Abstract. Non-pressure compensating drip hose is widely used for irrigation of vegetables and orchards. One limitation is that the lateral line length must be short to maintain uniformity due to head loss and slope. Any procedure to increase the length is appropriate because it represents low initial cost of the irrigation system.

The hypothesis of this research is that it is possible to increase the lateral line length combining two points: using a larger spacing between emitters at the beginning of the lateral line and a smaller one after a certain distance; and allowing a higher pressure variation along the lateral line under an acceptable value of distribution uniformity. To evaluate this hypothesis, a nonlinear programming model (NLP) was developed. The input data are: diameter, roughness coefficient, pressure variation, emitter operational pressure, relationship between emitter discharge and pressure. The output data are: line length, discharge and length of the each section with different spacing between drippers, total discharge in the lateral line, multiple outlet adjustment coefficient, head losses, localized head loss, pressure variation, number of emitters, spacing between emitters, discharge in each emitter, and discharge per linear meter.

The mathematical model developed was compared with the lateral line length obtained with the algebraic solution generated by the Darcy-Weisbach equation. The NLP model showed the best
results since it generated the greater gain in the lateral line length, maintaining the uniformity and the flow variation under acceptable standards. It had also the lower flow variation, so its adoption is feasible and recommended.

**Keywords.** emitters spacing, trickle irrigation, emission uniformity, optimization.
Introduction

In drip irrigation systems water is applied directly in the root system region, with high efficiency, but this system has the disadvantage of possible emitters clogging and its installation has a high cost (Mantovani et al., 2009). Basically the emitters can be compensating or non-pressure compensating. The compensating drippers provide constant flow rate under pressure variations along the lateral line, allowing longer lengths but they are more expensive. Using non-pressure compensating emitters, the flow rate decreases as the pressure is reduced, resulting in shorter lateral lines in order to obtain the desired uniformity.

Non-pressure compensating drip hose is widely used for irrigation of vegetables and orchards. One limitation of this kind of emitter is the lateral line length must be short to maintain uniformity due to head loss and slope.

It is important to study procedures and criteria to obtain longer lateral lines when using non-pressure compensating emitters. Andrade (2009) considered it is possible to extend the lateral line length using two emitters spacing in different section. He assumed that the spacing changing point would be at 40% of the total length, because this is approximately the location of the average flow according with Bliesner & Keller (1990). Talens (2002) found that, for practical purposes, the average pressure is located at 40% of the length of the lateral line and that until this point it has already consumed 75% of total head loss (hf).

Wu (1997) proposes the use of a 30% $\Delta q$ and he found that this value resulted in a distribution uniformity over 80%. Andrade (2009) hypothesized that the use of two spacing between emitters would be an alternative to get longer lateral line. He adopted the spacing changing point at 40% of the total lateral line length, which is not necessarily the best solution. In this case, the system design consists in the determination of the two emitters spacing utilized and the changing point between spacing.

In the non-pressure compensating emitters in the design usually is adopted a flow variation ($\Delta q$) of 10% and a corresponding pressure variation ($\Delta H$) of 20%, allowing uniformity distribution between 95 and 98% (Wu, 1997; Talens, 2002). It is possible to obtain uniformity coefficient higher than 80% even if discharge variation of 30% is used (Wu, 1997). To evaluate the irrigation uniformity two indicators can be used: distribution uniformity (DU) which is the ratio between the average 25% lower flow values and the average, expressed as a percentage (Clemmens & Solomon, 1997; Styles et al., 2008); and the emission uniformity (EU), which considers the emitters characteristics and the operational unit hydraulic configuration (Marcussi & Saad, 2006).

The design criterion for non-pressure compensating drip hose is normally to have 10% of flow variation ($\Delta q$) in the lateral line, corresponding to 20% of head pressure variation ($\Delta H$). Longer lateral lines in drip irrigation systems using conventional drippers provide cost reduction, but it is necessary to obtain the uniformity of irrigation (Andrade, 2009). The use of $\Delta q$ higher levels can provide longer lateral lines.

The lateral line head loss (hf) determination can be performed by the empirical equations of Hazen-Williams or Darcy-Weisbach. It is important to estimate the localized head loss at dripper ($hfe$), which is integrated inside the hose, reducing the flow section and causing a partial flow obstruction (Andrade, 2009; Rettore Neto et al., 2009).

The emitter flow can be characterized empirically as a function of the operational pressure, according to equation 1 (Howell and Hiler, 1974; Howell et al., 1983).
\[ q = K \ H^x \]  \hfill (1)

where:
- \( q \) = emitter flow (L h\(^{-1}\));
- \( K \) = proportionality factor;
- \( H \) = emitter pressure (m.c.a.);
- \( X \) = exponent of flow which characterizes the flow regime.

The design should be optimized and it can be obtained with the use of mathematical optimization models based on Operations Research techniques, as it is the case of Nonlinear Programming (NLP).

Maximizing the lateral line length with two spacing and the definition of spacing changing point is typically an optimization problem that can be characterized and solved by a nonlinear programming model.

This study aimed to evaluate the possibility of increasing the lateral line length of an irrigation system using non-pressure compensating drip hose with different spacing between emitters but, maintaining irrigation uniformity at appropriate levels. For this, a comparison was carried out between the NLP model and the usual design procedure.

**Methodology**

A mathematical model using Nonlinear Programming was developed for comparison with the usual methodology to define the lateral line maximum length, which is based on the Darcy-Weisbach. As an example a commercial non-pressure compensating drip hose was adopted, the characteristics are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service pressure (m.c.a.)</td>
<td>10</td>
</tr>
<tr>
<td>Minimum pressure (m.c.a.)</td>
<td>6</td>
</tr>
<tr>
<td>Pipeline diameter (mm)</td>
<td>16</td>
</tr>
<tr>
<td>K coefficient</td>
<td>0.46297</td>
</tr>
<tr>
<td>Exponent (x)</td>
<td>0.503</td>
</tr>
<tr>
<td>Drip hose section with dripper (mm²)</td>
<td>188.73</td>
</tr>
<tr>
<td>Emitter coefficient of manufacturing variation (Cv)</td>
<td>0.0353</td>
</tr>
</tbody>
</table>

The developed model is the total lateral length maximization using two spacing in different sections as described in the objective function (eq. 2).

\[ \text{Máx} \ldots L = t_1 + t_2 \]  \hfill (2)
where:
\( t_1 \)- length of the lateral line section using dripper spacing 1, m;
\( t_2 \)- length of the lateral line section using dripper spacing 2, m.

For each section the model provides a pressure variation (\( \Delta h_{1} \) and \( \Delta h_{2} \)) and the sum of them must be equal to the \( \Delta h \) informed on the input data. With these data it is possible to obtain the head loss in each section, \( h_f \), which is used to obtain the length by the equations 3 and 4.

\[
h_{f1} = \left( \frac{10,646 \cdot Q_1^{1,85} \cdot C^{1,85}}{D^{4,87}} \cdot (t_1 + t_2) \right) - \left( \frac{10,646 \cdot Q_2^{1,85} \cdot t_2 \cdot f_2}{C^{1,85} \cdot D^{4,87}} \right)
\]

\[
h_{f2} = \left( \frac{10,646 \cdot Q_2^{1,85} \cdot t_2 \cdot f_2}{C^{1,85} \cdot D^{4,87}} \right)
\]

where:
\( Q_1 \)= total lateral line discharge (L h\(^{-1}\));
\( Q_2 \)= section 2 discharge (L h\(^{-1}\));
\( f_1 \)= multiple outlet adjustment coefficient for the total lateral line;
\( f_2 \)= multiple outlet adjustment coefficient for the second section.

The data used in the calculations are shown in Table 2. As the model in GAMS ® uses nonlinear equations is necessary the definition of the allowable variation of some variables in order to avoid division by zero during the calculations (Table 3).

Table 2. Input data for the GAMS developed model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Valor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (mm)</td>
<td>16</td>
</tr>
<tr>
<td>Friction coefficient (Hazen-Williams equation)</td>
<td>140</td>
</tr>
<tr>
<td>Pressure variation (%)</td>
<td>20 e 40</td>
</tr>
<tr>
<td>Inlet pressure (m.c.a.)</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 3. Range of variables used in the Gams model.

<table>
<thead>
<tr>
<th>Variable (por trecho)</th>
<th>Lower limit</th>
<th>Upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge (m(^{3})s(^{-1}))</td>
<td>2.78x10(^{-07})</td>
<td>2.78x10(^{04})</td>
</tr>
<tr>
<td>Lateral line section length (m)</td>
<td>1</td>
<td>1000</td>
</tr>
<tr>
<td>Multiple outlet adjustment coefficient</td>
<td>0.3</td>
<td>1</td>
</tr>
<tr>
<td>Dripper number</td>
<td>1</td>
<td>10000</td>
</tr>
<tr>
<td>Dripper spacing (m)</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>Discharge per linear meter (m(^{3})s(^{-1}))</td>
<td>8.33x10(^{07})</td>
<td>1.39x10(^{06})</td>
</tr>
</tbody>
</table>

The model provides several output data: lateral line total length, flow and length of each lateral line section with different spacing between emitters, total number of emitters, emitter spacing, flow rate per linear meter, head loss, pressure variation, multiple outlet adjustment coefficient, discharge in each emitter, and emitter average discharge.
The NLP model was compared with the usual procedure for lateral line length estimation, which consists in calculating the maximum length of the lateral line from Darcy-Weisbach equation associated with Blasius equation. In the case of non-compensating emitters used in orchards, the lateral lines works on level. Thus, any pressure variation is due to the total head loss (in the pipeline and located in the emitters). The lateral line diameter adopted in this study is 16 mm, the most used commercially. Thus, the lateral line length becomes the only variable to be defined.

The software GAMS was selected to solve the NLP model by its great number of solvers for nonlinear problems.

The usual procedure to determine the lateral line is based on the equations of Darcy-Weisbach and Blasius, considering \( Q = \frac{L}{Se} \). \( Q \) and making appropriate mathematical arrangements, obtaining the equation 5.

\[
L = \left( \frac{12811650554 \cdot hf'}{q^{1.75} \cdot F \cdot Se^{1.75} \cdot D^{4.75}} \right)^{\frac{1}{2.75}}
\]  

(5)

where:
- \( Hf' \) = total head loss in the lateral line;
- \( Se \) = emitters spacing (m);
- \( D \) = pipe diameter (m);
- \( q \) = emitter discharge (L h\(^{-1}\));
- \( F \) = Multiple outlet adjustment coefficient.

To compare the NLP model and the usual procedure, two pressure variations were used, 20 and 40%.

**Results and Discussion**

The NLP model allowed the lateral line design using two sections with different spacing. The usual method calculated the lateral line to a single spacing between drippers (0.4 m).

The emitter average flow was located at 37.85 and 38.64% of lateral line length, with head loss until this point of 73.61 and 75.20% of the total lateral line head loss for the pressure variation of 20 and 40%, respectively (Table 4). These values are consistent with those found by Talens (2002).
Table 4. Average flow location and head loss until the average flow location for the evaluated procedures and pressure variations.

<table>
<thead>
<tr>
<th>( \Delta H ) (%)</th>
<th>GAMS®</th>
<th>Conventional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average flow location (% of L)</td>
<td>Cumulative head loss at average flow location (% of total head loss)</td>
</tr>
<tr>
<td>20</td>
<td>38.64</td>
<td>74.10</td>
</tr>
<tr>
<td>40</td>
<td>37.85</td>
<td>73.61</td>
</tr>
</tbody>
</table>

The \( \Delta h \) and \( \Delta q \) values found agree with those obtained by Wu & Yue (1993), who suggest that \( \Delta q \) can be related to \( \Delta h \), obtaining to the \( \Delta h \) of 20 and 40%, \( \Delta q \) of 10 and 22% respectively (Table 5).

Table 5. Total length, discharge (Q), distribution uniformity (DU), emission uniformity (EU), flow variation per meter (\( \Delta q_l \)) and flow variation (\( \Delta q \)) for the pressure variation of 20 and 40%.

<table>
<thead>
<tr>
<th>Method</th>
<th>( \Delta H ) (%)</th>
<th>L (m)</th>
<th>Q (L.h(^{-1}))</th>
<th>DU (%)</th>
<th>EU (%)</th>
<th>( \Delta q_l ) (%)</th>
<th>( \Delta q ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLP model</td>
<td>20</td>
<td>152.04</td>
<td>454.07</td>
<td>96.98</td>
<td>92.58</td>
<td>8.71</td>
<td>10.87</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>194.43</td>
<td>575.68</td>
<td>93.04</td>
<td>88.72</td>
<td>18.40</td>
<td>22.82</td>
</tr>
<tr>
<td>Usual</td>
<td>20</td>
<td>130.80</td>
<td>450.02</td>
<td>97.50</td>
<td>93.09</td>
<td>9.02</td>
<td>9.02</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>168.00</td>
<td>543.12</td>
<td>95.04</td>
<td>90.69</td>
<td>16.72</td>
<td>16.72</td>
</tr>
</tbody>
</table>

The use of higher pressure variation (40%), when compared to lower pressure variation (20%), in the lateral line showed a lower distribution uniformity, a lower emission uniformity, a higher emitters flow variation and a greater discharge variation per linear meter (Table 5). However, for the NLP model the length gain was of 42.39 m, compared to the \( \Delta h \) 20%, and keeping the \( \Delta q \) and the DU values within the limits offered by Wu (1997).

Comparing the NLP model and the usual procedure it was found that the DU had difference of 2% (to 40% DH) and 0.52% (to 20% DH). The difference in the EU was 1.97% (to 40% DH) and 0.5% (to 20% DH). The variation in flow rate per meter showed a difference of 0.31% (20% DH) and 1.68% (40% DH). The lowest flow variation per linear meter was found in the model developed in GAMS®, with \( \Delta h \) of 20%.

Comparing the uniformity indexes, the EU showed lower values compared to DU, because it is more restrictive when considering the lowest emitter flow. However, the values are higher than 88% (Table 5).

The values obtained for the emitter number, lateral line length, and length and total flow for the \( \Delta h \) evaluated are shown in Table 6. The usual procedure underestimated the lateral line length because it did not use all permissible pressure variation.
Emitter spacing obtained in the NLP model were between 0.39 and 0.47 m (Table 6). In all situations evaluated the spacing in the initial section was higher than in the final. This is because the emitter pressures, which were non-pressure compensating was higher in the first section, and generated higher flow rate per emitter, allowing greater spacing while maintaining the flow rate per meter. As the pressure decreases along the lateral line, the emitter discharge also decreases and the model select a smaller spacing for the second section aiming to keep the flow rate per meter uniform in the lateral line.

In the NLP model, the first section was at 44.51 and 46.11% of L, and at this point it was consumed 80.55 and 82.61% of the total head loss, to the $\Delta h$ of 20 and 40% respectively.

The NLP model was developed to design the lateral line optimizing the length using different spacing and assuring a uniform discharge per linear meter. This can be seen in Figures 1 and 2 for the $\Delta h$ 20 and 40%, respectively. In the spacing changing point, the discharge per linear meter modified from 2.87 to 3.03 L. h$^{-1}$ m$^{-1}$ and from 2.7 to 3.07 L. h$^{-1}$ m$^{-1}$ for the pressure variation of 20 and 40%, respectively. The highest discharge per linear meter occurred in the beginning of the lateral line, 3.14 (to 20%) and 3.29 L. h$^{-1}$ m$^{-1}$ (to 40%) and the lowest values occurred at the spacing changing point. At the lateral line end, the discharge per linear meter was of 2.97 and 2.92 L. h$^{-1}$ m$^{-1}$ for a $\Delta h$ of 20 and 40%, respectively.

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### Table 6. Inlet pressure (H ent.), pressure at lateral line final (H fim.), emitter spacing (Esp.), dripper number, lateral line length (L), and lateral line initial discharge (Q).

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Delta H$ (%)</th>
<th>Initial pressure (m.c.a)</th>
<th>Final pressure (m.c.a)</th>
<th>Spacing (m)</th>
<th>Drip numbers</th>
<th>L (m)</th>
<th>Total Q (L h$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Initial</td>
<td>Final</td>
<td>Initial</td>
<td>Final</td>
<td>Initial</td>
<td>Final</td>
</tr>
<tr>
<td>NLP</td>
<td>20</td>
<td>10</td>
<td>8.00</td>
<td>0.47</td>
<td>0.44</td>
<td>144</td>
<td>190</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>10</td>
<td>6.00</td>
<td>0.45</td>
<td>0.39</td>
<td>201</td>
<td>268</td>
</tr>
<tr>
<td>Usual</td>
<td>20</td>
<td>10</td>
<td>8.29</td>
<td>0.47</td>
<td>0.44</td>
<td>144</td>
<td>190</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>10</td>
<td>6.95</td>
<td>0.40</td>
<td>0.40</td>
<td>327</td>
<td>168.00</td>
</tr>
</tbody>
</table>

Figure 1. Discharge per linear meter as a function of the lateral line length, for the 20% pressure variation.
The longest lateral line was obtained with the NLP model. The use of higher pressure variation resulted in longer lateral lines and maintained the uniformity under acceptable standards. The adoption of two spacing in the same lateral line showed advantages in relation to use of a single one.

In NLP model even with the lateral line showing higher $\Delta q$ and $\Delta ql$, the DU was under acceptable standards (over 93%). The system implementation low cost was the result of the lateral line increase.

**Conclusions**

The initial hypothesis that the adoption of two emitter spacing would increase the lateral line length was confirmed. For pressure variations of 20 and 40% it was obtained a length gain of 16.2 and 15.7%, respectively, compared to the conventional method that use a single spacing.

The spacing changing ideal location was approximately 45% of the total lateral line length.

The use high flow variations under acceptable uniformity standards allowed the best results.

NLP model showed the best results when compared with the usual procedure, generating gain in the lateral line length, keeping the uniformity and flow variation under acceptable standards.

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**References**


