

Soft CP violation in K -meson systemsJ. C. Montero,^{1,*} C. C. Nishi,^{1,†} V. Pleitez,^{1,‡} O. Ravinez,^{2,§} and M. C. Rodriguez^{3,||}¹*Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona, 145 01405-900– São Paulo, Brazil*²*Facultad de Ciencias, Universidad Nacional de Ingeniería UNI, Avenida Tupac Amaru S/N apartado 31139 Lima, Peru*³*Departamento de Física, Fundação Universidade Federal do Rio Grande/FURG, Av. Itália, km 8, Campus Carreiros 96201-900, Rio Grande, RS, Brazil*

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We consider a model with soft CP violation which accommodates the CP violation in the neutral kaons even if we assume that the Cabibbo-Kobayashi-Maskawa mixing matrix is real and the sources of CP violation are three complex vacuum expectation values and a trilinear coupling in the scalar potential. We show that for some reasonable values of the masses and other parameters the model allows us to explain all the observed CP violation processes in the K^0 - \bar{K}^0 system.

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I. INTRODUCTION

Until some time ago, the only physical system in which the violation of the CP symmetry was observed was the neutral kaon system [1]. Besides, only the indirect CP violation described by the ϵ parameter was measured in that system. Only recently has clear evidence for direct CP violation parametrized by the ϵ' parameter been observed in laboratory [2]. Moreover, the CP violation in the B -mesons system has been finally observed as well [3]. It is in fact very impressive that all of these observations are accommodated by the electroweak standard model with a complex Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [4,5] when QCD effects are also included. In the context of that model, the only way to introduce CP violation is throughout its hard violation due to complex Yukawa couplings, which imply a surviving phase in the charged current coupled to the vector boson W^\pm in the quark sector. In the neutral kaon system, despite the CKM phase being $O(1)$, the breakdown of that symmetry is naturally small because its effect involves the three quark families at the one loop level [6]. This is not the case of the B mesons where the three families are involved even at the tree level and the CP violating asymmetries are $O(1)$ [7].

Notwithstanding, if new physics does exist at the TeV scale it may imply new sources of CP violation. In this context the question if the CKM matrix is complex becomes nontrivial since at least part of the CP violation may come from the new physics sector [8]. For instance, even in the context of a model with $SU(2)_L \otimes U(1)_Y$ gauge symmetry, we may have spontaneous CP violation through the complex vacuum expectation values (VEVs). This is the case of the two Higgs doublets extension of the standard model if we do not impose the suppression of flavor-

changing neutral currents (FCNCs), as in Ref. [9]. The CP violation may also arise throughout the exchange of charged scalars if there are at least three doublets and no FCNCs [10]. Truly soft CP violation may also arise throughout a complex dimensional coupling constant in the scalar potential and with no CKM phase [11]. In fact, all these mechanisms can be at work in multi-Higgs extensions of the standard model [12]. Hence, in the absence of a general principle, all possible sources of CP violation must be considered in a given model. However, it is always interesting to see the potentialities of a given source to explain by itself all the present experimental data. This is not a trivial issue since, for instance, CP violation mediated by Higgs scalars in models without flavor changing neutral currents have been almost ruled out even by old data [13–17].

Among the interesting extensions of the standard model there are the models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry called 3-3-1 models for short [18–20]. These models have shown to be very predictive not only because of the relation with the generation problem, some representation content of these models allows three and only three families when the cancellation of anomalies and asymptotic freedom are used; they also give some insight about the observed value of the weak mixing angle [21]. The 3-3-1 models are also interesting context in which new theoretical ideas as extra dimensions [22] and the little Higgs mechanism can be implemented [23].

In the minimal 3-3-1 model [18] both mechanisms of CP violation, hard [24] and spontaneous [25] have already been considered. In this paper we analyze soft CP violation in the framework of the 3-3-1 model of Ref. [19] in which only three triplets are needed for breaking the gauge symmetry appropriately and give mass to all fermions. Although it has been shown that in this model pure spontaneous CP violation is not possible [25], we can still implement soft CP violation if, besides the three scalar VEVs, a trilinear parameter in the scalar potential is allowed to be complex. In this case a physical phase survives violating

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the CP symmetry. This mechanism was developed in Ref. [26] but there a detailed analysis of the CP observables in both kaons and B -mesons was not given. Here we will show that all the CP violating parameters in the neutral kaon system can be explained through this mechanism, leaving the case of the B -mesons for a forthcoming paper.

The outline of this paper is as follows. In Sec. II we briefly review the model of Ref. [26] in which we will study a mechanism for soft CP violation. In Sec. III we review the usual parametrization of the CP violating parameters of the neutral kaon system, ϵ and ϵ' , establishing what is in fact being calculated in the context of the present model. In Sec. IV we calculate ϵ , and in Sec. V we do the same for ϵ' . The possible values for those parameters in the context of our model are considered in Sec. VI, while our conclusions are in the last section. In the appendix we write some integrals appearing in box and penguin diagrams.

II. THE MODEL

Here we are mainly concerned with the doubly charged scalar and its Yukawa interactions with quarks since this is the only sector in which the soft CP violation arises in this model [26]. The interaction with the doubly charged vector boson will be considered when needed (Sec. V). As expected, there is only a doubly charged would be Goldstone boson, G^{++} , and a physical doubly charged scalar, Y^{++} , defined by

$$\begin{pmatrix} \rho^{++} \\ \chi^{++} \end{pmatrix} = \frac{1}{N} \begin{pmatrix} |v_\rho| & -|v_\chi|e^{-i\theta_\chi} \\ |v_\chi|e^{i\theta_\chi} & |v_\rho| \end{pmatrix} \begin{pmatrix} G^{++} \\ Y^{++} \end{pmatrix}, \quad (1)$$

where $N = (|v_\rho|^2 + |v_\chi|^2)^{1/2}$; the mass square of the Y^{++} field is given by

$$m_{Y^{++}}^2 = \frac{A}{\sqrt{2}} \left(\frac{1}{|v_\chi|^2} + \frac{1}{|v_\rho|^2} \right) - \frac{a_8}{2} (|v_\chi|^2 + |v_\rho|^2), \quad (2)$$

where we have defined $A \equiv \text{Re}(fv_\eta v_\rho v_\chi)$ with f a complex parameter in the trilinear term $\eta\rho\chi$ of the scalar potential and a_8 is the coupling of the quartic term $(\chi^\dagger\rho) \times (\rho^\dagger\chi)$ in the scalar potential. For details and notation see Ref. [26]. Notice that since $|v_\chi| \gg |v_\rho|$, it is ρ^{++} which is almost Y^{++} .

In Ref. [26] it was shown that all CP violation effects arise from the singly and/or doubly charged scalar-exotic quark interactions. Notwithstanding, the CP violation in the singly charged scalar is avoided by assuming the total leptonic number L (or $B + L$, see below) conservation and, in this case, only two phases survive after the redefinition of the phases of all fermion fields in the model: a phase of the trilinear coupling constant f and the phase of a vacuum expectation value, say v_χ . Among these phases, actually only one survives because of the constraint equation

$$\text{Im}(fv_\chi v_\rho v_\eta) = 0, \quad (3)$$

which implies $\theta_\chi = -\theta_f$.

Let us briefly recall the representation content of the model [26] with a little modification in the notation. In the quark sector we have $Q_{iL} = (d_i, u_i, j_i)_L^T \sim (\mathbf{3}, \mathbf{3}^*, -1/3)$, $i = 1, 2$ $Q_{3L} = (u_3, d_3, J)_L^T \sim (\mathbf{3}, \mathbf{3}, 2/3)$; $U_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, 2/3)$, $D_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, -1/3)$, $\alpha = 1, 2, 3$, $j_{iR} \sim (\mathbf{3}, \mathbf{1}, -4/3)$ and $J_R \sim (\mathbf{3}, \mathbf{1}, 5/3)$, and the Yukawa interactions are written as:

$$\begin{aligned} -\mathcal{L} = & \sum_{i\alpha} \overline{Q}_{iL} (F_{i\alpha} \rho^* U_{\alpha R} + \tilde{F}_{i\alpha} D_{\alpha R} \eta^*) + \overline{Q}_{3L} (F_{3\alpha} U_{\alpha R} \eta \\ & + \tilde{F}_{3\alpha} D_{\alpha R} \rho) + \sum_{im} \lambda_{im} \overline{Q}_{iL} j_{mR} \chi^* \\ & + \lambda_3 \overline{Q}_{3L} J_R \chi + H.c., \end{aligned} \quad (4)$$

where all couplings in the matrices F , \tilde{F} , and λ 's are in principle complex. Although the fields in Eq. (4) are symmetry eigenstates we have omitted a particular notation. Here we will assume that all the Yukawa couplings in Eq. (4) are real in such a way that we may be able to test to what extension only the phase θ_χ can describe the CP violation parameters in the neutral kaon system, ϵ and ϵ' .

In order to diagonalize the mass matrices coming from Eq. (4), we introduce real and orthogonal left- and right-handed mixing matrices defined as

$$U'_{L(R)} = \mathcal{O}_{L(R)}^u U_{L(R)}, \quad D'_{L(R)} = \mathcal{O}_{L(R)}^d D_{L(R)}, \quad (5)$$

with $U = (u, c, t)^T$ etc.; the primed fields denote symmetry eigenstates and the unprimed ones mass eigenstates, being the Cabibbo-Kobayashi-Maskawa matrix defined as $V_{\text{CKM}} = \mathcal{O}_{L'}^{uT} \mathcal{O}_L^d$.

In terms of the mass eigenstates the Lagrangian interaction involving exotic quarks, the known quarks, and doubly charged scalars is given by [26]:

$$\begin{aligned} -\mathcal{L}_Y = & \sqrt{2} \tilde{J} \left[e^{-i\theta_\chi} \frac{|v_\chi|}{N} \frac{M_\alpha^d}{|v_\rho|} R - e^{+i\theta_\chi} \frac{|v_\rho|}{N} \frac{m_J}{|v_\chi|} L \right] \\ & \times (\mathcal{O}_L^d)_{3\alpha} d_\alpha Y^{++} + \text{h.c.}, \end{aligned} \quad (6)$$

where N is the same parameter appearing in Eq. (1), i.e., $N = (|v_\rho|^2 + |v_\chi|^2)^{1/2}$ and now, unlike Eq. (4), all fields are mass eigenstates, $L = (1 - \gamma_5)/2$, $R = (1 + \gamma_5)/2$, with $m_J = \lambda_3 |v_\chi| / \sqrt{2}$. In writing the first term of Eq. (6) we have used $\tilde{F}_{3\alpha} = \sqrt{2} (\mathcal{O}_L^d M^d \mathcal{O}_R^{dT})_{3\alpha} / |v_\rho|$, where M^d is the diagonal mass matrix in the d -quark sector and we have omitted the summation symbol in α so that $d_\alpha = d, s, b$. The Eq. (6) contains all CP violation in the quark sector once we have assumed that all the Yukawa couplings are real. Unlike in multi-Higgs extensions of the standard model [9–17] there is no Cabibbo suppression since in this model only one quark, J , contributes in the internal line, i.e., we have the replacement $u, c, t, \rightarrow J$.

Notice that in Eq. (6) the suppression of the mixing angle in the sector of the doubly charged scalars [see Eq. (1)] has been written explicitly. We will use as illustrative values $|v_\rho| \leq 246$ GeV and $|v_\chi| \geq 1$ TeV. In this situation the CP violation in the neutral kaon system will impose constraints only upon the masses m_J , m_Y , and, in principle, on m_U the mass of the doubly charged vector boson. Although \mathcal{O}_L^j has free parameters since the masses $m_{j_{1,2}}$ are not known, the exotic quarks $j_{1,2}$, do not play any role in the CP violation phenomena of K mesons.

We should mention that it was implicit in the model of Ref. [26] the conservation of the quantum number $B + L$ defined in Refs. [19,20]. Only in this circumstance (or by introducing appropriately a Z_2 symmetry) we can avoid terms like $\epsilon(\Psi_{aL})^c \Psi_{bL} \eta$ and $(l_{aL})^c E_{bR}$, where Ψ_L , l_R and E_R denote the left-handed lepton triplet, and the usual right-handed components for usual and exotic leptons. These interactions imply mixing among the left- and right-handed components of the usual charged leptons with the exotic ones [27]. The quartic term $\chi^\dagger \eta \rho^\dagger \eta$ in the scalar potential which would imply CP violation throughout the single charged scalar exchange is also avoided by imposing the $B + L$ conservation. In fact, this model has the interesting feature that when a Z_2 symmetry is imposed, the Peccei-Quinn $U(1)$, the total lepton number, and the baryon number are all automatic symmetries of the classic Lagrangian [28].

III. CP VIOLATION IN THE NEUTRAL KAONS

First of all let us say that in the present model there are tree level contributions to the mass difference $\Delta M_K = 2\text{Re}M_{12}$ (where $M_{12} = \langle K^0 | \mathcal{H}_{\text{eff}} | \bar{K}^0 \rangle / 2m_K$). This is because the existence of the flavor changing neutral currents in the model in both the scalar sector and in the couplings with the Z^0 . The H^0 's contributions to ΔM_K have been considered in Ref. [25]. For $m_H \sim 150$ GeV the constraint coming from the experimental value of ΔM_K implies $(\mathcal{O}_L^d)_{dd}(\mathcal{O}_L^d)_{ds} \leq 0.01$. There are also tree-level contributions to ΔM_K coming from the Z' exchange which were considered in Ref. [18,29]. However, since there are 520 diagrams contributing to ΔM_K , we will use in this work the experimental value for this parameter. In this vain *a priori* there is no constraints on the matrix elements of \mathcal{O}_L^d .

The definition for the relevant parameters in the neutral kaon system is the usual one [30–33]:

$$\begin{aligned} \epsilon' &= \frac{e^{i(\delta_2 - \delta_0 + \pi/2)}}{\sqrt{2}} \frac{\text{Re}A_2}{\text{Re}A_0} \left[\frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right], \\ \epsilon &= \frac{e^{i\pi/4}}{\sqrt{2}} \left[\frac{\text{Im}A_0}{\text{Re}A_0} + \frac{\text{Im}M_{12}}{\Delta M_K} \right]. \end{aligned} \quad (7)$$

We shall use the $\Delta I = 1/2$ rule for the nonleptonic decays which implies that $\text{Re}A_0/\text{Re}A_2 \simeq 22.2$ and that the phase $\delta_2 - \delta_0 \simeq -\frac{\pi}{4}$ is determined by hadronic parameters following Ref. [34] and it is, therefore, model independent.

The ϵ parameter has been extensively measured and its value is reported to be [33]

$$|\epsilon_{\text{exp}}| = (2.284 \pm 0.014) \times 10^{-3}. \quad (8)$$

More recently, the experimental status for the ϵ'/ϵ ratio has stressed the clear evidence for a nonzero value and, therefore, the existence of direct CP violation. The present world average (wa) is [33]

$$|\epsilon'/\epsilon|_{\text{wa}} = (1.67 \pm 0.26) \times 10^{-3}, \quad (9)$$

where the relative phase between ϵ and ϵ' is negligible [35]. These values of $|\epsilon|$ and $|\epsilon'/\epsilon|$ imply

$$|\epsilon'_{\text{exp}}| = 3.8 \times 10^{-6}. \quad (10)$$

On the other hand, we can approximate

$$|\epsilon| \approx \frac{1}{\sqrt{2}} \left| \frac{\text{Im}M_{12}}{\Delta M_K} \right|, \quad (11a)$$

$$|\epsilon'| \approx \frac{1}{22.2\sqrt{2}} \left| \frac{\text{Im}A_0}{\text{Re}A_0} \right|. \quad (11b)$$

In the prediction of ϵ'/ϵ , $\text{Re}A_0$ and ΔM_K are taken from experiments, whereas $\text{Im}A_0$ and $\text{Im}M_{12}$ are computed quantities [36]. The experimental values used in this work are $\text{Re}A_0 = 3.3 \times 10^{-7}$ GeV and $\Delta M_K = 3.5 \times 10^{-15}$ GeV.

Let us finally consider the condition with which we will calculate the parameters ϵ and ϵ' . The main $\Delta S = 1$ contribution for the ϵ' parameter comes from the gluonic penguin diagram in Fig. 1 that exchanges a doubly charged scalar. The electroweak penguin is suppressed as in the SM and will not be considered. On the other hand the $\Delta S = 2$ and CP violating parameter ϵ has only contributions coming from box diagrams involving two doubly charged scalars Y^{++} [see Fig. 2(a)] and box diagrams involving one doubly charged scalar and one vector boson U^{++} [see Fig. 2(b)]. The relevant vertices for the calculations are

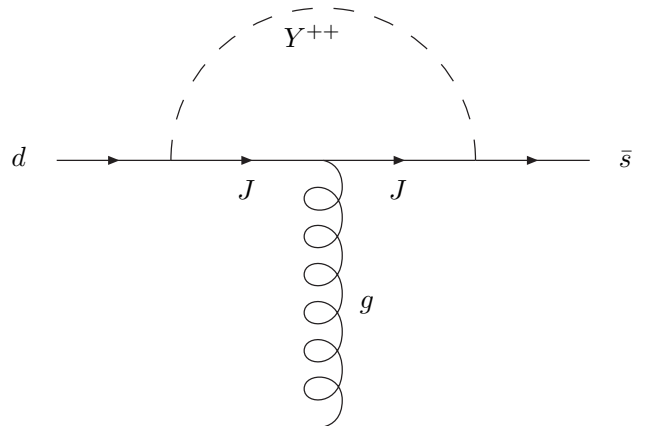


FIG. 1. Dominant CP violating penguin diagram contributing to the decay $K^0 \rightarrow \pi\pi$.

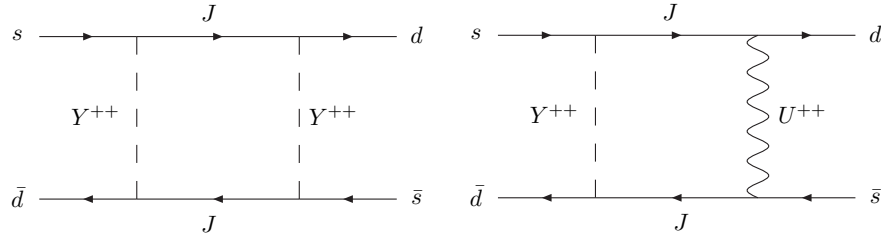


FIG. 2. (a) One of the box diagrams responsible for the transition $\bar{K}^0 \rightarrow K^0$ that involves the exchange of two doubly charged scalars Y^{++} . (b) One of the box diagrams responsible for the transition $\bar{K}^0 \rightarrow K^0$ that involves the exchange of a doubly charged scalar Y^{++} and a doubly charged vector boson U^{++} .

given in Eq. (6) and we will use the unitary gauge in our calculations. In other renormalizable R_ξ gauges we must take into account the would be Goldstone contributions and notice that, according to Eq. (1), the component of $\chi^{++} \sim O(1)G^{++}$.

The hadronic matrix elements will be taken from literature and whenever possible we also take, for the reasons we expose at the beginning of this section, from the experimental data or as free parameters. One of the features of this model is that there is no GIM mechanism since the only CP violation source comes from the vertices involving a d -type quark, an exotic quark, and a single doubly charged scalar.

IV. DIRECT CP VIOLATION

The dominant contributions to the ϵ' parameter come from the penguin diagram showed in Fig. 1 [32,37]. The part of the Lagrangian that takes into account this amplitude is obtained from Eq. (6) and the corresponding imaginary effective interaction is given by

$$\text{Im}\mathcal{L}_{\epsilon'} = \frac{g_s}{16\pi^2 N^2} C_{ds} m_s \left[\bar{s} \sigma^{\mu\nu} \frac{\lambda_a}{2} \left(L - \frac{m_d}{m_s} R \right) d \right] G_{\mu\nu}^a \times \frac{1}{2} [h(x) - xh'(x)] \sin 2\theta_\chi, \quad (12)$$

where we have defined $C_{ds} = (\mathcal{O}_L^d)_{3d} (\mathcal{O}_L^s)_{3s}$, and $G_{\mu\nu}^a$ in the context of the effective interactions is just $G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a$, $x = m_Y^2/m_J^2$ and the function $h(x)$ is given in the appendix, and the prime denotes first derivative.

Neglecting the γ, Z contributions, i.e., the amplitudes with $I = 2$, and using the values for the other parameters given above, Eq. (11b) leads to

$$|\epsilon'| \approx \frac{1}{\sqrt{2}} \frac{1}{22.2} \frac{|\text{Im}A_0|}{|\text{Re}A_0|} \approx 9.6 \times 10^4 \frac{|\text{Im}A_0|}{1 \text{ GeV}}, \quad (13)$$

where we have used $\text{Re}A_0 = 3.3 \times 10^{-7} \text{ GeV}^{-1}$, with,

$$|\text{Im}A_0| = \sqrt{3} \frac{g_s}{16\pi^2} \frac{m_s}{N^2} C_{ds} \left| \frac{1}{2} (h(x) - xh'(x)) \right| \times \left| P_L - \frac{m_d}{m_s} P_R \right| \sin 2\theta_\chi. \quad (14)$$

We can write $|\epsilon'|$ as follows:

$$\frac{|\epsilon'|}{|\epsilon'_{\text{exp}}|} = C_{ds} A(x) \sin 2\theta_\chi, \quad (15)$$

$$A(x) = \sqrt{3} \frac{g_s}{(4\pi)^2} \frac{m_s}{|\epsilon'_{\text{exp}}| N^2} \left| P_L - \frac{m_s}{m_d} P_R \right| \times \left| \frac{1}{2} (h(x) - xh'(x)) \right| \frac{9.6 \times 10^4}{1 \text{ GeV}}, \quad (16)$$

where we have defined the matrix elements

$$P_L = \langle \pi\pi(I=0) | \left(\bar{s} \sigma^{\mu\nu} L \frac{\lambda^a}{2} d \right) G_{\mu\nu}^a | K^0 \rangle, \quad (17)$$

$$P_R = \langle \pi\pi(I=0) | \left(\bar{s} \sigma^{\mu\nu} R \frac{\lambda^a}{2} d \right) G_{\mu\nu}^a | K^0 \rangle.$$

Using the bag model (BM) it has been obtained that $P_L = -0.5 \text{ GeV}^2$ [15]. The other term in Eq. (14) with the matrix element P_R is negligible [even if $|P_R| \approx O(|P_L|)$]

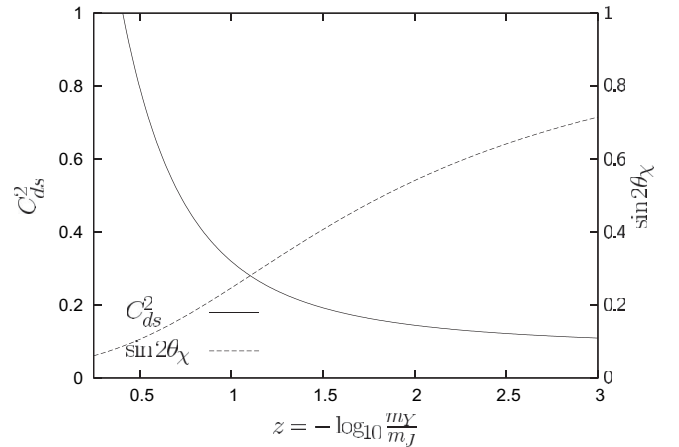


FIG. 3. Using Eq. (28) and (29) we studied the x -dependence of C_{ds}^2 (left scale) and $\sin 2\theta_\chi$ (right scale) on z , respectively, with z defined by $10^{-z} = m_Y/m_J = \sqrt{x}$. We have used $P_L = (1/2)P_L(\text{BM})$ and $B_L \rightarrow 3B_L(\text{VI})$, where BM indicates the value of P_L in the bag model, and VI means the vacuum insertion value of B_L . We have also used $N = 0.7 \text{ TeV}$ and $m_U/m_J = 1$. Notice that $\sin 2\theta_\chi$ does not depend on N .

since it has a m_d factor. We will also use the following values: $m_K = 498$ MeV, $m_d/m_s = 1/20$, $m_s = 120$ MeV, and $\alpha_s = 0.2$. The function $|h(x) - xh'(x)|$ has its maximum equal to one at $x = 0$. Both P_L and P_R matrix elements can be considered as free parameters, for instance in Fig. 3 we use $P_L = (1/2)P_L(BM)$. Of course, there is also a solution if we use the bag model value of P_L .

V. INDIRECT CP VIOLATION

The contributing diagrams for the ϵ parameter are of two types, one with the exchange of two Y^{++} and the other with

one U^{++} and one Y^{++} . They are shown in the Figs. 2(a) and 2(b), respectively. The imaginary part for this class of diagrams has been derived in Refs. [16,17]. The Higgs scalar-quark interaction is given in Eq. (6) and the gauge boson-quark Lagrangian interaction is

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\bar{J}(\mathcal{O}_L^d)_{3\alpha}\gamma^\mu L d_\alpha U_\mu^{++} + H.c. \quad (18)$$

The contributions to the effective Lagrangian of diagrams like that shown in Fig. 2(a) are given by

$$\begin{aligned} \text{Im}\mathcal{L}_\epsilon^{YY} &= \frac{C_{ds}^2}{(4\pi)^2} \frac{2m_K^2}{N^2} \frac{m_s^2}{N^2} \left\{ \left[\frac{\sin 4\theta_\chi}{m_K^2} \left[(\bar{s}Ld)^2 - \frac{m_d^2}{m_s^2} (\bar{s}Rd)^2 \right] \right] g_0(x) - \frac{|v_\rho|}{4|v_\chi|} \sin 2\theta_\chi \left[\bar{s}\gamma^\mu L i\vec{\partial}_\mu d \bar{s} \left(L - \frac{m_d}{m_s} R \right) d \right] \right. \\ &\quad \left. \times \left[5g_0(x) + \frac{3}{2}xg_0'(x) \right] + \bar{s} \left(L - \frac{m_d}{m_s} R \right) i\vec{\partial}_\mu d \bar{s} \gamma^\mu L d \left[g_0(x) + \frac{3}{2}xg_0'(x) \right] \right\}, \end{aligned} \quad (19)$$

where $g_0(x)$ is given in the appendix.

On the other hand, the contributions to the effective Lagrangian of diagrams like that shown in Fig. 2(b) are given by

$$\begin{aligned} \text{Im}\mathcal{L}_\epsilon^{UY} &= \frac{C_{ds}^2}{(4\pi)^2} \frac{2m_K^2}{N^2} \frac{m_s^2}{N^2} \left(\frac{g^2}{2} \frac{N^2}{4m_j^2} \right) \frac{\sin 2\theta_\chi}{m_s m_K^2} \left\{ \left[\bar{s}\gamma^\mu \gamma^\nu \left(L - \frac{m_d}{m_s} R \right) i\vec{\partial}_\mu d \right] (\bar{s}\gamma_\nu L d) E_1(x, y) \right. \\ &\quad + \bar{s}\gamma_\nu L i\vec{\partial}_\mu d \bar{s}\gamma^\mu \gamma^\nu \left(L - \frac{m_d}{m_s} R \right) d E_2(x, y) + \left[\bar{s} \left(L - \frac{m_d}{m_s} R \right) i\vec{\partial}_\mu d (\bar{s}\gamma^\mu L d) + \bar{s}\gamma^\mu L i\vec{\partial}_\mu d \bar{s} \left(L - \frac{m_d}{m_s} R \right) d \right] E_3(x, y) \\ &\quad - \left[i\partial_\mu \left[\bar{s}\gamma^\mu \gamma^\nu \left(L - \frac{m_d}{m_s} R \right) d \right] \bar{s}\gamma_\nu L d + i\partial_\mu (\bar{s}\gamma_\nu L d) \bar{s}\gamma^\mu \gamma^\nu \left(L - \frac{m_d}{m_s} R \right) d \right] E_4(x, y) \\ &\quad \left. - \left(i\partial_\mu \left[\bar{s} \left(L - \frac{m_d}{m_s} R \right) d \right] \bar{s}\gamma^\mu L d + i\partial_\mu (\bar{s}\gamma^\mu L d) \bar{s} \left(L - \frac{m_d}{m_s} R \right) d \right) E_5(x, y) \right\}, \end{aligned} \quad (20)$$

where $y = m_U^2/m_j^2$ and the functions $E_{1,2,3,4,5}$ are defined in the appendix.

Taking into account both contributions in Eqs. (19) and (20) and using

$$\text{Im}M_{12} = \frac{\text{Im}\langle \bar{K}^0 | \mathcal{L}_\epsilon(0) | K^0 \rangle}{2m_K}, \quad (21)$$

we obtain

$$\begin{aligned} \text{Im}M_{12} &= -\frac{C_{ds}^2}{(4\pi)^2} \frac{m_K^2}{N^2} \frac{f_K^2}{2N^2} \left(1 + \frac{m_d}{m_s} \right)^{-2} \left\{ \frac{5}{6} \sin 4\theta_\chi \left(1 - \frac{m_d^2}{m_s^2} \right) g_0(x) - \frac{|v_\rho|}{2|v_\chi|} \sin 2\theta_\chi \left[\frac{5}{12} \left(5g_0(x) - \frac{3}{2}g_0'(x) \right) \right. \right. \\ &\quad \left. \left. - \frac{1}{3} \left[1 + \frac{1}{4} \left(\frac{m_s + m_d}{m_K} \right)^2 \right] \left(g_0(x) + \frac{3}{2}xg_0'(x) \right) \right] + \frac{g^2}{2} \frac{N^2}{2m_j^2} \sin 2\theta_\chi \left[\frac{2}{3} (E_1'(x, y) + E_3(x, y)) - \frac{2}{3} \left(\frac{m_s + m_d}{m_K} \right)^2 \right. \right. \\ &\quad \left. \left. \times [E_1(x, y) + E_4(x, y)] + \frac{1}{12} \left[1 - \left(\frac{m_s + m_d}{m_K} \right)^2 \right] (E_2(x, y) + E_4(x, y)) \right] \right\}. \end{aligned} \quad (22)$$

Thus, we can calculate $|\epsilon|$ from Eq. (11a) using $f_K = 161.8$ MeV and $\Delta M_K = 3.5 \times 10^{-15}$ GeV [33]. We have used the vacuum insertion (VI) approximation, and obtained:

$$\begin{aligned}
 B_L &= \langle \bar{K}^0 | [\bar{s}L(R)d]^2 | K^0 \rangle = -\frac{5}{12} \frac{m_K^4 f_K^2}{(m_s + m_d)^2}, \\
 \langle \bar{K}^0 | [\bar{s}\gamma^\mu R(L)d]^2 | K^0 \rangle &\simeq \frac{2}{3} f_K^2 m_K^2 \langle \bar{K}^0 | \bar{s}Rd\bar{s}Ld | K^0 \rangle = \frac{1}{2} \frac{m_K^4 f_K^2}{(m_s + m_d)^2} + \frac{1}{12} m_K^2 f_K^2, \\
 \langle \bar{K}^0 | \bar{s}\gamma^\mu Ld\bar{s}L(i\partial_\mu)d | K^0 \rangle &= -\frac{5}{6} \frac{m_d m_K^4 f_K^2}{(m_s + m_d)^2}, \\
 \langle \bar{K}^0 | \bar{s}\gamma^\mu Ld(-i\partial_\mu)\bar{s}Ld | K^0 \rangle &= \frac{1}{3} \frac{m_s m_K^4 f_K^2}{(m_s + m_d)^2} + \frac{1}{12} m_s m_K^2 f_K^2, \\
 \langle \bar{K}^0 | (-i\partial_\mu)\bar{s}\gamma_\nu Ld\bar{s}\gamma^\mu u\gamma^\nu Ld | K^0 \rangle &= -\frac{2}{3} \frac{m_s m_K^4 f_K^2}{(m_s + m_d)^2}.
 \end{aligned} \tag{23}$$

We have verified that the main contribution to the box diagrams in Eqs. (19) comes from the matrix element denoted by B_L . Thus, in order not to be restricted to the VI approximation, B_L can be considered a free parameter and, for instance in Fig. 3, we have used $B_L = 3B_L(VI)$, but there is also a solution using the VI value of B_L .

VI. FITTING THE EXPERIMENTAL VALUES

In order to compare the prediction of the model with the experimental data for the CP violation in the neutral kaon system we use Eqs. (13)–(17) for $|\epsilon'|$ and rewrite Eq. (22) for $|\epsilon|$ as

$$\frac{|\epsilon|}{|\epsilon_{\text{exp}}|} = C_{ds}^2 B(x) \left(\frac{1}{2} \sin 4\theta_\chi - b(x, y) \sin 2\theta_\chi \right). \tag{24}$$

where

$$\begin{aligned}
 B(x) &= \frac{1}{(4\pi)^2 |\epsilon_{\text{exp}}|} \frac{m_K^2}{N^2} \frac{f_K^2}{2N^2} \frac{\sqrt{2}m_K}{\Delta M_K} \left(1 + \frac{m_d}{m_s} \right)^{-2} \\
 &\times \left(1 - \frac{m_d^2}{m_s^2} \right) \frac{5}{6} g_0(x) \\
 &\approx 1.34 \times \left(\frac{1 \text{ TeV}}{N} \right)^4 g_0(x)
 \end{aligned} \tag{25}$$

$$\begin{aligned}
 b(x, y) &= \frac{6}{5} \left(1 - \frac{m_d^2}{m_s^2} \right)^{-1} \frac{1}{g_0(x)} \left\{ \frac{|v_\rho|}{4|v_\chi|} \left[\frac{5}{12} \left[5g_0(x) - \frac{3}{2} x g'_0(x) \right] \right. \right. \\
 &\quad \left. \left. - \frac{1}{3} \left[1 + \frac{1}{4} \left(\frac{m_s + m_d}{m_K} \right)^2 \right] \left[g_0(x) + \frac{3}{2} x g'_0(x) \right] \right] \right\} \\
 &\quad - \frac{g^2}{2} \frac{N^2}{4m_J^2} \left\{ \frac{2}{3} \left[E_2(x, y) + E_4(x, y) \right] - \frac{2}{3} \left(\frac{m_s + m_d}{m_K} \right)^2 \right. \\
 &\quad \times \left[E_1(x, y) + E_4(x, y) \right] + \frac{1}{12} \left[1 - \left(\frac{m_s + m_d}{m_K} \right)^2 \right] \\
 &\quad \left. \times (E_3(x, y) + E_5(x, y)) \right\}.
 \end{aligned} \tag{26}$$

Next we use the constraints

$$\left| \frac{\epsilon'(C_{ds}, x, \theta_\chi)}{\epsilon'_{\text{exp}}} \right| = 1, \quad \left| \frac{\epsilon(C_{ds}, x, y, \theta_\chi)}{\epsilon_{\text{exp}}} \right| = 1. \tag{27}$$

Notice that the above conditions are the strongest since we are not considering the experimental error.

After some algebraic manipulations the constraints in Eqs. (27) imply

$$C_{ds}^2 = \frac{D^4(x)}{D^2(x) - \frac{b^2(x, y)}{A^2(x)} - \frac{b(x, y)A(x)}{B(x)}} \leq 1, \tag{28}$$

and

$$C_{ds} \sin \theta_\chi = \frac{1}{A(x)}, \tag{29}$$

where we have defined

$$D^2(x) = \frac{1}{A^2(x)} + \frac{A^2(x)}{B^2(x)}, \tag{30}$$

with $A(x)$ defined in Eq. (16), and $B(x)$ and $b(x, y)$ were defined in Eqs. (25) and (26), respectively.

It is interesting to note that

$$C_{ds} \sin 2\theta_\chi \geq \frac{1}{A(0)} = 0.072 \left(\frac{N}{1 \text{ TeV}} \right)^2, \tag{31}$$

where we have used the value of the parameters as discussed below Eq. (17).

We have study numerically Eq. (28) and (29) and verified that they are sensible to the values of the matrix elements P_L in Eq. (17) and B_L defined in Eq. (23).

The curves in Fig. 3 are curves of compatibility with experimental data according to the constraints in Eq. (27). The dashed curve shows all the allowed values for $\sin 2\theta_\chi$ while the continue curve shows the allowed values for C_{ds} as a function of x . However, these values are not independent from one another if we want to satisfy both constraints at the same time. The compatibility with the experimental data is obtained by drawing a vertical line for a given value of z . For instance using $z = 2$ (i.e., $m_J = 100m_Y$) we found $\sin 2\theta_\chi \approx 0.15$ and $C_{ds}^2 \approx 0.6$, for $z = 1$ we obtain $\sin 2\theta_\chi \approx 0.25$ and $C_{ds}^2 \approx 0.3$. Notice that from Fig. 3 we see that we have solution in the range $2.5^\circ \lesssim \theta_\chi \lesssim 22.5^\circ$.

VII. CONCLUSIONS AND DISCUSSIONS

The study showed that the 3-3-1 model considered here can account for the direct and indirect CP violation present in the $K^0 - \bar{K}^0$ system for sensible values of the unknown parameters. Within the approximations used, $N \leq 1$ TeV there are infinitely many possible values for C_{ds} , θ_χ and $m_{\tilde{\nu}}^2/m_j^2$ allowed by the experimental data. Although they are not all independent and the constraint $|C_{ds}| < 1$ implies a very small upper bound for the ratio $m_{\tilde{\nu}}^2/m_j^2$. Such bound becomes smaller as N becomes greater. Thus very large values of N leads to unrealistically small values of the ratio. Notice also that the constraints used in Eq. (27) are very strong. However, weaker constraints arise if a detailed analysis which take into account the experimental error in both ϵ and ϵ' is done. Of course, it is clear that in this case there will exist a solution as well.

The model implies also some contributions to the neutron electric dipole moment (EDM) as in Ref. [26]

$$d_n \simeq 4.9 \times 10^{-22} \left[\sum_{\alpha} G_{\alpha 1} (\mathcal{O}_R^u)_{1\alpha} (\mathcal{O}_L^u)_{11} \sin\theta_\chi \right] \text{ e cm}, \quad (33)$$

and we see that a value compatible with the experimental bound of [38]

$$|d_n| < 6.3 \times 10^{-26} \text{ e cm} \quad (90\% \text{CL}), \quad (34)$$

is obtained for practically any value of the phase θ_χ , if $\sum_{\alpha} G_{\alpha 1} (\mathcal{O}_R^u)_{1\alpha} (\mathcal{O}_L^u)_{11} \sim 10^{-5}$. The EDM of the charged leptons also produces results compatible with the experimental limit for a large range of the parameters of the model. In addition this model allows magnetic dipole moments for massive neutrinos in the range 10^{-13} – $10^{-11} \mu_B$ almost independently of the neutrino mass [39], which is near the experimental upper limit for the electron neutrino magnetic moment [40]

$$\mu_e < 10^{-11} \mu_B (90\% \text{CL}). \quad (35)$$

Moreover, as in the standard model the lepton charge asymmetry in the K_{l3} decay, δ_L , which has the experimental value (the weighted average of $\delta(\mu)$ and $\delta(e)$ [33]) $\delta_L = (3.27 \pm 0.12) \times 10^{-3}$, is also automatically fitted in the present model because $|A(K^0 \rightarrow \pi^- e^+ \nu_e)| = |A(\bar{K}^0 \rightarrow \pi^+ e^- \bar{\nu}_e)|$ is still valid.

Recent analysis on CP violation indicates that the phase of the CKM matrix, which is $O(1)$, is the dominant contribution to the CP violation in both K and B mesons so, new phases coming from physics beyond the standard model must be small perturbations. The CKM mechanism is also at work in the present model but we switch it off in order to study the possibilities of the extra phase of the model. Concerning the K meson and EDM for elementary particles it seems that the model does well. Presently we are working out the case of B decays; if the model is not able to fit these data it implies that CKM phase must be

switched on. It is still possible that new phases may be at work if decays based on $b \rightarrow s$ gluonic dominated transition really need new physics [7]. Either way, the extra phase in the model could be important for other CP violation parameters like the EDM or if new CP violation observables in B -mesons will not be fitted by the CMK mechanism.

Finally, some remarks concerning the masses of the extra particles in 3-3-1 models. First, let us consider the Z' vector boson it contributes to the ΔM_K at the tree level so that there is a constraint over the quantity [41,42]

$$(\mathcal{O}_L^d)_{3d} (\mathcal{O}_L^d)_{3s} \frac{M_Z}{M_{Z'}}, \quad (36)$$

which must be of the order of 10^{-4} to have compatibility with the measured ΔM_K . This can be achieved with $M_{Z'} \sim 4$ TeV if we assume a Fritzsche-structure $\mathcal{O}_{Lij}^d = \sqrt{m_j/m_i}$ or, since there is no *a priori* reason for \mathcal{O}_L^d having the Fritzsche-structure, it is possible that the product of the mixing angles saturates the value 10^{-4} [41], in this case Z' can have a mass near the electroweak scale. However, in 3-3-1 models there are flavor-changing neutral currents in the scalar sector implying new contributions to ΔM_K which are of the form

$$(\mathcal{O}_L^d)_{d3} \Gamma_{3\beta}^d (\mathcal{O}_R)_{\beta s} \frac{M_Z}{M_H} \quad (37)$$

that involve the mass of the scalar M_H , the unknown matrix elements \mathcal{O}_R^d , and also the Yukawa coupling Γ^d , so their contributions to ΔM_K can have opposite sign relative to that of the Z' contribution. This calculation has not been done in literature, where only the latter contribution has been taken into account [41,42]. The model has also doubly charged scalars that are important in the present CP violating mechanism. The lower limit for the mass of doubly charged scalars is a little bit above 100 GeV [43]. Concerning the doubly charged vector boson, if they have masses above 500 GeV they can be found (if they really do exist) by measuring left-right asymmetries in lepton-lepton scattering [44]. Fermion pair production at LEP, and lepton flavor violating of the charged leptons suggest a low bound of 750 GeV for the U^{--} mass [45]. In e^+e^- , $e\gamma$, $\gamma\gamma$ colliders the detection of bileptons with masses between 500 GeV and 1 TeV [46] is favored, while if their masses are of the order of ≤ 1 TeV they could be also observed at hadron colliders like LHC [47]. Muonium-antimuonium transitions would imply a lower bound of 850 GeV on the masses of the doubly charged gauge bileptons, U^{--} [48]. However, this bound depends on assumptions on the mixing matrix in the lepton charged currents coupled to U^{--} and also it does not take into account that there are in the model doubly charged scalar bileptons which also contribute to that transition [49]. The muonium fine structure only implies $m_U/g > 215$ GeV assuming only the vector bilepton contributions [50]. Concerning the exotic quark

masses, there is no lower limit for their masses but if they are in the range of 200–600 GeV they may be discovered at the LHC [51]. A search for free stable color triplets quarks has been carried out in a $p\bar{p}$ collider at an energy of 1.8 GeV, excluding these particles in the range 50–139 GeV, 50–116 GeV, and 50–140 GeV for the electric charges of +1, 2/3, and 4/3, respectively [52]. We can conclude that the masses for the extra degrees of freedom which distinguish 3-3-1 models with respect to the standard model may be accessible at the energies of the colliders of the next generations.

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APPENDIX: INTEGRALS

$$g_0(x, y) = -\frac{1}{x-y} \left[\left(\frac{x}{x-1} \right)^2 \ln x - \left(\frac{y}{y-1} \right)^2 \ln y - \frac{1}{x-1} + \frac{1}{y-1} \right] \quad (\text{A1})$$

$$g_1(x, y) = -\frac{1}{x-y} \left[\frac{x}{(x-1)^2} \ln x - \frac{y}{(y-1)^2} \ln y - \frac{1}{x-1} + \frac{1}{y-1} \right] \quad (\text{A2})$$

$$g_0(x) = \lim_{y \rightarrow x} g_0(x, y) = -\frac{2}{(x-1)^2} + \frac{x+1}{(x-1)^3} \ln x \quad (\text{A3})$$

$$h(x) = -\frac{x}{(x-1)^2} \ln x + \frac{1}{x-1}, \quad (\text{A4})$$

$$h'(x) = \frac{2-2x+(1+x)\ln x}{(x-1)^3},$$

$$E_1(x, y) = \left(\frac{5}{2} + \frac{1}{4}(x\partial_x + y\partial_y) \right) g_0(x, y) - \frac{5}{8y} g_1(x, y) \quad (\text{A5})$$

$$E_2(x, y) = \left(\frac{1}{2} + \frac{1}{4}(x\partial_x + y\partial_y) \right) g_0(x, y) - \frac{1}{8y} g_1(x, y) \quad (\text{A6})$$

$$E_3(x, y) = \frac{3}{4y} (1 + x\partial_x + y\partial_y) g_1(x, y). \quad (\text{A7})$$

$$E_4(x, y) = \frac{1}{4} \left[(x\partial_x - y\partial_y) g_0(x, y) + \frac{1}{2y} g_1(x, y) \right] \quad (\text{A8})$$

$$E_5(x, y) = -\frac{1}{4y} (3 + x\partial_x - y\partial_y) g_1(x, y) \quad (\text{A9})$$

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