

Zero mode effect generalization for the electromagnetic current in the light frontAlfredo Takashi Suzuki,^{1,3} Jorge Henrique Sales,² and Luis Alberto Soriano¹¹*Instituto de Física Teórica, UNESP-Universidade Estadual Paulista,**Rua Dr. Bento Teobaldo Ferraz 271, 01140-070 São Paulo, São Paulo, Brazil*²*Universidade Estadual de Santa Cruz, Km 16 Rodovia Jorge Amado, 45662-900 Ilhéus, Bahia, Brazil*³*Department of Physics and Engineering, Southern Adventist University, Collegedale, Tennessee 37315, USA*

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We consider in this work the electromagnetic current for a system composed of two charged bosons and show that it has a structure of many bodies even in the impulse approximation, when described in the light-front time x^+ . In terms of the two-body component for the bound state, the current contains two-body operators. We consider the photon interacting with two bosons and the process of pair creation connected to this interaction, interpreting it as a zero mode contribution to the current and discuss the consequences of this pair creation to the components of currents in the light front.

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I. INTRODUCTION

In the traditional approach to restore covariance of the electromagnetic current in the light front [1], an *ad hoc* prescription of dislocating the pole is employed [2]. However, this procedure of “pole dislocation” has no physical grounds, and arriving at the correct result is just fortuitous. On the other hand, as we integrate in the light-front “energy” variable k^- , considering carefully all the possible domains of the allowed longitudinal momentum component k^+ , we arrive at the conclusion that the light-front Fock space of positive quantum solutions is incomplete and that, as a consequence, the nontriviality of the light-front vacuum turns out to be a mandatory feature in the new scenario.

In a recent article Bakker *et al.* [3] studied how the behavior of the contour integration in the k^- complex plane could influence the results of integrations in the light front. They showed that in the diagrams with loops such as self-energy and triangular ones, the integration on the arc in the contour of the k^- complex plane is nonvanishing and the inclusion of this arc contribution is fundamental to restore the covariant results obtained through well-established techniques. Our work fundamentally differs from theirs in the physical processes studied, though in essence and ultimately we also look for the covariance of the end result. Since the physical process we analyze is different from what they study, we do not obtain the same kinds of subtle and tricky arc contributions in order to recover covariance of the results from the light-front calculations because these arc contributions are absent as we deal with “ladder”-type diagrams. In our process and approach, as we evaluate all the relevant ranges of integration for k^+ variables, we get the correct terms equivalent to the results of computations obtained via covariant Minkowski calculations. Since this is our present case, it is well worth noting that in our “ladder” diagram calculation, happily, no subtle or difficult points like those tricky arc contributions arise. And, the covariance of the electromagnetic current can be recovered just by careful consideration of those many different domains of k^+ integration that are

allowed and defined from the nontrivial results stemming from the light-front energy (k^-) integration.

As stated above, it is not difficult to see why in our case the arc contributions do not appear; this is due to the fact that ladder-type diagrams involve more momentum power in the denominator of the integrands coming from the presence of more virtual particle propagators than in the case of self-energy or triangle diagrams considered by Bakker *et al.* Moreover, in our work we demonstrate that in this curious ladder diagram in a background field in which we consider the contribution of an external photon over two noninteracting bosons, we have seen that the pair creation contribution is needed to restore covariance of the current J^- . However, if more interacting photons are present, this does not occur; that is, there is no pair contribution to restore covariance of the current J^- . It is also interesting to note that in the case of two bosons exchanging a virtual boson in a background field with one photon, there is no pair correction either. However, for two photons, we need the correction coming from the pair creation. In the method we use, we always consider the ranges of integration in k^+ ; there are ranges for which the integration does not vanish, as in the case of Eq. (13).

This means that restoration of covariance in our case does not come from “hidden” nonvanishing arc contributions that have been forgotten in the calculation, but, rather, it comes from considering all the allowed ranges of integration in the longitudinal momentum component, in which some peculiar situation entails the conclusion that pair contributions are necessary or suppressed in the process considered. In this sense, our methodology of calculation entails a more physically grounded approach than the earlier forced “pole dislocation” prescription for which one finds no physical basis except for giving the desired and expected end results.

In order to demonstrate this we calculate in a specific example the matrix element for the electromagnetic current in Breit’s reference frame for $q^+ \rightarrow 0$ and $\vec{q}_\perp \neq 0$. To this

end we use a constant vertex for the bound state of two bosons in the light front. Such a calculation agrees with the results obtained through the computation of the triangular diagram for the electromagnetic current of a composite boson whose vertex is constant [2].

Sawicki [4] has shown that in Breit's reference frame, the $J^+ = J^0 + J^3$ component of the electromagnetic current for the bound state of two bosons, obtained from the triangular Feynman diagram after integration in the k^- component of the loop momentum, has no pair production contribution from the photon. As a consequence, the electromagnetic form factor, calculated in the light front, starting from J^+ , is identical to the one obtained in the covariant calculation. By covariant calculation of an amplitude we mean the computation of a momentum loop integrated directly without transformation into a light-front momentum.

The problem that appears when integrating in the light-front coordinates in momentum loops was studied by Chang and Yan [5] and more recently discussed in Refs. [2,6–8]. In the Chang and Yan works, although they pointed out the difficulty in the k^- integration for certain amplitudes and suggested a possible solution to the problem, in our view two distinct aspects are mingled together, which are the renormalization question and the problem of integration in the light-front coordinates. Our emphasis here is on the covariance restoration for the electromagnetic current through a careful integration in the k^- of the loop momentum in finite diagrams. We know that for the J^+ component of the electromagnetic current for a particle of spin 1, there are terms that correspond to pair production in the light-front formalism for $q^+ = 0$ [2,8]. In the case of the vector meson ρ , the rotational invariance of the current J^+ is broken when we use the light-front formalism, unless pair production diagrams are duly considered [9,10].

In Refs. [4,11–13] the electromagnetic current in the light front for a composite system is obtained from the triangular diagram (impulse approximation) when it is integrated in the internal loop momentum component $k^- = k^0 - k^3$. This integration in k^- by Cauchy's theorem uses the pole of the spectator particle in the process of photon absorption for $q^+ = 0$. Using the current J^+ the process of pair creation by the photon is, in principle, eliminated [4,11]. In general, covariance is preserved under kinematic transformations, but the current loses this physical property under a more general transformation, such as rotations and parity transformations. We show how the pair production is necessary for the complete calculation of the current's J^- component in the Drell-Yan reference frame ($q^+ = 0$).

The paper is organized as follows: Section II introduces the propagator in a background field with interacting bosons. In Sec. III we consider the relevant operator components for the electromagnetic current. We study the possible contributions to the order g^2 that might contribute to the zero modes in Sec. IV and conclude in Sec. V. In Appendix A we consider the case of a free propagator in the light front,

which serves as a notational convention used throughout this work, while in Appendix B we consider, in some detail, the pole structure in the light-front energy variables and the integrations in their corresponding complex planes.

II. PROPAGATOR IN A BACKGROUND FIELD WITH INTERACTING BOSONS

In a recent article [14] we considered, in the zeroth order of perturbative coupling, the calculation of the electromagnetic current in the light-front coordinates for scalar bosons in the electromagnetic background field. The calculation was considered only in the region $0 < k_2^+ < k_i^+ < k_4^+ < k_f^+$ and its combinations. The same result is found in the article by Marinho, Frederico and Sauer [15], using a different technique.

The Lagrangian density for interacting scalar and electromagnetic fields is given by

$$\begin{aligned} \mathcal{L} &= D_\mu \phi_1 D^\mu \phi_1^* - m^2 \phi_1^* \phi_1 - m^2 \phi_2^* \phi_2 \\ &\quad + g \phi_1^* \phi_1 \sigma + g \phi_2^* \phi_2 \sigma \\ &= \partial_\mu \phi_1 \partial^\mu \phi_1^* - m^2 \phi_1^* \phi_1 - m^2 \phi_2^* \phi_2 \\ &\quad - ieA^\mu (\phi_1 \partial_\mu \phi_1^* - \phi_1^* \partial_\mu \phi_1) \\ &\quad + g \phi_1^* \phi_1 \sigma + g \phi_2^* \phi_2 \sigma + e^2 A^\mu A_\mu \phi_1 \phi_1^*. \end{aligned} \quad (1)$$

In the calculation of the propagator for a particle in a background field we use the interaction Lagrangian of a scalar field and an electromagnetic field. The Lagrangian (1) immediately shows that there are two types of vertices. The first term corresponds to a vertex containing a photon and two scalar particles. The second vertex contains two photons and two scalar particles.

In this framework we construct the electromagnetic current operator for the system composed of two free bosons in the light front. The technique we use to deduce such operators is to define the global propagators in the light front when an electromagnetic background field acts on one of the particles. Although we are in fact calculating the global propagator for two bosons in an electromagnetic background field, we extrapolate the language using terms such as ‘‘current operator’’ and ‘‘current’’ to designate such operations. We show that for the J^- case the two free boson propagators in a background field have a contribution from the process of the photon's pair production, which is crucial to restore the current's covariance.

The normalized generating functional is given by

$$Z[J] = \frac{\int \mathcal{D}\phi \exp[iS + i \int dx J\phi]}{\int \mathcal{D}\phi e^{iS}}, \quad (2)$$

where $S = \int \mathcal{L} dx$ is the relevant action. In the Appendix we have the expression $Z_0[J]$ for the free particle and the corresponding propagator (A7). So, we can find the propagators, or Green's functions, in an electromagnetic field.

Equation (2) indicates the propagation of two bosons S_1 and S_2 from $x^+ = 0$ to $x^+ > 0$ interacting with an external

electromagnetic field $A^\mu(x^+)$ at \bar{x}_3^+ and with the exchange of two intermediate bosons σ between \bar{x}_1^+ and \bar{x}_2^+ . The propagator $S_3(\bar{x}_3^+ - \bar{x}_1^+)$, which is the propagation of a boson after the emission of the boson σ at \bar{x}_1^+ , later interacts with the external field at \bar{x}_3^+ . The propagator S_5 is the boson propagation after the interaction with the external field. The propagator S_4 is the boson propagation after the absorption of the intermediate σ boson. Therefore the correction to the free propagator of two bosons in the light front with a background field in the ladder diagram is

$$S(x^+) = (-ie)(ig)^2 \int d\bar{x}_1^+ d\bar{x}_2^+ d\bar{x}_3^+ dq^- A^\mu(q^-) e^{-\frac{i}{2}q^- \bar{x}_3^+} \\ \times S_1(\bar{x}_1^+) S_2(\bar{x}_2^+) S_4(x^+ - \bar{x}_2^+) S_\sigma(\bar{x}_2^+ - \bar{x}_1^+) \\ \times \left[S_3 \frac{\partial S_5}{\partial \bar{x}_3^\mu} - \frac{\partial S_3}{\partial \bar{x}_3^\mu} S_5 \right], \quad (3)$$

where $A^\mu(q^-)$ is the Fourier transform and μ indicates the components $-$, $+$, \perp .

The diagram in Fig. 1 shows the perturbative correction to the propagator with a source up to the order of $\mathcal{O}(g^2)$, i.e., with an intermediate boson exchange. This intermediate boson σ propagates between the time intervals $\bar{x}_2^+ - \bar{x}_1^+$, and the source term q couples to the field at the point \bar{x}_3^+ . The indices 1, 2, 3, 4, and 5 label the initial momenta k_1 and k_2 , the internal momentum k_3 , and the final momenta k_4 and k_5 , respectively.

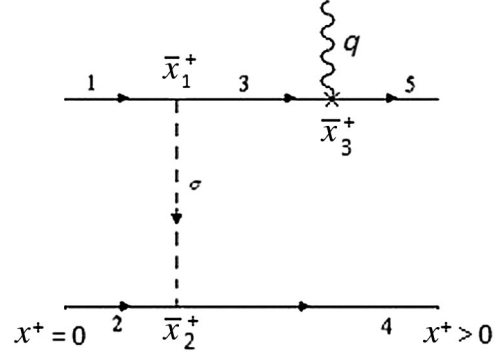


FIG. 1. The $\mathcal{O}(g^2)$ correction to the propagator with a source and an intermediate scalar boson created at \bar{x}_1^+ and absorbed at \bar{x}_2^+ . The external photon interacts at the light-front time \bar{x}_3^+ . Initial and final light-front times $x^+ = 0$ and $x^+ > 0$ are located on the left and right, respectively. These in general—and henceforth—are omitted in the diagrams. However, this ordering is important to bear in mind for all time-ordered diagrams that we consider.

The part of the interaction Lagrangian that contains the vector-scalar-scalar vertex can be redefined as

$$\mathcal{L}_{\text{Interaction}} = -ieA^\mu(\phi_1 \partial_\mu \phi_1^* - \phi_1^* \partial_\mu \phi_1) = J_\mu A^\mu. \quad (4)$$

So, we observe that the operator component J^μ is obtained from the operator \mathcal{O}^μ which we identified with the help of Eqs. (3) and (4) such that

$$\mathcal{O}^\mu = (-ie)(ig)^2 \int d\bar{x}_1^+ d\bar{x}_2^+ d\bar{x}_3^+ e^{-\frac{i}{2}q^- \bar{x}_3^+} S_1(\bar{x}_1^+) S_2(\bar{x}_2^+) S_4(x^+ - \bar{x}_2^+) S_\sigma(\bar{x}_2^+ - \bar{x}_1^+) \left[S_3 \frac{\partial S_5}{\partial \bar{x}_3^\mu} - \frac{\partial S_3}{\partial \bar{x}_3^\mu} S_5 \right], \quad (5)$$

where the Greek index μ indicates the light-front components $+$, $-$, or \perp .

Therefore, using the definition for the propagator and making explicit the integration and the k^- , k^+ and k_\perp components, we have the following:

$$S_1(x^+) = \frac{i}{2(2\pi)^4} \int \frac{dk_1^- dk_1^+ d^2 k_{1\perp}}{k_1^+} \frac{e^{-ik_1^- \bar{x}_1^+} e^{-ik_1^+ \bar{x}_1^-} e^{i\vec{k}_{1\perp} \cdot \vec{x}_{1\perp}}}{\left[k_1^- - \frac{k_{1\perp}^2 + m^2}{2k_1^+} + \frac{i\epsilon}{2k_1^+} \right]}, \\ S_2(x^+) = \frac{i}{2(2\pi)^4} \int \frac{dk_2^- dk_2^+ d^2 k_{2\perp}}{k_2^+} \frac{e^{-ik_2^- \bar{x}_2^+} e^{-ik_2^+ \bar{x}_2^-} e^{i\vec{k}_{2\perp} \cdot \vec{x}_{2\perp}}}{\left[k_2^- - \frac{k_{2\perp}^2 + m^2}{2k_2^+} + \frac{i\epsilon}{2k_2^+} \right]}, \\ S_3(x^+) = \frac{i}{2(2\pi)^4} \int \frac{dk_3^- dk_3^+ d^2 k_{3\perp}}{k_3^+} \frac{e^{-ik_3^- (\bar{x}_3^+ - \bar{x}_1^+)} e^{-ik_3^+ (\bar{x}_3^- - \bar{x}_1^-)} e^{i\vec{k}_{3\perp} \cdot (\vec{x}_3 - \vec{x}_1)_\perp}}{\left[k_3^- - \frac{k_{3\perp}^2 + m^2}{2k_3^+} + \frac{i\epsilon}{2k_3^+} \right]}, \\ S_4(x^+) = \frac{i}{2(2\pi)^4} \int \frac{dk_4^- dk_4^+ d^2 k_{4\perp}}{k_4^+} \frac{e^{-ik_4^- (x^+ - \bar{x}_2^+)} e^{-ik_4^+ (x^- - \bar{x}_2^-)} e^{i\vec{k}_{4\perp} \cdot (\vec{x} - \vec{x}_2)_\perp}}{\left[k_4^- - \frac{k_{4\perp}^2 + m^2}{2k_4^+} + \frac{i\epsilon}{2k_4^+} \right]}, \\ S_5(x^+) = \frac{i}{2(2\pi)^4} \int \frac{dk_5^- dk_5^+ d^2 k_{5\perp}}{k_5^+} \frac{e^{-ik_5^- (x^+ - \bar{x}_3^+)} e^{-ik_5^+ (x^- - \bar{x}_3^-)} e^{i\vec{k}_{5\perp} \cdot (\vec{x} - \vec{x}_3)_\perp}}{\left[k_5^- - \frac{k_{5\perp}^2 + m^2}{2k_5^+} + \frac{i\epsilon}{2k_5^+} \right]}, \\ S_\sigma(x^+) = \frac{i}{2(2\pi)^4} \int \frac{dk_\sigma^- dk_\sigma^+ d^2 k_{\sigma\perp}}{k_\sigma^+} \frac{e^{-ik_\sigma^- (\bar{x}_2^+ - \bar{x}_1^+)} e^{-ik_\sigma^+ (\bar{x}_2^- - \bar{x}_1^-)} e^{i\vec{k}_{\sigma\perp} \cdot (\vec{x}_2 - \vec{x}_1)_\perp}}{\left[k_\sigma^- - \frac{k_{\sigma\perp}^2 + m^2}{2k_\sigma^+} + \frac{i\epsilon}{2k_\sigma^+} \right]}. \quad (6)$$

For the bosons identified by the labels 3 and 5, we also need the derivatives with respect to \bar{x}_3^μ as follows:

(i) With respect to component \bar{x}_3^+ :

$$\frac{\partial S_5}{\partial \bar{x}_3^+} = -\frac{1}{2(2\pi)^4} \int \frac{dk_5^- dk_5^+ d^2 k_{5\perp}}{k_5^+} \frac{k_5^- e^{-ik_5^-(x^+ - \bar{x}_3^+)} e^{-ik_5^+(x^- - \bar{x}_3^-)} e^{ik_{5\perp} \cdot (\bar{x} - \bar{x}_3)_\perp}}{\left[k_5^- - \frac{k_{5\perp}^2 + m^2}{2k_5^+} + \frac{i\epsilon}{2k_5^+} \right]},$$

$$\frac{\partial S_3}{\partial \bar{x}_3^+} = \frac{1}{2(2\pi)^4} \int \frac{dk_3^- dk_3^+ d^2 k_{3\perp}}{k_3^+} \frac{k_3^- e^{-ik_3^-(\bar{x}_3^+ - \bar{x}_1^+)} e^{-ik_3^+(\bar{x}_3^- - \bar{x}_1^-)} e^{ik_{3\perp} \cdot (\bar{x}_3 - \bar{x}_1)_\perp}}{\left[k_3^- - \frac{k_{3\perp}^2 + m^2}{2k_3^+} + \frac{i\epsilon}{2k_3^+} \right]}.$$
(7)

(ii) With respect to component \bar{x}_3^- :

$$\frac{\partial S_5}{\partial \bar{x}_3^-} = -\frac{1}{2(2\pi)^4} \int \frac{dk_5^- dk_5^+ d^2 k_{5\perp}}{k_5^+} \frac{k_5^+ e^{-ik_5^-(x^+ - \bar{x}_3^+)} e^{-ik_5^+(x^- - \bar{x}_3^-)} e^{ik_{5\perp} \cdot (\bar{x} - \bar{x}_3)_\perp}}{\left[k_5^- - \frac{k_{5\perp}^2 + m^2}{2k_5^+} + \frac{i\epsilon}{2k_5^+} \right]},$$

$$\frac{\partial S_3}{\partial \bar{x}_3^-} = \frac{1}{2(2\pi)^4} \int \frac{dk_3^- dk_3^+ d^2 k_{3\perp}}{k_3^+} \frac{k_3^+ e^{-ik_3^-(\bar{x}_3^+ - \bar{x}_1^+)} e^{-ik_3^+(\bar{x}_3^- - \bar{x}_1^-)} e^{ik_{3\perp} \cdot (\bar{x}_3 - \bar{x}_1)_\perp}}{\left[k_3^- - \frac{k_{3\perp}^2 + m^2}{2k_3^+} + \frac{i\epsilon}{2k_3^+} \right]}.$$
(8)

(iii) With respect to component $\bar{x}_{3\perp}$:

$$\frac{\partial S_5}{\partial \bar{x}_{3\perp}} = \frac{1}{2(2\pi)^4} \int \frac{dk_5^- dk_5^+ d^2 k_{5\perp}}{k_5^+} \frac{k_{5\perp} e^{-ik_5^-(x^+ - \bar{x}_3^+)} e^{-ik_5^+(x^- - \bar{x}_3^-)} e^{ik_{5\perp} \cdot (\bar{x} - \bar{x}_3)_\perp}}{\left[k_5^- - \frac{k_{5\perp}^2 + m^2}{2k_5^+} + \frac{i\epsilon}{2k_5^+} \right]},$$

$$\frac{\partial S_3}{\partial \bar{x}_{3\perp}} = -\frac{1}{2(2\pi)^4} \int \frac{dk_3^- dk_3^+ d^2 k_{3\perp}}{k_3^+} \frac{k_{3\perp} e^{-ik_3^-(\bar{x}_3^+ - \bar{x}_1^+)} e^{-ik_3^+(\bar{x}_3^- - \bar{x}_1^-)} e^{ik_{3\perp} \cdot (\bar{x}_3 - \bar{x}_1)_\perp}}{\left[k_3^- - \frac{k_{3\perp}^2 + m^2}{2k_3^+} + \frac{i\epsilon}{2k_3^+} \right]}.$$
(9)

Substituting Eq. (6) and the relevant derivatives above in Eq. (3) and performing the integrations over $d\bar{x}_1^+ d\bar{x}_2^+ d\bar{x}_3^+$, we can evaluate the Fourier transform $\tilde{S}(k_f^-) = \int dx^+ e^{ik_f^- x^+} S(x^+)$ with the help of the following momentum conservation relations:

$$\begin{aligned} k_i &= k_1 + k_2, & k_f &= k_4 + k_5, & k_f &= k_i + q, & k_3 &= k_i - k_4, \\ q &= k_5 - k_3, & k_\sigma &= k_4 - k_2, & k_\sigma &= k_1 - k_3, \end{aligned}$$
(10)

where k_i is the total initial momentum and k_f the total final momentum. These momentum conservation equations can be checked, for example, using Fig. 1.

The final propagator can therefore be written as a function of only two momenta, and in this case we choose ‘‘spectator’’ particles with respect to the current, those labeled as 2 and 4:

$$\begin{aligned} \tilde{S}(k_f^-) &= -\frac{ie(ig)^2}{2^6(2\pi)^2} \int dq^- A^\mu(q^-) \left\{ \int \frac{dk_2^- dk_4^-(k_f^\mu + k_i^\mu - 2k_4^\mu)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \right. \\ &\quad \times \frac{1}{\left[k_2^- - k_i^- + (k_i - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_2)^+} \right] \left[k_2^- - k_2^- + \frac{i\epsilon}{2k_2^+} \right] \left[k_4^- - k_i^- + (k_i - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_4)^+} \right] \left[k_4^- - k_4^- + \frac{i\epsilon}{2k_4^+} \right]} \\ &\quad \left. \times \frac{1}{\left[k_2^- - k_4^- + (k_4 - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_4 - k_2)^+} \right] \left[k_4^- - k_f^- + (k_f - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_f - k_4)^+} \right]} \right\}, \end{aligned}$$
(11)

where

$$\begin{aligned}
k_{2\text{on}} &= \frac{k_{2\perp}^2 + m^2}{2k_2^+}, & (k_i - k_2)_{\text{on}} &= \frac{(k_i - k_2)_{\perp}^2 + m^2}{2(k_i^+ - k_2^+)}, & (k_i - k_4)_{\text{on}} &= \frac{(k_i - k_4)_{\perp}^2 + m^2}{2(k_i^+ - k_4^+)}, \\
k_{4\text{on}} &= \frac{k_{4\perp}^2 + m^2}{2k_4^+}, & (k_f - k_4)_{\text{on}} &= \frac{(k_f - k_4)_{\perp}^2 + m^2}{2(k_f^+ - k_4^+)}, & (k_4 - k_2)_{\text{on}} &= \frac{(k_4 - k_2)_{\perp}^2 + m_{\sigma}^2}{2(k_4^+ - k_2^+)}.
\end{aligned} \tag{12}$$

The propagator Eq. (11) in the momentum space representation will be solved using the Cauchy integral formula. Each of the single poles may be located at either the lower half or the upper half of the complex k^- plane, depending on the values of k_2^+ , k_4^+ , k_i^+ and k_f^+ . Altogether there are 24 different possibilities or regions (numbered from 1 to 24) where the propagator may exist (see Table I).

Integrating first in k_2^- we find three poles:

$$\begin{aligned}
1) \quad k_2^- &= k_i^- - (k_i - k_2)_{\text{on}} + \frac{i\varepsilon}{2(k_i - k_2)^+} \\
2) \quad k_2^- &= k_{2\text{on}} - \frac{i\varepsilon}{2k_2^+} \\
3) \quad k_2^- &= k_4^- - (k_4 - k_2)_{\text{on}} + \frac{i\varepsilon}{2(k_4 - k_2)^+}.
\end{aligned} \tag{13}$$

Depending on the region chosen from Table I, these poles will locate themselves either in the lower half or the upper half of the complex k^- plane. Analyzing the possibilities one by one, we see that, upon integrating first in k_2^- , eight of such regions give vanishing contributions (all poles are located on the same half of the complex k^- plane), leaving us with 16 regions yet to be analyzed. Table II summarizes the relevant results for the k_2^- pole locations.

As can be seen from this table, poles 1, 2 and 3 of Eq. (13) are all located in the lower half of the complex

k_2^- plane for the eight regions 5, 6, 19, 20, 21, 22, 23 and 24 (cf. Table II) so that all eight regions yield a vanishing contribution to the k_2^- integration.

The next step is the integration in k_4^- , but here things get complicated because, although the number of regions decreased to 16, now there are many more poles. To make this transparent we will consider conveniently defined parts. The following poles arise within the corresponding 16 possible regions for integration in k_4^- :

(i) For the eight regions 1, 3, 7, 9, 10, 13, 15 and 16 there are four poles:

$$\begin{aligned}
1) \quad k_4^- &= k_i^- - (k_i - k_4)_{\text{on}} + \frac{i\varepsilon}{2(k_i - k_4)^+} \\
2) \quad k_4^- &= k_{4\text{on}} - \frac{i\varepsilon}{2k_4^+} \\
3) \quad k_4^- &= k_f^- - (k_f - k_4)_{\text{on}} + \frac{i\varepsilon}{2(k_f - k_4)^+} \\
4) \quad k_4^- &= k_{2\text{on}} + (k_4 - k_2)_{\text{on}} - \frac{i\varepsilon}{2(k_4 - k_2)^+}.
\end{aligned} \tag{14}$$

Table III summarizes the relevant results for the k_4^- pole locations. From this table, it is clear that regions 13, 15 and 16 yield vanishing contributions to the k_4^- integration.

(ii) For the four regions 2, 8, 11 and 12 there are four more poles:

TABLE I. The 24 possible domains or regions defined by the longitudinal momentum intervals where the propagator Eq. (11) may, in principle, exist.

1) $0 < k_2^+ < k_4^+ < k_i^+ < k_f^+$	9) $0 < k_2^+ < k_f^+ < k_4^+ < k_i^+$	17) $0 < k_i^+ < k_f^+ < k_2^+ < k_4^+$
2) $0 < k_4^+ < k_2^+ < k_i^+ < k_f^+$	10) $0 < k_f^+ < k_2^+ < k_4^+ < k_i^+$	18) $0 < k_f^+ < k_i^+ < k_2^+ < k_4^+$
3) $0 < k_2^+ < k_i^+ < k_4^+ < k_f^+$	11) $0 < k_4^+ < k_f^+ < k_2^+ < k_i^+$	19) $0 < k_4^+ < k_i^+ < k_f^+ < k_2^+$
4) $0 < k_i^+ < k_2^+ < k_4^+ < k_f^+$	12) $0 < k_f^+ < k_4^+ < k_2^+ < k_i^+$	20) $0 < k_i^+ < k_4^+ < k_f^+ < k_2^+$
5) $0 < k_4^+ < k_i^+ < k_2^+ < k_f^+$	13) $0 < k_2^+ < k_i^+ < k_f^+ < k_4^+$	21) $0 < k_4^+ < k_f^+ < k_i^+ < k_2^+$
6) $0 < k_i^+ < k_4^+ < k_2^+ < k_f^+$	14) $0 < k_i^+ < k_2^+ < k_f^+ < k_4^+$	22) $0 < k_f^+ < k_4^+ < k_i^+ < k_2^+$
7) $0 < k_2^+ < k_4^+ < k_f^+ < k_i^+$	15) $0 < k_2^+ < k_f^+ < k_i^+ < k_4^+$	23) $0 < k_i^+ < k_f^+ < k_4^+ < k_2^+$
8) $0 < k_4^+ < k_2^+ < k_f^+ < k_i^+$	16) $0 < k_f^+ < k_2^+ < k_i^+ < k_4^+$	24) $0 < k_f^+ < k_i^+ < k_4^+ < k_2^+$

TABLE II. Location of poles 1, 2 and 3 of Eq. (13) in the complex k_2^- plane.

Upper half of k_2^- plane	Lower half of k_2^- plane	Regions
Pole 3	Poles 1 and 2	4, 14, 17 and 18
Pole 1	Poles 2 and 3	2, 8, 11 and 12
Poles 1 and 3	Pole 2	1, 3, 7, 9, 10, 13, 15 and 16
No pole	Poles 1, 2 and 3	5, 6, 19, 20, 21, 22, 23 and 24

TABLE III. Location of poles 1, 2, 3 and 4 of Eq. (14) in the complex k_4^- plane.

Upper half of k_4^- plane	Lower half of k_4^- plane	Regions
Poles 1 and 3	Poles 2 and 4	1 and 7
Pole 3	Poles 1, 2 and 4	3
Pole 1	Poles 2, 3 and 4	9 and 10
No pole	Poles 1, 2, 3 and 4	13, 15 and 16

$$\begin{aligned}
5) \quad k_4^- &= k_i^- - (k_i - k_4)_{\text{on}} + \frac{i\varepsilon}{2(k_i - k_4)^+} \\
6) \quad k_4^- &= k_{4_{\text{on}}} - \frac{i\varepsilon}{2k_4^+} \\
7) \quad k_4^- &= k_f^- - (k_f - k_4)_{\text{on}} + \frac{i\varepsilon}{2(k_f - k_4)^+} \\
8) \quad k_4^- &= k_i^- - (k_i - k_2)_{\text{on}} + (k_4 - k_2)_{\text{on}} \\
&\quad - \frac{i\varepsilon}{2(k_4 - k_2)^+}.
\end{aligned} \tag{15}$$

Table IV summarizes the relevant results for the k_4^- pole locations. Here all the pertinent regions yield nonvanishing contributions to the k_4^- integration.

(iii) For the last four regions, 4, 14, 17 and 18, there are five more poles:

$$\begin{aligned}
9) \quad k_4^- &= k_i^- - (k_i - k_2)_{\text{on}} + (k_4 - k_2)_{\text{on}} + \frac{i\varepsilon}{2(k_i - k_2)^+} \\
10) \quad k_4^- &= k_{2_{\text{on}}} + (k_4 - k_2)_{\text{on}} - \frac{i\varepsilon}{2k_2^+} \\
11) \quad k_4^- &= k_i^- - (k_i - k_4)_{\text{on}} + \frac{i\varepsilon}{2(k_i - k_4)^+} \\
12) \quad k_4^- &= k_{4_{\text{on}}} - \frac{i\varepsilon}{2k_4^+} \\
13) \quad k_4^- &= k_f^- - (k_f - k_4)_{\text{on}} + \frac{i\varepsilon}{2(k_f - k_4)^+}.
\end{aligned} \tag{16}$$

Table V summarizes the relevant results for the k_4^- pole locations. From this table, it is clear that

TABLE IV. Location of poles 5, 6, 7 and 8 of Eq. (15) in the complex k_4^- plane.

Upper half of k_4^- plane	Lower half of k_4^- plane	Regions
Poles 5, 7 and 8	Pole 6	2, 8 and 11
Pole 5 and 8	Poles 6 and 7	12

TABLE V. Location of poles 9, 10, 11, 12 and 13 of Eq. (16) in the complex k_4^- plane.

Upper half of k_4^- plane	Lower half of k_4^- plane	Regions
Pole 13	Poles 9, 10, 11 and 12	4
No pole	Poles 9, 10, 11, 12 and 13	14, 17 and 18

regions 14, 17 and 18 are such that k_4^- integration is zero since the upper half-plane is analytic and the residues there are zero.

More details of these steps are given in the Appendix. Tables III through V indicate that the ten nonvanishing contributions arise from regions 1, 2, 3, 4, 7, 8, 9, 10, 11 and 12. The nonvanishing contributions are represented in Figs. 2–7 according to their specific set of regions.

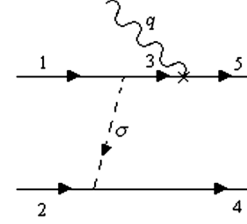


FIG. 2. Light-front time-ordered diagram related to regions 2, 8 and 11 of the longitudinal momentum interval possibilities as given in Table I.

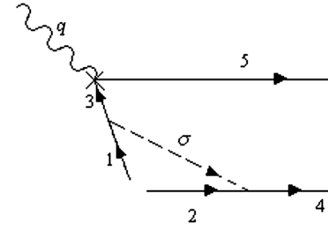


FIG. 3. Light-front time-ordered diagram related to region 4 of the longitudinal momentum interval possibilities as given in Table I.

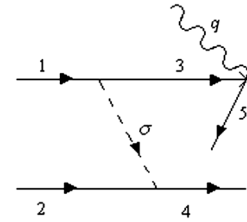


FIG. 4. Light-front time-ordered diagram related to regions 9 and 10 of the longitudinal momentum interval possibilities as given in Table I.

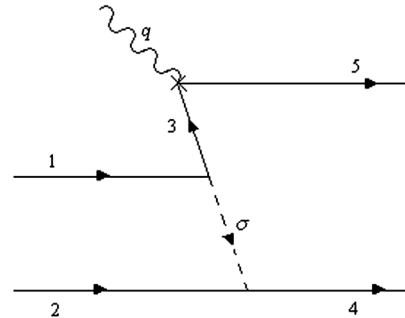


FIG. 5. Light-front time-ordered diagram related to region 3 of the longitudinal momentum interval possibilities as given in Table I.

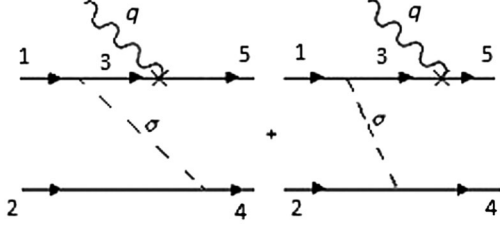


FIG. 6. Light-front time-ordered diagram related to regions 1 and 7 of the longitudinal momentum interval possibilities as given in Table I. Note that here we have two possibilities: In the first diagram the background field is associated with the propagator of three bodies (particle 3, intermediate sigma boson and particle 2). In the second diagram we have the association with two bodies (particles 3 and 4). These diagrams are representative of the detailed calculation in the Appendix resulting in Eq. (B10).

We begin our discussion with an illustrative example, where the pair term appears. To this end we use the “Z-graph,” that is, region 3 in Fig. 5, for which the various longitudinal momenta k^+ are defined within the range of possibilities $0 < k_2^+ < k_i^+ < k_4^+ < k_f^+$ (see Table I). In this example we show that the current’s J^-

$$\begin{aligned} \mathcal{O}_3^\mu = & -\frac{ie(ig)^2}{2^6(2\pi)^2} \int \frac{dk_2^- dk_4^-(k_f^\mu + k_i^\mu - 2k_4^\mu)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \\ & \times \frac{1}{\left[k_2^- - k_i^- + (k_i - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_2)^+} \right] \left[k_2^- - k_2_{\text{on}} + \frac{i\epsilon}{2k_2^+} \right]} \frac{1}{\left[k_4^- - k_i^- + (k_i - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_4)^+} \right] \left[k_4^- - k_4_{\text{on}} + \frac{i\epsilon}{2k_4^+} \right]} \\ & \times \frac{1}{\left[k_2^- - k_4^- + (k_4 - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_4 - k_2)^+} \right] \left[k_4^- - k_f^- + (k_f - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_f - k_4)^+} \right]}, \end{aligned} \quad (17)$$

where k_i^μ and k_f^μ are the initial and final four-momentum of the system and m is the mass of the boson. The integration in Eq. (17), using the Cauchy integral formula over k_2^- and k_4^- , has ten nonvanishing contributions for the residue calculation, but for our example, we concentrate on a specific region, that is, the range of momenta satisfying $0 < k_2^+ < k_i^+ < k_4^+ < k_f^+$, which corresponds to the Z-graph or what we call region 3. Thus we are ready to

$$\begin{aligned} \mathcal{O}_3^- = & \frac{ie(ig)^2}{2^6} \frac{\theta(k_i^+ - k_2^+) \theta(k_2^+) \theta(k_4^+ - k_i^+) \theta(k_4^+) \theta(k_f^+ - k_4^+) \theta(k_4^+ - k_2^+)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \frac{\left[k_f^- - k_i^- - \frac{(k_f - k_4)_1^2 + m^2}{(k_f - k_4)^+} \right]}{\left[k_i^- - \frac{(k_i - k_2)_1^2 + m^2}{2(k_i - k_2)^+} - \frac{k_2^2 + m^2}{2k_2^+} \right]} \\ & \times \frac{1}{\left[k_f^- - \frac{k_2^2 + m^2}{2k_2^+} - \frac{(k_f - k_4)_1^2 + m^2}{2(k_f - k_4)^+} - \frac{(k_4 - k_2)_1^2 + m^2}{2(k_4 - k_2)^+} \right]} \frac{1}{\left[k_f^- - k_i^- + \frac{(k_i - k_4)_1^2 + m^2}{2(k_i - k_4)^+} - \frac{(k_f - k_4)_1^2 + m^2}{2(k_f - k_4)^+} \right] \left[k_f^- - \frac{k_4^2 + m^2}{2k_4^+} - \frac{(k_f - k_4)_1^2 + m^2}{2(k_f - k_4)^+} \right]}. \end{aligned} \quad (18)$$

The physical process represented by Eq. (18) is the pair creation due to the interacting photon. The denominator

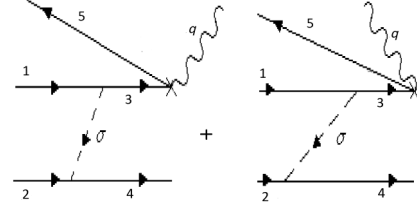


FIG. 7. Light-front time-ordered diagrams related to region 12 of the longitudinal momentum interval possibilities as given in Table I. Observe that in the diagram on the left, $q^- < 0$, which means we have an antiparticle for the photon.

component does not have a contribution from the pair production in the Drell-Yan reference frame [16], that is, in the limit $q^+ = q^- = 0$. For all the other nine significant regions, the calculations follow suit; thus we limit ourselves to considering detailed steps only for the region pertinent to Fig. 5 as an illustrative case for all of them.

Then, for region 3 of Fig. 5, we look for the components of the current operator J^μ , which, as we mentioned before, will be obtained from the operator \mathcal{O}^μ represented by the square brackets in Eq. (3), so

work out the distinct components of the operator, which we detail in the next section.

III. OPERATOR COMPONENTS $\mathcal{O}^{-,+, \perp}$

After performing the relevant integrals via Cauchy’s residue theorem in Eq. (17), we have for the $-$ component

$$\left[k_f^- - k_i^- + \frac{(k_i - k_4)_\perp^2 + m^2}{2(k_i^+ - k_4^+)} - \frac{(k_f - k_4)_\perp^2 + m^2}{2(k_f^+ - k_4^+)} \right]^{-1}$$

corresponds to the propagation in the intermediate state of a pair particle-antiparticle, composed of the initial bound state, the particle and the antiparticle produced by the photon. The denominators

$$\left[k_i^- - \frac{(k_i - k_2)_\perp^2 + m^2}{2(k_i^+ - k_2^+)} - \frac{k_{2\perp}^2 + m^2}{2k_2^+} \right]^{-1},$$

$$\left[k_f^- - \frac{k_{2\perp}^2 + m^2}{2k_2^+} - \frac{(k_f - k_4)_\perp^2 + m^2}{2(k_f^+ - k_4^+)} - \frac{(k_4 - k_2)_\perp^2 + m^2}{2(k_4^+ - k_2^+)} \right]^{-1},$$

and

$$\left[k_f^- - \frac{k_{4\perp}^2 + m^2}{2k_4^+} - \frac{(k_f - k_4)_\perp^2 + m^2}{2(k_f^+ - k_4^+)} \right]^{-1}$$

are the intermediate states of two and three particles that propagate forward in time x^+ .

In a similar way, we obtain for the components \mathcal{O}_3^+ and \mathcal{O}_3^\perp

$$\mathcal{O}_3^\beta = -\frac{ie(ig)^2}{2^6} \frac{\theta(k_i^+ - k_2^+)\theta(k_2^+)\theta(k_4^+ - k_i^+)\theta(k_4^+)\theta(k_f^+ - k_4^+)\theta(k_4^+ - k_2^+)}{(k_i^+ - k_2^+)k_2^+(k_i^+ - k_4^+)k_4^+(k_f^+ - k_4^+)(k_4^+ - k_2^+)} \frac{[k_f^\beta + k_i^\beta - 2k_4^\beta]}{\left[k_i^- - \frac{(k_i - k_2)_\perp^2 + m^2}{2(k_i^+ - k_2^+)} - \frac{k_{2\perp}^2 + m^2}{2k_2^+} \right]}$$

$$\times \frac{1}{\left[k_f^- - \frac{k_{2\perp}^2 + m^2}{2k_2^+} - \frac{(k_f - k_4)_\perp^2 + m^2}{2(k_f^+ - k_4^+)} - \frac{(k_4 - k_2)_\perp^2 + m^2}{2(k_4^+ - k_2^+)} \right]} \frac{1}{\left[k_f^- - k_i^- + \frac{(k_i - k_4)_\perp^2 + m^2}{2(k_i^+ - k_4^+)} - \frac{(k_f - k_4)_\perp^2 + m^2}{2(k_f^+ - k_4^+)} \right]} \frac{1}{\left[k_f^- - \frac{k_{4\perp}^2 + m^2}{2k_4^+} - \frac{(k_f - k_4)_\perp^2 + m^2}{2(k_f^+ - k_4^+)} \right]}, \quad (19)$$

where we have introduced the notation $\beta = +, \perp$. The difference between the operators \mathcal{O}_3^- and \mathcal{O}_3^β is in the numerators of Eqs. (18) and (19), which have components $+$ and \perp instead of $-$.

IV. ZERO MODE CONTRIBUTION AT $\mathcal{O}(g^2)$

To calculate the electromagnetic current generated by the diverse configurations, we must have the matrix elements $J^{\cdot,+,\perp} = \langle \Gamma | \mathcal{O}^{\cdot,+,\perp} | \Gamma \rangle$, where Γ is the constant vertex and $\mathcal{O}^{\cdot,+,\perp}$ are the current operator components, which we can obtain directly from the sum of the final results in each region. Introducing the unit resolution into the matrix element we have

$$\begin{aligned} \langle \Gamma | \mathcal{O}^{\cdot,+,\perp} | \Gamma \rangle &= \int dk_j^+ d^2k_{j\perp} \langle \Gamma | k_j^+, \vec{k}_{j\perp} \rangle \langle k_j^+, \vec{k}_{j\perp} | \mathcal{O}^{\cdot,+,\perp} \int dk_j'^+ d^2k_{j\perp}' | k_j'^+, \vec{k}_{j\perp}' \rangle \langle k_j'^+, \vec{k}_{j\perp}' | \Gamma \rangle \\ &= \Gamma \int dk_j^+ d^2k_{j\perp} dk_j'^+ d^2k_{j\perp}' \langle k_j^+, \vec{k}_{j\perp} | \mathcal{O}^{\cdot,+,\perp} | k_j'^+, \vec{k}_{j\perp}' \rangle \Gamma \\ &= \Gamma^2 \int dk_j^+ d^2k_{j\perp} dk_j'^+ d^2k_{j\perp}' \delta(k_j^+ - k_j'^+ - q^+) \delta(\vec{k}_{j\perp} - \vec{k}_{j\perp}' - \vec{q}_\perp) \langle k_j^+, \vec{k}_{j\perp} | \mathcal{O}^{\cdot,+,\perp} | k_j'^+, \vec{k}_{j\perp}' \rangle \\ &= \Gamma^2 \int dk_2^+ d^2k_{2\perp} dk_4^+ d^2k_{4\perp} \mathcal{O}^{\cdot,+,\perp}. \end{aligned} \quad (20)$$

Thus, for the example of Fig. 5 the electromagnetic current $J_3^{\cdot,+,\perp}$, pertinent to region 3 with momentum range $0 < k_2^+ < k_i^+ < k_4^+ < k_f^+$, is obtained by introducing in the integrand above the current operator components \mathcal{O}_3^- given in Eq. (18) and \mathcal{O}_3^β given in Eq. (19).

Our next step is to perform the remaining momentum integration over k_2^+ and k_4^+ and take the limit $q^+ \rightarrow 0$. To calculate the momentum integrations we make two changes of variables that will facilitate our job of integrating them,

$$x = \frac{k_i^+ - k_2^+}{q^+} \quad y = \frac{k_f^+ - k_4^+}{q^+}. \quad (21)$$

On the other hand, taking advantage of the momentum conservation relations in Eq. (10) we get

$$k_1^+ = xq^+ \quad k_3^+ = (y-1)q^+ \quad k_5^+ = yq^+ \quad k_\sigma^+ = (x-y+1)q^+. \quad (22)$$

Now it is just a matter of putting things together. We begin by taking the $-$ component.

(i) *Current J_3^-* Substituting Eqs. (21) and (22) into Eq. (20) we get

$$J_3^- = \langle \Gamma | \mathcal{O}_3^- | \Gamma \rangle = \Gamma^2 \int dk_2^+ d^2 k_{2\perp} dk_4^+ d^2 k_{4\perp} \mathcal{O}^- = \Gamma^2 \int d^2 k_{2\perp} d^2 k_{4\perp} \left\{ (q^+)^2 \int dx dy \mathcal{O}_3^- \right\}, \quad (23)$$

where the operator contribution \mathcal{O}_3^- takes the following form:

$$\begin{aligned} \mathcal{O}_3^- &= \frac{ie(ig)^2}{2^6} \frac{\theta(k_i^+ - k_2^+) \theta(k_2^+) \theta(k_4^+ - k_i^+) \theta(k_4^+) \theta(k_f^+ - k_4^+) \theta(k_4^+ - k_2^+)}{xq^+(k_i^+ - xq^+)(y-1)q^+(k_f^+ - yq^+)yq^+(x-y+1)q^+} \frac{\left[k_f^- - k_i^- - \frac{(k_f - k_4)_\perp^2 + m^2}{yq^+} \right]}{\left[k_i^- - \frac{(k_i - k_2)_\perp^2 + m^2}{2xq^+} - \frac{k_{2\perp}^2 + m^2}{2(k_i^+ - xq^+)} \right]} \\ &\times \frac{1}{\left[k_f^- - \frac{k_{2\perp}^2 + m^2}{2(k_i^+ - xq^+)} - \frac{(k_f - k_4)_\perp^2 + m^2}{2yq^+} - \frac{(k_4 - k_2)_\perp^2 + m_\sigma^2}{2(x-y+1)q^+} \right]} \frac{1}{\left[k_f^- - k_i^- + \frac{(k_i - k_4)_\perp^2 + m^2}{2(y-1)q^+} - \frac{(k_f - k_4)_\perp^2 + m^2}{2yq^+} \right]} \\ &\times \frac{1}{\left[k_f^- - \frac{k_{4\perp}^2 + m^2}{2(k_f^+ - yq^+)} - \frac{(k_f - k_4)_\perp^2 + m^2}{2yq^+} \right]}, \end{aligned} \quad (24)$$

which can be written in a more convenient form, factoring out all the relevant factors of q^+ to make more evident how this particular operator component depends on q^+ :

$$\begin{aligned} \mathcal{O}_3^- &= \frac{ie(ig)^2}{2^6} \left(\frac{1}{q^+} \right) \frac{\theta(k_i^+ - k_2^+) \theta(k_2^+) \theta(k_4^+ - k_i^+) \theta(k_4^+) \theta(k_f^+ - k_4^+) \theta(k_4^+ - k_2^+)}{x(k_i^+ - xq^+)(y-1)(k_f^+ - yq^+)y(x-y+1)} \frac{\left[(k_f^- - k_i^-)q^+ - \frac{(k_f - k_4)_\perp^2 + m^2}{y} \right]}{\left[k_i^- q^+ - \frac{(k_i - k_2)_\perp^2 + m^2}{2x} - \frac{k_{2\perp}^2 + m^2}{2(k_i^+ - xq^+)} q^+ \right]} \\ &\times \frac{1}{\left[k_f^- q^+ - \frac{k_{2\perp}^2 + m^2}{2(k_i^+ - xq^+)} q^+ - \frac{(k_f - k_4)_\perp^2 + m^2}{2y} - \frac{(k_4 - k_2)_\perp^2 + m_\sigma^2}{2(x-y+1)} \right]} \\ &\times \frac{1}{\left[(k_f^- - k_i^-)q^+ + \frac{(k_i - k_4)_\perp^2 + m^2}{2(y-1)} - \frac{(k_f - k_4)_\perp^2 + m^2}{2y} \right] \left[k_f^- q^+ - \frac{k_{4\perp}^2 + m^2}{2(k_f^+ - yq^+)} q^+ - \frac{(k_f - k_4)_\perp^2 + m^2}{2y} \right]}. \end{aligned} \quad (25)$$

We observe that substitution of Eq. (25) into Eq. (23) implies that the matrix element for J_3^- is now proportional to q^+ . Then, in the Drell-Yan reference frame [16], $q^+ = q^- \rightarrow 0$, the current component J_3^- vanishes.

(ii) *Current J_3^+*

In a similar way the component J_3^+ can be worked out and it yields

$$J_3^+ = \langle \Gamma | \mathcal{O}_3^+ | \Gamma \rangle = \Gamma^2 \int dk_2^+ d^2 k_{2\perp} dk_4^+ d^2 k_{4\perp} \mathcal{O}^+ = \Gamma^2 \int d^2 k_{2\perp} d^2 k_{4\perp} \left\{ (q^+)^2 \int dx dy \mathcal{O}_3^+ \right\}, \quad (26)$$

where the operator factor \mathcal{O}_3^+ is now

$$\begin{aligned} \mathcal{O}_3^+ &= \frac{i(ig)^2}{2^6} \frac{\theta(k_i^+ - k_2^+) \theta(k_2^+) \theta(k_4^+ - k_i^+) \theta(k_4^+) \theta(k_f^+ - k_4^+) \theta(k_4^+ - k_2^+)}{x(k_i^+ - xq^+)(y-1)(k_f^+ - yq^+)y(x-y+1)} \frac{q^+(2y-1)}{\left[k_i^- q^+ - \frac{(k_i - k_2)_\perp^2 + m^2}{2x} - \frac{k_{2\perp}^2 + m^2}{2(k_i^+ - xq^+)} q^+ \right]} \\ &\times \frac{1}{\left[k_f^- q^+ - \frac{k_{2\perp}^2 + m^2}{2(k_i^+ - xq^+)} q^+ - \frac{(k_f - k_4)_\perp^2 + m^2}{2y} - \frac{(k_4 - k_2)_\perp^2 + m_\sigma^2}{2(x-y+1)} \right]} \\ &\times \frac{1}{\left[(k_f^- - k_i^-)q^+ + \frac{(k_i - k_4)_\perp^2 + m^2}{2(y-1)} - \frac{(k_f - k_4)_\perp^2 + m^2}{2y} \right] \left[k_f^- q^+ - \frac{k_{4\perp}^2 + m^2}{2(k_f^+ - yq^+)} q^+ - \frac{(k_f - k_4)_\perp^2 + m^2}{2y} \right]}, \end{aligned} \quad (27)$$

where we have used Eqs. (21) and (22) and found that $[k_f^+ + k_i^+ - 2k_4^+] = q^+(2y - 1)$. We observe that in the substitution of Eq. (27) into Eq. (26) the current component J_3^+ is proportional to $(q^+)^3$. Then, in the Drell-Yan frame of reference, the current J^+ also vanishes.

(iii) *Current J_3^\perp*

In the same manner, for J_3^\perp we have

$$J_3^\perp = \langle \Gamma | \mathcal{O}_3^\perp | \Gamma \rangle = \Gamma^2 \int dk_2^+ d^2 k_{2\perp} dk_4^+ d^2 k_{4\perp} \mathcal{O}^\perp = \Gamma^2 \int d^2 k_{2\perp} d^2 k_{4\perp} \left\{ (q^+)^2 \int dx dy \mathcal{O}_3^\perp \right\}, \quad (28)$$

where the operator factor \mathcal{O}_3^\perp has the following form:

$$\begin{aligned} \mathcal{O}_3^\perp &= \frac{i(ig)^2}{2^6} \frac{\theta(k_i^+ - k_2^+) \theta(k_2^+) \theta(k_4^+ - k_i^+) \theta(k_4^+) \theta(k_f^+ - k_4^+) \theta(k_4^+ - k_2^+)}{x(k_i^+ - xq^+)(y-1)(k_f^+ - yq^+)y(x-y+1)} \frac{[k_f^\perp + k_i^\perp - 2k_4^\perp]}{\left[k_i^- q^+ - \frac{(k_i - k_2)_\perp^2 + m^2}{2x} - \frac{k_2^2 + m^2}{2(k_i^+ - xq^+)} q^+ \right]} \\ &\times \frac{1}{\left[k_f^- q^+ - \frac{k_{2\perp}^2 + m^2}{2(k_i^+ - xq^+)} q^+ - \frac{(k_f - k_4)_\perp^2 + m^2}{2y} - \frac{(k_4 - k_2)_\perp^2 + m_{\sigma}^2}{2(x-y+1)} \right]} \frac{1}{\left[(k_f^- - k_i^-) q^+ + \frac{(k_i - k_4)_\perp^2 + m^2}{2(y-1)} - \frac{(k_f - k_4)_\perp^2 + m^2}{2y} \right]} \\ &\times \frac{1}{\left[k_f^- q^+ - \frac{k_{4\perp}^2 + m^2}{2(k_f^+ - yq^+)} q^+ - \frac{(k_f - k_4)_\perp^2 + m^2}{2y} \right]}. \end{aligned} \quad (29)$$

We observe now that in the substitution of Eq. (29) into Eq. (28) the current component J_3^\perp is proportional to $(q^+)^2$. Therefore, in the Drell-Yan reference frame, the current component J_3^\perp also vanishes.

In this manner we have shown that for the current components J_3^- , J_3^+ and J_3^\perp , only the contribution for the residue corresponding to the composite system survives in the integration over k^- via Cauchy's formula method. However, these components do not contribute to the pair production in the limit $q^+ \rightarrow 0$.

Having achieved this result, we want to generalize to J^- for this specific example, counting the terms that bear q^+ , that is, performing a power counting on the factors q^+ for a quick analysis of the result in the frame $q^+ \rightarrow 0$:

$$\begin{aligned} \text{Momentum integration } \int dk_2^+ dk_4^+ &\Rightarrow (q^+)^2 \int dx dy \\ \frac{1}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} &\Rightarrow \frac{1}{(q^+)^4} \\ \text{Legs of the type } \frac{1}{\left[a + \frac{b}{cq^+} + \frac{d}{eq^+} + \dots \right]^4} &\Rightarrow (q^+)^4 \\ \text{Numerator of the type } (a + k_{2\text{on}}) \text{ or } (a + k_{4\text{on}}) &\Rightarrow (q^+)^0 = 1 \\ \text{Numerator of the type } (a + k_{j\text{on}}) \text{ with } j \neq 2 \text{ or } 4 &\Rightarrow \frac{1}{q^+}, \end{aligned} \quad (30)$$

where a represents the momenta k_i , k_f , $k_f - k_i$, etc. and b , d , etc. are $k_{2\text{on}}$, $k_{4\text{on}}$ or $k_{j\text{on}}$. Note that as we multiply all these factors together there will always remain at least a $(q^+)^1$, which, in the limit for $q^+ \rightarrow 0$, makes the current in all regions vanish. What we conclude here is that the introduction of a virtual boson, in comparison to the configuration considered in [14], does not alter the current because the factors in the second and third lines of Eq. (30) cancel each other. The important factor is the photon vertex, since it increases the power in the numerator; only with correct factors of $\frac{1}{q^+}$ can we cancel the factor coming from the change of variables.

In a general manner we can count the terms in q^+ for n intermediate bosons and with one external source; that is, for n bosons and one photon, we have

$$\begin{aligned}
 &\text{Momentum integration } \int \prod_{j=1}^{n+1} dk_{2j}^+ \Rightarrow (q^+)^{n+1} \int \prod_{j=1}^{n+1} dx_j \\
 &\prod_{j=0}^n \frac{1}{(k_i^+ - k_{2j+2}^+)k_{2j+2}^+} \prod_{j=1}^n \frac{1}{(k_{2j+2}^+ - k_{2j}^+)(k_f^+ - k_{2j+2}^+)} \Rightarrow \frac{1}{(q^+)^{2n+2}} \\
 &\text{Legs of the type } \frac{1}{[a + \frac{b}{cq^+} + \frac{d}{eq^+} + \dots]} \Rightarrow (q^+)^{2n+2} \\
 &\text{Numerator of the type } (a + k_{2\text{on}}) \text{ or } (a + k_{4\text{on}}) \Rightarrow (q^+)^0 = 1 \\
 &\text{Numerator of the type } (a + k_{j\text{on}}) \text{ with } j \neq 2, 4, \dots, 2n+2 \Rightarrow \frac{1}{q^+}.
 \end{aligned}$$

Again we perceive that, as we multiply these factors, a term that is proportional to $(q^+)^n$ will always remain, and in the limit $q^+ \rightarrow 0$, this makes the current for all regions vanish. Therefore, adding more intermediate bosons makes no difference since the powers in q^+ that are given by the second and third lines of formulas written above will always cancel each other.

Now, finally, if we have m external sources for n interacting bosons, we obtain

$$\begin{aligned}
 &\text{Momentum integration } \int \prod_{j=1}^{n+1} dk_{2j}^+ \Rightarrow (q^+)^{n+1} \int \prod_{j=1}^{n+1} dx_j \\
 &\frac{1}{(k_i^+ - k_2^+) \dots (k_{2n+2}^+ - k_{2n}^+)(k_f^+ - k_{2n+2}^+)(k_f^+ - k_{2j+2}^+)} \Rightarrow \frac{1}{(q^+)^{2n+m+2}} \\
 &\text{Legs of the type } \frac{1}{[a + \frac{b}{cq^+} + \frac{d}{eq^+} + \dots]} \Rightarrow (q^+)^{2n+m+2} \\
 &\text{Numerator of the type } (a + k_{2\text{on}}) \text{ or } (a + k_{4\text{on}}) \Rightarrow (q^+)^0 = 1 \\
 &\text{Numerator of the type } (a + k_{j\text{on}}) \text{ with } j \neq 2, 4, \dots, 2n+2 \Rightarrow \frac{1}{(q^+)^m}. \tag{31}
 \end{aligned}$$

In this manner it is only possible to observe the contributions of antiparticles when we put more energy into the system of two interacting bosons. We can check this in the case shown previously: In the second order of the coupling constant for a virtual boson, this results in no observation of antiparticle contributions for $q^+ \rightarrow 0$ in a background field. However, in the expression Eq. (31) we have a case of two external sources ($m = 2$) and one interacting intermediate boson ($n = 1$) in which we obtain a cancellation

of the factors $\frac{(q^+)^{n+1}}{(q^+)^m} = 1$. As a consequence, in this case we will have a nonvanishing contribution from the diagrams of antiparticles. Therefore, as we increase the number of photons (more energy put into the system) on the n bosons, we will encounter nonvanishing contributions from pair production diagrams in the limit $q^+ \rightarrow 0$.

We have plotted in Figs. 8–10 the pair production contribution in the components of the electromagnetic current in the light front for the propagation of two scalar bosons

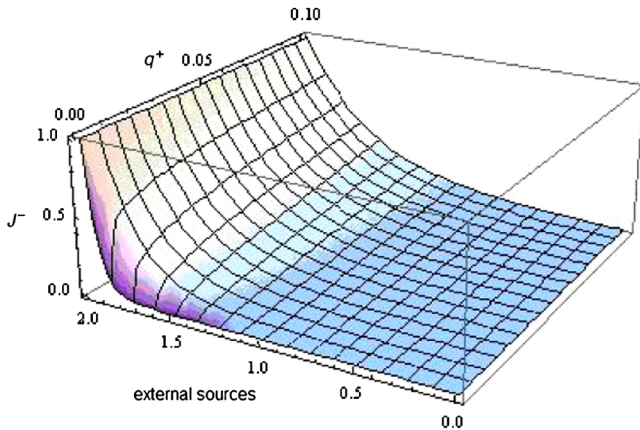


FIG. 8 (color online). J^- Current component in the light front.

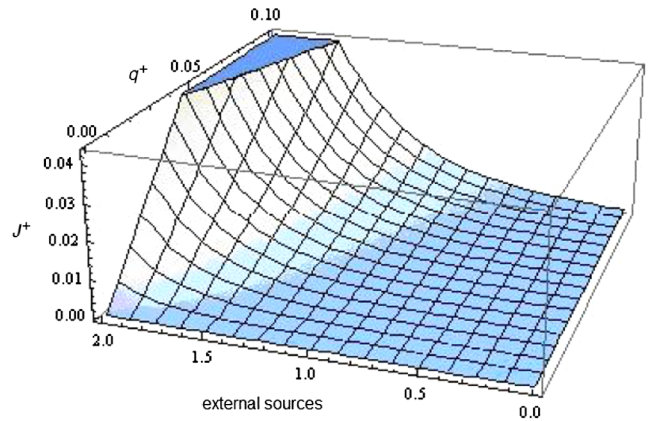


FIG. 9 (color online). J^+ Current component in the light front.

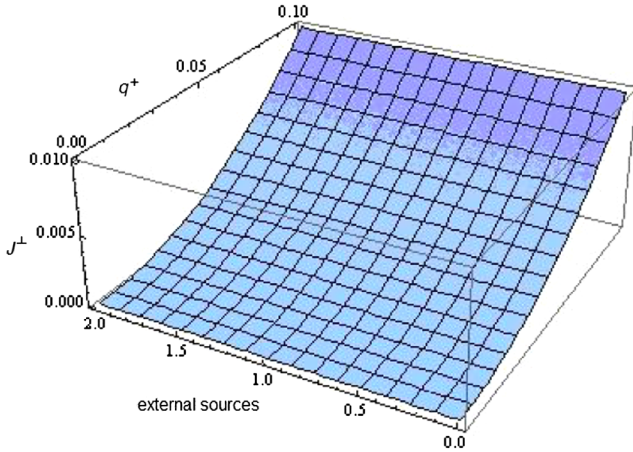


FIG. 10 (color online). J^\perp Current component in the light front.

with one scalar boson exchange. Over this system there is a background field up to two photons (labeled external sources). We can see that for one external photon there is no pair contribution for any of the components of the current. With two photons, there is a pair contribution in the J^- component of the current. The other components, (J^+, J^\perp) , have no contribution in any order up to order 2 in the background field.

V. CONCLUSION

We have demonstrated that the propagator of two bosons in a background field has a nonvanishing contribution coming from the pair creation by the photon. In particular, in an example of a bound state with a constant vertex, we demonstrated that the J^- current component in Breit's reference frame ($q^+ = 0$) has a nonzero contribution from the process of pair creation by the photon. This conclusion is reached as long as we first have q^+ different from zero, integrating in k^- and then taking the limit $q^+ \rightarrow 0$. The integration in k^- and the limit $q^+ \rightarrow 0$ does not commute in general.

In the process of these calculations it has been pointed out that the emergence of a nonvanishing contribution from pair production by the interacting photon is naturally achieved by extending the region of allowed quantum solutions in the light front, that is, extending the Fock space of positive quanta to include relevant solutions from the Fock space of negative quanta. This also means that the myth of a light-front trivial vacuum must be forever abandoned.

We have demonstrated that the inclusion of the pair production term in the light-front formalism is of capital importance for the validity of rotational symmetry for the electromagnetic current of a bound state of two bosons in the model of a constant vertex [2]. In the case of components J^+ and \vec{J}_\perp we concluded that the pair creation term does not contribute in the limit $q^+ \rightarrow 0$. For the J^- component,

however, we have shown that we must take into account the pair production so that rotational symmetry is satisfied in the limit $q^+ \rightarrow 0$. This result has been known for a while in the light-front *milieu*, but with our new approach we have shown that the result which has been reached before via *ad hoc* mathematical techniques can be achieved on the basis of physical grounds.

We also show that the method of “dislocating the integration pole” is nothing more than a particular case of our approach, so that such an *ad hoc* prescription can be better understood as we deal with the whole Fock space. With this we can also prepare to deal with cases involving interactions.

In this work we performed the calculations for corrections to the propagator in a background field up to second order in the coupling. We obtained more diagrams than those considered in a recent article [15], just those in which antiparticles appear. The Z-graph appears naturally in our approach. Yet, in Breit's reference frame these diagrams do not contribute to the current in order g^2 .

For orders in g^n , perhaps it may be possible to devise a recipe for how to correctly introduce the orders in q^+ so that the results in some regions survive, as in [14] in Breit's frame.

We also point out that there is a relation between the number of interacting bosons n and the number of external photon fields m for which the pair creation contribution is nonvanishing in the J^- current, and this is given by $m \geq 1 + n$.

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APPENDIX A: CURRENT FOR TWO FREE BOSONS

To describe the electromagnetic current for a system composed of two free bosons, we study the process in which two bosons of the same mass m propagate forward in time, and in a given instant in the light front \bar{x}^+ one of them interacts with an electromagnetic field. In the following we calculate the components of two noninteracting boson currents in an external electromagnetic field, with total momenta before and after the absorption of the photon being $K_i^+ > 0$ and $K^+ > 0$, respectively.

The Lagrangian density that involves the scalar field and electromagnetic field in the interaction is given by

$$\mathcal{L} = D_\mu \phi D^\mu \phi^* - m^2 \phi^* \phi. \quad (\text{A1})$$

The derivative between the scalar field and the electromagnetic field is contained in the covariant derivative $D_\mu \phi$.

In the calculation of the propagator for a particle in a background field we use the interaction Lagrangian of a scalar field and an electromagnetic field. As we have already mentioned, the interaction between the scalar and electromagnetic fields is contained in the first term of (A1), so that the interaction Lagrangian is

$$\mathcal{L}_1 = ieA^\mu(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi) + e^2A^\mu A_\mu\phi\phi^*. \quad (\text{A2})$$

The Lagrangian (A2) shows immediately that there are two types of vertices. The first term corresponds to a vertex containing a photon and two scalar particles. The second vertex contains two photons and two scalar particles.

Using the concept of a generating functional $Z[J]$, or a vacuum-vacuum transition amplitude in the presence of an external source $J(x)$, we write

$$Z[J] = \int \mathcal{D}\phi e^{i\int d^4x[\mathcal{L}(\phi) + J(x)\phi + \frac{i}{2}\phi^2]} \propto \langle 0, \infty | 0, -\infty \rangle^J, \quad (\text{A3})$$

where $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$ and

$$\mathcal{L}_0 = \partial_\mu\phi\partial^\mu\phi^* - m^2\phi^*\phi.$$

The Green functions are the expectation values of the time-ordered product of field operators in vacuum and can be written in terms of functional derivatives of the generating functional $Z_0[J]$. That is,

$$G(x_1, \dots, x_n) = \langle 0 | T(\phi(x_1) \dots \phi(x_n)) | 0 \rangle, \quad (\text{A4})$$

which are the n -point Green functions of the theory, where

$$\langle 0 | T(\phi(x_1) \dots \phi(x_n)) | 0 \rangle = \frac{1}{i^n} \frac{\delta^n Z_0[J]}{\delta J(x_1) \dots \delta J(x_n)} \Big|_{J=0}. \quad (\text{A5})$$

Green functions for field theories are extremely important because they are intimately related to the matrix elements of the scattering matrix S from which we can calculate quantities measured directly in the experiments such as scattering processes where the cross section for a given reaction is measured, decay of a particle into two or more where we can measure the mean life of particles involved, etc.

The propagator is associated with the Green function equation as

$$G(t - t') = -iS(t - t'). \quad (\text{A6})$$

The Green function or the propagator describes completely the evolution for the quantum system. In this present case we are using the propagator for ‘‘future times.’’ We could also have defined the propagator ‘‘backwards’’ in time.

The propagation of a free particle with spin zero in four-dimensional space-time is represented by the covariant Feynman propagator

$$S(x^\mu) = \int \frac{d^4k}{(2\pi)^4} \frac{ie^{-ik^\mu x_\mu}}{k^2 - m^2 + i\epsilon}, \quad (\text{A7})$$

where the coordinate x^0 represents the time and k^0 the energy. We are going to calculate this propagation in the light front, that is, for times x^+ .

We show the projection of the propagator for a boson in time associated with the light front [17–20], rewriting the coordinates in terms of the time coordinate x^+ and the position coordinates (x^- and \vec{x}_\perp). With these, the momenta are given by k^- , k^+ and \vec{k}_\perp , and therefore we have

$$S(x^+) = \frac{1}{2} \int \frac{dk_1^-}{(2\pi)} \frac{ie^{\frac{i}{2}k_1^- x^+}}{k_1^+ \left(k_1^- - \frac{k_{1\perp}^2 + m^2}{k_1^+} + \frac{i\epsilon}{k_1^+} \right)}. \quad (\text{A8})$$

The Jacobian of the transformation $k^0, \vec{k} \rightarrow k^-, k^+, \vec{k}_\perp$ is equal to $\frac{1}{2}$, and k^+, k_\perp are momentum operators.

Evaluating the Fourier transform, we obtain

$$\tilde{S}(k^-) = \int dx^+ e^{\frac{i}{2}k^- x^+} S(x^+), \quad (\text{A9})$$

where we have used

$$\delta\left(\frac{k^- - k_1^-}{2}\right) = \frac{1}{2\pi} \int dx^+ e^{\frac{i}{2}(k^- - k_1^-)x^+} \quad (\text{A10})$$

and the property of Dirac’s delta ‘‘function’’

$$\delta(ax) = \frac{1}{a} \delta(x), \quad (\text{A11})$$

and we get

$$\tilde{S}(k^-) = \frac{i}{k^+ \left(k^- - \frac{k_\perp^2 + m^2}{k^+} + \frac{i\epsilon}{k^+} \right)}, \quad (\text{A12})$$

which describes the propagation of a particle forward in time and of an antiparticle backwards in time. This can be observed by the denominator which hints that for $x^+ > 0$ and $k^+ > 0$ we have the particle propagating forward in the light front time. On the other hand, for $x^+ < 0$ and $k^+ < 0$ we have an antiparticle propagating backwards in time.

APPENDIX B: POLE STRUCTURE AND INTEGRATIONS IN THE k_2^- AND k_4^- COMPLEX PLANE

The propagator in the momentum representation is given by Eq. (11), and the integrations over the k_2^- and k_4^- components are done according to the 24 different regions of the longitudinal momenta in Table I.

As mentioned in the main text, we start by performing the k_2^- integration first, for which there are three relevant poles. According to Table II eight regions, numbered 5, 6, 19, 20, 21, 22, 23, and 24, yield a vanishing result for this integration. For the nonvanishing integrations, we conveniently choose the circuit that encloses only one pole, going around either the upper hemisphere or the lower hemisphere of the complex k_2^- plane. Then we get the following results.

(i) For regions 1, 3, 7, 9, 10, 13, 15 and 16:

$$\begin{aligned} \tilde{S}(k_f^-) &= \frac{-i(ig)^2(-2\pi i)}{2^6(2\pi)^2} \int \frac{dq^- A(q^-) dk_4^-(k_f^- + k_i^- - 2k_4^-)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \frac{1}{[k_i^- - k_{2\text{on}} - (k_i - k_2)_{\text{on}}]} \\ &\times \frac{1}{[k_4^- - k_i^- + (k_i - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_4)^+}]} \frac{1}{[k_4^- - k_{4\text{on}} + \frac{i\epsilon}{2k_4^+}]} \frac{1}{[k_4^- - k_{2\text{on}} - (k_4 - k_2)_{\text{on}} + \frac{i\epsilon}{2(k_4 - k_2)^+}]}, \\ &\times \frac{1}{[k_4^- - k_f^- + (k_f - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_f - k_4)^+}]} \end{aligned} \quad (\text{B1})$$

(ii) For regions 2, 8, 11 and 12:

$$\begin{aligned} \tilde{S}(k_f^-) &= -\frac{ig^2(2\pi i)}{2^6(2\pi)^2} \int \frac{dq^- A(q^-) dk_4^-(k_f^- + k_i^- - 2k_4^-)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \frac{1}{[k_i^- - (k_i - k_2)_{\text{on}} - k_{2\text{on}}]} \\ &\times \frac{1}{[k_4^- - k_i^- + (k_i - k_4)_{\text{on}} - \frac{i\epsilon}{(k_i - k_4)^+}]} \frac{1}{[k_4^- - k_{4\text{on}} + \frac{i\epsilon}{k_4^+}]} \frac{1}{[k_4^- - k_i^- + (k_i - k_2)_{\text{on}} - (k_4 - k_2)_{\text{on}} + \frac{i\epsilon}{(k_4 - k_2)^+}]}, \\ &\times \frac{1}{[k_4^- - k_f^- + (k_f - k_4)_{\text{on}} - \frac{i\epsilon}{(k_f - k_4)^+}]} \end{aligned} \quad (\text{B2})$$

(iii) For regions 4, 14, 17 and 18:

$$\begin{aligned} \tilde{S}(k_f^-) &= \frac{i(ig)^2(2\pi i)}{2^6(2\pi)^2} \int \frac{dq^- A(q^-) dk_4^-(k_f^- + k_i^- - 2k_4^-)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \\ &\times \frac{1}{[k_4^- - k_i^- + (k_i - k_2)_{\text{on}} - (k_4 - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_2)^+}]} \frac{1}{[k_4^- - k_{2\text{on}} - (k_4 - k_2)_{\text{on}} + \frac{i\epsilon}{2k_2^+}]} \\ &\times \frac{1}{[k_4^- - k_i^- + (k_i - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_4)^+}]} \frac{1}{[k_4^- - k_{4\text{on}} + \frac{i\epsilon}{2k_4^+}]} \frac{1}{[k_4^- - k_f^- + (k_f - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_f - k_4)^+}]} \end{aligned} \quad (\text{B3})$$

1. Integration in k_4^-

The next step is to perform the k_4^- integration, for which we have thirteen different poles and six more vanishing integrations, corresponding to the regions numbered 13, 15, 16, 14, 17, and 18 (see Table V); therefore, from the original 24 possibilities, there remain now only 10. Later on we shall consider the case of regions in which it is not possible to choose a circuit that encloses only a single pole (regions 1, 7 and 12). For all the other regions in which we can conveniently choose a circuit of integration that encloses only a single pole, the integration is quite straightforward to perform.

(i) Regions 2, 8 and 11: The result is

$$\begin{aligned} \tilde{S}(k_f^-) &= \frac{-i(ig)^2}{2^6} \int \frac{dq^- A(q^-) (k_f^- + k_i^- - 2k_{4\text{on}})}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \frac{1}{[k_i^- - (k_i - k_2)_{\text{on}} - k_{2\text{on}}]} \frac{1}{[k_i^- - (k_i - k_4)_{\text{on}} - k_{4\text{on}}]} \\ &\times \frac{1}{[k_i^- - (k_i - k_2)_{\text{on}} - k_{4\text{on}} + (k_4 - k_2)_{\text{on}}]} \frac{1}{[k_f^- - k_{4\text{on}} - (k_f - k_4)_{\text{on}}]} \end{aligned} \quad (\text{B4})$$

(ii) Region 4: The result is

$$\begin{aligned} \tilde{S}(k_f^-) &= \frac{-i(ig)^2}{2^6} \int \frac{dq^- A(q^-) [k_f^- - k_i^- - 2(k_f - k_4)_{\text{on}}]}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \\ &\quad \times \frac{1}{[k_f^- - k_i^- + (k_i - k_2)_{\text{on}} - (k_f - k_4)_{\text{on}} - (k_4 - k_2)_{\text{on}}]} \\ &\quad \times \frac{1}{[k_f^- - k_{2_{\text{on}}} - (k_f - k_4)_{\text{on}} - (k_4 - k_2)_{\text{on}}][k_f^- - k_{4_{\text{on}}} - (k_f - k_4)_{\text{on}}]} \\ &\quad \times \frac{1}{[k_f^- - k_i^- + (k_i - k_4)_{\text{on}} - (k_f - k_4)_{\text{on}}]}. \end{aligned} \quad (\text{B5})$$

(iii) Regions 9 and 10: Here the result is

$$\begin{aligned} \tilde{S}(k_f^-) &= \frac{i(ig)^2}{2^6} \int \frac{dq^- A(q^-) [k_i^- - k_f^- - 2(k_i - k_4)_{\text{on}}]}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \frac{1}{[k_i^- - (k_i - k_2)_{\text{on}} - k_{2_{\text{on}}}] [k_i^- - (k_i - k_4)_{\text{on}} - k_{4_{\text{on}}}] } \\ &\quad \times \frac{1}{[k_i^- - k_{2_{\text{on}}} - (k_i - k_4)_{\text{on}} - (k_4 - k_2)_{\text{on}}]} \frac{1}{[k_i^- - k_f^- - (k_i - k_4)_{\text{on}} + (k_f - k_4)_{\text{on}}]}. \end{aligned} \quad (\text{B6})$$

(iv) Region 3: The result is

$$\begin{aligned} \tilde{S}(k_f^-) &= \frac{i(ig)^2}{2^6} \int \frac{dq^- A(q^-) [k_f^- - k_i^- - 2(k_f - k_4)_{\text{on}}]}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \\ &\quad \times \frac{1}{[k_i^- - (k_i - k_2)_{\text{on}} - k_{2_{\text{on}}}] [k_f^- - k_{2_{\text{on}}} - (k_f - k_4)_{\text{on}} - (k_4 - k_2)_{\text{on}}]} \\ &\quad \times \frac{1}{[k_f^- - k_i^- + (k_i - k_4)_{\text{on}} - (k_f - k_4)_{\text{on}}] [k_f^- - k_{4_{\text{on}}} - (k_f - k_4)_{\text{on}}]}. \end{aligned} \quad (\text{B7})$$

For the cases of circuits that enclose two poles, the integration may become simpler or more complex depending on which contour we choose. These cases turn out to be manageable only after separating the poles using the helpful trick of disentangling them by using the partial fractioning of the denominators, that is, using the identity

$$\frac{1}{(x - A)(x - B)(x - C)(x - D)} = \frac{1}{(A - B)(x - C)(x - D)} \times \left\{ \frac{1}{(x - A)} - \frac{1}{(x - B)} \right\}. \quad (\text{B8})$$

The key point is to choose the poles conveniently in order to do this. Let us see how this can be done.

(i) For regions 1 and 7: Here we choose A as pole 3 and B as pole 2 and use partial fractioning. Then we get

$$\begin{aligned} \tilde{S}(k_f^-) &= -\frac{i(ig)^2(-2\pi i)}{2^6(2\pi)^2} \int \frac{dq^- A(q^-) dk_4^- (k_f^- + k_i^- - 2k_4^-)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \\ &\quad \times \frac{1}{[k_i^- - k_{2_{\text{on}}} - (k_i - k_2)_{\text{on}}] [k_f^- - k_{4_{\text{on}}} - (k_f - k_4)_{\text{on}}]} \frac{1}{[k_4^- - k_i^- + (k_i - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_4)^+}] } \\ &\quad \times \left\{ \frac{1}{[k_4^- - k_{2_{\text{on}}} - (k_4 - k_2)_{\text{on}} + \frac{i\epsilon}{2(k_4 - k_2)^+}] } \left[\frac{1}{[k_4^- - k_f^- + (k_f - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_f - k_4)^+}] } - \frac{1}{[k_4^- - k_{4_{\text{on}}} + \frac{i\epsilon}{2k_4^+}] } \right] \right\}. \end{aligned} \quad (\text{B9})$$

We may localize the following poles from one side,

$$\begin{aligned}
1: k_4^- &= k_i^- - (k_i - k_4)_{\text{on}} + \frac{i\epsilon}{2(k_i - k_4)^+} \\
2: k_4^- &= k_{2_{\text{on}}} + (k_4 - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_4 - k_2)^+} \\
3: k_4^- &= k_f^- - (k_f - k_4)_{\text{on}} + \frac{i\epsilon}{2(k_f - k_4)^+},
\end{aligned}$$

and from the other side,

$$\begin{aligned}
1: k_4^- &= k_i^- - (k_i - k_4)_{\text{on}} + \frac{i\epsilon}{2(k_i - k_4)^+} \\
2: k_4^- &= k_{2_{\text{on}}} + (k_4 - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_4 - k_2)^+} \\
3: k_4^- &= k_{4_{\text{on}}} - \frac{i\epsilon}{2k_4^+}.
\end{aligned}$$

It is not difficult to see that the choice of poles 1 and 2 leads us to the same result.

$$\begin{aligned}
\tilde{S}(k_f^-) &= \frac{i(ig)^2}{2^6} \int \frac{dq^- A(q^-)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \\
&\times \frac{1}{[k_i^- - k_{2_{\text{on}}} - (k_i - k_2)_{\text{on}}][k_f^- - k_{4_{\text{on}}} - (k_f - k_4)_{\text{on}}]} \times \frac{1}{[k_i^- - k_{2_{\text{on}}} - (k_i - k_4)_{\text{on}} - (k_4 - k_2)_{\text{on}}]} \\
&\times \left\{ \frac{[k_f^- + k_i^- - 2k_{2_{\text{on}}} - 2(k_4 - k_2)_{\text{on}}]}{[k_f^- - k_{2_{\text{on}}} - (k_f - k_4)_{\text{on}} - (k_4 - k_2)_{\text{on}}]} + \frac{[k_f^- - k_i^- + 2(k_i - k_4)_{\text{on}}]}{[k_i^- - (k_i - k_4)_{\text{on}} - k_{4_{\text{on}}}]} \right\}. \tag{B10}
\end{aligned}$$

This is the final result for the calculation.

(ii) Region 12: Again, it is not possible to obtain a direct result from poles 1 and 4 or 2 and 3. Using partial fractioning by setting $A = 3$ and $B = 4$ we get

$$\begin{aligned}
\tilde{S}(k_f^-) &= \frac{-i(ig)^2(2\pi i)}{2^6(2\pi)^2} \int \frac{dq^- A(q^-) dk_4^- (k_f^- + k_i^- - 2k_4^-)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \\
&\times \frac{1}{[k_i^- - (k_i - k_2)_{\text{on}} - k_{2_{\text{on}}}] [k_i^- - k_f^- - (k_i - k_2)_{\text{on}} + (k_f - k_4)_{\text{on}} + (k_4 - k_2)_{\text{on}}]} \\
&\times \frac{1}{[k_4^- - k_i^- + (k_i - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_i - k_4)^+}] [k_4^- - k_{4_{\text{on}}} + \frac{i\epsilon}{2k_4^+}]} \left\{ \frac{1}{[k_4^- - k_f^- + (k_f - k_4)_{\text{on}} - \frac{i\epsilon}{2(k_f - k_4)^+}]} \right. \\
&\left. - \frac{1}{[k_4^- - k_i^- + (k_i - k_2)_{\text{on}} - (k_4 - k_2)_{\text{on}} + \frac{i\epsilon}{2(k_4 - k_2)^+}]} \right\} \tag{B11}
\end{aligned}$$

and the poles are localized at

$$\begin{aligned}
1: k_4^- &= k_i^- - (k_i - k_4)_{\text{on}} + \frac{i\epsilon}{2(k_i - k_4)^+} \\
2: k_4^- &= k_{4_{\text{on}}} - \frac{i\epsilon}{2k_4^+} \\
3: k_4^- &= k_f^- - (k_f - k_4)_{\text{on}} + \frac{i\epsilon}{2(k_f - k_4)^+}
\end{aligned}$$

and

$$\begin{aligned}
1: k_4^- &= k_i^- - (k_i - k_4)_{\text{on}} + \frac{i\epsilon}{2(k_i - k_4)^+} \\
2: k_4^- &= k_{4\text{on}} - \frac{i\epsilon}{2k_4^+} \\
3: k_4^- &= k_i^- - (k_i - k_2)_{\text{on}} + (k_4 - k_2)_{\text{on}} - \frac{i\epsilon}{2(k_4 - k_2)^+}.
\end{aligned}$$

Finally, the choice of poles 1 and 2 yields the following result:

$$\begin{aligned}
\tilde{S}(k_f^-) &= \frac{-i(ig)^2}{2^6} \int \frac{dq^- A(q^-)}{(k_i - k_2)^+ k_2^+ (k_i - k_4)^+ k_4^+ (k_f - k_4)^+ (k_4 - k_2)^+} \\
&\times \frac{1}{[k_i^- - (k_i - k_2)_{\text{on}} - k_{2\text{on}}][k_i^- - (k_i - k_4)_{\text{on}} - k_{4\text{on}}]} \times \frac{1}{[k_i^- - k_f^- - (k_i - k_2)_{\text{on}} + (k_f - k_4)_{\text{on}} + (k_4 - k_2)_{\text{on}}]} \\
&\times \left\{ \frac{k_i^- - k_f^- - 2(k_i - k_4)_{\text{on}}}{[k_i^- - k_f^- - (k_i - k_4)_{\text{on}} + (k_f - k_4)_{\text{on}}]} - \frac{k_f^- + k_i^- - 2k_{4\text{on}}}{[k_i^- - (k_i - k_2)_{\text{on}} - k_{4\text{on}} + (k_4 - k_2)_{\text{on}}]} \right\}. \tag{B12}
\end{aligned}$$

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