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QUANTIFICATION OF PARAMETRIC UNCERTAINTIES EFFECTS IN STRUCTURAL FAILURE CRITERIA

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TÍTULO DA DISSERTAÇÃO: Quantification of parametric uncertainties effects in structural failure criteria

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Abstract

Failure theory is the investigation of predicting circumstances under which solid materials under the processing of external loads. The theories of failure are known as different failure criteria such as von Mises and Tresca which are the most famous of these for certain materials. Additionally, this master dissertation intends to show a comparison between Tresca and von Mises failure criterions, taking into account the underlying uncertainties in the constitutive equations and stress analysis. In order to exemplify the comparison, some numerical simulations are performed using a simple plate, simple deflection problem and a frame of the formula car. Due to the complexity of frame of the formula car, different kind of probabilistic steps are used as a response surface method and parameters correlation. Results show that several random input variables effect the random output variables in various ways, and there is no such a big difference between the von Mises and Tresca failure criterions when uncertainties are assumed in the formulation for stress analysis.
Keywords: Failure criterions. Stress analysis. Uncertainty quantification. Parametric probabilistic approach. Monte-Carlo method.

Resumo

Critérios de falhas realizam a predição de circunstâncias nas quais materiais sólidos estão sobre ação de carregamentos externos. As teorias de falhas são conhecidas como diferentes critérios de falhas, como von Mises e Tresca, os quais são os mais famosos para determinados materiais. Além disso, esta dissertação de mestrado pretende mostrar a comparação entre os critérios de falha de Tresca e von Mises, levando em conta incertezas subjacentes nas equações constitutivas e na análise de tensão. Para exemplificar a comparação, algumas simulações são realizadas usando uma placa simples, um problema de deflexão simples,e a estrutura de um carro do formula SAE. Devido à complexidade deste sistema, diferentes tipos de etapas probabilísticas são utilizadas, como o método de superfície de resposta e a correlação de parâmetros. Os resultados mostram que várias variáveis aleatórias de entrada afetam em maneiras diferentes as variáveis aleatórias de saída e que não há uma diferença grande entre os critérios de falha de von Mises e Tresca quando incertezas são assumidas na formulação para a análise de tensão.
Palavras-chave: Critérios de falha. Análise de tensão. Quantificação de incerteza. Probabilística paramétrica. Método de Monte-Carlo.

List of Symbols

${F_s}^{vm}$	-	Safety factor for von Mises failure criterion
${F_s}^{tr}$	-	Safety factor for Tresca failure criterion
$p_X(x)$	-	Probability density function
$\mathbb{R}$	-	Real numbers set
N_{p_f}	-	Number of simulations
$conv(.)$	-	Monte Carlo convergence function
$M(x)$	-	Deterministic moment equation considering variable x
F	-	Deterministic force
E	-	Deterministic elastic modulus
I	-	Deterministic moment of inertia
ℓ	-	Deterministic length of the beam
C_1	-	First constant value of integral equation
C_2	-	First constant value of integral equation
$v(x)$	-	Deterministic deflection function considering variable x
$\frac{dv}{dx}$	-	Deterministic slope function considering variable x
G	-	Deterministic shear modulus
b	-	Deterministic width of the beam
h	-	Deterministic height of the beam
$h(x)$	-	Deterministic map considering variable x
a	-	Regression coefficient for approximation function
b_i	-	Regression coefficient for approximation function
c_i	-	Regression coefficient for approximation function

Greek Letters

σ	-	Stress tensor
σ_x	-	Normal stress for x component
σ_y	-	Normal stress for y component
σ_z	-	Normal stress for z component
τ_{xy}	-	Shear stress for xy component
τ_{xz}	-	Shear stress for xz component
τ_{yz}	-	Shear stress for yz component
σ_1	-	First principal stress
σ_2	-	Second principal stress
σ_3	-	Third principal stress
σ_{vm}	-	Stress for von Mises failure criterion
σ_{tr}	-	Stress for Tresca failure criterion
S_Y	-	Yield stress
μ_x	-	Mean value
σ_x	-	Variance value
λ	-	Deterministic lambda modulus
γ_{xy}	-	Deterministic shear strain for xy component
γ_{yz}	-	Deterministic shear strain for yz component
γ_{xz}	-	Deterministic shear strain for xz component
ν	-	Deterministic Poisson's ratio
ϵ_{xx}	-	Deterministic strain for x component
ϵ_{yy}	-	Deterministic strain for y component
ϵ_{zz}	-	Deterministic strain for z component
ϵ_{xy}	-	Deterministic strain for xy component
ϵ_{yz}	-	Deterministic strain for yz component
ϵ_{xz}	-	Deterministic strain for xz component
u_x	-	Displacement for x component
u_y	-	Displacement for y component
u_z	-	Displacement for z component
ν_{p_f}	-	Variation index

List of Acronyms

MC	-	*Monte Carlo simulations*
LHS	-	*Latin Hypercube sampling*
RSM	-	*Response surface method*
PDF	-	*Probability density function*

List of Figures

List of Tables

Contents

1 Introduction

This chapter regards the introduction of this dissertation. Section 1.1 demonstrates the general informations about failure criteria as a von Mises, Tresca and Coulomb-Mohr and their safety factors. Section 1.2 gives the information about failure criteria in structural analysis, also this section indicates historical perspective and using of failure theory in engineering. Section 1.3 describes the problem of interest applied in various stress analysis considering the presence of uncertainties. Section 1.4 presents the main objectives and contributions of the dissertation. Finally, section 1.5 presents the outline of the rest of the text.

1.1 Failure criteria and safety factors

Well-designed failure theories differentiate safe states of stress in materials from states of certain failure, depend on calibration by a minimal number of failure-type properties (YU; YU; YU, 2004). In the work to follow and the historical reason, the expression of "failure" or "failure characterization" will be utilized in included as implementing to either or both of the conditions of yielding or actual failure. All thoughts about failure criterions are generally with homogeneous and isotropic materials, where the historical aim was to discover a general failure criterion that would cover the full range from ductile to brittle types of materials (CHRISTENSEN, 2013).
Presumably, the most misinterpreted but definitely the most significant of all failure concepts is that of the Coulomb-Mohr type. This failure theory was application for isotropic materials under three dimensional stress conditions. A failure plane was postulated to be activated by shear stress but with a possible linear dependence of the critical shear stress level upon the transverse normal stress component acting upon the failure plane (MOHR, 1900). Its history go back to nearly the beginning of all of mechanics, and its popular reception continues up to the present day. For brittle materials, the Coulomb-Mohr foretells that uniaxial and equibiaxial tensile failure are at the same levels, but that equitriaxial tensile failure is much stronger (COULOMB, 1773). Also, in testing geological materials, von Karman (KÁRMÁN, 1910), and also Böker (BOKER, 1915) indicated Coulomb-Mohr criterion is only in part successful when the stress are some-

what compressive, and not at all successful otherwise, either in the very compressive or slightly compressive region. Furthermore, another two-parameter failure criterion was given much later by Drucker and Prager. The Mises criterion is considered as cylindrical surface in principal stress space. The Drucker-Prager form generalized the Mises criterion to take the form of a cone in principal stress space. Although both the Coulomb-Mohr and Drucker-Prager concepts widely utilized in soil mechanics, both failed to gain use for general engineering materials (DRUCKER; PRAGER, 1952).

After the many tries to cast the Coulomb-Mohr hypothesis into a generally realistic, accessible concept never gave good results, and the quest for a general energy criterion necessarily came to an unsuccessful outcome, a degree of pessimism impacted the further efforts to discover a general criterion. The consideration of the Coulomb-Mohr criterion had expanded over more than a hundred years and still had not gave the desired outcome (MOHR, 1900). In spite of the long, difficult history, there stays the possibility that the extending knowledge base of the modern period may supply the advantage for a more successful formulation of failure theory (CHRISTENSEN, 2013).

There have been presented such famous failure criteria as the Tresca, von Mises and Coulomb-Mohr. The Tresca and von Mises criteria usually accepted as failure criteria for metals, and the Coulomb-Mohr adopted as a criterion for granular materials such as soils (MATSUOKA; NAKAI, 1985). A physical explanation of von Mises criterion proposing that yielding begins when the elastic energy of distortion gets into a critical value. Because of this reason, the von Mises criterion also acknowledged as the maximum distortion strain energy criterion (FORD; ALEXANDER, 1963). Moreover, the von Mises stress is used to forecast the yielding of materials under any loading condition from results of simple uniaxial tensile strength tests. The von Mises stress satisfies the property that two stress states with equal distortion energy have equal von Mises stress (HILL, 1998).

The study of yielding was influenced by the wish to foresee mechanical failure of materials. Yielding is taken into account as the beginning of a process which will finally guide to fracture, characterized by the breaking of the bonds between atoms and separation of the material. It is important to highlight that the stress required to break the atom bonds is roughly one third of the material's Young's modulus. However, ductile materials fail with stress values far smaller than this estimate (HOULSBY, 1986). This phenomenon is connected to defects and the way they move inside the materials. Consequently, failure in ductile materials is effected by shear deformations. Thus, it is logical to establish a yield criterion in terms of the amount of shear stress a material is able to sustain, this highlights the principle of the Tresca theory (LANCE; ROBINSON, 1971). In generally, the literature on this subject says that the Tresca theory is more conservative than the von Mises theory and the essential and striking differences between the von Mises and Tresca failure criteria are the corners that occur in the Tresca form and their complete absence in the Mises form (CHRISTENSEN, 2013). It foresees a narrower elastic region. The Tresca criterion can be safer from the design point of view, but it could lead the engineer to consider unnecessary measures to prevent an unlikely failure. The criterion choice depends on the type of design and personal taste of the designer (LIN; ITO, 1966). Coulomb-Mohr failure theory is required to relate the available strength of a soil as a function of measurable properties and the imposed stress conditions. Coulomb-Mohr main hypothesis is based on the premise that a combination of normal and shear stresses creates a more critical limiting state than would be found if only the major principal stress or maximum shear stress were to be considered individually (LABUZ; ZANG, 2012).

1.2 Failure criteria in the structural analysis

The aim behind the failure criteria is to predict or estimate the yield and failure of machine parts and basic individuals. However, the most part of the common failure criteria, as von Mises and Tresca, are demonstrated together with little proposal or discrimination between them in all mechanics of materials books. Furthermore, the primary explanation of the Mises criterion is that it shows a critical value of the distortion energy kept in the isotropic material while the Tresca criterion is that of a crucial value of the maximum shear stress in the isotropic material (BURNS, 2015). Verifiable, the Tresca form is thought to be more fundamental comparing to von Mises criterion, however the Mises form is viewed as an appealing, scientifically helpful estimation to it. Currently, both are normally expressed one next to the other with almost no inclination (ERASLAN, 2002). The conditions allow the uniform definition of various groups of materials for which quite various forms of the failure criteria have been utilized to date (PODGÓRSKI, 1985).

1.3 Problem of interest and contributions

In general, literature say that Tresca failure criterion is more conservative compared to von Mises failure criterion. Nevertheless, this is not always true in the scenario with different kind of variabilities. Furthermore, there are some situations that von Mises failure criterion has a same characteristic as Tresca failure criterion considering common possible uncertainty parameters (CHRISTENSEN, 2013).

In this dissertation, the quantification of security factor uncertainties is addressed through

a parametric probabilistic approach, using Monte Carlo simulation as a stochastic solver. The goal is to verify the effect of the parametric uncertainties in the safety factor obtained with basis on the failure criteria von Mises and Tresca.
Furthermore, to the best of my knowledge, it is the first time that the uncertainties are assumed to compute the safety factors using both classical criterions. This dissertation indicates the effects of different type of probabilistic analyses and random input variable on random output variables.

1.4 Objectives

The main goal of this dissertation is that to show a critical comparison between the Tresca and von Mises failure criterion assuming uncertainties in the constitutive equations and stress analysis. Furthermore, this dissertation compares the different kind of probabilistic results in various stress analysis, and examines the effect of different types of probabilistic analysis and indicates the effect of random input variables which are force components, shear and normal stresses and Young's modulus, on random output variables as a safety factor for von Mises and Tresca failure criterions.

1.5 Outline

This dissertation is organized in the following chapters:

- **Chapter 1 - Introduction:** This chapter indicates the introduction of the dissertation and gives the information about failure criterions as a von Mises, Tresca and Coulomb-Mohr. The main objectives and contributions of research

- **Chapter 2 - Literature Review:** This chapter gives the informations about uncertainty quantification, probabilistic model of uncertainties and stochastic solvers as a crude Monte-Carlo simulation, Latin Hypercube sampling method, response surface method and their advantages and drawbacks.

- **Chapter 3 - Criterions of Failure Assuming Uncertainty Quantification:** This chapter shows the explanation of equilibrium of an elastic body for isotropic, linear and elastic materials. Also, the chapter gives information about the state of tension in a material point, von Mises and Tresca failure criterions. Furthermore, this chapter indicates explanation of uncertainty quantification, maximum entropy principle and mean-square convergence method. Moreover, the chapter gives information about Monte-Carlo simulation.

- **Chapter 4 - Experiments with Simple Structural Systems:** This chapter shows the probabilistic results of simple plate, simple deflection problem considering uncertainty quantification and discussion about probabilistic results.

- **Chapter 5 - Experiments with a Complex Structural System:** This chapter demonstrates the different kind of probabilistic finite element analyzes as a response surface method, parameters correlation, optimization process, Six-Sigma analysis and probabilistic analysis on the examination of frame of the formula car. Additionally, this chapter shows interpretation about results and graphics of the comparison between failure criterions with a complex structural system.

- **Chapter 6 - Final Remarks:** This chapter presents the the main results about the present dissertation and discussion about contributions, the future works and the continuity of the research.

2 Literature Review

The chapter of dissertation is organized as follows. Section 2.1 reviews the basic notions on uncertainty quantification. Section 2.2 indicates the fundamental aspects of probabilistic modeling of uncertainties. Finally, section 2.3, section 2.4 and section 2.5 present three methodologies that are crude Monte-Carlo simulation, Latin Hypercube sampling method and response surface method as a stochastic solver, respectively. Also, section 2.6 demonstrates the conclusions.

2.1 Basic notions on uncertainty quantification

Most of the predictions that are essential for decision making in engineering, and sciences are made in with aid of computer models. These models depend on assumptions that could be not in accordance with reality. Therefore, a model can have uncertainties on its forecasts, because of conceivable wrong hypothesis made during its originations (SOIZE, 2017; CUNHA, 2017). If the input variables affecting the attitude of a design are uncertain, after the fundamental mission of an uncertainty analysis is to quantify how much the outcome parameters describing the product behavior are influenced by those uncertainties (REH et al., 2006). Additionally, these arbitrary variabilities may rise from an diversity of sources including the geometry of the issue, material characteristics, limit conditions, starting conditions, or excitations forced on the framework (BAE; GRANDHI; CANFIELD, 2004).

2.2 Probabilistic modeling of uncertainties

The parametric probabilistic approach comprises in modeling the uncertain parameters of the computational model by random variables, in order to construct a stochastic model to deal with the underlying variabilities. This kind of approach is highly fit and extremely effective to consider the uncertainties on the computational model parameters (GHANEM; DOOSTAN; RED-HORSE, 2008). Two primary type of uncertainties are encountered. The first type is known as data uncertainty, due to variabilities in parameters of the model (SOIZE, 2013a). This type of uncertainty can not be reduced, only better characterized. The second type of uncertainty is epistemic (model) uncertainty, due to lack of knowledge about the phenomena of interest. This can be reduce increasing the knowledge about the phenomena of interest (SOIZE, 2009; SOIZE; GHANEM, 2009; LANGLEY, 2000; MACE; WORDEN; MANSON, 2005; SCHUËLLER, 2001; SCHUELLER, 2007; SCHUËLLER, 2006; SCHUELLER, 2007; SCHUËLLER; JENSEN, 2008; SCHUËLLER; PRADLWARTER, 2009; SCHUELLER; PRADLWARTER, 2009).

The stochastic models assume a critical function in clarifying numerous regions of engineering sciences. Stochastic procedures which are methods for measuring the dynamic relationships of series of aleatory circumstances. They can be utilized to examine the versatility natural in medical and biological procedures, to manage uncertainties influencing administrative choices and, to ensure new points of view, strategy, and to help in other mathematical and scientific investigations (TAYLOR; KARLIN, 2014).

Firstly, the modeling mistakes and the estimations faults are at the same time considered and can not be truly be independently defined. Take note of that if there is no useful experimental information,such a technique can not be utilized. Because, there are no data for building the stochastic model of such a noise (SOIZE, 2012). The second one depends on the non parametric probabilistic approximation of modeling uncertainties (modeling errors) which has been suggested in as an another technique to the output expectation error strategy owing to consider modeling errors (BECK; KATAFYGIOTIS, 1998; SOIZE, 2000; SOIZE, 2001; SOIZE, 2005). A stochastic model forecasts a group of potential consequences considered by their probabilities or chances (TAYLOR; KARLIN, 2014). Different kind of processes exist for assessing uncertainties in a model. These processes are deterministic or probabilistic processes. Nowadays, it is well known that the probabilistic approach of uncertainties need to be utilized once the probability theory can be implemented (LANGLEY, 2000; SCHUELLER, 2007).

In case of model-parameter uncertainties, the primary process is attributed on the utility of the parametric probabilistic approach which has largely been improved in the last years. Furthermore, the parametric probabilistic which is still in improvement and which allows the uncertain model parameters of the mean model to be considered through the introduction of a main probability model of parameters (GHANEM; SPANOS, 1991; BABUŠKA; TEMPONE; ZOURARIS, 2005; SCHUËLLER; JENSEN, 2008; SOIZE, 2006; BABUŠKA; NOBILE; TEMPONE, 2007; NOUY et al., 2008).

2.3 Monte Carlo method

Once a computational model is constructed, it is necessary investigate how the uncertainties propagate from model parameters to the response. This process can be done via Monte Carlo method (METROPOLIS; ULAM, 1949). The Monte Carlo strategy is the most widely recognized technique for stochastic calculation, because of its simpleness and great factual outcomes. Nevertheless, its computational expense is to a exceedingly, and, inhibitive. Luckily the Monte Carlo calculation is effectively parallelizable, which permits its utilization in simulations where the calculation of a individual realization expensive (CUNHA et al., 2014). This technique generates several realizations (samples) of the random parameters according to their distributions (stochastic model). Each of these realizations defines a deterministic problem, which is solved (processing) using a deterministic technique, generating an amount of data. Then, all of these data are combined through statistics to access the response of the random system under analysis (CUNHA et al., 2014). The Monte Carlo technique does not necessitate that one applies another computer program to reproduce a stochastic model. Whether a deterministic code to simulate a same deterministic model is accessible, the stochastic simulate can be implemented by running the deterministic program a few times, modifying just the parameters that are indiscriminately created. In principle, this technique is an algorithm in which a few realizations of the haphazard parameters of the stochastic model are produced considering the possibility distribution that is different to them a priority. Each one of these realizations describes a deterministic issue, which is resolved utilizing a deterministic method, creating an number of information. At that point, these information are associated through statistics to get to the response of the random system under examination (KROESE; TAIMRE; BOTEV, 2013).

Concerning the computational usage, the Monte Carlo method has a non-intrusive characteristic, it does not necessitate a another code to simulate a stochastic model. Whether a deterministic code to simulate a same deterministic model is existing, the stochastic simulation can be directed by running the deterministic program a several times, changing just the value of the parameters that are haphazardly created (KROESE; TAIMRE; BOTEV, 2013). Unluckily, Monte Carlo is a very time-consuming technique,which makes impossible its utilization for complicated computer simulations, when the operation time of a only realization is big scale or the number of realizations to an correct result is enormous. Luckily the algorithm is effectively parallelizable, permitting overcome this deficit (CUNHA et al., 2014). Furthermore, if the Monte Carlo simulation is implemented for a extensive number of examples, it totally defines the statistical conduct of the random

system (CAFLISCH, 1998).
From the way of thinking of one's approach to the values taken into consideration, structural reliability analyses can be categorized in two categories that are deterministic analyses and stochastic analyses (JANAS; KREJSA, 2002; KRÁLIK; JR, 2006; MAREK, 2001). Different forms of analyses (probabilistic analysis, statistical analysis, sensitivity analysis) can be applied considering the stochastic approach. Most of these methods are based on the integration of Monte Carlo (MC) simulations. Three categories of method are presented in this chapter.
The advantages of the Monte-Carlo simulations are that easily apprehensible and transparent. The Monte-Carlo simulation converges to the true and accurate probabilistic outcomes due to increasing number of samples. The Monte-Carlo simulation is hence extensively used a the benchmark to confirm the correctness of other probabilistic methods. Another advantage of the Monte-Carlo simulation is the fact that the required number of simulations is not a function of the number of input variables (LOW; TANG, 1997).
Also, the drawbacks of the Monte-Carlo simulation are slow calculation system of complex design and its computational cost. Thus, if design of the system of failure probability has low values, after the required number of samples may be large number of simulations (REH et al., 2006).

2.4 Latin Hypercube Sampling Method

A significant part of the analysis of engineering structure is to figure out the probabilities of failure or unacceptable structural performance. Latin hypercube sampling process which is very effective for predicting mean values and standard deviations in stochastic structural analysis (OLSSON; SANDBERG, 2002; OLSSON, 1999; SANDBERG; OLSSON, 1999). Furthermore, Latin hypercube sampling process is only little more effective than the Monte-Carlo simulation processing for predicting small probabilities (PEBESMA; HEUVELINK, 1999). According to different numerical samples, it is indicated that more than 50 percent of the computer working process can be saved by using Latin hypercube sampling instead of simple Monte Carlo simulation in importance sampling. Nevertheless the precise savings are connected on details in the use of Latin hypercube sampling process on the form of the failure surface problems. The decrease of number of simulations expresses a valuable advantage from this method compared to the crude Monte Carlo simulation (OLSSON; SANDBERG; DAHLBLOM, 2003).

2.5 Response Surface Method

Response surface methods prevent from the drawbacks of Monte-Carlo simulations by changing the true input–output relationship by an approximation function. The number of experiments is also described by the number of unknown coefficients in the response surface function (KAYMAZ; MCMAHON, 2005).
The advantages of the response surface method are that it can be simulated considerably less number of simulations comparing with the crude Monte Carlo method. Particular simulations in response surface method are independent from each other, also parallel calculations can be utilized in this method (KIM; NA, 1997).
Furthermore, the disadvantages of response surface method are that the number of simulations are contingent on the number of variable input parameters. The method is inefficient considering large number of input parameters (LIU; MOSES, 1994).
In this master dissertation, the quantification of security factor uncertainties is addressed through a parametric probabilistic approach, using Monte Carlo simulation as stochastic solver. The goal is to verify the effect of the parametric uncertainties in the safety factor obtained with basis on the failure criteria of von Mises and Tresca.

2.6 Conclusions

This chapter is presented as a literature review and gives brief description of uncertainty quantification, probabilistic model of uncertainties and stochastic solvers as a crude Monte-Carlo simulation, Latin Hypercube sampling method, response surface method and their advantages and disadvantages. In the next sections, these stochastic solvers and methods will be used in experiments with simple and complex structural systems as a plane stress, simple deflection problem and model of the frame of formula car.

3 Uncertainty Quantification in Structural Failure Theory

This chapter of dissertation is formed as follows. Section 3.1 shows the information about theory of elasticity which includes equations of equilibrium, constitutive equations and superposition principle for isotropic, linear and elastic materials. Section 3.2 gives the information about the state of tension in a material point according to solid mechanical system. Also, section 3.2 presents the general information about von Mises and Tresca failure criterions. Section 3.3 gives the general information about uncertainty quantification, and section 3.4 presents the conclusions.

3.1 Equilibrium of an elastic body

A generic isotropic, linear and elastic body is illustrated in Figure 1. This generic elastic body which is subjected to action with external forces. The external force components are indicated as a F (LEKSZYCKI et al., 2018).
Equations of equilibrium for isotropic, linear and elastic materials are given by (JOHNSON; KENDALL; ROBERTS, 1971; LEKHNITSKIJ, 1977).

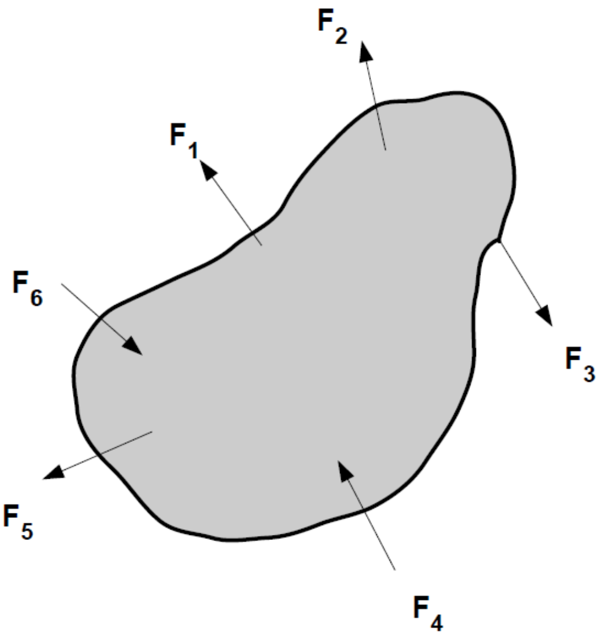
$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0, \tag{1}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0, \tag{2}$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = 0, \tag{3}$$

where σ_{xx}, σ_{yy}, σ_{zz} are the normal stresses and τ_{xy}, τ_{xz} and τ_{yz} are shear stresses in different axis.

Figure 1 – Body subject to action with external forces



Source: Prepared by the author

Furthermore, relationships between displacements and deformations are indicated by

$$\epsilon_{xz} = \frac{\partial u_x}{\partial x} \qquad \epsilon_{xy} = \frac{1}{2}\gamma_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right), \tag{4}$$

$$\epsilon_{yy} = \frac{\partial u_y}{\partial y} \qquad \epsilon_{xz} = \frac{1}{2}\gamma_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right), \tag{5}$$

$$\epsilon_{zz} = \frac{\partial u_z}{\partial z} \qquad \epsilon_{yz} = \frac{1}{2}\gamma_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right), \tag{6}$$

where u_x, u_y and u_z are displacements. Also, ϵ_{xx}, ϵ_{yy}, ϵ_{zz} and ϵ_{xy}, ϵ_{yz}, ϵ_{xz} are the strains. Constitutive equations which relate stress and strains quantities are described by

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \qquad \epsilon_{xy} = \frac{1}{2}\gamma_{xy} = \frac{\tau_{xy}}{2G}, \tag{7}$$

$$\epsilon_{yy} = \nu\frac{\sigma_{xx}}{E} + \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E} \qquad \epsilon_{xz} = \frac{1}{2}\gamma_{xz} = \frac{\tau_{xz}}{2G}, \tag{8}$$

$$\epsilon_{zz} = -\nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \frac{\sigma_{zz}}{E} \qquad \epsilon_{yz} = \frac{1}{2}\gamma_{yz} = \frac{\tau_{yz}}{2G}. \tag{9}$$

Moreover, γ_{xy}, γ_{yz} and γ_{xz} are the shear strains. G and E are shear modulus and elastic modulus, respectively. Finally, the equations of elasticity are given and there is a symmetry between shear stress components.

3.2 Main stresses and their planes of action

The state of tension in a material point of a solid mechanical system is defined by the stress tensor

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}. \tag{10}$$

From the stress tensor in a point, principal stresses are obtained through the solution of a eigenvalue problem

$$(\boldsymbol{\sigma} - \lambda I\ )\, \upsilon = 0, \tag{11}$$

where I is the identity tensor and (λ, υ) is an eigenpair for $\boldsymbol{\sigma}$. The obtained eigenvalues which are equal to the principal stress values σ_1, σ_2 and σ_3.

3.2.1 Von Mises failure criterion

Moreover, the von Mises stress is determined as (XIE; STEVEN, 1993; KARAOGLU; KURALAY, 2002).

$$\sigma_{vm} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}. \tag{12}$$

The relationship between S_Y and σ_{vm} is determined shown below

$$\sigma_{vm} < S_Y, \tag{13}$$

where S_Y is the yield stress of the material. Thus, the safety factor for von Mises failure criterion is defined as

$$F_s^{\,vm} = \frac{S_Y}{\sigma_{vm}}. \tag{14}$$

3.2.2 Tresca failure criterion

Furthermore, the Tresca stress is given by (MATSUOKA; HOSHIKAWA; UENO, 1990).

$$\tau_{max} = \frac{\sigma_{tr}}{2} = \frac{|\sigma_3 - \sigma_1|}{2}, \tag{15}$$

where τ_{max} is maximum shear stress. The relationship between S_Y and σ_{tr} is determined shown below

$$\sigma_{tr} < \frac{S_Y}{2}, \tag{16}$$

while the safety factor for Tresca failure criterion is given by

$$F_s^{tr} = \frac{S_Y}{\sigma_{tr}}. \tag{17}$$

3.3 Uncertainty quantification

This section indicates probabilistic framework and the sampling approach to uncertainty quantification. Furthermore, this section presents the maximum entropy principal and the mean-square convergence analysis and gives the information about Monte Carlo simulation, respectively.

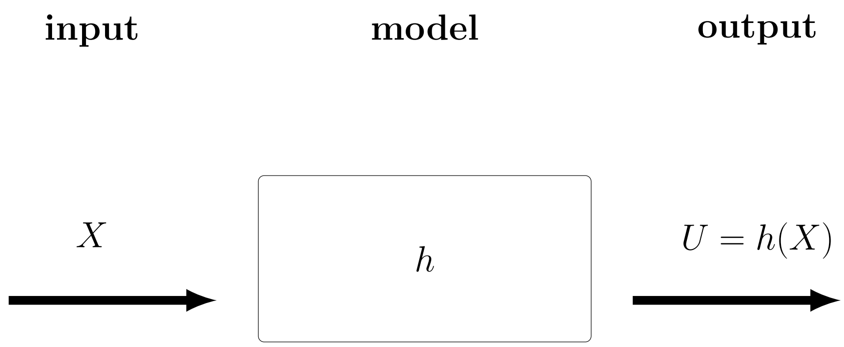
3.3.1 Probabilistic framework

In this dissertation, a stochastic version of failure criterions are defined through a parametric probabilistic approach (SOIZE, 2012; SOIZE, 2005). The parameters of model oriented to uncertainties are defined as a random processes or variables, which utilizes on the probability space (Θ, Σ, P), where Θ is a sample space, Σ is a σ-algebra over Θ, also P which is a probability measure.
Here in Figure 2, the uncertain variables are generated according to a user-prescribed specification and subsequently propagated through the model, h, yielding a set of samples of output(s), U as displayed in Figure 2 for a given stochastic input, X (SOIZE, 2017).
In this probabilistic setting, it is presumed that any random variable X is described in (Θ, Σ, P), with a probability distribution on $P_X(\mathrm{dx})$ on $\mathbb{R}$. The mean value of random variable $X : \Theta \mapsto \mathbb{R}$ is defined by

$$E(X) = \int_{\mathbb{R}} x p_X(x) dx, \tag{18}$$

where p_X(x) is the probability density function of X.

Figure 2 – Generic system subjected to an uncertainty input



Source: Prepared by the author

3.3.2 Probabilistic model

The probability density function associated with the random variables corresponding to the chosen uncertain parameters will be constructed using by maximum entropy principle (JAYNES, 1957).

The maximum entropy principle constructs the probability density functions consist in maximizing the entropy. The Shannon entropy of random variable X is described as

$$S(p_X) = \int_b^a p_X(x) \ln p_X(x) dx. \tag{19}$$

The main object is to maximize the entropy considering constraints defined by the following equations

$$\int_b^a p_X(x) dx = 1, \tag{20}$$

and

$$\int_b^a p_X(x) g_i(x) dx = a_i, \; i = 1, ..., m, \tag{21}$$

where the functions g_i and the real numbers a_i are given respectively.

The parameters can be modeled as a random variable with values in $\mathbb{R}$ and the mean value μ_x and the variance value σ_x are known. Typically, these parameters represent statistical moments. Then, the support of the probability density function which is [a,b], the mean value which is such that E[X] $= \mu_x$. The second-order moment is $\sigma_x^2 + \mu_x^2$ where σ_x is the standard deviation. The probability density function p_X have then to verify the following constraint

$$\int_b^a p_X(x)dx = 1, \tag{22}$$

$$\int_b^a x p_X(x)dx = \mu_x, \tag{23}$$

$$\int_b^a x^2 p_X(x)dx = \mu_x^2 + \sigma_x^2. \tag{24}$$

This probability density function of X is given by

$$p_X(x) = e^{-\lambda_0} e^{-\lambda_1 x - \lambda_2 x^2}, \tag{25}$$

where λ_0, λ_1 and λ_2 are the parameters of this distribution.

3.3.3 Propagation of uncertainties

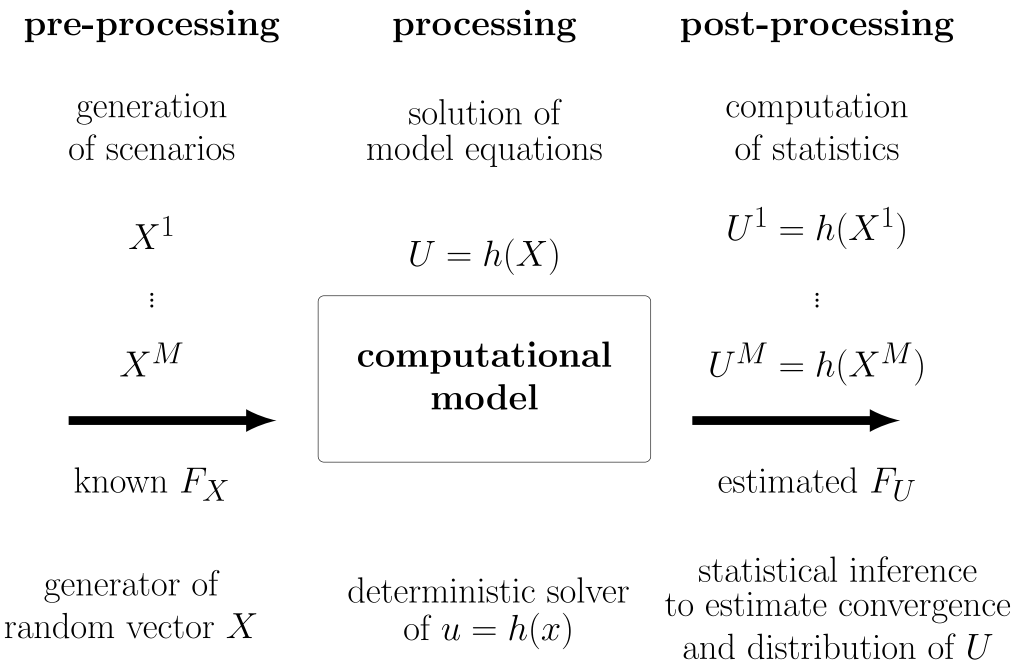
Monte Carlo (MC) simulation is utilized to evaluate the propagation of uncertainties of the random parameters through the computational model (KROESE; TAIMRE; BOTEV, 2013).
The uncertain parameter x is indicated by the random variable X, map of given given inputs can be described as h for the estimation of distribution.
In this stochastic process, Figure 3 (CUNHA; SAMPAIO, 2012) indicates that the steps of Monte Carlo simulation are defined, M independent samples of X are defined according to the distribution $p_X(x)dx$, where independent observations of X is given by

$$X^1,\ X^2, \ldots,\ X^M. \tag{26}$$

Each of these scenarios for x is indicated as input to the map of given input $x \longmapsto U = h(x)$, where set of possible realizations is determined shown below

$$h(X^1),\ h(X^2), \ldots,\ h(X^M). \tag{27}$$

Figure 3 – Monte Carlo simulation schematic



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Furthermore, the mean-square convergence analysis (CATALDO et al., 2009) with respect to independent realizations U_1, U_2..., U_m of the random variables are carried out studying the function $m \longmapsto conv(m)$ defined by

$$conv(m) = \frac{1}{m}\sum_{j=1}^{m} {U_j}^2. \tag{28}$$

3.4 Conclusions

This chapter demonstrates the equations of equilibrium, constitutive equations and superposition principle for isotropic, linear and elastic materials. Also, the chapter clarifies the information about von Mises and Tresca failure criterions. After giving these informations, this chapter explains how uncertainty quantification samples and Monte Carlo simulation work and how to construct probabilistic model with using maximum entropy principle. In the next chapters, these stochastic models and methods will be used in experiments with simple and complex structural systems as a plane stress, simple deflection problem and model of the frame of formula car to compare von Mises and Tresca failure criterions and prove that there is no big differences between them.

4 Analysis of Uncertainty Propagation in Simple Structural Systems

This chapter presents some numerical simulations with simple structural systems to explain the comparison between von Mises and Tresca failure criterions. First of all, section 4.1 shows the explanation of simple plate problem considering one uncertainty and the results. Section 4.2 demonstrates the definition of simple plate problem considering two uncertainty and the results. Moreover, section 4.3 indicates the simple deflection problem considering uncertainty quantification and results. Section 4.4 presents the conclusions of each problem, respectively.

4.1 Example in plane stress problem considering one uncertainty

In this part of section, model parameters subjected to uncertainties are described as random variables and the system response also becomes a random variable. A simple plate applying with normal and shear stresses is described in Figure 4, where the values of normal stresses σ_y and σ_x are chosen for illustration as 100 MPa and 90 MPa in the Eq. 10, respectively. Also, the value of S_y yield stress of material is determined as 250 MPa. After giving this value, random values are utilized by applying uncertainty quantification method considering shear stress τ_{xy}. Thus, these parameters are examined in the uniform and truncated-exponential obtained from maximum entropy principle, respectively (PAVON; FERRANTE, 2013).

Figure 4 – Illustration of simple plate problem applying with normal and shear stresses

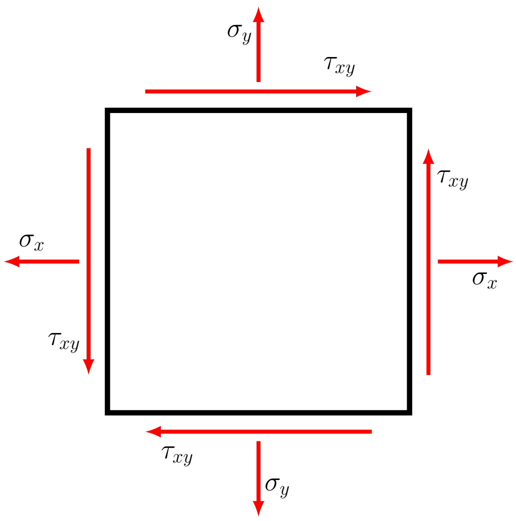

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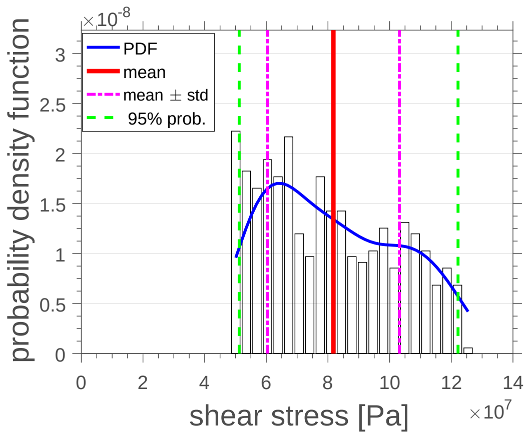
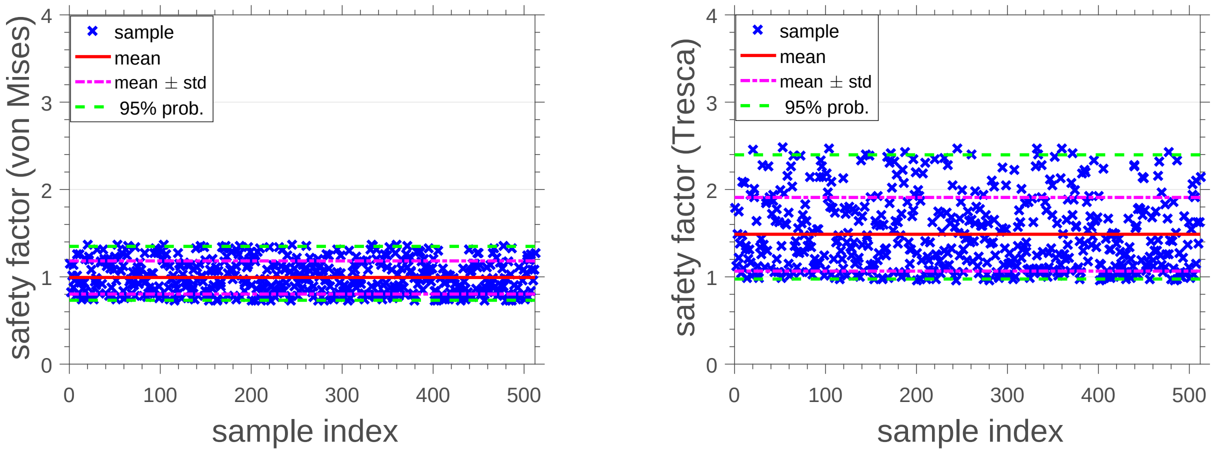
Figure 5 – Illustration of shear stress distribution with given by a truncated exponential with 3 parameters



Source: Prepared by the author

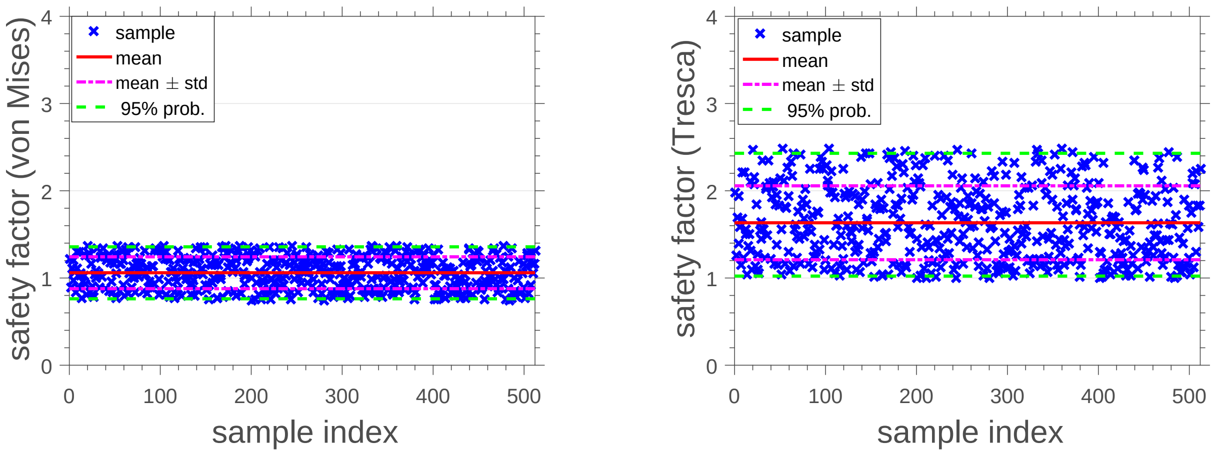
According to Figure 5, it shows that statistical results of input variable when the random variable is considered as a shear stress applying with truncated exponential distribution. Minimum support and maximum support of the distribution are considered as 50 MPa and 130 MPa, respectively. Also, the mean value of input variable is given as 85 MPa. Coefficient of variation is considered as 0.2 and shear stress distribution is constructed.

Figure 6 – Comparison between safety factor samples when applied shear stress distribution is an uniform: a)von Mises; b)Tresca



Source: Prepared by the author

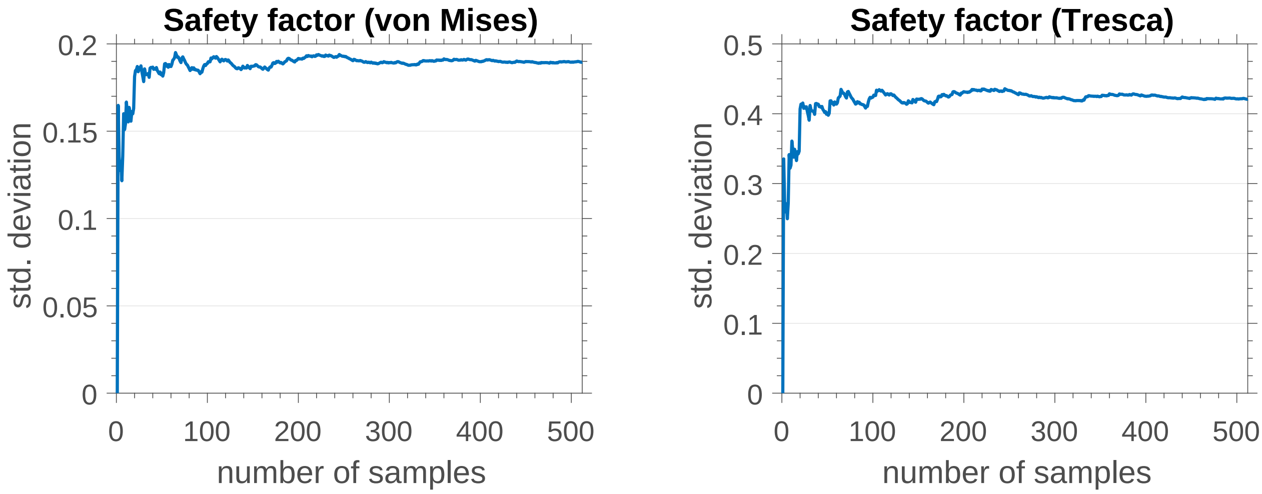
Figure 7 – Comparison between safety factor samples when applied shear stress distribution is a truncated exponential with 3 parameters: a)von Mises; b)Tresca



Source: Prepared by the author

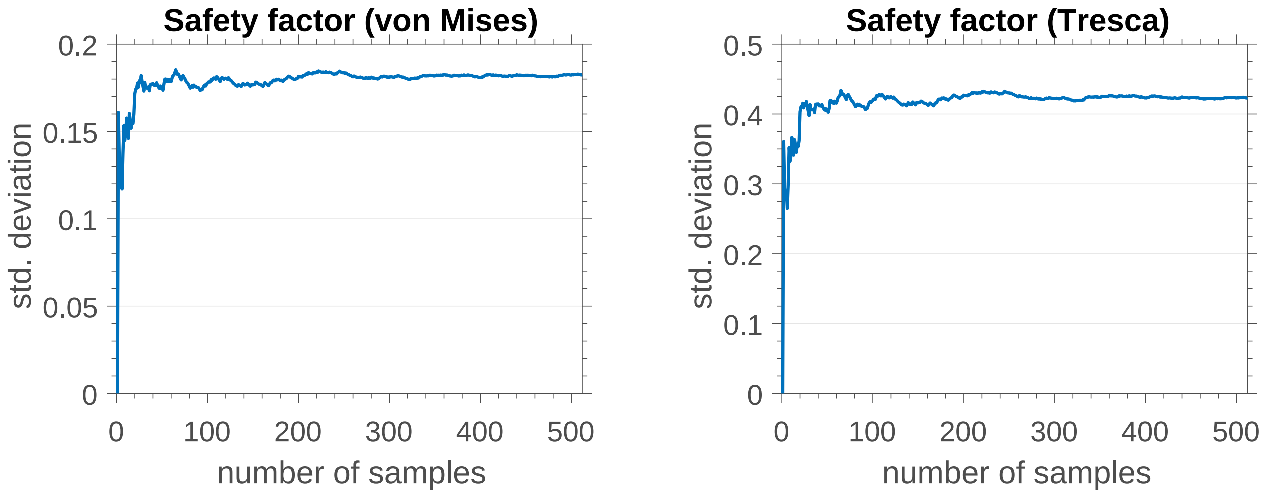
The results in Figures 6 and 7 indicate that the comparison between safety factor samples of von Mises and Tresca failure criterions when the random variable is considered as a shear stress applying with uniform and truncated exponential distribution, respectively. Also, the cross shape figure signifies each sample that is used for force distribution, and the red line, pink dashed line, green dashed line point out the mean, mean plus and minus standard deviation, 95 percent probability boundary lines, respectively in Figures 6 and 7.

Figure 8 – Convergence study of simple plate problem for the safety factor, standard deviation as a function of the number of statistical samples considering an uniform distribution



Source: Prepared by the author

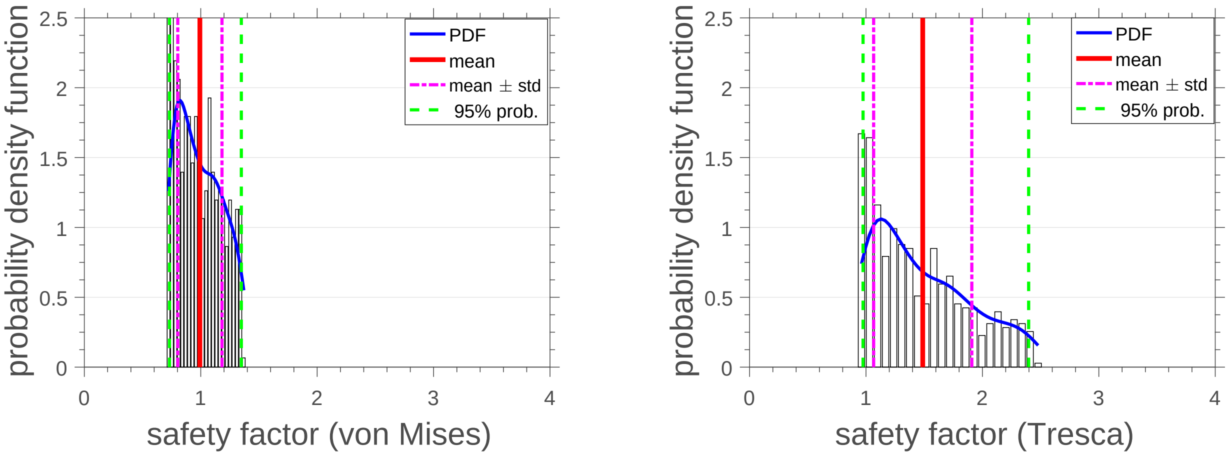
Figure 9 – Convergence study of simple plate problem for the safety factor, standard deviation as a function of the number of statistical samples considering a truncated exponential



Source: Prepared by the author

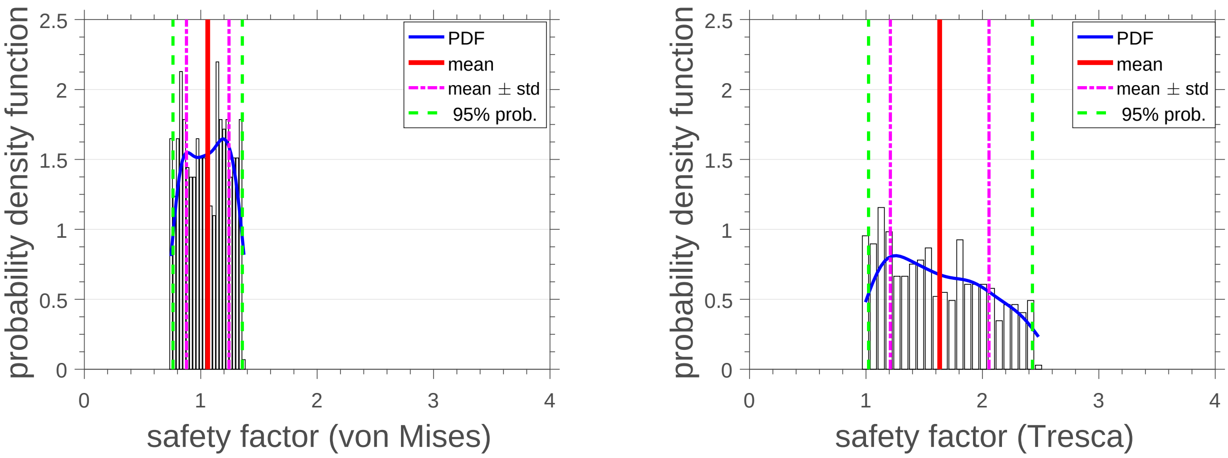
Figures 8 and 9 demonstrate that the convergence plots of simple plate problem for Tresca and von Mises failure criterions which begin to converge after at a value of 200 number of samples nearly. At a value of 512 number of samples, these graphs are approximately stable and converged. The purpose of these convergence plots is that how close to this exact balance is acceptable. Also, converged standard deviation value varies from uniform distribution to truncated exponential distribution.

Figure 10 – Statistical results of probability density function when applied shear stress distribution is an uniform: a)von Mises; b)Tresca



Source: Prepared by the author

Figure 11 – Statistical results of probability density function when applied shear stress distribution is a truncated exponential with 3 parameters: a)von Mises; b)Tresca

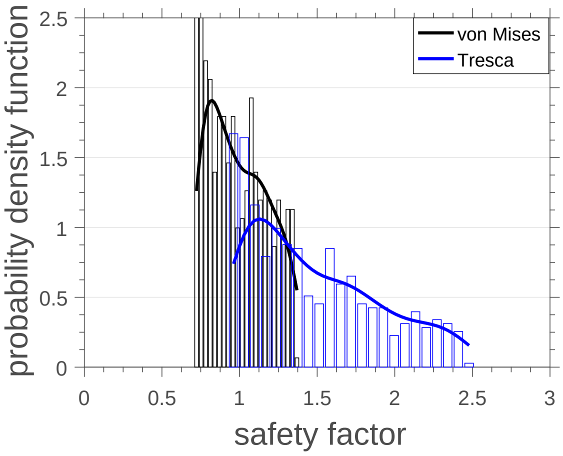


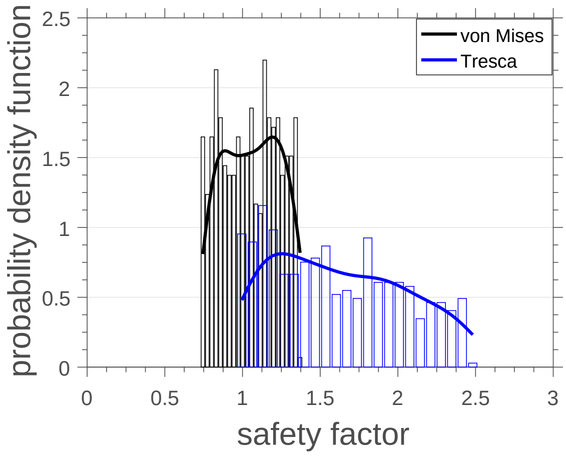
Source: Prepared by the author

Moreover, while obtaining random parameters in this problem, 512 number of samples are utilized by considering different distributions. Figures 10 and 11 show that statistical results of probability density function and distribution when implemented shear stress distributions are uniform and truncated exponential for von Mises and Tresca failure criterions. Moreover, the blue line, red line, pink dashed line and green dashed line represent probability density function (PDF), mean value, mean plus and minus standard deviation value and 95 percent probability boundary lines, respectively in Figures 10 and 11.

The comparison between von Mises and Tresca failure criterions when the shear stress distributions are implemented with uniform and truncated exponential, is indicated in Figure 12. The black line and blue line point out von Mises and Tresca failure criteri-

Figure 12 – Comparison of failure criterions when the shear stress distributions are applied with: a)uniform ; b)truncated exponential





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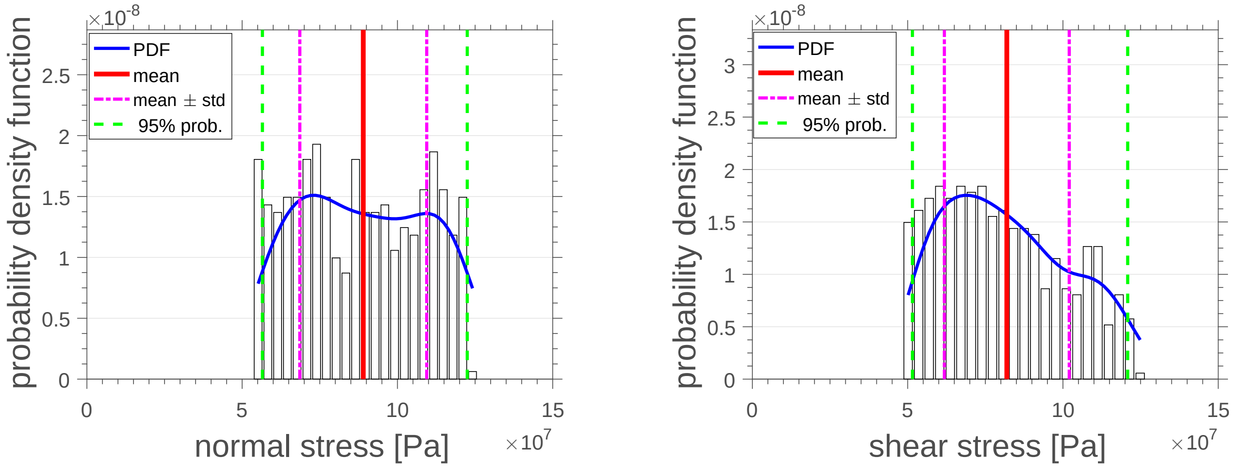
ons, respectively in Figure 12. Furthermore, the closest match between von Mises and Tresca failure criterions which is in the range of 1.35 F_s (Safety Factor), approximately. Furthermore, there is no common intersection area between von Mises and Tresca failure criterion in the range of 0.7-1.0 F_s (Safety Factor) and 1.45-2.5 F_s (Safety Factor). As a result, von Mises and Tresca failure criterion have many common intersection area in the range of 1.0-1.45 F_s (Safety Factor) and the superposition intervals are the same in both distributions in Figure 12.

4.2 Example in plane stress problem considering two uncertainties

In this part of section, a simple plate applying with normal and shear stresses is described in Figure 4, where the value σ_y is chosen for illustration as 100 MPa in the Eq. 10. After giving this value, random values are used by applying uncertainty quantification method considering τ_{xy} and σ_x. Thus, these parameters are uncorrelated and examined in the uniform and truncated-exponential distributions, respectively.

Figure 13 represents that statistical results of input variable when the random variable is considered as a shear stress and normal stress applying with truncated exponential distribution. Minimum support and maximum support of the distribution for shear stress are considered as 50 MPa and 130 MPa, respectively. Also, the mean value of input variable is given as 85 MPa. Furthermore, minimum support and maximum support of the distribution for normal stress are considered as 55 MPa and 125 MPa, respectively.

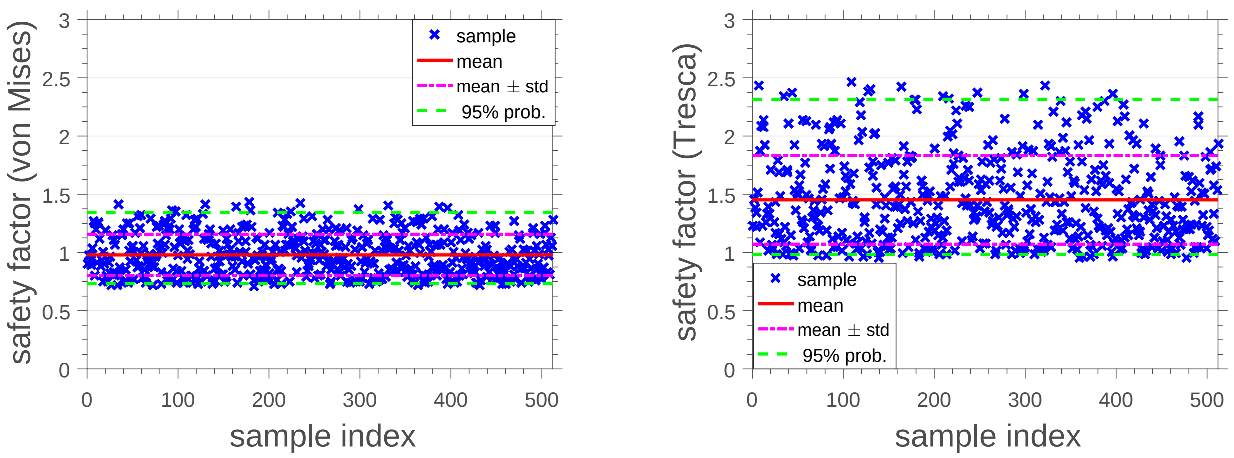
Figure 13 – Illustration of shear stress and normal stress distribution with given by truncated exponential with 3 parameters: a)normal stress; b)shear stress



Source: Prepared by the author

Also, the mean value of input variable is given as 90 MPa. Coefficient of variation for both random variable is considered as 0.2 and shear stress and normal stress distributions are constructed.

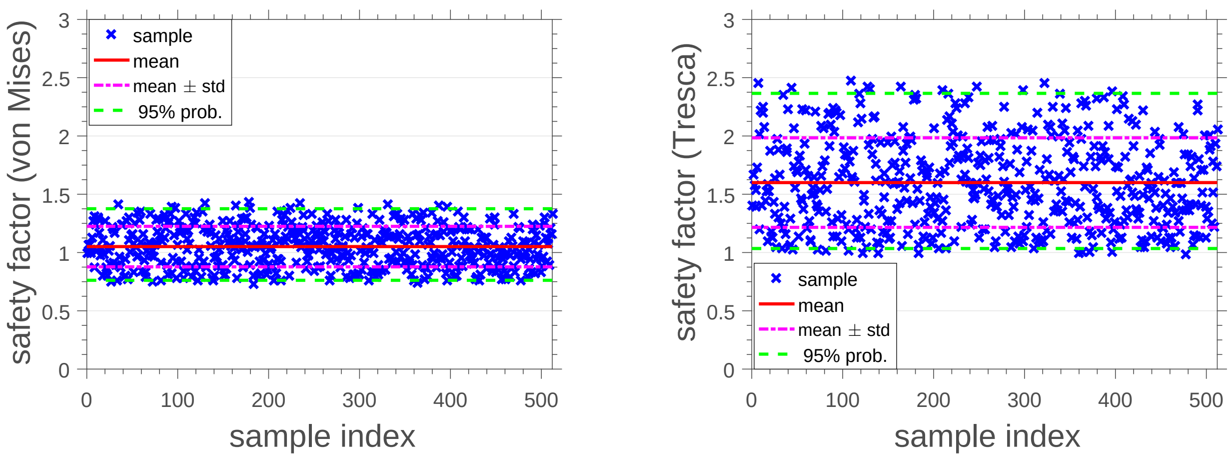
Figure 14 – Comparison between safety factor samples when applied shear stress and normal stress distribution are an uniform: a)von Mises; b)Tresca



Source: Prepared by the author

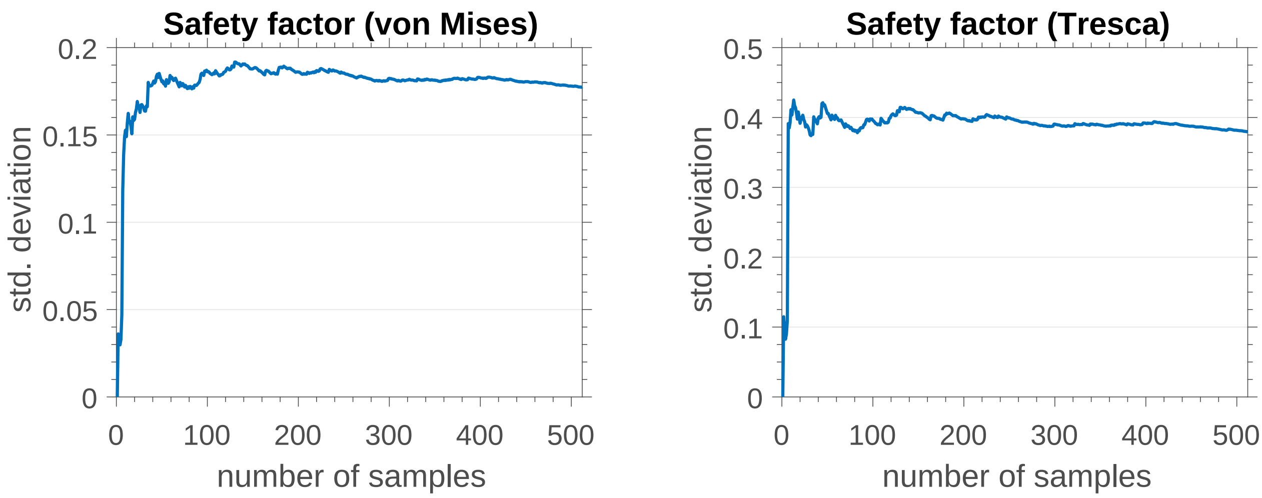
Figures 14 and 15 represent that the comparison between safety factor samples of von Mises and Tresca failure criterions when the random variable is considered as a shear stress and normal stress applying with uniform and truncated exponential distributions, respectively.

Figure 15 – Comparison between safety factor samples when applied shear stress and normal stress distribution are a truncated exponential with 3 parameters: a)von Mises; b)Tresca



Source: Prepared by the author

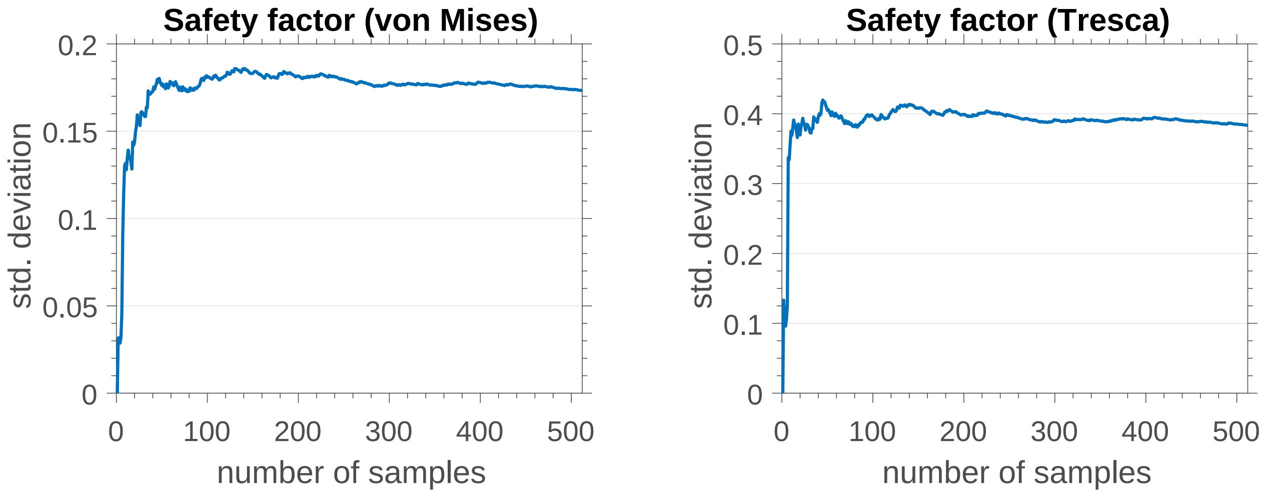
Figure 16 – Convergence study of simple plate problem with two uncertainties, standard deviation as a function of the number of statistical samples considering an uniform distribution



Source: Prepared by the author

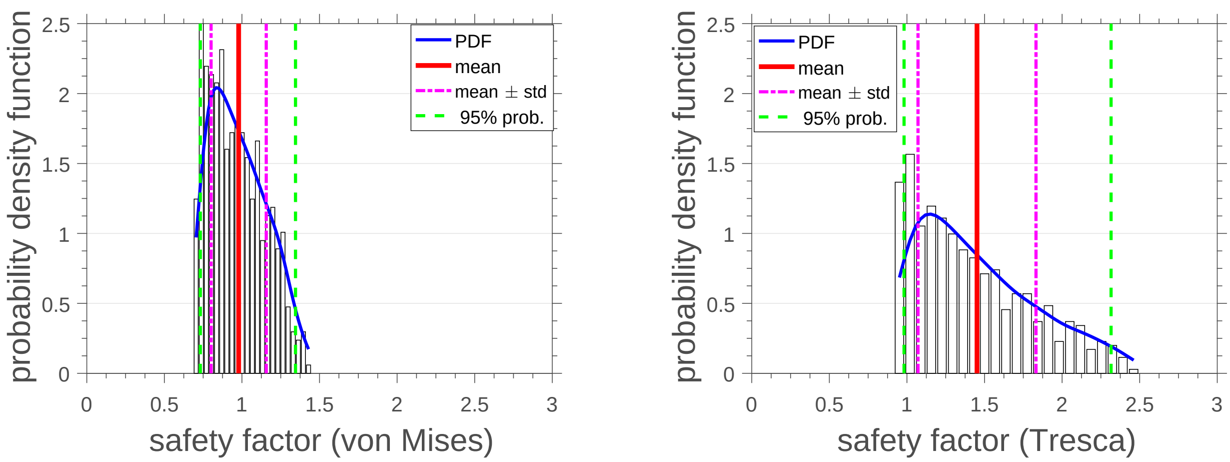
Figures 16 and 17 indicate that the convergence plots of simple plate problem for Tresca and von Mises failure criterions considering the random variables as shear and normal stress. The convergence plots of Tresca and von Mises failure criterions are shown in Figures 16 and 17 and we can see that they start to converge after at a value of 175 number of samples nearly. At a value of 512 number of samples, these graphs are approximately stable and converged.

Figure 17 – Convergence study of simple plate problem with two uncertainties, standard deviation as a function of the number of statistical samples considering a truncated exponential



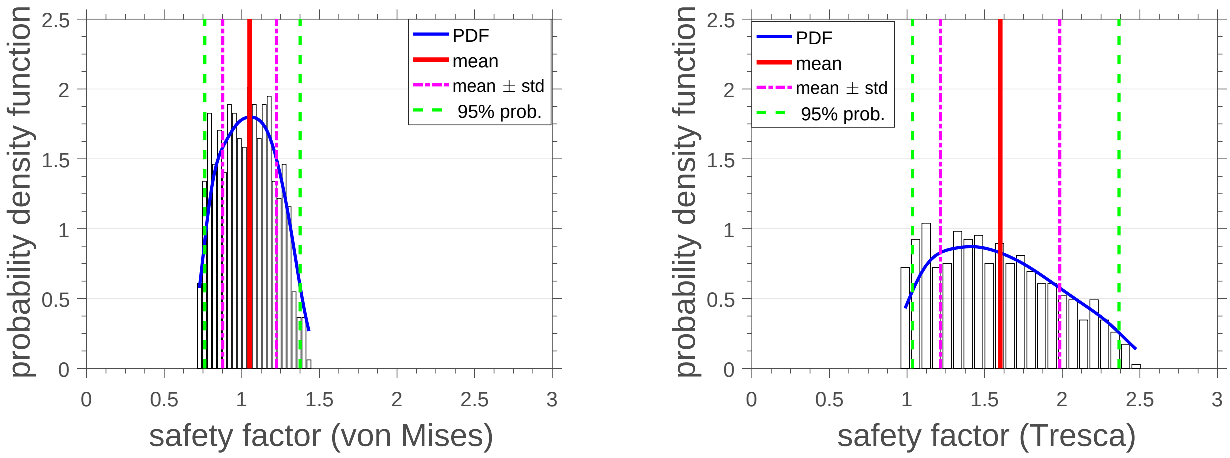
Source: Prepared by the author

Figure 18 – Statistical results of probability density function when applied shear stress and normal stress distribution are an uniform with: a)von Mises; b)Tresca



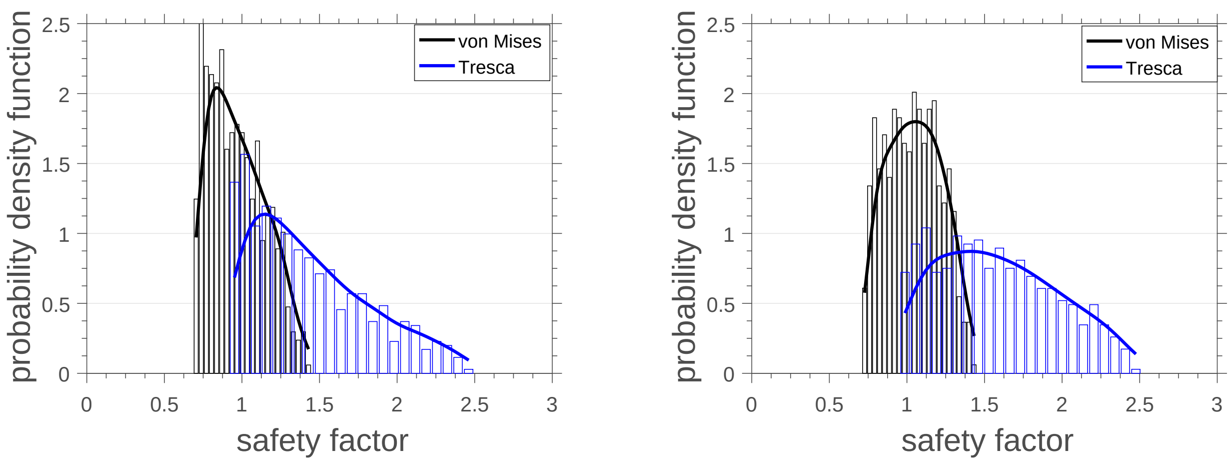
Source: Prepared by the author

Figure 19 – Statistical results of probability density function when applied shear stress and normal stress distribution are a truncated exponential with 3 parameters: a)von Mises; b)Tresca



Source: Prepared by the author

Figure 20 – Comparison of failure criterions when the shear stress and normal stress distributions are applied with: a)uniform ; b)truncated exponential



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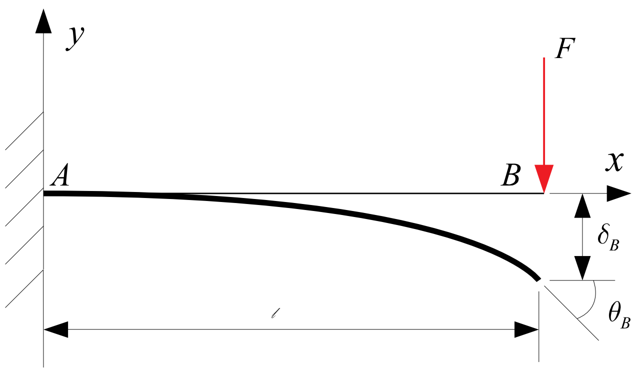
The random parameters τ_{xy} and σ_x are independent in each other. Figures 18 and 19 demonstrate that statistical results of probability density function when utilized shear stress distribution and normal stress distributions are uniform and truncated exponential for von Mises and Tresca failure criterions. The comparison between von Mises and Tresca failure criterions when the shear stress distribution and normal stress distributions are implemented with uniform and truncated exponential, is indicated in Figure 20. The closest match between von Mises and Tresca failure criterions is in the range of 1.35 F_s (Safety Factor), approximately. Besides, there is no common intersection area between von Mises and Tresca failure criterion in the range of 0.7-1.0 F_s (Safety Factor) and 1.45-

2.5 F_s (Safety Factor). As a result of Figure 20, von Mises and Tresca failure criterion have many common intersection area in the range of 1.0-1.45 F_s (Safety Factor).
The main interpretation of the comparison of these results considering one uncertainty and two uncertainties in a plane stress state is that the effect of the σ_x random variable is too less to compare in both situation and the superposition intervals are the same in both distributions in Figure 20.

4.3 The simple deflection problem considering uncertainty quantification

In this part of the section, failure criterions are examined in the design of a cantilever beam with cross-section rectangular shape (CARRERA et al., 2010; GRUTTMANN; WAGNER, 2001). Figure 21 illustrates the simple deflection problem of cross-section cantilever beam subject to a vertical load F at the end of the beam. truncated exponential, uniform, distributions are utilized. The load F which is considered as a random variable. Afterwards, random variables τ_{xy} and σ_x are considered for stress matrix. The

Figure 21 – Deflection beam



Source: Prepared by the author

parameters of simple deflection problem are the cantilever beam's elastic modulus E, inertia moment of area I, vertical load F and cantilever beam's width b, height h, length ℓ and also poisson's ratio ν, yield strength S_y of material, respectively. According to slope function, the value of E and S_y are given as 210 GPa and 250 MPa for solving the equation in a deterministic way, respectively.
The equation of the elastic curve and the deflection and slope at A are determined as shown below

$$\frac{d^2v}{dx^2} = \frac{M(x)}{EI}. \tag{29}$$

Using the free-body diagram of the portion AB of the beam according to Figure 21, if the equation is determined as shown below

$$M(x) = F(\ell - x). \tag{30}$$

Substituting for M into Eq. 29 and multiplying both members by the constant EI, if the equation is determined as shown below

$$EI\frac{d^2v}{dx^2} = (F\ell - Fx). \tag{31}$$

If the equation integrate considering variable x, the equation will be obtained as shown below

$$EI\frac{dv}{dx} = F\ell x - \frac{1}{2}Fx^2 + C_1. \tag{32}$$

It is observed that at the fixed end of B, v(0) = 0 and θ = dv/dx = 0. Substituting these values into Eq. 32 and solving for C_1, then C_1 is determined as shown below

$$C_1 = 0. \tag{33}$$

Then which we carry back into Eq. 32, then the slope function is determined as shown below

$$EI\frac{dv}{dx} = F\ell x - \frac{1}{2}Fx^2. \tag{34}$$

Integrating both members of Eq. 34, then the equation is determined as shown below

$$EIv = \frac{1}{2}F\ell x^2 - \frac{1}{6}Fx^3 + C_2. \tag{35}$$

But, at B we have $x = \ell$, $y = 0$. Substituting into Eq. 35, then the equation is determined as shown below

$$\frac{1}{2}F\ell x^2 - \frac{1}{6}Fx^3 + C_2 = 0, \tag{36}$$

$$C_2 = 0. \tag{37}$$

Carrying the value of C_2 back into Eq. 35, then the equation of the deflection function is determined as shown below

$$v(x) = \frac{1}{EI}\left(\frac{1}{2}F\ell x^2 - \frac{1}{6}Fx^3\right). \tag{38}$$

The boundary conditions are that the displacement and slope are both zero at clamped and from which the two constant of integration can be obtained. Then, the slope function will be maximum at $x = \ell$, the equation of slope function is determined as shown below

$$\frac{dv}{dx} = \frac{1}{EI}\left(F\ell x - \frac{1}{2}Fx^2\right), \tag{39}$$

$$\frac{dv}{dx} = \epsilon_x = \frac{1}{EI}\left(\frac{1}{2}F\ell^2\right). \tag{40}$$

In the Hooke-Lamé's Law in Cartesian Coordinates, shear (G) and lambda (λ) modulus are determined as shown below for explain to σ_x normal stress

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \tag{41}$$

$$G = \frac{E}{2(1+\nu)}, \tag{42}$$

where the parameter σ_x normal stress in the state of tension for simple deflection problem is described as

$$\sigma_x = (\lambda + 2G)\ \epsilon_x, \tag{43}$$

$$\sigma_x = \frac{1}{2}\frac{(1-\nu)F\ell^2}{(1+\nu)(1-2\nu)I}. \tag{44}$$

Furthermore, the parameter τ_{xy} shear stress in the state of tension for simple deflection problem is determined as

$$\tau_{xy} = \frac{1}{2bh}\left(\frac{F\ell^3}{EI}\right). \tag{45}$$

Also, moment of inertia of cantilever beam is described as shown below

$$I = \int y^2 dA = \int_{-h/2}^{+h/2} y^2 dy = \frac{bh^3}{12}. \tag{46}$$

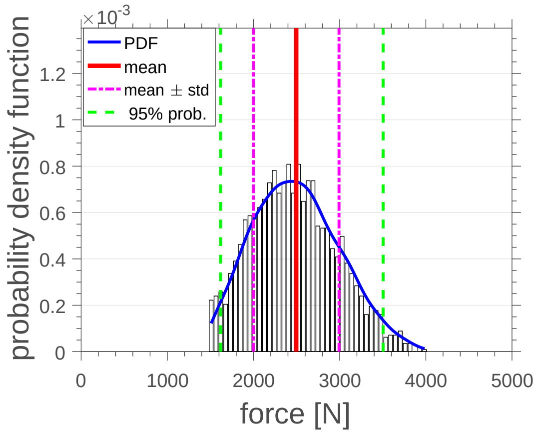
Table 1 – The parameters used in simple deflection problem.

E [GPa]	ν	h [mm]	b [mm]	ℓ [mm]	S_y [MPa]
210	0.3	200	100	2300	250

Source: Prepared by the author

it is indicated that statistical results of input variable when the random variable is considered as a force applying with truncated exponential distribution in Figure 22. Also, minimum support and maximum support of the distribution graph are considered as 1500

Figure 22 – Illustration of force distribution with given by truncated exponential with 3 parameters



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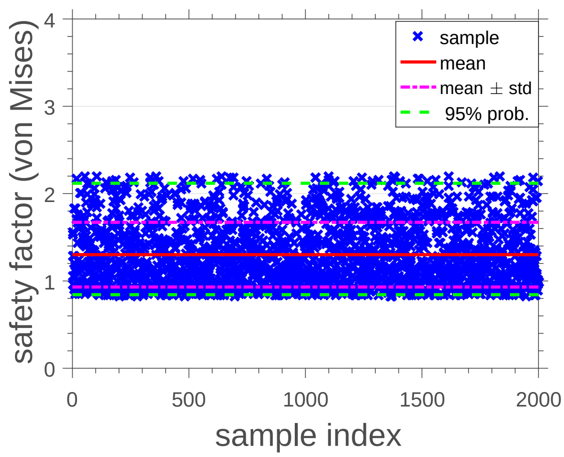
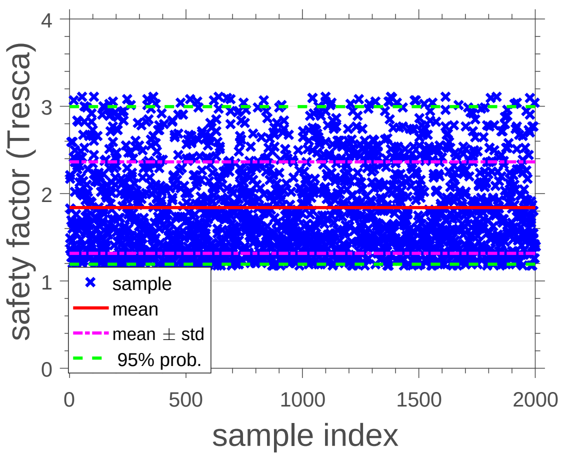
N and 4000 N, respectively. Also, the mean value of input variable is given as 2500 N. Coefficient of variation is considered as 0.2 and force distribution is constructed in Figure 22. Figures 23 and 24 indicate that the comparison between safety factor samples of

Figure 23 – Comparison between safety factor samples when applied force distribution is an uniform: a)von Mises; b)Tresca





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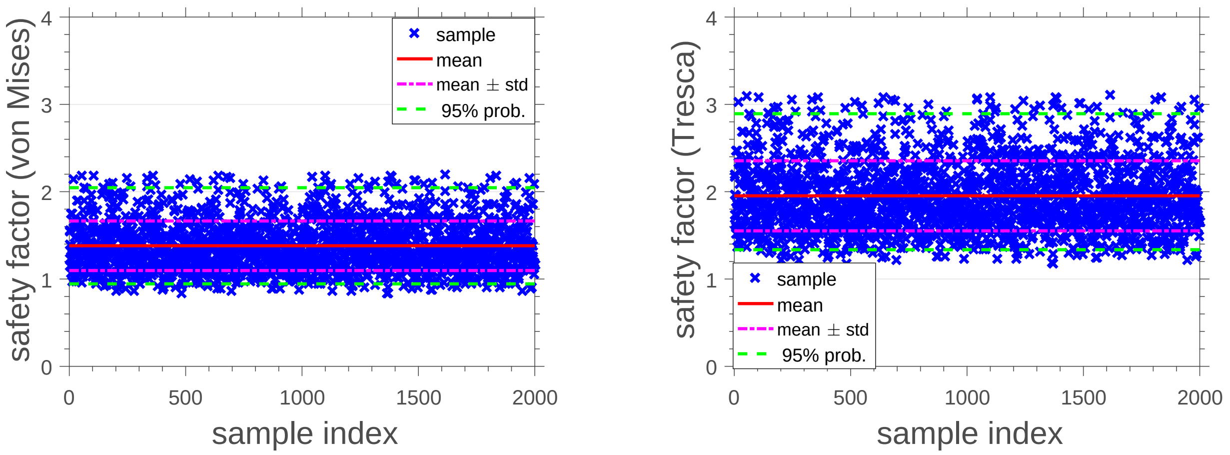
von Mises and Tresca failure criterions when the random variable is considered as a force applying with uniform and truncated exponential distribution, respectively.

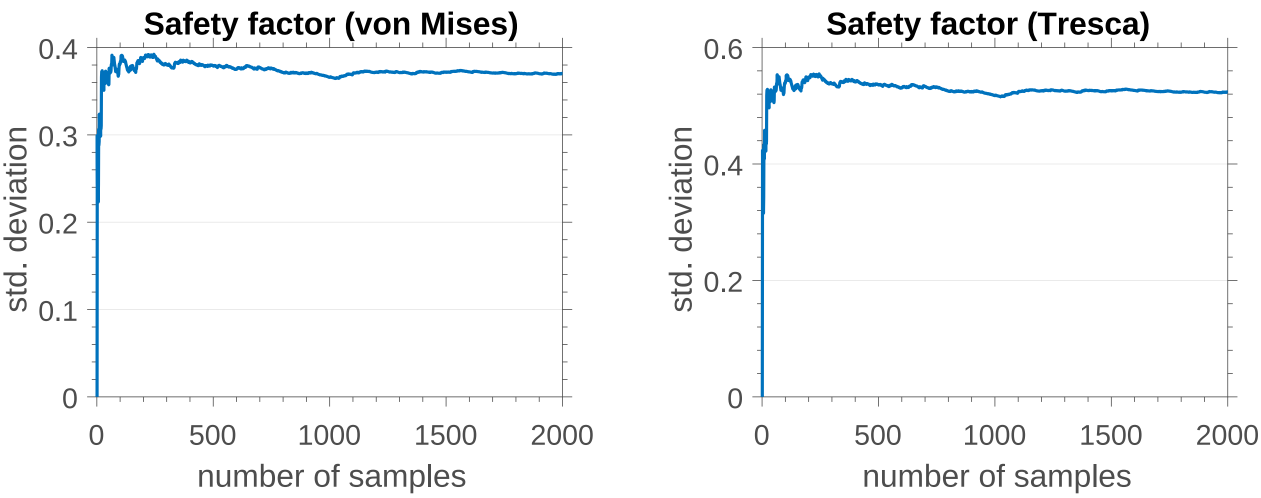
Figures 25 and 26 indicate that the convergence plots of simple plate problem for Tresca and von Mises failure criterions considering the random variables as a force. The convergence plots of Tresca and von Mises failure criterions are demonstrated in Figures 25 and 26 and they begin to converge after 400 number of samples nearly. At a value of

Figure 24 – Comparison between safety factor samples when applied force distribution is a truncated exponential with 3 parameters: a)von Mises; b)Tresca



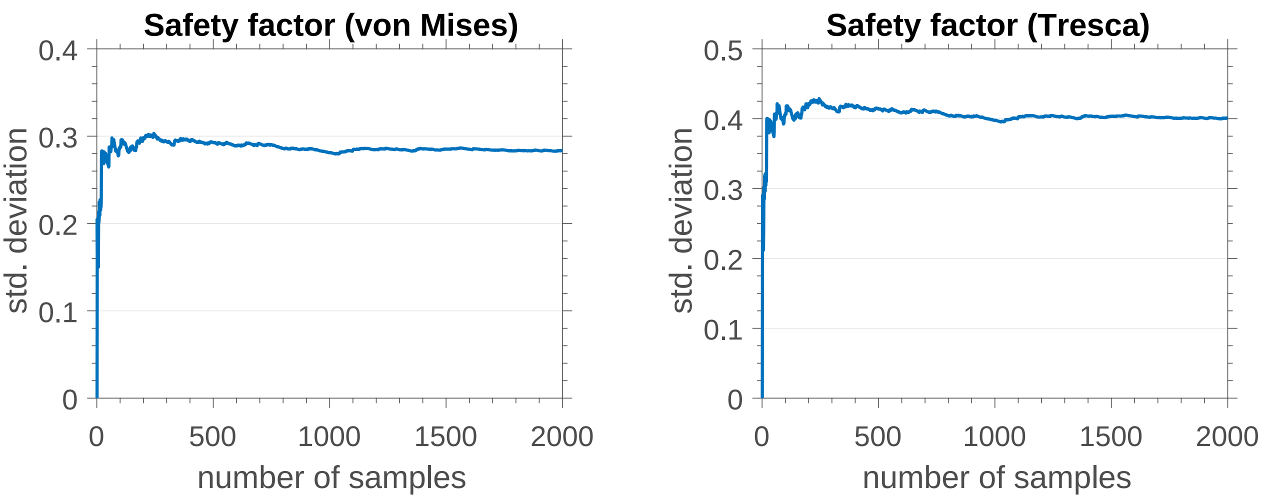
Source: Prepared by the author

Figure 25 – Convergence study for the safety factor, standard deviation as a function of the number of statistical samples considering an uniform distribution



Source: Prepared by the author

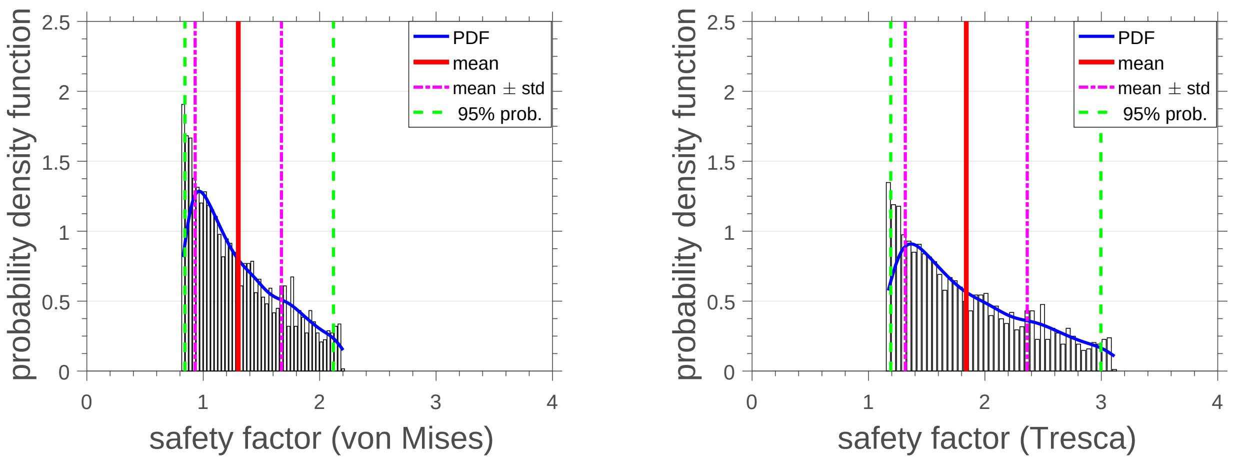
Figure 26 – Convergence study for the safety factor, standard deviation as a function of the number of statistical samples considering a truncated exponential



Source: Prepared by the author

2000 number of samples, these graphs are approximately stable and converged.

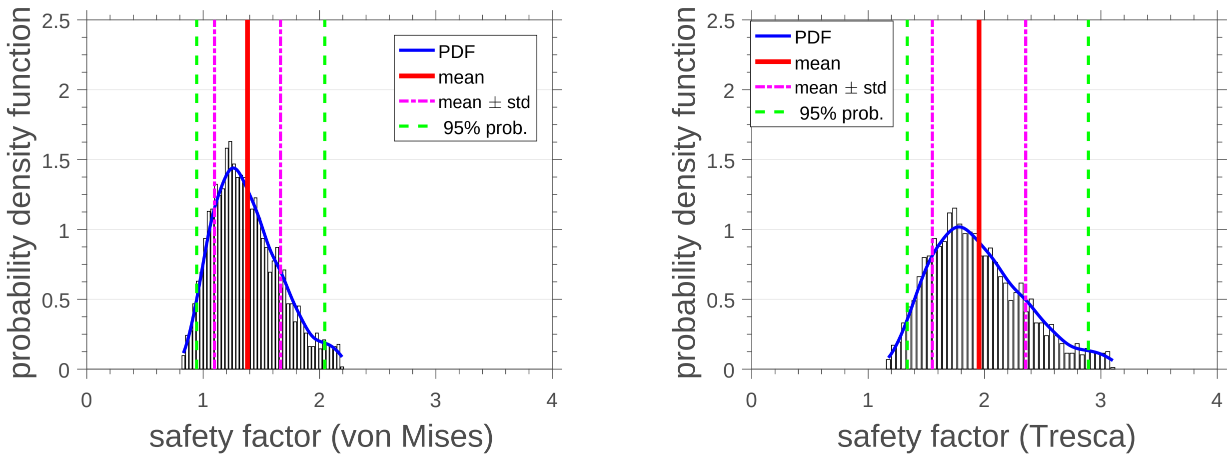
Figure 27 – Statistical results of probability density function when applied force distribution is an uniform: a)von Mises; b)Tresca



Source: Prepared by the author

In addition, Figures 27 and 28 indicate that statistical results of probability density function when utilized force distribution is uniform and truncated exponential for von Mises and Tresca failure criterions. Moreover, Figure 29 indicates that the comparison between von Mises and Tresca failure criterions when the force distribution is implemented with uniform and truncated exponential. These plots show the statistical results of comparison between von Mises and Tresca failure criterions considering different distributions. Besides, the closest match between von Mises and Tresca failure criterion is in the range of 1.6 F_s (Safety Factor). Also, there is no common intersection area between

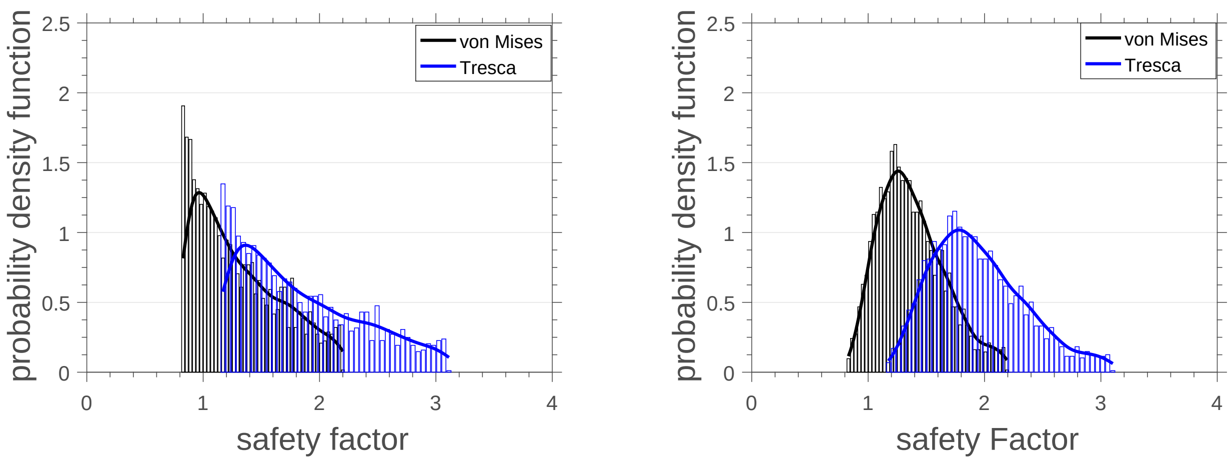
Figure 28 – Statistical results of probability density function when applied force distribution is a truncated exponential with 3 parameters: a)von Mises; b)Tresca



Source: Prepared by the author

von Mises and Tresca failure criterion in the range of 0.8-1.2 F_s (Safety Factor) and 2.2-3.1 F_s (Safety Factor). As a result, von Mises and Tresca failure criterion have many common intersection area in the range of 1.2-2.2 F_s (Safety Factor) and it is seen that von Mises and Tresca failure criterions are not different from each other in superposition area according to Figure 29, when the force is considered as a random variable.

Figure 29 – Comparison of failure criterions when the force distributions are applied with: a)uniform ; b)truncated exponential



Source: Prepared by the author

Furthermore, the main interpretation of the difference between the comparison of these results is that the superposition area is different in each distribution because of different probability values. Nevertheless, the superposition interval in each distribution remains the same.

4.4 Conclusions

Based on the results shown, two different kind of models were analyzed by taking into the consideration with Monte Carlo simulations and maximum entropy principle. These examinations are simple deflection problem and simple plate problem. The aim of this dissertation is to demonstrate that there are some cases that von Mises and Tresca failure criterions have the same characteristic qualification considering with various kind of variabilities. When the results of the comparison of failure criterions obtained by uniform and truncated exponential distributions are examined, It is important to highlight that the distribution graphs have the different kind of behavior considering the superposition area. The uniform distribution generally have greater area comparing to truncated exponential distribution. Nevertheless, these distribution graph does not affect the superposition interval. The superposition interval remains the same in each examination. Also, it is possible to see that the total number of uncertainties affect the behavior of distribution graph in the last examination.

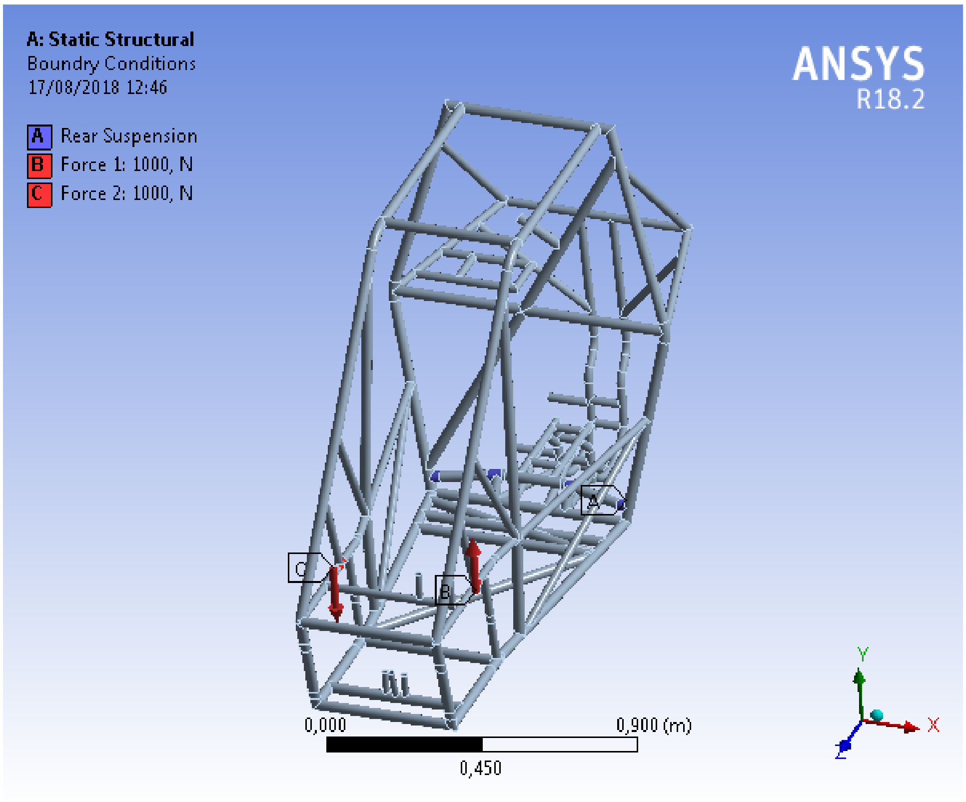
5 Analysis of Uncertainty Propagation in a Complex Structural System

This chapter describes application of the response surface method on the examination of the frame of formula car, optimization process on this examination, definition of the parameters correlation, probabilistic analysis and Six Sigma analysis assuming data uncertainties with Latin Hypercube simulations in the DesignXplorer section in ANSYS. In examination of the frame of formula car design, truncated-Gaussian and log-normal distribution are utilized for solving examination of frame of formula car design. Most of simulations are based on the integration of Monte Carlo (MC) simulations. Two kind of probabilistic methods have been utilized in this chapter for probabilistic examination of frame of the formula car. These two methodologies that are Latin Hypercube sampling method and response surface method. Furthermore, the results of these two methods are interpreted in each other considering their advantages and drawbacks.
This chapter is organized as follows. Section 5.1 indicates the definition about deterministic model of the frame of formula car. Section 5.2 gives a information about response surface method on the examination of the frame of formula car. Section 5.2.1 presents the definition of the parameters correlation on the examination of the frame of formula car. Besides, section 5.2.2 indicates the optimization process considering the examination of the frame of formula car. Finally, section 5.2.3 presents the results about probabilistic analysis on the examination of the frame of formula car.
Hence, the general informations about interpretation of probabilistic results are explained. Also, the main purpose of this chapter is to compare the von Mises and Tresca failure criterions on complex designs with using probabilistic response analysis. Consequently, this chapter introduces probabilistic design outcomes of complex design in the DesignXplorer section in ANSYS. A summary on the data is provided by different kind of graphics in the next sections.

5.1 Description of the deterministic model of the frame of formula car

The geometry of the frame of formula car is built, designed and totally meshed in ANSYS. Also, all material properties as well as static boundary conditions are real numbers. In this part of work, one type of finite element analysis is performed, namely the frame of formula car is quantified with a static analysis using the maximum deflection, von Mises stress, minimum principal stress, maximum principal stress and maximum shear stress. The frame of formula car are shown in Figure 30. The boundary conditions are demonstrated on point A where the tires of formula car locate and the force components are also indicated on point B and C in Figure 30. Furthermore, there are four design variables to optimize the frame of formula car frame, namely the forces, Young's modulus, minimum principal stress, maximum principal stress and von Mises stress.

Figure 30 – The probabilistic result of Matlab



Source: Prepared by the author

Also, a table 3 summarizes parameters used to the frame of formula car design in ANSYS.

Table 2 – Parameters used for the frame of formula car in deterministic way

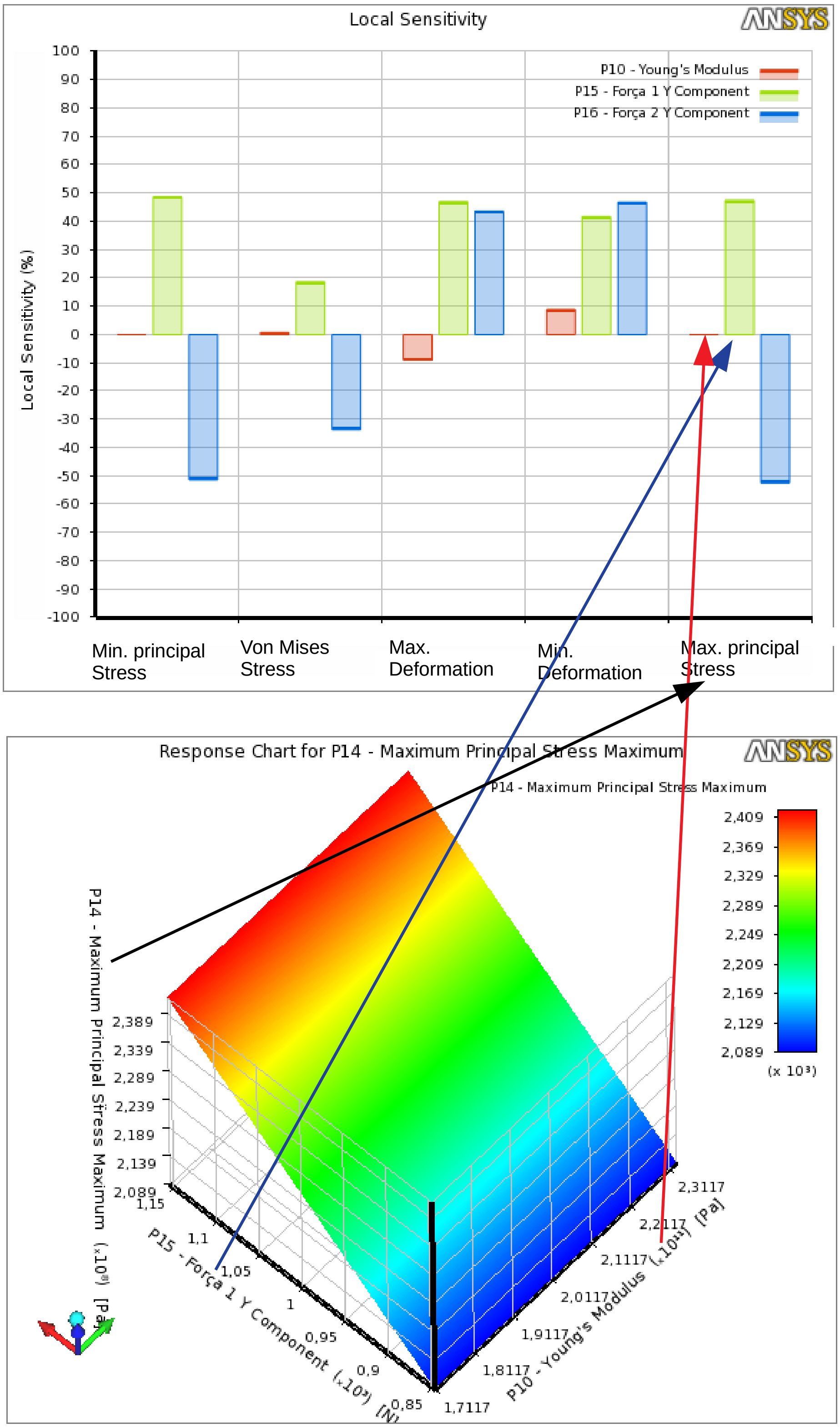
Parameter List		
Parameter type	Parameter	Baseline input
Material	Young's modulus	210×10^9 [Pa]
	Bulk modulus	166×10^9 [Pa]
	Poisson's ratio	0.3
	Tensile yield strength	25×10^7 [Pa]
	Tensile ultimate strength	46×10^7 [Pa]
	Density	7850 $[kg/m^3]$
Geometry	Length X	0.87421 [m]
	Length Y	1.3916 [m]
	Length Z	1.9476 [m]
Force	Force 1	1000 N Y axis
	Force 2	−1000 N Y axis

Source: Prepared by the author

5.2 Description of the response surface method on the examination of the frame of formula car

Response surface method is an efficient way to get the variation of a given performance with respect to input parameters as mentioned before.
Besides, local sensitivity chart and the generated response surface considering the term of the force and Young's modulus are shown in Figure 31. Local sensitivity chart is plotted to notice the effect of the input parameters as the force and Young's modulus on output parameters. This process calculates change of the output parameters based on change of input parameters severally at the available value of each input parameter. The bigger change of the output parameters, the more important is the role of the input parameters that are varied. Moreover, negative sensitivity means that if the output parameter increases, while input parameter decreases. If output parameter increases, while input parameter increases, the sign is positive. It can be noticed that the different two component of the forces (input parameter) have maximum influence on maximum principal stress output parameters comparing to Young's modulus.

Figure 31 – The comparison between local sensitivity chart and the generated response surface considering the term of the force and Young's modulus



Source: Prepared by the author

Furthermore, Young's modulus has a maximum impact on maximum deformation and minimum deformation comparing to other output parameters. These sensitive parameters can be treated considering to reduce critical influence of individual input parameters. It also calls attention to local sensitivity curve indicates important influence on the frame of formula car and output parameters (min principal stress, maximum principal stress, von Mises stress, minimum and maximum deformation). Consequently, it is critical to carefully design each section of the frame of formula car for better structural performance and for structural robustness.

5.2.1 Definition of the parameters correlation on the examination of the frame of formula car

The parameters correlation is based on frame component of formula car. The correlation sorts input and output parameters by importance. This section defines the selected input parameters and their variation range, the selected correlation type, and the generated correlation matrix and charts. The parameters correlation process is performed simulations based on a random sampling of the design space for using (Latin Hypercube sampling). Also, there are two correlation calculation methods (Pearson's linear correlation and Spearman's rank correlation) that are provided. Between two of these parameters correlation, Pearson's linear correlation is utilized because this method provides linear connection between two variables as it is a measure of the linear dependence between two variables. Furthermore, there are two major input parameters (force 1, force 2) considered as a correlation parameters. When the parameters correlation method is performed, 120 number of sample is selected. Also, mean value accuracy and standard deviation accuracy are chosen as 0.01 and 0.02 respectively.

Table 3 – Parameters used for correlation processing

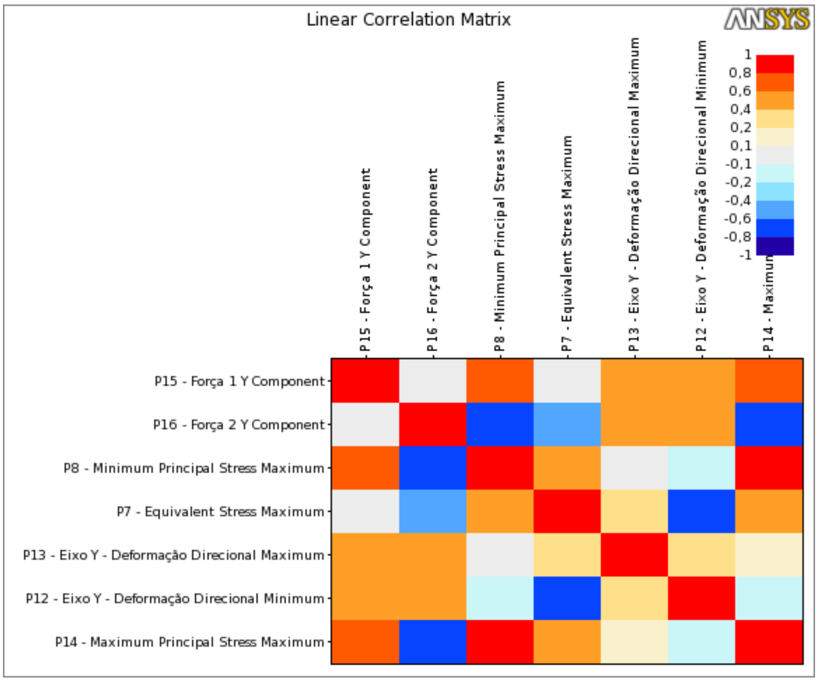
Correlation Parameters		
Parameter Type	Lower Bound	Upper Bound
Force 1	800 N	1200 N
Force 2	−1200 N	−800 N

Source: Prepared by the author

As a results of the linear correlation matrix are essentially relationship between changing in the input parameters and changing in the output parameters in the system. The different kind of colors significant the relationship between output and input parameters

according to type of the color. Dark red and dark blue colors represent the strong correlation between parameters, the gray color significant the zero or weak correlation between parameters considering the linear correlation matrix.
Furthermore, the linear correlation matrix for the probabilistic design is shown in Figure 32

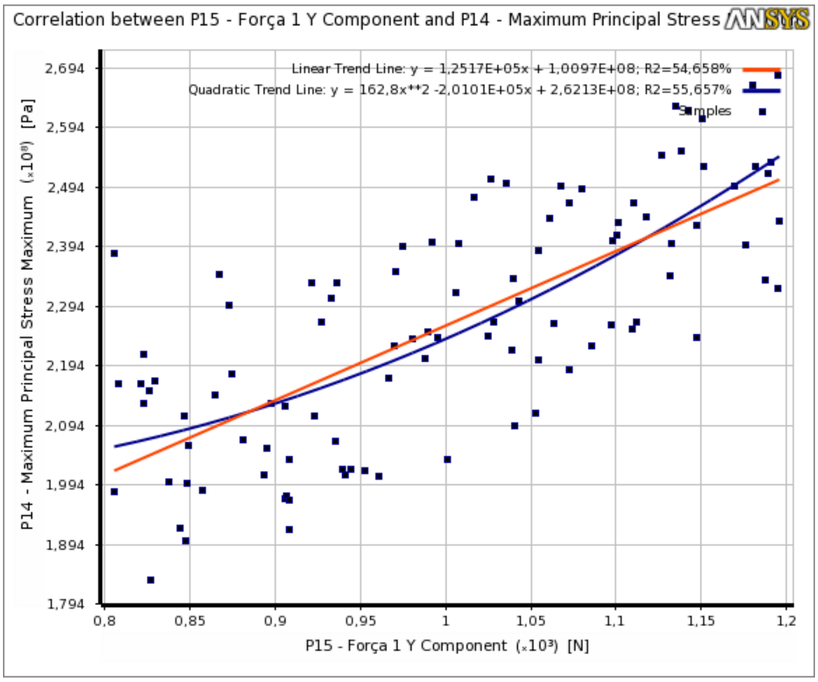
Figure 32 – The linear correlation matrix



Source: Prepared by the author

The closer the R-square is to 100 percent, the better data will fit to linear/quadratic curves. If R-square value is small, then the variation in the output parameter is not well defined by the input parameter variation. Also, R1 and R2 are 54 and 55 percent respectively, according to Figure 33

Figure 33 – The correlation scatter chart



Source: Prepared by the author

5.2.2 Optimization process on the examination of the frame of formula Car

The design optimization process is solved and generated by response surface method. Based on the created responses, 1000 number of samples and 3 design candidates are generated within the minimum and maximum values for variable parameters. Moreover, the screening optimization method that uses a simple approach based on sampling and sorting, is utilized in the optimization. It supports multiple objectives and constraints as well as all types of input parameters.

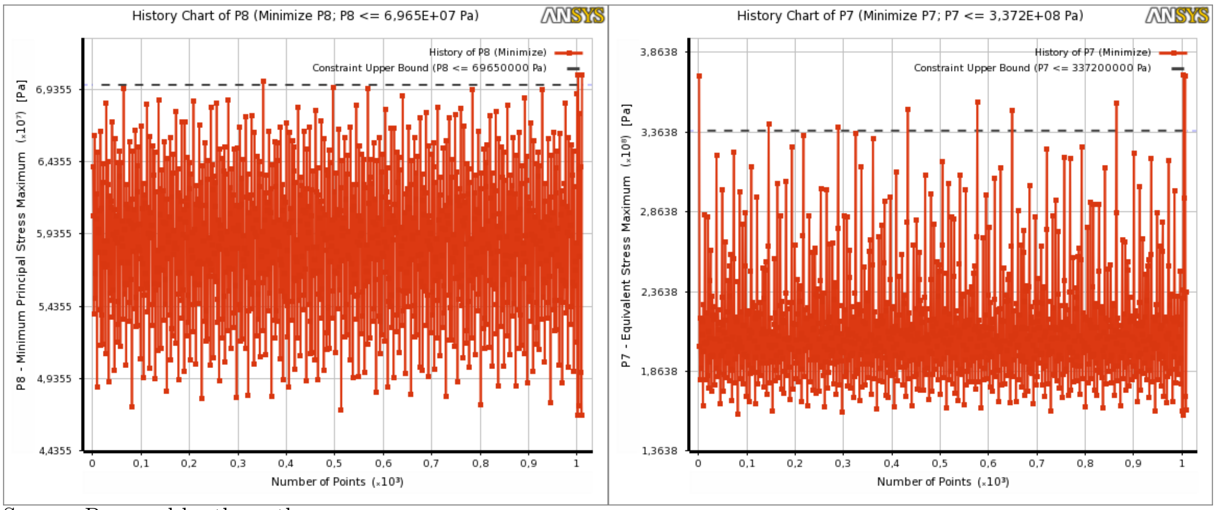
Table 4 – Candidate points that are utilized for the optimization process.

	Young's modulus	Force 1	Force 2
Candidate point 1	16×10^{10} [Pa]	821.74 N	-800 N
Candidate point 2	17.52×10^{10} [Pa]	839.26 N	-817.91 N
Candidate point 3	18.96×10^{10} [Pa]	832.23 N	-809.13 N

Source: Prepared by the author

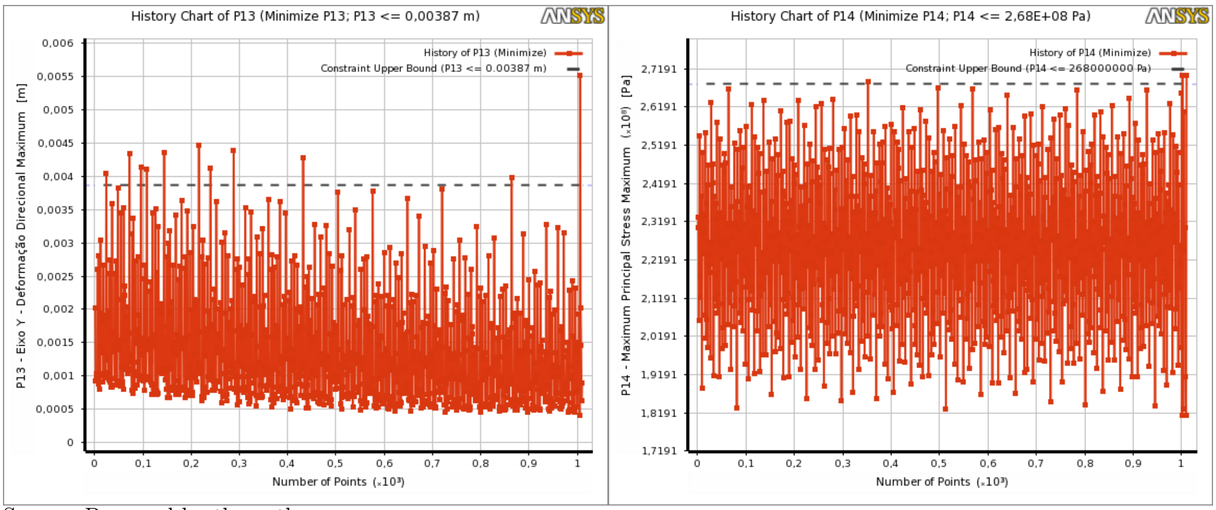
These candidate points are utilized in optimization process in order to maximize the force component and minimize the stress values on the frame of formula car. Usually it is used for preliminary design, which may lead you to apply other methods for more

Figure 34 – History chart of minimum principal stress and maximum equivalent stress maximum



Source: Prepared by the author

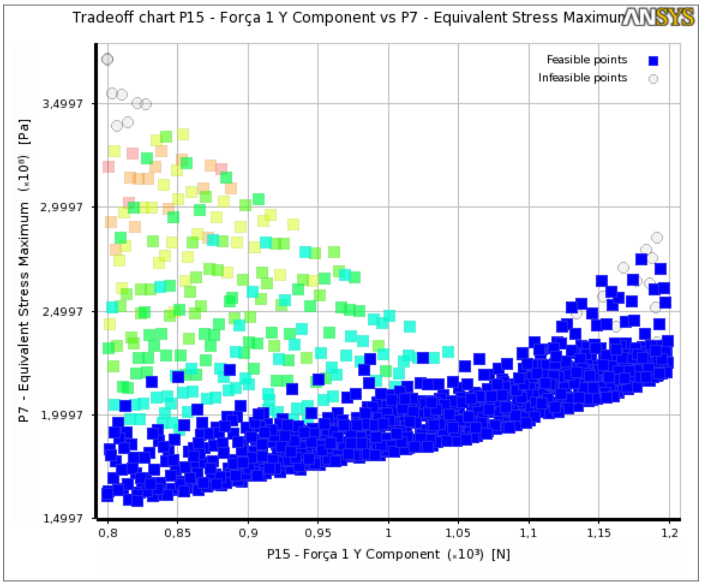
Figure 35 – History chart of maximum deformation in Y axis and maximum pricipal stress maximum



Source: Prepared by the author

refined optimization results. This situation provides an improved response quality and fits higher order variations of the output parameters. Furthermore, all design candidates are analyzed. Figures 34 and 35 show the values of objective parameters at each design point. Figure 36 shows trade-off chart for two objectives, these are von Mises stress and force 1 component. It can be observed that von Mises stress and force 1 component of the frame of formula car are depend on each other. It also demonstrates the feasible and infeasible points on the trade-off chart.
Furthermore, the trade-off chart for the optimization process is shown in Figure 36 according to comparison between von Mises stress and force 1 component.

Figure 36 – The trade-off chart considering the term of the force 1 component and von Mises stress



Source: Prepared by the author

5.2.3 Description of the probabilistic analysis on the examination of the frame of formula car

Probabilistic design can be utilized to define the outcome of one or more variables on the effect of the analysis. The frame of formula car is designed and analyzed in the section of DesignXplorer in ANSYS. Firstly, design of experiment process is utilized and different kind of design points are chosen according to Latin Hypercube sampling design. Also, totally 120 number of design points are utilized for each random input variables. Afterwards, response surface method is used for probabilistic design. Some characteristic material properties as a Young's modulus, force 1 and force 2 component of the frame of formula car are considered as random variables as well, leading to a total of 3 random input variables for frame of the formula car.

Additionally, tables 5 and 6 summarize the random input variables and their distributions.

Table 5 – Random input parameters used for the examination of the frame of formula car.

Parameter type	Input variable	Distribution type
Material	Young's modulus [Pa]	Log-normal distribution
Force	Force 1 [N]	Truncated-Gaussian distribution
Force	Force 2 [N]	Truncated-Gaussian distribution

Source: Prepared by the author

The reliability of the frame of formula car is calculated with two different probabilistic methods implemented in the DesignXplorer section in ANSYS, namely the Latin Hypercube sampling design and the response surface method. Also, 180,000 Latin Hypercube sampling runs on the response surfaces took only about 30 seconds to complete, which illustrates the computational advantage of the response surface method. From Figure 37 to Figure 44 indicate the cumulative distribution function and distribution graphs for each random input variable and random output variable, respectively.

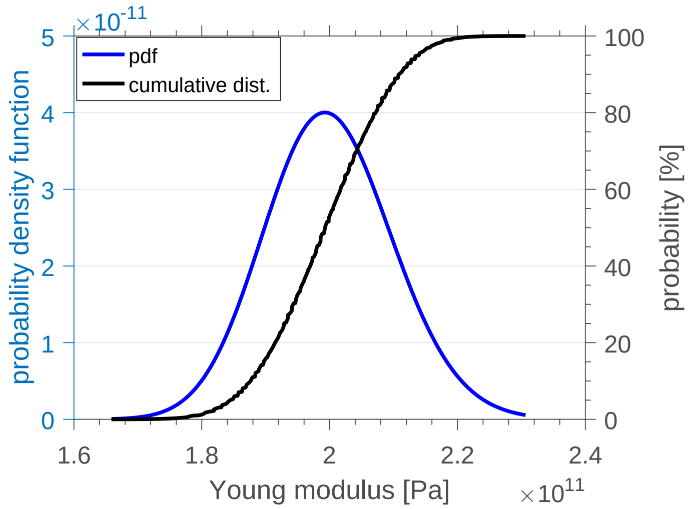
Table 6 – Attributes of the random input variables

Random input variables				
Input variable	Lower bound	Upper bound	Mean value	Standard deviation
Young's modulus [Pa]	0	$+Infinite$	26.02×10^9	0.0499×10^9
Force 1 [N]	800	1200	1000	50
Force 2 [N]	−1200	−800	−1000	50

Source: Prepared by the author

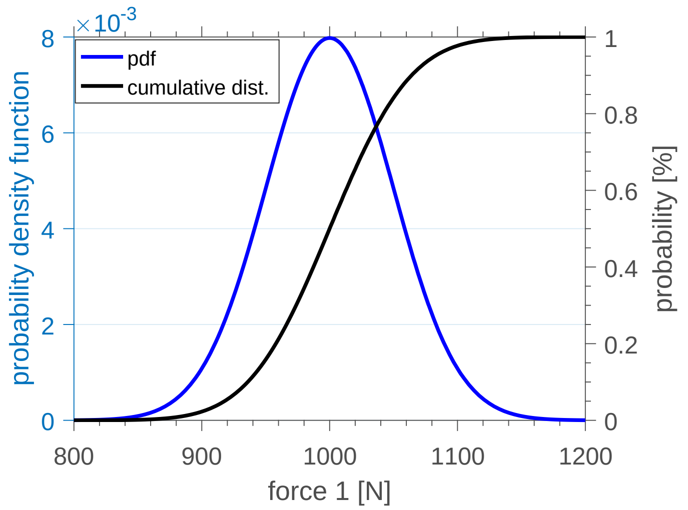
From Figure 37 to Figure 44, these graphics are implemented and constructed considering the data from analysis on the examination of the frame of formula car in ANSYS.

Figure 37 – Log-normal distribution graph for Young's Modulus



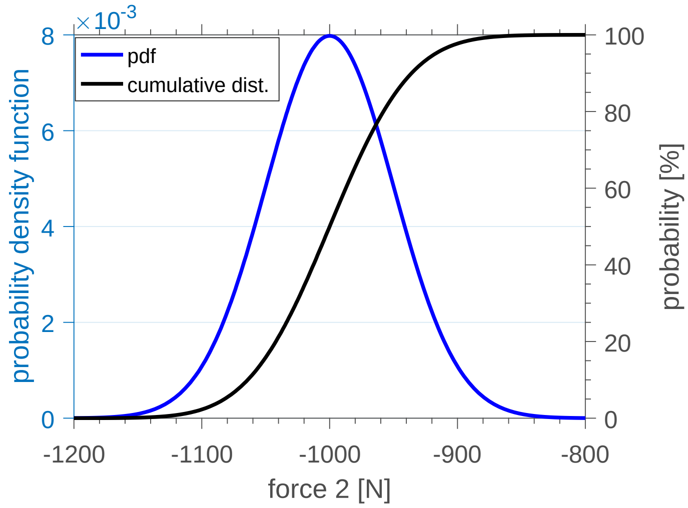
Source: Prepared by the author

Figure 38 – Truncated-Gaussian distribution graph for force 1



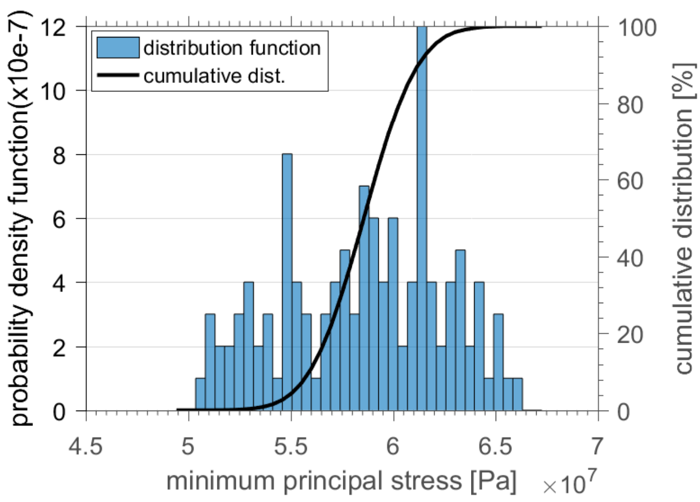
Source: Prepared by the author

Figure 39 – Truncated-Gaussian distribution graph for force 2



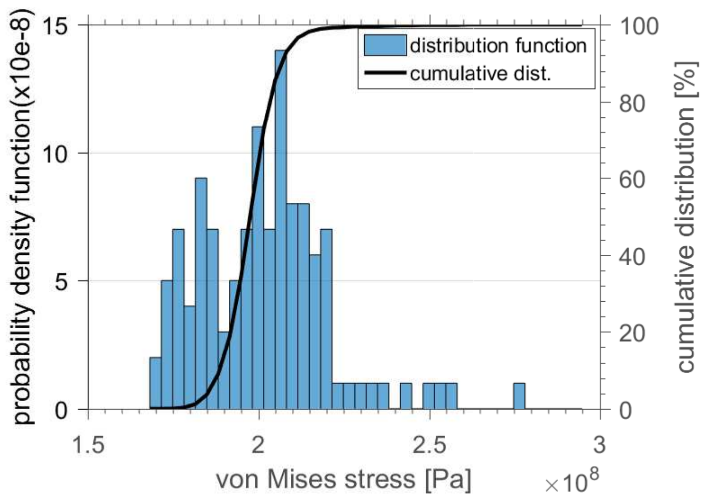
Source: Prepared by the author

Figure 40 – Distribution graph for minimum principal stress



Source: Prepared by the author

Figure 41 – Distribution graph for von Mises stress



Source: Prepared by the author

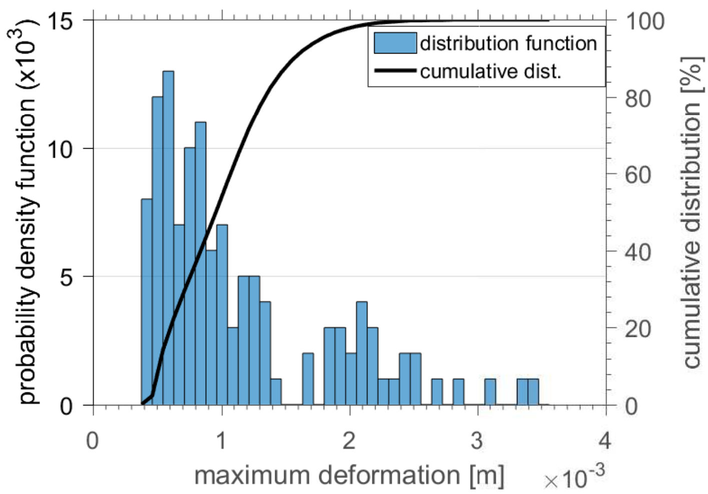
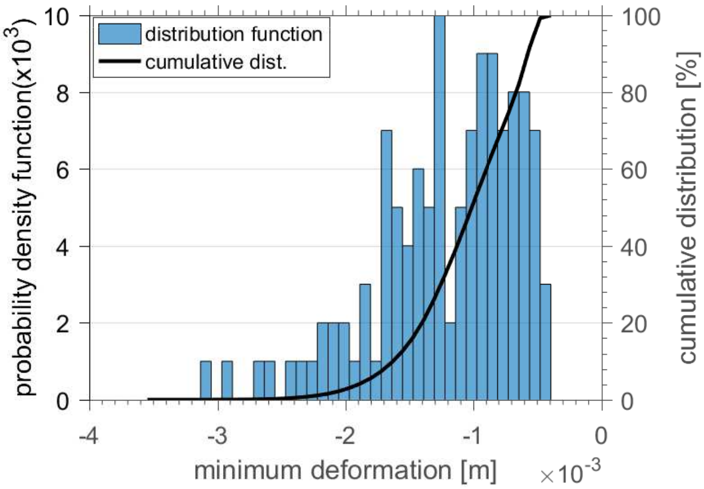


Figure 42 – Distribution graph for maximum deformation in Y axis

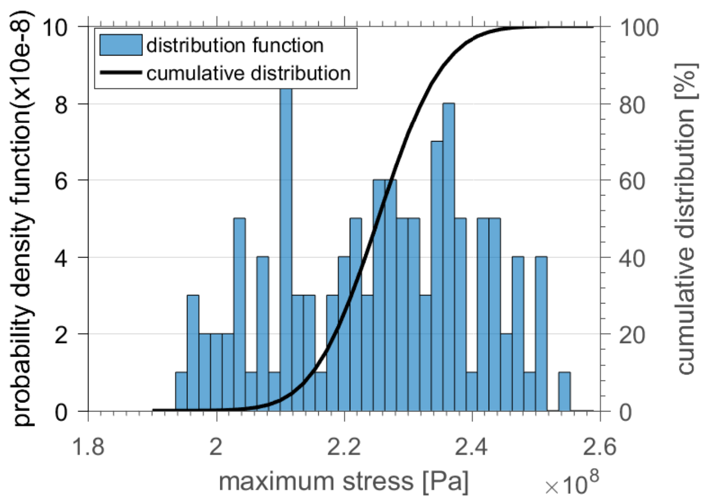
Source: Prepared by the author

Figure 43 – Distribution graph for minimum deformation in Y axis



Source: Prepared by the author

Figure 44 – Distribution graph for maximum principal stress



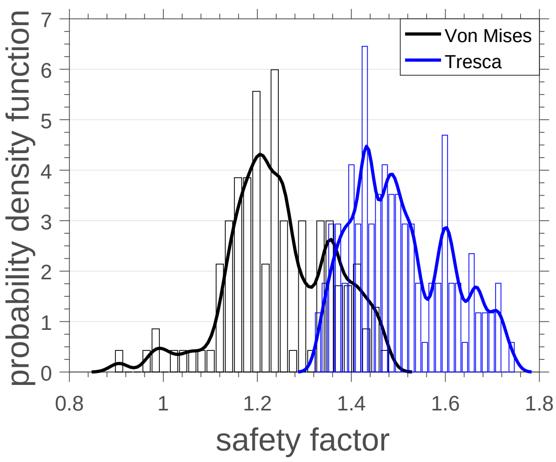
Source: Prepared by the author

After obtaining the results from the section DesignXplorer in ANSYS, the maximum principal stress, minimum stress and von Mises stress values are utilized for the equations for von Mises and Tresca failure criterion. According to the probability density function graph in Figure 45, the F_s values are obtained and von Mises and Tresca failure criterion methods are compared. In this case, while obtaining random input variables, 120 number of samples for each random input variable are described by using Latin Hypercube sampling method in order to compare both probabilistic results.

In addition, Figure 45 shows the closest match between von Mises and Tresca failure criterion is in the range of 1.37 F_s (Safety Factor). Furthermore, there is no common intersection area between von Mises and Tresca failure criterion in the range of 0.84-1.28 F_s (Safety Factor) and 1.53-1.79 F_s (Safety Factor). As a result, von Mises and Tresca failure criterion have many common intersection areas in the range of 1.28-1.53 F_s (Safety Factor).

On the example of frame of formula car, response surface method, parameters correlation, optimization process and probabilistic analysis are examined according to random input variables. From these analyses results, the response surface method is the most effective method in the case of limited input variable parameters. In the case of the random input variable data, Latin Hypercube analysis is the most efficient method for solution of the structural reliability. In summary, the probabilistic methods implemented on the examination of frame of the formula car are discussed and the advantages and disadvantages are compared.

Figure 45 – The probabilistic result of ANSYS



Source: Prepared by the author

5.2.4 Conclusions

This chapter has shown probabilistic design of the examination of frame of the formula car, estimated based on Latin Hypercube sampling design and response surface method, to describe the critical comparison between failure criteria considering uncertainties. There are two probabilistic methods (Latin Hypercube sampling method and response surface method) are analyzed from the point of the accuracy and effectiveness in comparison with the probabilistic methods. Also, it can be seen that von Mises criterion has the same characteristic as Tresca failure criterion in the superposition interval. The effect of the random input variables has shown adequate to allow identifying the critical comparison between failure criteria in an uncertainty scenario.

6 Final Remarks

This final chapter presents in the section 6.1 the main results about the approach and the outcomes showed in this work. Additionally, section 6.2 and 6.3 indicate contributions and the suggestion for future works and applications in the research, respectively.

6.1 Conclusions

Considering the results obtained and presented in this work, methodologies were presented for the comparison of structural criteria of failure by taking into the consideration uncertainty quantification. The description about failure analysis in structural mechanics and general explanation of uncertainty quantification are presented in chapter 3, respectively. The probabilistic results of simple structural systems that are simple plate and simple deflection problem, illustrated in chapter 4. Moreover, the probabilistic outcomes and optimization process of complex structural system is presented in chapter 5, some remarks can be shown:

- The using of von Mises and Tresca failure criteria can help the investigation of predicting the circumstances under which solid materials under the processing of external loads. Moreover, simple plate and simple deflection problem are analyzed in deterministic way and probabilistic way. Based on the results indicated, when the results of the comparison of failure criterions obtained by uniform and truncated exponential distributions are interpreted, it is important to emphasize that the distribution graphs have different kind of behaviour on the superposition area. The uniform distribution generally have greater area comparing to truncated exponential distribution in all examinations. However, these distribution graph does not affect the superposition interval. The superposition interval remains the same in each examination with different distributions. Also, it can be seen that the total number of uncertainties influences the behaviour of distributions on the probabilistic examination. Based on the results of simple structural system, it can be seen that von Mises and Tresca failure criterions have the same behavior in the superposition area with using uncertainties.

- Based on the outcomes of a complex structural design, the effect of random input variables varies from one to the other. Young's modulus almost does not have any effect on random input variables comparing to force components. The dissertation also summarizes different kind of probabilistic methods (crude Monte-Carlo method, Latin Hypercube sampling method, response surface method) affect just the parameters as a computational cost, number of simulations on the same examination.

- Considering the results obtained from a complex structural design, the response surface method is the most useful method with limited random input variables. Also, Latin Hypercube analysis is the most efficient method for solution of the structural reliability, it makes to possible to do same examination with less number of simulations comparing to crude Monte-Carlo simulations. According to results, it can be seen that different kind of random input variables influence random output variables in different way. Unlike Tresca failure criterion, von Mises criterion can have the conservative characterization in the case of uncertainties. In summary, it is important to emphasize that von Mises and Tresca failure criterions have the same characteristic on the complex structural design in case of some areas with various kind of random input variables.

6.2 Contributions

A few examples and applications performed during the master study with the subject of this thesis were published in the following papers for conference and journal:

- Yanik et al.(2018): 'Uncertainty quantification in the comparison of structural criterions of failure' here is the article mentioned (YANIK; SILVA; CUNHA, 2018);

- Also, the following paper was sent for publication: "On the influence of parametric uncertainties in the performance of structural failure criteria" submitted to the International Latin American Journal of Solids and Structures in a partnership with Prof. Dr. Samuel da Silva and Prof. Dr. Americo Barbosa da Cunha Jr.

6.3 Suggestions for future works

Analyzing this work it is possible to define a few future research topics in order to improve the current results and also explore new applications:

- Application of the experimental setup of complex problem to examine the and compare von Mises and Tresca failure criteria.
- Using the more complex structure and adding different kind of random input variables in order to do probabilistic examination.

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