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Dynamical twisting and the b ghost in the pure spinor formalism

Nathan Berkovits

ICTP South American Institute for Fundamental Research
Instituto de Física Teórica, UNESP - Univ. Estadual Paulista,
Rua Dr. Bento T. Ferraz 271, 01140-070, São Paulo, SP, Brasil

E-mail: nberkovi@ift.unesp.br

Abstract: After adding an RNS-like fermionic vector ψ^m to the pure spinor formalism, the non-minimal b ghost takes a simple form similar to the pure spinor BRST operator. The N=2 superconformal field theory generated by the b ghost and the BRST current can be interpreted as a "dynamical twisting" of the RNS formalism where the choice of which spin $\frac{1}{2}$ ψ^m variables are twisted into spin 0 and spin 1 variables is determined by the pure spinor variables that parameterize the coset SO(10)/U(5).

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Contents

1 Introduction

The pure spinor formalism for the superstring [1] has the advantage over the Ramond-Neveu-Schwarz (RNS) formalism of being manifestly spacetime supersymmetric and has the advantage over the Green-Schwarz (GS) formalism of allowing covariant quantization. However, the worldsheet origin of the pure spinor formalism is mysterious since its BRST operator and b ghost do not arise in an obvious manner from gauge-fixing. Although there have been various suggestions [2–4], there is still no convincing derivation of the pure spinor formalism from a worldsheet reparameterization-inviarant theory.

In the non-minimal pure spinor formalism, the BRST current and b ghost can be interpreted as twisted $\hat{c} = 3$ N=2 superconformal generators [5]. But when expressed in terms of the d=10 superspace variables and the non-minimal pure spinor variables, the b ghost and the resulting N=2 superconformal transformations are extremely complicated. In fact, the nilpotency of the b ghost was only recently verified [6, 7]. An unusual feature of the b ghost in the pure spinor formalism is its dependence on inverse powers of the pure spinor variables which require regularization in superstring amplitudes above two-loops [8]. This multilooop regularization procedure is not yet well-understood and a better understanding of the b ghost might relate these multiloop subtleties in the pure spinor formalism with the multiloop subtleties recently found in the RNS formalism involving nonsplit supermoduli [9–11].

In this paper, it will be shown that the b ghost dramatically simplifies when expressed in terms of a fermionic vector ψ^m that is defined in terms of the other worldsheet variables. If one treats the ten ψ^m variables as independent variables, 5 of the 16 θ^α variables of d=10 superspace (and their conjugate momenta) can be eliminated [12]. The remaining 11 θ^α variables and their conjugate momenta transform as the worldsheet superpartners of the pure spinor variables. The resulting N=2 superconformal field theory generated by the b ghost and the BRST current can be interpreted as a "dynamically twisted" version of the RNS formalism.

In this dynamically twisted superconformal field theory, the N=2 generators are

$$\begin{aligned} T &= -\frac{1}{2}\partial x^m \partial x_m - \frac{(\lambda\gamma_m\gamma_n\overline{\lambda})}{2(\lambda\overline{\lambda})}\psi^m\partial\psi^n + \dots, \\ b &= \frac{(\lambda\gamma_m\gamma_n\overline{\lambda})}{2(\lambda\overline{\lambda})}\psi^m\partial x^n + \dots, \\ j_{\text{BRST}} &= -\frac{(\lambda\gamma_m\gamma_n\overline{\lambda})}{2(\lambda\overline{\lambda})}\psi^n\partial x^m + \dots, \\ J &= -\frac{(\lambda\gamma_m\gamma_n\overline{\lambda})}{2(\lambda\overline{\lambda})}\psi^m\psi^n + \dots, \end{aligned} \tag{1.1}$$

where λ^α and $\overline{\lambda}_\alpha$ are the non-minimal pure spinor ghosts whose projective components parameterize the coset SO(10)/U(5) that describes different twistings. The remaining terms ... in (1.1) are determined by requiring that $(\lambda^\alpha, \overline{\lambda}_\alpha)$ and their worldsheet superpartners transform in an N=2 supersymmetric manner.

So the resulting N=2 superconformal field theory is the sum of a dynamically twisted RNS superconformal field theory with an N=2 superconformal field theory for the pure spinor variables. This interpretation of the BRST operator and the b ghost as coming from dynamical twisting of an N=1 superconformal field theory will hopefully lead to a better geometrical understanding of the pure spinor formalism.

In section 2, the non-minimal pure spinor formalism is reviewed. In section 3, the b ghost in the pure spinor formalism is shown to simplify when expressed in terms of an RNS-like ψ^m variable. In section 4, dynamical twisting of the RNS formalism will be defined and the resulting twisted N=2 superconformal generators will be related to the b ghost and BRST current in the pure spinor formalism. And in section 5, the results will be summarized.

2 Review of non-minimal pure spinor formalism

As discussed in [5], the left-moving contribution to the worldsheet action in the non-minimal pure spinor formalism is

$$S = \int d^2z \left[-\frac{1}{2}\partial x^m \overline{\partial} x_m - p_\alpha \overline{\partial}\theta^\alpha + w_\alpha \overline{\partial}\lambda^\alpha + \overline{w}^\alpha \overline{\partial}\overline{\lambda}_\alpha - s^\alpha \overline{\partial} r_\alpha \right] \tag{2.1}$$

where x^m and θ^α are d=10 superspace variables for $m = 0$ to 9 and $\alpha = 1$ to 16, p_α is the conjugate momentum to θ^α, λ^α and $\overline{\lambda}_\alpha$ are bosonic Weyl and anti-Weyl pure spinors constrained to satisfy $\lambda\gamma^m\lambda = 0$ and $\overline{\lambda}\gamma^m\overline{\lambda} = 0$, and r_α is a fermionic spinor constrained to satisfy $\overline{\lambda}\gamma^m r = 0$. Because of the constraints on the pure spinor variables, their conjugate momenta w_α, $\overline{w}^\alpha$ and s^α can only appear in gauge-invariant combinations such as

$$N^{mn} = \frac{1}{2}(w\gamma^{mn}\lambda), \quad J_\lambda = (w\lambda), \quad S^{mn} = \frac{1}{2}(s\gamma^{mn}\overline{\lambda}), \quad S = (s\overline{\lambda}), \tag{2.2}$$

which commute with the pure spinor constraints.

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The d=10 superspace variables satisfy the free-field OPE's

$$x^m(y)x^n(z) \to -\eta^{mn}\log|y-z|^2, \quad p_\alpha(y)\theta^\beta(z) \to (y-z)^{-1}\delta_\alpha^\beta, \tag{2.3}$$

and, as long as the pure spinor conjugate momenta appear in gauge-invariant combinations and normal-ordering contributions are ignored, one can use the free-field OPE's of pure spinor variables

$$w_\alpha(y)\lambda^\beta(z) \to (y-z)^{-1}\delta_\alpha^\beta, \quad \overline{w}^\alpha(y)\overline{\lambda}_\beta(z) \to (y-z)^{-1}\delta_\beta^\alpha, \quad s^\alpha(y)r_\beta(z) \to (y-z)^{-1}\delta_\beta^\alpha. \tag{2.4}$$

It is convenient to define the spacetime supersymmetric combinations

$$\Pi^m = \partial x^m + \frac{1}{2}(\theta\gamma^m\partial\theta), \quad d_\alpha = p_\alpha - \frac{1}{2}\left(\partial x^m + \frac{1}{4}(\theta\gamma^m\partial\theta)\right)(\gamma_m\theta)_\alpha \tag{2.5}$$

which satisfy the OPE's

$$d_\alpha(y)d_\beta(z) \to -(y-z)^{-1}\Pi_m\gamma^m_{\alpha\beta}, \quad d_\alpha(y)\Pi^m(z) \to (y-z)^{-1}(\gamma^m\partial\theta)_\alpha. \tag{2.6}$$

As shown in [5], the non-minimal BRST current forms a twisted $\hat{c}=3$ N=2 superconformal algebra with the stress tensor, a composite b ghost, and a U(1) ghost-number current. These twisted N=2 generators are

$$T = -\frac{1}{2}\partial x^m\partial x_m - p_\alpha\partial\theta^\alpha + w_\alpha\partial\lambda^\alpha + \overline{w}^\alpha\partial\overline{\lambda}_\alpha - s^\alpha\partial r_\alpha, \tag{2.7}$$

$$\begin{aligned} b = {} & s^\alpha\partial\overline{\lambda}_\alpha + \frac{\overline{\lambda}_\alpha\left(2\Pi^m(\gamma_m d)^\alpha - N_{mn}(\gamma^{mn}\partial\theta)^\alpha - J_\lambda\partial\theta^\alpha - \frac{1}{4}\partial^2\theta^\alpha\right)}{4(\overline{\lambda}\lambda)} \\ & - \frac{(\overline{\lambda}\gamma^{mnp}r)(d\gamma_{mnp}d + 24N_{mn}\Pi_p)}{192(\overline{\lambda}\lambda)^2} + \frac{(r\gamma_{mnp}r)(\overline{\lambda}\gamma^m d)N^{np}}{16(\overline{\lambda}\lambda)^3} \\ & - \frac{(r\gamma_{mnp}r)(\overline{\lambda}\gamma^{pqr}r)N^{mn}N_{qr}}{128(\overline{\lambda}\lambda)^4}, \end{aligned} \tag{2.8}$$

$$j_{\text{BRST}} = \lambda^\alpha d_\alpha - \overline{w}^\alpha r_\alpha, \tag{2.9}$$

$$J_{\text{ghost}} = w_\alpha\lambda^\alpha - s^\alpha r_\alpha - 2(\lambda\overline{\lambda})^{-1}[(\lambda\partial\overline{\lambda}) + (r\partial\theta)] + 2(\lambda\overline{\lambda})^{-2}(\lambda r)(\overline{\lambda}\partial\theta). \tag{2.10}$$

The terms $-\frac{1}{16}(\lambda\overline{\lambda})^{-1}\partial^2\theta^\alpha$ in (2.8) and $-2(\lambda\overline{\lambda})^{-1}[(\lambda\partial\overline{\lambda}) + (r\partial\theta)] + 2(\lambda\overline{\lambda})^{-2}(\lambda r)(\overline{\lambda}\partial\theta)$ in (2.10) are higher-order in α' and come from normal-ordering contributions. To simplify the analysis, these normal-ordering contributions will be ignored throughout this paper. However, it should be possible to do a more careful analysis which takes into account these contributions.

3 Simplification of b ghost

In this section, the complicated expression of (2.8) for the b ghost will be simplified by including an auxiliary fermionic vector variable which will be later related to the RNS ψ^m variable. The trick to simplifying the b ghost is to observe that the terms involving d_α in (2.8) always appear in the combination

$$\overline{\Gamma}^m = \frac{1}{2}(\lambda\overline{\lambda})^{-1}(\overline{\lambda}\gamma^m d) - \frac{1}{8}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma^{mnp}r)N_{np}. \tag{3.1}$$

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Note that only five components of $\overline{\Gamma}^m$ are independent since $\overline{\Gamma}^m(\gamma_m\overline{\lambda})^\alpha = 0$. In terms of $\overline{\Gamma}^m$,

$$b = \Pi^m\overline{\Gamma}_m - \frac{1}{4}(\lambda\overline{\lambda})^{-1}(\lambda\gamma^{mn}r)\overline{\Gamma}_m\overline{\Gamma}_n + s^\alpha\partial\overline{\lambda}_\alpha + w_\alpha\partial\theta^\alpha - \frac{1}{2}(\lambda\overline{\lambda})^{-1}(w\gamma_m\overline{\lambda})(\lambda\gamma^m\partial\theta) \quad (3.2)$$

where terms coming from normal-ordering are being ignored and the identity

$$\delta^\gamma_\beta\delta^\delta_\alpha = \frac{1}{2}\gamma^m_{\alpha\beta}\gamma^{\gamma\delta}_m - \frac{1}{8}(\gamma^{mn})^\gamma_\alpha(\gamma_{mn})^\delta_\beta - \frac{1}{4}\delta^\gamma_\alpha\delta^\delta_\beta \quad (3.3)$$

has been used.[1]

It is useful to treat (3.1) as a first-class constraint where $\overline{\Gamma}^m$ is a new worldsheet variable which carries $+1$ conformal weight and satisfies the constraint $\overline{\Gamma}^m(\gamma_m\overline{\lambda})^\alpha = 0$. Its conjugate momentum will be defined as Γ_m of conformal weight zero and can only appear in combinations invariant under the gauge tranformation generated by the constraint of (3.1). Note that $\overline{\Gamma}^m$ and Γ_m satisfy the OPE $\overline{\Gamma}^m(y)\ \Gamma^n(z) \to (y-z)^{-1}\eta^{mn}$ and have no singular OPE's with the other variables.

One can easily verify that the b ghost of (3.2) is gauge-invariant since it has no singularity with (3.1). Furthermore, any operator $\mathcal{O}$ which is independent of Γ_m can be written in a gauge-invariant manner by defining $\mathcal{O}_{inv} = e^R\ \mathcal{O}\ e^{-R}$ where

$$R = \int \Gamma_m \left[\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\overline{\lambda}\gamma^m d) - \frac{1}{8}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma^{mnp}r)N_{np}\right]. \quad (3.4)$$

For example, the gauge-invariant version of the BRST current is

$$\begin{aligned} G^+ &= e^R\ (\lambda^\alpha d_\alpha - \overline{w}^\alpha r_\alpha)e^{-R} = \lambda^\alpha d_\alpha - \overline{w}^\alpha r_\alpha \qquad (3.5)\\ &-\frac{1}{2}\Gamma^m(\lambda\overline{\lambda})^{-1}\left[(\overline{\lambda}\gamma_m\gamma_n\lambda)\Pi^n - (r\gamma_n\gamma_m\lambda)\overline{\Gamma}^n\right]\\ &+\frac{1}{4}\Gamma^m\Gamma^n[(\lambda\overline{\lambda})^{-1}(\overline{\lambda}\gamma_{mn}\partial\theta) - (\lambda\overline{\lambda})^{-2}(\overline{\lambda}\partial\theta)(\overline{\lambda}\gamma_{mn}\lambda)]\\ &+\frac{1}{8}\Gamma^m\Gamma^n(\lambda\overline{\lambda})^{-2}[(\overline{\lambda}\gamma_{mnp}r)\Pi^p + (r\gamma_{mnp}r)\overline{\Gamma}^p]\\ &-\frac{1}{24}\Gamma^m\Gamma^n\Gamma^p\left[2(\lambda\overline{\lambda})^{-3}(\overline{\lambda}\partial\theta)(\overline{\lambda}\gamma_{mnp}r) - (\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}\partial\overline{\lambda})\right] \end{aligned}$$

where the constraint of (3.1) has been used to substitute $\overline{\Gamma}^m$ for $\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\overline{\lambda}\gamma^m d) - \frac{1}{8}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma^{mnp}r)N_{np}$.

One can also compute the gauge-invariant version of the stress tensor and U(1) current of (2.7) and (2.10) which are

$$\begin{aligned} T &= e^R\left(-\frac{1}{2}\partial x^m\partial x_m - p_\alpha\partial\theta^\alpha + w_\alpha\partial\lambda^\alpha - s^\alpha\partial r_\alpha + \overline{w}^\alpha\partial\overline{\lambda}_\alpha\right)\ e^{-R} \qquad (3.6)\\ &= -\frac{1}{2}\partial x^m\partial x_m - p_\alpha\partial\theta^\alpha + w_\alpha\partial\lambda^\alpha - s^\alpha\partial r_\alpha + \overline{w}^\alpha\partial\overline{\lambda}_\alpha - \overline{\Gamma}^m\partial\Gamma_m \end{aligned}$$

[1]When expressed in terms of $\overline{\Gamma}^m$, the b ghost no longer has poles when $\lambda^\alpha \to 0$. However, the definition of $\overline{\Gamma}^m$ in (3.1) is singular in this limit, so a multiloop regularization procedure such as [8] will probably still be necessary.

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and

$$J = e^R\,(w_\alpha\lambda^\alpha + r_\alpha s^\alpha)\,e^{-R} = w_\alpha\lambda^\alpha + r_\alpha s^\alpha + \Gamma_m\overline{\Gamma}^m. \tag{3.7}$$

The operators of (3.6), (3.2), (3.5) and (3.7) form a set of twisted N=2 superconformal generators which preserve the first-class constraint of (3.1). The resulting N=2 superconformal field theory will be related to a dynamical twisting of the RNS formalism where the RNS fermionic vector variable ψ^m is defined as

$$\psi^m = \overline{\Gamma}^m + \frac{1}{2}(\lambda\overline{\lambda})^{-1}\Gamma_n(\lambda\gamma^m\gamma^n\overline{\lambda}). \tag{3.8}$$

Note that ψ^m satisfies the usual OPE $\psi^m(y)\psi^n(z) \to (y-z)^{-1}\eta^{mn}$ and commutes with the constraint $\overline{\Gamma}^m(\gamma_m\overline{\lambda})^\alpha = 0$. Since this constraint eliminates half of the $\overline{\Gamma}^m$ variables and can be used to gauge-fix half of the Γ_m variables, the remaining 10 variables of $\overline{\Gamma}^m$ and Γ_m can be expressed in terms of ψ^m.

Although w_α and $\overline{w}^\alpha$ have singular OPE's with ψ^m, one can define variables w'_α and $\overline{w}'^\alpha$ which have no singular OPE's with ψ^m as

$$w_\alpha = w'_\alpha - \frac{1}{4}\psi_m\psi_n[(\lambda\overline{\lambda})^{-1}(\gamma^{mn}\overline{\lambda})_\alpha - \overline{\lambda}_\alpha(\lambda\overline{\lambda})^{-2}(\lambda\gamma^{mn}\overline{\lambda})], \tag{3.9}$$

$$\overline{w}^\alpha - \frac{1}{2}\overline{\Gamma}^m\Gamma^n(\lambda\overline{\lambda})^{-1}(\gamma_m\gamma_n\lambda)^\alpha = \overline{w}'^\alpha - \frac{1}{4}\psi_m\psi_n[(\lambda\overline{\lambda})^{-1}(\gamma^{mn}\lambda)^\alpha - \lambda^\alpha(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma^{mn}\lambda)].$$

Note that $\overline{w}^\alpha$ always appears in the combination $\overline{w}^\alpha - \frac{1}{2}\overline{\Gamma}^m\Gamma^n(\lambda\overline{\lambda})^{-1}(\gamma_m\gamma_n\lambda)^\alpha$ since it is this combination which commutes with the constraint $\overline{\Gamma}^m(\gamma_m\overline{\lambda})^\alpha = 0$.

When expressed in terms of ψ^m, w'_α and $\overline{w}'^\alpha$, the twisted N=2 generators of (3.6), (3.2), (3.5) and (3.7) take the form

$$\begin{aligned} T &= -\frac{1}{2}\partial x^m\partial x_m - p_\alpha\partial\theta^\alpha + w'_\alpha\partial\lambda^\alpha - s^\alpha\partial r_\alpha + \overline{w}'^\alpha\partial\overline{\lambda}_\alpha \\ &\quad -\frac{1}{2}\psi^m\partial\psi_m - \frac{1}{4}\partial[(\lambda\overline{\lambda})^{-1}(\lambda\gamma_m\gamma_n\overline{\lambda})\psi^m\psi^n], \\ G^- &= \frac{1}{2}(\lambda\overline{\lambda})^{-1}(\lambda\gamma_m\gamma_n\overline{\lambda})\psi^m\Pi^n + s^\alpha\partial\overline{\lambda}_\alpha + w'_\alpha\partial\theta^\alpha - \frac{1}{2}(\lambda\overline{\lambda})^{-1}(w'\gamma^m\overline{\lambda})(\lambda\gamma_m\partial\theta) \\ &\quad +\frac{1}{4}\psi_m\psi_n(\lambda\overline{\lambda})^{-1}\Big[(\overline{\lambda}\gamma^{mn}\partial\theta) + (\lambda\overline{\lambda})^{-1}(\overline{\lambda}\partial\theta)(\lambda\gamma^{mn}\overline{\lambda}) \\ &\qquad\qquad + (r\gamma^{mn}\lambda) + (\lambda\overline{\lambda})^{-1}(r\lambda)(\lambda\gamma^{mn}\overline{\lambda})\Big], \\ G^+ &= -\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\lambda\gamma_m\gamma_n\overline{\lambda})\psi^n\Pi^m + \lambda^\alpha d_\alpha - \overline{w}'^\alpha r_\alpha \\ &\quad +\frac{1}{4}\psi_m\psi_n(\lambda\overline{\lambda})^{-1}\Big[(\overline{\lambda}\gamma^{mn}\partial\theta) + (\lambda\overline{\lambda})^{-1}(\overline{\lambda}\partial\theta)(\lambda\gamma^{mn}\overline{\lambda}) \\ &\qquad\qquad + (r\gamma^{mn}\lambda) + (\lambda\overline{\lambda})^{-1}(r\lambda)(\lambda\gamma^{mn}\overline{\lambda})\Big], \\ &\quad +G^-\left[\frac{1}{24}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p\right], \\ J &= -\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\lambda\gamma_{mn}\overline{\lambda})\psi^m\psi^n + w'_\alpha\lambda^\alpha + r_\alpha s^\alpha, \end{aligned} \tag{3.10}$$

where $G^-\left[\frac{1}{24}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p\right]$ denotes the single pole in the OPE of G^- with $\frac{1}{24}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p$ and is equal to the last two lines of (3.5).

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Except for the extra term $G^-\left[\frac{1}{24}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p\right]$ in G^+, the generators of (3.10) have a very symmetric form. This asymmetry in G^+ and G^- can be removed by performing the similarity transformation $\mathcal{O}\to e^R\,\mathcal{O}\,e^{-R}$ on all operators where

$$R=-\frac{1}{24}\int(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p. \tag{3.11}$$

This similarity transformation leaves G^+ of (3.10) invariant but transforms T, G^- and J as

$$\begin{aligned} T &\to T+\frac{1}{24}\partial((\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p), \\ G^- &\to G^-+G^-\left[\frac{1}{24}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p\right], \\ J &\to J+\frac{1}{12}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p. \end{aligned} \tag{3.12}$$

It also transforms the constraint of (3.1) into the constraint

$$\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\lambda\gamma^n\gamma^m\overline{\lambda})\psi_n=\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\overline{\lambda}\gamma^m d)-\frac{1}{8}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma^{mnp}r)N'_{np} \tag{3.13}$$

where $N'_{np}=\frac{1}{2}w'\gamma_{np}\lambda$.

After performing the similarity transformation of (3.11), the twisted N=2 generators preserve the constraint of (3.13) and take the symmetrical form

$$\begin{aligned} T &= -\frac{1}{2}\partial x^m\partial x_m-\frac{1}{2}\psi^m\partial\psi_m-p_\alpha\partial\theta^\alpha+\frac{1}{2}(w'_\alpha\partial\lambda^\alpha-\lambda^\alpha\partial w'_\alpha) \\ &\quad -\frac{1}{2}(s^\alpha\partial r_\alpha+r_\alpha\partial s^\alpha)+\overline{w}'^\alpha\partial\overline{\lambda}_\alpha+\frac{1}{2}\partial J, \\ -G^++G^- &= \psi_m\Pi^m-\lambda^\alpha d_\alpha+\overline{w}'^\alpha r_\alpha+s^\alpha\partial\overline{\lambda}_\alpha+w'_\alpha\partial\theta^\alpha-\frac{1}{2}(\lambda\overline{\lambda})^{-1}(w'\gamma^m\overline{\lambda})(\lambda\gamma_m\partial\theta), \\ J &= -\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\lambda\gamma_{mn}\overline{\lambda})\psi^m\psi^n+\frac{1}{12}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p+w'_\alpha\lambda^\alpha+r_\alpha s^\alpha, \\ G^++G^- &= [-G^++G^-,J] \\ &= \psi_m\Pi_n(\lambda\overline{\lambda})^{-1}(\lambda\gamma^{mn}\overline{\lambda})+\lambda^\alpha d_\alpha-\overline{w}'^\alpha r_\alpha+s^\alpha\partial\overline{\lambda}_\alpha+w'_\alpha\partial\theta^\alpha \\ &\quad -\frac{1}{2}(\lambda\overline{\lambda})^{-1}(w'\gamma^m\overline{\lambda})+\frac{1}{2}\psi_m\psi_n(\lambda\overline{\lambda})^{-1}\Big[(\overline{\lambda}\gamma^{mn}\partial\theta)+(\lambda\overline{\lambda})^{-1}(\overline{\lambda}\partial\theta)(\lambda\gamma^{mn}\overline{\lambda}) \\ &\qquad +(r\gamma^{mn}\lambda)+(\lambda\overline{\lambda})^{-1}(r\lambda)(\lambda\gamma^{mn}\overline{\lambda})\Big] \\ &\quad +\frac{1}{4}\psi^m\psi^n\left[(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\Pi^p+\frac{1}{2}(\lambda\overline{\lambda})^{-3}(r\gamma_{mnp}r)(\overline{\lambda}\gamma^p\gamma^q\lambda)\psi_q\right] \\ &\quad +\frac{1}{12}\psi^m\psi^n\psi^p[-2(\lambda\overline{\lambda})^{-3}(\overline{\lambda}\partial\theta)(\overline{\lambda}\gamma_{mnp}r)+(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}\partial\overline{\lambda})], \end{aligned} \tag{3.14}$$

where the last two lines in G^++G^- is $G^-\left[\frac{1}{12}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p\right]$. These N=2 generators of (3.14) will now be related to a dynamically twisted version of the RNS formalism.

4 Dynamical twisting of the RNS formalism

In this section, the RNS formalism will be "dynamically twisted" to an N=2 superconformal field theory by introducing bosonic pure spinor variables λ^α and $\overline{\lambda}_\alpha$ and their fermionic worldsheet superpartners. The corresponding twisted N=2 superconformal generators will then be related to the twisted N=2 generators of (3.14) in the pure spinor formalism.

Twisting the N=1 RNS superconformal generators

$$T = -\frac{1}{2}\partial x^m \partial x_m - \frac{1}{2}\psi^m \partial \psi_m, \quad G = \psi^m \partial x_m \tag{4.1}$$

into N=2 superconformal generators usually involves choosing a U(5) subgroup of the Wick-rotated SO(10) Lorentz group and splitting the ten x^m and ψ^m variables into five complex pairs $(x^a, \overline{x}^{\overline{a}})$ and $(\psi^a, \overline{\psi}^{\overline{a}})$ for $a = 1$ to 5. One then defines the twisted N=2 superconformal generators as

$$\begin{aligned} T_{\mathrm{RNS}} &= -\partial x^a \partial \overline{x}^{\overline{a}} - \overline{\psi}^{\overline{a}} \partial \psi^a, \\ G^-_{\mathrm{RNS}} &= \overline{\psi}^{\overline{a}} \partial x^a, \quad G^+_{\mathrm{RNS}} = -\psi^a \partial \overline{x}^{\overline{a}}, \\ J_{\mathrm{RNS}} &= -\overline{\psi}^{\overline{a}} \psi^a, \end{aligned} \tag{4.2}$$

which satisfy the OPE $G^+(y)G^-(z) \to (y-z)^{-2}J(z) + (y-z)^{-1}T(z)$.

To dynamically twist, one instead introduces pure spinor worldsheet variables λ^α and $\overline{\lambda}_\alpha$ satisfying

$$\lambda\gamma^m\lambda = 0, \quad \overline{\lambda}\gamma^m\overline{\lambda} = 0, \tag{4.3}$$

whose projective components parameterize the coset SO(10)/U(5). The N=2 superconformal generators of (4.2) can then be written in a Lorentz-covariant manner as

$$\begin{aligned} T_{\mathrm{RNS}} &= -\frac{1}{2}\partial x^m \partial x_m - \frac{1}{2}\psi^m \partial \psi_m - \frac{1}{4}\partial[(\lambda\overline{\lambda})^{-1}(\lambda\gamma^m\gamma^n\overline{\lambda})\psi_m\psi_n], \\ G^-_{\mathrm{RNS}} &= \frac{1}{2}(\lambda\overline{\lambda})^{-1}(\lambda\gamma^m\gamma^n\overline{\lambda})\psi_m\partial x_n, \quad G^+_{\mathrm{RNS}} = -\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\lambda\gamma^n\gamma^m\overline{\lambda})\psi_m\partial x_n, \\ J_{\mathrm{RNS}} &= -\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\lambda\gamma^m\gamma^n\overline{\lambda})\psi_m\psi_n. \end{aligned} \tag{4.4}$$

The next step is to introduce the fermionic worldsheet superpartners of the pure spinor variables $(\lambda^\alpha, \overline{\lambda}_\alpha)$ and their conjugate momenta $(w'_\alpha, \overline{w}'^\alpha)$. The fermionic superpartners of λ^α and w'_α will be denoted $\tilde{\theta}^\alpha$ and $\tilde{p}_\alpha$, and the fermionic superpartners of $\overline{\lambda}_\alpha$ and $\overline{w}'^\alpha$ will be denoted r_α and s^α. They are constrained to satisfy

$$\lambda\gamma^m\partial\tilde{\theta} = 0, \quad \overline{\lambda}\gamma^m r = 0, \tag{4.5}$$

which will be the worldsheet supersymmetry transformation of the pure spinor constraints of (4.3). Because of the constraint $\lambda\gamma^m\partial\tilde{\theta} = 0$, $\tilde{\theta}^\alpha$ is a constrained version of θ^α which only contains eleven independent non-zero modes. The corresponding twisted N=2 superconformal generators for these pure spinor multiplets are defined as

$$\begin{aligned} T_{\mathrm{pure}} &= w'_\alpha \partial\lambda^\alpha - \tilde{p}_\alpha \partial\tilde{\theta}^\alpha + \overline{w}'^\alpha \partial\overline{\lambda}_\alpha - s^\alpha \partial r_\alpha, \\ G^-_{\mathrm{pure}} &= w'_\alpha \partial\tilde{\theta}^\alpha + s^\alpha \partial\overline{\lambda}_\alpha, \quad G^+_{\mathrm{pure}} = \lambda^\alpha \tilde{p}_\alpha - \overline{w}'_\alpha r^\alpha, \\ J_{\mathrm{pure}} &= w'_\alpha \lambda^\alpha + r^\alpha s_\alpha, \end{aligned} \tag{4.6}$$

which preserve the pure spinor constraints of (4.3) and (4.5).

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Finally, one adds the N=2 superconformal generators of (4.4) and (4.6) in a manner that preserves the N=2 algebra. This can be done by defining T, J and $-G^+ + G^-$ as the sum

$$\begin{aligned} T &= T_{\rm RNS} + T_{\rm pure}, \quad J = J_{\rm RNS} + J_{\rm pure}, \\ -G^+ + G^- &= (-G^+ + G^-)_{\rm RNS} + (-G^+ + G^-)_{\rm pure}, \end{aligned} \tag{4.7}$$

and then defining $G^+ + G^-$ using the commutator algebra

$$G^+ + G^- = [-G^+ + G^-, J].$$

Since $G^+_{\rm pure}$ and $G^-_{\rm pure}$ do not commute with $J_{\rm RNS}$, $G^+ + G^-$ is not the sum of $(G^+ + G^-)_{\rm RNS}$ and $(G^+ + G^-)_{\rm pure}$.

The resulting N=2 superconformal generators for the dynamically twisted RNS formalism are

$$\begin{aligned} T &= -\frac{1}{2}\partial x^m \partial x_m - \frac{1}{2}\psi^m \partial\psi_m - \tilde{p}_\alpha \partial\tilde{\theta}^\alpha + \frac{1}{2}(w'_\alpha \partial\lambda^\alpha - \lambda^\alpha \partial w'_\alpha) \\ &\quad -\frac{1}{2}(s^\alpha \partial r_\alpha + r_\alpha \partial s^\alpha) + \overline{w}'^\alpha \partial\overline{\lambda}_\alpha + \frac{1}{2}\partial J, \\ -G^+ + G^- &= \psi^m \partial x_m - \lambda^\alpha \tilde{p}_\alpha + \overline{w}'^\alpha r_\alpha + s^\alpha \partial\overline{\lambda}_\alpha + w'_\alpha \partial\tilde{\theta}^\alpha, \\ J &= -\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\lambda\gamma_{mn}\overline{\lambda})\psi^m\psi^n + w'_\alpha\lambda^\alpha + r_\alpha s^\alpha, \\ G^+ + G^- &= [-G^+ + G^-, J] \\ &= \psi_m \partial x_n (\lambda\overline{\lambda})^{-1}(\lambda\gamma^{mn}\overline{\lambda}) + \lambda^\alpha \tilde{p}_\alpha - \overline{w}'^\alpha r_\alpha + s^\alpha \partial\overline{\lambda}_\alpha + w'_\alpha \partial\tilde{\theta}^\alpha \\ &\quad + \frac{1}{2}\psi_m\psi_n(\lambda\overline{\lambda})^{-1}\Big[(\overline{\lambda}\gamma^{mn}\partial\tilde{\theta}) + (\lambda\overline{\lambda})^{-1}(\overline{\lambda}\partial\tilde{\theta})(\lambda\gamma^{mn}\overline{\lambda}) \\ &\qquad + (r\gamma^{mn}\lambda) + (\lambda\overline{\lambda})^{-1}(r\lambda)(\lambda\gamma^{mn}\overline{\lambda})\Big]. \end{aligned} \tag{4.8}$$

The N=2 superconformal generators of (4.8) are obviously closely related to the N=2 generators of (3.14) in the pure spinor formalism, but there are three important differences. Firstly, the generators of (4.8) are not manifestly spacetime supersymmetric since they involve ∂x^m and $\tilde{p}_\alpha$ instead of Π^m and d_α. Secondly, the U(1) generator J of (4.8) does not include the term $\frac{1}{12}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p$. And thirdly, the $\tilde{\theta}^\alpha$ variable in (4.8) is constrained to satisfy $\lambda\gamma^m\partial\tilde{\theta} = 0$.

The first difference is easily removed by performing the similarity transformation $\mathcal{O} \to e^R\, \mathcal{O}\, e^{-R}$ on all operators in (4.8) where

$$R = \frac{1}{2}\int (\lambda\gamma^m\tilde{\theta})\psi_m. \tag{4.9}$$

This similarity transformation does not affect T or J of (4.8) but transforms $-G^+ + G^-$ into the manifestly spacetime supersymmetric expression

$$-G^+ + G^- = \psi^m\tilde{\Pi}_m - \lambda^\alpha\tilde{d}_\alpha + \overline{w}'^\alpha r_\alpha + s^\alpha\partial\overline{\lambda}_\alpha + w'_\alpha\partial\tilde{\theta}^\alpha \tag{4.10}$$

where $\tilde{\Pi}^m = \partial x^m + \frac{1}{2}(\tilde{\theta}\gamma^m\partial\tilde{\theta})$ and $\tilde{d}_\alpha = \tilde{p}_\alpha - \frac{1}{2}\left(\partial x^m + \frac{1}{4}(\tilde{\theta}\gamma_m\partial\tilde{\theta})\right)(\gamma_m\tilde{\theta})_\alpha$, and transforms the $\psi_m\partial x_n(\lambda\overline{\lambda})^{-1}(\lambda\gamma^{mn}\overline{\lambda})$ term in $G^+ + G^-$ into $\psi_m\tilde{\Pi}_n(\lambda\overline{\lambda})^{-1}(\lambda\gamma^{mn}\overline{\lambda})$.

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The second difference in the generators can be removed by modifying the definition of dynamical twisting in (4.4) so that the appropriate term is added to J. The generator $-G^+ + G^- = (-G_+ + G^-)_{\mathrm{RNS}} + (-G^+ + G^-)_{\mathrm{pure}}$ and the untwisted stress tensor $T - \frac{1}{2}\partial J = \left(T - \frac{1}{2}\partial J\right)_{\mathrm{RNS}} + \left(T - \frac{1}{2}\partial J\right)_{\mathrm{pure}}$ of (4.8) will be left unchanged. But J will be modified so that after performing the similarity transformation of (4.9), the new J includes the term $\frac{1}{12}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma_{mnp}r)\psi^m\psi^n\psi^p$. And to preserve the N=2 algebra, $G^+ + G^-$ will be defined as the commutator $[-G^+ + G^-, J]$ using the new J.

Since $e^{-R}\ \psi^m\ e^R = \psi^m - \frac{1}{2}(\lambda\gamma^m\tilde{\theta})$, this means one should modify J in (4.8) to

$$
\begin{aligned}
J = & -\frac{1}{2}(\lambda\overline{\lambda})^{-1}(\lambda\gamma^{mn}\overline{\lambda})\psi_m\psi_n + w'_\alpha\lambda^\alpha + r_\alpha s^\alpha \\
& + \frac{1}{12}(\lambda\overline{\lambda})^{-2}(\overline{\lambda}\gamma^{mnp}r)\left(\psi_m - \frac{1}{2}(\lambda\gamma_m\tilde{\theta})\right)\left(\psi_n - \frac{1}{2}(\lambda\gamma_n\tilde{\theta})\right)\left(\psi_p - \frac{1}{2}(\lambda\gamma_p\tilde{\theta})\right).
\end{aligned}
\tag{4.11}
$$

Although this modification of J looks unnatural, it has the important consequence of breaking the abelian shift symmetry $\tilde{\theta}^\alpha \to \tilde{\theta}^\alpha + c^\alpha$ where c^α is any constant. This shift symmetry leaves invariant the generators of (4.8), but has no corresponding symmetry in the pure spinor formalism and should not be a physical symmetry.

After modifying J in this manner and performing the similarity transformation of (4.9), the generators of (4.8) coincide with the generators of (3.14) except for the restriction that $\lambda\gamma^m\partial\tilde{\theta} = 0$. This final difference between the generators can be removed by interpreting $\lambda\gamma^m\partial\tilde{\theta} = 0$ as a partial gauge-fixing condition for the symmetry generated by the first-class constraint of (3.13). After relaxing the restriction $\lambda\gamma^m\partial\tilde{\theta} = 0$ and adding the term $-\frac{1}{2}(\lambda\overline{\lambda})^{-1}(w'\gamma^m\overline{\lambda})(\lambda\gamma^m\partial\theta)$ to G^-, the generators of (4.8) coincide with those of (3.14) and therefore preserve the constraint of (3.13).

Since the generators preserve (3.13), it is consistent to interpret (4.8) as a partially gauge-fixed version of (3.14) where the symmetry generated by (3.13) is used to gauge-fix $\lambda\gamma^m\partial\theta = 0$. On the other hand, the original N=2 generators of (2.7)–(2.10) of the pure spinor formalism can be interpreted as a gauge-fixed version of (3.14) where the gauge-fixing condition is $(\lambda\gamma^m\gamma^n\overline{\lambda})\psi_n = 0$. This is easy to see since $(\lambda\gamma^m\gamma^n\overline{\lambda})\psi_n = 0$ implies that $R = 0$ in the similarity transformations of (3.5), (3.6) and (3.7).

5 Summary

In section 2, the b ghost of the pure spinor formalism was simplified by introducing the fermionic vector variable $\overline{\Gamma}^m$ of (3.1). After expressing $\overline{\Gamma}^m$ in terms of the RNS variable ψ^m using (3.8), the b ghost and BRST current form a symmetric set of twisted N=2 generators (3.14) which preserve the constraint of (3.13).

In section 3, the corresponding N=2 superconformal field theory was interpreted as a dynamically twisted version of the RNS formalism in which the pure spinors λ^α and $\overline{\lambda}_\alpha$ parameterize the SO(10)/U(5) choices of twisting. The dynamically twisted RNS generators are obtained from (3.14) using the constraint of (3.13) to gauge-fix $\lambda\gamma^m\partial\theta = 0$. And the twisted N=2 generators of the original pure spinor formalism are obtained from (3.14) using the constraint of (3.13) to gauge-fix $(\lambda\gamma^m\gamma^n\overline{\lambda})\psi_n = 0$.

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