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A unified model for the long and high jump

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A simple model based on the maximum energy that an athlete can produce in a small time interval is used to describe the high and long jump. Conservation of angular momentum is used to explain why an athlete should run horizontally to perform a vertical jump. Our results agree with world records.
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I. INTRODUCTION

A few years ago, William Harris asked how the kinetic energy acquired from running is converted in the long and high jump events.[1] The question was whether athletes running horizontally can change their velocity into one that forms an angle of 45° with the horizontal without changing their speed. The three negative answers[2–4] were that athletes cannot convert their initial horizontal velocity into a vertical one,[3] because it is impossible to generate the necessary power required by the task,[4] or equivalently, athletes cannot sustain the necessary acceleration to acquire the vertical velocity at takeoff.[2]

If the answer were positive, the center of mass of a world class athlete running at 10 m/s and taking off at 45° would go a horizontal distance of about 10.2 m. Because the athlete's center of mass is forward of the front edge of the runway at takeoff and behind the point where her heels hit the ground at landing (see Sec. III), the actual total jump length would be about 11 m. For the high jump, the athlete's center of mass would attain a maximum height of 3.5 m. These results are much greater than actual records. We conclude that athletes cannot totally change their horizontal velocity into a vertical one, as stated by the negative answers.

The negative answers bring some new questions. If the magnitude of the velocity is assumed to be unchanged and thus the kinetic energy, why is extra power necessary? Why does a high jumper run horizontally to jump vertically? How do runners change their horizontal velocity into a velocity with a vertical component? What is conserved and what is changed at takeoff?

II. WHAT LIMITS AN ATHLETE'S PERFORMANCE?

There are three systems that any animal uses to produce energy.[5] For humans and activities whose duration is longer than a few minutes, aerobic energy is produced at a low rate by burning carbohydrates or fats. For activities that last a few tens of seconds, the main source of energy is the breakdown of glycogen in the absence of oxygen and the production of lactic acid. For very short duration activities the body uses the adenosine triphosphate (ATP) stored in muscles, and energy is supplied immediately, following the conversion of ATP into adenosine diphosphate (ADP).

The static force generated from muscle contraction can be very high. However the magnitude of the force is limited by the rate that the muscles can produce energy, which is limited by the rate that ATP is depleted (ATP depletion is completed in a few seconds). For example, after a few seconds sprinters no longer accelerate, but decelerate,[6] because they cannot overcome the force due to air resistance. ATP is the source of energy in all explosive sports, such as jumps, sprints, and weightlifting. Thus, if we produce a force with a nonvanishing velocity, that is, we produce mechanical power, the force is limited by the rate that our muscles can convert ATP into ADP.

To discuss the performance of a jumper, we use the results of two observations. One is the maximum force that athletes can produce in a small time interval with their legs while producing work. We will take this information from observations of the squat. In this exercise a weight is rested on the shoulders of an athlete. He starts from the upright position, and bends his ankles, knees, and hips as if sitting. At the lowest position, when his thighs are horizontal, he returns to the upright position. The maximum weight an athlete can lift doing a squat is about 5000 N. For a first class male athlete with mass 100 kg, the maximum force that can be made with just one leg is about 3000 N. (In the high and long jump, athlete's use just one leg to push their center of mass.) This force is different from a static force.

In both the high and long jump, the height of the runner's center of mass remains almost constant until the last stride. In the last stride the center of mass rises about 25 cm before the runner takes off and his foot loses contact with the ground.

From these two observations we conclude that an 80 kg runner (assuming 80 kg athlete can produce the same 3000 N force with each leg) can add about $(3000-800)\mathrm{N}\times 0.25$ m $=550$ J to the kinetic energy. We will use this result in the following.

An equivalent result can be obtained from an analysis of an elite 100 m runner. At 6 m/s, an elite runner has an acceleration of about 5 $\mathrm{m/s^2}$ (see, for instance, Fig. 1 in Ref. 6). For an 80 kg athlete this acceleration corresponds to a mechanical power of 2400 W. At every stride, the runner's center of mass rises about 5 cm. At a rate of 5 strides/s (a typical rate in the 100 m dash), the up-down movement of the center of mass corresponds to 200 W of additional power. Thus at each stride, the runner produces $(2400+200)$W $\times 0.2$ s$=520$ J. If we also include a small contribution due to air resistance, this result is consistent with the previous estimate of 550 J.

For female runners, the time of the 100 m dash is about 10% greater, and the velocity about 10% smaller than the male records. Thus, we estimate that the acceleration and, hence the force, is about 20% smaller than that for male

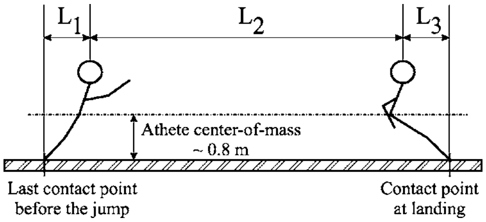

Fig. 1. Schematic of the long jump. The distance of the long jump is the sum of the takeoff (L_1), the flight (L_2), and the landing (L_3) distances.

runners. Thus, for events that depend on velocity and force—as do the long and high jumps—we expect that female performance to be between 10% and 20% smaller.

III. LONG JUMP

The model for the long jump is simple. After running a few seconds the runner produces a big vertical force against the ground in the last stride. Some details of the movement of the athlete are shown in Fig. 1. At takeoff, the runner's center of mass is about 0.4 m forward of the front edge of the runway, L_1, in Fig. 1. At landing, the runner's center of mass is behind the point where the heels hit the sand ($L_3 \simeq 0.4$ m). The difference between the vertical height of the runner's center of mass at takeoff and at landing, which would lead to a small difference between L_1 and L_3,[7] will be neglected. The length of the jump is $L_1+L_2+L_3$, where L_2 is the flight distance.

The horizontal velocity of the runner at the last stride, v_0, is about 10 m/s. (Usually, long jumpers are good sprinters.) The best a runner can do is to use the additional energy of 550 J to obtain a vertical velocity. For an 80 kg athlete, the vertical velocity gained is $\Delta v = 3.7$ m/s. Thus, L_2 is given by

$$L_2 = \frac{2v_0\Delta v}{g} = 7.6 \text{ m}, \tag{1}$$

and $L_1+L_2+L_3=(0.4+7.6+0.4)\text{m}=8.4$ m. The latter result agrees with observations between 7.4 and 8.8 m for elite male long jumpers and 6.5 and 7.5 m for elite female jumpers.[7] The takeoff angle can be calculated from the velocity diagram in Fig. 2, and we find $\theta = \arctan(\Delta v/v_0) \sim 20°$. This result agrees with observations of $\theta \sim 20° \pm 2°$.[7]

IV. HIGH JUMP

Before jumping, a high jumper runs a few meters and attains a speed of about 7.5 m/s. Why does the jumper run horizontally before jumping vertically? What is the mechanism used by the jumper to change her horizontal velocity into a velocity with a vertical component? (A detailed description of the high jump can be found in Ref. 8.)

To answer these questions, consider a bar representing a jumper of length 2ℓ, which has a horizontal velocity v_0 and

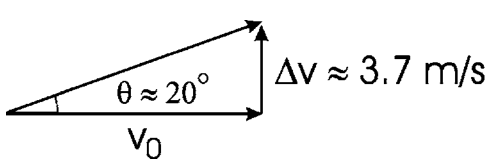

Fig. 2. The long jump velocities at takeoff.

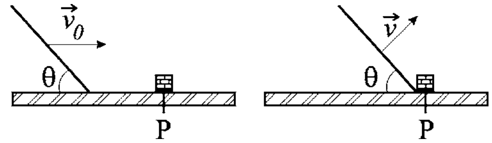

Fig. 3. A bar with a horizontal velocity v_0 is suddenly stopped at point P and begins to rotate, changing its initial horizontal velocity into a new one with a vertical component (adapted from Fig. 2 of Ref. 8).

is inclined backward, as shown in Fig. 3. At point P, the bottom end of the bar stops suddenly, and the bar begins to rotate around P. Due to the impact, the bar's velocity will change, but the angular momentum with respect to P is conserved. Conservation of angular momentum at the instant the bottom end stops gives

$$v = \tfrac{3}{4}v_0 \sin\theta, \tag{2}$$

where v is the bar's center of mass velocity immediately after the impact and the velocity is perpendicular to the bar. Thus, the initial horizontal velocity has changed to a velocity with a vertical component.

The horizontal, v_h, and vertical, v_v, projections of the velocity just after the bottom end of the bar stops are

$$v_h = \tfrac{3}{4}v_0 \sin^2\theta, \tag{3}$$

$$v_v = \tfrac{3}{4}v_0 \sin\theta\cos\theta. \tag{4}$$

If the bar stops the rotation immediately after hitting P and if the horizontal and vertical velocities at takeoff are given by Eqs. (3) and (4), the center of mass of the bar will attain a maximum height (calculated by energy conservation) given by

$$h = \frac{1}{2g}\left(v_v^2 + \frac{2}{m}550 \text{ J}\right) + \ell\sin\theta. \tag{5}$$

If we use $v_0=7.5$ m/s and a 2 m($\ell=1$ m) bar, we obtain the maximum h for $\theta=56°$.

Suppose that the runner is represented by a bar moving at 7.5 m/s inclined backward at 56° (in Sec. V we will show how this motion is possible). Then the runner begins to rotate just for a moment around point P. As in the long jump, she can add 550 J to the total kinetic energy. The best the higher jumper can do is to gain an additional velocity in the vertical direction, which can be calculated from

$$\tfrac{1}{2}m(v_h^2+v_v^2) + 550 \text{ J} = \tfrac{1}{2}m(v_h^2+v_v'^2), \tag{6}$$

giving $v_v'=4.5$ m/s. The increase in her vertical velocity is $\Delta v = v_v' - v_v = 1.9$ m/s.

If the runner takes off with a vertical velocity equal to 4.5 m/s, the runner's center of mass will rise about 1.0 m. If we add this value to the initial position of the runner's center of mass at takeoff, which is about 1.1 m, we obtain a total height of 2.1 m. Because we know the horizontal and vertical velocities, we can calculate the takeoff angle as 49° (see Fig. 4). The world record is 2.45 m for men and 2.09 m for women and a typical takeoff angle is 50°. These values suggest that the model is reasonable.

Despite the simplification of representing a person by a bar to explain the high jump, the pole vault is even simpler to analyze. The pole vaulter runs about 15 m and reaches a velocity of about 10 m/s. Her kinetic energy is changed to

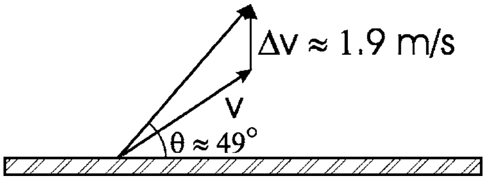

Fig. 4. The high jump velocities at takeoff.

elastic potential energy of the bar, and then the energy is changed to the vaulter's gravitational potential energy. If we assume that the vaulter's center of mass at the beginning of the jump is about 1.1 m above the ground, we can use energy conservation to calculate that the height reached by a pole vaulter is 6.2 m. The records are about 5 and 6 m for men and women, respectively. This estimate is only approximate because it does not take into account that the vaulter's center of mass passes significantly under the bar, and it assumes a 100% efficiency of horizontal to vertical jump energy conversion.

V. DISCUSSION AND CONCLUSION

Can a high jumper run with his body inclined backward, as assumed in the model? The answer is that it is not possible. Like everyone else, the jumper runs in a vertical position. However, in the last strides, the high jumper makes a circular curve with a radius of about 10 m. Due to the centripetal force, the jumper leans to the center of the circle and the trajectory of the center of mass differs from the trajectory of the footprints. These trajectories are illustrated in Fig. 5. At the beginning of the takeoff, corresponding to the instant that the bottom end of the bar stops, the jumper's center of mass is tilted toward the center of the curve, and his velocity is along the arrow shown in Fig. 5.[9] During the next 0.2 s, he changes his body orientation, tilting it outside the circle, and produces a very strong vertical force against the ground. (This model is an oversimplification of the detailed model of Ref. 8, but it retains the essence of the athlete's movement at takeoff as illustrated in Fig. 1 of Ref. 8.)

In our model we assumed that the long jumper does not change her initial velocity into a new velocity using the same trick used by the high jumper because fixing one foot against the ground would cause some loss of the horizontal velocity, which is very important to the long jumper.

We note that the high jump differs from the vertical standing jump, which has been studied in Ref. 10. In the standing jump the athlete uses both legs to jump. In the vertical standing jump the athlete does not run before jumping and the

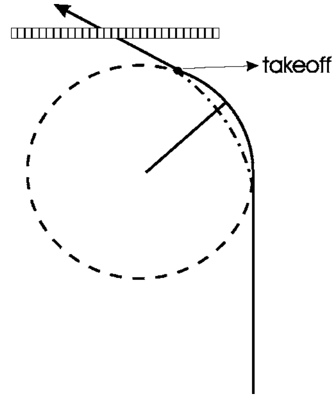

Fig. 5. Trajectory developed by the athlete before high jumping. The solid line in the circumference is the trajectory of the athlete's feet, the dot-dashed line is the trajectory of her center of mass.

only source of energy comes from the leg muscles. As can be estimated from the figures of Ref. 10, the mechanical energy gained at takeoff by a nonathlete is about 550 J when using both legs; our hypothesis that an elite athlete can add 550 J when using only one leg seems reasonable.

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