Chaos Suppression in NEMs Resonators by Using Nonlinear Control Design

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Abstract. In this work the chaotic behavior of a micro-mechanical resonator with electrostatic forces on both sides is suppressed. The aim is to control the system in an orbit of the analytical solution obtained by the Method of Multiple Scales. Two control strategies are used for controlling the trajectory of the system, namely: State Dependent Riccati Equation (SDRE) Control and Optimal Linear Feedback Control (OLFC). The controls proved effectiveness in controlling the trajectory of the system. Additionally, the robustness of each strategy is tested considering the presence of parametric errors and measurement noise in control.

Keywords: Optimal Control, SDRE Control, Chaos, MEMS.
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INTRODUCTION

Currently a great deal of research has been performed to report chaotic behavior in MEMS resonators [1, 2]. In order to drive the chaotic movement to a stable orbit, an Optimal Linear Feedback Control (OLFC) is used in [3], SDRE Control is used in [4] Robust Adaptive Fuzzy Control in [5], Fuzzy Sliding Mode Control Design in [6].

The SDRE strategy first proposed by [7] and later expanded by [8], was independently studied by [9] and alluded to by [10]. The SDRE strategy is an effective algorithm for synthesizing nonlinear feedback controls by allowing nonlinearities in the system states, while additionally offering great design flexibility through state-dependent weighting matrices [11].

The Optimal Linear Feedback Control was proposed by [12]. In [12] the quadratic nonlinear Lyapunov function was proposed to resolve the optimal nonlinear control design problem. The theorem formulated by [12] explicitly expresses the form of minimized functional and gives the sufficient conditions that allow using the Linear Feedback Control for nonlinear systems [13, 14].

The micromechanical resonator system studied in this work is depicted in Figure 1. Considering the device of Figure 1 as consisting of two fixed plates and a movable plate between them, to which is applied a voltage $V(t)$ composed of a polarization voltage *(DC)* V_p, and alternating voltage *(AC)* $V_i \sin(\omega t)$.

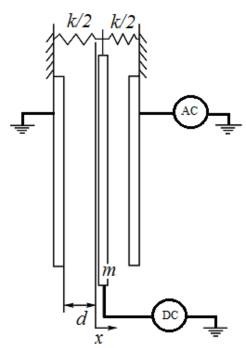

FIGURE 1. Micromechanical resonator.

where: d (distance between the plates), x (lateral movement), m (front panel mass), DC (polarization voltage V_p) and AC (alternating voltage $V_i \sin(\omega t)$).

MATHEMATICAL MODEL OF MICRO ELECTROMECHANICAL RESONATOR

The equation of motion of the plates is given by:

$$m\ddot{x} = -F_k - F_c + F_e \tag{1}$$

where: F_k is the conservative force of the spring, F_c the damping force of the elastic term and F_e the electric force.

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According to [4] the forces F_k and F_c can be defined as:

$$F_k = k_1 x + k_3 x^3 \tag{2}$$
$$F_c = c\dot{x} \tag{3}$$

According to [5, 15] the force F_e can be defined as:

$$F_e = \frac{1}{2}\frac{C_0}{(d-x)^2}\left(V_p + V_i\sin(\omega t)\right) - \frac{1}{2}\frac{C_0}{(d-x)^2}V_p^2 \tag{4}$$

where: C_0 (capacitance of the parallel-plate actuator).

Substituting (2), (3) and (4) in (1) we obtain the equation of motion:

$$m\ddot{x} + k_1 x + k_3 x^3 + c\dot{x} = + \frac{1}{2}\frac{C_0}{(d-x)^2}\left(V_p + V_i\sin(\omega t)\right) - \frac{1}{2}\frac{C_0}{(d-x)^2}V_p^2 \tag{5}$$

According to [5] the equation (5) can be represented in nondimensional form:

$$\ddot{u} + \mu\dot{u} + u + \alpha_3 u^3 = \gamma\left(\frac{1}{(1-u)^2} - \frac{1}{(1+u)^2}\right) + \frac{\sigma}{(1-u)^2}\sin(wT) \tag{6}$$

where: $w = \dfrac{\omega}{\omega_0}$, $T = \omega_0 t$, $\omega_0^2 = \dfrac{k_1}{m}$, $u = \dfrac{x}{d}$, $\mu = \dfrac{c}{m\omega_0}$, $\alpha_3 = \dfrac{k_3 d^2}{m\omega_0^2}$, $\gamma = \dfrac{C_0 \mathrm{V}_\mathrm{p}^2}{2m\omega_0^2 d^3}$, and $\sigma = \dfrac{2\gamma V_i}{V_p}$.

Rewriting equation (6) in state space:

$$\begin{aligned} y'_1 &= y_2 \\ y'_2 &= -\mu y_2 - y_1 - \alpha_3 y_1^3 + \gamma\left(\frac{1}{(1-y_1)^2} - \frac{1}{(1+y_1)^2}\right) + \frac{\sigma}{(1-y_1)^2}\sin(wT) \end{aligned} \tag{7}$$

where: $y_1 = u$ and $y_2 = \dot{u}$.

In Figure 2, the displacement, the phase portrait diagram, the Lyapunov exponent and the Poincare map are shown considering the parameters: $\mu = 0.01$, $\alpha_3 = 12$, $\gamma = 0.338$, $\sigma = 0.03558$ and $w = 0.5$.

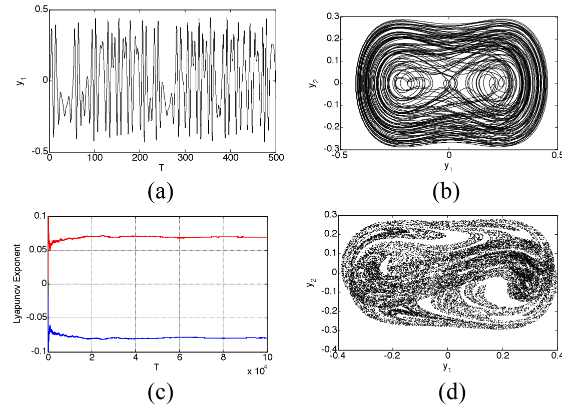


FIGURE 2. (a): The displacement of AFM without control. (b): Phase portrait of atomic force microscope. (c): Exponents of Lyapunov: $\lambda_1 = 0.0694$ and $\lambda_2 = -0.0794$. (d): Poincare map.

As can be observed in Figure 2c, the system has a positive Lyapunov exponent. The chaotic behavior also can be observed in the phase portrait in Figure 4b and Poincare map in Figure 4d.

ANALYTICAL APPROXIMATE SOLUTIONS OBTAINED THROUGH THE PERTURBATION METHOD

This procedure is used in order to obtain an approximate analytical solution [16]. Considering first the rational substitution of the term of the equation (6): $\left[\dfrac{1}{(1-u)^2} - \dfrac{1}{(1+u)^2}\right]$ and $\dfrac{1}{(1-u)^2}$ by a polynomial function: $P_4(u) = \delta_0 + \delta_1 u + \delta_2 u^2 + \delta_3 u^3 + \delta_4 u^4$, where $-0.5 \le u \le 0.5$.

According to [17] one can approximate the two functions by least squares method minimizing the error:

$$\int_{-0.5}^{0.5}\left[\left(f(u)\right) - \left(P_4(u)\right)\right]^2 du \tag{8}$$

Resulting in the following approximation:

$$\left[\frac{1}{(1-u)^2} - \frac{1}{(1+u)^2}\right] = 0.0674u^2 + 11.0263u^3 \tag{9}$$

$$\frac{1}{(1-u)^2} = 0.9585 + 1.8525u + 4.498u^2 + 6.426u^3 \tag{10}$$

Substituting (9) and (10) in (6) we obtain the following differential equation:

$$\ddot{u} + \mu\dot{u} + u + a_2u^2 + a_3u^3 = a_4u^3\sin(wT) + a_5u^2\sin(wT) + a_6u\sin(wT) + f\sin(wT) \tag{11}$$

where: $a_2 = -0.0674$, $a_3 = 0.9737$, $a_4 = 0.2286$, $a_5 = 0.1772$, $a_6 = 0.0659$, $f = 0.0341$ and $\mu = 0.01$.

Now, we will use the method of multiple scales to find analytically an approximate analytical solution to the above governing equation, this is done for a balance of order as follows. Therefore the equation is:

$$\ddot{u} + u + \varepsilon\mu\dot{u} + \varepsilon^2a_2u^2 + \varepsilon a_3u^3 = \varepsilon^2a_4u^3\sin(wT) + \varepsilon^2a_5u^2\sin(wT) + \varepsilon^2a_6u\sin(wT) + f\sin(wT) \tag{12}$$

Where ε is the parameter responsible for this balance [16]. Introducing the scales $T_0 = T$ and $T_1 = \varepsilon T$. Seeking solutions in the following way:

$$u = u_0(T_0, T_1) + \varepsilon u_1(T_0, T_1) + \ldots \tag{13}$$

As the original independent variable (time scale T) was substituted by independent scales T_0 and T_1, derivatives with respect to T should be expressed in terms of partial derivatives in respect of T_n such that:

$$\begin{aligned} \frac{d}{dT} &= D_0 + \varepsilon D_1 + \ldots \\ \frac{d^2}{dT^2} &= D^2{}_0 + 2\varepsilon D_0D_1 + \ldots \end{aligned} \tag{14}$$

Substituting (13) in (12) and considering the derivatives (14), (12) is represented in form perturbed:

$$\begin{aligned} &(D^2{}_0 + 2\varepsilon D_0D_1)(u_0 + \varepsilon u_1) + \varepsilon\mu(D_0 + \varepsilon D_1)(u_0 + \varepsilon u_1) \\ &+ (u_0 + \varepsilon u_1) + \varepsilon^2a_2(u_0 + \varepsilon u_1)^2 + \varepsilon a_3(u_0 + \varepsilon u_1)^3 \\ &- \varepsilon^2a_4(u_0 + \varepsilon u_1)^3\sin(wT_0) - \varepsilon^2a_5(u_0 + \varepsilon u_1)^2\sin(wT_0) \\ &- \varepsilon^2a_6(u_0 + \varepsilon u_1)\sin(wT_0) - f\sin(wT_0) = 0 \end{aligned} \tag{15}$$

Separating the terms in relation with the potential for ε^0 and ε^1 we have:

$$\varepsilon^0 : D_0^2u_0 + u_0 = f\sin(wT_0) \tag{16}$$

$$\varepsilon^1 : D_0^2u_1 + u_1 = -2D_0D_1u_0 - \mu D_0u_0 - a_3u_0^3 \tag{17}$$

one possible solution for (16) in polar form is:

$$u_0 = A(T_1)e^{iT_0} + \Lambda e^{iwT_0} + cc \tag{18}$$

where:

$$A = \frac{1}{2}ae^{i\beta} \text{ and } \Lambda = \frac{f}{2(1-w^2)} \tag{19}$$

substituting (18) in (17) we obtain:

$$\begin{aligned} &D_0^2u_1 + au_1 = -i(2A' + \mu A)e^{iT_0} + i(2\bar{A}' + \mu\bar{A})e^{-iT_0} \\ &- i\mu we^{iwT_0} + i\mu we^{-iwT_0} \\ &- a_3\left[A(T_1)e^{iT_0} + \Lambda e^{iwT_0} + \bar{A}(T_1)e^{-iT_0} + \bar{\Lambda}e^{-iwT_0}\right]^3 \end{aligned} \tag{20}$$

Eliminating the secular terms, of the equation (20) as follows:

$$2iA' + \mu iA + 3a_3(A\bar{A} + 2A^2)A = 0 \tag{21}$$

Substituting (19) into (21) and separating real and imaginary parts gives:

$$\begin{cases} a' = 0 \\ \beta' = \dfrac{3a_3a^2}{8} + \dfrac{3a_3a^2}{2}\cos(2\beta) \end{cases} \tag{22}$$

β is calculated numerically integrating equation (22). One possible solution for u is:

$$u = a\cos(T + \beta) + \frac{4f}{3}\sin(wT) \tag{23}$$

where: $a = 0.2$, $\beta = 0.9117$, $f = 0.0341$ and $w = 0.5$.

NONLINEAR CONTROL DESIGN

The objective is to determine a signal control U, which carries the system (6) from any initial state to final state:

$$e(\infty) = 0 \tag{24}$$

where:

$$e = \left[y_1 - \tilde{y}_1 \quad y_2 - \tilde{y}_2\right]^T \tag{25}$$

Where $\tilde{y}$ is the vector of desired orbits.

Application of Optimal Linear Feedback Control

The equations that describe the motion of the system with the control law U are described by the following nonlinear equations:

$$
\begin{aligned}
y'_1 &= y_2 \\
y'_2 &= -\mu y_2 - y_1 - \alpha_3 y_1^3 \\
&+ \frac{\left(4\gamma + 2\sigma\sin(wt) + y_1\sigma\sin(wt)\right)y_1}{\left(1 - y_1^2\right)^2} \\
&+ \frac{\sigma}{\left(1 - y_1^2\right)^2}\sin(wt) + U
\end{aligned} \tag{26}
$$

with:

$$U = \tilde{u}_o + u_{of} \tag{27}$$

Where u_{of} is the feedback control, and $\tilde{u}_o$ is the feedforward control, for optimal control, given by:

$$
\begin{aligned}
\tilde{u}_0 &= \tilde{y}'_2 + \mu\tilde{y}_2 + \tilde{y}_1 + \alpha_3\tilde{y}_1^3 \\
&- \frac{\left(4\gamma + 2\sigma\sin(wT) + \tilde{y}_1\sigma\sin(wT)\right)\tilde{y}_1}{\left(1 - \tilde{y}_1^2\right)^2} - \frac{\sigma}{\left(1 - \tilde{y}_1^2\right)^2}\sin(wT)
\end{aligned} \tag{28}
$$

Replacing (28) into (26) and considering the deviations (25) we obtain:

$$
\begin{aligned}
e'_1 &= e_2 \\
e'_2 &= -\mu e_2 - e_1 - \alpha_3(y_1 - \tilde{y}_1)^3 + \alpha_3\tilde{y}_1^3 \\
&+ \frac{\left(4\gamma + 2\sigma\sin(wT) + (y_1 - \tilde{y}_1)\sigma\sin(wT)\right)(y_1 - \tilde{y}_1)}{\left(1 - (y_1 - \tilde{y}_1)^2\right)^2} \\
&- \frac{\left(4\gamma + 2\sigma\sin(wT) + y_1\sigma\sin(wT)\right)\tilde{y}_1}{\left(1 - \tilde{y}_1^2\right)^2} \\
&+ \frac{\sigma}{\left(1 - (y_1 - \tilde{y}_1)^2\right)^2}\sin(wT) - \frac{\sigma}{\left(1 - \tilde{y}_1^2\right)^2}\sin(wT) + u_{of}
\end{aligned} \tag{29}
$$

Considering the system (29) written in the following way:

$$e' = Ae + G(e, \tilde{y}) + Bu_{of} \tag{30}$$

where:

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix},\ A = \begin{bmatrix} 0 & 1 \\ -1 & -\mu \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and}$$

$$
G(e, \tilde{y}) = \begin{bmatrix} 0 \\ -\alpha_3(y_1 - \tilde{y}_1)^3 + \alpha_3\tilde{y}_1^3 + \\ \dfrac{\left(\dfrac{4\gamma}{\sigma} + \sin(wT)(2 + (y_1 - \tilde{y}_1))\right)\sigma(y_1 - \tilde{y}_1)}{\left(1 - (y_1 - \tilde{y}_1)^2\right)^2} \\ -\dfrac{\left(4\gamma + 2\sigma\sin(wT) + y_1\sigma\sin(wT)\right)\tilde{y}_1}{\left(1 - \tilde{y}_1^2\right)^2} \\ +\left(\dfrac{\sigma}{\left(1 - (y_1 - \tilde{y}_1)^2\right)^2} - \dfrac{\sigma}{\left(1 - \tilde{y}_1^2\right)^2}\right)\sin(wT) \end{bmatrix} \tag{31}
$$

According to [13, 14], if there are an error weighted matrix Q, and the control weighted matrix R, positive definite symmetric matrix, and a matrix Riccati P, such that the matrix:

$$\tilde{Q} = Q - G^T(e, x^*)P - PG(e, x^*) \tag{32}$$

is positive definite matrix G restricted, then the control u_{of} is optimal and transfers the non-linear systems from any initial state, to the final state:

$$e(\infty) = 0 \tag{33}$$

minimizing the functional:

$$J = \int_0^\infty (e^T\tilde{Q}e + u_{of}{}^T R u_{of})dt \tag{34}$$

Then control u_{0f} can be found by solving the equation:

$$u_{of} = -R^{-1}B^T Pe = -ke \tag{35}$$

Since the symmetric matrix P, can be obtained from the Riccati algebraic equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{36}$$

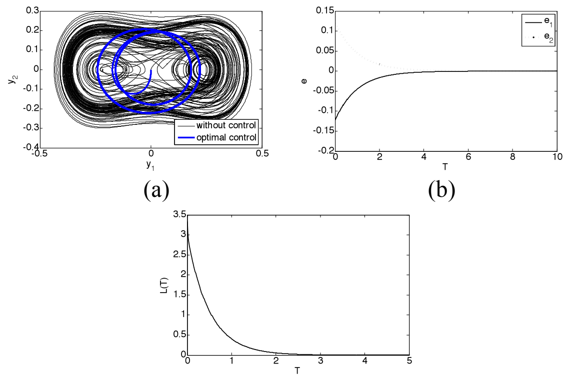
Using the matrices A and B (31), choosing:

$$Q = \begin{bmatrix} 10^2 & 0 \\ 0 & 10^2 \end{bmatrix} \text{ and } R = \left[10^{-2}\right] \tag{37}$$

and using the command $k = lqr(A, B, Q, R)$ from Matlab[r], we get:

$$u_{of} = -99.005e_1 - 100.975e_2 \tag{38}$$

For the optimal control verification (38), the function (32) is numerically calculated with $L(T)=e^{T}\widetilde{Q}e$ [13, 14]. The next figure shows the trajectory of the periodic function, considering the application of control, and the desired orbit ($\widetilde{y}_1$) the equation (23).



(a) (b)

(c)

FIGURE 3. (a): Phase portrait, chaotic (black) and controlled orbit (blue) (b): Signal deviations (c): $L(T)$ calculated in optimal

In Figure 3, it can be seen that the control was effective to move the system from a chaotic state to a periodic orbit (23).

Application SDRE Control

The dynamic system defined by equation (26) can be parameterized as a first order state equation and written in the state-dependent coefficient (SDC) and non state-dependent coefficient in the following way:

$$y' = A(y)y + BU_s + F(y,T) \tag{39}$$

Where $y=\begin{bmatrix} y_1 & y_2 \end{bmatrix}^T$ is state time dependent, $y' \in R^2$ is the vector of the first order time derivates of the states. $U_s = u_{sf} + \widetilde{u}_s$, where u_{sf} the feedback control, $\widetilde{u}_s$ is the feedforward control, and $F(y,T)$ is the nonlinear vector.

And the complete system by:

$$A(y)=\begin{bmatrix} 0 & 1 \\ -\alpha_1-\alpha_3 y_1^2+\dfrac{\left(4\gamma+2\sigma\sin(wT)+y_1\sigma\sin(wT)\right)}{\left(1-y_1^2\right)^2} & 0 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } F(y,T)=\begin{bmatrix} 0 \\ \dfrac{\sigma\sin wT}{\left(1-y_1^2\right)^2} \end{bmatrix} \tag{40}$$

A state feedback instead of output feedback is adopted to enhance the control performance. The cost function for the regulator problem is given by:

$$J=\frac{1}{2}\int_{t_0}^{\infty}\left[y^T Q(y)y + u_{sf}^T R(y)u_{sf}\right]dt \tag{41}$$

Where $Q(y)$ is semi-positive-definite matrix and $R(y)$ positive definite. Assuming full state feedback, the control law is given by:

$$u_{sf} = -R^{-1}(y)B^T(y)P(y)y = -k(y)y \tag{42}$$

The estate-dependent Riccati equation to obtain $P(y)$, is given by:

$$A^T(y)P(y)+P(y)A(y)-P(y)B(y)R^{-1}(y)B^T(y)P(y) + Q(y)=0 \tag{43}$$

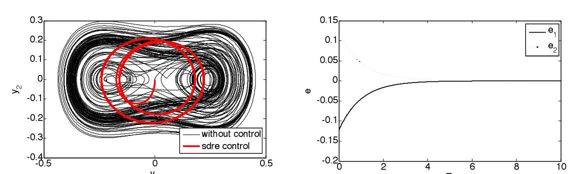
Defining the feedforward control as:

$$\widetilde{u}_s=\begin{bmatrix} 0 \\ -\dfrac{\sigma\sin wT}{\left(1-y_1^2\right)^2} \end{bmatrix} \tag{44}$$

Replacing (44) into (39), the system (39) can be represented in the form:

$$y' = A(y)y + Bu_{sf} \tag{45}$$

The next figure shows the trajectory of the periodic function considering the application of command $k = lqr(A,B,Q,R)$, using the matrices A and B (40), choosing Q and R (37), and desired orbit $\widetilde{y}_1$ (23).



(a) (b)

FIGURE 4. (a): Phase portrait, chaotic (black) and controlled orbit (red). (b): Signal deviations.

We can see in Figure 4, that the SDRE control also was effective to take the system from a chaotic state to a periodic orbit (23).

Control System Behavior In The Presence Of Parametric Errors And Measurement Noise

The parameters used in the control were obtained from a data set. The data set provided parametric errors, as measurement errors or model uncertainties. To consider the effect of parameter uncertainties on the performance of the controller *U*, the parameters used in the control will be considered as a random error of ±20% [18].

To consider the effect of measurement noise on the performance of the controller, a sinusoidal noise with random frequency and amplitude of noise is added:

$$e_{noise} = \psi(T)\sin(\varpi T) \tag{46}$$

$\psi(T) = 0.05 \pm 0.002 * r(T)$, $\varpi(T) = 0.5 \pm 0.02 * r(T)$ and $r(t)$ are normally distributed random functions.

In Figure 5, we observe the robustness of the control to maintain the system in the desired orbit (23).

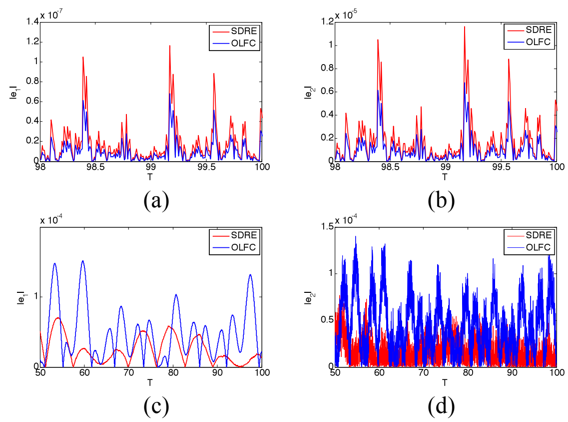


FIGURE 5. (a): Signal deviations $|e_1|$ without parameter uncertainties and measurement noise, (b): Signal deviations $|e_2|$ without parameter uncertainties and measurement noise. (c): Signal deviations $|e_1|$ with parameter uncertainties and measurement noise. (d): Signal deviations $|e_2|$ with parameter uncertainties and measurement noise.

Can be seen in Figure 5 the control (CLFO) proved to be most indicated for case of the control was not subject to uncertainties. In the case of the control subject to uncertainties SDRE control proved to be indicated.

CONCLUSIONS

Two control strategies were used, suppressing the chaotic trajectory and leading the system to a desired periodic orbit, obtained by the application of the multiple scales method. A comparison of the obtained results showed that both controls are efficient. An interesting contribution of these controls is that they do not need linearization or lose the nonlinearity of the considered systems and show the robustness of the controls when the system has measurement noise.

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