

STOCHASTIC FIELD PROCESSES IN THE MATHEMATICAL
MODELLING OF DAMPED TRANSMISSION LINE VIBRATIONS

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Abstract. Wind-excited vibrations in the frequency range of 10 to 50 Hz due to vortex shedding often cause fatigue failures in the cables of overhead transmission lines. Damping devices, such as the Stockbridge dampers, have been in use for a long time for suppressing these vibrations. The dampers are conveniently modelled by means of their driving point impedance, measured in the lab over the frequency range under consideration. The cables can be modelled as strings with additional small bending stiffness. The main problem in modelling the vibrations does however lay in the aerodynamic forces, which usually are approximated by the forces acting on a rigid cylinder in planar flow. In the present paper, the wind forces are represented by stochastic processes with arbitrary crosscorrelation in space; the case of a Kármán vortex street on a rigid cylinder in planar flow is contained as a limit case in this approach. The authors believe that this new view of the problem may yield useful results, particularly also concerning the reliability of the lines and the probability of fatigue damages.

Keywords. Stochastic processes, transmission lines, vibrations

INTRODUCTION

Different vibration phenomena due to aerodynamic forces occur in overhead transmission lines. In the present paper, we only deal with vibrations caused by vortex shedding, which can often be observed in the frequency range of 10 to 50 Hz. Vibrations of the "galloping" type, which are of much lower frequency, are not considered.

Since approximately 1930 dampers of the Stockbridge type or of similar construction have been used successfully for the suppression of these vibrations, in previous papers we have studied the optimal tuning of the dampers and estimated the bending strains in the cable (Hagedorn, 1980, 1982). The weakest point in these and in similar calculations lies in the model of the wind forces, which are usually assumed as in a Kármán vortex street. Moreover, they are assumed all in phase with the local displacement of the cable section, and this leads to contradictions, as was shown by Schäfer (1984) and also explained by Hagedorn (1985a).

In the present paper, a different model is used for the wind forces: they are modelled by means of stochastic field processes. In this manner, the main physical properties of a vortex street can still be maintained, but the flow is no longer assumed to be planar. It is known, that even in careful laboratory experiments the vortices shed by a cylinder at relative axial distances of the order of a few diameters are not in phase but are almost uncorrelated, unless special precautions are taken. This fact can easily be included in the stochastic model by considering an appropriate crosscorrelation of the excitation process.

The final goal is of course to estimate the useful lifetime of the cables of a line, depending on the local weather conditions and on the damping devices used. If such estimates were available, the utility operating the line could for example evaluate if additional costs related to sophisti-

cated damping devices is or not economical. At the moment, the necessary data on the wind forces is not at hand, so that these calculations can not be carried out completely. The authors do however believe that the stochastic model here presented for the wind forces will be helpful in future calculations of transmission line vibrations.

THE MECHANICAL MODEL AND THE EXCITATION

In a typical transmission line the span is of the order of a few hundred meters, while the sag is of a few percent of the span only. The wavelengths of the vibrations under consideration being small compared to the span, the sag can be completely disregarded in first approximation and the cable can conveniently be represented by a beam under a high tensile force. Although the cable vibrations are not strictly planar they occur predominantly in a vertical plane. In what follows, we consider a single cable under steady transverse wind oscillating purely in the vertical plane, described by the differential equation

$$EI w''''(x,t) - T w''(x,t) + m \ddot{w}(x,t) = q(x,t) + d(w, \dot{w}, t) \quad (1)$$

In (1), $w(x,t)$ is the transverse displacement at the location x of the cable and at time t (see Fig. 1), m is the mass per unit length, $q(x,t)$ represents the forces acting on the cable due to the wind loading and vortex shedding and $d(w, \dot{w}, t)$ stands for the cable's self damping.

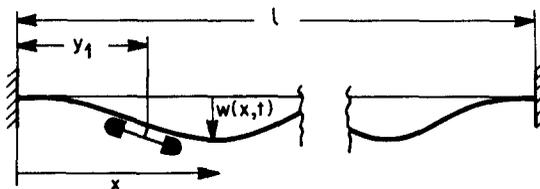


FIG. 1. Overhead transmission line with damper

The primes indicate differentiation with respect to x , while the dots stand for differentiation with respect to t . Equation (1) is valid for $x \neq l_1$; at this point the damper force has to be taken into account. The equation has to be solved for the boundary conditions

$$\begin{aligned} w(0,t) = 0, \quad w(l,t) = 0, \\ w'(0,t) = 0, \quad w'(l,t) = 0, \end{aligned} \tag{2}$$

which means that the suspension clamps are assumed as fixed during the vibrations. This is not necessarily so during the actual vibrations, but it certainly is a case which may occur in reality (due to symmetric spans for example) and it is therefore taken as a reference in the calculations. Only single cables will be treated in this paper and the case of bundled conductors will be omitted.

As shown in Hagedorn (1980), the bending stiffness in cables is very small, i. e. $EI/l_1^2 \ll 1$, so that the bending stiffness EI in (1) can be omitted in the calculation of the vibration levels. This considerably simplifies the problem, since the order of the differential equation is reduced (of course the boundary conditions in w' are dropped in this part of the calculation). The bending strains - which are responsible for material fatigue - can be calculated a posteriori. The maximum bending strains - which are proportional to the curvature - occur at the suspension clamps at the ends of the cable and at the damper clamp, and they can be calculated by means of simple formulae from the values of w' obtained for the string, i. e. with $EI = 0$, using singular perturbations. At $x = 0$, for example in first approximation the curvature is

$$w''_{EI}(0,t) = (l/EI) w'_0(0^+, t), \tag{3}$$

where the index zero refers to the string with zero bending stiffness and the index EI to the beam (see Hagedorn, 1980).

We will calculate the system's response using Green's function (Nascimento, 1984). For the string without damper, this function is easily obtained from the steady state solution of

$$-T \underline{w}''(x,t) + m \underline{\ddot{w}}(x,t) + d \underline{\dot{w}}(x,t) = \delta(x-y) e^{j\omega t}, \tag{4}$$

$$\underline{w}(0,t) = 0, \quad \underline{w}(l,t) = 0,$$

which is of course complex (here and in what follows we underline complex quantities); it is of the form

$$\underline{w}(x,t) = \underline{H}(y,x,\omega) e^{j\omega t}, \tag{5}$$

where $\underline{H}(y,x,\omega)$ is the frequency response for the damped string. In (4), linear external damping was assumed for the cable and this equation describes the problem of the vibrations of a damped string, forced by a concentrated unit force acting at $x = y$ with angular frequency ω .

Separation of variables leads to

$$\underline{H}''(y,x,\omega) + (m\omega^2 - j\omega d) \frac{1}{T} \underline{H}(y,x,\omega) = -\frac{1}{T} \delta(x-y) \tag{6}$$

and $\underline{H}(y,x,\omega)$ is given by

$$\underline{H}(y,x,\omega) := \begin{cases} A_1 e^{j\alpha x} + A_2 e^{-j\alpha x}, & 0 \leq x < y, \\ A_3 e^{j\alpha x} + A_4 e^{-j\alpha x}, & y < x \leq l, \end{cases} \tag{7}$$

where

$$\alpha := \sqrt{\frac{m\omega^2 - j\omega d}{T}} \tag{8}$$

and the A_i , $i = 1,2,3,4$ are obtained from

$$\begin{aligned} \underline{H}(y,0,\omega) = 0, \quad \underline{H}(y,l,\omega) = 0, \\ \underline{H}(y,y^-, \omega) = \underline{H}(y,y^+, \omega), \\ \underline{H}'(y,y^-, \omega) - \underline{H}'(y,y^+, \omega) = -1/T. \end{aligned} \tag{9}$$

The coefficients A_i , $i = 1,2,3,4$ are functions of y and ω , and the prime in (6) and (9) again denotes differentiation with respect to x . The function $\underline{H}(y,x,\omega)$ for $d = 0$ is depicted in Fig. 2.

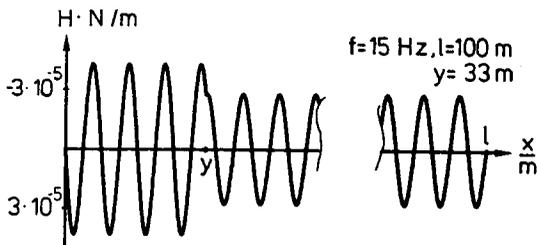


FIG. 2. Forced vibrations of a string, Green's function

For the computation of the vibrations of a cable with Stockbridge damper we do however need the frequency response not only for the cable, but for the cable including the damper. For the calculation of this function, consider the cable with damper of Fig. 1, executing steady state forced vibrations excited by a concentrated force $\hat{p} e^{j\omega t}$ with frequency ω acting at $x = y$.

If the damper force in these steady state oscillations is $\hat{p}_1 e^{j\omega t}$, then due to linearity the steady state solution $\underline{w}(x,t)$ can be written as

$$\underline{w}(x,t) = [\underline{H}(y_1,x,\omega)\hat{p}_1 + \underline{H}(y,x,\omega)\hat{p}] e^{j\omega t}, \tag{10}$$

the damper being located at $x = y_1$. Suppose now that the damper impedance $\underline{Z}(\omega)$ is known, then one has

$$\hat{p}_1 = -\underline{Z}(\omega)\hat{w}(y_1), \tag{11}$$

where $\hat{w}(y_1)$ is the complex velocity amplitude at $x = y_1$. Substituting

$$\underline{w}(x,t) = \hat{w}(x) e^{j\omega t} \tag{12}$$

in (10) and using $\hat{w}(y_1) = j\omega \hat{w}(y_1)$ in (11) gives

$$\hat{p}_1 = -\underline{Z}(\omega)j\omega [\underline{H}(y_1,y_1,\omega)\hat{p}_1 + \underline{H}(y,y_1,\omega)\hat{p}] \tag{13}$$

and therefore

$$\hat{p}_1 = -j \underline{Z}(\omega) \frac{\underline{H}(y,y_1,\omega)}{1 + j \underline{Z}(\omega)\underline{H}(y_1,y_1,\omega)} \hat{p} \tag{14}$$

and

$$\underline{w}(x,t) = [\underline{H}(y,x,\omega) - \frac{j\omega \underline{Z}(\omega) \underline{H}(y,y_1,\omega)}{1 + j\omega \underline{Z}(\omega) \underline{H}(y_1,y_1,\omega)} \underline{H}(y_1,x,\omega)] \hat{p} e^{j\omega t} \tag{15}$$

or

$$\underline{w}(x,t) = \underline{H}_D(y,x,\omega)\hat{p} e^{j\omega t}, \tag{16}$$

$$\underline{H}_D(y,x,\omega) = \underline{H}(y,x,\omega) -$$

$$\frac{j\omega \underline{Z}(\omega) \underline{H}(y,y_1,\omega)}{1 + j\omega \underline{Z}(\omega) \underline{H}(y_1,y_1,\omega)} \underline{H}(y_1,x,\omega). \tag{17}$$

The function $H_D(y,x,\omega)$ is exactly the frequency response for the cable with Stockbridge damper, i. e. it gives the cable's complex vibration amplitude at the location x for a unit exciting force with angular frequency ω at the location y , a damper with impedance $Z(\omega)$ being attached to the cable at the location y_1 (the argument y_1 is considered as a parameter only since it remains constant in most of the following calculations). $H_D(y,x,\omega)$ is shown in Fig. 3.

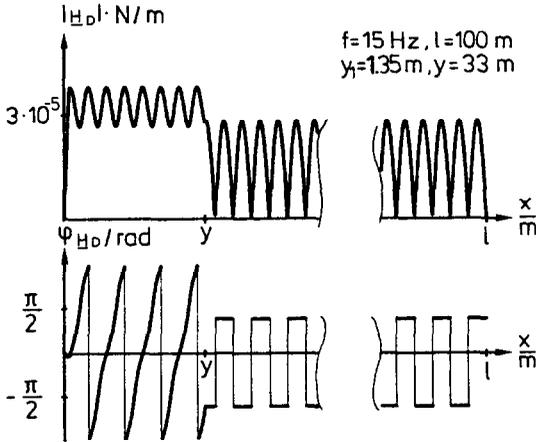


FIG. 3. Green's function for the cable with damper

Suppose now that in (1) an arbitrary distributed force $q(x,t)$ acts on the cable. If $Q(x,\omega)$ is the Fourier transform of $q(x,t)$, i. e.

$$q(x,t) \rightarrow Q(x,\omega), \quad (18)$$

then the Fourier transform of the corresponding solution $w(x,t)$ is

$$W(x,\omega) = \int_0^1 H_D(y,x,\omega) Q(y,\omega) dy \quad (19)$$

and $w(x,t)$ can easily be obtained as

$$w(x,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W(x,\omega) e^{j\omega t} d\omega = \frac{1}{\pi} \int_0^{\infty} \text{Re}[W(x,\omega) e^{j\omega t}] d\omega, \quad (20)$$

since $W(x,\omega)$ is the Fourier transform of a real function. On the other hand, if $h(y,x,t)$ is the response at the location x and at time t to a unit impulse at $t = 0$ at the location y , i. e. if

$$h(y,x,t) \rightarrow H_D(y,x,\omega) \quad (21)$$

then $w(x,t)$ can be obtained directly in the time domain as

$$w(x,t) = \int_{y=0}^1 \int_{\tau=-\infty}^{\infty} h(y,x,t-\tau) q(y,\tau) d\tau dy. \quad (22)$$

In the present paper however we wish to model the wind forces by means of an ergodic field process $q(x,t)$, which is completely characterized by its mean value

$$E[q(x,t)] = m_Q(x) := \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T q(x,t) dt \quad (23)$$

and its crosscorrelation function

$$E[q(y,t)q(x,\bar{t})] = k_Q(y,x,\tau) := \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T q(y,t)q(x,t+\tau) dt \quad (24)$$

with $\tau := \bar{t} - t$, for $0 \leq x, y \leq 1$ provided the process is Gaussian. The spectral-power-density $S_Q(y,x,\omega)$ can be used instead of $k_Q(y,x,\tau)$ to characterize the exciting process, it is related to the cross-correlation through

$$k_Q(y,x,\tau) \rightarrow S_Q(y,x,\omega). \quad (25)$$

For a known spectral-density $S_Q(y,x,\omega)$ of the wind forces and known frequency response $H_D(y,x,\omega)$ of the cable with damper, the cross-power-density $S_W(y,x,\omega)$ of the response process $w(x,t)$ can be obtained (Nascimento, 1984) as

$$S_W(y,x,\omega) = \int_0^1 \int_0^1 H_D^*(z_1,x,\omega) S_Q(z_1,z_2,\omega) H_D(z_1,y,\omega) dz_1 dz_2 \quad (26)$$

(the asterisk means "complex conjugate"). The correlation between the processes $w(x,t)$ and $w(y,t)$ is then

$$m_W(x,y) = \frac{1}{\pi} \int_0^{\infty} \text{Re}[S_W(x,y,\omega)] d\omega \quad (27)$$

and the variance of the displacement at the location x is

$$\sigma_W^2(x) = m_W(x,x). \quad (28)$$

In transmission line vibrations we are however not so much interested in the displacement amplitude, but more in the strain amplitudes, since these are directly related to the material fatigue. As explained previously, the bending strains can only be calculated if the bending stiffness is taken into account in the mathematical model of the cable, which was not the case in the frequency response calculated with (6). An excellent approximation to the bending strain is however given by

$$\epsilon = \frac{D}{2} w''_{EI}(0^+,t) = \frac{D}{2} \frac{\partial}{\partial t} w'_0(0^+,t) \quad (29)$$

with D as the cable's diameters (compare (3)), so that the variance of $w'(0,t)$ in the string can be used as a measure for the variance of the bending strains at the suspension clamp at $x = 0$. It can be calculated according to

$$\sigma_W^2(0) = \frac{1}{\pi} \int_0^1 \int_0^1 \int_0^{\infty} \text{Re} \left[H_D^*(z_1,0,\omega) S_Q(z_1,z_2,\omega) H_D'(z_2,0,\omega) \right] dz_1 dz_2 d\omega, \quad (30)$$

where the prime stands for differentiation with respect to the second argument of $H(\dots)$.

As shown by Nascimento (1984) the crosscorrelation function $k_Q(x,y,\tau)$ of the excitation process has to satisfy certain conditions which are automatically fulfilled if it is of the form

$$k_Q(x,y,\tau) = f(x)f(y)k(\tau) \frac{1}{\mu\sqrt{\pi}} \exp \left[-\frac{(x-y)^2}{\mu^2} \right], \quad (31)$$

where $f(x)$ is an arbitrary function representing the intensity of the process, $k(\tau)$ is an auto-correlation function of a suitable scalar process and μ has the meaning of coefficient representing the correlation in space. It was shown (Hagedorn and Nascimento, 1985) that this coefficient can have an important influence on the system's response in general and Nascimento (1984) considered different types of functions $f(x)$. In the present case of transmission lines the field of the aerodynamic forces can probably be considered as homogeneous in space, so that we set

$$f(x) := B = \text{const.} \quad (32)$$

Instead of specifying directly $k(\tau)$, it will be more convenient to specify its Fourier transform $\underline{S}(\omega)$. In the usual, deterministic model of transmission line vibrations the exciting forces are due to vortex shedding, so that in a realistic stochastic model the power density $\underline{S}(\omega)$ for a constant transverse wind velocity will be that of a narrow band process; for simplicity one can assume a spectral density as shown in Fig. 4.

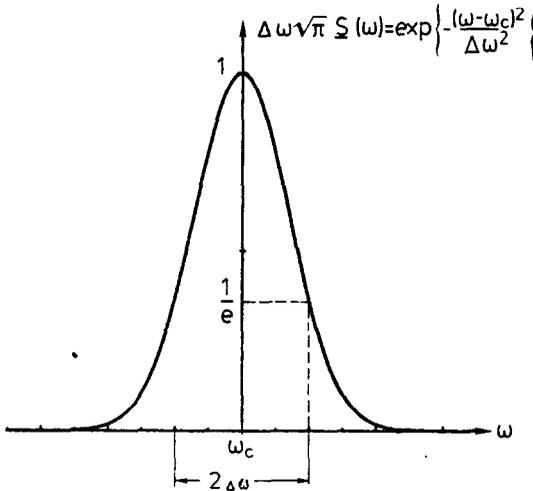


FIG. 4. Input spectral power density $\underline{S}(\omega)$

Its central frequency is chosen as the Strouhal frequency

$$\omega_c = 2 \pi f_c = 2 \pi 0.2 \frac{v}{D}, \quad (33)$$

D being the cable's diameter and v the wind speed, and the bandwidth $2\Delta\omega$ is probably of the order of a few percent of ω_c . Since the intensity of the wind forces can be represented by the constant B in (32), $\underline{S}(\omega)$ can always be normalized so that

$$\int_{-\infty}^{+\infty} \underline{S}(\omega) d\omega = 1.$$

Estimates of B can be obtained from the experiments carried out with rigid cylinders in planar flow (see Staubli, 1983, Farquharson and Mc Hugh, 1956), by comparing the power of the stochastic process to the power measured (for the maximal amplitudes) in the experiments. Little data is available on the correlation coefficient μ , it is however expected that it is of the order of a few cable diameters only. If these parameters are known for a certain wind speed, the variance of w' can be computed, and due to the assumptions mentioned the calculations simplify to the evaluation of

$$\sigma_{w'}^2(0) = B^2 \frac{1}{\pi \sqrt{\pi} \mu} \int_0^{\infty} \text{Re} \left[\int_0^1 \int_0^1 \underline{H}_D^*(z_1, 0, \omega) \exp \left[-\frac{(z_1 - z_2)^2}{\mu^2} \right] \underline{H}_D(z_2, 0, \omega) dz_2 dz_1 \right] \underline{S}(\omega) d\omega. \quad (34)$$

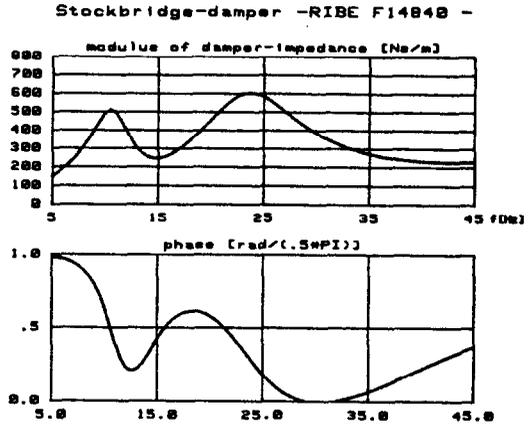


FIG. 5. Impedance of Stockbridge damper

THE SYSTEM'S RESPONSE, CONCLUSIONS

Calculations were performed for a damper of the type RIBE F14840 located at $y_1 = 1.35$ m with impedance as given in Fig. 5 and a cable of the Finch type with $D = 33$ mm, $m = 2.12$ kg/m, $EI = 52$ Nm². The span was $l = 400$ m and the tensile force $T = 35$ kN. The spectral density of the system's response was obtained for these data. From these results, information can also be gained on the cable's lifetime with regard to material fatigue. Details of the calculations as well as an evaluation of the results can be found in the paper by Schmidt and Hagedorn (1986).

The authors believe that this new view of the problem of wind-excited cable vibrations may yield useful results concerning the reliability of the lines and the probability of fatigue damage.

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