88	0	1	0	0	107.5	0	300
43	88	0	1	1	0	-203.8	-203.8	250
14	31	2	1	0	500.0	250.0	0	250
19	66	1	1	1	296.6	296.6	296.6	350

Source: Created by the author.

The improved transport model in a disjunctive version presents a investment cost of $v = US\$315.35$, and uses the next additions:

$n_{52-88} = 1$	$n_{43-88} = 2$	$n_{57-81} = 1$	$n_{14-31} = 1$
$n_{55-84} = 1$	$n_{55-62} = 1$	$n_{19-66} = 2$	$n_{68-86} = 1$

Source: Created by the author.

5.2 IMPROVED DISJUNCTIVE HYBRID MODEL

In this part, we show a comparative analysis for the hybrid model in the two versions: the traditional disjunctive hybrid model and the improved disjunctive hybrid model. We use an improved disjunctive version of the hybrid model because this model presents a better result than the traditional hybrid model. This improved version of the hybrid model presents intermediate results between the traditional hybrid model and the DC model. This model uses the KFL for the existing and expansion corridors, and the SKL for the existing circuits exclusively: see Section (3.4). This model is used in this work to identify the orientations of the active power flows in the construction of bus-angle difference cuts.

When the transmission planning problem uses the traditional hybrid model, in the disjunctive version, it presents different active power flows between existing circuits and expansion circuits in parallel, and of the same corridor. Table 4 shows the results of four corridors when we solve the Colombian test system of 93 bus bar and 155 corridors with this model.

Table 4 – Active power flows in traditional hybrid model.

i	j	n^o_{ij}	$\omega_{i,j,1}$	$\omega_{i,j,2}$	P^o_{ij}	$P_{i,j,1}$	$P_{i,j,2}$	$P_{ij}max$
52	88	0	1	0	0	133.4	0	300
43	88	0	1	1	0	-183.4	-250.0	250
14	31	2	1	1	133.5	250.0	250.0	250
15	18	1	1	0	-360.7	-389.6	0	450

Source: Created by the author.

This table presents different active power flows for parallel circuits in corridors $(43,88)$, $(14,31)$ and $(15,18)$. The traditional hybrid model in a disjunctive version is solved in 10.23 s (10865.25 ticks), presents a investment cost of $v = US\$470.36$, and uses the following additions:

$n_{52-88}=1$	$n_{43-88}=2$	$n_{57-81}=1$	$n_{14-31}=1$	$n_{15-18}=1$	$n_{55-84}=1$	$n_{55-62}=1$
$n_{69-70}=1$	$n_{09-69}=1$	$n_{60-69}=3$	$n_{31-72}=3$	$n_{19-22}=1$	$n_{19-58}=1$	$n_{27-64}=1$
$n_{19-66}=3$	$n_{34-70}=1$	$n_{08-71}=1$	$n_{50-54}=1$	$n_{68-86}=1$		

Source: Created by the author.

The *ticks* are a computer-independent measure of how much algorithmic work is required to obtain a provable optimum, independently of the computer on which it is run.

When we use the improved disjunctive hybrid model, the active power flows are the same. Table 5 shows the results of four new corridors when we solve the Colombian test system of 93 busbars and 155 corridors with the improved model.

Table 5 – Active power flows in improved hybrid model.

i	j	n_{ij}^o	$\omega_{i,j,1}$	$\omega_{i,j,2}$	P_{ij}^o	$P_{i,j,1}$	$P_{i,j,2}$	$P_{ij}max$
43	88	0	1	1	0	-150.0	-150.0	250
15	18	1	1	0	-310.2	-310.2	0	450
45	81	1	1	0	335.1	335.1	0	450
19	82	1	1	1	368.0	368.0	368.0	450

Source: Created by the author.

This Table, presents the same active power flows for parallel circuits in corridors $(43,88)$, $(15,18)$, $(45,81)$ and $(19,82)$. The improved disjunctive hybrid model is solved in 33.86 s (33909.57 ticks), presents a investment cost of $v = US\$536.15$, and uses the following additions:

$n_{43-88}=2$	$n_{57-81}=1$	$n_{27-89}=1$	$n_{74-89}=1$	$n_{15-18}=1$	$n_{55-57}=1$	$n_{55-84}=1$
$n_{55-62}=1$	$n_{28-29}=1$	$n_{62-73}=1$	$n_{45-81}=1$	$n_{19-82}=2$	$n_{82-85}=1$	$n_{68-86}=1$

Source: Created by the author.

5.3 LOW EFFORT METHOD

In this work, the test of the Brazilian north-northeastern system was made with the angular cuts methodology explained in Chapter 4 using maximum circuits per corridor as shown in Table 6, option 2. Table 6, Option 1 presents the worst case, which was used to reduce the search space of the TEP problem.

Table 6 – Options for the maximum number of lines per corridor.

i	j	n_{ij}^0	DTM	DHM	LowEff	BKSa	BKSb	Opc1	Opc2	Opc3
1	2	2	1	1	0.7	1	1	1	1	1
2	4	0	0	0	0.4	0	1	0	1	2
2	60	0	1	1	0.6	0	0	1	1	2
2	87	1	0	0	0	1	0	1	1	2
3	71	0	0	0	0.2	0	0	0	1	0
3	81	0	0	0	0.1	0	0	0	1	2
3	83	0	0	0	0.2	0	0	0	1	2
3	87	0	0	0	1.2	0	0	1	2	3
4	5	1	2	0	0	4	4	4	5	6
4	6	0	1	0	0.6	0	0	1	1	2
4	32	0	0	0	0	0	0	0	1	0
4	60	0	0	0	0.4	0	0	0	1	1
4	68	0	1	0	1.4	1	0	1	2	2
4	69	0	0	0	0.2	0	0	0	1	0
4	81	0	3	6	2.5	3	3	6	6	6
4	87	1	0	0	0.3	0	0	0	0	0
5	6	1	0	0	0.1	0	0	0	0	0
5	38	2	0	0	0.3	0	0	0	0	0
5	56	0	0	0	1.0	1	1	1	2	1
5	58	0	3	3	0.7	3	4	4	5	5
5	60	0	1	1	0.8	0	0	1	1	1
5	68	0	0	1	0.5	0	0	1	1	1
5	70	0	0	0	0.3	0	0	0	0	1
5	80	0	0	0	0	0	0	0	0	0
6	7	1	0	0	0.2	0	0	0	0	0
6	37	1	0	0	0	1	0	1	2	1
6	67	0	0	2	0.3	0	0	2	2	2
6	68	0	0	0	0.4	0	0	0	1	1
6	70	0	0	0	0.4	0	0	0	1	0
6	75	0	0	0	0.4	0	1	1	1	1
7	8	1	0	0	0	0	0	0	1	0
7	53	0	0	0	0.4	0	0	0	1	0
7	62	0	0	0	0.3	0	0	0	1	0
8	9	1	0	0	0	0	0	0	0	0
8	12	0	0	0	0.1	0	0	0	0	0
8	17	0	0	0	0.3	0	0	0	0	0
8	53	1	0	0	0	0	0	0	0	0
8	62	0	0	0	0.3	0	0	0	0	0
8	73	0	0	0	0.5	0	0	1	1	1
9	10	1	0	0	0	0	0	0	0	0
10	11	1	0	0	0	0	0	0	0	0
11	12	1	0	0	0	0	0	0	0	0
11	15	1	0	0	0	0	0	0	0	0
11	17	1	0	0	0	0	0	0	0	0
11	53	1	0	0	0	0	0	0	0	0
12	13	1	0	0	0	0	0	0	0	0
12	15	1	0	0	0	1	0	1	2	2
12	17	1	0	1	0	0	0	1	1	2

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i	j	n_{ij}^0	DTM	DHM	LowEff	BKSa	BKSb	Opc1	Opc2	Opc3
12	35	2	0	0	0	0	0	0	0	0
12	84	0	0	0	2.2	0	0	2	3	3
13	14	0	0	1	1.3	0	0	1	2	2
13	15	0	4	2	1.0	4	4	4	5	5
13	17	0	0	1	0.7	0	0	1	1	2
13	45	1	0	0	0	0	0	0	1	0
13	59	1	0	0	0	0	0	0	1	0
14	17	0	0	0	0.3	0	0	0	1	0
14	45	0	1	0	0.7	1	0	1	2	2
14	59	0	0	1	0.8	0	1	1	2	2
15	16	2	4	1	0	4	3	4	5	5
15	45	0	0	0	0.5	0	0	1	1	1
15	46	1	0	1	0.8	1	1	1	2	2
15	53	0	0	0	0.2	0	0	0	1	1
16	44	4	6	6	4.7	6	6	6	7	6
16	45	0	0	0	0.8	0	0	1	1	1
16	61	0	1	1	0.9	2	0	2	3	3
16	77	0	0	3	3.0	0	0	3	4	4
17	18	2	0	2	0.2	0	0	2	2	2
17	59	0	0	0	0.4	0	0	1	1	1
18	50	4	11	11	9,7	11	11	11	11	11
18	59	1	0	0	0	0	0	0	1	1
18	74	0	6	3	3.7	6	5	6	7	7
19	20	1	0	1	0.7	0	0	1	1	0
19	22	1	0	0	0	0	0	0	0	0
20	21	1	2	1	0	0	0	2	2	2
20	21	1	0	2	0	0	0	2	2	2
20	38	2	2	2	0	0	0	2	2	2
20	56	0	0	0	0.4	0	0	0	1	1
20	66	0	0	0	1.2	0	0	1	2	2
21	57	0	0	0	1.2	2	2	2	3	3
22	23	1	1	1	0	1	2	2	3	3
22	37	2	0	0	0	2	0	2	2	2
22	58	0	2	2	1.3	0	3	2	3	3
23	24	1	0	0	0	0	1	1	2	2
24	25	1	0	0	0	0	0	0	0	0
24	43	0	1	1	1.3	0	0	1	2	2
25	26	2	0	0	0	0	0	0	0	0
25	26	3	0	0	0	0	0	0	0	0
25	55	0	3	3	2.7	4	4	4	5	5
26	27	2	0	0	0	0	0	0	0	0
26	27	3	0	0	0	0	0	0	0	0
26	29	1	2	2	0	0	0	2	2	2
26	54	0	0	0	1.1	1	0	1	2	2
27	28	3	0	0	0	0	0	0	0	0
27	35	2	0	0	0	0	0	0	0	0
27	53	1	0	1	0	1	0	1	2	2

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i	j	n_{ij}^0	DTM	DTM	LowEff	BKSa	BKSb	Opc1	Opc2	Opc3
28	35	3	0	0	0	0	0	0	0	0
29	30	1	2	2	0	1	0	2	3	3
30	31	1	0	0	0.3	2	1	2	2	2
30	63	0	0	0	1.8	2	2	2	2	2
31	34	1	0	0	0	0	0	0	1	0
32	33	0	0	0	0	0	0	0	0	0
33	67	0	0	0	0	0	0	0	0	0
34	39	2	0	0	0	0	0	0	1	0
34	39	2	0	0	0	0	0	0	1	0
34	41	2	0	0	0	0	0	0	1	0
35	46	4	0	0	0	0	1	1	2	2
35	47	2	0	0	1.0	0	0	1	1	1
35	51	3	0	0	0	2	2	2	3	3
36	39	2	0	0	0	1	0	1	2	2
36	46	2	0	1	0	3	2	3	4	4
39	42	1	0	1	0.8	0	1	1	1	1
39	86	0	4	3	2.3	0	2	4	4	4
40	45	1	2	1	1.7	2	1	2	3	3
40	46	3	0	2	0	0	2	2	3	3
41	64	0	2	2	2.5	2	2	3	3	3
42	44	2	1	0	0	0	1	1	2	2
42	85	2	0	0	0	0	0	0	1	1
43	55	0	2	2	0.5	2	2	2	3	3
43	58	0	2	2	0.9	2	2	2	3	3
44	46	3	0	0	0	0	0	0	1	1
47	48	2	0	0	1.0	0	0	1	1	1
48	49	1	2	1	2.2	1	1	2	3	3
48	50	2	0	0	0	0	0	0	1	1
48	51	2	0	0	0.7	0	0	1	1	1
49	50	1	3	4	2.5	4	4	4	5	5
51	52	2	0	0	0.7	0	0	1	1	1
52	59	1	1	1	1.5	1	1	2	2	2
53	54	0	0	0	0.4	0	0	0	1	1
53	70	0	0	0	0.2	0	0	0	0	0
53	76	0	0	0	0.4	0	0	0	1	1
53	86	0	1	1	1.1	0	1	1	2	2
54	55	0	0	0	1.1	0	0	1	1	0
54	58	0	0	0	0.3	1	1	1	2	2
54	63	0	0	0	1.1	1	1	1	2	2
54	70	0	0	0	0.5	0	0	1	0	0
54	79	0	0	0	1.3	0	0	1	1	0
56	57	0	0	0	0.7	1	1	1	2	2
58	78	0	0	0	0.7	0	0	1	2	2
60	66	0	0	0	0.7	0	0	1	0	0
60	87	0	0	0	0	0	0	0	0	0
61	64	0	1	1	0.2	1	0	1	2	2

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i	j	n_{ij}^0	DTM	DHM	LowEff	BKSa	BKSb	Opc1	Opc2	Opc3
61	85	0	2	2	3.4	3	2	3	4	4
61	86	0	0	0	0.4	0	1	0	2	2
62	67	0	0	0	0.3	0	0	0	0	0
62	68	0	0	0	0.3	0	0	0	0	0
62	72	0	0	0	0	0	0	0	0	0
63	64	0	0	0	0.6	0	1	1	2	2
65	66	0	0	0	0	0	0	0	0	0
65	87	0	0	0	0	0	0	0	0	0
67	68	0	1	0	0	0	1	1	2	2
67	69	0	1	1	0.5	2	1	2	2	2
67	71	0	3	0	0.9	3	3	3	4	4
68	69	0	0	0	0.4	0	0	0	1	0
68	83	0	0	0	1.3	0	0	1	2	2
68	87	0	0	0	0	0	0	0	0	0
69	87	0	0	1	0.3	1	1	1	2	2
70	82	0	0	0	0	0	0	0	0	0
71	72	0	1	0	0.2	1	1	1	2	2
71	75	0	0	0	0.3	0	0	0	1	1
71	83	0	0	0	0	0	0	0	0	0
72	73	0	1	0	0.4	1	1	1	2	2
72	83	0	0	0	0.2	0	0	0	0	0
73	74	0	2	1	0.7	2	2	2	3	2
73	75	0	1	1	0.3	1	1	1	2	2
73	84	0	0	0	0.2	0	0	0	1	0
74	84	0	0	0	0.7	0	0	1	1	0
75	76	0	0	1	0.3	0	0	1	2	2
75	81	0	1	2	0.3	1	1	2	2	2
75	82	0	0	0	0.4	0	0	0	2	2
75	83	0	0	0	0.3	0	0	0	1	1
76	77	0	0	1	0.4	0	0	1	2	2
76	82	0	0	0	0.2	0	0	0	2	1
76	84	0	0	0	0	0	0	0	0	0
77	79	0	0	0	0.2	0	0	0	2	2
77	84	0	0	0	0.5	0	0	1	1	1
78	79	0	0	0	0.3	0	0	0	2	2
78	80	0	0	0	0.6	0	0	1	2	2
79	82	0	0	0	0.4	0	0	0	2	2
80	81	0	0	0	0.4	0	0	0	2	2
80	82	0	0	0	0.3	0	0	0	0	0
80	83	0	0	0	0.4	0	0	0	1	1
81	83	0	0	0	0.3	0	0	0	0	0
82	84	0	0	0	0.1	0	0	0	0	0

Source: Created by the author.

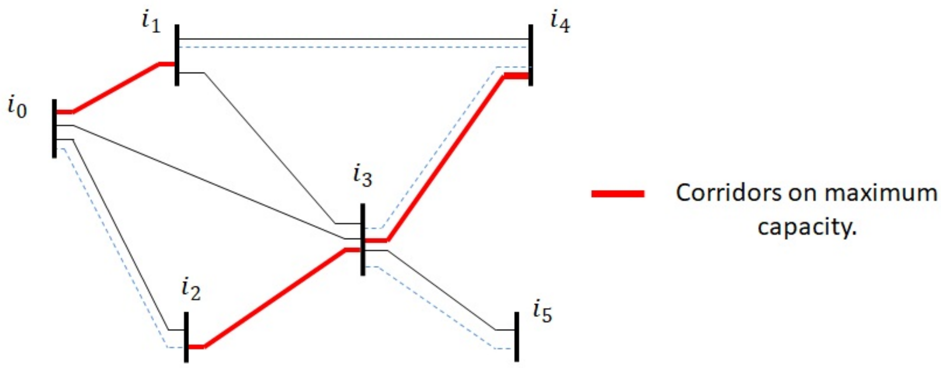
In Table 6 we present the corridors of the Brazilian North-northeasterntest system, the existing circuits n_{ij}^0, and the solutions obtained using the disjunctive transport model (DTM), The disjunctive hybrid model (DHM), the low effort method (LowEff) , the best known solution presented in Rahmani, Romero and Rider (2013) (BKSa), the best known solution presented in Escobar, Gallego and Toro (2010) (BKSb), and the maximum investment options are used in this work.

In Table 6, it is considered that some circuits must be retired from the analysis and not be part of the solution process of the method. There is a concern that the output of these circuits can cut the solution space and the optimal solution of the Brazilian North-northeastern System problem, the same problem as that of the GRASP -CP, as shown on Rahmani, Romero and Rider (2013). The analysis looks into every solution and if a circuit is used at least once, even when the minimum effort algorithm does not have an important flow on it, then this circuit is added with the option of maximum lines with the value on the solution it was found. However, if the circuit is not even used in the low effort, then this conclusion is made by the power flow found on it, in which case if it is lower than 0.5, it is considered that the system does not need it. In this case, the circuit will have a value of 0, thus showing that this circuit is out of the option. It is important to obtain different options where these same circuits have a quantitatively lower value than the most important or most used circuits for the system on the solution methods found to date for the Brazilian North-northeastern System, so the search space of the TEP problem is reduced, while taking care that the value of each line corresponds to the behavior that is present in all solutions, thus having a set of circuits in the system that will have values greater than 2, those being the most important circuits and a set of less important lines that will have an upper value of 1, these being these in the list to be excluded from the analysis.

5.4 IMPROVED DISJUNCTIVE TRANSPORT MODEL AND BASE CYCLES METHOD

When we use the improved disjunctive transport model (IDTM) and Base Cycles Method, the problem is solved with the transport model, with the purpose of finding all the corridors that present circuits with power flow in the maximum capacity. Using this information, the corridors that present this characteristic will represent the starting point in the analysis. First, we visualize which corridors have maximum active power flow in the results, found by solving the transport model. Then, these corridors are identified in the power system to define which corridors can close a minimum cycle with it, and lastly, with this cycle, new constraints are created to be added to the improved transport model. For example, in Figure 11, corridor (i_0, i_1) is at its capacity limit. Using it as a guide mark, the cycle corresponds to the corridors $(i_0, i_1), (i_1, i_3), (i_0, i_3)$, this will be the base to write the new constraint, if this is a minimum cycle: cycle with the link (i_0, i_1) and lower sum of the capacity-reactance products.

Figure 11 – Corridors in the maximum capacity.



Source: Created by the author.

5.4.1 Southern Brazilian test system

The IDTM and the base cycles are used in the Southern Brazilian test system. In this case, the number of critical cycles is 11. Table 7 shows the minimum cycles found to create the new constraints added to the model, implemented on AMPL and solved using the solver CPLEX. Cycle 1 presented in Table 7 involves two candidate lines and corresponds to a disjunctive constraint that can be obtained by combining the two parts, cycle 1a, and cycle 1b. When the methodology is applied, 11 cycles are found, and when added to the improved disjunctive transportation model it allows us to obtain the best-known solution for this system: US \$ 72,870x10^{6} with an execution time of 0.51 s and a computational effort of 482.74 ticks The linear disjunctive DC model obtained this solution in 1.17 s and with 923.46 ticks. It is important to highlight the improvement found when the SKL is replaced by a new set of constraints, finding the optimal global solution in less time and with less computational effort: 53.2 % of the linear disjunctive DC model. This can also provide in high complexity TEP problems without a known optimal global solution, by finding a locally optimal solution as a starting point to reduce the search space and finally ease the process to find better optimal solutions.

Table 7 – Critical cycles for the Southern Brazilian test system

cycle $1a^*$	$-\left(\frac{P^0_{19-46}}{n^0_{19-46}}\right)x_{19-46}-\left(\frac{P^0_{18-19}}{n^0_{18-19}}\right)x_{18-19}-\left(\frac{P^0_{13-18}}{n^0_{13-18}}\right)x_{13-18}-\left(\frac{P^0_{8-13}}{n^0_{8-13}}\right)x_{8-13}$ $-\left(\frac{P^0_{5-8}}{n^0_{5-8}}\right)x_{5-8}+\left(P_{5-6,1}\right)\cdot x_{5-6}+\left(P_{6-46,1}\right)\cdot x_{6-46}\leq\left(2-y_{5-6,1}-y_{6-46,1}\right)M$
cycle $1b^*$	$-\left(\frac{P^0_{19-46}}{n^0_{19-46}}\right)x_{19-46}-\left(\frac{P^0_{18-19}}{n^0_{18-19}}\right)x_{18-19}-\left(\frac{P^0_{13-18}}{n^0_{13-18}}\right)x_{13-18}-\left(\frac{P^0_{8-13}}{n^0_{8-13}}\right)x_{8-13}$ $-\left(\frac{P^0_{5-8}}{n^0_{5-8}}\right)x_{5-8}+\left(P_{5-6,1}\right)\cdot x_{5-6}+\left(P_{6-46,1}\right)\cdot x_{6-46}\geq\left(2-y_{5-6,1}-y_{6-46,1}\right)M$
cycle 2	$\left(\frac{P^0_{1-2}}{n^0_{1-2}}\right)x_{1-2}+\left(\frac{P^0_{2-5}}{n^0_{2-5}}\right)x_{2-5}+\left(\frac{P^0_{5-8}}{n^0_{5-8}}\right)x_{5-8}-\left(\frac{P^0_{7-8}}{n^0_{7-8}}\right)x_{7-8}-\left(\frac{P^0_{1-7}}{n^0_{1-7}}\right)x_{1-7}=0$
cycle 3	$\left(\frac{P^0_{4-5}}{n^0_{4-5}}\right)x_{4-5}+\left(\frac{P^0_{5-9}}{n^0_{5-9}}\right)x_{5-9}-\left(\frac{P^0_{4-9}}{n^0_{4-9}}\right)x_{4-9}=0$
cycle 4	$\left(\frac{P^0_{5-9}}{n^0_{5-9}}\right)x_{5-9}+\left(\frac{P^0_{9-14}}{n^0_{9-14}}\right)x_{9-14}+\left(\frac{P^0_{14-18}}{n^0_{14-18}}\right)x_{14-18}-\left(\frac{P^0_{13-18}}{n^0_{13-18}}\right)x_{13-18}+$ $-\left(\frac{P^0_{8-13}}{n^0_{8-13}}\right)x_{8-13}-\left(\frac{P^0_{5-8}}{n^0_{5-8}}\right)x_{5-8}=0$
cycle 5	$\left(\frac{P^0_{14-22}}{n^0_{14-22}}\right)x_{14-22}+\left(\frac{P^0_{22-26}}{n^0_{22-26}}\right)x_{22-26}-\left(\frac{P^0_{14-26}}{n^0_{14-26}}\right)x_{14-26}=0$
cycle 6	$\left(\frac{P^0_{16-17}}{n^0_{16-17}}\right)x_{16-17}+\left(\frac{P^0_{17-19}}{n^0_{17-19}}\right)x_{17-19}+\left(\frac{P^0_{19-46}}{n^0_{19-46}}\right)x_{19-46}-\left(\frac{P^0_{16-46}}{n^0_{16-46}}\right)x_{16-46}=0$
cycle 7	$\left(\frac{P^0_{13-18}}{n^0_{13-18}}\right)x_{13-18}+\left(\frac{P^0_{18-20}}{n^0_{18-20}}\right)x_{18-20}-\left(\frac{P^0_{13-20}}{n^0_{13-20}}\right)x_{13-20}=0$
cycle 8	$\left(\frac{P^0_{18-19}}{n^0_{18-19}}\right)x_{18-19}+\left(\frac{P^0_{19-21}}{n^0_{19-21}}\right)x_{19-21}-\left(\frac{P^0_{20-21}}{n^0_{20-21}}\right)x_{20-21}-\left(\frac{P^0_{18-20}}{n^0_{18-20}}\right)x_{18-20}=0$
cycle 9	$\left(\frac{P^0_{14-26}}{n^0_{14-26}}\right)x_{14-26}+\left(\frac{P^0_{26-27}}{n^0_{26-27}}\right)x_{26-27}+\left(\frac{P^0_{27-38}}{n^0_{27-38}}\right)x_{27-38}+\left(\frac{P^0_{38-42}}{n^0_{38-42}}\right)x_{38-42}+\left(\frac{P^0_{42-43}}{n^0_{42-43}}\right)x_{42-43}+$ $-\left(\frac{P^0_{32-43}}{n^0_{32-43}}\right)x_{32-43}-\left(\frac{P^0_{19-32}}{n^0_{19-32}}\right)x_{19-32}-\left(\frac{P^0_{18-19}}{n^0_{18-19}}\right)x_{18-19}-\left(\frac{P^0_{14-18}}{n^0_{14-18}}\right)x_{14-18}=0$
cycle 10	$\left(\frac{P^0_{24-34}}{n^0_{24-34}}\right)x_{24-34}-\left(\frac{P^0_{33-34}}{n^0_{33-34}}\right)x_{33-34}-\left(\frac{P^0_{24-33}}{n^0_{24-33}}\right)x_{24-33}=0$
cycle 11	$\left(\frac{P^0_{20-23}}{n^0_{20-23}}\right)x_{20-23}+\left(\frac{P^0_{23-24}}{n^0_{23-24}}\right)x_{23-24}+\left(\frac{P^0_{24-34}}{n^0_{24-34}}\right)x_{24-34}+\left(\frac{P^0_{34-35}}{n^0_{34-35}}\right)x_{34-35}+\left(\frac{P^0_{35-38}}{n^0_{35-38}}\right)x_{35-38}+$ $+\left(\frac{P^0_{14-18}}{n^0_{14-18}}\right)x_{14-18}+\left(\frac{P^0_{18-20}}{n^0_{18-20}}\right)x_{18-20}-\left(\frac{P^0_{27-38}}{n^0_{27-38}}\right)x_{27-38}-\left(\frac{P^0_{26-27}}{n^0_{26-27}}\right)x_{26-27}-\left(\frac{P^0_{14-26}}{n^0_{14-26}}\right)x_{14-26}=0$

Source: Created by the author.

5.4.2 Colombian test system

When applying the methodology to the Colombian system, using the IDTM and base cycles, the number of critical cycles increase to 50, which will be added to the model, later on. This process allows us to find the optimal solution of this system of $US\$562,417$. The execution time was 45.32 s with 46302.67 ticks. The traditional linear disjunctive DC model obtained this solution in 84.26 s and with 87211.64 ticks. It is important to highlight the improvement found when the SKL is replaced by a new set of constraints, finding the optimal global solution in less time and with less computational effort: 53.1% of the linear disjunctive DC model. Table 8 shows the minimum cycles found to create the new constraints added to the model. In this table we present the sequence of busbars for each cycle. The cycles 48, 49 and 50, use expansion corridors and require disjunctive restrictions.

Table 8 – Critical cycles for the Colombian test system.

Cycle 1	$\langle 27,44,80,27 \rangle$	Cycle 2	$\langle 27,35,44,27,44 \rangle$
Cycle 3	$\langle 29,31,72,73,74,64,29 \rangle$	Cycle 4	$\langle 29,31,72,30,64,29 \rangle$
Cycle 5	$\langle 30,64,65,30 \rangle$	Cycle 6	$\langle 1,93,92,11,1 \rangle$
Cycle 7	$\langle 1,8,59,1 \rangle$	Cycle 8	$\langle 8,9,77,79,87,8 \rangle$
Cycle 9	$\langle 2,9,83,2 \rangle$	Cycle 10	$\langle 31,33,34,31 \rangle$
Cycle 11	$\langle 31,32,34,31 \rangle$	Cycle 12	$\langle 31,72,73,62,60,31 \rangle$
Cycle 13	$\langle 31,60,69,70,34,31 \rangle$	Cycle 14	$\langle 13,14,18,20,13 \rangle$
Cycle 15	$\langle 18,21,22,18 \rangle$	Cycle 16	$\langle 16,18,21,16 \rangle$
Cycle 17	$\langle 18,22,19,58,18 \rangle$	Cycle 18	$\langle 15,76,17,23,24,15 \rangle$
Cycle 19	$\langle 13,23,24,15,20,13 \rangle$	Cycle 20	$\langle 15,76,17,15 \rangle$
Cycle 21	$\langle 45,50,54,45 \rangle$	Cycle 22	$\langle 19,82,62,60,69,66,19 \rangle$
Cycle 23	$\langle 37,61,68,37 \rangle$	Cycle 24	$\langle 19,61,68,86,19 \rangle$
Cycle 25	$\langle 1,3,71,1 \rangle$	Cycle 26	$\langle 2,9,69,70,34,4,2 \rangle$
Cycle 27	$\langle 9,69,60,62,82,85,83,9 \rangle$	Cycle 28	$\langle 27,28,29,27 \rangle$
Cycle 29	$\langle 2,4,5,6,3,71,8,9,2 \rangle$	Cycle 30	$\langle 1,59,67,68,61,19,82,85,91,92,93,1 \rangle$
Cycle 31	$\langle 1,8,71,1 \rangle$	Cycle 32	$\langle 31,33,72,31 \rangle$
Cycle 33	$\langle 45,50,54,56,81,45 \rangle$	Cycle 34	$\langle 18,66,19,22,18 \rangle$
Cycle 35	$\langle 27,28,29,64,27 \rangle$	Cycle 36	$\langle 2,4,5,6,10,78,7,90,3,71,8,9,83,2 \rangle$
Cycle 37	$\langle 8,9,83,85,82,19,61,68,67,59,8 \rangle$	Cycle 38	$\langle 19,82,55,62,60,69,66,19 \rangle$
Cycle 39	$\langle 15,20,18,21,16,23,24,15 \rangle$	Cycle 40	$\langle 14,60,62,73,72,31,14 \rangle$
Cycle 41	$\langle 4,34,31,29,27,35,36,4 \rangle$	Cycle 42	$\langle 1,3,90,91,92,93,1 \rangle$
Cycle 43	$\langle 12,75,24,15,76,12 \rangle$	Cycle 44	$\langle 25,28,27,29,25 \rangle$
Cycle 45	$\langle 26,28,29,64,27,26 \rangle$	Cycle 46	$\langle 15,24,23,16,21,18,15 \rangle$
Cycle 47	$\langle 13,14,60,62,82,19,22,18,20,13 \rangle$	Cycle 48*	$\langle 19,61,68,39,43,88,52,47,54,56,57,55,82,19 \rangle$
Cycle 49*	$\langle 56,57,81,56 \rangle$	Cycle 50*	$\langle 27,29,64,74,89,27 \rangle$

Source: Created by the author.

As a partial conclusion, the DC model is considered the ideal model to solve the TEP problem, which can be replaced by the improved disjunctive transportation model, plus the set of critical cycles of the system. In medium-size systems and great complexity systems, this shows to be a promising technique to reduce the computation time and effort.

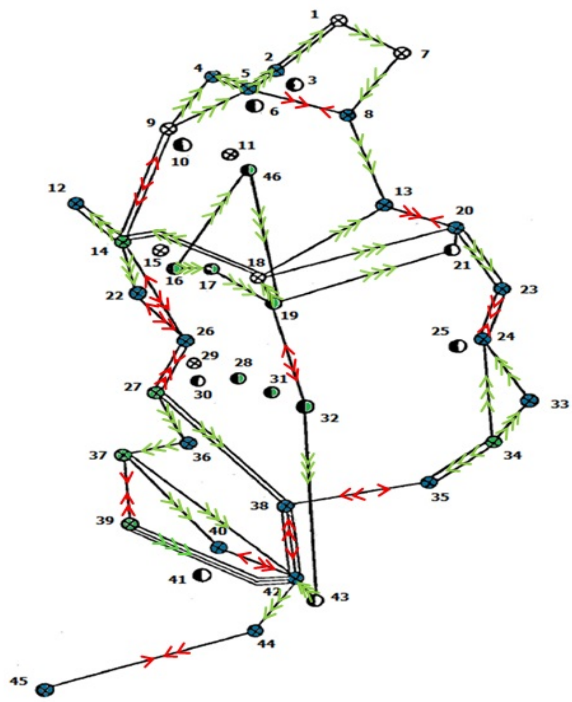
5.5 ANGULAR CUTS IN THE TRANSMISSION EXPANSION PLANNING PROBLEM

Since there are several types of representations of the angular cuts, and improved constraints, the test will be developed whith each one separately and then a small combination of them. First, the low-effort heuristic method and the improved transport disjunctive model is applied to have a guide for creating the cuts the solution flows from the chosen relaxations are then used to find single paths of same-direction flows of maximum length, using a breadth-first search algorithm. For larger instances, the maximum length of each path and the maximum number of paths starting from each bus are capped to prevent memory issues. Paths with the same initial and final bus are combined to form parallel paths. Once all or, in the case of the particularly large instances, the maximum allowed number of single paths and parallel paths are found, cuts are added to the model from those lists in a random order since the ordering of cuts has a strong impact on the solution time in CPLEX. In medium instances, we use the low-effort heuristic method, the improved transport disjunctive model and the improved disjunctive hybrid model to guide the process.

5.5.1 Southern Brazilian test system

The methodology is proven on the Southern Brazilian test system to validate the methodology, considering angular cuts in combination with the reduced linear disjunctive model implemented on AMPL and solved using the solver CPLEX. The system is solved with and without strong capacity-reactance constraints. Using the low-effort algorithm in combination with the improved disjunctive transport model and the improved disjunctive hybrid model, the active power flows are compared to find the possible flow behavior in the future network. If the three models present the same direction as the active power flows, this information will be used to create bus-angle difference cuts. On the corridors in which the 3 different solutions do not have the same directions, we consider that the future flow direction on the future transmission system is unknown, as seen in Figure 12.

Figure 12 – Resulting power flow directions for the Southern Brazilian test system.



Source: Created by the author.

The Southern Brazilian test system has a total of 10 bus-angle difference cuts, of which 4 shown better results and are presented in Tables 9 and 10.

Table 9 – Critical cycles found on the Southern Brazilian test system

Angular cut 1	$(\theta_2 - \theta_8) \leq \overline{P}_{1-2} x_{1-2} + \overline{P}_{1-7} x_{1-7} + \overline{P}_{7-8} x_{7-8}$
Angular cut 2	$(\theta_3 2 - \theta_{42}) \leq \bar{P}_{32-43} x_{32-43} + \overline{P}_{42-43} x_{42-43}$
Angular cut 3	$(\theta_{19} - \theta_{14}) \leq \overline{P}_{18-19} x_{18-19} + \overline{P}_{14-18} x_{14-18}$
Angular cut 4	$\frac{P^o_{33-34}}{n^o_{33-34}} x_{33-34} + \frac{P^o_{24-33}}{n^o_{24-33}} x_{24-33} \leq \overline{P}_{24-34} x_{24-34}$

Source: Created by the author.

Table 10 – Southern Brazilian test system with and without bus-angle difference cuts

	Ticks (units)	Ticks (%)	MIPs Simplex iterations	Branch & Bound nodes
Base Case (BC)	731,73	100	31389	1432
Base Case + constraint 2	629,57	86,1	26210	926
Base Case + constraint 1 and 2	491,74	67,2	19536	973
Base Case + constraint 3	437,57	**59,8**	12912	571
Base Case + constraint 4	544,47	74,4	18777	926

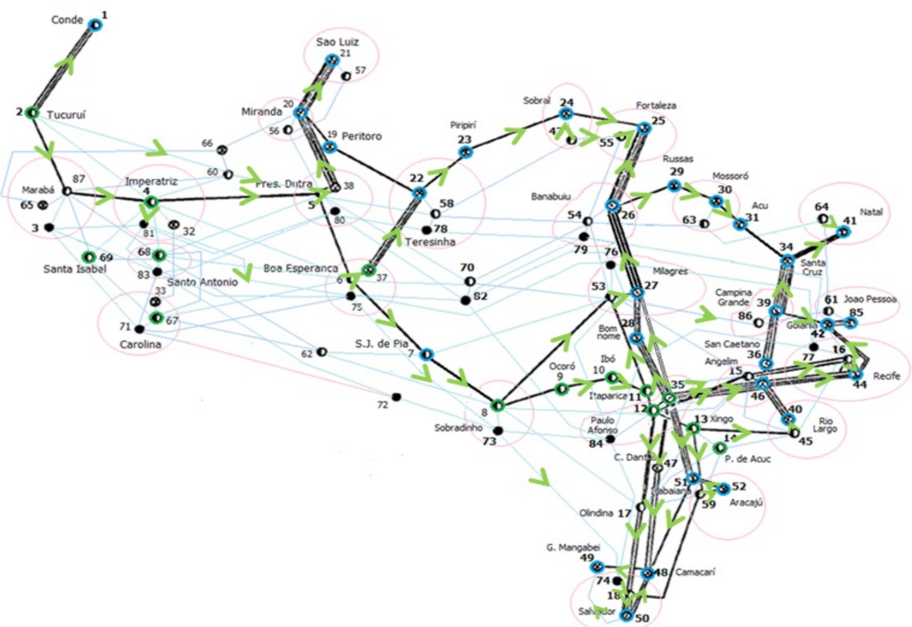
Source: Created by the author.

As a result, the base case is used to compare the traditional solution with the addition of the new constraints to the model, and then analyzed to verify if the results show a better behavior, as shown in the table below. Ticks is a measure of how much algorithmic work is required to obtain a provably optimum (Ticks) independently of the computer on which it is run as shown in the first part of Table 10, followed by the number of iterations and nodes created in the branch and cut process, which is the key point to verify the methodology. Constraint 1 limits the maximum angular distance between the buses 2 and 8 in all possible parallel connections with the series (1, 2), (1, 7) and (7, 8); constraint 2 limits the capacity-reactance sum of the circuits in corridors $(33-34)$ and $(24-33)$; constraint 3 fixes a lower bound for the angular distance of buses 32 and 42; and constraint 4 fixes a lower bound for the angular distance of buses 19 and 14. The base case (BC) resolved the MILP problem without strong capacity-reactance constraints. In the other cases, one or two strong constraints are added. In the cases with strong capacity-reactance constraints, the ticks are less than the base case. The most effective strong capacity-reactance constraint was constraint 3 which reduces the algorithmic work to 59,8%.

5.5.2 Brazilian North-northeastern test system

The Brazilian North-northeasternSystem is used as the second case study. This system consists of 87 buses and 183 circuits. The system data is provided in Appendix A. This system represents a benchmarkin the transmissionplanning problem due to its high complexity and the unknown global optimal solution. There are two levels of demand, one considered for 2002 (P1) with a level of 20316 MW and the other for 2008 (P2) with a level of 29748 MW. Considering the minimumeffort criterionshown in Chapter 3 in combinationwith the angular cuts explained in section 5.1, as for the study of the Southern Brazilian test using the 3 different solutions to find the behavior of the flow in the future as shown in Figure 13 the same flow directions are found for this test system. Twenty-two angular cuts were obteided by the end of the tests, 4 of them presented in Table 11 in combinationwith the reduced linear disjunctive model, find the best-known solution to the test system.

Figure 13 – Resulting power flow directions for the Brazilian North-northeasterntest system.



Source: Created by the author.

Table 11 – Critical cycles found on the North-northeasterntest system

Angular cut 1	$(\theta_2 - \theta_5) \leq \overline{P}_{2-87} x_{2-87} + \overline{P}_{4-87} x_{4-87} + \overline{P}_{4-5} x_{4-5}$
Angular cut 2	$(\theta_4 - \theta_{74}) \leq \bar{P}_{4-81} x_{4-81} + \overline{P}_{75-81} x_{75-81} + \overline{P}_{73-75} x_{73-75} + \overline{P}_{73-74} x_{73-74}$
Angular cut 3	$(\theta_{12} - \theta_{48}) \leq \overline{P}_{12-17} x_{12-7} + \overline{P}_{17-18} x_{17-18} + \overline{P}_{18-50} x_{18-50} + \overline{P}_{48-50} x_{48-50}$
Angular cut 4	$\frac{P^o_{6-7}}{n^o_{6-7}} x_{6-7} + \frac{P^o_{7-8}}{n^o_{7-8}} x_{7-8} + \frac{P^o_{8-53}}{n^o_{8-53}} x_{8-53} + \frac{P^o_{27-53}}{n^o_{27-53}} x_{27-53} + \frac{P^o_{26-27}}{n^o_{26-27}} x_{26-27} + \frac{P^o_{25-26}}{n^o_{25-26}} x_{25-26}$ $\leq \overline{P}_{6-37} x_{6-37} + \overline{P}_{22-37} x_{22-37} + \overline{P}_{22-23} x_{22-23} + \overline{P}_{23-24} x_{23-24} + \overline{P}_{24-25} x_{24-25}$

Source: Created by the author.

It is important to highlight that the best known solution for the Brazilian north-northeastern system, reported in Rahmani, Romero and Rider (2013), has a total cost of US$2,546,417x10 [3] while the solution obtained in this work requires an investment of US$2,538,075x10 [3], which is US$8,348x10 [3] less, and it proposes the following additions:

$n_{01-02} = 1$	$n_{02-87} = 1$	$n_{04-05} = 3$	$n_{04-81} = 3$	$n_{05-06} = 1$	$n_{05-58} = 3$
$n_{05-68} = 1$	$n_{06-37} = 1$	$n_{12-15} = 1$	$n_{13-15} = 4$	$n_{14-59} = 1$	$n_{15-16} = 4$
$n_{15-46} = 1$	$n_{16-44} = 6$	$n_{16-61} = 1$	$n_{18-50} = 11$	$n_{18-74} = 6$	$n_{21-57} = 2$
$n_{22-37} = 1$	$n_{24-43} = 2$	$n_{25-55} = 4$	$n_{26-29} = 2$	$n_{29-30} = 2$	$n_{34-39} = 1$
$n_{35-51} = 1$	$n_{36-46} = 1$	$n_{39-86} = 3$	$n_{40-45} = 1$	$n_{40-46} = 2$	$n_{41-64} = 2$
$n_{42-44} = 2$	$n_{42-85} = 1$	$n_{43-55} = 3$	$n_{43-58} = 3$	$n_{49-50} = 5$	$n_{52-59} = 1$
$n_{53-86} = 1$	$n_{56-57} = 1$	$n_{61-64} = 1$	$n_{61-85} = 2$	$n_{67-69} = 2$	$n_{67-71} = 3$
$n_{69-87} = 2$	$n_{71-72} = 1$	$n_{72-73} = 1$	$n_{73-74} = 2$	$n_{73-75} = 1$	$n_{75-81} = 1$

Source: Created by the author.

The solution found in Rahmani, Romero and Rider (2013) has a gap of 5% from the optimal solution after the program ran for 15 days. The optimal solution was found after 55 hours considering the full space of the problem. However, the CPLEX branch and bound could not find a better solution. The investment found in this work was found with a gap of 0%and the process concluded in a total of 4.51 hours, considering the full space of the problem. Although the new solution found on this work is better and the program is able to close the process with a gap of 0%, it is not possible to ensure that the new solution is the global optimal solution of the Brazilian North-northeasternsystem, since the low effort method remove some lines from the maximum corridor parameter. The test does not use the GRASP-CP method used on Rahmani (2013), in this work the minimummethod in combination with other solutions found for this test system is used to find the best amount of maximum lines used in every corridor. when one of the two method are not used this causes the Brazilian north-northeasternsystem to do not have an investment solution, ending the process with the Termination code 9,an error on the solver when the memory is insufficient. It is important to understand that this process was run on the LAPSEE server with a total of 35 threads which means that, in this study case, the use of the angular cuts not only finds a better solution in a shorter amount of time but it also is the only way the model in combination with the solver is able to close with a 0% gap this can be observed in the test performed for the Southern Brazilian test system(BNN) where the number of ticks or the algorithmic work required to obtain a provable optimum (Ticks) is significantly reduced when using the angular cuts methodology.

Table12 – Brazilian North-northeasterntest system with strong capacity-reactance constraints

System	Time (hr)	Time BKS (hr)	Angular Cuts (Active)	Cost $x10^3$	Best Known $x10^3$ US$
BNN	**4,51**	55	4	**2,538,075**	2,546,417

Source: Created by the author.

5.6 VALID INEQUALITIES WITH POWER FLOWS IN THE TRANSMISSION EXPANSION PLANNING PROBLEM

In this section we present the test using the valid inequality or bus-angle difference cuts type 1, 2, 3, and 4, using the formulation based in active power flows only. The test will be developed whith each one separately and then a combination of them. First, the low-effort method, the DTM and the DHM are applied as a guide to create the cuts.

5.6.1 Southern Brazilian test system

The test was made using the valid inequalities with the different types or, as they are typically referred to, as cutting planes. We tested several variations on the number of cuts added and type of cuts before solving. Some tests were made manually this leads the process to take several months to be accomplished. The cases considered were:

- Adding all possible cuts.
- Adding at most 3 of each cut using only existing lines (using Theorems 1 and 3).

- Adding at most 3 of each cut using at least one candidate line (using Theorems 2 and 4).
- Adding at most 3 of each cut using only single paths.
- Adding at most 3 of each cut using only parallel paths.

The results are analyzed primarily by comparing the average solve time of each case with that of the model with no cuts added. The average computational time for all the cases was 5.9 seconds, this includes the time needed to apply the low-effort heuristic, and to solve TEP with the added cuts, compared to 6.19 seconds on average to solve the original model. Table 13 summarizes the distribution in the number of cuts found per type according to which combination of relaxation models was utilized in the low-effort heuristic. Since in some cases some of the added valid inequalities increase the computational effort by adding more points to be analyzed when the search space is cut, less cuts were added. This was the impetus to run four different test cases with a maximum of three cuts per type. To show these results, the configuration denoted $(3,3,0,0)$ represents a maximum of three cuts each based on Types 1 and 2, with no cuts added from Types 3 and 4. Similarly, the other tests were made with the configurations $(0,0,3,3),(3,0,3,0), and (0,3,0,3)$. as can be found in Table 13.

Table 13 – Comparative Results for each Configuration

Configuration	Type Used	Average Over All Low-effort Heuristic Models			
	(1,2,3,4)	Solution	Path Search	Relax Time	C+P+R
I	(3,3,0,0)	5.19	0.01	1.55	6.75
II	(0,0,3,3)	3.83	0.01	1.34	5.17
III	(3,0,3,0)	4.15	0.01	1.34	5.50
IV	(0,3,0,3)	3.79	0.01	1.34	**5.13**

Source: Created by the author.

We analyze the solution times found after solving all three relaxation models for the low-effort heuristic. Configuration IV produced the greatest improvement in solution time, 5.13 seconds, a 1.06 second reduction from the original solve time. Configuration II produced a comparable reduction, a 5.17 second solve time, only .04 seconds worse than the best case. It is important to note that, the trade-off for each cut added using Type 3 or 4: an additional binary variable. The increase in the number of binary variables may reduce the effectiveness of those cuts. In fact, the only configuration that did not produce improvement was configuration I, which uses only Types 1 and 2 valid inequalities.

6 CONCLUSIONS & FUTURE WORKS

This chapter presents conclusive remarks regarding the new methods and improved models presented. Besides, new lines of research are suggested to continue in the improvement of the Valid inequalities, cycles method and new mathematical representation.

6.1 CONCLUSIONS

This work presents a new mathematical framework that uses a mixed-integer linear programming model, valid inequalities, improved models and a low-effort heuristic method for solving TEP. The objective is to reduce the total computational effort of planning. This work shows a significant improvement of the preliminary studies carried out in previously, in which the solutions were found after manual analysis of the test system, creation of cuts using two of the valid inequalities introduced in this work (specifically from Type 1 and 2), which at that time had had a basic structure as angular cuts, and tests made with different cut combinations. However, this work not only shows an improvement on the computational effort when using these new constraints, but also an improvement of the traditional models building the path to find better solutions. The methodology based on valid inequalities finds high quality solutions that improve the BKS published in the specialized literature for the Brazilian north-northeast system, with 0% of gap, which shows an important improvement for this type of instances, since this gap has been searched for years with different methods and models.

The proposed new relaxed models help reduce the solution space without eliminating the best known solutions for the test systems. The critical-cycle method in combination with the improved disjunctive transport model finds the optimal solution of the test systems used without using Kirchhoff's second law and with a shorter computation time.

Computational tests show the effectiveness of the 3 methods and improved models in reducing the solution time of TEP, as the improved disjunctive transportation model and the improved disjunctive hybrid model represent a novel idea to describe the TEP with out the complications of using Kirchhoff's Second Law with great results. This lead to the conclusion of the DC model considered as the ideal model to solve the TEP problem, can be replaced by the improved disjunctive transportation model, plus the set of critical cycles of the system. In medium size systems and great complexity systems, this shows to be a promising technique to reduce the computation time and effort.

6.2 FUTURE WORKS

In future work, we will conduct further studies to determine the most effective use of the Cycles method in combination with the improve models. As the size of a system increases, the number of critic cycles increases at an exponential rate. Finding and adding all these inequalities takes significant time, although the use of the new models can lead to new optimal solution and possible global solutions for big size systems that do not have a know global solution. Thus, additional testing is planned to determine how to select an ideal subset of single path and parallel path inequalities to help decrease total solution time, particularly in large systems.

Some specific ideas for future works are as follows:

1. Apply the ideas of valid inequalities and critical cycles in problems related to the transmission expansion planning on networks that include contingencies, energy losses and multistage planning.

2. Automate the process of creating valid inequalities and critical cycles, since the process takes a lot of manual effort and time, and the test shown a great improvement with high complexity problems.

3. Improve the idea of cycles for improved models, especially the improved disjunctive hybrid.

The original idea of angular cuts has also been the starting point for the development of new algorithms on other thesis and papers, as is the case for the student J. Kyle Skolfield were the valid inequalities ideas will be mathematically analyzed and proven on his thesis in the future.

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APPENDIX A -- SYSTEMS DATA FOR THE TEP PROBLEM

A.1 SOUTHERN BRAZILIAN SYSTEM DATA

Table 14 – Southern Brazilian bus system data

Node	Generation (MW)	Demand (MW)
1	0	0
2	0	443,1
3	0	0
4	0	300,7
5	0	238
6	0	0
7	0	0
8	0	72,2
9	0	0
10	0	0
11	0	0
12	0	511,9
13	0	185,8
14	1257	0
15	0	0
16	2000	0
17	1050	0
18	0	0
19	1670	0

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Node	Generation	Demand
	(MW)	(MW)
20	0	1091,2
21	0	0
22	0	81,9
23	0	458,1
24	0	478,2
25	0	0
26	0	231,9
27	220	0
28	800	0
29	0	0
30	0	0
31	700	0
32	500	0
33	0	229,1
34	748	0
35	0	216
36	0	90,1
37	300	0
38	0	216
39	600	0
40	0	262,1
41	0	0
42	0	1607,9
43	0	0
44	0	79,1
45	0	86,7
46	700	0

Source: LAPSEE 2012.

Table 15 – Southern Brazilian system lines data

node i	node j	n_{ij}^0	x_{ij} (p.u.)	Capacity (MW)	Cost
1	7	1	0,0616	270	4349
1	2	2	0,1065	270	7076
4	9	1	0,0924	270	6217
5	9	1	0,1173	270	7732
5	8	1	0,1132	270	7480
7	8	1	0,1023	270	6823
4	5	2	0,0566	270	4046
2	5	2	0,0324	270	2581
8	13	1	0,1348	240	8793
9	14	2	0,1756	220	11267
12	14	2	0,074	270	5106
14	18	2	0,1514	240	9803
13	18	1	0,1805	220	11570
13	20	1	0,1073	270	7126
18	20	1	0,1997	200	12732
19	21	1	0,0278	1500	32632
16	17	1	0,0078	2000	10505
17	19	1	0,0061	2000	8715
14	26	1	0,1614	220	10409
14	22	1	0,084	270	5712
22	26	1	0,079	270	5409
20	23	2	0,0932	270	6268
23	24	2	0,0774	270	5308
26	27	2	0,0832	270	5662
24	34	1	0,1647	220	10611
24	33	1	0,1448	240	9399
33	34	1	0,1265	270	8288
27	36	1	0,0915	270	6167
27	38	2	0,208	200	13237
36	37	1	0,1057	270	7025
34	35	2	0,0491	270	3591
35	38	1	0,198	200	12631

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node i	node j	n_{ij}^0	x_{ij} (p.u.)	Capacity (MW)	Cost
37	39	1	0,0283	270	2329
37	40	1	0,1281	270	8389
37	42	1	0,2105	200	13388
39	42	3	0,203	200	12934
40	42	1	0,0932	270	6268
38	42	3	0,0907	270	6116
32	43	1	0,0309	1400	35957
42	44	1	0,1206	270	7934
44	45	1	0,1864	200	11924
19	32	1	0,0195	1800	23423
46	19	1	0,0222	1800	26365
46	16	1	0,0203	1800	24319
18	19	1	0,0125	600	8178
20	21	1	0,0125	600	8178
42	43	1	0,0125	600	8178
2	4	0	0,0882	270	5965
14	15	0	0,0374	270	2884
46	10	0	0,0081	2000	10889
4	11	0	0,2246	240	14247
5	11	0	0,0915	270	6167
46	6	0	0,0128	2000	16005
46	3	0	0,0203	1800	24319
16	28	0	0,0222	1800	26365
16	32	0	0,0311	1400	36213
17	32	0	0,0232	1700	27516
19	25	0	0,0325	1400	37748
21	25	0	0,0174	2000	21121
25	32	0	0,0319	1400	37109
31	32	0	0,0046	2000	7052
28	31	0	0,0053	2000	7819
28	30	0	0,0058	2000	8331

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node i	node j	n_{ij}^0	x_{ij} (p.u.)	Capacity (MW)	Cost
27	29	0	0,0998	270	6672
26	29	0	0,0541	270	3894
28	41	0	0,0339	1300	39283
28	43	0	0,0406	1200	46701
31	41	0	0,0278	1500	32632
32	41	0	0,0309	1400	35957
41	43	0	0,0139	2000	17284
40	45	0	0,2205	180	13994
15	16	0	0,0125	600	8178
46	11	0	0,0125	600	8178
24	25	0	0,0125	600	8178
29	30	0	0,0125	600	8178
40	41	0	0,0125	600	8178
2	3	0	0,0125	600	8178
5	6	0	0,0125	600	8178
9	10	0	0,0125	600	8178

Source: LAPSEE 2012.

A.2 BRAZILIAN NORTH-NORTHEAST TEST SYSTEM DATA

Table 16 – Brazilian north-northeastsystem bus data

Bus Number	Type	Load	Generation	Load	Generation
		(2002)	(2002)	(2008)	(2008)
1	0	1857	0	2747	0
2	1	0	4048	0	4550
3	0	0	0	0	0
4	1	0	517	0	6422
5	0	0	0	0	0
6	0	0	0	0	0
7	0	31	0	31	0
8	1	0	403	0	82
9	1	0	465	0	465
10	1	0	538	0	538
11	1	0	2200	0	2260
12	1	0	2257	0	4312
13	2	0	4510	0	5900
14	1	0	542	0	542
15	0	0	0	0	0
16	0	0	0	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	86	0	125	0
20	0	125	0	181	0
21	0	722	0	1044	0
22	0	291	0	446	0
23	0	58	0	84	0
24	0	159	0	230	0
25	0	1502	0	2273	0
26	0	47	0	68	0
27	0	378	0	546	0
28	0	189	0	273	0
29	0	47	0	68	0

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Bus Number	Type	Load	Generation	Load	Generation
30	0	189	0	273	0
31	0	110	0	225	0
32	0	0	0	0	0
33	0	0	0	0	0
34	0	28	0	107	0
35	1	0	1635	0	1531
36	0	225	0	325	0
37	1	0	169	0	114
38	0	0	0	0	0
39	0	186	0	269	0
40	0	1201	0	1738	0
41	0	520	0	752	0
42	0	341	0	494	0
43	0	0	0	0	0
44	0	4022	0	5819	0
45	0	0	0	0	0
46	0	205	0	297	0
47	0	0	0	0	0
48	0	347	0	432	0
49	0	777	0	1124	0
50	0	5189	0	7628	0
51	0	290	0	420	0
52	0	707	0	1024	0
53	0	0	0	0	0
54	0	0	0	0	0
55	0	0	0	0	0
56	0	0	0	0	0
57	0	0	0	0	0
58	0	0	0	0	0
59	0	0	0	0	0
60	0	0	0	0	0
61	0	0	0	0	0

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Bus Number	Type	Load	Generation	Load	Generation
62	0	0	0	0	0
63	0	0	0	0	0
64	0	0	0	0	0
65	0	0	0	0	0
66	0	0	0	0	0
67	1	0	1242	0	1242
68	1	0	888	0	888
69	1	0	902	0	902
70	0	0	0	0	0
71	0	0	0	0	0
72	0	0	0	0	0
73	0	0	0	0	0
74	0	0	0	0	0
75	0	0	0	0	0
76	0	0	0	0	0
77	0	0	0	0	0
78	0	0	0	0	0
79	0	0	0	0	0
80	0	0	0	0	0
81	0	0	0	0	0
82	0	0	0	0	0
83	0	0	0	0	0
84	0	0	0	0	0
85	0	487	0	705	0
86	0	0	0	0	0
87	0	0	0	0	0

Source: Mohsen 2013.

Table 17 – Brazilian north-northeastsystem lines data

circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^0	Capacity (MW)	Cost $(US\$ \times 10^3)$	n_{max-ij}
1	1	2	0.0374	2	1000	44056	16
2	2	4	0.0406	0	1000	48880	16
3	2	60	0.0435	0	1000	52230	16
4	2	87	0.0259	1	1000	31192	16
5	3	71	0.0078	0	3200	92253	16
6	3	81	0.0049	0	3200	60153	16
7	3	83	0.0043	0	3200	53253	16
8	3	87	0.0058	0	1200	21232	16
9	4	5	0.0435	1	1000	52230	16
10	4	6	0.0487	0	1000	58260	16
11	4	32	0.0233	0	300	7510	16
12	4	60	0.0215	0	1000	26770	16
13	4	68	0.007	0	1000	10020	16
14	4	69	0.0162	0	1000	20740	16
15	4	81	0.0058	0	1200	21232	16
16	4	87	0.0218	1	1000	26502	16
17	5	6	0.0241	1	1000	29852	16
18	5	38	0.0117	2	600	8926	16
19	5	56	0.0235	0	1000	29182	16
20	5	58	0.022	0	1000	27440	16
21	5	60	0.0261	0	1000	32130	16
22	5	68	0.0406	0	1000	48880	16
23	5	70	0.0464	0	1000	55580	16
24	5	80	0.0058	0	1200	21232	16
25	6	7	0.0288	1	1000	35212	16
26	6	37	0.0233	1	300	7510	16
27	6	67	0.0464	0	1000	55580	16
28	6	68	0.0476	0	1000	56920	16
29	6	70	0.0371	0	1000	44860	16
30	6	75	0.0058	0	1200	21232	16
31	7	8	0.0234	1	1000	29048	16

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circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^0	Capacity (MW)	Cost $(US\$ \times 10^3)$	n_{max-ij}
32	7	53	0.0452	0	1000	54240	16
33	7	62	0.0255	0	1000	31460	16
34	8	9	0.0186	1	1000	23420	16
35	8	12	0.0394	0	1000	47540	16
36	8	17	0.0447	0	1000	53570	16
37	8	53	0.0365	1	1200	44190	16
38	8	62	0.0429	0	1000	51560	16
39	8	73	0.0058	0	1200	21232	16
40	9	10	0.0046	1	1000	7340	16
41	10	11	0.0133	1	1000	17390	16
42	11	12	0.0041	1	1200	6670	16
43	11	15	0.0297	1	1200	36284	16
44	11	17	0.0286	1	1200	35078	16
45	11	53	0.0254	1	1000	31326	16
46	12	13	0.0046	1	1200	7340	16
47	12	15	0.0256	1	1200	31594	16
48	12	17	0.0246	1	1200	30388	16
49	12	35	0.0117	2	600	8926	16
50	12	84	0.0058	0	1200	21232	16
51	13	14	0.0075	0	1200	10690	16
52	13	15	0.0215	0	1200	26770	16
53	13	17	0.0232	0	1200	28780	16
54	13	45	0.029	1	1200	35480	16
55	13	59	0.0232	1	1200	28780	16
56	14	17	0.0232	0	1200	28780	16
57	14	45	0.0232	0	1200	28780	16
58	14	59	0.0157	0	1200	20070	16
59	15	16	0.0197	2	1200	24760	16
60	15	45	0.0103	0	1200	13906	16
61	15	46	0.0117	1	600	8926	16
62	15	53	0.0423	0	1000	50890	16
63	16	44	0.0117	4	600	8926	16

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circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^0	Capacity (MW)	Cost $(US\$ \times 10^3)$	n_{max-ij}
64	16	45	0.022	0	1200	27440	16
65	16	61	0.0128	0	1000	16720	16
66	16	77	0.0058	0	1200	21232	16
67	17	18	0.017	2	1200	21678	16
68	17	59	0.017	0	1200	21678	16
69	18	50	0.0117	4	600	8926	16
70	18	59	0.0331	1	1200	40170	16
71	18	74	0.0058	0	1200	21232	16
72	19	20	0.0934	1	170	5885	16
73	19	22	0.1877	1	170	11165	16
74	20	21	0.0715	1	300	6960	16
75	20	21	0.1032	1	170	6435	16
76	20	38	0.1382	2	300	12840	16
77	20	56	0.0117	0	600	8926	16
78	20	66	0.2064	0	170	12210	16
79	21	57	0.0117	0	600	8926	16
80	22	23	0.1514	1	170	9130	16
81	22	37	0.2015	2	170	11935	16
82	22	58	0.0233	0	300	7510	16
83	23	24	0.1651	1	170	9900	16
84	24	25	0.2153	1	170	12705	16
85	24	43	0.0233	0	300	7510	16
86	25	26	0.1073	2	300	29636	16
87	25	26	0.1691	3	170	10120	16
88	25	55	0.0117	0	600	8926	16
89	26	27	0.1404	2	300	25500	16
90	26	27	0.2212	3	170	12760	16
91	26	29	0.1081	1	170	6710	16
92	26	54	0.0117	0	600	8926	16
93	27	28	0.0826	3	170	5335	16
94	27	35	0.1367	2	300	25000	16

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circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^0	Capacity (MW)	Cost ($US\$ \times 10^3$)	n_{max-ij}
95	27	53	0.0117	1	600	8926	16
96	28	35	0.1671	3	170	9900	16
97	29	30	0.0688	1	170	4510	16
98	30	31	0.0639	1	170	4235	16
99	30	63	0.0233	0	300	7510	16
100	31	34	0.1406	1	170	8525	16
101	32	33	0.1966	0	170	11660	16
102	33	67	0.0233	0	300	7510	16
103	34	39	0.116	2	170	7150	16
104	34	39	0.2968	2	80	6335	16
105	34	41	0.0993	2	170	6215	16
106	35	46	0.2172	4	170	12705	16
107	35	47	0.1327	2	170	8085	16
108	35	51	0.1602	3	170	9625	16
109	36	39	0.1189	2	170	7315	16
110	36	46	0.0639	2	170	4235	16
111	39	42	0.0973	1	170	6105	16
112	39	86	0.0233	0	300	7510	16
113	40	45	0.0117	1	600	8926	16
114	40	46	0.0875	3	170	5500	16
115	41	64	0.0233	0	300	7510	16
116	42	44	0.0698	2	170	4565	16
117	42	85	0.0501	2	170	3465	16
118	43	55	0.0254	0	1000	31326	16
119	43	58	0.0313	0	1000	38160	16
120	44	46	0.1671	3	170	10010	16
121	47	48	0.1966	2	170	11660	16
122	48	49	0.0757	1	170	4895	16
123	48	50	0.0256	2	170	2090	16
124	48	51	0.2163	2	170	12760	16
125	49	50	0.0835	1	170	5335	16
126	51	52	0.056	2	170	3795	16

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circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^0	Capacity (MW)	Cost $(US\$ \times 10^3)$	n_{max-ij}
127	52	59	0.0117	1	600	8926	16
128	53	54	0.027	0	1000	32120	16
129	53	70	0.0371	0	1000	44860	16
130	53	76	0.0058	0	1200	21232	16
131	53	86	0.0389	0	1000	46870	16
132	54	55	0.0206	0	1000	25028	16
133	54	58	0.051	0	1000	60940	16
134	54	63	0.0203	0	1000	25430	16
135	54	70	0.036	0	1000	43520	16
136	54	79	0.0058	0	1200	21232	16
137	56	57	0.0122	0	1000	16050	16
138	58	78	0.0058	0	1200	21232	16
139	60	66	0.0233	0	300	7510	16
140	60	87	0.0377	0	1000	45530	16
141	61	64	0.0186	0	1000	23420	16
142	61	85	0.0233	0	300	7510	16
143	61	86	0.0139	0	1000	18060	16
144	62	67	0.0464	0	1000	55580	16
145	62	68	0.0557	0	1000	66300	16
146	62	72	0.0058	0	1200	21232	16
147	63	64	0.029	0	1000	35480	16
148	65	66	0.3146	0	170	18260	16
149	65	87	0.0233	0	300	7510	16
150	67	68	0.029	0	1000	35480	16
151	67	69	0.0209	0	1000	26100	16
152	67	71	0.0058	0	1200	21232	16
153	68	69	0.0139	0	1000	18060	16
154	68	83	0.0058	0	1200	21232	16
155	68	87	0.0186	0	1000	23240	16
156	69	87	0.0139	0	1000	18060	16
157	70	82	0.0058	0	1200	21232	16
158	71	72	0.0108	0	3200	125253	16

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circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^{0}	Capacity (MW)	Cost ($US\$ \times 10^3$)	n_{max-ij}
159	71	75	0.0108	0	3200	125253	16
160	71	83	0.0067	0	3200	80253	16
161	72	73	0.01	0	3200	116253	16
162	72	83	0.013	0	3200	149253	16
163	73	74	0.013	0	3200	149253	16
164	73	75	0.013	0	3200	149253	16
165	73	84	0.0092	0	3200	107253	16
166	74	84	0.0108	0	3200	125253	16
167	75	76	0.0162	0	3200	185253	16
168	75	81	0.0113	0	3200	131253	16
169	75	82	0.0086	0	3200	101253	16
170	75	83	0.0111	0	3200	128253	16
171	76	77	0.013	0	3200	149253	16
172	76	82	0.0086	0	3200	101253	16
173	76	84	0.0059	0	3200	70953	16
174	77	79	0.0151	0	3200	173253	16
175	77	84	0.0115	0	3200	132753	16
176	78	79	0.0119	0	3200	137253	16
177	78	80	0.0051	0	3200	62253	16
178	79	82	0.0084	0	3200	98253	16
179	80	81	0.0101	0	3200	117753	16
180	80	82	0.0108	0	3200	125253	16
181	80	83	0.0094	0	3200	110253	16
182	81	83	0.0016	0	3200	23253	16
183	82	84	0.0135	0	3200	155253	16

Source: Mohsen 2013.

A.3 COLOMBIAN TEST SYSTEM

Table 18 – Colombian test system bus data

Bus number	Load (MW)	Generation (MW)
1	0.00	241
2	486.66	165
3	587.08	0
4	0	0
5	351.42	40
6	0	34
7	448.03	136
8	505.87	230
9	519.69	0
10	88.84	0
11	220.15	108
12	0	47
13	260.08	0
14	0	0
15	562.84	0
16	351.9	0
17	203	35
18	54.1	539
19	29.28	1340
20	302.27	45
21	277.44	0
22	79.17	200
23	302.27	0
24	0	150
25	0	86
26	0	70
27	396.71	0
28	486.39	14
29	505.96	618
30	199.55	0

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Bus number	Load (MW)	Generation (MW)
31	391.88	189
32	188.33	0
33	247.24	0
34	115.81	0
35	256.86	200
36	167.29	44
37	176.3	138
38	129.72	15
39	268.19	15
40	0	305
41	81.85	100
42	152.39	0
43	52.9	0
44	384.64	23
45	0	1208
46	181.62	150
47	61.6	0
48	896.26	885
49	193.27	0
50	632.75	240
51	190.45	0
52	55.6	0
53	0	320
54	114.19	0
55	333.59	40
56	0	0
57	336.94	130
58	0	190
59	0	160
60	0	1216
61	0	155
62	0	0

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Bus number	Load (MW)	Generation (MW)
63	52.77	1090
64	132.35	280
65	197.58	0
66	0	300
67	397.98	474
68	0	0
69	106.61	0
70	0	180
71	471.21	424
72	0	0
73	0	0
74	0	0
75	0	0
76	0	40
77	82.85	0
78	54.07	0
79	146.87	300
80	88.34	0
81	0	0
82	0	0
83	0	0
84	0	500
85	0	0
86	0	850
87	0	0
88	0	300
89	0	0
90	0	0
91	0	0
92	0	0
93	0	0

Source: Escobar 2012.

Table 19 – Colombian test system lines data

circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^0	Capacity (MW)	Cost $(US\$ \times 10^6)$	n_{max-ij}
1	52	88	0.098	0	300	34.19	5
2	43	88	0.1816	0	250	39.56	5
3	57	81	0.0219	0	550	58.89	5
4	73	82	0.0374	0	550	97.96	5
5	27	89	0.0267	0	450	13.27	5
6	74	89	0.0034	0	550	14.57	5
7	73	89	0.0246	0	550	66.65	5
8	79	83	0.0457	0	350	15.4	5
9	8	67	0.224	0	250	29.2	5
10	39	86	0.0545	0	350	9.88	5
11	25	28	0.0565	1	320	9.77	5
12	25	29	0.057	1	320	9.88	5
13	13	14	0.0009	2	350	3.9	5
14	13	20	0.0178	1	350	5.74	5
15	13	23	0.0277	1	350	7.01	5
16	14	31	0.1307	2	250	18.62	5
17	14	18	0.1494	2	250	20.23	5
18	14	60	0.1067	2	300	15.98	5
19	2	4	0.0271	2	350	6.66	5
20	2	9	0.0122	1	350	5.28	5
21	2	83	0.02	1	570	5.97	5
22	9	83	0.02	1	400	5.97	5
23	15	18	0.0365	1	450	7.93	5
24	15	17	0.0483	1	320	9.42	5
25	15	20	0.0513	1	320	9.65	5
26	15	76	0.0414	1	320	9.88	5
27	15	24	0.0145	1	350	5.28	5
28	37	61	0.0139	1	350	4.94	5
29	19	61	0.1105	2	250	16.09	5
30	61	68	0.0789	1	250	12.41	5
31	37	68	0.0544	1	320	9.65	5

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circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^0	Capacity (MW)	Cost $(US\$ \times 10^6)$	n_{max-ij}
32	40	68	0.132	1	320	18.16	5
33	12	75	0.0641	1	320	11.49	5
34	24	75	0.0161	1	350	5.51	5
35	35	36	0.2074	1	250	27.36	5
36	27	35	0.1498	1	250	22.07	5
37	35	44	0.1358	2	250	20.35	5
38	38	68	0.0389	1	350	7.93	5
39	38	39	0.03	1	350	6.32	5
40	27	80	0.0242	1	350	7.01	5
41	44	80	0.1014	1	250	17.59	5
42	56	81	0.0114	1	550	32.86	5
43	45	54	0.0946	1	320	13.56	5
44	45	50	0.007	2	350	4.36	5
45	10	78	0.0102	1	350	4.94	5
46	7	78	0.0043	1	350	4.13	5
47	30	64	0.1533	1	250	20.58	5
48	30	65	0.091	1	250	13.68	5
49	30	72	0.0173	2	350	5.51	5
50	55	57	0.0174	1	600	46.81	5
51	57	84	0.0087	1	600	26.66	5
52	55	84	0.0087	1	600	26.66	5
53	56	57	0.024	2	600	62.62	5
54	9	77	0.019	1	350	5.86	5
55	77	79	0.0097	1	350	5.17	5
56	1	59	0.0232	2	350	6.2	5
57	59	67	0.118	2	250	16.67	5
58	8	59	0.1056	2	250	15.4	5
59	1	3	0.104	1	250	15.86	5
60	3	71	0.0136	1	450	5.17	5
61	3	6	0.0497	1	350	9.42	5
62	55	62	0.0281	1	550	70.99	5

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circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^0	Capacity (MW)	Cost $(US\$ \times 10^6)$	n_{max-ij}
63	47	52	0.0644	1	350	10.57	5
64	51	52	0.0859	1	250	12.87	5
65	29	31	0.1042	2	250	32.98	5
66	41	42	0.0094	1	350	4.71	5
67	40	42	0.0153	1	350	5.17	5
68	46	53	0.1041	2	250	14.6	5
69	46	51	0.1141	1	250	16.32	5
70	69	70	0.0228	2	350	6.2	5
71	66	69	0.1217	2	250	17.13	5
72	9	69	0.1098	2	350	15.75	5
73	60	69	0.0906	2	350	13.68	5
74	31	32	0.0259	1	350	6.55	5
75	32	34	0.054	1	350	9.77	5
76	16	18	0.0625	1	350	10.92	5
77	16	23	0.0238	1	350	6.89	5
78	16	21	0.0282	1	350	6.89	5
79	31	34	0.0792	1	250	12.41	5
80	31	33	0.0248	2	350	6.43	5
81	31	60	0.1944	2	250	25.98	5
82	31	72	0.0244	2	350	6.32	5
83	47	54	0.1003	2	250	14.25	5
84	47	49	0.0942	2	250	13.56	5
85	18	58	0.0212	2	350	5.74	5
86	18	20	0.0504	1	350	9.54	5
87	18	66	0.0664	2	350	11.38	5
88	18	21	0.0348	1	350	7.47	5
89	18	22	0.0209	1	350	6.43	5
90	19	22	0.0691	1	350	11.72	5
91	4	5	0.0049	3	350	4.25	5
92	5	6	0.0074	2	350	4.48	5
93	17	23	0.0913	1	250	12.99	5

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circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^0	Capacity (MW)	Cost $(US\$ \times 10^6)$	n_{max-ij}
94	17	76	0.002	1	350	3.9	5
95	12	17	0.0086	1	350	4.71	5
96	1	71	0.0841	2	250	14.37	5
97	1	8	0.081	1	250	13.22	5
98	1	11	0.0799	1	250	12.53	5
99	4	36	0.085	2	250	13.56	5
100	19	58	0.0826	1	320	11.72	5
101	27	64	0.028	1	350	6.78	5
102	27	28	0.0238	1	350	6.2	5
103	27	44	0.0893	1	250	16.32	5
104	26	27	0.0657	1	350	10.92	5
105	27	29	0.0166	1	350	5.05	5
106	19	66	0.0516	1	350	9.31	5
107	73	74	0.0214	1	600	58.28	5
108	64	65	0.0741	1	350	11.84	5
109	29	64	0.0063	1	350	4.36	5
110	4	34	0.1016	2	270	14.94	5
111	34	70	0.0415	2	350	8.27	5
112	33	34	0.1139	1	320	16.32	5
113	8	71	0.0075	1	400	4.48	5
114	54	63	0.0495	3	320	9.08	5
115	48	63	0.0238	1	350	6.32	5
116	67	68	0.166	2	250	22.07	5
117	39	68	0.0145	1	350	5.28	5
118	8	9	0.0168	1	350	5.97	5
119	79	87	0.0071	1	350	4.48	5
120	8	87	0.0132	1	350	5.17	5
121	39	43	0.1163	1	250	16.55	5
122	41	43	0.1142	1	250	16.32	5
123	23	24	0.0255	1	350	6.32	5

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circuit No.	node i	node j	x_{ij} (p.u.)	n_{ij}^0	Capacity (MW)	Cost $(US\$ \times 10^6)$	n_{max-ij}
124	21	22	0.0549	1	350	9.88	5
125	26	28	0.0512	1	350	9.31	5
126	28	29	0.0281	1	350	6.78	5
127	6	10	0.0337	1	350	7.58	5
128	33	72	0.0228	1	350	6.2	5
129	39	40	0.102	2	250	16.21	5
130	12	76	0.0081	1	350	4.71	5
131	48	54	0.0396	3	350	8.04	5
132	50	54	0.0876	2	250	12.87	5
133	62	73	0.0272	1	750	73.16	5
134	49	53	0.1008	2	250	14.25	5
135	40	41	0.0186	1	350	5.74	5
136	45	81	0.0267	1	450	13.27	5
137	64	74	0.0267	1	500	13.27	5
138	54	56	0.0267	3	450	13.27	5
139	60	62	0.0257	3	450	13.27	5
140	72	73	0.0267	2	500	13.27	5
141	19	82	0.0267	1	450	13.27	5
142	55	82	0.029	1	550	77.5	5
143	62	82	0.0101	1	600	31	5
144	83	85	0.0267	2	450	13.27	5
145	82	85	0.0341	1	700	89.9	5
146	19	86	0.1513	1	300	20.92	5
147	68	86	0.0404	1	350	8.27	5
148	7	90	0.005	2	350	4.25	5
149	3	90	0.0074	1	350	4.59	5
150	90	91	0.0267	1	550	13.27	5
151	85	91	0.0139	1	600	40.3	5
152	11	92	0.0267	1	450	13.27	5
153	1	93	0.0267	1	450	13.27	5
154	92	93	0.0097	1	600	30.07	5
155	91	92	0.0088	1	600	27.59	5

Source: Antonio 2012.

APPENDIX B -- ARTICLES PUBLISHED

Escobar L.M., Escobedo A.R., Skolfield K. “Bus-Angle Difference Valid Inequalities algorithm for DC Power Transmission Expansion Planning”, submitted and under review. 2018.

Escobar, L. M., Romero, R. “Angular Cuts for Expansion Paths in Transmission System Planning”. *IEEE PES Transmission & Distribution Conference and Exposition*, vol 1, pp. 1-6, 2018.

Escobedo A.R., Escobar, L. M. “Generation of Angular Valid Inequalities for Transmission Expansion Planning”, *XXIII International Symposium on Matematical Programing*, vol 1, pp. 1-6, 2018.

Escobar, L. M., Escobedo A.R., Escobar, D., Romero, R. “Bus-Angle Difference Structural Cuts for Transmission System Expansion Planning with L-1 Reliability”, *IEEE Canada Electrical Power and Energy Conference*.vol 1, pp. 1-6, 2018.

Escobar, L. M., Romero, R. “A Methodology for Transmission Expansion Planning Problem Considering Reduction of the Search Space Based on Angular Cuts and Minimum Effort Criterion”, *XXI Conference of the International Federation of Operational Research Societies*, vol 1, pp. 1-6, 2018.

Escobar, L. M., Romero, R. “Angular cuts applied to the long-term transmission expansion planning problem” *XLIX Simpósio Bra-sileiro de Pesquisa Operacional*,vol. 1, pp. 1–6, 2017.

Escobar, L. M., Romero, R. “Long Term Transmission Expansion Planning considering Generation-Demand Scenarios and HVDC lines”, *IEEE PES Transmission & Distribution Conference and Exposition*,vol. 1, pp. 1–6, 2016.

Escobar, A.H., Escobar, D., Escobar L.M., Gallego, R.A. “Ubicación óptima de puntos de carga para redesde vehículos eléctricos”, *Congreso ASOCIO 2015*, vol. 1, pp. 1–6, 2015.