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ROBUST $\mathscr{H}_\infty$ SWITCHED STATIC OUTPUT FEEDBACK CONTROL DESIGN FOR LINEAR SWITCHED SYSTEMS SUBJECT TO ACTUATOR SATURATION

Ilha Solteira
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ROBUST $\mathscr{H}_\infty$ SWITCHED STATIC OUTPUT FEEDBACK CONTROL DESIGN FOR LINEAR SWITCHED SYSTEMS SUBJECT TO ACTUATOR SATURATION

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Prof. Dr. Marcelo Carvalho Minhoto Teixeira
Advisor

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"What we know is a drop, what we don’t know is an ocean." Isaac Newton

ABSTRACT

This thesis is devoted to the study of the robust $\mathscr{H}_\infty$ control problem of continuous-time switched linear systems subject to actuator saturation with polytopic uncertainties, considering an output-dependent switching law and a switched static output feedback controller. The proposed method offers new sufficient conditions based on linear matrix inequalities (LMIs) for designing the switched controllers using parameter-dependent Lyapunov functions. The method is based on a static output feedback $\mathscr{H}_\infty$ control design recently presented in the literature that avoids linear matrices equalities (LMEs) and the need to impose any constraints on output system matrices, that is, the output matrices of the system are allowed to be of non-full row rank. In order to extend those results, the actuator saturation constraint is also studied. Theoretical analyses and simulation results show that these new procedures are less conservative than recent methods available in the literature. The conditions of the proposed methods are a particular class of Bilinear Matrix Inequalities (BMIs), which contain some bilinear terms as the product of a matrix and a scalar, related to a suitable convex combination and scalars parameters to provide extra free dimensions in the solution space. The hybrid algorithm Differential Evolution-Linear Matrix Inequality (DE-LMI), is proposed for obtaining feasible solutions of this particular NP-hard problem. Examples show that the proposed methodologies reduce the design conservatism of two recent known procedures for solving the presented control problems. In particular, an example presents an implementation of the switched controllers in an Active Suspension System manufactured by Quanser®.

Keywords: Robust $\mathscr{H}_\infty$ switching control. Switched static output feedback. Linear matrix inequalities. Actuator saturation.

RESUMO

Este trabalho dedica-se ao estudo do problema de controle robusto envolvendo custo $\mathscr{H}_\infty$ para sistemas lineares chaveados no tempo contìnuo, sujeitos à saturação no atuador e com incertezas politópicas, considerando leis de chaveamento e controladores chaveados dependentes da saída da planta. Os métodos propostos oferecem novas condições baseadas em Desigualdades Matriciais Lineares (LMIs - do inglês, *Linear Matrix Inequalities*) para o projeto de controladores chaveados utilizando funções de Lyapunov dependentes de parâmetros. O método é baseado em um resultado recentemente introduzido na literatura para o projeto de controle $\mathscr{H}_\infty$ de saída o qual evita igualdades matriciais lineares (LMEs - do inglês, *Linear Matrix Equalities*) e a necessidade de impor restrições nas matrizes de saída do sistema, isto é, as matrizes de saída do sistema podem ser de posto linha incompleto. Com o objetivo de estender estes resultados, a restrição de saturação no atuador é estudada. Análises teóricas e resultados de simulações mostram que os novos procedimentos são menos conservativos quando comparados a métodos publicados recentemente na literatura. No método proposto, as condições são uma classe particular de desigualdades matriciais bilineares (BMIs - do inglês, *Bilinear Matrix Inequalities*), as quais contêm alguns termos bilineares devido à multiplicação de matrizes por escalares. Estes termos estão relacionados à combinação convexa das matrizes de chaveamento bem como a outros parâmetros escalares que proporcionam dimensões extras livres no espaço de solução. Para tanto, o algoritmo híbrido denominado DE-LMI (do inglês, *Differential Evolution-Linear Matrix Inequality*) é proposto a fim de encontrar soluções factíveis para este problema *NP-hard*. Exemplos mostram que as metodologias propostas reduzem o conservadorismo de dois procedimentos recentes presentes na literatura para resolver os problemas de controle tratados. Em particular, um exemplo apresenta a implementação do controle chaveado em um sistema de suspensão ativa fabricado pela Quanser®.

Keywords: Controle $\mathscr{H}_\infty$ chaveado e robusto . Controle chaveado estático com realimentação da saída. Desigualdades Matriciais Lineares. Saturação no atuador.

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LIST OF ABBREVIATIONS AND ACRONYMS

LMIs	Linear Matrix Inequalities
BMIs	Bilinear Matrix Inequalities
SOF	Static Output Feedback
T-S	Takagi-Sugeno
DE	Differential Evolution
DE-LMI	Differential Evolution-Linear Matrix Inequality based

LIST OF SYMBOLS

$\mathbb{R}$	Set of real numbers.
$\mathbb{R}^n$	Set of the vectors with $n \times 1$ real elements.
$\mathbb{R}^{n\times m}$	Set of the matrices with $n \times m$ real elements.
$\mathbb{K}_N$	Set of the first N positive integers $\{1,2,\ldots,r\}$.
$M^{'}$	Transpose of the real matrix M.
$M > (\geq) 0$	M is a symmetric positive definite (semi-definite) matrix.
$M < (\leq) 0$	M is a symmetric negative definite (semi-definite) matrix..
I	Identity matrix.
$\|z\|$	Absolute value of a real number z.
$\|\|x\|\|$	Euclidean norm of the vector $x \in \mathbb{R}^n : \|\|x\|\| = \sqrt{x^T x}$.
$\mathrm{co}(\mathcal{M})$	Convex hull of a set $\mathcal{M}$
$\mathscr{L}_2$	Set of all finite vector $\xi(t)$ such that $\int_0^\infty \xi(t)'\xi(t)\mathrm{d}t < \infty$.
Λ_r	Set $\Lambda_r = \left\{\alpha \in \mathbb{R}^n : \alpha_j \geq 0,\ j \in \mathbb{K}_r,\ \sum_{j=1}^r \alpha_j = 1\right\}$.
A_λ	Set $A_\lambda = \sum_{j=1}^r \lambda_i A_i,\ \lambda = \begin{bmatrix}\lambda_1 & \lambda_2 & \ldots & \lambda_r\end{bmatrix}' \in \Lambda_r$.

TABLE OF CONTENTS

1 INTRODUCTION

This chapter aims to familiarise the reader with the topic of investigation, exploiting the results presented in the literature and setting the problem statement and the objectives. Additionally, the thesis outline and the notation used throughout it are presented.

1.1 BACKGROUND

The hybrid system concept arises when the dynamical systems manifest continuous and discrete behaviours. Continuous switched systems are a special case of the hybrids one, which are composed of a family of continuous subsystems where a switching rule or strategy (discrete behaviour) defines the active subsystem at each instant of time (LIBERZON, 2003).

Recently, the designing of control laws for switched systems have received lots of attention (YU; WU, 2015; ZHANG; ZHUANG; BRAATZ, 2016). The growing interest in this topic is mainly due their widespread practical applications, such as power electronics (CARDIM *et al.*, 2009; DEAECTO *et al.*, 2010), embedded systems (ZHANG; HU, 2008), road traffic control strategies (PAPAGEORGIOU *et al.*, 2003), among others. A significant result concerning the stability of switched linear systems was presented in Wicks, Peleties and DeCarlo (1994): it was demonstrated that if there exists a Hurwitz convex combination of the subsystems matrices, then there exists a state switching rule that stabilises the switched linear system. Regarding the concepts of robust stabilisation, Zhai, Lin and Antsaklis (2003) proposed a quadratic stabilisation rule for uncertain switched linear systems based on LMIs. In Lin and Antsaklis (2007) were developed two necessary and sufficient conditions for providing global stability for a class of switched linear systems with time-variant parametric uncertainties. Concerning stability and stabilisability of switched linear systems, in Lin and Antsaklis (2009) can be found a survey of available results and a proposed necessary and sufficient condition for asymptotic stabilisability.

Additionally, Daafouz, Riedinger and Iung (2002) proposed two different LMI-based conditions. In the first one, it is presented a classical method while the second incorporates slack variables in order to relax the conditions. Moreover, Ding and Yang (2009) describe more relaxed conditions through of Finsler's Lemma and piecewise quadratic Lyapunov functions for Static Output Feedback (SOF) control. For robust stabilisation of switched linear systems, when all subsystems matrices are not Hurwitz, in Yu and Wu (2015) are presented sufficient conditions, under some assumptions, for stability using the invariant subspace theory and average dwell time method. Considering some hypotheses, the authors in Yu and Zhao (2016) developed a necessary condition of stability for discrete-time switched linear systems.

With regard to the output feedback control design problem for uncertain switched linear systems, its solution is among one of the most challenging problems in literature, due to their non-convex characteristic (SADABADI; PEAUCELLE, 2016; SYRMOS *et al.*, 1997). Nevertheless, in recent years, the design of SOF controllers has been scrutinised by several authors, mainly due to the use of output feedback techniques results in simpler implementation routines for practical applications. In Peaucelle and Arzelier (2005), the authors proposed a two-step iterative algorithm focused on $\mathscr{H}_2$ optimisation. In doing so, Agulhari, Oliveira and Peres (2010) presented an extension of a previous method considering polynomial Lyapunov functions.

Concerning a performance criterion, in this case, the $\mathscr{H}_\infty$ cost, several authors have proposed LMI-based conditions considering an output feedback strategy. Crusius and Trofino (1999) presented sufficient conditions for SOF controllers, adopting linear matrices equalities and inequalities and imposing constraints on the output systems matrices. In this approach, the output systems matrices are required to be full-row rank. Aiming to overcome these drawbacks, Dong and Yang (2013) presented new LMI conditions, for cases where the output matrix is not required to be full rank. In sequence, Chang, Park and Zhou (2015) have extended the flexibility of conditions for robust SOF $\mathscr{H}_\infty$ controller design. More specifically, the developed method is applicable for uncertain systems relaxing the constraint in system matrices.

Furthermore, in Shi *et al.* (2017), the authors investigated the dynamic output feedback $\mathscr{H}_\infty$ control for a class of switched systems with mode-dependent average dwell time switching. In (WU *et al.*, 2017), it was proposed a sliding mode control (SMC) for stochastic systems via output feedback considering among others, the exogenous disturbance constraint in SMC design. An extended state observer (ESO) was used in order to reject external disturbance considering SMC for power converters (LIU *et al.*, 2017). Additionally, in Ban *et al.* (2018), it was designed a technique for robust $\mathscr{H}_\infty$ finite-time control for discrete-time polytopic uncertain switched linear systems.

The actuator saturation constraint plays an important role regarding the design of controllers due to practical applications limitations. Besides compromises the closed-loop performance, considering that the controller was not designed taking into account that constraint, if the closed-loop system is under saturation it may become an unstable system (CAO; LIN, 2003). To deal with the problem of design controllers under actuator saturation restriction Hu, Lin and Chen (2002) employs an improved condition based on convex hull representation by means of adding auxiliary matrices.

Concerning switched controllers and actuator saturation constraint, Alves *et al.* (2016) provided conditions to the design of smoothing switched controllers for uncertain nonlinear systems subject to actuator saturation. To cope with the robust $\mathscr{H}_\infty$ control design for Takagi-Sugeno (T-S) fuzzy system subject to actuator saturation, Oliveira *et al.* (2018) propose conditions to obtain switched controllers. Moreover, the bounded energy disturbance approach allowed a significant result, thus, this concept is adopted in the present work.

1.2 PROBLEM STATEMENT AND CONTRIBUTIONS

In practical applications, the state vector may not be completely available. In this situation, it is important to aim at strategies for switching based on the measured output of the plant. Nowadays, to the best of the author's knowledge, there are not available in the literature papers which consider switched SOF $\mathscr{H}_\infty$ controllers design for uncertain switched linear systems subject to actuator saturation with output-depending switching. Regarding the aforementioned researches, usually, papers on this subject consider full or reduced order output feedback controllers through of estimated state-dependent switching or state feedback.

The major contribution proposed is an exclusively output-dependent switching strategy jointly with the design of switched SOF $\mathscr{H}_\infty$ controllers to cope with the actuator saturation constraint. It is important to highlight that the proposed methodology also provides conditions to design switched output feedback $\mathscr{H}_\infty$ controllers for plants with only one dynamic subsystem. These two different situations are detailed in numerical examples.

Two different strategies to design output feedback controllers considering output dependent switching strategy for uncertain switched linear systems are presented. Firstly, it is considered that there does not exist exogenous input either control input. In sequence, the results presented in Mainardi Júnior *et al.* (2015) are relaxed and the inclusion of guaranteed cost performance and decay rate criterion is approached. Following, novel conditions are proposed based on less conservative results available in Liu and Zhang (2003), Teixeira, Assunção and Avellar (2003), Mozelli and Palhares (2011).

Besides that, novel and less conservative conditions for switching SOF $\mathscr{H}_\infty$ control of continuous-time switched linear systems are proposed. The conditions are based on a recent SOF $\mathscr{H}_\infty$ control design presented in Chang, Park and Zhou (2015) that avoids linear matrices equalities and does not impose any constraints on output systems matrices, as treated in Crusius and Trofino (1999). The results presented in Chang, Park and Zhou (2015) are relaxed considering the inclusion of switched output feedback $\mathscr{H}_\infty$ controllers jointly with an output-dependent switching strategy.

Furthermore, as a step to achieve the main result, conditions design to robust SOF $\mathscr{H}_\infty$ design for system subject to actuator saturation, based on Chang, Park and Zhou (2015), are proposed.

Finally, the main result that concerns conditions to design robust $\mathscr{H}_\infty$ switched static output feedback control design for linear switched systems subject to actuator saturation, using parameter-dependent Lyapunov function, is stated. This result takes into account the contributions of (ALVES *et al.*, 2016) and (OLIVEIRA *et al.*, 2018) concerning switched controllers under saturation, operation region and bounded energy disturbance approach.

Moreover, some bilinear terms appear in conditions of the proposed theorems. The conditions of the proposed methods are a special class of BMIs (Bilinear Matrix Inequalities), which contain some bilinear terms as the product of a matrix and a scalar, related to a suitable convex combination and two scalar parameters to provide extra free dimensions in the solution space. Currently, to the best of the authors' knowledge, there are not available solvers (deterministic methods) able to find the optimum solution for non-convex problems. Thus, the proposed design method of the output gains in order to stabilise an uncertain switched linear system is an NP-hard problem (LIN; ANTSAKLIS, 2009). Therefore, it is proposed the use of a hybrid metaheuristic technique, called DE-LMI (Differential Evolution - Linear Matrix Inequality) (STORN; PRICE, 1997) for finding quasi-optimum parameters values and/or a suitable convex combination in the design of SOF gains (SANDOU, 2013). The proposed procedure can also be used for designing robust controllers for uncertain plants subject to structural failures, considering the plant uncertainties and the structural failures as polytopic uncertainties (SILVA *et al.*, 2013).

1.3 OUTLINE AND NOTATION

- Chapter 2 presents the initial concepts involving dynamical systems, Lyapunov theory and a general definition of polytopic uncertain switched linear systems.

- Chapter 3 addresses the first and second problems statement and relaxation results for robust SOF control design for uncertain switched linear systems with an output dependent switching law introduced in Mainardi Júnior *et al.* (2015) jointly with a performance criterion and the decay rate. Based on the relaxation concepts available in Liu and Zhang (2003), Teixeira, Assunção and Avellar (2003) and Mozelli and Palhares (2011) are proposed novel conditions for stability of uncertain switched linear systems. A theoretical analysis shows if the conditions given in Mainardi Júnior *et al.* (2015) hold, then the novel proposed conditions also hold. A numerical example illustrates the flexibility obtained through of these less conservative conditions comparing feasible area and guaranteed cost obtained in the theorems proposed in this chapter.

- Chapter 4 presents the results for robust SOF $\mathscr{H}_\infty$ control developed in Chang, Park and Zhou (2015). The third problem is stated and it is developed novel and less conservative conditions for robust switching SOF $\mathscr{H}_\infty$ control of continuous time switched linear systems. Furthermore, the results available in Chang, Park and Zhou (2015) are generalised through a switched output $\mathscr{H}_\infty$ controller. A theoretical analysis shows that these new conditions hold when the conditions presented in Chang, Park and Zhou (2015) hold. Finishing the contributions of this chapter, it is showed that the proposed methodology to design robust SOF $\mathscr{H}_\infty$ switched controllers can be directly applied to non-switched linear systems. In Example III (Chapter 8) is shown that there exist cases where the conditions from Theorem 7 are less conservative than the conditions from Theorem 2.

- Chapter 5 deals with the problem of design output-dependent $\mathscr{H}_\infty$ controllers for linear systems subject to actuator saturation. The conditions presented in Chang, Park and Zhou (2015) are exploited to obtain conditions to cope with the $\mathscr{H}_\infty$ problem considering operation region for linear systems subject to actuator saturation.

- Chapter 6 addresses the main contribution, aiming the design of switched static output feedback controllers to cope with the $\mathscr{H}_\infty$ control problem for linear switched systems subject to actuator saturation and considering operation region.

- Chapter 7 introduces the DE-LMI algorithm and briefly describes how it is applied in order to solve the proposed control problem.

- Six examples in Chapter 8 illustrate the effectiveness of the proposed methods. The first example is related to switched systems without either control input or disturbance. The following examples show, in some cases, that proposed conditions hold while the conditions present in literature do not hold. Furthermore, a practical application and an implementation example are presented.

- Finally, in Chapter 9 the conclusions and suggestions for future work are discussed.

1.3.1 Notation

The notation used in this document are described as follows. For real matrices or vectors $(')$ indicates transpose. The set composed by the first N positive integers $\{1,\ldots,N\}$ is represented by $\mathbb{K}_N$. The set of all vectors $\lambda = [\lambda_1 \ \ldots \ \lambda_N]'$ such that $\lambda_i \geq 0$, $i \in \mathbb{K}_N$ and $\lambda_1 + \lambda_2 + \ldots + \lambda_N = 1$ is designated by Λ_N. The convex combination of a set of matrices $(A_1,\ldots,A_N)$ is denoted by $A_\lambda = \sum_{i=1}^{N} \lambda_i A_i$, where $\lambda \in \Lambda_N$. In addition, an asterisk $(*)$ will be used in matrix expressions to express the transpose of the symmetric element. Moreover, for in-line expressions, the symbol $(*)$ represents the transpose of the left side term. The notation $\mathrm{He}(M)$ refers to $M + M'$. The set of all finite $\zeta(t)$ trajectories, such that $\int_0^\infty \zeta(t)'\zeta(t)dt < \infty$ is denoted by $\mathscr{L}_2$. For simplicity of notation, $\sigma(t) = \sigma$. Abusing of the notation already defined as Λ_N, the following one denotes the convex combination of the vector $\alpha \in \mathbb{K}_r$,

$$\alpha = [\alpha_1 \ \alpha_2 \ \ldots \ \alpha_r]^T \in \Lambda_r = \left\{ \alpha \in \mathbb{R}^r : \alpha_i \geq 0, i \in \mathbb{K}_r, \sum_{i=1}^{r} \alpha_i = 1 \right\}. \tag{1}$$

9 CONCLUSIONS AND FUTURE RESEARCH

This chapter is devoted to draw the conclusions and discuss the future work perspectives.

9.1 CONCLUSIONS

Initially, in this work was presented in Theorem 4 a strategy to design an exclusive output-dependent switching strategy for controlling linear time-invariant continuous-time uncertain switched linear systems.

Theorem 5 shows that, if the known conditions of Theorem 3 hold, then the conditions proposed in Theorem 4 also hold. Furthermore, from simulations results (Examples I and II), the conditions proposed in Theorem 4 present a greater feasible region and reduce the guaranteed cost when compared with the conditions of Theorem 3. Therefore, the conditions proposed in Theorem 4 are less conservative than that presented in Theorem 3. The second control problem studied in this work was the robust switching static output feedback $\mathscr{H}_\infty$ control of continuous-time switched linear time-invariant systems. For a particular case of switched systems with only one subsystem, a proof in Theorem 8 shows that if the known conditions of Theorem 6 hold, then the conditions proposed in Theorem 7 also hold.

Additionally, from simulations results (Example III), the conditions proposed in Theorem 7 present a greater feasible region and reduce the $\mathscr{H}_\infty$ cost when compared with the conditions of Theorem 6. Therefore, the conditions proposed in Theorem 7 are less conservative than that presented in Theorem 6.

The conditions of the proposed methods are a special class of BMIs (Bilinear Matrix Inequalities), which contain some bilinear terms as the product of a matrix and a scalar, related to a suitable convex combination and two scalar parameters to provide extra free dimensions in the solution space. The hybrid algorithm DE-LMI is proposed for obtaining feasible solutions of this particular NP-hard problem.

In Example IV, it was presented a practical application on a semi-active suspension system. It was possible to observe a dynamic response improvement considering the reduction of the guaranteed cost, when compared with the results obtained considering the procedure presented in (CARDIM *et al.*, 2016). A second study regarding this problem, considering an uncertain bounded mass and a fault in the spring, confirms the effectiveness of the proposed approach.

Regarding actuator saturation and switched controllers, Examples V and VI explore the conditions of Theorem 9 and 10. These examples shows that for some cases the proposed conditions, considering switched controllers, hold while the conditions proposed in Chang, Park and Zhou (2015) does not. Moreover, it is possible to observe that the switched controllers method yields a better $\mathscr{H}_\infty$ bound (γ).

Finally, Example VII presents the robust $\mathscr{H}_\infty$ switched controllers designing, based on Theorem 10, for a switched linear system subject to actuator saturation. This examples shows that, even under actuator saturation, the systems stability is ensured for all $x(t) \in \mathcal{X}(\mathcal{N}_h)$ and the set constraints $\mathscr{E}(P(\alpha^*), \varphi^{-1}\varepsilon) \subset \mathcal{X}(\mathcal{N}_h)$ and $\mathscr{E}(P(\alpha^*), \varphi^{-1}\varepsilon) \subset \mathcal{G}_{ic}(\alpha^*)$ hold.

It is important to highlight that the proposed methods are LMI-based and consider a parameter-dependent Lyapunov function. Furthermore, based on recent result presented in the literature the design avoids linear matrices equalities and the need to impose any constraints on system matrices

9.2 FUTURE RESEARCH DIRECTIONS

As futures research directions, the following proposes are listed:

- Generalize the control design for uncertain linear discrete-time systems.
- Extend this work to cope with for uncertain nonlinear systems described by T–S fuzzy systems subject to actuator saturation.

9.3 PUBLICATIONS

- **CARNIATO, LEONARDO ATAIDE**; CARNIATO, ALEXANDRE ATAIDE; TEIXEIRA, MARCELO CARVALHO MINHOTO; CARDIM, RODRIGO; MAINARDI JUNIOR, EDSON ITALO; ASSUNÇÃO, EDVALDO . Output control of continuous-time uncertain switched linear systems via switched static output feedback. INTERNATIONAL JOURNAL OF CONTROL, v. 1, p. 1-20, 2018. https://doi.org/10.1080/00207179.2018.1495341
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