125 GeV scalar boson and $SU(N_{\text{TC}}) \otimes SU(3)_L \otimes U(1)_X$ models

A. Doff$^1$ and A. A. Natale$^{2,3}$

$^1$Universidade Tecnológica Federal do Paraná-UTFPR-DAFIS, Avenida Monteiro Lobato Km 04, 84016-210 Ponta Grossa, Paraná, Brazil
$^2$Centro de Ciências Naturais e Humanas, Universidade Federal do ABC, 09210-170 Santo André, São Paulo, Brazil
$^3$Instituto de Física Teórica, UNESP, Rua Dr. Bento T. Ferraz, 271, Bloco II, 01140-070 São Paulo, SP, Brazil

(Received 16 March 2013; published 10 May 2013)

We verify that $SU(N_{\text{TC}}) \otimes SU(3)_L \otimes U(1)_X$ models, where the gauge symmetry breaking is totally dynamical and promoted by the non-Abelian technicolor group and the strong Abelian interactions, are quite constrained by the LHC data. The theory contains a $T$ quark self-energy involving the mixing between the neutral gauge bosons, which introduces the coupling between the light and heavy composite scalar bosons of the model. We determine the lightest scalar boson mass for these models from an effective action for composite operators, assuming details about the dynamics of the strong interaction theories. Comparing the value of this mass with the ATLAS and CMS observation of a new boson with a mass $M_{\phi} \sim 125$ GeV and considering the lower bound determined by the LHC Collaboration on the heavy neutral gauge boson ($Z'$) present in these models, we can establish constraints on the possible models. For example, if $SU(N_{\text{TC}}) = SU(2)_\text{TC}$, with technifermions in the fundamental representation, the model barely survives the confrontation with the LHC data.

DOI: 10.1103/PhysRevD.87.095004 PACS numbers: 12.60.Nz, 12.60.Rc

I. INTRODUCTION

The Standard Model (SM) of electroweak interactions is in excellent agreement with the experimental data and has explained many features of particle physics throughout the years. Despite its success there are some points in the model, for instance, the enormous range of masses between the lightest and heaviest fermions and other peculiarities, that could be better explained at a deeper level assuming the introduction of new fields or symmetries.

Recently the ATLAS and CMS collaborations reported the observation of a new boson with a mass $M_{\phi} \sim 125$ GeV which is suspected to be the SM Higgs boson. The current data on this boson diphoton event rate exhibit a signal strength about 1.5–2 times larger than the one expected for the Standard Model Higgs boson. There are already many SM extensions trying to explain this possible enhancement of the $\gamma\gamma$ decay. In particular, the increase of this decay rate is natural in the context of a 3-3-1 model [1,2] and its alternative version with exotic leptons [3], due to the presence of an extra charged vector boson and a doubly charged one as discussed in Ref. [4].

This class of models predicts interesting new physics at the TeV scale [5] and addresses some fundamental questions that cannot be explained in the framework of the Standard Model [6,7]. These models also contain a set of fundamental scalar bosons, with many parameters and clearly suffering from the problems of naturalness and hierarchy [8,9]. However, in Refs. [10,11] it was suggested that the gauge symmetry breaking in some versions of the 3-3-1 model [3] could be promoted dynamically, because at the scale of a few TeVs the $U(1)_X$ coupling constant becomes strong and the exotic quark $T$ that appears in the model forms a condensate breaking $SU(3)_L \otimes U(1)_X$ to the SM electroweak symmetry. This is a very interesting feature and peculiar to this class of models. Unfortunately the SM gauge symmetry still remains intact, and the nice characteristics of the model could be missed with the introduction of an elementary scalar field in order to break the electroweak gauge symmetry, leading to an unpleasant system of composite and elementary fields responsible for the gauge symmetry breaking.

In Ref. [12] the full realization of the dynamical symmetry breaking of an $SU(3)_L \otimes U(1)_X$ extension of the SM [3] was explored. This was accomplished assuming the gauge symmetry $SU(2)_\text{TC} \otimes SU(3)_L \otimes U(1)_X$, where the electroweak symmetry is broken dynamically by a technifermion condensate generated by the $SU(2)_\text{TC}$ technicolor (TC) gauge group; i.e., besides the exotic $T$ quark condensate and respective composite scalar, we now have another composite scalar boson formed by $SU(2)_\text{TC}$ technifermions. This symmetry breaking also occurs when we exchange the $SU(2)_\text{TC}$ group by the $SU(N_{\text{TC}})$ group, as well as when we deal with different technifermion representations [13].

In 3-3-1 models where the gauge symmetry breaking is promoted by elementary scalar fields, the many parameters in the scalar potential can be varied in a large range leaving space to scape, up to now, to the LHC experimental constraints. However, in the case where the gauge symmetry breaking is totally dynamical, once we describe the possible dynamics of the theory, we may already have some limitation on the possible models. The study of possible constraints in this class of models is the main motivation of this work. We compute the effective potential for composite operators of a class of 3-3-1 models where the gauge symmetry is dynamically broken, with
the main purpose of determining the composite scalar masses. If, for instance, we consider $SU(N)_{TC} \cong SU(2)_{TC}$, we verify that it is quite difficult to generate a scalar boson mass of 125 GeV, assuming that it is a composite scalar that has been observed at the LHC, obeying, at the same time, the lower limit on the $Z'$. The composite scalar system of these 3-3-1 models has a mixing related to the $Z$ and $Z'$ mixing, which is present in the exotic $T$ quark self-energy. This mixing will appear in the calculation of the effective potential for composite operators [14], which, when minimized, supply the physical scalar masses, and it is important to know its amount because the scalar masses may be modified by this effect. There are other possible contributions to this mixing, that because the scalar masses may be modified by this effect.

II. $SU(N)_{TC} \otimes SU(3)_L \otimes U(1)_X$ MODELS

Below we describe the main features of the models, which are similar to those proposed in Ref. [12]; the fermionic content has the following form:

$$Q_{3L} = \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L \sim (1, 3, 2/3), \quad t_R \sim (1, 1, 2/3),$$

$$b_R \sim (1, 1, -1/3), \quad T_R \sim (1, 1, 5/3),$$

$$Q_{aL} = \begin{pmatrix} D \\ u \\ a \end{pmatrix}_a \sim (1, 3^*, -1/3), \quad u_{aR} \sim (1, 1, 2/3),$$

$$d_{aR} \sim (1, 1, -1/3), \quad D_{aR} \sim (1, 1, -4/3),$$

where $\alpha = 1, 2$ is the family index and we represent the third quark family by $Q_{3L}$. In these expressions $(1, 3, X)$ or $(1, 1, X)$ denote the transformation properties under $SU(N)_{TC} \otimes SU(3)_L \otimes U(1)_X$ and $X$ is the corresponding $U(1)_X$ charge. The leptonic sector includes, besides the conventional charged leptons and their respective neutrinos, the charged heavy leptons $E_a$ [3].

$$l_{aL} = \begin{pmatrix} \nu_a \\ l_a \\ E_a \end{pmatrix}_L \sim (1, 3, 0),$$

where $a = 1, 2, 3$ is the family index and $l_{aL}$ transforms as triplets under $SU(3)_L$. Moreover, we have to add the corresponding right-handed components, $l_{aR} \sim (1, 1, -1)$ and $E_{aR} \sim (1, 1, +1)$.

The fermionic content associated with the TC sector has the form

$$\Psi_{1L} = \begin{pmatrix} U_1 \\ D_1 \\ U_1' \end{pmatrix}_L \sim (N_{TC}, 3, 1/2), \quad U_{1R} \sim (N_{TC}, 1, 1/2), \quad D_{1R} \sim (N_{TC}, 1, -1/2), \quad U'_R \sim (N_{TC}, 1, 3/2),$$

$$\Psi_{2L} = \begin{pmatrix} D_2' \\ U_2 \\ D_2 \end{pmatrix}_L \sim (N_{TC}, 3^*, -1/2), \quad U_{2R} \sim (N_{TC}, 1, 1/2), \quad D_{2R} \sim (N_{TC}, 1, -1/2), \quad D'_R \sim (N_{TC}, 1, -3/2),$$

(3)

where 1 and 2 label the first and second techniquark families; $U'$ and $D'$ can be considered as exotic techniquarks making an analogy with quarks $T$ and $D$ that appear in the ordinary fermionic content of the model. The model is anomaly-free if we have equal numbers of triplets and antitriplets, counting the color of $SU(3)_L$. Therefore, in order to make the model anomaly-free two of the three quark generations transform as $3^*$, and the third quark family and the three lepton generations transform as $3$. It is easy to check that all gauge anomalies cancel out in this model; in the TC sector the triangular anomaly cancels between the two generations of technifermions. In the present version of the model the technifermions are singlets of $SU(3)_c$.

As pointed out in Refs. [10,11], one interesting feature of the versions [1–3] of 3-3-1 models is the following.
relationship among the coupling constants $g$ and $g'$ associated with the gauge group $SU(3)_L \otimes U(1)_X$, 

$$ t^2 = \frac{g'^2}{g^2} = \frac{\alpha_x}{\alpha} = \frac{\sin^2 \theta_W(\mu)}{1 - 4 \sin^2 \theta_W(\mu)} \quad (4) $$

where $\alpha = g^2/4\pi$, $\alpha' = g'^2/4\pi$, and $\theta_W$ is the electroweak mixing angle. According to the discussion presented in [10,11], it is precisely this feature of the model that allows the gauge symmetry breaking of this version of the $SU(3)_L \otimes U(1)_X$ model to the SM symmetry, because at the scale of a few TeVs the $U(1)_X$ coupling constant becomes strong as we approach the peak existent in Eq. (4). Therefore, in the model described in this section the exotic quark $T$ will form a condensate breaking $SU(3)_L \otimes U(1)_X$ to the electroweak gauge symmetry, while the SM gauge symmetry will be broken dynamically by a technifermion condensate.

In order to compute the effective action generated for the composite scalar bosons resulting from the two symmetry breaking stages, in the next section we discuss the T quark self-energy, which is related to the mixing between the standard model neutral gauge boson $Z$ with the $Z'$ boson.

### III. (Z'-Z) MIXING AND SELF-ENERGIES

In the models that we consider here, there is a mixing between the Standard Model neutral gauge boson $Z$ with the $Z'$ boson; the mass eigenstates are [15]

$$ Z_1 = Z \cos \theta - Z' \sin \theta, \quad (5) $$

$$ Z_2 = Z' \cos \theta + Z \sin \theta, \quad (6) $$

where the mixing angle ($\theta$) is given by [15]

$$ \tan \theta = \frac{M_{Z'} - M_Z^2}{M_{Z'} - M_Z^2}. \quad (7) $$

In this case ($Z_1$) represents the SM neutral boson and ($Z_2$) corresponds to the additional 3-3-1 heavy neutral boson. Therefore, assuming this mixing we can write in the Euclidean space the following linearized gap equation $[\Sigma_T(p^2)]$ for the T quark:

$$ \Sigma_T(p^2) = a \cos \theta \int dk^2 k^2 \frac{\Sigma_T(k^2)}{[k^2 + \mu_X^2]} \frac{1}{[(p-k)^2 + M_T^2]} $$

$$ + a \sin \theta \int dk^2 k^2 \frac{\Sigma_T(k^2)}{[k^2 + \mu_X^2]} \frac{1}{[(p-k)^2 + M_T^2]} \quad (8) $$

where

$$ a = \frac{3 g^3_X X_L X_R}{16 \pi^2}. $$

$$ \mu_X $$ is the energy scale where the $U(1)_X$ interaction becomes sufficiently strong to break dynamically the $SU(3)_L \otimes U(1)_X$ to $SU(2)_L \otimes U(1)_Y$, $g^3_X$ is the $U(1)_X$ coupling constant, and $X_L$ and $X_R$ are, respectively, $U(1)_Y$ charges attributed to the chiral components of the exotic quark $T$.

Besides the condensate and composite states (scalar and pseudoscalar) associated with Eq. (8), we have similar entities due to the $SU(N)_T$ group condensation at the scale $\mu_T$, generated by a nontrivial technifermion self-energy $[\Sigma_{TC}(p^2)]$. As discussed in Ref. [12] the technifermion multiplets $\Psi$ and $\Psi_2$ described in Eq. (3) lead to the formation of composite scalar bosons ($\phi_1$ and $\phi_2$) that are equivalent to the set of fundamental scalar fields, $\rho$ and $\eta$ [1,3], so, in order to obtain a structure of the scalar potential similar the one described in [1,3], we will assume that

$$ \phi = \phi_1 + \phi_2. \quad (9) $$

This normalization results from the fact that $f^2_T \propto \phi^2$ [as we shall describe in Eq. (29)], and from this it is possible to verify that

$$ \phi^2 = \phi_1^2 + \phi_2^2, \quad (10) $$

once $f^2_T = f^2_{Z_1} + f^2_{Z_2}$, and ($\phi_1$) = ($\phi_2$), typical of the two technifermion generations of Eq. (3). The $f_{\pi_i}$ are the technipion decay constants that can be computed through the linearized Pagels and Stokar relation [16]

$$ f^2_{\pi_i} = \frac{N_{TC}}{4\pi^2} \int \frac{d^3p}{(2\pi)^3} \frac{\Sigma^2_{TC}(p^2)}{(p^2 + \mu_{TC}^2)^2}, \quad (11) $$

whereas the pseudoscalar decay constant associated with $T$ quark self-energy, $\Sigma_T(p^2)$ will be written as

$$ F^2_{\pi_i} = \frac{1}{4\pi^2} \int \frac{d^3p}{(2\pi)^3} \frac{\Sigma_{TC}(p^2)}{(p^2 + \mu_{TC}^2)^2}. \quad (12) $$

To compute the effective potential for composite operators, [14] we need to know the self-energies of the strongly interacting fermions: the $T$ quark and the fermions with $SU(N)_T$ charges. Equation (8) has two possible solutions, and in the program developed in Refs. [17,18] it was verified that the solution falling slowly with the momentum is the dominating one if suitable new interactions are assumed to be relevant at the scale of the (UV) cutoff. In this case the gauge boson mass integrals receive significant contributions from a very large range of loop momenta, and the SM gauge boson masses $M_W$ and $M_Z$ turn out to be of similar magnitude when compared to the top quark mass. This is exactly the situation that we have in the approach proposed to promote the gauge symmetry breaking of $SU(3)_L \times U(1)_X$ to the electroweak symmetry, where at the scale of a few TeVs the $U(1)_X$ coupling constant becomes strong as we approach the peak existent in Eq. (4). In Ref. [10], after the numerical calculation of $M_T$, it was found that the magnitudes of $M_Z$ and $M_T$ are the same order. Therefore, considering the above comments we will assume that the solution of Eq. (8) is given by

$$ 095004-3 $$
\[
\Sigma_T(p^2) \approx \Sigma_T'(p^2) \left(1 - h(\omega) \ln \left(\frac{p^2}{\mu_X^2}\right) \tan \theta \right),
\]
(13)

and it is the one that will be used for the \( T \) quark self-energy to determine the effective potential (\( \Omega_T \)). To write Eq. (13) we define the following quantities:

\[
\Sigma_T'(p^2) = \mu_X \left(\frac{p^2}{\mu_X^2}\right)^{-\omega},
\]

where \( \omega = \sqrt{1 - 4A}, A = a \cos \theta \), and

\[
h(\omega) = \left(1 - \frac{1}{\omega} + \frac{A}{\omega}\right).
\]
(14)

In the case of fermions with \( SU(N)_{TC} \) charges the self-energy will be given by

\[
\Sigma_{TC}(p^2) \sim \mu_{TC}[1 + \beta g^2 \ln (p^2/\mu_{TC}^2)]^{-\gamma},
\]
(15)

where \( \mu_{TC} \) is the \( SU(N)_{TC} \) characteristic scale of mass generation,

\[
\gamma = 3c/16\pi^2 b
\]
(16)

and \( c = \frac{1}{2}[C_2(R_1) + C_2(R_2) - C_2(R_3)] \) where \( C_2(R_i) \) are the Casimir operators for fermions in the representations \( R_1 \) and \( R_2 \) that condense in the representation \( R_3 \), \( b = (11N - 2N_f)/48\pi^2 \) for the \( SU(N)_{TC} \) group with \( N_f \) flavors, and \( g_{TC}^2 \) is the coupling constant for which we assume the expression

\[
g_{TC}^2(k^2) = \frac{1}{b \ln\left([k^2 + 4m_T^2]/\Lambda^2\right]},
\]
(17)

where \( m_T \) is an infrared dynamical gauge boson mass, whose phenomenologically preferred value is \( m_T \approx 2\Lambda \) [19,20], and we will set \( \Lambda = \mu_{TC} \). Note that, using the above coupling constant, we are assuming that non-Abelian gauge theories generate dynamical masses for their gauge bosons [19,21]. As a consequence, it is expected that confinement should be necessary to generate nontrivial fermionic self-energies [22,23], and the expression for the self-energy is the one of Eq. (15), as discussed at length in Ref. [24]. The main features of Eq. (15) are that it causes the decoupling of heavier degrees of freedom in models where there is an interaction connecting different fermionic families [24], it leads to the deepest minimum of energy, with a vacuum expectation value proportional to \( 1/g^2 \) [25–27], it is the only self-energy able to naturally explain fermion masses as heavy as the top quark [28], and it is the unique possible form of solution that may generate a light composite scalar boson [29,30].

In the self-energies that we discussed above the characteristic scales \( \mu_X \) and \( \mu_{TC} \) have not been determined up to now. However, they should be constrained by the value of the \( SU(3)_L \otimes U(1)_X \) gauge boson masses. In order to do so we notice that, for \( M_Z \gg M_T \), it is possible to show that [15]

\[
\tan \theta \approx \frac{1}{2\sqrt{3}e^2} \frac{M_Z^2}{M_T^2}.
\]
(18)

Assuming the result described in Ref. [12] we obtain the masses

\[
M_Z^2 = \frac{g_\pi^2}{4} \left[ f_\pi^2 + f_\pi^2 \right] \left(1 + 4r^2 \right) = \frac{g_\pi^2}{4} \left[ f_\pi^2 + 4r^2 \right] \left(1 + 3r^2 \right). \]
(19)

\[
M_Z^2 = \frac{g_\pi^2}{4} \left[ f_\pi^2 + 4r^2 \right] \left[ \frac{4}{3} + 4r^2 \right]. \]
(20)

With these masses and Eq. (18), we can write Eq. (13) in the form

\[
\Sigma_T(p^2) = \Sigma_T'(p^2) \left(1 + A(\omega) \frac{\mu_X^2}{\mu_{TC}^2} \ln \left(\frac{p^2}{\mu_{TC}^2}\right)\right), \]
(21)

where for \( SU(N)_{TC} \)

\[
A(\omega) = -\frac{\sqrt{3}N_{TC}}{16\pi^2} \left(1 + 4r^2\right) \left(1 + 3r^2\right)^{1/2} h(\omega).
\]

The \( f_\pi \) decay constant is related to the SM vacuum expectation value (vev) through

\[
f_\pi^2 = \left( f_\pi^2 + f_\pi^2 \right) = v^2 = \frac{4M_Z^2}{g_\pi^2} = (246 \text{ GeV})^2, \]
(22)

and in the case of \( T \) quark self-energy, since there is no evidence of the \( Z' \) boson, we just assume \( F_{L} \sim O(\mu_X) \sim O(\text{TeV}) \). Equations (15) and (21) are the main ingredients to compute the effective action for the model described in Sec. II.

**IV. THE EFFECTIVE ACTION FOR COMPOSITE SCALAR BOSONS OF THE SU(N)_{TC} \otimes SU(3)_L \otimes U(1)_X MODEL**

The effective potential for composite operators [14] is a function of the Green’s functions of the theory; in particular, it can be written as a function of the complete fermion (S) and gauge boson (D) propagators as

\[
V(S,D) = -i \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left( \ln S_0^{-1} S - S_0^{-1} S + 1 \right) + V_2(S,D),
\]
(23)

where \( S_0 \) (and \( D_0 \)) stands for the bare fermion (gauge boson) propagator and \( V_2(S,D) \) is the sum of all two-particle irreducible vacuum diagrams. The physically meaningful quantity that we must compute is the vacuum energy density given by

\[
\Omega_V = V(S,D) - V(S_0, D_0),
\]
(24)

where we are subtracting the symmetric part of the potential from the potential that admits condensation in the scalar channel, which is denoted by \( V(S_0, D_0) \) and is a function of the perturbative propagators (\( S_0 \) and \( D_0 \)).
The vacuum energy density, if we remove all indices and integrations, can be written as \[14,25\]

\[
\Omega_V = -i \text{Tr}(\ln S_0^{-1} S - S_0^{-1} S + 1) + i \text{Tr} \Sigma(S - S_0) + \frac{1}{2} i \text{Tr}(\Gamma S \Gamma S - \Gamma S_0 \Gamma S_0) D. \tag{25}
\]

The self-energies described by Eqs. (15) and (21) enter into the definition of \(S\).

How the effective action for composite scalar bosons emerges from Eq. (25) in the case of a dynamically broken gauge theory including the kinetic term has been detailed by Cornwall and Shellard \[31\], and is also discussed in Refs. \[25,32\]. Here we will skip lengthy details and follow closely the work of Ref. \[32\], where it was shown that the effective action generated for a TC model could be written in the following way:

\[
\Omega_{\text{TC}} = \int d^4 x \left[ \frac{1}{2Z_{\text{TC}}} \partial_\mu \phi \partial^\mu \phi - \frac{\lambda_{4\text{TC}}}{4} \phi^4 - \frac{\lambda_{6\text{TC}}}{6} \phi^6 - \cdots \right]. \tag{26}
\]

The effective scalar field \(\phi(x)\) acts like a dynamical effective scalar field with anomalous dimension 2\(\gamma\) and is related to the bilinear self-energy \(\Sigma(k, p) \propto \phi(k)\Sigma(p)\); it is seen as a variational parameter, and the kinetic term for \(\phi(x)\) is related to the bilinear self-energy \(\Sigma\).

In the above equation

\[
Z_{\text{TC}} = \frac{4\pi^2 \beta(2\gamma - 1)}{N_{\text{TC}} N_f} \tag{27}
\]

and

\[
\lambda_{4\text{TC}} = \frac{N_{\text{TC}} N_f}{4\pi^2} \left( \frac{1}{\beta(4\delta - 1)} + \frac{1}{2} \right), \tag{28}
\]

\[
\lambda_{6\text{TC}} = -\frac{N_{\text{TC}} N_f}{4\pi^2} \left[ \frac{1}{\mu_{\text{TC}}^2} \right].
\]

In these expressions, \(\beta = g_{\text{TC}}^2\), \(N_f\) denotes the number of technifermions; \(\gamma\) has been defined in Eq. (16) and is calculated for the respective TC representations.

In Eq. (26) a term proportional to \(\phi^2\) does not appear because we assume that the self-energies are exact solutions of the linearized gap equations \[14\]; also, odd terms in \(\phi\) do not appear because we do not have current fermion masses. The constant \(Z_{\text{TC}}\) arises when the contribution of the kinetic term is included in the calculation of the effective action \[25,31,32\], and this acts as a normalization constant. This contribution is important in our calculation because it will give the correct normalization of the effective fields, \(\phi\) and \(\phi_T\), as discussed in Refs. \[25,31,32\].

In terms of these fields we can also write the decay constants for the TC and \(T\) fermions as

\[
f_\pi^2 = \frac{N_{\text{TC}} \phi^2}{4\pi^2} \frac{1}{\beta(2\gamma - 1)}, \tag{29}
\]

while for the self-energy, \(\Sigma_T(p^2)\), we obtain the following relation,

\[
F_{\Pi}^2 = \frac{\phi_T^2}{4\pi^2} \frac{1}{2a'}, \tag{30}
\]

and in this case \(\langle \phi_T \rangle \approx \mu_x\).

We can now present the contribution to the effective action due to the composite scalar boson formed by the strong interaction of the exotic \(T\) quark. Below we show the \(\phi_T^2\) and \(\phi_T^6\) terms of the effective potential \(\Omega_T\) and the corrections \(\Delta \Omega_T\) assuming the mixture in Eq. (13) and the comments leading to Eq. (21):

\[
\Omega_T = \int d^4 x \left[ \frac{1}{2Z_T} \partial_\mu \phi_T \partial^\mu \phi_T - \frac{\lambda_{4T}}{4} \phi_T^4 - \frac{\lambda_{6T}}{6} \phi_T^6 \right. \\
\left. - \frac{\Delta \lambda_{4T}}{4} \phi_T^4 \phi^2 - \frac{\Delta \lambda_{6T}}{6} \phi_T^4 \phi^2 - \cdots \right]. \tag{31}
\]

where we identify

\[
Z_T = 8\pi^2 a, \quad \lambda_{4T} = \frac{1}{4\pi^2} \left( \frac{1}{4a} + \frac{1}{4} \right), \tag{32}
\]

\[
\lambda_{6T} = -\frac{1}{4\pi^2} \frac{1}{\mu_x^2}, \quad \Delta \lambda_{4T} = \frac{a \Delta \lambda_4}{\pi^2} \left( \frac{1}{4a} + \frac{1}{4} \right),
\]

\[
\Delta \lambda_{6T} = -\frac{\Delta \lambda_4}{4\pi^2} \frac{24a^2}{1 + 2a} \frac{1}{\mu_x^2},
\]

\[
\Delta \lambda_4 = \frac{\sqrt{3} N_{\text{TC}}}{16\beta t^2} \left( \frac{1}{(2\gamma - 1)(1 - 2a)} \right) \left( 1 + 4t^2 \right). \tag{33}
\]

In these expressions we assume the existence of just one exotic quark that condenses in the most attractive channel \[33\].

In order to reproduce a standard scalar effective field theory we introduce in our effective Lagrangian the normalized fields

\[
\Phi(x) = Z_{\text{TC}}^{-1/2} \phi(x), \tag{34}
\]

\[
\Phi_T(x) = Z_T^{-1/2} \phi_T(x). \tag{35}
\]

Now, considering Eq. (9), where we see that the field \(\phi\) actually represents two fields (\(\phi_1\) and \(\phi_2\)), and adding Eq. (26) to Eq. (31) we can write down the full effective action in terms of the normalized fields \(\Phi_T\) and \(\Phi(x)\) (composed of \(\Phi_1\) and \(\Phi_2\)).
indeed observing a new scalar boson with a mass for this purpose we assume that ATLAS and CMS are in order to exclude possible candidates for TC models. Note where the normalized couplings for the composite fields, \(\Phi_4\), and \(\Phi_2\), are given, respectively, by

\[
\lambda_{\Phi_4}^R = \frac{Z_\Phi^2}{4\pi^2} \left( \frac{1}{4a} + 1 \right), \quad \lambda_{\Phi_2}^R = \frac{N_{TC} N_{fZ}}{4\pi^2} \left( \frac{1}{B(4\gamma - 1)} + \frac{1}{2} \right)
\]

where the minimum value in the potential, we obtain

\[
M^2_{\Phi_4} = \lambda_{\Phi_4}^{R(a)} \left( \frac{\lambda_{\Phi_4}^{R(a)}}{\lambda_{\Phi_4}^{R(b)}} \right) + 2 \lambda_{\Phi_2}^{R(b)} \left( \frac{\lambda_{\Phi_2}^{R(b)}}{\lambda_{\Phi_2}^{R(c)}} \right).
\]

Assuming the above equation, with couplings \(\lambda_{\Phi_4}^{R(a)}\), \(\lambda_{\Phi_4}^{R(b)}\), and \(\lambda_{\Phi_2}^{R(c)}\) defined by Eq. (37), we find that the Higgs mass to the range 120 GeV < \(M_H\) < 130 GeV, in order to exclude possible candidates for TC models. Note that for this purpose we assume that ATLAS and CMS are indeed observing a new scalar boson with a mass \(M_{\phi} \sim 125\) GeV, and we consider also the strong limit on the Z' mass announced by these collaborations [34].

The extra Z' boson is predicted in many extensions of the Standard Model at the TeV mass scale, as in the Sequential Standard Model (SSM) [35], with SM-like couplings. With LHC data at \(\sqrt{s} = 8\) TeV, currently the ATLAS and CMS collaborations placed strong constraints on the mass of these particles [34]. These constraints depend on the knowledge of the coupling of this boson with SM fermions. In the case of the Z_ssm model with SM-like couplings, the Z' mass can be excluded below 2.49 TeV. This limit can also be taken as a lower limit on the Z' mass of the models discussed here. In this particular case, if \(M_{Z'} > 2.49\) TeV the energy scale \(\mu_X\) should be limited to \(\mu_X > 1.1\) TeV. In Fig. 1 we present the allowed region of parameters for the \(SU(2)_T, SU(3)_T\), and \(SU(4)_T\) cases, with a scalar composite “Higgs” mass range 120 GeV < \(M_H\) < 130 GeV. The solid black line corresponds to the lower limit on \(\mu_X\), and from this figure we verify that if \(SU(N)_T = SU(2)_T\) the model barely survives the confrontation with the LHC data.

There are possible corrections to the scalar mass values that we discussed here. As pointed out recently in Ref. [36] radiative corrections induced by the effective scalar coupling to the top quark may decrease the scalar mass. These corrections give a contribution to the scalar mass with a negative signal typical of fermion loops, allowing a
lighter scalar and possibly alleviating the mixing with the heavier scalar boson. They are not easy to compute at a fundamental level once there is a form factor in the effective coupling that is difficult to determine and may decrease the amount of this contribution. On the other hand, it is quite possible that the lighter scalar boson would also mix with other scalar states of the TC sector, as scalar techniglueballs, as well as the scalar mass, may receive contributions from technipion loops, which increase the scalar mass, implying a small overall effect. However, it is interesting to note that if the LHC provides new data, with even a small improvement in the lower bound on the $Z'$ mass, the type of models discussed here may be in trouble. This is because it will be quite difficult to generate a composite scalar system in which the scalars mix among themselves and, at the same time, give very different masses to the $Z$ and $Z'$ bosons.

In order to complement the analysis, in Fig. 2 we show the interval of parameters in the $SU(3)_{TC}$ case assuming $N_f = 8$ and $N_f = 10$. As can be noticed in Fig. 2, the range of values that the parameters $\mu_{TC}$ and $\mu_X$ can assume decreases when the number of technifermions is increased. The $N_f$ dependence on the kinetic term is important; it is transferred to the $\lambda_i$'s couplings and results in a decrease of the scalar mass.

We can write Eq. (39) in the following approximate form,

$$M_{\theta_i} \approx \delta_1 \sqrt{\beta(2\gamma - 1)} \mu_{TC} + \delta_2 \sqrt{\beta(2\gamma - 1)} \mu_X.$$  

(40)

where $\delta_1$ and $\delta_2$ are constants. The increase of the number of fermions implies an increase of $\sqrt{\beta(2\gamma - 1)}$; then, to keep the interval to the Higgs mass $120 \text{ GeV} < M_H < 130 \text{ GeV}$, we observe a decrease in the area $[\mu_{TC} \times \mu_X]$ in the parameter space. This effect is the same as appears in the normalization of the Bethe-Salpeter wave function discussed in Ref. [29].

In Fig. 3 we show the interval of parameters in the $SU(2)_{TC}$ case assuming $N_f = 7$ (in green), and again we verify that the $SU(2)_{TC}$ case with this number of fermions can be ruled out, in confrontation with the LHC data.

**VI. CONCLUSIONS**

In this work we computed an effective action for the composite scalar boson system, $\Phi_T$, $\Phi_1$ and $\Phi_2$, formed by the fermions and technifermions $Q_3, \Psi_1$, and $\Psi_3$, described in Eqs. (1) and (3). We include in this calculation the kinetic term of the effective theory. This term is important because it provides a normalization factor for the effective scalar boson Lagrangian. The effective Lagrangian is then normalized in order to reproduce a standard scalar effective field theory, leading to a nontrivial set of scalar self-couplings. From this Lagrangian we can determine the scalar boson masses of the theory.

To compute the effective action for the model described in Sec. II, we first determined the correction to the $T$ quark self-energy that results from the mixing between the standard model neutral gauge boson $Z$ with the $Z'$ boson. We show that this correction is responsible for introducing the coupling between the composite scalars, $\Phi_T$ and $\Phi_{1,2}$, associated, respectively, with the $T$ quark and with the technifermions. In Sec. III we discussed the self-energies used to compute the effective potential ($\Omega_T$), where we assumed that the interaction $U(1)_Y$ plays a role analogous to the ultraviolet dynamical symmetry breaking program.
We also assumed a TC self-energy that decays slowly with the momentum whose origin is due to the introduction of confinement in the gap equation, as discussed in Ref. [24], that is typical of the gauged Nambu-Jona-Lasinio type of models, where the anomalous dimension is \( \gamma_m = 2 \). Note that it is hardly possible to generate light scalar composite bosons without this particular choice [29,30]. We finally determined an effective scalar boson Lagrangian, and from it we obtained the scalar boson masses associated with these models.

As already discussed in the Introduction, the models that we consider here are interesting due to the particular form of their anomaly cancellation and due to the fact that they have a naturally strong Abelian theory at the TeV scale, capable of producing a dynamical symmetry breaking of the model to the SM symmetry, whereas the SM gauge symmetry is broken by a TC condensate. Within this class of models we can also explain the larger decay rate of the 125 GeV boson into photons that is observed by the LHC experimental results with the mass values that we obtained for these quantities. We verified that the models are strongly constrained, showing that it is rather difficult to have light scalar composites and at the same time generate masses for neutral gauge bosons where one of them (the \( Z' \)) is quite heavy. It is possible that in models with the presence of fundamental scalar bosons such a difficulty is not present due to the many parameters that can be adjusted in the scalar potential; however, in this case, we may also foresee that this adjustment may lead to unnatural values of the coupling constants, unless some discrete symmetries are introduced by hand in order to avoid undesirable terms in the scalar potential.

ACKNOWLEDGMENTS

This research was partially supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES).
