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On the stability of hypothetical satellites coorbital to Mimas or Enceladus

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ABSTRACT

In the present work, we study the stability of hypothetical satellites that are coorbital with Enceladus and Mimas. We performed numerical simulations of 50 particles around the triangular Lagrangian equilibrium points of Enceladus and Mimas taking into account the perturbation of Mimas, Enceladus, Tethys, Dione, Titan and the oblateness of Saturn. All particles remain on tadpole orbits after 10 000 yr of integration. Since in the past the orbit of Enceladus and Mimas expanded due to the tidal perturbation, we also simulated the system with Enceladus and Mimas at several different values of semimajor axes. The results show that in general the particles remain on tadpole orbits. The exceptions occur when Enceladus is at semimajor axes that correspond to 6:7, 5:6 and 4:5 resonances with Mimas. Therefore, if Enceladus and Mimas had satellites librating around their Lagrangian triangular points in the past, they would have been removed if Enceladus crossed one of these first-order resonances with Mimas.

Key words: planets and satellites: formation – planets and satellites: individual: Saturn – planets and satellites: individual: Mimas – planets and satellites: individual: Enceladus.

1 INTRODUCTION

Coorbital systems are composed of objects that oscillate around the equilibrium Lagrangian stable points, L_4 and/or L_5, of a determinate body called the secondary, which is in orbit around another and more massive body called the primary. A well-known example of coorbital system is the Trojan asteroids, which are coorbitals of Jupiter. There are asteroids coorbital with Mars (Tabachnik & Evans 1999) and four asteroids coorbital with Neptune (Chiang et al. 2003; Sheppard & Trujillo 2006). Recently Morbidelli et al. (2005) discussed a new plausible capture mechanism for Jupiter and Neptune Trojans due to planetary migration of the giant planets.

Saturn is the only planet known to have coorbital satellite systems. Tethys has Telesto oscillating around L_4 and Calypso around L_5. Dione has Helene around its L_4 equilibrium point, and recent images from Cassini show the existence of a small satellite around L_5 (Murray et al. 2005; Porco et al. 2005). Roddier et al. show a thin ring near Enceladus. According to authors, this ring probably link and it is seed by Enceladus itself. Roddier et al. suggest this ring is formed by collision with Enceladus. There is also one pair of coorbital satellites which have comparable masses, i.e. Janus and Epimetheus. In a rotating coordinates system, their orbits have the shape of horseshoes. On the other hand, the Saturnian system has some other mean motion resonances (Roy & Ovenden 1954). The coorbital systems are also involved in mean motion resonances with other satellites.

For the planet–asteroid problem, Zhang & Innanen (1988) studied the stability of the Saturnian Lagrangian points taking into account the perturbation of Jupiter. They integrated for 10^5 yr and found that the regions close to the equilibrium points are unstable, while there are stable regions for large tadpole orbits. Holman & Wisdom (1993) numerically explored the stability of bodies coorbital to Jupiter, Saturn, Uranus and Neptune. They integrated 2×10^7 yr confirming the results of Zhang & Innanen (1988) and found it to be stable in the cases of Jupiter, Uranus and Neptune. Considering planetary migration, Gomes (1998) studied the coorbitals of Jupiter, Saturn, Uranus and Neptune during the radial migration of these planets. In such dynamics, the planets crossed some mean motion resonances. Gomes explored the 1:2 resonance Jupiter–Saturn, showing that it is unstable for coorbitals of Jupiter and Saturn. Gomes also showed an asymmetry in L_4 to L_5 survival time when Jupiter is in resonance 2:1 with Saturn. Michtchenko, Beaugé & Roig (2001) explored the stability of the coorbital region of Jupiter and Saturn at several mean motion resonances. Nesvorný & Dones (2002) investigated the dynamical evolution stability of coorbitals of Jupiter, Saturn, Uranus and Neptune using maximum Lyapunov characteristic exponents and numerical simulations. They did not find the asymmetry described by Gomes (1998) and concluded that the instability of Saturn's coorbitals is due to the high eccentricity in the secular dynamics.

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The inner Saturnian system has several satellite coorbitals, but none associated with Mimas or Enceladus. At the time this work was done, there was some evidence for possible, undetected, coorbitals of Mimas (Gordon, Murray & Beurle 1996). The question remained 'why not Enceladus?' Enceladus–Dione are in a 2:1 resonance and Mimas–Tethys in a 4:2 inclination-type resonance. In both the cases, the precession of the longitudes due to Saturn oblateness is extremely relevant (Greenberg 1973a). On the evolution of Saturn's satellites, the tidal perturbation is considered to be the most important effect to bring the satellites to the resonant configuration (Goldreich 1965; Sinclair 1972, 1974; Peale 1976). Greenberg (1973b) shows that Enceladus migration was dominant in the evolution of the pair Enceladus–Dione. Sinclair (1984) used an analytic approach to examine the consequences of tidal and resonant perturbations on actual or hypothetical coorbitals of Mimas, Enceladus, Tethys and Dione. He concluded that tidal perturbations would have an insignificant effect on coorbital stability. For Enceladus coorbitals, he found that perturbations due to the Dione 2:1 resonance (the current configuration), while not sufficient to remove such bodies, are sufficient to produce high probabilities of collisions if there are multiple coorbitals. He further 'speculated' that the erratic perturbations of semimajor axis and eccentricity on particles in the tadpole region due to the 2:1 resonance with Dione could have prevented the accretion of particles to form a satellite.

Using disturbing function, Sinclair (1984) found that due to the 2:1 resonance between Enceladus and Dione the maximum amplitude of the semimajor axis and eccentricity of hypothetical coorbital to Enceladus are

$$\Delta a_{\rm res} = 0.4 \times 10^{-4} a_E \text{ and } \Delta e_{\rm res} = 50 \times 10^{-4}, \tag{1}$$

where $a_{\rm E}$ is the semimajor axis of Enceladus. Using the equations given in Dermott & Murray (1981), Sinclair found that the maximum width for stable tadpole and for horseshoe orbits is given by

$$\Delta a_{\rm tad} = (8m_e/3)^{1/2} a_e = 5.8 \times 10^{-4} a_e \tag{2}$$

$$\Delta a_{\rm horse} = \left(0.4 m_e^{1/3}\right) a_e = 40 \times 10^{-4} a_e, \tag{3}$$

where m_e is the Enceladus' mass. Then, Sinclair compare the values of Δa due the 2:1 resonance with Dione with these maximum amplitudes for horseshoe and tadpole orbits. He verified that the perturbation due to the resonance with Dione is only 10 per cent of the maximum amplitude for a tadpole orbit and 1 per cent of the maximum amplitude for a horseshoe orbit. However, Sinclair (1984) believes that perturbations in eccentricity $\Delta e_{\rm res}$ would be decisive to explain the non-existence of coorbitals to Enceladus. He argues that these variations of eccentricity would imply in orbital radius variations in tadpole and horseshoe regions, so collisions between particles would prevent the coexistence of tadpole and horseshoe orbits, avoiding the formation of satellites coorbital to Enceladus.

In spite of the Mimas–Thetis resonance being of inclination type, Sinclair (1984) suggests that the inclination effect is not important and analyses the system using the planar case. Considering the effect due to an exact 2:1 resonance between Mimas and Thetys, Sinclair computed the maximum amplitudes of the semimajor axis and eccentricity of hypothetical coorbitals to Mimas. He found $\Delta a_{\rm res} = 0.5 \times 10^{-4}$ and $\Delta e_{\rm res} = 12 \times 10^{-4}$. Therefore, he verified that the perturbation due to the resonance with Thetys is not enough to remove possible coorbitals of Mimas. So, Sinclair (1984) speculated that the high proper eccentricity of Mimas is the most important factor to explain the non-existence of its coorbitals, because similar to Enceladus case, it would provoke collisions that would avoid the coorbital's satellite formation.

On the other hand, Sinclair (1984) do not discuss whether these collisions are constructive or not. It could provoke loss of energy that would lead to the stabilization of bodies in coorbital regions. Moreover, it would be possible the existence of coorbital satellites system if we consider that the triangular equilibrium points oscillate together with the satellite (Namouni & Murray 2000). It would not mean that the coorbital would be crossing the tadpole and horseshoe limit region.

In the present work, we study the stability of particles librating around L_4 or L_5 of Enceladus or Mimas through numerical simulations. We examine stability for the current configuration of the Saturnian system and for various configurations that could have occurred during the orbital migration of Enceladus and Mimas. Our integrations include perturbations due to Mimas, Tethys, Dione, Titan and the oblateness of Saturn. The methodology of our work is presented in the next section, where we describe the details of the criterion of stability adopted and the procedure of the numerical simulations. The numerical results are presented in Section 3. In the last section, we make some final comments.

2 METHODOLOGY

2.1 Stability criterion

In this section, we present the criterion used to determine whether a test particle remained a tadpole coorbital during the integration. We used an approach described by Yoder et al. (1983). For the coorbital problem, Yoder et al. (1983) found a constant motion, neglecting the radial oscillations, given by

$$E = -\frac{1}{6}\dot{\phi} - \mu_1 n_1^2 f(x) > 0, \tag{4}$$

where ϕ is the angular displacement between the two coorbitals, μ_1 is the relative mass, n_1 is the mean motion and

$$f(x) = \frac{1+4x^3}{2x}, \quad \text{where} \quad x = \sin\frac{\phi}{2}. \tag{5}$$

At maximum angular amplitude in the separatrix, we have $\phi_{\max} = 180^\circ$, that implies $x = 1$. In this case, we have E given by

$$E = -2.5\mu_1 n_1^2. \tag{6}$$

Substituting this value of E in Equation (4) results in

$$-2.5\mu_1 n_1^2 = -\mu_1 n_1^2 \frac{1+4x^3}{2x} \Rightarrow 1 - 5x + 4x^3 = 0. \tag{7}$$

One can find three roots for x:

$$x_1 = 1, \; x_2 = -\frac{1}{2} - \frac{1}{2}\sqrt{2} \approx -1.2071 \text{ and}$$

$$x_3 = -\frac{1}{2} + \frac{1}{2}\sqrt{2} \approx 0.20711. \tag{8}$$

From equation (2), we have that the first value of x corresponds to $\phi = \phi_{\max} = 180^\circ$. The second is a non-real value, and the last value of x corresponds to the minimum angular distance between the coorbitals in the separatrix orbit. The x_3 value corresponds to $\phi_{\min} = 23.906^\circ$. Therefore, we define an arc of $47^\circ.812$ centred on Mimas and other on Enceladus. When the angular position of the particle gets inside the arc of its respective satellite, we do not consider this particle in a stable tadpole coorbital orbit (Fig. 1). In addition to this criterion, we examined the amplitude of oscillation of the angles relative to the respective satellite for each particle. This step was necessary in order to identify any particles which escape abruptly from the coorbital region without entering the angular region.

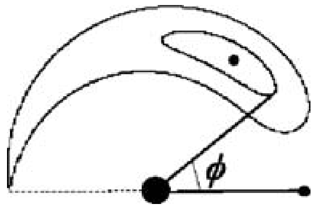

Figure 1. The criterion used to consider a stable tadpole coorbital particle depends on the angle ϕ. When the angle ϕ is less than 23°.906, the orbit changes from tadpole to horseshoe.

2.2 Numerical simulations

In this work, we numerically integrated particles initially located near the triangular equilibrium points of Enceladus and Mimas. The numerical simulations were performed using an integrator (Cordeiro et al. 1997) based on the MVS method developed by Wisdom & Holman (1991). In order to double check our results, we performed some numerical integrations using the Bulirsch–Stoer code from Mercury (Chambers 1999). The results gave confidence in using the MVS integrator.

We spread 25 particles around each one of the triangular Lagrangian points of Enceladus and Mimas. The particles were initially given the same orbital parameters as the reference satellites, but offset in longitude. The initial longitudes were uniformly distributed within $\pm 5^\circ$ from L_4 or L_5, forming tadpole orbits in the restricted three-body problem with maximum libration amplitude of about 10° (Fleming & Hamilton 2000).

We performed the integration of these systems for 10 000 yr. This time interval is more than 1.5×10^6 orbital periods of Enceladus. During the integration, we checked whether each particle was librating around L_4 or L_5 of the reference coorbital satellite using the method described in Section 2.1.

During the orbital evolution of the saturnian satellite system, before it reaches its present configuration, the satellites might have crossed mean motion resonances different from those they are nowadays. The evolution of the separation between Mimas and Enceladus, due to their orbital migration caused by tidal effects, may have followed one of the following possibilities.

(i) Enceladus migrated diverging from Mimas. That would have occurred if Mimas reached the 4:2 resonance with Thetis before Enceladus be locked to the 2:1 resonance with Dione. Therefore, Mimas would be with its present semimajor axis while Enceladus would be with a semimajor axis smaller than its present one.

(ii) Mimas migrated converging towards Enceladus. That would have occurred if Enceladus reached the 2:1 resonance with Dione before Mimas be locked to the 4:2 resonance with Thetis. Therefore, Enceladus would be with its present semimajor axis while Mimas would be with a semimajor axis smaller than its present one.

Actually what happened is unknown, but Dermott, Malhotra & Murray (1988) suggest that if all satellites migrated together during the age of the Solar system, then Mimas and Enceladus crossed the resonances given in the first hypothesis, since the migration of Mimas would be faster because its semimajor axis is smaller.

Considering these two possibilities for the evolution of the separation between Mimas and Enceladus, there is a set of mean motion resonances that they could eventually have crossed (Figs 2 and 3).

Therefore, in order to simulate the system in the past, for the possibilities (i) and (ii), we considered two groups of initial conditions.

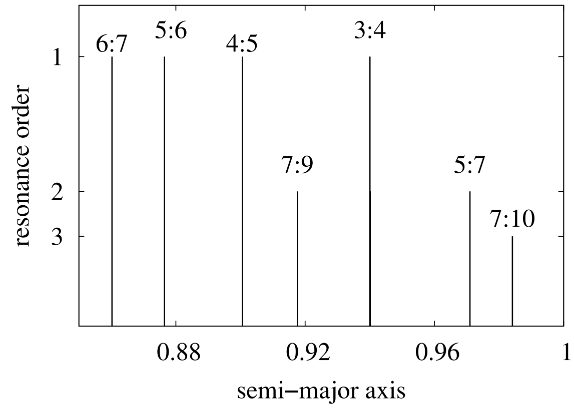

Figure 2. Mean motion resonances between Mimas and Enceladus as a function of hypothetical semimajor axis of Mimas, in the range of 85–100 per cent of its present value. The semimajor axis of current Mimas is normalized as unitary.

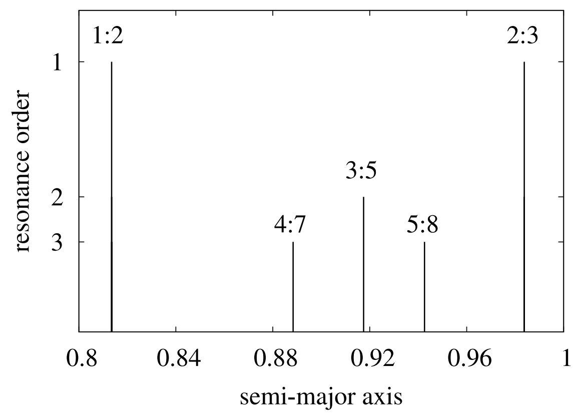

Figure 3. Mean motion resonances between Mimas and Enceladus as a function of hypothetical semimajor axis of Enceladus, in the range of 80–100 per cent of its present value. The semimajor axis of current Enceladus is normalized as unitary.

(i) We changed the value of Enceladus semimajor axis to a value in the range between 85 and 100 per cent its present value. Such range includes the location of first-order mean motion resonances 6:7, 5:6, 4:5 and 3:4 between Enceladus and Mimas (Fig. 2). Therefore, we numerically simulated the system at each one of these resonant configurations and several other non-resonant configurations.

(ii) We changed the value of Mimas semimajor axis to a value in the range between 80 and 100 per cent of its present value. Such range includes the location of first-order mean motion resonances 1:2 and 2:3 between Enceladus and Mimas (Fig. 3). Therefore, we numerically simulated the system at each one of these resonant configurations and several other non-resonant configurations.

3 RESULTS

In the case of the present configuration, with Enceladus locked in the 2:1 resonance with Dione and Mimas in the 4:2 resonance with Thetys, all particles remained in tadpole coorbital orbits during the entire integration period. Therefore, we conclude that the current configuration is stable for Enceladus and Mimas coorbitals,

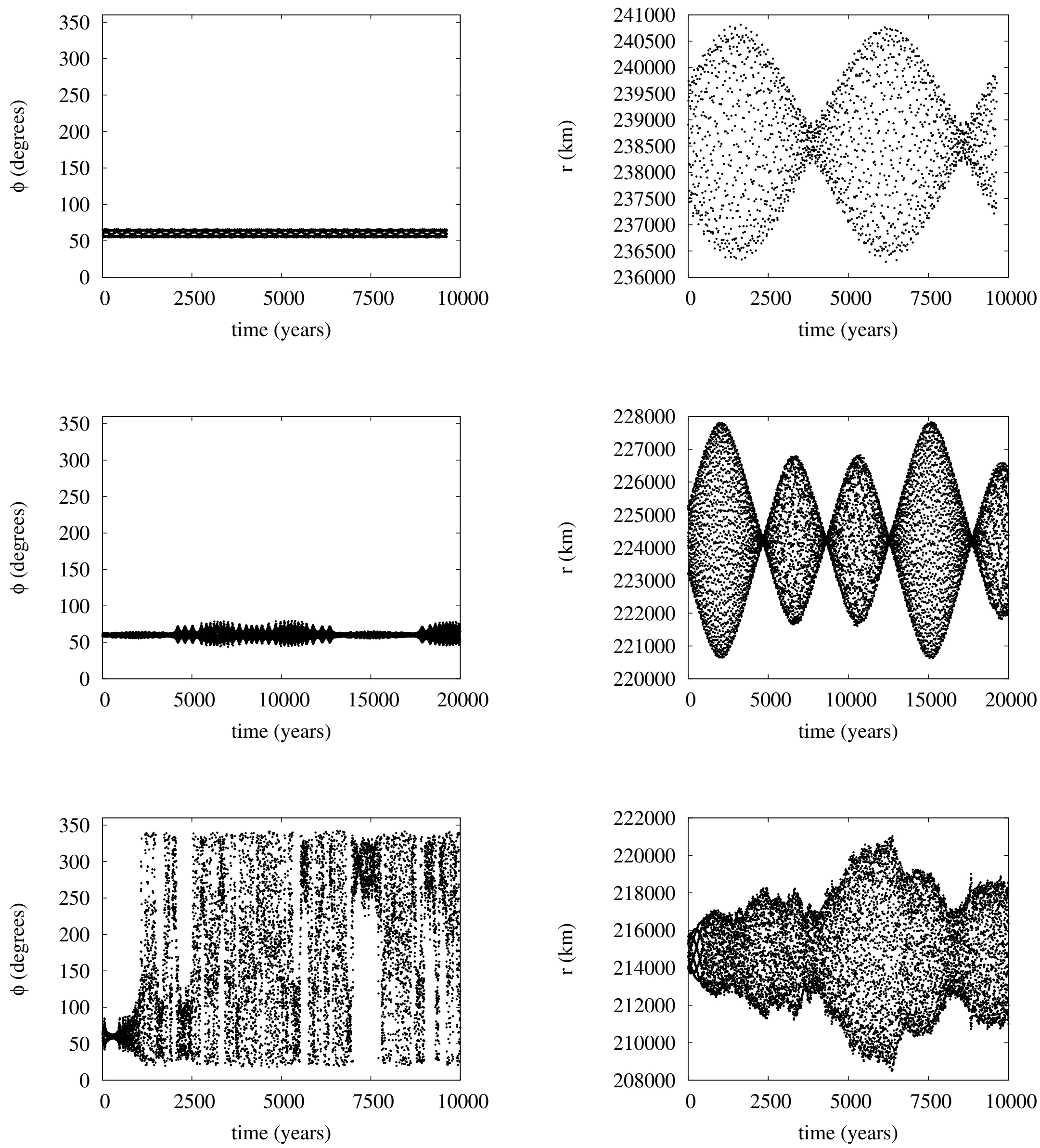

Figure 4. Sample of typical angular (left-hand side) and radial (right-hand side) evolution of particles initially placed near L_4 point of Enceladus. In each case, Enceladus is at a different semimajor axis. Top: present semimajor axis; middle: semimajor axis in resonance 3:4 with Mimas and bottom: semimajor axis in resonance 4:5 with Mimas.

confirming the results of Sinclair (1984). Fig. 4 (top) shows the typical angular and radial evolution for these particles.

A similar result was found for more than 20 simulations with Enceladus at another semimajor axes between 85 and 125 per cent of its present value. In all these cases, the particles remained in stable tadpole orbits; this included the external resonances, the second- and third-order internally and externally and the non-resonant positions. The only exceptions occurred when Enceladus was located at internal first-order mean motion resonances with Mimas.

In case of the 3:4 resonance, we noted some irregularity in the angular and radial evolution of the particles [see an example in Fig. 4 (middle)]. However, all particles remained on tadpole orbits during the integration time of 2×10^4 yr. The other three resonances, 4:5, 5:6 and 6:7, proved to be unstable. A typical example of the angular and radial evolution of these particles is presented in Fig. 4 (bottom). We do not note any case of collision or escape from the coorbital region.

Fig. 4 (bottom) depicts a representative example of the orbital evolution of a test particle that escaped the tadpole region and oscillates intermittently between horseshoe and tadpole orbits.

All evolve chaotically, but continue to librate about the respective equilibrium points. Many of them show an intermittent behaviour, changing between tadpole and horseshoe orbits of different amplitudes. In Figs 5 and 6, we show the histograms of the evolution of the surviving tadpole coorbitals for each of these three resonances.

4 FINAL COMMENTS

Sinclair (1984) showed that the resonant perturbation of Dione and Thetis with Mimas or Enceladus, respectively, would produce $\Delta a_{\rm res}$

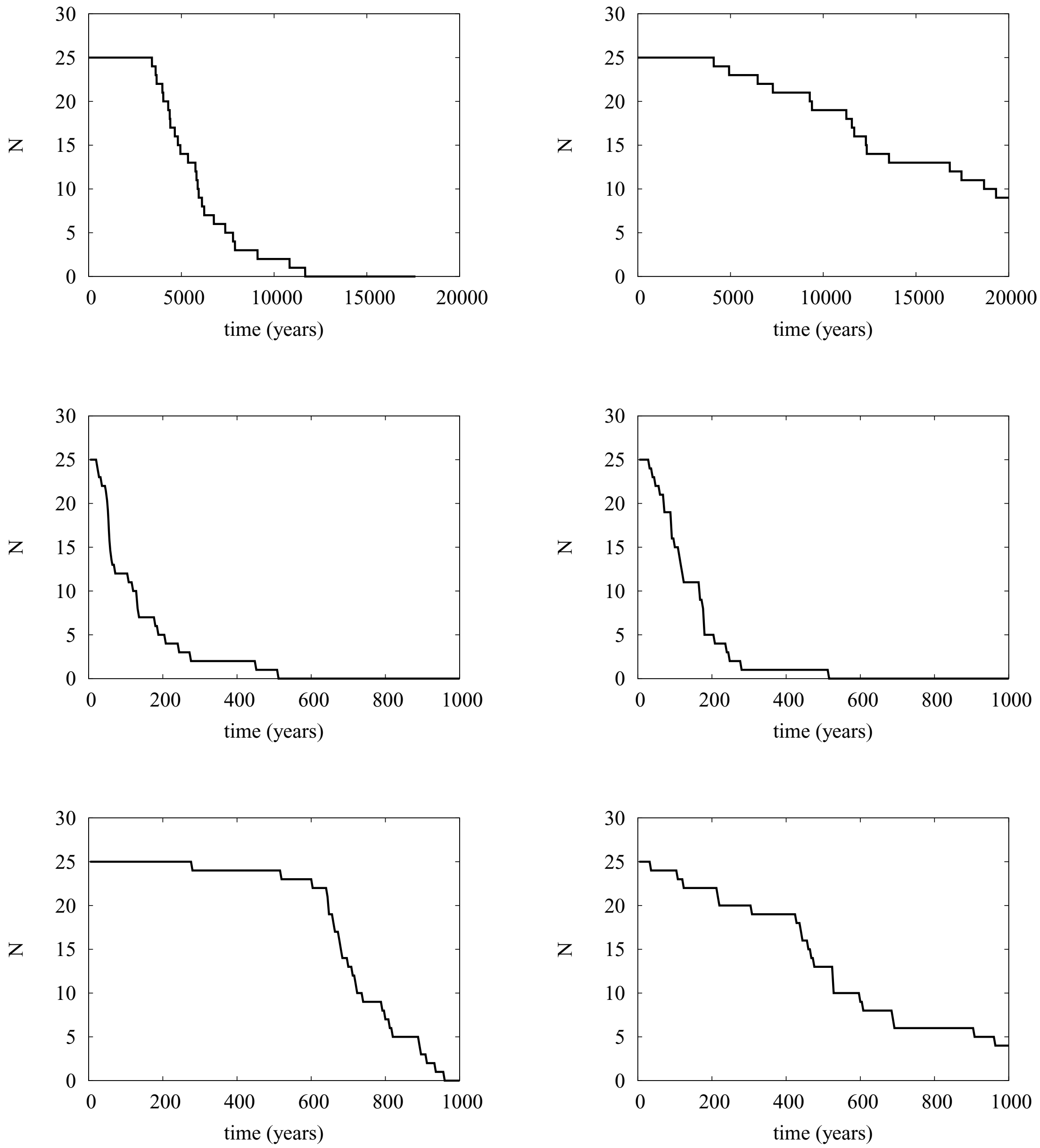

Figure 5. Histograms of the number of remaining tadpole coorbitals of Enceladus as a function of the time: around L_4 (left-hand side) and around L_5 (right-hand side). In each case, Enceladus is located at a given resonance with Mimas: 4:5 (top), 5:6 (middle) and 6:7 (bottom).

and Δe_{res} too small to remove coorbitals from its tadpole orbit. These results are corroborated by our numerical simulations.

Similarly to Sinclair (1984), we computed the maximum amplitude of oscillation in semimajor axis (Δa) due to several first-order resonances, $j : j + 1$. The results are given in Table 1. Comparing the values in this table, we note that Δa_{res} increases with the value of j. That could be the reason why the coorbitals are unstable for higher values of j, resonances 4:5, 5:6 and 6:7.

In this paper, a study of the stability of hypothetical satellites coorbital with Mimas and Enceladus is presented. We carried out numerical simulations of particles around the triangular Lagrangian points L_4 and L_5 of Mimas and Enceladus. The stability was investigated considering those orbits that left the tadpole region where they started unstable.

Since in the past Mimas and Enceladus migrated outwards due to tidal effects, we also studied several hypothetical configurations where Mimas or Enceladus are at semimajor axis smaller than they are today.

A general comment to be made is that all the results are similar for both satellites. In the present configuration of the saturnian satellite system, objects in tadpole coorbital orbits with Mimas or Enceladus are stable. The same is true for almost all the other configurations considered here. The only exceptions occur when Mimas and Enceladus are placed at semimajor axis that correspond to the first-order mean-motion resonances 4:5, 5:6 and 6:7, between them. In those configurations, the particles show an unstable behaviour. They evolve chaotically. Most of them present an intermittent temporal evolution between tadpole around one of the Lagrangian points, horseshoe and tadpole around the other Lagrangian point (Fig. 4, bottom). Such instability for a long period of time or any small perturbation would probably remove such objects from the coorbital region.

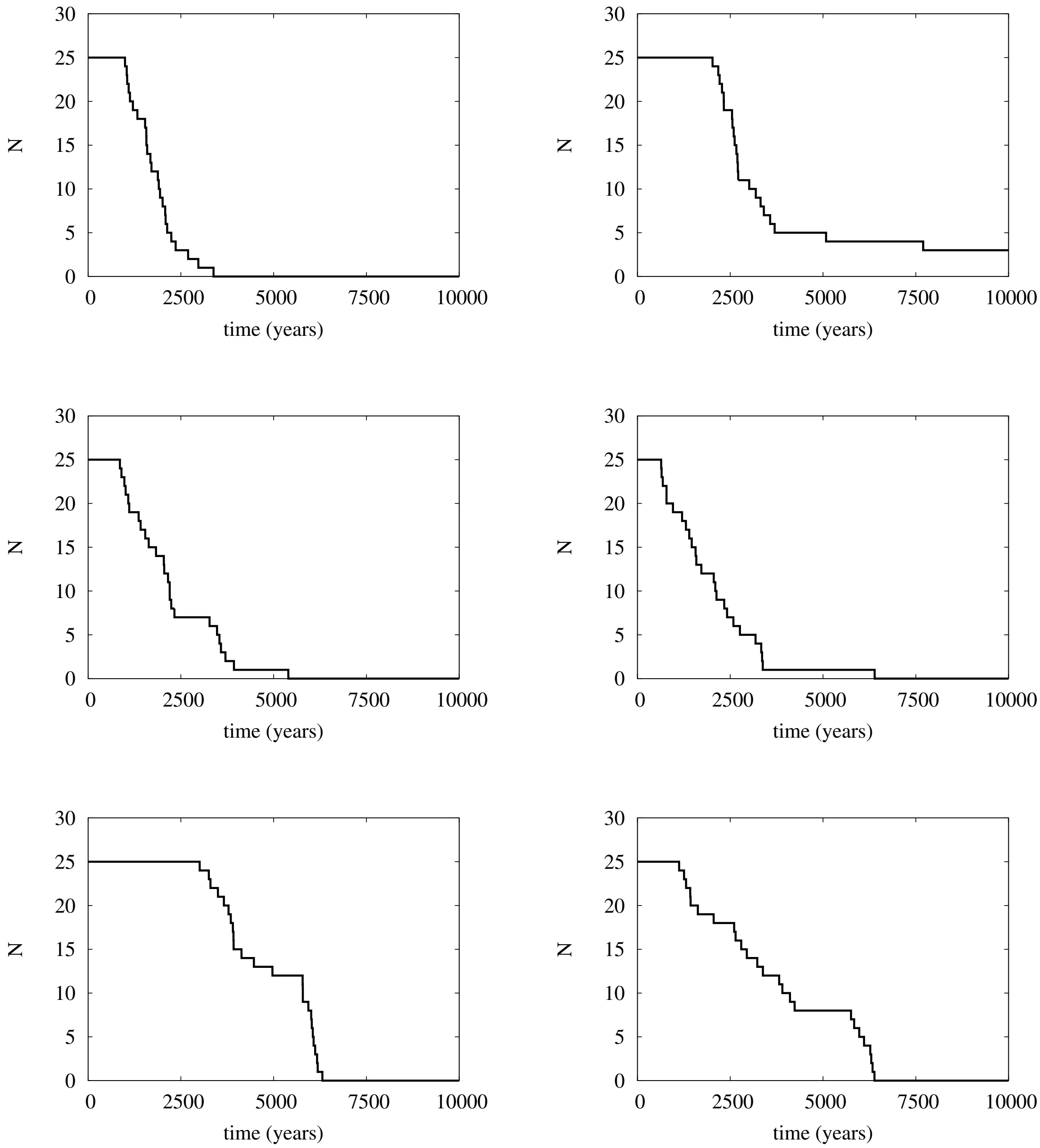

Figure 6. Histograms of number of remaining tadpole coorbitals of Enceladus as a function of the time: around L_4 (left-hand side) and around L_5 (right-hand side). In each case, Enceladus is located at a given resonance with Mimas: 4:5 (top), 5:6 (middle) and 6:7 (bottom).

Table 1. Maximum amplitude of oscillation of the semimajor axis due to first-order resonances between Mimas and Enceladus.

Resonance	Semimajor axis	Δa_{res}
1:2	0.629961	0.0814E-03
2:3	0.763143	0.1408E-03
3:4	0.825482	0.1869E-03
4:5	0.861774	0.2251E-03
5:6	0.885540	0.2584E-03

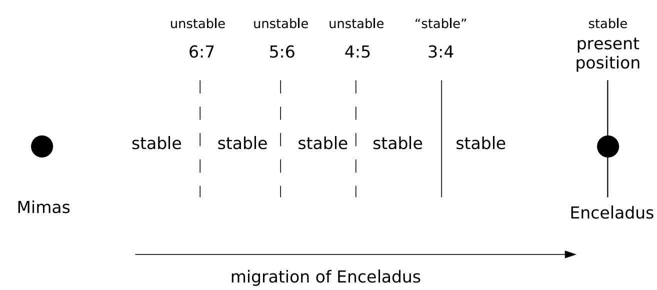

Figure 7. Diagram illustrating the stability of tadpole coorbital particles of Enceladus and Mimas. In this case, Enceladus is considered at different locations. Note that, in general, the particles are unstable only when Enceladus is placed in a first-order mean-motion resonance with Mimas.

The diagrams presented in Figs 7 and 8 summarize all these results. They represent the hypothetical configurations that could have ccurred along the orbital migration of Mimas and Enceladus indicating the first-order resonances they would have crossed.

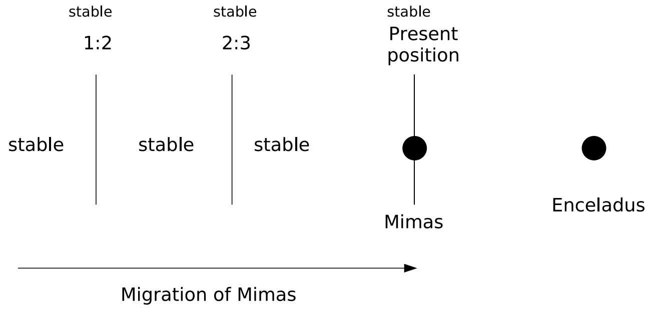

Figure 8. Diagram illustrating the stability of tadpole coorbital particles of Enceladus and Mimas. In this case, Mimas is considered at different locations. Note that the particles are stable even when Mimas is placed in a first-order mean motion resonance with Enceladus.

Therefore, if Mimas and Enceladus had any tadpole coorbital bodies in the past, they would have been lost in the case Mimas and Enceladus had crossed the mean motion resonances 4:5, 5:6 or 6:7.

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