AIP | Journal of Mathematical Physics

Propagator, tree-level unitarity and effective nonrelativistic potential for higher-derivative gravity theories in D dimensions

Antonio Accioly, Abel Azeredo, and Hatsumi Mukai

Citation: Journal of Mathematical Physics **43**, 473 (2002); doi: 10.1063/1.1415743
View online: http://dx.doi.org/10.1063/1.1415743
View Table of Contents: http://scitation.aip.org/content/aip/journal/jmp/43/1?ver=pdfcov
Published by the AIP Publishing

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 43, NUMBER 1 JANUARY 2002

Propagator, tree-level unitarity and effective nonrelativistic potential for higher-derivative gravity theories in *D* dimensions

Antonio Accioly[a] and Abel Azeredo
Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona 145, 01405-900 São Paulo, SP, Brazil

Hatsumi Mukai
Departamento de Física, Fundação Universidade Estadual de Maringá, Av. Colombo 5790, 87020-900 Maringá, PR, Brazil

(Received 5 September 2000; accepted for publication 17 September 2001)

A prescription for computing the propagator for D-dimensional higher-derivative gravity theories, based on the Barnes–Rivers operators, is presented. A systematic study of the tree-level unitarity of these theories is developed and the agreement of their linearized versions with Newton's law is investigated by computing the corresponding effective nonrelativistic potential. Three-dimensional quadratic gravity with a gravitational Chern–Simons term is also analyzed. A discussion on the issue of light bending within the framework of both D-dimensional quadratic gravity and three-dimensional quadratic gravity with a Chern–Simons term is provided as well. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1415743]

I. INTRODUCTION

Undoubtedly, Einstein's field theory accounts very well for all known macroscopic gravitational phenomena. However, as a quantum theory it is less satisfactory: it has an S matrix which, despite being finite at the one-loop level,[1] diverges at the two-loop order.[2] This is not surprising, since one of the most difficult field theories when it comes to quantization is certainly the theory of the space–time structure itself. It is not known what the correct quantum theory of gravitation is.

Now, as is well known, the four-dimensional space–time is the most problematic place for a quantum field theory to live. Indeed, quantum field theories are notorious for being ill defined in four-dimensional space–time but can often be handled in space–times of a different dimensionality. Dimensional regularization is a common example of such a procedure: results that are divergent in four dimensions are convergent for $D>4$. The divergences of the four-dimensional theory are removed by considering our space–time as a limit $D\rightarrow 4$ of higher-dimensional space–times.[3] On the other side of the dimensionality spectrum, there are field theories in space–times with $2\leqslant D<4$. In some cases, such quantum models are exactly soluble and provide a valuable insight into the clockwork of quantum field theory.

In this vein, here we study—at the tree level—various higher-derivative gravity theories. First, we shall consider gravitational theories which will be called, for short, D-dimensional higher-derivative gravity theories. These theories share some basic points with general relativity:

(i) General covariance and
(ii) the action is extremized under variation of the metric.

On the other hand, they differ from general relativity in the following respects:

(i) The field equations for the metric are of fourth order.

[a]Electronic mail: accioly@ift.unesp.br

(ii) Space–time is allowed to have any number of dimensions, which is a condition absolutely necessary in the context of string theory.

Quadratic gravity in four dimensions is the most typical example of such a theory.[4–6] At this point it is reasonable to pose the question: What is the use of probing into these theories? Before answering this question, let us comment, in passing, on some results recently obtained concerning quadratic gravity in four dimensions. In a series of papers on the photon propagation around a massive body in quadratic theories of gravitation it was shown that, unlike Einstein's gravity, quadratic gravity produces dispersive photon propagation.[7–9] To be more specific, quadratic gravity produces energy-dependent photon scattering. An interesting consequence of this fact is that gravity's rainbows and higher-derivative gravity can coexist without conflict.[9] In this sense quadratic gravity is closer to quantum electrodynamics than any currently known gravitational theory. In fact, dispersive photon propagation is a trivial phenomenon in the context of QED. Based on the fact that the rainbow effect which is present in quadratic gravity is undetectable, nowadays it is possible to find a new constraint on the value of the contribution of the quadratic part. This is a very important result given the scarcity of observational constraints on gravitational theories. In addition, it was also found that the gravitational deflection predicted by quadratic gravity is always smaller than that predicted by Einstein's theory.[7–9] It is worth mentioning that the R^2 sector of the theory of gravitation with higher derivatives does not contribute anything to the gravitational deflection.[10,11] After this little digression we return to the question raised previously. Our motivation for studying D-dimensional higher-derivative gravity theories is to try to answer, among other things, the following questions:

(i) Which is the lowest dimension in which these theories make sense?
(ii) Are these D-dimensional theories unitary at the tree level?
(iii) Is the massless excitation a dynamical degree of freedom in any dimension?
(iv) Does D-dimensional linearized quadratic gravity agree with Newton's theory in the limiting case of motion at low velocity in a weak gravitational field?
(v) Is the gravitational deflection within the context of the theories mentioned above greater or smaller than that related to the corresponding Einstein's theory?

Second, we shall analyze three-dimensional quadratic gravity with a gravitational Chern–Simons term. Now the action is extremized *à la* Palatini (independently varied connection). In other words, the propagation of the metric and affine structures of space–time are independent. Our goal here is, first of all, to find out how the nature of the field excitations is affected by the addition of a gravitational Chern–Simons term to the Lagrangian concerning three-dimensional quadratic gravity. Second, we shall study the tree-level unitarity of the above theory, the nonrelativistic limit of its linearized version and light bending within the framework of the same.

The plan of this work is as follows. In Sec. II we find the appropriate Lagrangian for computing the propagator concerning D-dimensional quadratic gravity. In Sec. III we present an algorithm for computing the propagator for D-dimensional higher-derivative gravity theories based on the Barnes–Rivers operators.[12–16] Using this prescription we get the propagator for D-dimensional quadratic gravity in an unconventional gauge. From this result we obtain in a straightforward way the propagator in a series of interesting gauges which for $D=4$ reduce to well known results which are broadly used in the literature.[17] In Sec. IV we study in a systematic way the tree-level unitarity of D-dimensional higher-derivative gravity theories. It is shown that for $D>2$ quadratic gravity is nonunitary at the tree level. On the other hand, we get that D-dimensional $R+R^2$ gravity is unitary at the tree level for $D>2$. Section V is devoted to the study of the effective nonrelativistic potential for D-dimensional quadratic gravity. An expression for computing this potential is obtained for $D>2$ and from that we get, in particular, the potential for three-dimensional linearized quadratic gravity. Unlike the Newtonian potential, this potential is well behaved: it is finite at the origin and zero at infinity. In Sec. VI we analyze the gravitational deflection in the framework of D-dimensional quadratic gravity. Section VII deals with three-dimensional quadratic gravity with a gravitational Chern–Simons term. Finally, a summary of the main results is presented in Sec. VIII.

In our notation the signature is $(+-\cdots-)$. The curvature tensor is defined by $R^{\alpha}_{\beta\gamma\delta} = -\partial_\delta \Gamma^{\alpha}_{\beta\gamma} + \cdots$, the Ricci tensor by $R_{\mu\nu} = R^{\alpha}_{\mu\nu\alpha}$, and the curvature scalar by $R = g^{\mu\nu}R_{\mu\nu}$, where $g_{\mu\nu}$ is the metric tensor. Natural units are used throughout.

II. FINDING THE APPROPRIATE LAGRANGIAN FOR COMPUTING THE PROPAGATOR CONCERNING *D*-DIMENSIONAL QUADRATIC GRAVITY

The action for quadratic gravity in $D>1$ dimensions is given by

$$I = \int d^D x \sqrt{(-1)^{D-1} g} \left[\frac{2R}{\kappa^2} + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R^2_{\mu\nu} + \frac{\gamma}{2} R^2_{\mu\nu\rho\sigma} + \frac{\delta}{2} \Box R \right], \tag{1}$$

where κ^2 is a suitable constant with dimension L^{D-2} which in four dimensions is equal to $32\pi G$, with G being Newton's constant, and α, β, γ and δ are constants with dimension L^{4-D}. The $\Box R$ term in this action is manifestly a total covariant divergence and can be ignored. For $D=1$ the space is flat and all the tensors $R_{\mu\nu\alpha\beta}$, $R_{\mu\nu}$, R identically vanish. Of course, there can be no dynamics in a space which does not possess both a spacelike and a timelike dimension. The lowest dimension in which the quadratic theory makes sense is thus $D=2$.

Proposition 1: We can drop out the $R^2_{\mu\nu\rho\sigma}$ term of the linearized Lagrangian related to gravity with higher derivatives in dimensions higher than second.

Proof: Let $\mathcal{L}^1(\mathcal{L}^2)$ be the Lagrangian corresponding to the gravity theory without (with) the $R^2_{\mu\nu\rho\sigma}$ term, namely,

$$\mathcal{L}^1 \equiv \sqrt{(-1)^{D-1} g} \left[\frac{2R}{\kappa^2} + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R^2_{\mu\nu} \right], \tag{2}$$

$$\mathcal{L}^2 \equiv \sqrt{(-1)^{D-1} g} \left[\frac{2R}{\kappa^2} + \frac{\alpha}{2} R^2 + \frac{\beta}{2} R^2_{\mu\nu} + \frac{\gamma}{2} R^2_{\mu\nu\rho\sigma} \right]. \tag{3}$$

Decomposing the metric, $g_{\mu\nu}$, as

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \tag{4}$$

where $\eta_{\mu\nu}$ is the Minkowski metric, and inserting (4) into (2) and (3) yields

$$\mathcal{L}^1_{lin.} = \frac{b}{4} [\Box h_{\mu\nu} \Box h^{\mu\nu} - (A^{\mu}_{,\mu})^2 - F^2_{\mu\nu} + (1+4c)(A^{;\alpha}_{\alpha} - \Box \phi)^2] - \frac{1}{2} [h^{\mu\nu} \Box h_{\mu\nu} + A^2_\nu + (A_\nu - \phi_{,\nu})^2], \tag{5}$$

$$\mathcal{L}^2_{lin.} = \left(\frac{b}{4} + d \right) \left[\Box h_{\mu\nu} \Box h^{\mu\nu} - (A^{\mu}_{,\mu})^2 - F^2_{\mu\nu} + \frac{(b/4)(1+4c)}{b/4+d} (A^{;\alpha}_{\alpha} - \Box \phi)^2 \right] - \frac{1}{2} [h^{\mu\nu} \Box h_{\mu\nu} + A^2_\nu + (A_\nu - \phi_{,\nu})^2],$$

where $A^{\mu} \equiv h^{\mu\nu}_{,\nu}$, $\phi \equiv h$, $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$, $b \equiv \beta\kappa^2/2$, $c \equiv \alpha/\beta$, and $d \equiv \gamma\kappa^2/2$. Indices are lowered (raised) using $\eta_{\mu\nu}(\eta^{\mu\nu})$. □

It is worth mentioning that we could have arrived at the conclusion that the $R^2_{\mu\nu\rho\sigma}$ term need never be considered in calculating the propagator by simply noting that the linearized Gauss–Bonet invariant is a total derivative in any space–time dimension, the restriction to $D=4$ coming in only when we take the full nonlinear structure into account.

Thus, we come to the conclusion that the appropriate Lagrangian for computing the propagator concerning quadratic gravity in $D>2$ dimensions is

$$\overline{\mathcal{L}}=\sqrt{(-1)^{D-1}g}\left[\frac{2R}{\kappa^2}+\frac{\alpha}{2}R^2+\frac{\beta}{2}R^2_{\mu\nu}\right]. \tag{6}$$

Proposition 2: In two dimensions

$$\frac{\alpha}{2}R^2+\frac{\beta}{2}R^2_{\mu\nu}+\frac{\gamma}{2}R^2_{\mu\nu\rho\sigma}=\left(\frac{\alpha}{2}+\frac{\beta}{4}+\frac{\gamma}{2}\right)R^2.$$

Proof: In two dimensions both the Riemann tensor and the Ricci tensor can be expressed in terms of the curvature scalar. Indeed,[18]

$$R_{\mu\nu\rho\sigma}=\tfrac{1}{2}R(g_{\mu\sigma}g_{\nu\rho}-g_{\mu\rho}g_{\nu\sigma}),$$

and

$$R_{\mu\nu}=\tfrac{1}{2}Rg_{\mu\nu}.$$

Therefore,

$$\frac{\alpha}{2}R^2+\frac{\beta}{2}R^2_{\mu\nu}+\frac{\gamma}{2}R^2_{\mu\nu\rho\sigma}=\frac{\alpha}{2}R^2+\frac{\beta}{2}\frac{R^2}{2}+\frac{\gamma}{2}R^2=\left(\frac{\alpha}{2}+\frac{\beta}{4}+\frac{\gamma}{2}\right)R^2.$$

□

Obviously, for $D=2$ the suitable Lagrangian for calculating the propagator related to quadratic gravity is $\overline{\mathcal{L}}=\sqrt{-g}[2R/\kappa^2+(\alpha/2)R^2]$. Nonetheless, we will not discuss this theory here. From now on we shall assume that $D>2$.

III. FINDING THE PROPAGATOR FOR *D*-DIMENSIONAL HIGHER-DERIVATIVE GRAVITY THEORIES

We begin by describing a prescription for computing the propagator for gravity theories with higher derivatives in $D>2$ dimensions. The algorithm is used afterward to get the propagator for D-dimensional quadratic gravity in an unconventional gauge. From this result we obtain the propagator in a series of gauges which in the $D=4$ case reduce to well known gauges that are widely used in the literature.[17]

A. The prescription

Let $\overline{\mathcal{L}}$ be the Lagrangian for any metric theory of gravity with higher derivatives. To compute the graviton propagator we need the bilinear part of this Lagrangian. The latter is obtained by decomposing the metric, $g_{\mu\nu}$, as in (4), and inserting (4) into $\overline{\mathcal{L}}$. Let $\mathcal{L}_g$ be the resulting Lagrangian. In the specific case of gauge-invariant theories, we add to $\mathcal{L}_g$ a gauge-fixing Lagrangian $\mathcal{L}_{gf}$. Accordingly, $\mathcal{L}=\mathcal{L}_g+\mathcal{L}_{gf}$ can be written as

$$\mathcal{L}=\tfrac{1}{2}h^{\mu\nu}\mathcal{O}_{\mu\nu,\rho\sigma}h^{\rho\sigma}. \tag{7}$$

In performing these calculations it is extremely convenient to work in terms of the Barnes–Rivers operators[12–16] in the space of symmetric rank-two tensors. The complete set of D-dimensional operators is given by

$$P^1_{\mu\nu,\rho\sigma}=\tfrac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma}+\theta_{\mu\sigma}\omega_{\nu\rho}+\theta_{\nu\rho}\omega_{\mu\sigma}+\theta_{\nu\sigma}\omega_{\mu\rho}),$$

$$P^2_{\mu\nu,\rho\sigma}=\frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma}+\theta_{\mu\sigma}\theta_{\nu\rho})-\frac{1}{D-1}\theta_{\mu\nu}\theta_{\rho\sigma},$$

$$P^0_{\mu\nu,\rho\sigma}=\frac{1}{D-1}\theta_{\mu\nu}\theta_{\rho\sigma},$$

$$\overline{P^0}_{\mu\nu,\rho\sigma}=\omega_{\mu\nu}\omega_{\rho\sigma},$$

$$\bar{\bar{P}}^0_{\mu\nu,\rho\sigma}=\theta_{\mu\nu}\omega_{\rho\sigma}+\omega_{\mu\nu}\theta_{\rho\sigma},$$

where $\theta_{\mu\nu}$ and $\omega_{\mu\nu}$ are the usual transverse and longitudinal vector projection operators

$$\theta_{\mu\nu}=\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2},\quad \omega_{\mu\nu}=\frac{k_\mu k_\nu}{k^2},$$

which satisfy the relations

$$\theta_{\mu\rho}\theta^\rho_\nu=\theta_{\mu\nu},\quad \omega_{\mu\rho}\omega^\rho_\nu=\omega_{\mu\nu},\quad \theta_{\mu\rho}\omega^\rho_\nu=0.$$

Here k_μ is the momentum of the graviton exchanged and $k^2\equiv k_\mu k^\mu$.

The set of operators $\{P^1,P^2,P^0,\overline{P^0}\}$ is a complete set of projection operators for symmetric rank-two tensors, i.e., they are idempotent, mutually orthogonal and satisfy the completness relation

$$[P^1+P^2+P^0+\overline{P^0}]_{\mu\nu,\rho\sigma}=\tfrac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})\equiv I_{\mu\nu,\rho\sigma}.$$

In the rest frame of a massive tensor field, the family of operators $\{P^1,P^2,P^0,\overline{P^0}\}$ project out the spin-1, spin-2, and two spin-0 parts of the field. The operator $\bar{\bar{P}}^0$, in turn, is nothing but the sum of two spin-0 transfer operators, namely,

$$\bar{\bar{P}}^0_{\mu\nu,\rho\sigma}\equiv[P^{\theta\omega}+P^{\omega\theta}]_{\mu\nu,\rho\sigma},$$

where $P^{\theta\omega}_{\mu\nu,\rho\sigma}\equiv\theta_{\mu\nu}\omega_{\rho\sigma}$ and $P^{\omega\theta}_{\mu\nu,\rho\sigma}\equiv\omega_{\mu\nu}\theta_{\rho\sigma}$. Its multiplicative table is given by

$$\bar{\bar{P}}^0P^1=P^1\bar{\bar{P}}^0=\bar{\bar{P}}^0P^2=P^2\bar{\bar{P}}^0=O,$$

$$(\bar{\bar{P}}^0)^2=(D-1)(P^0+\overline{P^0}),$$

$$P^0\bar{\bar{P}}^0=\bar{\bar{P}}^0\overline{P^0}=P^{\theta\omega},$$

$$\overline{P^0}\bar{\bar{P}}^0=\bar{\bar{P}}^0P^0=P^{\omega\theta},$$

where O is the null operator.

The expansion of the operator $\mathcal{O}$ in the basis $\{P^1,P^2,P^0,\overline{P^0},\bar{\bar{P}}^0\}$ is trivially obtained using the following tensorial identities:

$$\tfrac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})=[P^1+P^2+P^0+\overline{P^0}]_{\mu\nu,\rho\sigma},$$

$$\eta_{\mu\nu}\eta_{\rho\sigma}=[(D-1)P^0+\overline{P^0}+\bar{\bar{P}}^0]_{\mu\nu,\rho\sigma},$$

$$\frac{1}{k^2}(\eta_{\mu\rho}k_\nu k_\sigma+\eta_{\mu\sigma}k_\nu k_\rho+\eta_{\nu\rho}k_\mu k_\sigma+\eta_{\nu\sigma}k_\mu k_\rho)=[2P^1+4\overline{P^0}]_{\mu\nu,\rho\sigma}, \tag{8}$$

$$\frac{1}{k^2}(\eta_{\mu\nu}k_\rho k_\sigma+\eta_{\rho\sigma}k_\mu k_\nu)=[\bar{\bar{P}}^0+2\overline{P^0}]_{\mu\nu,\rho\sigma},$$

$$\frac{1}{k^4}(k_\mu k_\nu k_\rho k_\sigma)=\overline{P^0}_{\mu\nu,\rho\sigma}.$$

The identities,

$$P^2_{\mu\nu,\rho\sigma}=\frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})-\frac{1}{D-1}\eta_{\mu\nu}\eta_{\rho\sigma}-\left[P^1+\frac{D-2}{D-1}\overline{P^0}-\frac{1}{D-1}\bar{\bar{P}}^0\right]_{\mu\nu,\rho\sigma},$$
$$P^0_{\mu\nu,\rho\sigma}=\frac{1}{D-1}\eta_{\mu\nu}\eta_{\rho\sigma}-\frac{1}{D-1}[\overline{P^0}+\bar{\bar{P}}^0]_{\mu\nu,\rho\sigma}, \tag{9}$$

in turn, greatly facilitate the task of casting the propagator in a form wherein the terms proportional to the graviton momentum are omitted, which in practice widely simplifies computations involving conserved currents.

We will not display the demonstrations of (8) and (9) since they follow straightforwardly from the very definition of the operators $P^1,\dots,\bar{\bar{P}}^0$.

We are now ready to find the propagator, $\mathcal{O}^{-1}$. To accomplish this we have to invert the operator $\mathcal{O}$. Expanding the latter in the basis $\{P^1,P^2,P^0,\overline{P^0},\bar{\bar{P}}^0\}$ with the help of the identities (8), we get

$$\mathcal{O}=x_1P^1+x_2P^2+x_0P^0+\overline{x_0}\,\overline{P^0}+\bar{\bar{x}}_0\bar{\bar{P}}^0.$$

Assume then that $\mathcal{O}^{-1}=y_1P^1+y_2P^2+y_0P^0+\overline{y_0}\,\overline{P^0}+\bar{\bar{y}}_0\bar{\bar{P}}^0$, where $y_1,y_2,\dots,\bar{\bar{y}}_0$ are parameters to be determined. Since $\mathcal{O}\mathcal{O}^{-1}=I$, we promptly obtain the following set of simultaneous equations:

$$\begin{aligned} x_1y_1&=1,\\ x_2y_2&=1,\\ x_0y_0+(D-1)\bar{\bar{x}}_0\bar{\bar{y}}_0&=1,\\ \overline{x_0}\,\overline{y_0}+(D-1)\bar{\bar{x}}_0\bar{\bar{y}}_0&=1,\\ \bar{\bar{x}}_0y_0+\overline{x_0}\bar{\bar{y}}_0&=0,\\ \bar{\bar{x}}_0\overline{y_0}+x_0\bar{\bar{y}}_0&=0. \end{aligned} \tag{10}$$

Before going on we need a lemma.

Lemma: If $x_1\neq 0$, $x_2\neq 0$, *and* $[x_0\overline{x_0}-(D-1)\bar{\bar{x}}_0]\neq 0$, *then (10) has one and only one solution.*

Proof: Row reducing the argumented matrix of the system (10) to echelon form yields

$$\begin{bmatrix} x_1 & 0 & 0 & 0 & 0 & 1\\ 0 & x_2 & 0 & 0 & 0 & 1\\ 0 & 0 & x_0 & 0 & (D-1)\bar{\bar{x}}_0 & 1\\ 0 & 0 & \bar{\bar{x}}_0 & 0 & \overline{x_0} & 0\\ 0 & 0 & 0 & \overline{x_0} & (D-1)\bar{\bar{x}}_0 & 1\\ 0 & 0 & 0 & \bar{\bar{x}}_0 & x_0 & 0 \end{bmatrix} \sim \begin{bmatrix} x_1 & 0 & 0 & 0 & 0 & 1\\ 0 & x_2 & 0 & 0 & 0 & 1\\ 0 & 0 & x_0 & 0 & (D-1)\bar{\bar{x}}_0 & 1\\ 0 & 0 & 0 & \overline{x_0} & (D-1)\bar{\bar{x}}_0 & 1\\ 0 & 0 & 0 & 0 & [x_0\overline{x_0}-(D-1)\bar{\bar{x}}_0^2] & -\bar{\bar{x}}_0\\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

□

Therefore, the propagator is given by

$$\mathcal{O}^{-1}=\frac{1}{x_1}P^1+\frac{1}{x_2}P^2+\frac{1}{x_0\overline{x_0}-(D-1)\bar{\bar{x}}_0^2}[\overline{x_0}P^0+x_0\overline{P^0}-\bar{\bar{x}}_0\bar{\bar{P}}^0]. \tag{11}$$

In summary, the prescription for computing the propagator consists of the following procedures.

(1) Linearize the original Lagrangian using (4).
(2) Add to the previous result a suitable gauge-fixing Lagrangian. Obviously, we only do this in the case of gauge-invariant theories.
(3) Cast the resulting Lagrangian into the bilinear form $\mathcal{L}=\frac{1}{2}h^{\mu\nu}\mathcal{O}_{\mu\nu,\rho\sigma}h^{\rho\sigma}$.
(4) Find the coefficients $x_1,x_2,\ldots,\bar{\bar{x}}_0$ by expanding the operator $\mathcal{O}$ in the basis $\{P^1,P^2,P^0,\overline{P^0},\bar{\bar{P}}^0\}$ with the help of the identities (8).
(5) Insert these coefficients in (11).

B. Propagator for *D*-dimensional quadratic gravity in an unconventional gauge

Let us then find the propagator by means of the prescription developed in Sec. III A. Of course, the linearization of (6) leads to (5). So, $\mathcal{L}_g\equiv\mathcal{L}^1_{lin.}$. Lagrangian (5) is invariant under the infinitesimal coordinate transformation $x^\mu\rightarrow x^\mu+\kappa\xi^\mu(x)$, where $\xi^\mu(x)$ is an infinitesimal vector field. It must be infinitesimal to avoid inconsistency with (4). Under this transformation we have from (4)

$$h_{\mu\nu}(x)\rightarrow h_{\mu\nu}(x)-\xi_{\mu,\nu}-\xi_{\nu,\mu}. \tag{12}$$

The presence of the local gauge symmetry (12) requires the addition of a gauge-fixing term, $\mathcal{L}_{gf}$, to Lagrangian (5). It is common practice to choose a linear combination of A_μ and $\phi_{,\mu}$ as gauge functions. However, looking at (5) we clearly see the presence not only of this linear combination but also of its curl ($F_{\mu\nu}$) and its divergence ($A^\mu_{,\mu}-\Box\phi$). Hence, we choose the following unconventional gauge-fixing Lagrangian,

$$\mathcal{L}_{gf}=\lambda_1(A_\nu-\lambda\phi_{,\nu})^2+\frac{b}{4}[\lambda_2(A^\mu_{,\mu}-\lambda\Box\phi)^2+\lambda_3F^2_{\mu\nu}],$$

where λ, λ_1, λ_2, and λ_3 are suitable gauge parameters. Casting the Lagrangian, $\mathcal{L}=\mathcal{L}_g+\mathcal{L}_{gf}$, into the bilinear form $\mathcal{L}=\frac{1}{2}h^{\mu\nu}\mathcal{O}_{\mu\nu,\rho\sigma}h^{\rho\sigma}$, and expanding the operator $\mathcal{O}$ in the basis $\{P^1,P^2,\ldots,\bar{\bar{P}}^0\}$ with the help of (8), we obtain

$$\mathcal{O}=x_1P^1+x_2P^2+x_0P^0+\overline{x_0}\,\overline{P^0}+\bar{\bar{x}}_0\bar{\bar{P}}^0,$$

whereupon

$$x_1\equiv\frac{b}{2}\left(\lambda_3k^4+\frac{2\lambda_1k^2}{b}\right),$$

$$x_2\equiv\frac{b}{2}\left(k^4+\frac{2k^2}{b}\right),$$

$$x_0\equiv\frac{b}{2}\left[Dk^4-\frac{2(D-2)k^2}{b}+4(D-1)k^4c+(D-1)k^4\lambda_2\lambda^2+\frac{4(D-1)k^2\lambda_1\lambda^2}{b}\right],$$

$$\overline{x_0}\equiv\frac{b}{2}\left(k^4\lambda_2-2k^4\lambda\lambda_2+\frac{4k^2\lambda_1}{b}-\frac{8k^2\lambda\lambda_1}{b}+k^4\lambda_2\lambda^2+\frac{4k^2\lambda_1\lambda^2}{b}\right),$$

$$\bar{\bar{x}}_0\equiv\frac{b}{2}\left(\frac{4k^2\lambda_1\lambda^2}{b}+k^4\lambda_2\lambda^2-k^4\lambda\lambda_2-\frac{4k^2\lambda\lambda_1}{b}\right).$$

The propagator in momentum space is given by (11). From this result we can find the propagator in a series of interesting gauges, by judiciously choosing the parameters λ, λ_1, λ_2, and λ_3. We list below the most important covariant gauges that result from such choices.

(1) *Julve–Tonin gauge* $(\lambda=0)$*:*[19]

$$\mathcal{L}_{gf}=\lambda_1 A_\nu^2+\frac{b}{4}[\lambda_2(A^\mu_{,\mu})^2+\lambda_3 F^2_{\mu\nu}].$$

Propagator:

$$\mathcal{O}^{-1}=\frac{m_1^2}{k^2(m_1^2\lambda_1-\lambda_3 k^2)}P^1+\frac{m_1^2}{k^2(m_1^2-k^2)}P^2+\frac{m_0^2}{2k^2[k^2-[(D-2)/2]\,m_0^2]}P^0$$
$$+\frac{m_1^2}{(2m_1^2\lambda_1-\lambda_2 k^2)k^2}\overline{P}^0, \tag{13}$$

where

$$m_0^2\equiv\frac{2}{D\beta\kappa^2/4+(D-1)\kappa^2\alpha},\quad m_1^2\equiv-\frac{4}{\beta\kappa^2}.$$

Absence of tachyons requires $\beta<0$ and $(D-1)\alpha+\beta D/4>0$. Note that the choice $\lambda=0$ gives a propagator that only contains the spin-projection operators, i.e., P^1, P^2, P^0, $\overline{P^0}$, and it gives also a propagator all parts of which behave like k^{-4}.

(2) *de Donder gauge* $(\lambda_2=\lambda_3=0,\lambda=\frac{1}{2})$*:*

$$\mathcal{L}_{gf}=\lambda_1(A_\nu-\tfrac{1}{2}\partial_\nu\phi)^2.$$

Propagator:

$$\mathcal{O}^{-1}=\frac{1}{\lambda_1 k^2}P^1+\frac{m_1^2}{k^2(m_1^2-k^2)}P^2+\frac{m_0^2}{2k^2[k^2-[(D-2)/2]\,m_0^2]}P^0$$
$$+\left[\frac{2}{\lambda_1 k^2}+\frac{(D-1)m_0^2}{2k^2[k^2-[(D-2)/2]\,m_0^2]}\right]\overline{P^0}+\frac{m_0^2}{2k^2[k^2-[(D-2)/2]\,m_0^2]}\bar{\bar{P}}^0.$$

(3) *Feynman gauge* $(\lambda_2=\lambda_3=0,\lambda_1=1,\lambda=\frac{1}{2})$:

$$\mathcal{L}_{gf}=(A_\nu-\tfrac{1}{2}\partial_\nu\phi)^2.$$

Propagator:

$$\mathcal{O}^{-1}=\frac{1}{k^2}P^1+\frac{m_1^2}{k^2(m_1^2-k^2)}P^2+\frac{m_0^2}{2k^2[k^2-[(D-2)/2]\,m_0^2]}P^0$$
$$+\left[\frac{2}{k^2}+\frac{(D-1)m_0^2}{2k^2[k^2-[(D-2)/2]\,m_0^2]}\right]\overline{P^0}+\frac{m_0^2}{2k^2[k^2-[(D-2)/2]\,m_0^2]}\bar{\bar{P}}^0.$$

IV. A SYSTEMATIC STUDY OF TREE-LEVEL UNITARITY

Now we present a method for analyzing the unitarity at the tree level of D-dimensional higher-derivative gravity theories.

In order to verify whether ghosts and tachyons are absent in a given theory of gravity with higher derivatives we require that the corresponding propagator has only first poles at k^2-M^2

$=0$ with real masses M (no tachyons) and with positive residues (no ghosts).[20–22] Therefore, to probe the tree-level unitarity of D-dimensional higher-derivative gravity theories we couple the propagator to external conserved currents, $T^{\mu\nu}$, compatible with the symmetries of the theory, and afterward we examine the current-current amplitude at the poles. The transition amplitude in momentum space, in turn, can be cast in the form

$$\mathcal{A}=\mathsf{g}^2 T^{\mu\nu}\mathcal{O}^{-1}_{\mu\nu,\rho\sigma}T^{\rho\sigma}, \tag{14}$$

where g is the effective coupling constant of the theory. Note that only the spin-projectors P^2 and P^0 will give a non-null contribution to the current-current amplitude since $k_\mu T^{\mu\nu}=0$.

Let us then expand the sources in a suitable basis. The set of independent vectors in momentum space,

$$k^\mu \equiv (k^0,\mathbf{k}),\quad \tilde{k}^\mu \equiv (k^0,-\mathbf{k}),\quad \varepsilon_i^\mu \equiv (0,\vec{\epsilon}_i),\quad i=1,\ldots,D-2,$$

where $\vec{\epsilon}_1,\ldots,\vec{\epsilon}_{D-2}$ are mutually orthogonal unit vectors which are also orthogonal to $\mathbf{k}$, serves our purpose. Accordingly, the symmetric current tensor $T^{\mu\nu}(k)$ can be written as

$$T^{\mu\nu}=ak^\mu k^\nu+b\tilde{k}^\mu\tilde{k}^\nu+c^{ij}\varepsilon_i^{(\mu}\varepsilon_j^{\nu)}+dk^{(\mu}\tilde{k}^{\nu)}+e^i k^{(\mu}\varepsilon_i^{\nu)}+f^i\tilde{k}^{(\mu}\varepsilon_i^{\nu)}. \tag{15}$$

The current conservation, $k_\mu T^{\mu\nu}=0$, gives the following constraints for the coefficients a, b, d, e^i and f^i

$$ak^2+(k_0^2+\mathbf{k}^2)\frac{d}{2}=0, \tag{16}$$

$$b(k_0^2+\mathbf{k}^2)+d\frac{k^2}{2}=0, \tag{17}$$

$$e^i k^2+f^i(k_0^2+\mathbf{k}^2)=0. \tag{18}$$

If we saturate the indices of $T^{\mu\nu}$ with momenta k_μ, we obtain the equation $k_\mu k_\nu T^{\mu\nu}=0$, which yields a consistency relation for the coefficients a, b, and d:

$$ak^4+b(k_0^2+\mathbf{k}^2)^2+dk^2(k_0^2+\mathbf{k}^2)=0. \tag{19}$$

Now, all we have to do is to compute the residue of $\mathcal{A}$ at each first pole of the propagator and verify whether its sign is positive.

Proposition 3: Higher-derivative gravity is nonunitary at the tree level in $D>2$ dimensions. If $m_0^2>0$ $[D\beta/4+(D-1)\alpha>0]$ and $m_1^2>0(-\beta>0)$, the theory is nontachyonic and has one normal massless spin-2 particle, one massive spin-2 ghost and one normal massive spin-0 particle. The massless excitation is not a dynamical degree of freedom in $D=3$.

Proof: From (14) and (13) we promptly obtain

$$\begin{aligned}\mathcal{A}&=\mathsf{g}^2T^{\mu\nu}\left\{\frac{m_1^2}{k^2(m_1^2-k^2)}P^2+\frac{m_0^2}{2k^2[k^2-(D-2)m_0^2/2]}P^0\right\}T^{\rho\sigma}\\&=\mathsf{g}^2\left\{\frac{T_{\mu\nu}T^{\mu\nu}-T^2/(D-2)}{k^2}-\frac{T_{\mu\nu}T^{\mu\nu}-T^2/(D-1)}{k^2-m_1^2}\right.\\&\quad\left.+\frac{T^2}{(D-1)(D-2)[k^2-(D-2)m_0^2/2]}\right\},\end{aligned} \tag{20}$$

where $T=\eta_{\mu\nu}T^{\mu\nu}$. Thus, we have two poles for the spin-2 sector, i.e., $k^2=0$, $k^2=m_1^2$, and one pole for the spin-0 sector, namely, $k^2=(D-2)m_0^2/2$. Let us then find the sign of the residue at these poles. We assume that D-dimensional quadratic gravity has no tachyons, which implies $m_0^2>0$ and $m_1^2>0$.

• *Pole* $k^2=0$. From (16)–(18) and (20), we get that the residue of $\mathcal{A}$ at the pole $k^2=0$ is

$$\text{Res}\,\mathcal{A}|_{k^2=0}=\mathsf{g}^2\left[(c^{ij})^2-\frac{(c^{ii})^2}{D-2}\right]_{k^2=0}.$$

Therefore, the massless excitation is not a dynamical degree of freedom in three dimensions. For $D>3$ the result above tells us that $\text{Res}\,\mathcal{A}|_{k^2=0}>0$.

• *Pole* $k^2=(D-2)m_0^2/2$. In this case

$$\text{Res}\,\mathcal{A}|_{k^2=(D-2)m_0^2/2}=\left[\frac{\mathsf{g}^2T^2}{(D-1)(D-2)}\right]_{k^2=(D-2)m_0^2/2},$$

which implies that the residue of the current-current amplitude at the pole $k^2=(D-2)m_0^2/2$ is always positive for $D>2$. Thence, the scalar massive particle is a physical one.

• *Pole* $k^2=m_1^2$. The residue of the transition amplitude at the pole $k^2=m_1^2$ is given by

$$\begin{aligned}\text{Res}\,\mathcal{A}|_{k^2=m_1^2}=&-\mathsf{g}^2\bigg\{ab(k_0^2+\mathbf{k}^2)^2+b^2k^4+bdk^2(k_0^2+\mathbf{k}^2)+(c^{ij})^2-\frac{1}{2}(k_0^2+\mathbf{k}^2)e^if^i-\frac{k^2}{2}(f^i)^2\\&-\frac{1}{D-1}[ak^2+bk^2-c^{ii}+d(k_0^2+\mathbf{k}^2)]^2\bigg\}_{k^2=m_1^2},\\=&-\mathsf{g}^2\left\{[(a-b)k^2]^2+(c^{ij})^2+\frac{k^2}{2}[(e^i)^2-(f^i)^2]-\frac{1}{D-1}[(b-a)k^2-c^{ii}]^2\right\}_{k^2=m_1^2},\end{aligned}$$

where use has been made of (16)–(19). This expression can also be written as

$$\begin{aligned}\text{Res}\,\mathcal{A}|_{k^2=m_1^2}=&-\mathsf{g}^2\bigg\{\frac{D-2}{D-1}[(a-b)k^2]^2+\left[(c^{ij})^2-\frac{(c^{ii})^2}{D-1}\right]+\frac{k^2}{2}[(e^i)^2-(f^i)^2]\\&-\frac{2}{D-1}(a-b)k^2c^{ii}\bigg\}_{k^2=m_1^2}.\end{aligned}$$

Now, assuming as usual that $T\geqslant 0$, we get that $c^{ii}\leqslant 0$, which implies that $\text{Res}\,\mathcal{A}|_{k^2=m_1^2}<0$ for $D>2$. So, we have a nontachyonic massive spin-2 ghost in the propagator of higher-derivative gravity. In conclusion we way say that D-dimensional higher-derivative gravity is nonunitary at the tree level. □

Corollary 1: D-dimensional $R+R^2$ gravity is unitary at the tree level for $D>2$. In three dimensions the massless excitation is not a dynamical degree of freedom.

Proof: Clear. □

V. EFFECTIVE NONRELATIVISTIC POTENTIAL

In principle any relativistic theory of gravitation ought to agree with Newton's theory in the limiting case of motion at low velocity in a weak gravitational field. Accordingly it is worthwhile to probe whether D-dimensional linearized quadratic gravity leads to the right nonrelativistic law for gravitational interactions. To do this we compute the effective nonrelativistic potential for the interaction of two identical massive bosons of zero spin via a graviton exchange. The expression for the potential is

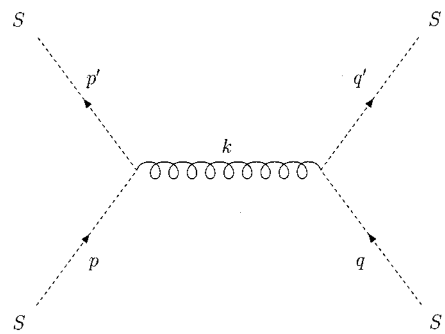

FIG. 1. One-graviton-exchange contribution to the scattering of two identical spinless massive bosons. S denotes a scalar particle with mass m.

$$U(r)=\frac{1}{4m^2}\frac{1}{(2\pi)^{D-1}}\int d^{D-1}\mathbf{k}\mathcal{M}_{N.R.}e^{-i\mathbf{k}\cdot\mathbf{r}}, \tag{21}$$

whereupon $\mathcal{M}_{N.R.}$ is the nonrelativistic limit of the Feynman amplitude for the process $S+S \rightarrow S+S$, where S stands for a spinless boson of mass m. The corresponding Feynman diagram is shown in Fig. 1.

The Lagrangian for the interaction of gravity with a free, massive scalar field $\tilde{\phi}$ is

$$\mathcal{L}_{int}=-\frac{\kappa h^{\mu\nu}}{2}\left[\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}-\frac{1}{2}\eta_{\mu\nu}(\partial_\alpha\tilde{\phi}\partial^\alpha\tilde{\phi}-m^2\tilde{\phi}^2)\right].$$

From the previous expression the Feynman rule for the elementary vertex may readily be deduced. It is shown in Fig. 2. The invariant amplitude for the process shown in Fig. 1 is

$$\begin{aligned}\mathcal{M}=&\frac{m_1^2}{k^2(m_1^2-k^2)}\frac{\kappa^2}{2}\Big\{(p.q)(p'.q')+(p.q')(p'.q)+(p.p')(m^2-q.q')+(q.q')(m^2-p.p')\\&+\frac{D}{2}(m^2-p.p')(m^2-q.q')-\frac{1}{2(D-1)}[Dm^2-(D-2)p.p'][Dm^2-(D-2)q.q']\Big\}\\&+\frac{m_0^2}{k^2(k^2-[(D-2)/2]m_0^2)}\frac{\kappa^2}{8(D-1)}[Dm^2-(D-2)p.p'][Dm^2-(D-2)q.q'].\end{aligned}$$

In the nonrelativistic limit this expression reduces to

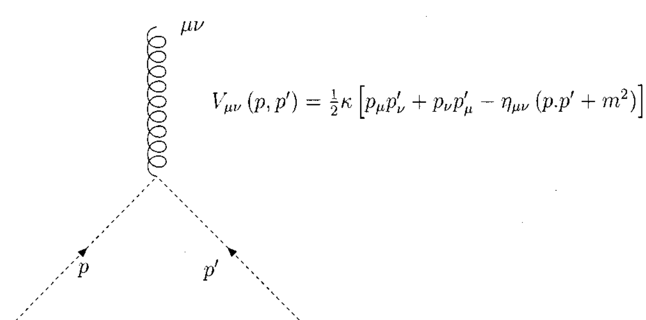

FIG. 2. The relevant Feynman rule for boson-boson interaction.

$$\mathcal{M}_{N.R.} = -\frac{D-2}{D-1}\frac{\kappa^2 m^4 m_1^2}{\mathbf{k}^2(\mathbf{k}^2+m_1^2)} + \frac{\kappa^2 m^4 m_0^2}{2(D-1)\mathbf{k}^2(\mathbf{k}^2+[(D-2)/2]\,m_0^2)}. \tag{22}$$

Substituting (22) into (21) we obtain[23]

$$U(r) = I_1 + I_2,$$

where

$$I_1 \equiv -\frac{\kappa^2 m^2 m_1^2}{4(2\pi)^{D-1}}\frac{D-2}{D-1}\int_0^\infty\int_0^\pi\cdots\int_0^\pi\int_0^\pi\int_0^{2\pi}\left[\frac{e^{-i\mathbf{k}.\mathbf{r}}}{\mathbf{k}^2(\mathbf{k}^2+m_1^2)}|\mathbf{k}|^{D-2}d|\mathbf{k}|\sin^{D-3}\right.$$
$$\left.\times\,\theta_{D-2}d\theta_{D-2}\cdots\sin^2\theta_3 d\theta_3\sin\theta_2\, d\theta_2 d\theta_1\right], \tag{23}$$

$$I_2 \equiv \frac{\kappa^2 m^2 m_0^2}{8(2\pi)^{D-1}}\frac{1}{D-1}\int_0^\infty\int_0^\pi\ldots\int_0^\pi\int_0^\pi\int_0^{2\pi}\left[\frac{e^{-i\mathbf{k}.\mathbf{r}}}{\mathbf{k}^2(\mathbf{k}^2+[(D-2)/2]\,m_0^2)}|\mathbf{k}|^{D-2}d|\mathbf{k}|\sin^{D-3}\right.$$
$$\left.\times\,\theta_{D-2}d\theta_{D-2}\cdots\sin^2\theta_3 d\theta_3\sin\theta_2 d\theta_2 d\theta_1\right]. \tag{24}$$

Hence, the problem of computing the effective nonrelativistic potential was reduced to quadratures. We give in the following two examples to illustrate the efficacy of the method: linearized quadratic gravity in three and four dimensions, respectively.

• *Three-dimensional linearized quadratic gravity*. For $D=3$ the expressions (23) and (24) tell us that

$$U(r) = \frac{\kappa^2 m^2}{8(2\pi)^2}\int_0^\infty\left(\int_0^{2\pi}e^{-i|\mathbf{k}|r\cos\theta}d\theta\right)\left(\frac{1}{\mathbf{k}^2+m_1^2}-\frac{1}{\mathbf{k}^2+m_0^2/2}\right)|\mathbf{k}|d|\mathbf{k}|$$
$$= \frac{\kappa^2 m^2}{16\pi}\int_0^\infty\left(\frac{1}{\mathbf{k}^2+m_1^2}-\frac{1}{\mathbf{k}^2+m_0^2/2}\right)J_0(|\mathbf{k}|r)|\mathbf{k}|d|\mathbf{k}|,$$

where J_0 is the Bessel function of the first kind of order zero. Now, from a mathematical point of view, $\int_0^\infty[xJ_0(ax)/(x^2+b^2)]\,dx$ only makes sense if[24] $a>0$, $\mathrm{Re}\,b>0$. Accordingly, we assume that $m_0^2>0(\frac{3}{4}\beta+2\alpha>0)$ and $m_1^2>0(-\beta>0)$, which is nothing but the condition for absence of tachyons (both positive and negative energy) in the dynamical field. Performing the integration yields

$$U(r) = 2\bar{G}m^2[K_0(m_1 r)-K_0(m_0 r/\sqrt{2})],$$

where K_0 is the modified Bessel function of the order of zero and $\bar{G}\equiv\kappa^2/32\pi$.

Therefore, the potential is given by

$$V(r) = 2\bar{G}m[K_0(m_1 r)-K_0(m_0 r/\sqrt{2})].$$

Note that $V(r)$ behaves as $2\bar{G}m\ln(m_0/m_1\sqrt{2})$ at the origin and as $2\bar{G}m[\sqrt{\pi/2m_1 r}e^{-m_1 r} - \sqrt{\pi/m_0 r\sqrt{2}}e^{-m_0 r/\sqrt{2}}]$ asymptotically.[25]

Three comments are in order here:

(i) Unlike the Newtonian potential, $V_N = 2Gm \ln r_0/r$, which has a logarithmic singularity at the origin and is unbounded at infinity, the potential concerning three-dimensional linearized quadratic gravity is extremely well behaved: it is finite at the origin and zero at the infinity.

(ii) $V(r) \to 0$ as m_0 and $m_1 \to \infty$, confirming in this way the well known fact that the standard correspondence of three-dimensional linearized Einstein's theory with Newton's theory breaks down.[26]

(iii) Recently it was shown that the solution of the linearized field equations concerning three-dimensional linearized quadratic gravity, having as source a static point like mass m, is given by[27]

$$h_{00} = \frac{\kappa m}{8\pi}[K_0(m_1 r) - K_0(m_0 r/\sqrt{2})],$$

$$h_{11} = h_{22} = \frac{\kappa m}{8\pi}[2 \ln r + K_0(m_1 r) + K_0(m_0 r/\sqrt{2})].$$

Since $V = \kappa h_{00}/2$, we see that this independent computation of the potential leads to the same result as that obtained via the effective nonrelativistic potential. The former is, in a sense, a major test of our semiclassical computation.

• *Four-dimensional linearized quadratic gravity*. The effective nonrelativistic potential is computed in this case as follows:

$$U(r) = \frac{\kappa^2 m^2}{6(2\pi)^2} \int_0^\infty \left(\int_0^\pi e^{-i|\mathbf{k}| r \cos\theta} \sin\theta d\theta \right) \left[-\frac{3}{4}\frac{1}{\mathbf{k}^2} + \frac{1}{\mathbf{k}^2 + m_1^2} - \frac{1}{4(\mathbf{k}^2 + m_0^2)} \right] \mathbf{k}^2 d|\mathbf{k}|$$
$$= \frac{\kappa^2 m^2}{6(2\pi)^2} \frac{1}{r} \mathrm{Im}\left[\int_{-\infty}^{+\infty} e^{ix} \left(-\frac{3}{4x} + \frac{x}{x^2 + m_1^2 r^2} - \frac{1}{4}\frac{x}{x^2 + m_0^2 r^2} \right) dx \right]. \tag{25}$$

Now, the condition for the existence of integrals like $\int_{-\infty}^{+\infty}[(x \sin x)/(x^2 + a^2)]dx$ $(a \neq 0)$ is that $a > 0$. In this case they can be easily evaluated by the method of contour integration. Therefore, assuming that $3\alpha + \beta > 0$ and $-\beta > 0$, which corresponds to the absence of tachyons in the dynamical field, we promptly obtain from (25)

$$U(r) = Gm^2\left[-\frac{1}{r} + \frac{4}{3} e^{-m_1 r} - \frac{1}{3} e^{-m_0 r} \right].$$

So, the potential for linearized higher-derivative gravity is given by the expression[4,7]

$$V(r) = Gm\left[-\frac{1}{r} + \frac{4}{3}\frac{e^{-m_1 r}}{r} - \frac{1}{3}\frac{e^{-m_0 r}}{r} \right],$$

which agrees asymptotically with Newton's law. At the origin it tends to the finite value $Gm((m_0 - 4m_1)/3)$.

From the computation of $V(r)$ for linearized quadratic gravity in three and four dimensions we learned that the existence of the potential is related to the absence of tachyons in the dynamical field. Consequently, we may conjecture that this is so in any dimension $D > 2$. This issue will be treated elsewhere.

VI. GRAVITATIONAL DEFLECTION

Using the mathematical apparatus developed in the last sections we consider now the issue of light bending within the framework of D-dimensional quadratic gravity. Accordingly, let us consider the interaction between a fixed source and a light ray. The associated energy-momentum tensors will be designated respectively as $T^{\mu\nu}$ and $F^{\mu\nu}$. The current-current amplitude for this process is given by

$$\mathcal{A}=\mathsf{g}^2 T_{\mu\nu}\mathcal{O}^{-1}_{\mu\nu,\rho\sigma}F^{\rho\sigma}.$$

But, on mass shell, $k_\mu T^{\mu\nu}=0$ and $k_\mu F^{\mu\nu}=0$, implying that only P^2 and P^0 will give a non-null contribution to the current-current amplitude. Therefore,

$$\mathcal{A}=\mathsf{g}^2 T^{\mu\nu}F^{\rho\sigma}\left[\frac{m_1^2}{k^2(m_1^2-k^2)}P^2+\frac{m_0^2}{2k^2[k^2-[(D-2)/2]m_0^2]}P^0\right]_{\mu\nu,\rho\sigma}.$$

Now, taking (9) into account and recalling that the energy-momentum tensor for light (electromagnetic radiation) is traceless, while $T^{\mu\nu}=\eta^{\mu 0}\eta^{\nu 0}T^{00}$ for a static source, we promptly obtain

$$\mathcal{A}=\mathsf{g}^2 T^{00}F^{00}\left[\frac{1}{k^2}-\frac{1}{k^2-m_1^2}\right].$$

Since $\mathsf{g}^2 T^{00}F^{00}/k^2$ is precisely the current-current amplitude for the interaction between a fixed source and a light ray in the context of D-dimensional linearized general relativity, we come to the conclusion that the gravitational deflection predicted by D-dimensional linearized quadratic gravity is always smaller than that predicted by D-dimensional linearized Einstein's theory.

Let us then discuss in more detail the result above in the particular case $D=3$. Proposition 3 tell us that the massless excitation is not a dynamical degree of freedom in three dimensions. Consequently, only $-\mathsf{g}^2 T^{00}F^{00}/(k^2-m_1^2)$ will contribute to light deflection. Thus, we come to the surprising result that in the framework of three-dimensional linearized quadratic gravity a light ray is deflected upward instead of downward as its four-dimensional counterpart. This is, perhaps, a property peculiar to quadratic gravity in $D=3$. It is worth mentioning that a classical computation of the gravitational deflection[27] assures us of the correctness of our result which is based on a semiclassical approach.

VII. THREE-DIMENSIONAL QUADRATIC GRAVITY WITH A GRAVITATIONAL CHERN–SIMONS TERM

To conclude we consider quadratic gravity with a gravitational Chern–Simons term. The topological Chern–Simons Lagrangian is given by

$$\mathcal{L}_{C.S.}=\frac{1}{2\mu}\varepsilon^{\mu\nu\lambda}\left(R^{\alpha}_{\beta\mu\nu}\Gamma^{\beta}_{\alpha\lambda}-\frac{2}{3}\Gamma^{\alpha}_{\beta\mu}\Gamma^{\beta}_{\gamma\nu}\Gamma^{\gamma}_{\alpha\lambda}\right)=\frac{\varepsilon^{\lambda\mu\nu}}{\mu}\Gamma^{\rho}_{\sigma\lambda}\left(\partial_\mu\Gamma^{\sigma}_{\rho\nu}+\frac{2}{3}\Gamma^{\sigma}_{\omega\mu}\Gamma^{\omega}_{\nu\rho}\right), \tag{26}$$

where μ is a dimensionless parameter. Linearizing (26), we obtain

$$\mathcal{L}_{C.S.lin.}=\frac{1}{2}\frac{1}{M}h^{\mu\nu}\mathcal{P}_{\mu\nu,\rho\sigma}h^{\rho\sigma}, \tag{27}$$

where[28]

$$\mathcal{P}_{\mu\nu,\rho\sigma}\equiv\frac{\Box\partial^{\lambda}}{4}[\varepsilon_{\mu\lambda\rho}\theta_{\nu\sigma}+\varepsilon_{\mu\lambda\sigma}\theta_{\nu\rho}+\varepsilon_{\nu\lambda\rho}\theta_{\mu\sigma}+\varepsilon_{\nu\lambda\sigma}\theta_{\mu\rho}], \tag{28}$$

and $M\equiv\mu/\kappa^2$. In order to have a complete basis for the operator space of the field equations concerning higher-derivative gravity theories in three dimensions with a Chern–Simons term, we

TABLE I. Multiplication table for the three-dimensional operators P^1, P^2, P^0, $\overline{P^0}$, $\bar{\bar{P}}^0$, and P.

	P^1	P^2	P^0	$\overline{P^0}$	$\bar{\bar{P}}^0$	P
P^1	P^1	O	O	O	O	O
P^2	O	P^2	O	O	O	P
P^0	O	O	P^0	O	$P^{\theta\omega}$	O
$\overline{P^0}$	O	O	O	$\overline{P^0}$	$P^{\omega\theta}$	O
$\bar{\bar{P}}^0$	O	O	$P^{\omega\theta}$	$P^{\theta\omega}$	$2(P^0+\overline{P^0})$	O
P	O	P	O	O	O	$-k^6P^2$

include in the collection of three-dimensional operators $\{P^1,P^2,P^0,\overline{P^0},\bar{\bar{P}}^0\}$ (see Sec. III A) the operator P (28). The multiplication table for these operators is displayed in Table I.

We are now ready to compute the propagator. Expanding both operators $\mathcal{O}$ and $\mathcal{O}^{-1}$ in the basis $\{P^1,P^2,P^0,\overline{P^0},\bar{\bar{P}}^0\}$, we obtain

$$\mathcal{O}=x_1P^1+x_2P^2+x_0P^0+\overline{x_0}\,\overline{P^0}+\bar{\bar{x}}_0\bar{\bar{P}}^0+pP,$$

$$\mathcal{O}^{-1}=y_1P^1+y_2P^2+y_0P^0+\overline{y_0}\,\overline{P^0}+\bar{\bar{y}}_0\bar{\bar{P}}^0+qP.$$

With the help of Table I and taking into account that $\mathcal{O}\mathcal{O}^{-1}=I$, we find that $y_1=1/x_1$, $y_2=x_2/(x_2^2-p^2k^6)$, $y_0=\overline{x_0}/(x_0\overline{x_0}-2\bar{\bar{x}}_0^2)$, $\overline{y_0}=x_0/(x_0\overline{x_0}-2\bar{\bar{x}}_0^2)$, $\bar{\bar{y}}_0=-\bar{\bar{x}}_0(x_0\overline{x_0}-2\bar{\bar{x}}_0^2)$, and $q=-p/(x_2^2-p^2k^6)$, while the propagator is given by

$$\mathcal{O}^{-1}=\frac{1}{x_1}P^1+\frac{x_2}{x_2^2-p^2k^6}P^2+\frac{\overline{x_0}}{x_0\overline{x_0}-2\bar{\bar{x}}_0^2}P^0+\frac{x_0}{x_0\overline{x_0}-2\bar{\bar{x}}_0^2}\overline{P^0}-\frac{\bar{\bar{x}}_0}{x_0\overline{x_0}-2\bar{\bar{x}}_0^2}\bar{\bar{P}}^0-\frac{p}{x_2^2-p^2k^6}P. \tag{29}$$

Accordingly, let us then find the propagator for quadratic gravity with a Chern–Simons term. This theory is defined by the Lagrangian

$$\overline{\mathcal{L}}=-\frac{2R\sqrt{g}}{\kappa^2}+\frac{\varepsilon^{\mu\nu\lambda}}{\mu}\Gamma^{\rho}_{\sigma\lambda}\left(\partial_\mu\Gamma^{\sigma}_{\rho\nu}+\frac{2}{3}\Gamma^{\sigma}_{\omega\mu}\Gamma^{\omega}_{\nu\rho}\right)+\left(\frac{\alpha}{2}R^2+\frac{\beta}{2}R^2_{\mu\nu}\right)\sqrt{g}.$$

In the Julve–Tonin gauge the operator $\mathcal{O}$ has the form

$$\mathcal{O}=-k^2\left(\lambda_1+\lambda_3\frac{b}{2}k^2\right)P^1+k^2\left(k^2\frac{b}{2}-1\right)P^2+\left[k^2+bk^4\left(\frac{3}{2}+4c\right)\right]P^0$$
$$-k^2\left(\frac{b}{2}\lambda_2k^2+2\lambda_1\right)\overline{P^0}+\frac{P}{M},$$

and the propagator is given by

$$\mathcal{O}^{-1}=\frac{-2}{k^2[2\lambda_1+b\lambda_3k^2]}P^1+\left[-\frac{1}{k^2}+\frac{1}{1+(bM_2^2/2)}\frac{1}{k^2-M_2^2}+\frac{1}{1+(bM_1^2/2)}\frac{1}{k^2-M_1^2}\right]P^2$$
$$+\left[\frac{1}{k^2}-\frac{1}{k^2-m^2}\right]P^0-\frac{1}{k^2[2\lambda_1+\lambda_2(b/2)k^2]}\overline{P^0}$$
$$-\left[\frac{4}{b^2M(M_1^2-M_2^2)}\left(\frac{1}{k^2-M_1^2}-\frac{1}{k^2-M_2^2}\right)\frac{1}{k^4}\right]P, \tag{30}$$

where

$$M_1^2 \equiv \left(\frac{2}{b^2M^2}\right)[1+bM^2+\sqrt{1+2bM^2}],$$

$$M_2^2 \equiv \left(\frac{2}{b^2M^2}\right)[1+bM^2-\sqrt{1+2bM^2}],$$

$$m^2 \equiv \frac{-1}{b\left(\frac{3}{2}+4c\right)}.$$

If we do not want tachyons in the dynamical field, we may choose, for instance, $b>0$ and $(\frac{3}{2}+4c)<0$. In this case the theory is causal at the tree level. In this vein we assume from now on m^2, M_1^2, M_2^2, and $M^2>0$.

We discuss in the following tree-level unitarity, nonrelativistic limit and gravitational deflection for quadratic-Chern-Simons gravity in $D=3$.

A. Tree-level unitarity

From (14) and (30) we obtain at once

$$\mathcal{A}=\mathrm{g}^2\left[\frac{2}{b^2}\frac{bM_2^2-2}{(M_2^2-M_1^2)M_2^2}\frac{T_{\mu\nu}T^{\mu\nu}-\frac{1}{2}T^2}{k^2-M_2^2}+\frac{2}{b^2}\frac{2-bM_1^2}{(M_2^2-M_1^2)M_1^2}\frac{T_{\mu\nu}T^{\mu\nu}-\frac{1}{2}T^2}{k^2-M_1^2}-\frac{\frac{1}{2}T^2}{k^2-m^2}\right.$$
$$\left.+\frac{4}{b^2M_1^2M_2^2}\frac{T^2-T_{\mu\nu}T^{\mu\nu}}{k^2}\right]. \tag{31}$$

Therefore, if $b>0$ and $(\frac{3}{2}+4c)<0$, (20) tells us that $\mathrm{Res}\,\mathcal{A}|_{k^2=M_1^2}>0$, $\mathrm{Res}\,\mathcal{A}|_{k^2=M_2^2}>0$, $\mathrm{Res}\,\mathcal{A}|_{k^2=m^2}<0$, and $\mathrm{Res}\,\mathcal{A}|_{k^2=0}=0$.

Proposition 4: Quadratic gravity with a gravitational Chern–Simons term is nonunitary at the tree level. If $b>0$ and $(\frac{3}{2}+4c)<0$, the theory in nontachyonic and has one normal massless spin-2 particle, one normal spin-2 particle of mass M_2, one normal spin-2 particle of mass M_1 and one spin-0 ghost of mass m. The massless excitation is not a dynamical degree of freedom.

Corollary 2: $R+R^2$ gravity with a Chern–Simons term is nonunitary at the tree level. If $\alpha<0$, the theory is nontachyonic and has one physical massless spin-2 particle, one physical particle of spin-2 and mass M and one spin-0 ghost of mass $m\equiv\sqrt{-1/2\alpha\kappa^2}$. The massless excitation is not a dynamical degree of freedom.

Proof: Clear. □

Therefore, we come to the conclusion that neither quadratic gravity with a Chern–Simons term nor $R+R^2$ gravity with a Chern–Simons term is unitary at the tree level.

A detailed comparison between three-dimensional quadratic gravity with quadratic gravity with a Chern–Simons term clearly shows that the harmless massive scalar mode of the former becomes a troublesome massive spin-0 ghost within the context of the latter, while the massive spin-2 ghost related to three-dimensional quadratic gravity is now replaced by two massive physical particles both of spin-2. On the other hand, if we make a comparison between three-dimensional $R+R^2$ gravity with $R+R^2$ gravity with a Chern–Simons term, we come to the conclusion that the gravitational Chern–Simons term is responsible for breaking down the unitarity of the former.

B. Nonrelativistic limit

In this case the invariant amplitude for the process $S+S\rightarrow S+S$, where S denotes a spinless boson of mass $\tilde{m}$, as well as for its nonrelativistic limit, are given respectively by

$$\mathcal{M}=\kappa^2\left[-\frac{1}{2k^2}+\frac{1}{bM_2^2+2}\,\frac{1}{k^2-M_2^2}+\frac{1}{b^2M_1^2+2}\,\frac{1}{k^2-M_1^2}\right]\times\left[(p\cdot q)(p'\cdot q')+(p\cdot q')(p'\cdot q)\right.$$
$$+p\cdot p'(\tilde{m}^2-q\cdot q')+(\tilde{m}^2-p\cdot p')q\cdot q'+\frac{3}{2}(\tilde{m}^2-q\cdot q')(\tilde{m}^2-p\cdot p')-\frac{1}{4}(3\tilde{m}^2-p\cdot p')$$
$$\left.\times(3\tilde{m}^2-q\cdot q')\right]+\frac{\kappa^2}{8}\left\{(3\tilde{m}^2-p\cdot p')(3\tilde{m}^2-q\cdot q')\left[\frac{1}{k^2}-\frac{1}{k^2-m^2}\right]\right\}$$

and

$$\mathcal{M}_{N.R.}=\kappa^2\tilde{m}^4\left[\frac{1}{2}\,\frac{1}{\mathbf{k}^2+m^2}-\frac{1}{2+bM_2^2}\,\frac{1}{\mathbf{k}^2+M_2^2}-\frac{1}{2+bM_1^2}\,\frac{1}{\mathbf{k}^2+M_1^2}\right]. \tag{32}$$

Inserting (32) into (21) and performing the integration yields

$$U(r)=2\tilde{m}^2\overline{G}\left[K_0(rm)-\frac{1}{1+(bM_1^2/2)}K_0(rM_1)-\frac{1}{1+(bM_2^2/2)}K_0(rM_2)\right].$$

As a result, the potential is given by the expression

$$V(r)=2\tilde{m}\overline{G}\left[K_0(rm)-\frac{1}{1+(bM_1^2/2)}K_0(rM_1)-\frac{1}{1+(bM_2^2/2)}K_0(rM_2)\right].$$

Note that $V(r)$ behaves as $2\tilde{m}\overline{G}\,\ln M_1^{1+(bM_1^2/2)}\,M_2^{1+(bM_2^2/2)}/m$ at the origin and as

$$2\tilde{m}\overline{G}\left[\sqrt{\frac{\pi}{2mr}}\,e^{-rm}-\frac{1}{1+(bM_1^2/2)}\sqrt{\frac{\pi}{2M_1r}}\,e^{-M_1r}-\frac{1}{1+(bM_2^2/2)}\sqrt{\frac{\pi}{2M_2r}}\,e^{-M_2r}\right]$$

asymptotically. Two comments are in order here:

(i) Unlike the Newtonian potential $V_N=2\overline{G}\tilde{m}\ln r_0/r_1$ which has a logarithmic singularity at the origin and is unbounded at infinity, the potential concerning linearized quadratic Chern–Simons gravity in $(2+1)D$ is extremely well behaved: it is finite at the origin and zero at infinity.

(ii) $V(r)\rightarrow 0$ as α and $\beta\rightarrow 0$, confirming in this way the fact that the standard correspondence of three-dimensional linearized Einstein–Chern–Simons gravity with Newton's theory breaks down.[29,30]

C. Light deflection

The current-current amplitude for the interaction of a light ray with a fixed source is given by

$$\mathcal{A}=\mathrm{g}^2T^{\mu\nu}F^{\rho\sigma}\left[\left(-\frac{1}{k^2}+\frac{1}{1+(bM_2^2/2)}\,\frac{1}{k^2-M_2^2}+\frac{1}{1+(bM_1^2/2)}\,\frac{1}{k^2-M_1^2}\right)P^2\right.$$
$$\left.+\left(\frac{1}{k^2}-\frac{1}{k^2-m^2}\right)P^0\right]_{\mu\nu,\rho\sigma}.$$

Taking (9) into account we can rewrite the expression above as

$$\mathcal{A}=\mathcal{A}_0+\mathcal{A}_1,$$

where

$$\mathcal{A}_0\equiv-\frac{\mathrm{g}^2T_{00}F_{00}}{k^2},$$

$$\mathcal{A}_1\equiv\mathrm{g}^2T_{00}F_{00}\left(\frac{1}{1+(bM_2^2/2)}\,\frac{1}{k^2-M_2^2}+\frac{1}{1+(bM_1^2/2)}\,\frac{1}{k^2-M_1^2}\right).$$

Of course, only $\mathcal{A}_1$ will contribute to light deflection since the massless excitation is not a degree of freedom. Hence, the light ray will be deflected downward as usual.

VIII. SUMMARY AND DISCUSSION

We proposed a prescription for finding the propagator concerning higher-derivative gravity theory in D dimensions based on the Barnes–Rivers operators. Using this algorithm, we computed the propagator for the latter in an unconventional gauge and, by a suitable choice of the gauge parameters, we reobtained the propagator in a series of gauges which are used in day-to-day physics.

A systematic study of the tree-level unitarity of D-dimensional higher-derivative gravity theory was presented afterward. It was shown that it is nonunitary at the tree level: the term quadratic in the Ricci tensor is the Achille's heel of the theory. This term is responsible for the presence of a massive spin-2 particle of negative residue, i.e., a ghost, in the bare propagator. Nevertheless, this may be a hasty conclusion. Indeed, as pointed out by Antoniadis and Tomboulis,[15] this excitation is unstable in four dimensions. Perhaps it would be unstable as well in any dimension. Therefore, D-dimensional quadratic gravity cannot yet be rejected as a viable possibility.

On the other hand, D-dimensional $R+R^2$ gravity is unitary at the tree level. We call attention to the fact that our discussion was confined to a particular style of variational principle. Perhaps there are richer unitary combinations in higher dimensions with the connection varied independently. If this is the case, that would be worth knowing. This matter will be the object of a future investigation.

The problem of computing the effective nonrelativistic potential for D-dimensional quadratic gravity was then reduced to quadratures. It seems that the existence of this potential in any dimension is related to the absence of tachyons (both positive and negative energy) in the dynamical field. It was also shown that, unlike three-dimensional linearized gravity which has no Newtonian limit, three-dimensional linearized quadratic gravity has a potential that, despite being rather different from the corresponding Newtonian one, is extremely well behaved.

It was shown afterward that the gravitational deflection is always smaller than that predicted by the corresponding Einstein's theory. This conclusion is totally independent of the value of D. For $D=3$ we arrive at the astonishing result that a light ray would be deflected upward instead of downward as its four-dimensional counterpart. It can be shown, in addition, that in $D=3$ a gravitational force is exerted on a slowly moving test particle.[31] This force greatly resembles that of a straight $U(1)$-gauge cosmic string in the framework of linearized quadratic gravity in four dimensions[32] and it is absent in three-dimensional general relativity.

Finally, we discussed three-dimensional quadratic gravity augmented by a Chern–Simons term. It was shown that neither the latter nor three-dimensional $R+R^2$ gravity with a Chern–Simons term are unitary at the tree level. On the other hand, as it was shown previously, light deflection has the ''wrong sign'' within the context of three-dimensional quadratic gravity. The addition of a topological massive term to the latter ''repairs'' the aforementioned sign. Is worth mentioning that ''antigravity'' is possible in the context of three-dimensional quadratic Chern–Simons gravity.[33]

To conclude we raise two interesting questions:

(i) Is it possible that gravity's rainbows and quadratic gravity theories can coexist without conflict in any dimension?

(ii) Is the photon propagation dispersive in the framework of three-dimensional quadratic gravity with a Chern–Simons term such as in four-dimensional quadratic gravity? These issues will be discussed elsewhere.

ACKNOWLEDGMENT

A.A. is very grateful to Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) for financial support.

[1] G.'t Hooft and M. Veltman, Ann. Inst. Henri Poincare, Sect. A **20**, 69 (1974).
[2] M. H. Goroff and A. Sagnotti, Phys. Lett. B **160**, 81 (1985).
[3] G.'t Hooft and M. Veltman, Nucl. Phys. B **44**, 189 (1972).
[4] K. S. Stelle, Gen. Relativ. Gravit. **9**, 353 (1978).
[5] A. J. Accioly and A. T. Gonçalves, J. Math. Phys. **28**, 1547 (1987).
[6] A. J. Accioly and G. E. A. Matsas, Phys. Rev. D **38**, 1083 (1988).
[7] A. Accioly, A. Azeredo, H. Mukai, and E. de Rey Neto, Prog. Theor. Phys. **104**, 103 (2000).
[8] A. Accioly, H. Mukai, and H. Blass, Nuovo Cimento B **115**, 1235 (2000).
[9] A. Accioly and H. Blas, Phys. Rev. D **64**, 067701 (2001).
[10] A. Accioly, A. D. Azeredo, E. C. de Rey Neto, and H. Mukai, Braz. J. Phys. **28**, 496 (1998).
[11] A. Accioly, S. Ragusa, E. C. de Rey Neto, and H. Mukai, Nuovo Cimento B **114**, 595 (1999).
[12] R. J. Rivers, Nuovo Cimento **34**, 387 (1964).
[13] P. van Nieuwenhuizen, Nucl. Phys. B **60**, 478 (1973).
[14] K. S. Stelle, Phys. Rev. D **16**, 953 (1977).
[15] I. Antoniadis and E. T. Tomboulis, Phys. Rev. D **33**, 2756 (1986).
[16] J. C. Alonso, F. Barbero, J. Julve, and A. Tiemblo, Class. Quantum Grav. **11**, 865 (1994).
[17] A. Accioly, S. Ragusa, H. Mukai, and E. de Rey Neto, Int. J. Theor. Phys. **39**, 1601 (2000).
[18] L. P. Eisenhart, *Riemannian Geometry* (Princeton U. P., Princeton, NJ, 1949), p. 92.
[19] J. Julve and M. Tonin, Nuovo Cimento **46**, 137 (1978).
[20] P. van Nieuwenhuizen, Nucl. Phys. B **60**, 478 (1973).
[21] F. C. P. Nunes and G. O. Pires, Phys. Lett. B **301**, 339 (1993).
[22] C. Pinheiro, G. O. Pires, and F. A. B. Rabelo de Carvalho, Braz. J. Phys. **27**, 14 (1997).
[23] A. Accioly, H. Mukai, and A. Azeredo, Class. Quantum Grav. **18**, L31 (2001).
[24] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed. (Academic, New York, 1994).
[25] A. N. Tikhonov and A. A. Samarskii, *Equations of Mathematical Physics* (Dover, New York, 1990).
[26] S. Giddings, J. Abbott, and K. Kuchăr, Gen. Relativ. Gravit. **16**, 751 (1984).
[27] A. Accioly, A. Azeredo, and H. Mukai, Phys. Lett. A **279**, 169 (2001).
[28] A. Accioly, H. Mukai, and A. Azeredo, Phys. Lett. B **495**, 394 (2000).
[29] S. Deser, R. Jackiw, and S. Templeton, Phys. Rev. Lett. **48**, 975 (1982).
[30] S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) **140**, 372 (1982).
[31] A. Accioly, H. Mukai, and A. Azeredo, Mod. Phys. Lett. A **16**, 1449 (2001).
[32] B. Linet and P. Teyssandier, Class. Quantum Grav. **9**, 159 (1992).
[33] A. Accioly, H. Mukai, and A. Azeredo, "Quadratic Gravity in (2+1)D with a Topological Chern-Simons Term," J. Phys. A (to appear).