

Constraints on singlet right-handed neutrinos coming from the  $Z^0$  width

C. O. Escobar

*Instituto de Física da Universidade de São Paulo, 01498-970 C.P. 20516-São Paulo, São Paulo, Brazil*

O. L. G. Peres and V. Pleitez

*Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona, 145, 01405-900-São Paulo, São Paulo, Brazil*

R. Zukanovich Funchal

*Instituto de Física da Universidade de São Paulo, 01498-970 C.P. 20516-São Paulo, São Paulo Brazil*

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We study the constraints on masses and mixing angles imposed by the measured  $Z^0$  invisible width, in a model in which a singlet right-handed neutrino mixes with all the standard model neutrinos.

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If neutrinos are massive an important question to be answered concerns the way the  $Z$ -pole observables constrain their masses and mixing parameters. In particular the measured  $Z$  invisible width  $\Gamma^{\text{inv}}$  implies that the number of families is compatible with three. On the other hand, it is well known that this number need not be an integer number if right-handed neutrinos transforming as singlets under  $SU(2)_L \otimes U(1)_Y$  are added to the particle content of the theory.

Experimental searches for sequential neutral leptons beyond the three generations exclude stable Dirac neutrinos below 41.8 GeV and stable Majorana neutrinos below 34.8 GeV. For the unstable case these values are 46.4 and 45.1 GeV, respectively [1]. However, it is worth stressing that these limits are valid for sequential leptons and do not apply to the case of singlets of right-handed neutrinos.

Here we will consider the simplest extension of the standard electroweak model [2] with the addition of one right-handed singlet neutral fermion [3], resulting in four physical neutrinos, two of them massless and two massive ones ( $\nu_P, \nu_F$ ). It has been argued that this simple extension allows the implementation of a seesaw mechanism [4] which would explain the smallness of the known neutrinos masses. Hence, from the theoretical point of view, usually this mechanism is assumed. In fact, within this view it has been claimed that three generations and a single right-handed singlet are ruled out by experiments ( $Z$  invisible width and  $\tau$  decay) [5] and for this reason a fourth generation must be added to the standard model other than the singlet [6]. However, one should not be limited by the seesaw framework, as ultimately the question of neutrino masses will be settled by experiments.

In fact in this Rapid Communication three mass regions will be considered for the massive neutrinos: (i)  $m_P, m_F < M_Z/2$ , (ii)  $m_P < M_Z/2$ , and  $M_Z/2 < m_F < M_Z$ , and (iii)  $m_P < M_Z/2$ ,  $m_F > M_Z$ .

We will take into account current experimental results for the  $Z^0$  invisible width in the particular model of Ref. [3]. Let us first briefly review the model we work with.

The matter fields are those in the standard model plus

a singlet right-handed neutrino. In this simple extension of the standard model the most general form of the neutrino mass term is

$$\mathcal{L}_\nu^M = - \sum_{j=1}^3 a_j \bar{\nu}'_{jL} \nu'_R - \frac{1}{2} M \bar{\nu}'_R \nu'_R + \text{H.c.}, \quad (1)$$

where the primed fields are not yet the physical ones. In this model, there are four physical neutrinos  $\nu_1, \nu_2, \nu_P$ , and  $\nu_F$ ; the first two are massless and the last two are massive Majorana neutrinos with masses

$$m_P = \frac{1}{2}(\sqrt{M^2 + 4a^2} - M), \quad m_F = \frac{1}{2}(\sqrt{M^2 + 4a^2} + M), \quad (2)$$

where  $a^2 = a_1^2 + a_2^2 + a_3^2$ . In terms of the physical fields the charged-current interactions are

$$\mathcal{L}^{\text{CC}} = (\bar{\nu}_1 \bar{\nu}_2 \bar{\nu}_P \bar{\nu}_F)_L \gamma^\mu \Phi R \begin{pmatrix} e \\ \mu \\ \tau \\ 0 \end{pmatrix}_L W_\mu^+ + \text{H.c.}, \quad (3)$$

where  $\Phi = \text{diag}(1, 1, i, 1)$  and  $R$  is the matrix

$$\begin{pmatrix} c_\beta & -s_\beta s_\gamma & -s_\beta c_\gamma & 0 \\ 0 & c_\gamma & -s_\gamma & 0 \\ c_\alpha s_\beta & c_\alpha c_\beta s_\gamma & c_\alpha c_\beta c_\gamma & -s_\alpha \\ s_\alpha s_\beta & s_\alpha c_\beta s_\gamma & s_\alpha c_\beta c_\gamma & c_\alpha \end{pmatrix}. \quad (4)$$

In Eq. (4)  $c$  and  $s$  denote the cosine and the sine of the respective arguments. The angles  $\alpha, \beta$ , and  $\gamma$  lie in the first quadrant and are related to the mass parameter as follows:

$$s_\alpha = \sqrt{m_P/(m_P + m_F)}, \quad (5)$$

$$s_\beta = a_1/a, \quad c_\beta s_\gamma = a_2/a, \quad c_\beta c_\gamma = a_3/a. \quad (6)$$

Notice that there are four parameters; we will choose two angles and the two Majorana masses.

The neutral-current interactions written in the physical basis read

$$\mathcal{L}^{\text{NC}} = (\bar{\nu}_1 \bar{\nu}_2 \bar{\nu}_P \bar{\nu}_F)_L \gamma^\mu \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_\alpha^2 & ic_\alpha s_\alpha \\ 0 & 0 & -ic_\alpha s_\alpha & s_\alpha^2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_P \\ \nu_F \end{pmatrix}_L Z_\mu + \text{H.c.} \quad (7)$$

The Majorana neutrinos have the property that their diagonal vector and nondiagonal axial-vector couplings to the  $Z^0$  vanish.

For the masses in region (i), both massive neutrinos contribute to the invisible width of the  $Z^0$  which is given by [3]

$$\Gamma^{\text{inv}}(Z \rightarrow \nu's) = \Gamma_0(2 + c_\alpha^4 \chi_{PP} + 2c_\alpha^2 s_\alpha^2 \chi_{PF} + s_\alpha^4 \chi_{FF}), \quad (8)$$

where  $\Gamma_0$  is the width for each massless neutrino pair in the standard model:

$$\chi_{ij} = \frac{\sqrt{\lambda(M_Z^2, m_i^2, m_j^2)}}{M_Z^2} X_{ij}, \quad (9)$$

$\lambda$  is the usual triangular function defined by

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc),$$

and  $X_{ij}$  include the mass dependence of the matrix elements. Explicitly,

$$\begin{aligned} X_{PP} &= 1 - 4 \frac{m_P^2}{M_Z^2}, & X_{FF} &= 1 - 4 \frac{m_F^2}{M_Z^2}, \\ X_{FP} &= 1 - \frac{\Delta m_{FP}^2}{2M_Z^2} - \frac{m_P^2 + 3m_F m_P}{M_Z^2} - \frac{(\Delta m_{FP}^2)^2}{4M_Z^4}, \end{aligned} \quad (10)$$

where we have defined  $\Delta m_{FP}^2 = m_F^2 - m_P^2$ . Thus,  $\chi_{ij}$  are bounded by unity whereby

$$\Gamma^{\text{inv}}(Z \rightarrow \nu's) \leq 3\Gamma_0. \quad (11)$$

The number of neutrinos  $N_\nu$ , defined as  $N_\nu = \Gamma^{\text{inv}}/\Gamma_0$ , as determined from recent data [7] from the CERN  $e^+e^-$  collider LEP, is

$$N_\nu = 3.00 \pm 0.05. \quad (12)$$

Next, it is interesting to see how this value of  $N_\nu$  constrains the allowed values for the neutrino masses  $m_P, m_F$ .

Let us isolate the nonstandard contribution in Eq. (8), and restrain it to a region compatible with (12). With the help of Eqs. (5), (9), and (10) we can write the relation

$$\frac{1}{(x+y)^2} [x^2 F(y) + y^2 F(x) + 2xy G(x, y)] = 1.00 \pm 0.05, \quad (13)$$

where we have defined

$$F(\zeta) = (1 - 4\zeta^2)^{\frac{3}{2}}, \quad (14)$$

$$\begin{aligned} G(x, y) &= \sqrt{1 + (x^2 - y^2)^2 - 2(x^2 + y^2)} \\ &\times \left[ 1 - \frac{x^2 + y^2}{2} - 3xy - \frac{(x^2 - y^2)^2}{4} \right], \end{aligned}$$

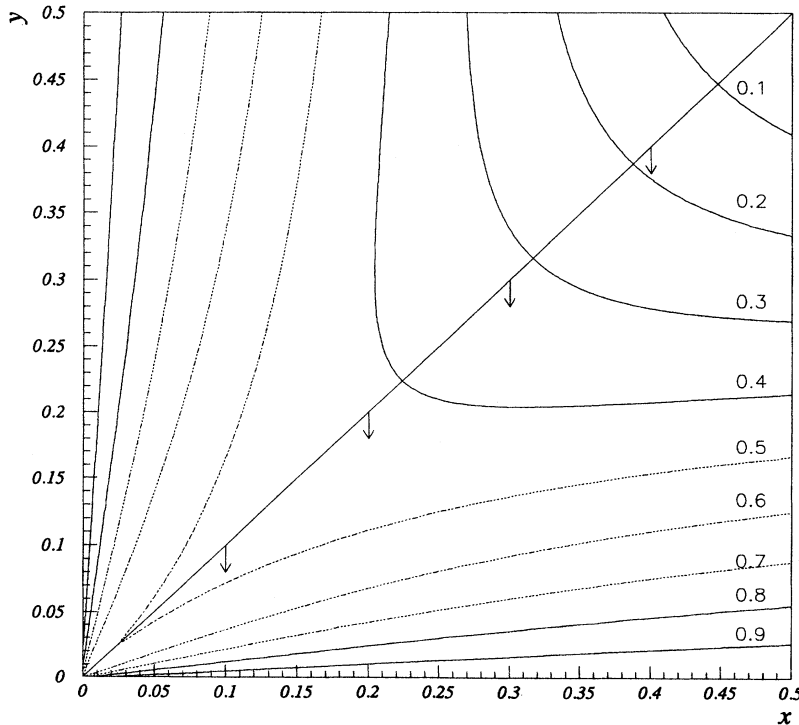


FIG. 1. Contour plot of function (13) where  $x = m_F/M_Z$  and  $y = m_P/M_Z$ . The 45° straight line limits the region where  $m_F > m_P$ .

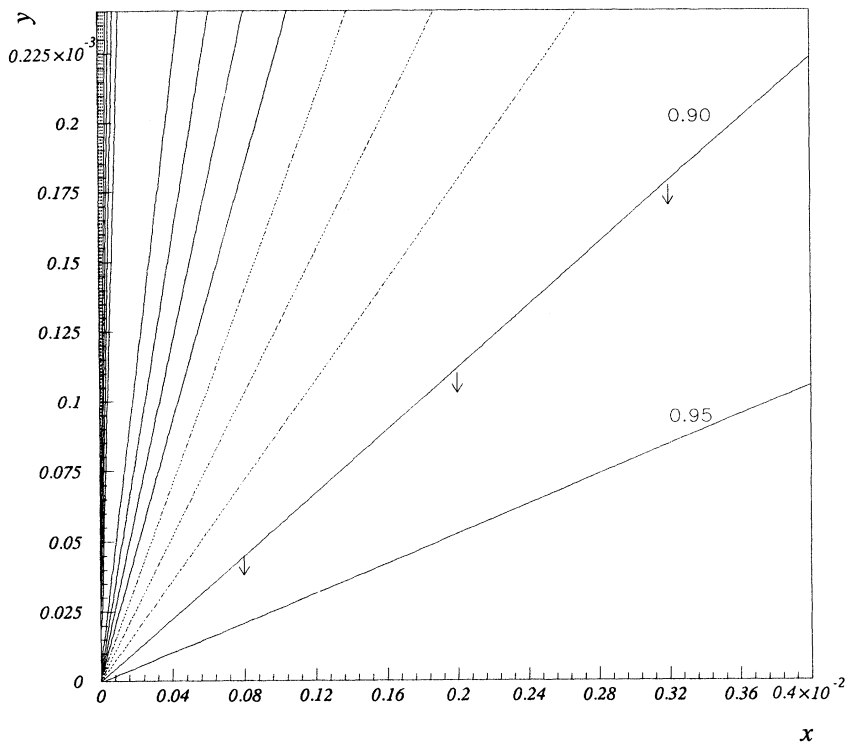


FIG. 2. This blowup of Fig. 1 shows the regions constrained by the measured  $Z^0$  invisible width within one and two standard deviations ( $1 - 0.05$ ,  $1 - 0.10$ ).

and we have used  $x = m_F/M_Z$  and  $y = m_P/M_Z$ .

We show in Fig. 1 the contour plot of the function on the left-hand side of Eq. (13). As can be seen from Eq. (2) one of the neutrinos must be heavier than the other. We choose the  $M$  parameter to be positive, i.e.,  $m_F > m_P$ . This limit appears in Fig. 1 as the region below the  $45^\circ$  straight line. Notice from Eq. (11) that in this type of model the neutrino counting will result in a number at most 3 even in the case, as here, of 3 families [3]. As a result the right-hand side of Eq. (13) should be limited from above by 1.00.

A blowup of Fig. 1 is shown in Fig. 2 highlighting the one and two standard deviation contours below the central value 1.00. From Eq. (5) for the mixing angle  $\alpha$ , we obtain  $y = x \tan^2 \alpha$ . This straight line has to lie below the before-mentioned contours in order to be consistent with experiments. This implies a constraint on the mixing angle  $\alpha$ ,  $\tan^2 \alpha < 0.055$ , i.e.,  $\sin \alpha < 0.23$ . In terms of the masses we have

$$m_F > 18.2 m_P. \quad (15)$$

In this model the mixing in the charged current is as

shown in Eqs. (3) and (4). From those couplings we see that  $\Gamma(\tau \rightarrow \nu_P e^- \bar{\nu}_e) \propto c_\alpha^2$  and  $\Gamma(\tau \rightarrow \nu_F e^- \bar{\nu}_e) \propto s_\alpha^2$ . It has been pointed out that in order to solve the puzzle of the leptonic  $\tau$  decays  $\sin \alpha$  must be between 0.1 and 0.3 [8]. Although this is compatible with our result we stress that as the two neutrinos can contribute to the  $\tau$  decay a more careful analysis must be done taking into account the respective phase space factors [9].

In the case of region (ii) all the masses in the kinematical region are allowed. For the third region (iii), we have found that the lightest neutrino mass has to be smaller than 9 GeV, which of course is not in conflict with the  $\nu_\tau$  mass limit.

In conclusion we see that singlet right-handed neutrinos need not necessarily come from a seesaw mechanism. In particular the region (i) has a rich phenomenology which deserves to be studied [9].

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