PHYSICAL REVIEW C VOLUME 45, NUMBER 3 MARCH 1992

Nuclear alignment: Classical dynamical model for the ^{238}U-^{238}U system

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(Received 12 September 1991)

Dynamical properties of the ^{238}U-^{238}U system at the classical turning point, specifically the distance of closest approach, the relative orientations of the nuclei, and deformations have been studied at the sub-Coulomb energy of $E_{\text{lab}}=6.07$ MeV/nucleon using a classical dynamical model with a variable moment of inertia. Probability of favorable alignment for anomalous positron-electron pair emission through vacuum decay is calculated. The calculated small favorable alignment probability value of 0.116 is found to be enhanced by about 16% in comparison with the results of a similar study using a fixed moment of inertia as well as the results from a semiquantal calculation reported earlier.

PACS number(s): 25.70.Bc, 24.10.−i

I. INTRODUCTION

The narrow peaks observed in positron spectra from the collision of two heavy nuclei at sub-Coulomb energies [1] are found to correspond to events where electrons and positrons are emitted back to back in the center of mass of the colliding system. The e^+ peaks are observed in a narrow range of bombarding energies at the Coulomb barrier (5.7–6.07 MeV/nucleon). Several possible explanations of the peak structure are discussed in Ref. [2]. Oberacker [3] has analyzed the effects of nuclear alignment on anomalous pair production using a semiquantal method for the ^{238}U-^{238}U system and found the probability for favorable alignment to be only about 10%. A purely classical dynamical study [4] of nuclear polarization effects, on the other hand, predicts the favorable alignment probability to be almost 30% larger than that predicted by Oberacker. In this paper, we present the results for the favorable alignment probability, calculated by using a classical dynamical model with a deformation-dependent semiclassical variable moment of inertia. Since at sub-Coulomb energies the excitation energy of a ^{238}U nucleus is only a few MeV, rotational degree of freedom turns out to be the most important collective mode of excitation. Using a variable moment of inertia, which is consistent with the energies of low-lying rotational energy states, one expects to account for the softness of the nucleus as it rotates. We also include the vibrational degree of freedom by including a deformation potential in the Hamiltonian.

II. FORMALISM

For the study of the dynamical evolution of the ^{238}U-^{238}U system, we consider the following Hamiltonian:

$$H_{\text{coll}}=T+V_n+V_c+V_\beta \ . \tag{1}$$

The kinetic energy T and the potential energy V are determined by eleven coordinates (q_ν, $\nu=1,\ldots,11$), that is, the coordinates of relative motion (R,Θ,Φ), the Euler angles defining the orientation of the intrinsic principal axes of the nucleus with respect to the laboratory frame, (θ_i,ϕ_i,ψ_i, $i=1,2$), and the quadrupole deformations (β_i, $i=1,2$) of the target and projectile nuclei. The orientation of colliding nuclei at the distance of closest approach is mainly determined by the infinite-range Coulomb force. By scattering of ^{238}U on ^{238}U at energies of $E_{\text{lab}}=6$ MeV/nucleon, one is probing only the outer part of the nuclear interaction potential V_n and it is seen to have little influence on the nuclear alignment already established at the turning point. As such, for determining the classical dynamical time evolution of the system under study, we use an expansion of the Coulomb interaction between homogeneous nonoverlapping charge distributions up to quadratic term in deformation parameter as given in Eq. (A17) of Ref. [5] and do not consider V_n. The deformation potential is given by

$$V_\beta=\tfrac{1}{2}\sum_{i=1}^{2} C_{\beta_i}(\beta_i-\beta_{0i})^2 \ . \tag{2}$$

The stiffness parameters C_β and collective mass parameters D_β defining the vibrational part of the kinetic energy are taken from microscopic vibrational model [6].

The rotational kinetic energy is determined by inertia parameters $\mathcal{J}(1)$ and $\mathcal{J}(2)$, associated with rotational degree of freedom, of the target and the projectile nuclei, respectively. We use the variable moment of inertia model [7] for the calculation of these parameters. In this model, for a given spin J the moment of inertia $\mathcal{J}_J$ of the nucleus is given by the real root of the cubic equation,

$$\mathcal{J}_J^3-\mathcal{J}_J^2\mathcal{J}_0-\sigma J(J+1)\mathcal{J}_0^3=0 \ , \tag{3}$$

where $\mathcal{J}_0$ is the ground-state moment of inertia and the parameter σ provides a measure of the softness of the nucleus. The values of parameters, $\mathcal{J}_0=67\hbar^2$/MeV and $\sigma=0.000\,91$, are taken from Ref. [7]. The variable moment of inertia model is mathematically equivalent to the two-parameter Harris model [8].

The time evolution of the coordinates is determined by

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the following set of classical Hamiltonian equations of motion:

$$\dot{q}_\nu = \frac{\partial H}{\partial p_\nu} \quad (\nu = 1, \ldots, 11) , \tag{4}$$

where p_ν are the momenta, conjugate to coordinates q_ν.

III. RESULTS AND DISCUSSION

For the numerical solution of coupled equations we have used the predictor-corrector method coupled to the Runge-Kutta approach. We ensure energy conservation up to two decimal places along the trajectory. The infinite range of Coulomb force implies a special care in the choice of the initial relative distance, R_i, between the target and projectile nuclei, at the time $t=0$. To get a quantitative estimate of the effect of the range of the Coulomb potential on the relevant coordinates at the distance of closest approach, we have studied the variation of these as a function of R_i, using a variable moment of inertia as well as a fixed moment of inertia for ^{238}U-^{238}U central collision ($l=0$). The calculation has been carried out for $E_{\rm lab} = 6.07$ MeV/nucleon, initial deformations ($\beta_{01} = \beta_{02} = 0.26$), and initial orientation angles $\theta_1 = \theta_2 = \theta_i = 40°$. The relative coordinate, R_c, the orientation angle of the nucleus with respect to the z axis, θ_c, and the deformation parameter, β_c, at the distance of closest approach between the target and the projectile have been plotted as a function of R_i as curve (1) in Figs. 1(a)–(c). We notice that the critical point values of R_c, β_c, and θ_c are quite sensitive to the choice of R_i. In particular, the range of variation of θ_c, as R_i varies from 25 to 700 fm, is very large. For example, the value of $\Delta\theta = \theta_c - \theta_i = 13.8°$ at $R_i = 700$ fm is almost twice as large as the $\Delta\theta$ value of 7° for $R_i = 30$ fm. Curve (2) shows a similar variation for the case when a fixed moment of inertia value of $\mathcal{J}_0 = 67\hbar^2/\text{MeV}$ (ground-state moment of inertia for ^{238}U) is used in the calculation. These results indicate that the initial relative distance should be larger than 300 fm or else the results obtained may be misleading. We choose $R_i = 700$ fm for the calculation of favorable orientation probability.

In Fig. 2(a), curves (1) and (2) show our results for the orientation angle at the classical turning point, θ_c, versus initial orientation, $\theta_i (= \theta_1 = \theta_2)$, for the ^{238}U-^{238}U central collision ($l=0$). These curves have been obtained using a parametrization as given in Ref. [4], that is, for the range $\theta_i = 0$ to $\pi/2$,

$$\theta_i = \theta_c + b\theta_c(\pi/2 - \theta_c) + c\theta_c(\pi/2 - \theta_c)^2 . \tag{5}$$

From our calculation with variable moment of inertia (VMI), we obtain the parameter values

$$b = \frac{-1.65}{\pi} , \tag{6}$$

$$c = \frac{1.86}{\pi^2} . \tag{7}$$

For the fixed moment of inertia calculation (FMI) we get

$$b = \frac{-1.83}{\pi} , \tag{8}$$

$$c = \frac{2.06}{\pi^2} . \tag{9}$$

Results from Ref. [3] and Ref. [4] are also shown, the pa-

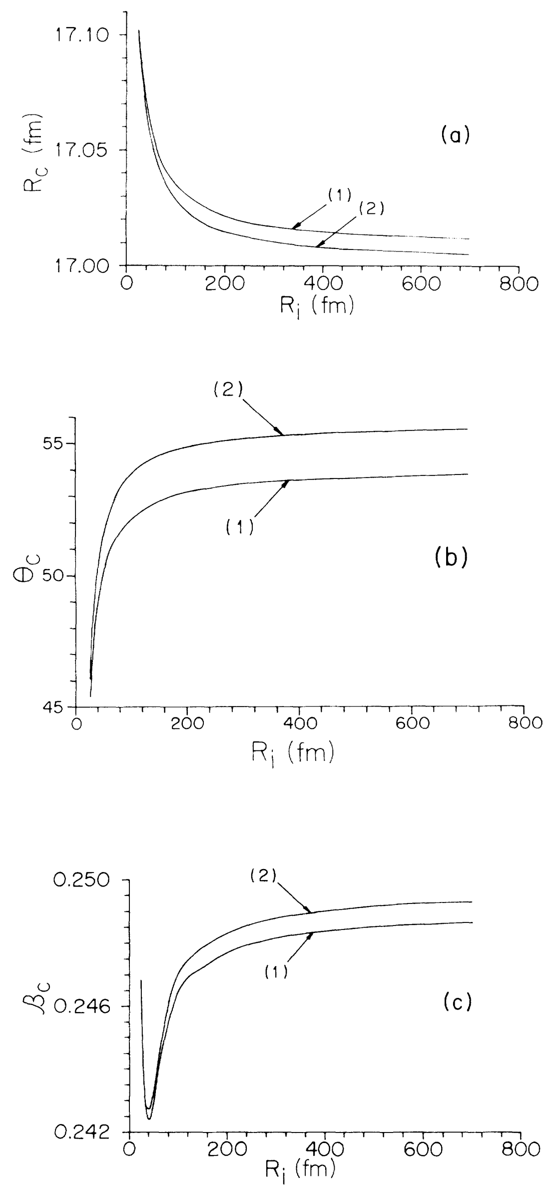

FIG. 1. (a) The distance of closest approach R_c vs the initial separation R_i between the target and the projectile nuclei for the ^{238}U-^{238}U central collision ($l=0$), at $E_{\rm lab} = 6.07$ MeV/nucleon. Initial orientation angle $\theta_1 = \theta_2 = \theta_i = 40°$ and initial deformation $\beta_1 = \beta_2 = \beta_i = 0.26$. Curve (1) is the variable moment of inertia calculation (VMI) while curve (2) is obtained by using a fixed moment of inertia (FMI). (b) The orientation angle at the classical turning point θ_c vs the initial separation R_i for the initial conditions as in (a). (c) The deformation at the classical turning point β_c vs the initial separation R_i for the initial conditions as in (a).

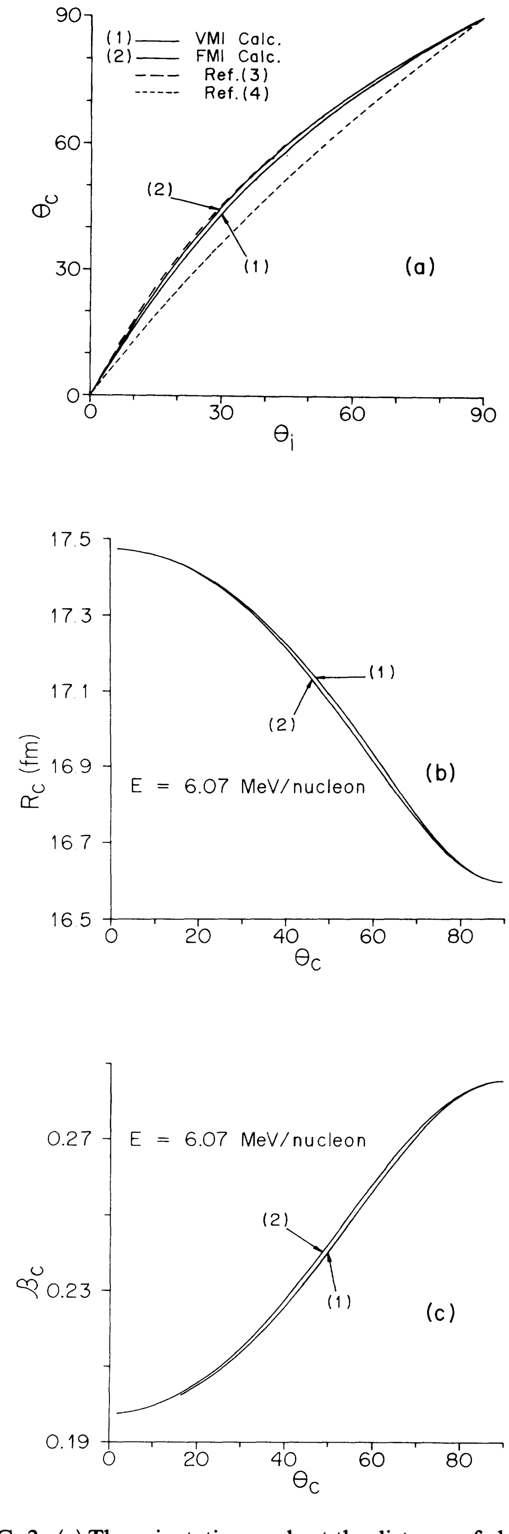

FIG. 2. (a) The orientation angle at the distance of closest approach θ_c vs the initial orientation, $\theta_i(=\theta_1=\theta_2)$, for the ^{238}U-^{238}U central collision ($l=0$), at $E_{\text{lab}}=6.07$ MeV/nucleon. The results are a plot of Eq. (5) with parameter sets described in the text. (b) The distance of closest approach R_c vs the orientation angle θ_c for initial orientations $\theta_i(=\theta_1=\theta_2)$ for the ^{238}U-^{238}U central collision ($l=0$). (c) The deformation at the distance of closest approach, β_c vs the orientation angle θ_c for initial orientations $\theta_i(=\theta_1=\theta_2)$ for the ^{238}U-^{238}U central collision ($l=0$).

rameters b and c for these having been taken from Ref. [4]. It is interesting to note that our FMI results, except for a small deviation at low values of θ_i, largely overlap those from the semiquantal calculation of Ref. [3]. The inclusion of variable moment of inertia is seen to diminish the maximum $\Delta\theta=15.5°$, at $\theta_i=40°$ for FMI by about 12%. As such, the favored orientation probability is expected to enhance somewhat when VMI is used. We may note that for $\theta_i=40°$ the average value of angular momentum associated with the rotational degrees of freedom at the turning point is $J_c \approx 18\hbar$ for the VMI calculation. The earlier conclusion that the nuclei tend to align themselves with $\theta_1=\theta_2=90°$ is confirmed.

Figures 2(b) and (c) present the distance of closest approach R_c and the deformation at the classical turning point β_c as a function of the orientation angle θ_c. A sensible degree of correlation between the orientation angle and the value of other system coordinates at the classical turning point is observed. The variation in the value of R_c, as θ_c varies from 0° to 90°, is of the order of 1 fm. The nuclear deformation, on the other hand, shows a strong dependence on θ_c being the largest for the belly-to-belly and nose-to-nose configurations of the target and the projectile nuclei, as observed in other calculations. The mutual polarizing effect of the Coulomb interaction between the target and projectile nuclei causes the multipole deformations at the classical turning point to be much different from the ground-state ones.

As in Ref. [4], we assume that for an initial orientation, θ_i, each nucleus has an orientation probability given by $dP=0.5\sin\theta_i\, d\theta_i$. Using Eq. [5], we obtain

$$dP=0.5\sin\theta_i\left[1+b\frac{\pi}{2}-2b\theta_c+c\left[\frac{\pi}{2}-\theta_c\right]^2 -2c\theta_c\left[\frac{\pi}{2}-\theta_c\right]\right]d\theta_c\ . \tag{10}$$

Our orientation probabilities, $dP/(\sin\theta_c\, d\theta_c)$, are plotted as a function of θ_c, along with those obtained from parametrization of the results [Eq. (5) and Eq. (10)] from Ref. [3] and Ref. [4], in Fig. 3. The relative probability of

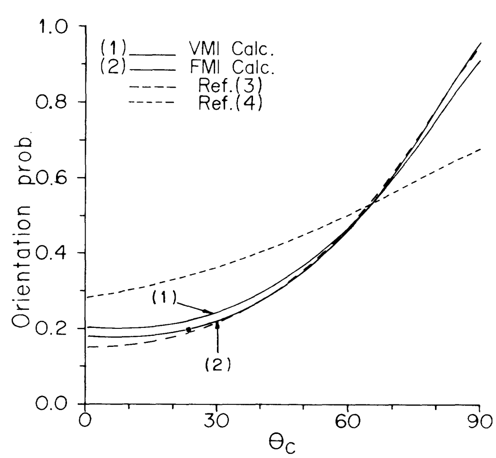

FIG. 3. Orientation probability, $dP/(\sin\theta_c\, d\theta_c)$ [calculated by using Eq. (10)] vs θ_c for initial orientations $\theta_i(=\theta_1=\theta_2)$ for ^{238}U-^{238}U central collision ($l=0$), at $E_{\text{lab}}=6.07$ MeV/nucleon.

0.91:0.20 from VMI calculation, for the occurrence of the belly-to-belly configuration and the nose-to-nose configuration, is smaller than the FMI prediction of 0.96:0.17 and the value 0.95:0.15 from Ref. [3]. It is, however, much larger than the estimated value 0.68:0.28 obtained in Ref. [4]. Considering that the favorable angles of approach for the nuclei must lie in the cones 0° to 30° and 145° to 180°, we integrate the orientation probability, Eq. (10), in these cones to get the total favorable alignment probability. It is found that our normalized result of 0.104 from FMI calculations, for one nucleus to be aligned favorably, matches closely with the value of about 0.1 reported by Oberacker, whereas the inclusion of a variable moment of inertia gives a value of 0.116 that is an enhancement of about 16% in comparison with the same. The VMI calculation prediction for favorable probability is, however, considerably lower than that given in Ref. [4].

We conclude therefore that the classical dynamical calculation with a deformation-dependent moment of inertia predicts a small enhancement in the favorable orientation probability at the classical turning point for the ^{238}U-^{238}U system. As a consequence, the predicted nuclear cross section for the e^+ peak in U+U is expected to be somewhat larger, but still much smaller than that from the experiment. A fixed moment of inertia calculation, on the other hand, is almost equivalent to the semiquantal calculation of Ref. [3]. These results are expected to also affect the cross-section estimates for the sub-Coulomb transfer of one neutron in ^{238}U-^{238}U, as these depend on the probability of occurrence at R_c of the various oriented configurations in collisions between various unpolarized ions.

S.S.S. acknowledges financial support from the CNPq.

[1] J. Schweppe, A. Gruppe, K. Bethge, H. Bokemeyer, T. Cowan, H. Folger, J. S. Greenberg, H. Grein, S. Ito, R. Schule, D. Schwalm, K. E. Stiebing, N. Trautmann, P. Vincent, and M. Waldschmidt, Phys. Rev. Lett. **51**, 2261 (1983); M. Clemente, E. Berdermann, P. Kienle, H. Tsertos, W. Wagner, C. Kozhuharov, F. Bosch, and W. Koenig, Phys. Lett. **137B**, 41 (1984); T. Cowan, H. Backe, M. Begemann, K. Bethge, H. Bokemeyer, H. Folger, J. S. Greenberg, H. Grein, A. Gruppe, Y. Kido, M. Kluver, D. Schwalm, J. Schweppe, K. E. Stiebing, N. Trautmann, and P. Vincent, Phys. Rev. Lett. **54**, 1761 (1985); H. Tsertos, F. Bosch, P. Kienle, W. Koenig, C. Kozhuharov, E. Berderman, S. Huchler, and W. Wagner, Z. Phys. A **326**, 235 (1987).

[2] *Physics of Strong Fields,* edited by W. Greiner (Plenum, New York, 1987).

[3] V. E. Oberacker, Phys. Rev. C **32**, 1793 (1985).

[4] R. K. Gupta and W. Scheid, Phys. Rev. C **36**, 1232 (1987).

[5] M. Münchow, D. Hahn, and W. Scheid, Nucl. Phys. **A388**, 381 (1982).

[6] W. Dudek, Acta Phys. Pol. B **5**, 725 (1974).

[7] M. A. J. Mariscotti, G. S. Goldhaber, and Brian Buck, Phys. Rev. **178**, 1864 (1969).

[8] S. M. Harris, Phys. Rev. **138**, B509 (1965).