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$SU(5) \otimes Z_{13}$ grand unification model

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We propose an $SU(5)$ grand unified model with an invisible axion and the unification of the three coupling constants which is in agreement with the values, at M_Z, of α, α_s, and $\sin^2\theta_W$. A discrete, anomalous, Z_{13} symmetry implies that the Peccei-Quinn symmetry is an automatic symmetry of the classical Lagrangian protecting, at the same time, the invisible axion against possible semiclassical gravity effects. Although the unification scale is of the order of the Peccei-Quinn scale the proton is stabilized by the fact that in this model the standard model fields form the $SU(5)$ multiplets completed by new exotic fields and, also, because it is protected by the Z_{13} symmetry.

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I. INTRODUCTION

The unification idea, mainly in $SU(5)$ [1], is still an interesting alternative for the physics beyond the standard model [2]. Unfortunately, the minimal nonsupersymmetric $SU(5)$ model has been ruled out by experimental data: (i) the proton is more stable than the prediction of the minimal model [3]; (ii) the value of weak mixing angle at the Z-peak $\sin^2\theta_W(M_Z) = 0.23122(15)$ [or alternatively $\alpha_s(M_Z)$] does not agree with experimental data [4]. It means that the three coupling constants do not meet at a single point if only the standard model particles are taken into account; (iii) the electron and d-like quark masses are equal at the unification scale, and (iv) last but not least, neutrinos are massless in the model. Moreover, the supersymmetric version, i.e., supersymmetric (SUSY) $SU(5)$, although it allows a unification of the coupling constants, it has serious problems with the proton decay [5] (however, see [6]) and probably also with the electroweak data [7]. Thus, it appears natural to ask ourselves if there are other options besides SUSY $SU(5)$ that yield convergence of the couplings, the observed value of the weak mixing angle, and the other parameters at the Z-pole, an appropriately stable proton and, at the same time, realistic fermion masses including neutrino masses. Another problem, not necessarily related to the previous one, concerns the existence of axions [8]. Recently, the interest in theories involving such particles has raised also due experiments devoted to the search of axionlike particles [9]. If the axion does exist it is important to know the realistic model in which the Peccei-Quinn symmetry (PQ) can be automatically implemented and how the axion parameters can be stabilized against possible semiclassical gravitational effects [10].

On the other hand, it was shown in Ref. [11] that in the context of the multi-Higgs extension of the standard model with an invisible axion proposed in Ref. [12] we have: (i) the unification of the three gauge coupling constants near the Peccei-Quinn scale; (ii) the model predicts the correct value of the weak mixing angle at the Z-peak; (iii) the axion and the nucleon are stabilized by the cyclic $Z_{13} \otimes Z_3$ discrete symmetries; finally, (iv) although neutrinos got an arbitrary Dirac mass, through the effective $d = 10$ operators $\Lambda_{PQ}^{-1}\Lambda^{-5}L\Phi_\nu L\Phi_\nu\phi^5$, the left-handed neutrinos also get a Majorana mass ≤ 2 eV and the right-handed neutrinos acquire a large Majorana mass term via $d = 7$ effective operator $\Lambda_{PQ}^{-3}\overline{\nu_{aR}^c}(M_R)_{ab}\nu_{bR}(\phi^*\phi)^2$, implementing in this way a seesaw mechanism at the PQ energy scale.

Here we will consider an $SU(5)$ grand unified theory which unifies the model of Ref. [12], in such a way that the partner of the standard model fields in $SU(5)$ multiplets are new heavy fields. This model allows a stable proton, unification of the three coupling constants, a natural Peccei-Quinn symmetry of the classical Lagrangian, and the axion being protected against semiclassical gravity effects.

The outline of the paper is as follows. In Sec. II we give the representation content of the model and the Z_{13} and PQ charge assignments of the several multiplets. Next, in Sec. III, we consider the running equations for the three gauge coupling constants related to the low energy $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry group. In Sec. IV we consider the proton stabilization and other phenomenological consequences concerning the model; finally the last section is devoted to our conclusions.

II. NON-SUSY $SU(5)$ GRAND UNIFIED THEORY

In Ref. [12] the representation content of the standard model was augmented by adding scalar fields and three right-handed neutrinos, in such a way that a discrete $Z_{13} \otimes Z_3$ symmetry was implemented in the model there. Explicitly, the particle content of the model is the following: $Q_L = (u, d)_L^T \sim (\mathbf{3}, \mathbf{2}, 1/3)$, $L_L = (\nu, l)_L^T \sim (\mathbf{1}, \mathbf{2}, -1)$ denote quark and lepton doublets, respectively; $u_R \sim (\mathbf{3}, \mathbf{1}, 4/3)$, $d_R \sim (\mathbf{3}, \mathbf{1}, -2/3)$, $l_R \sim (\mathbf{1}, \mathbf{1}, -2)$, $\nu_R \sim (\mathbf{1}, \mathbf{1}, 0)$ are the right-handed components. It was also as-

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sumed that each charged sector gains mass from a different scalar doublet: H_u, H_d, H_l, and H_ν which generate Dirac masses for u-like, d-like quarks, charged leptons, and neutrinos, respectively [all of them of the form $(\mathbf{1}, \mathbf{2}, +1) = (\varphi^+, \varphi^0)^T$]. Some other scalar fields were also considered in order to permit the full symmetry realization: a neutral complex singlet $\phi \sim (\mathbf{1}, \mathbf{1}, 0)$, a singly charged singlet $h^+ \sim (\mathbf{1}, \mathbf{1}, +2)$, and a triplet $\vec{\mathcal{T}} \sim (\mathbf{1}, \mathbf{3}, +2)$. Next, we wonder what is the simplest group embedding the above representation content. The answer is $SU(5)$. To achieve this, along with a Z_{13} symmetry, we have to add new fermions and scalar fields.

In this vein, the representation content of our $SU(5)$ model is as follows. For each family, the fermion representation content under $SU(5) \supset SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, there are two $\mathbf{5}^*$: $(\Psi^c)_{dL} = (d^c_1, d^c_2, d^c_3, E^-, -N)^T_L$, and $(\Psi^c)_{eL} = (D^c_1, D^c_2, D^c_3, e^-, -\nu_e)^T_L$ and two $\mathbf{10}$:

$$\Phi_{dL} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^c_3 & -u^c_2 & -u_1 & -d_1 \\ -u^c_3 & 0 & u^c_1 & -u_2 & -d_2 \\ u^c_2 & -u^c_1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -E^+ \\ d_1 & d_2 & d_3 & E^+ & 0 \end{pmatrix}_L, \quad (1)$$

and

$$\Phi_{eL} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & U^c_3 & -U^c_2 & -U_1 & -D_1 \\ -U^c_3 & 0 & U^c_1 & -U_2 & -D_2 \\ U^c_2 & -U^c_1 & 0 & -U_3 & -D_3 \\ U_1 & U_2 & U_3 & 0 & -e^+ \\ D_1 & D_2 & D_3 & e^+ & 0 \end{pmatrix}_L, \quad (2)$$

where E and N are heavy charged and neutral leptons, respectively, and U, D are heavy quarks having the same electric charge of the respective quarks u, d. Finally, in the fermion sector we have to add fermionic neutral singlets $(N^c)_L \equiv N^c_L$ and $(\nu^c)_L \equiv \nu^c_L$. We have used a notation in which the subindex $e(d)$ denotes the multiplet to which the known leptons (d-like quarks) belong to; on the other hand, the u-like quarks always belong to the decuplet Φ_d. Notice that since the known quarks and leptons belong to different representations of $SU(5)$, we have to impose that both quarks and leptons, and not quarks and antileptons, have gauge interactions through the left-handed components [13].

The scalars of the model are the usual $\mathbf{24}$, here denoted by ϕ_{24}, with vacuum expectation value (VEV) $\langle \phi_{24} \rangle = v_{24}\mathrm{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2})$; a complex singlet ϕ_0 which is almost the axion (we note that by considering a complex $\mathbf{24}$ it is possible to implement the axion in this model [14], however, this may introduce troubles with proton decay). In order to break the $SU(2) \otimes U(1)$ symmetry and generate the fermion's Dirac masses we use four Higgs multiplets: two $\mathbf{5}$, and two $\mathbf{45}^*$ to avoid the prediction $m_e(M_U) = m_d(M_U)$. (The using of $\mathbf{45}$ for avoiding this mass relation was done in Ref. [15].) Finally, we add a $\mathbf{10}$ (D_{10}) and a $\mathbf{15}$ (T_{15}) which contains, respectively, the singlet h^+ and the triplet $\mathcal{T}$ of Ref. [12]. T_{15} gives Majorana masses to the active neutrinos. We will denote the $\mathbf{5}$ as $H^5_a = (h^1_a, h^2_a, h^3_a, h^+_a, h^0_a)$ with $a = e$, d; and their VEVs are $\langle H^{5\alpha}_a \rangle = (v_{a5}/\sqrt{2})\delta^\alpha_5$; the $\mathbf{45}^*$ will be denoted by $H^{45}_a \equiv (H^{45}_a)^{\alpha\beta}_\rho$; $(H^{45}_a)^{\alpha\beta}_\rho = -(H^{45}_a)^{\beta\alpha}_\rho$; $(H^{45}_a)^{\alpha\beta}_\alpha = 0$, with $\langle H^{45}_a \rangle^{\alpha\beta}_\rho = (v_{a45}/\sqrt{2})(\delta^\alpha_\rho - 4\delta^4_\rho\delta^\alpha_4)\delta^\beta_5$; finally, $\langle T^{\alpha\beta}_{15} \rangle = (v_{15}/\sqrt{2})\delta^\alpha_5\delta^\beta_5$. The decuplet D_{10} does not necessarily get a VEV at lowest order. Since in this model all scalar's VEVs are of the order of the electroweak scale, except ϕ_{24} and ϕ_0 which have VEVs of the order of grand unified theory (GUT) and PQ scale, respectively, we have still the hierarchy problem. It is only ameliorated because the GUT scale is lower (as we will show below) than in other grand unification models.

Consider the following Yukawa interactions,

$$\begin{aligned} -\mathcal{L}_Y = {} & \overline{(\Psi_e)_R}[G_{e5}\Phi_{eL}H^{5*}_e + G_{e45}\Phi_{eL}H^{45}_e + G_\nu \nu^c_L H^5_e] \\ & + \overline{(\Phi^c_e)_R}\epsilon K_U \Phi_{eL} H^{45*}_e + \overline{(\Psi_d)_R}[G_{d5}\Phi_{dL}H^{5*}_d \\ & + G_{d45}\Phi_{dL}H^{45}_d + G_N N^c_L H^5_d] + \overline{(\Phi^c_d)_R}\epsilon K_d \Phi_{dL} H^{45*}_d \\ & - \overline{(\Phi^c_e)_R}\epsilon F_U \Phi_{eL} H^5_e - \overline{(\Phi^c)_{dR}}\epsilon F_d \Phi_{dL} H^5_d \\ & - \overline{(\Psi_e)_R} G_{e15} \Psi^c_{eL} T_{15} + \text{H.c.}, \end{aligned} \quad (3)$$

where G, K, F are 3×3 complex matrices but we have omitted generation and $SU(5)$ indices; ϵ denotes the $SU(5)$ fully antisymmetric tensor. With Eq. (3) we obtain the mass matrices (T denotes the transpose matrix)

$$\begin{aligned} M_e &= G^T_{e5}\frac{v^*_{e5}}{2} - 3G^T_{e45}v_{e45}, \\ M_D &= G_{e5}\frac{v^*_{e5}}{2} + G_{e45}v_{e45}, \\ M_U &= \sqrt{2}v_{e5}(F_U + F^T_U) + 2\sqrt{2}v^*_{e45}(K^T_U - K_U), \end{aligned} \quad (4)$$

and

$$\begin{aligned} M_E &= G^T_{d5}\frac{v^*_{d5}}{2} - 3G^T_{d45}v_{d45}, \\ M_d &= G_{d5}\frac{v^*_{d5}}{2} + G_{d45}v_{d45}, \\ M_u &= \sqrt{2}v_{d5}(F_d + F^T_d) + 2\sqrt{2}v^*_{d45}(K^T_d - K_d), \end{aligned} \quad (5)$$

$M^{\text{Dirac}}_\nu = (v_{e5}/\sqrt{2})G^T_\nu$, and $M^{\text{Dirac}}_N = (v_{d5}/\sqrt{2})G^T_N$. The left-handed neutrinos have a Majorana mass term coming from the T_{15}: $M^{\text{Majorana}}_\nu = (v_{15}/\sqrt{2})G^T_{e15}$. Both v_{a5} and v_{a45} are of the order of the electroweak scale, in fact $\sum_a(|v_{a5}|^2 + |v_{a45}|^2) + |v_{15}|^2 = (246\ \text{GeV})^2$, $a = e$, d, with $|v_{15}| < 3.89$ GeV [16]. For instance, using only one generation, if $v_{e5} = v_{e45} \equiv v_e$, assuming that these VEV are real and neglecting v_{15}, we have, from Eqs. (4) and (5), $M_e = (G^T_{e5}/2 - 3G^T_{e45})v_e$ and $M_D = (G_{e5}/2 + G_{e45})v_e$ (and similarly for M_E and M_d), so we can choose the Yukawa coupling constants must be such that $M_e \ll M_D$,

$M_u \ll M_U$ and $M_d \ll M_E$. In the context of three generations all these mass matrices are 3×3 matrices and those relations among the masses refer to the respective eigenvalues. Right-handed components of neutrinos and the neutral leptons N_R get also a Majorana mass term through the interactions with the axion [17].

We see that the representation content of the model implies that the vector bosons do not induce, at the tree level, the nucleon decay because these interactions involve the usual quarks and heavy leptons; or heavy quarks with the usual leptons. The same is true for the Yukawa interactions if they are given only by these in Eq. (3). This diminishes the importance of the constraints coming from nucleon decay on the leptoquarks masses. Thus, they may have a mass lower than the unification scale. Notwithstanding, when studying the evolution of the coupling constants, we will assume that all leptoquarks are heavy enough and do not consider them in the running of the couplings. Next, we will show that the Yukawa interactions in Eq. (3) are the only ones allowed by an appropriate discrete symmetry.

Let us use the fact that a Z_N symmetry with N being a prime number does not have any subgroup; in other words, it cannot be decomposed as $Z_p \otimes Z_q, (p, q < N)$, so that the Z_N symmetry may be a subgroup of a unique local group $U(1)$. In this vein, let us introduce the following Z_{13} symmetry in the Yukawa interactions in such a way that only these interactions in Eq. (3) are allowed. The fields of the model transform under Z_{13} as follows:

$$\begin{gathered}(\Psi^c)_{eL} \to \omega_3(\Psi^c)_{eL}, \qquad (\Psi^c)_{dL} \to \omega_1^{-1}(\Psi^c)_{dL},\\ \Phi_{eL} \to \omega_1^{-1}\Phi_{eL}, \qquad \Phi_{dL} \to \omega_4^{-1}\Phi_{dL},\\ \nu_L^c \to \omega_5^{-1}\nu_L^c, \qquad N_L^c \to \omega_6 N_L^c, \qquad H_e^5 \to \omega_2 H_e^5,\\ H_e^{45} \to \omega_2^{-1}H_e^{45}, \qquad H_d^5 \to \omega_5^{-1}H_d^5,\\ H_d^{45} \to \omega_5 H_d^{45}, \qquad D_{10} \to \omega_3 D_{10},\\ T_{15} \to \omega_6^{-1}T_{15}, \qquad \phi_{24} \to \omega_0\phi_{24}, \qquad \phi_0 \to \omega_4\phi_0.\end{gathered} \tag{6}$$

We have assumed that the three generations are replicas under Z_{13}. However, it could be interesting to consider the case when this is not the case.

The scalar potential has Hermitian quadratic terms $\mu_\chi^2\chi^\dagger\chi$ (where χ denotes any of the Higgs scalar multiplets of the model), which are needed to break the electroweak symmetry, trilinear and quartic Hermitian terms, and non-Hermitian self-interactions which are trilinears:

$$\begin{gathered}H_e^5 H_e^{45}\phi_{24}, \qquad H_d^5 H_d^{45}\phi_{24}, \qquad (H_d^5)^2 D_{10}^*,\\ H_d^5 H_d^{45*}D_{10}^*, \qquad (H_d^{45})^2 D_{10},\end{gathered} \tag{7}$$

and quartic:

$$\begin{aligned}&H_d^5H_d^{45}|T_{15}|^2, \quad H_d^5H_d^{45}\phi_{24}^2, \quad H_d^5H_d^{45*}D_{10}^*\phi_{24}, \quad H_d^5H_d^{45}H_e^{5*}H_e^{45*}, \quad H_d^5H_d^{45*}(H_d^{5*})^2, \quad (H_e^{45})^3T_{15}^*,\\ &T_{15}D_{10}^*\phi_{24}\phi_0^*, \quad (H_d^{45})^2D_{10}^*\phi_{24}, \quad (H_e^5)^2H_e^{45*}T_{15}, \quad (H_d^{45})^2T_{15}\phi_0^*, \quad (H_e^5)^2H_e^{5*}H_e^{45}, \quad (H_e^5)^2(H_e^{45})^2,\\ &(H_d^5)^2(H_d^{45})^2, \quad H_e^5H_e^{45}|H_d^5|^2, \quad H_e^5H_e^{45}|H_e^{45}|^2, \quad H_e^5H_e^{45}|H_d^{45}|^2, \quad H_e^5H_e^{45}|D_{10}|^2, \quad H_e^5H_e^{45}|T_{15}|^2,\\ &H_e^5H_e^{45}\phi_{24}^2, \quad |H_e^5|^2H_d^5H_d^{45}, \quad H_e^5(H_e^{45*})^2T_{15}, \quad H_e^5H_e^{45}H_d^5H_d^{45}, \quad (H_d^5)^2H_d^{5*}H_d^{45}, \quad (H_d^5)^2D_{10}^*\phi_{24},\\ &(H_d^5)^2T_{15}^*\phi_0, \quad H_d^5H_d^{45}|H_e^{45}|^2, \quad H_d^5H_d^{45}|H_d^{45}|^2, \quad H_d^5H_d^{45}|D_{10}|^2.\end{aligned} \tag{8}$$

Moreover, with the interactions in Eq. (3) and the non-Hermitian interactions in (7) and (8), allowed by the symmetry in Eq. (6), the Peccei-Quinn is an automatic symmetry and the PQ charges are shown between parenthesis in units of the PQ charge of Ψ_d:

$$\begin{gathered}(\Psi^c)_{eL}(-1/3), \qquad (\Psi^c)_{dL}(1), \qquad \Phi_{eL}(1/9),\\ \Phi_{dL}(-1/3), \qquad \nu_L^c(5/9), \qquad N_L^c(-5/3),\\ H_e^5(-2/9), \qquad H_e^{45}(2/9), \qquad H_d^5(2/3),\\ H_d^{45}(-2/3), \qquad D_{10}(4/3), \qquad T_{15}(2/3),\\ \phi_{24}(0), \qquad \phi_0(-2/3).\end{gathered} \tag{9}$$

As in the model of Ref. [12], the Z_{13} protect the axion against possible semiclassical gravity effects. The model has no domain wall problem [18].

III. EVOLUTION OF THE COUPLING CONSTANTS

In order to study the running of the coupling constants in a consistent way with the present model, we augmented the representation content of the model of Ref. [12]. Hence, we assume that the only extra degrees of freedom that are active at low energies, i.e., below the unification scale but above the electroweak scale, transforming under the standard model (SM) symmetries are (per family) $(N, E)_L^T \sim (\mathbf{3}, \mathbf{2}, -1)$, $(U, D)_L^T \sim (\mathbf{3}, \mathbf{2}, 1/3)$ and the respective singlets $E_R \sim (\mathbf{1}, \mathbf{2}, -2)$, $N_R \sim (\mathbf{1}, \mathbf{1}, 0)$, $U_R \sim (\mathbf{3}, \mathbf{2}, 2/3)$, and $D_R \sim (\mathbf{3}, \mathbf{1}, -1/3)$. In the Higgs boson sector we add also four scalar doublets, two of them H_d, H_l are those that belong to $\mathbf{5}$ and two others, say H_u and H_ν which belong to the $\mathbf{45}$; a triplet $\mathcal{T}$ belonging to T_{15} and the singlet h^+ which is part of the D_{10}. As in Ref. [11] only h^+ will be considered with mass of the order of the unification scale.

Let us look at the evolution equations at the 1-loop approximation, with all the new fermions entering only above an intermediate energy scale μ_{IS} which is certainly bigger than the electroweak scale. Some of the new fermions could have mass below the known heavier standard model particles, but we will not consider such possibility. Below we comment more on that (see Sec. V). Thus, the 1-loop equations are

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_Z)} - \frac{1}{2\pi}\left[b_i \ln\frac{\mu_{IS}}{M_Z} + b_i^{IS} \ln\frac{\mu}{\mu_{IS}}\right], \quad (10)$$

where $\alpha_i(M_Z) = g_i^2(M_Z)/4\pi$ are the usual gauge couplings defined for these equations, and b_i the well-known coefficients for a general $SU(N)$ gauge group, given by $b_i = (2/3)\sum T_{Ri}(F) + (1/3)\sum T_{Ri}(S) - (11/3)C_{2i}(G)$ for Weyl fermions (F) and complex scalars (S), and $T_R(I)\delta^{ab} = \mathrm{Tr}\{T^a(I), T^b(I)\}$ with $I = F,\ S$; $T_R(I) = 1/2$ for the fundamental representation, $C_2(G) = N$ when $N \geq 2$, for $U(1)$, $C_2(V) = 0$, and $T_{R1}(S_a, F_a) = (3/5)\,\mathrm{Tr}(Y_a^2/4)$. The same is valid for the b_i^{IS} with the counting extending to the exotic fermions representations. At the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ energy level with N_g fermion generations, N_H scalar doublets ($Y = \pm 1$) and N_T non-Hermitian scalar triplets ($Y = 2$), and N_s charged singlets, we have

$$\begin{aligned} b_1 &= \frac{4}{3}N_g + \frac{1}{10}N_H + \frac{3}{5}N_T + \frac{1}{5}N_s, \\ b_2 &= \frac{4}{3}N_g + \frac{1}{6}N_H + \frac{2}{3}N_T - \frac{22}{3}, \\ b_3 &= \frac{4}{3}N_g - 11, \end{aligned} \quad (11)$$

where a grand unification normalization factor $(3/5)$ for the hypercharge Y assignment is included in b_1. So that according the additional representations in the beginning of this section only the heavy fermions are activated above μ_{IS}, we have (with $N_s = 0$)

$$(b_1, b_2, b_3) = (5, -2, -7), \qquad (b_1^{IS}, b_2^{IS}, b_3^{IS}) = (9, 2, -3). \quad (12)$$

Note that there is no asymptotic freedom in α_2 at the one loop level. The grand unification mass scale and the weak mixing angle are given by

$$M_{\rm GUT} = \mu_{IS} \exp\left\{2\pi \frac{[\frac{3}{5}\alpha^{-1}(M_Z) - \frac{8}{5}\alpha_3^{-1}(M_Z)]}{(b_1^{IS} + \frac{3}{5}b_2^{IS} - \frac{8}{5}b_3^{IS})}\right\} \times \left(\frac{M_Z}{\mu_{IS}}\right)^{(b_1+(3/5)b_2-(8/5)b_3)/(b_1^{IS}+(3/5)b_2^{IS}-(8/5)b_3^{IS})}, \quad (13)$$

and

$$\sin^2\theta_W = \frac{3}{8} + \frac{5}{8}\frac{\alpha(M_Z)}{2\pi}\left[(b_1 - b_2)\ln\left(\frac{M_Z}{\mu_{IS}}\right) + (b_1^{IS} - b_2^{IS})\ln\left(\frac{\mu_{IS}}{M_{\rm GUT}}\right)\right]. \quad (14)$$

However, since

$$\begin{aligned} b_1 - b_2 &= b_1^{IS} - b_2^{IS}, \\ b_1 + \tfrac{3}{5}b_2 - \tfrac{8}{5}b_3 &= b_1^{IS} + \tfrac{3}{5}b_2^{IS} - \tfrac{8}{5}b_3^{IS}, \end{aligned} \quad (15)$$

$M_{\rm GUT}$, and $\sin^2\theta_W(M_Z)$ defined at the Z boson mass, do not depend on the scale μ_{IS} and we are left with

$$M_{\rm GUT} = M_Z \exp\left[2\pi\frac{\alpha^{-1}(M_Z) - \frac{8}{3}\alpha_3^{-1}(M_Z)}{\frac{5}{3}b_1 + b_2 - \frac{8}{3}b_3}\right], \quad (16)$$

and

$$\sin^2\theta_W(M_Z) = \frac{3}{8} + \frac{5}{8}\frac{\alpha(M_Z)}{2\pi}(b_1 - b_2)\ln\left(\frac{M_Z}{M_{\rm GUT}}\right). \quad (17)$$

Both $M_{\rm GUT}$ and $\sin^2\theta_W(M_Z)$, at the one loop level, are the same as in Ref. [11]. Using $M_Z = 91.1876$ GeV, $\alpha(M_Z) = 1/128$, and $\alpha_3(M_Z) = 0.1176$ [4], we obtain $M_{\rm GUT} = 2.86 \times 10^{13}$ GeV and $\sin^2\theta_W(M_Z) = 0.23100$, in agreement with the usual value [4]. Moreover, using the evolution equations in Eq. (10) we get $\alpha_{\rm GUT}^{-1} \simeq 23(21)$, if $\mu_{IS} \approx 1$ TeV ($\mu_{IS} = M_Z$). The inclusion of the scalar singlet at low energies ($N_s = 1$) gives worse values for this mixing angle, so it must be considered with mass near the unification scale. In Fig. 1 we show the evolution of the coupling constants at the one loop level in the present model. An analysis at the 2-loop can be done, but in general it does not lead to a prediction unless the top quark and all extra fermion and scalar fields are taken into account. This results in a large set of coupled equations [19] that deserve more careful study.

Notice that in this extension of the SM, the Yukawa interactions can be similar to those in Ref. [12], but it is worth noting that, if we want to avoid a general mixing in each charge sector the extra quark (lepton) generation must transform under Z_{13} in a different way from those of the usual lepton (quarks). However, we recall that getting a

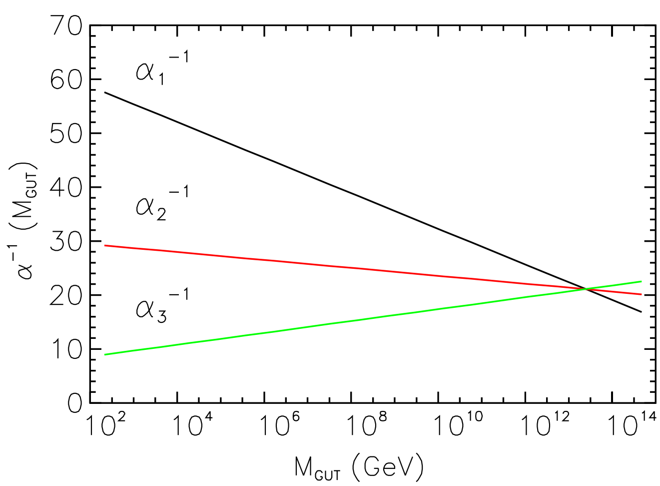

FIG. 1 (color online). In this figure we show the convergence point for the $SU(5)$ model here.

small mixing it can be interesting if in the future a departure from unitarity in the Cabibbo-Kobayashi-Maskawa mixing matrix would be observed [20]. In fact, even the usual three generations can be transformed under Z_{13} different from each other in such a way that predictive mass matrices can be obtained.

IV. STABILIZING THE PROTON

As we said before, in this model nucleon decays are forbidden at the tree level. Here we will discuss this point in more detail. The effective operators with dimension six [21], $d = 6$, that can induce the proton disintegration do not operate in our model because vector leptoquarks always mix the usual fermions with the heavy ones. However, without the Z_{13} symmetry, there are still dangerous $d = 4$ operators coming from the Yukawa couplings with the H^{45} Higgs scalars. For instance, without that discrete symmetry, Yukawa interactions like $\bar{\Psi}_{eR}\Phi_{dL}H_e^{45}$ and $\bar{\Phi}^c_{dR}\Phi_{dL}\epsilon H_e^{45*}$ are allowed. These terms induce the proton decay through interactions like $\bar{Q}^c_{mR}\epsilon\vec{\sigma}\cdot\vec{\eta}_m L$, and $\epsilon_{mnp}\bar{Q}^c_{mR}\epsilon\vec{\sigma}\cdot\vec{\eta}_n Q_{pL}$, respectively; here $\epsilon = i\sigma_2$, and m, n, p, are color indices, and $\vec{\eta}_m$ is the colored scalar triplet belonging to H_e^{45}. Once the Z_{13} symmetry is introduced the Yukawa interactions allowed are just those given in Eq. (3), and they only induce interactions like $\bar{Q}^c_{mR}\epsilon\vec{\sigma}\cdot\vec{\eta}_m L^\prime_L$ and $\epsilon_{mnp}\bar{Q}^c_{mR}\epsilon\vec{\sigma}\cdot\vec{\eta}_n Q^\prime_{pL}$, where the primed fields are heavy quarks, U, D, or heavy leptons, E, N. Hence, with the interactions in Eq. (3), independent of the mixing in the scalar sector, the nucleon is not allowed to decay at the tree level. The model is in this respect phenomenologically safe.

The Z_{13} symmetry introduced in this model allows effective interactions with flavor changing neutral current. For instance,

$$\frac{g^2_{Z_N}}{M^2_{Z_N}} h_{abcd}\overline{L_{aL}}\gamma^\mu L_{bL}\overline{L_{cL}}\gamma_\mu L_{dL}, \tag{18}$$

here a, b, c, d are family indices, and g_{Z_N} and M_{Z_N} denote the coupling constant of the $Z_N \subset U(1)_{\mathrm{local}}$ and the mass of the (heavy) vector boson associated with this symmetry, respectively, and h_{abcd} are dimensionless constants (there are also effective interactions induced by the heavy scalar that condensate at very high energies). The interactions in Eq. (18) induce rare transitions like $\mu \to eee$. Neglecting the electron masses we can write the width of this decay in terms of the muon decay width as follows:

$$\Gamma_{\mu\to 3e} = \left(\frac{g_{Z_N}}{g_2}\frac{M_W}{M_{Z_N}}\right)^4 \Gamma^{\mathrm{SM}}_{\mu\to e\nu\bar{\nu}}, \tag{19}$$

where g_2 and M_W are the well-known parameters of the standard model and we see that even if $g_{Z_N} \sim O(g)$ with $M_{Z_N} > 10^3 M_W$, we have already got a suppression factor of 10^{-12}. However, it is more natural that M_{Z_N} be of the order of the breakdown of the local $U(1)$ symmetry i.e., at least of the order of the PQ scale. It may be also interesting to assume that $g_{Z_N} \ll g$ at low energies, in such a way that, since g_{Z_N} which is not an asymptotic free parameter can fit with g at a high energy and the other coupling constant of the low energy model.

Just as another example, there are also interactions like

$$\frac{g^2_{Z_N}}{M^2_{Z_N}} h^\prime_{abcd}\overline{Q_{aL}}\gamma^\mu Q_{bL}\overline{F_{cL}}\gamma_\mu F_{dL}, \tag{20}$$

where $F = Q$, L, and $h^\prime$ is another dimensionless matrix. When $F = Q$ this interaction will induce a contribution to ΔM_K and other related parameters. Notice, however, that

$$\Delta M_K \propto \left(\frac{g_{Z_N}}{g}\frac{M_W}{M_{Z_N}}\right)^2 G_F B_K f_K^2 m_K. \tag{21}$$

We see that this contribution to ΔM_K is rather small for the same values of the Z_N parameters in Eq. (19).

In general discrete symmetries may be not free of anomalies. Although it is interesting looking for cyclic local discrete symmetries that are anomaly free, we would like to emphasize that it is not necessarily a loophole of models with anomalous Z_N symmetries. If $g_{Z_N} \le g$ the transition violating B and L conservation induced by the anomaly of the Z_N symmetry will be smaller than $e^{-16\xi\pi^2/g^2} \approx 10^{-117\xi}$ [22], with $\xi = O(1)$ a model dependent parameter. Although this transition is negligible at zero temperature it may be important in earlier ages of the Universe as a mechanism for leptogenesis generation through the decays of the heavy neutral leptons if CP violation is implemented in it. In fact, the model allows several ways to implement baryogenesis and leptogenesis [23,24] as we will show elsewhere.

V. CONCLUSIONS

Summarizing, we have obtained an $SU(5)$ extension of a previous model of Ref. [12], which is as good as SUSY $SU(5)$ with respect to the unification of the electroweak and strong interactions. We also have in this model that the proton is stable and the PQ an automatic symmetry of classical Lagrangian, with the invisible axion protected against gravitational effects by a local Z_{13} symmetry. The unification of the three coupling constants occurs at the PQ scale as in [11]. It is important to realize that the model does not admit supersymmetry at least at low energies, but we have seen that, in order to have unification, supersymmetry is not an indispensable factor anymore. As it was put forward in Ref. [17], the PQ energy scale can be related with the mass of the sterile neutrinos [17], and in the present model the PQ energy scale is related with the GUT scale. Put all this together and we have that in the present model it is possible to have $M_{\nu_R} \ll M_{PQ} \sim M_{\mathrm{GUT}}$ or $M_{\nu_R} \sim M_{PQ} \sim M_{\mathrm{GUT}}$, depending on the Z_{13} charges assignment.

Although the scalar and vector leptoquarks do not induce the nucleon decay at the tree level, their masses are of the order of the unification scale (they gain masses from the $\langle \mathbf{24} \rangle$). Since this scale is of the order of the PQ scale it means that both energy scales can be related to each other. On the other hand, the exotic quarks and leptons U, D, E gain mass from **5** and **45** which have VEVs of the order of the electroweak energy scale and, for this reason, they could not be very heavy. We recall that the experimental limits on the exotic leptons and quarks like E, N and U, D, respectively, are model dependent but since all of them gain mass from VEVs of the order of the electroweak scale they must not be very heavy indeed. For instance, from data we have lower bounds on the masses (in GeV) of a possible fourth family [4]: for sequential $E^{\pm}$ charged lepton, we have $m > 100.8$, C.L. $= 95\%$ (decay to νW); for stable charged heavy leptons $m > 102.6$, C.L. $= 95\%$; for stable neutral heavy lepton the limits are $m > 45.0$, C.L. $= 95\%$ (Dirac) and $m > 39.5$, C.L. $= 95\%$ (Majorana). Finally, for extra quarks of the b-type (b' fourth generation) the lower limits are $m > 190$, C.L. $= 95\%$ (quasistable b') or $m > 199$, C.L. $= 95\%$ (neutral currents); if it decays in $ll +$ jets, $l +$ jets, we have $m > 128$, C.L. $= 95\%$. Of course, these limits are strongly model dependent (in some models the fourth family is almost degenerate [25]). On the other hand, as we said before, the model has a general mixing among the fields of the same electric charge sector, thus the generalized unitarity triangle analysis of the Cabibbo-Kobayashi-Maskawa matrix [20] can be used for deriving upper bounds on the coefficients of the effective operators inducing such mixings [26].

The model has right-handed neutrinos, so it is possible that an $SO(10)$ would be more appropriate for the unification of the model of Ref. [12]. However since $SU(5) \subset SO(10)$ our $SU(5)$ model is already good enough for implementing a GUT theory for the extension of the standard model with proton stability.

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