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NOTE

A New Monohedral Hexamorphic Prototile

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In this paper we apply the fitting method and a result due to Martin (see ["Polynominoes—A Guide to Puzzles and Problems in Tiling," The Mathematical Association of American, New York, 1991] to get a hexamorphic prototile different from the only one we have known, presented by Fontaine and Martin in [*J. Combin. Theory Ser. A* **34** (1983), 119–121]. © 1997 Academic Press

By a monohedral tiling ϱ of the Euclidean plane we mean one in which each tile is congruent (directly or reflectively) to one fixed set P, called a prototile of ϱ. A prototile admitting—up to congruence—precisely r monohedral tilings is said to be r-morphic. For each positive integer r, $r \leqslant 10$, there exist a r-morphic prototile but the question about the existence of a r-morphic prototile for a prescribed value of r, if $r > 10$, remains open. Grünbaum and Shephard [7, 8], Harborth [9], and Fontaine and Martin [3–6] considered the possible numbers of distinct tilings admitted by a given prototile. To construct their trimorphic prototile Grünbaum and Shephard altered a pentagon by replacing its sides by zigzag fittings. Following them, Barbosa [1] developed a method for this transformation, and based on the difference among de enantiomorphic tilings we can construct dimorphic, trimorphic, and also original tetramorphic tiles [11, 12].

Our hexamorphic prototile (Fig. 1) is based on the transformation of a hypermorphic pentomino like Z, by using triangular, symmetric, and congruent fittings. (The Z-pentomino tiles the plane in infinitely many ways, if as usual both sides of the pentomino are allowed in the tilings. Indeed,

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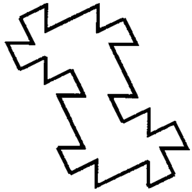

FIG. 1. An hexamorphic prototile obtained by using the fitting method.

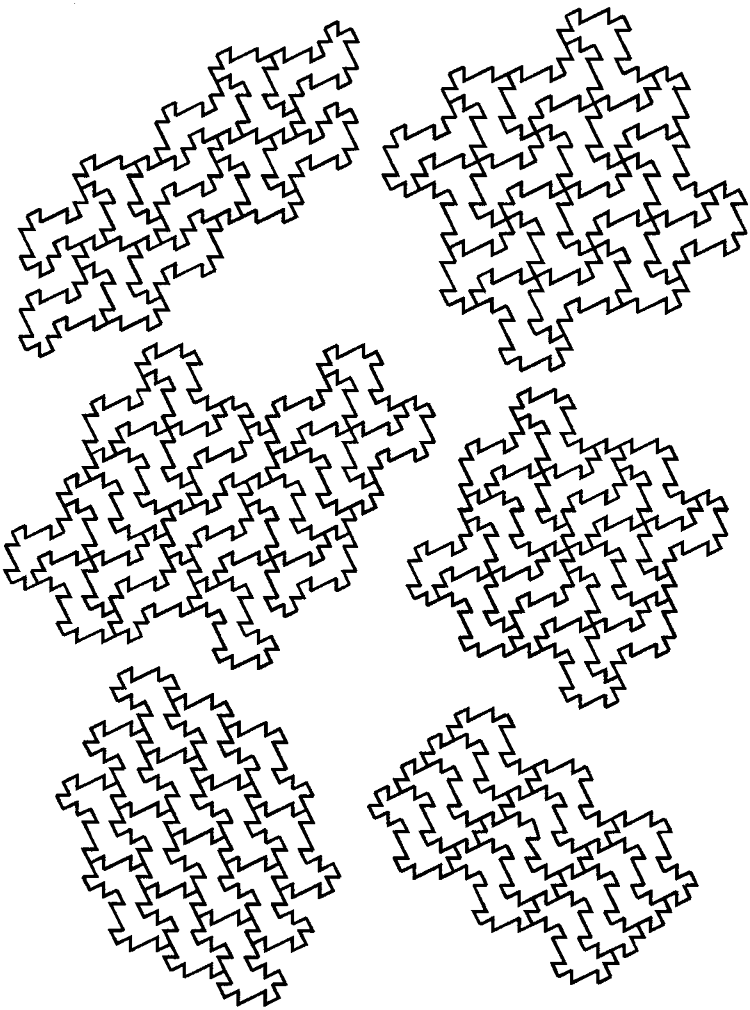

FIG. 2. The six tilings showed characterize the hypermorphic prototile.

the Z-pentomino tiles an infinite strip with parallel edges, having a symmetry that does not preserve the set of individual Z-pentominos in the strip. Therefore, copies of the strips can be pasted together in such way that we can form infinitely many tilings with the Z-pentomino.) The six tilings shown in Fig. 2 characterize the hypermorphic prototile. To show the hexamorphic condition we use the following remarkable property about the Z-pentomino: there are exactly six tilings of the plane that can be constructed without turning over the pentomino, that is, if only directly congruent copies of the Z-pentomino are allowed in the tilings [10]. A fundamental remark about the hexamorphic condition is that the transformed prototile preserves only the connections admitted for two direct congruent copies of basic prototile, since the reflection changes the fitting orientation. Thus, copies of infinite strips cannot be pasted together to form infinitely many tilings of the plane with direct and indirect replicas of the Z-transformed pentomino.

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