Five-dimensional $f(R)$ braneworld models

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After incorporating $f(R)$ gravity into the general braneworld sum rules scope, it is shown that some particular class of warped five-dimensional nonlinear braneworld models, which may be interesting for the hierarchy problem solution, still require a negative tension brane. For other classes of warp factors (suitable and not suitable for approaching the hierarchy problem) any negative brane tension in the compactification scheme is not necessary. In this vein, it is argued that, in the bulk $f(R)$ gravity context, some types of warp factors may be useful for approaching the hierarchy problem and for evading the necessity of a negative brane tension in the compactification scheme.

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I. INTRODUCTION

The idea that our Universe is a submanifold (the brane) embedded in a higher-dimensional spacetime led to a huge growth in the number of new braneworld models being proposed. Brane gravity theories in 11 dimensions were first introduced by Horava and Witten [1]. In the Randall and Sundrum (RS) [2,3] braneworld models, the spacetime has five dimensions—an effective Horava-Witten scenario without the 6D Calabi-Yau space—endowed with a warped structure. In the so-called RSI model [2] there are two 3-branes with opposite tensions placed at the end of an S^1/Z_2 orbifold. One of the branes has, typically, Planckian energy scales, and naturally, via the warp factor, the effective energy scale on the other brane is about TeV.

The RSI model line element is given by

$$ds^2_{\rm RSI} = e^{-2\kappa r_c|r|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 dr^2, \tag{1}$$

where $\eta_{\mu\nu}$ is the usual Minkowski metric and r_c is the compactification radius. The Planck brane is placed at $r = 0$, while the visible brane is placed at $r = \pi$. Therefore, it is easy to verify that any mass parameter on the visible brane will be dressed by the $e^{-2\kappa r_c\pi}$ factor. Consequently, a large hierarchy among the parameters is not necessary in order to obtain the TeV scale (the scale of the dressed masses) from the fundamental Planck scale [2].

In general, braneworld models are inspired in string theory, and it is expected that a considered model makes contact with some string theory limits. In this vein, it is important to study braneworld models in, for instance, modified Einstein-Hilbert gravity in the hopes that such a bottom-up approach turns out to be useful from the phenomenological point of view. Here we investigate how an $f(R)$ gravity theory in the bulk may act in the braneworld context [for recent reviews in $f(R)$ gravity see [4]]. Higher derivative gravity in the braneworld framework was analyzed in several contexts. For instance, in Ref. [5] it is shown how $f(R)$ theories may evade the fine-tuning problem present in the RSI model, while cosmological aspects of $f(R)$ braneworld gravity were studied in Ref. [6] (see also [7,8]).

In this work we are concerned with the following issue: The RSI model has one negative brane tension in the compactification scheme. It is indeed necessary in the model. However, a negative brane tension is not a gravitationally stable object by itself. Let us discuss this issue a little further. If the branes are allowed to vary their position (which is not the case in the RSI model), then it is possible to show that the resulting RSI-like scenario is free from gravitational instability, since the radion is massless and may be made heavy via the mechanism presented in Ref. [9]. By the same token, if the distance between the branes may vary, it is possible to show that the effective gravitational constant on the brane does not depend on the brane tension signal. However, if the distance between the branes is fixed *a priori* (i.e. the radion field is frozen), as in the RSI case, then the effective Newtonian constant on the brane depends linearly on the brane tension, leading to an ill-defined scenario for negative tension branes. The necessary conditions for circumventing the necessity of negative brane tensions were found in the context of 3-branes embedded into a five-dimensional bulk respecting Brans-Dicke gravity [10]. Roughly speaking, the presence of terms proportional to the scalar field ϕ (and their derivatives) relaxes the constraints imposed by the consistency conditions, allowing a braneworld setup without the necessity of a negative brane tension in the compactification scheme. In this vein, one may hope that, at least in some regime, $f(R)$ bulk gravity acts in the same way. In fact, keeping in mind the conformal equivalence between $f(R)$ and scalar-tensorial theories [4], the use of $f(R)$ in order to avoid a negative brane tension is expected to work. Note, however, that this equivalence requires a nonzero

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potential for the scalar field, and in Ref. [10] $V(\phi) = 0$. Therefore, this work concerns essentially a new case. Here, we show that for some classes of warp factors the previous considerations are correct; i.e., $f(R)$ theories indeed help us to evade the necessity of a negative brane tension even without varying the distance between the branes. However, for another class of warp factors (among them, the one usually used for the hierarchy problem solution) the scenario including $f(R)$ theory in the bulk still needs a negative brane tension.

This paper is structured as follows: In the next section we start with a short review on the consistency conditions obtained for the usual general relativity case, making explicit the necessity of a negative brane tension in the RSI framework. Then we move forward and generalize the consistency conditions to the bulk $f(R)$ gravity case. In Sec. III we conclude with a final discussion.

II. THE $f(R)$ BRANEWORLD MODELS

Here we shall first re-obtain, for completeness, the braneworld sum rules in general relativity. Gibbons, Kallosh, and Linde [11] prompted one to define a set of simple rules aimed at checking the consistency of five-dimensional models case by case. After that, a work by Leblond, Myers, and Winters [12] extended these *sum rules* for higher dimensions. We shall reproduce their arguments below.

We start by analyzing a D-dimensional bulk spacetime endowed with a nonfactorable geometry, whose metric is given by

$$ds^2 = G_{AB}dx^A dx^B = W^2(r)g_{\mu\nu}dx^\mu dx^\nu + g_{ab}(r)dr^a dr^b,$$

where $W^2(r)$ is the warp factor, x^A denotes the coordinates of the full D-dimensional spacetime, x^μ stands for the $(p+1)$ noncompact coordinates of the spacetime, and r^a labels the $(D-p-1)$ directions in the internal compact space. Note that this type of metric encodes the possibility of existing q-branes ($q > p$), so that the $(q-p)$ extra dimensions are compactified on the brane and constitute part of the internal space. As an example, if $D = 5$, $p = 3$, and $W(r) = e^{-2kr_c|r|}$, one arrives at the RS model.

The D-dimensional Einstein equations defined in the bulk are

$$R_{AB} = 8\pi G_D\left(T_{AB} - \frac{1}{D-2}G_{AB}T^M_M\right), \tag{2}$$

where G_D is the D-dimensional gravitational constant. We decompose the traced version of Ricci tensor components,

$$R^\mu_\mu = \frac{8\pi G_D}{D-2}[(D-p-3)T^\mu_\mu - (p+1)T^a_a], \tag{3}$$

where $T^\mu_\mu \equiv W^{-2}g^{\mu\nu}T_{\mu\nu}$ and

$$R^a_a = \frac{8\pi G_D}{D-2}[(p-1)T^a_a - (D-p-1)T^\mu_\mu], \tag{4}$$

both written in terms of the bulk stress-energy tensor.

Furthermore, the D-dimensional spacetime Ricci tensor can be related to the brane Ricci tensor and to the warp factor W by the equations

$$R_{\mu\nu} = \bar{R}_{\mu\nu} - \frac{g_{\mu\nu}}{(p+1)W^{p-1}}\nabla^2 W^{p+1} \tag{5}$$

and

$$R_{ab} = \tilde{R}_{ab} - \frac{p+1}{W}\nabla_a\nabla_b W, \tag{6}$$

where R_{ab}, ∇_a, and ∇^2 are, respectively, the Ricci tensor, the covariant derivative, and the Laplacian operator constructed by the internal space metric g_{ab}. $\bar{R}_{\mu\nu}$ is the Ricci tensor derived from $g_{\mu\nu}$. The three curvature scalars are denoted by $R = G^{AB}R_{AB}$, $\bar{R} = g^{\mu\nu}\bar{R}_{\mu\nu}$, and $\tilde{R} = g^{ab}\tilde{R}_{ab}$. Thus, the traces of Eqs. (5) and (6) yield the following equations:

$$\frac{1}{p+1}(W^{-2}\bar{R} - R^\mu_\mu) = pW^{-2}\nabla W\cdot\nabla W + W^{-1}\nabla^2 W \tag{7}$$

and

$$\frac{1}{p+1}(\tilde{R} - R^a_a) = W^{-1}\nabla^2 W, \tag{8}$$

where $R^\mu_\mu \equiv W^{-2}g^{\mu\nu}R_{\mu\nu}$ and $R^a_a \equiv g^{ab}R_{ab}$ (related by $R = R^\mu_\mu + R^a_a$). Supposing that α is an arbitrary constant, it is not difficult to see that

$$\nabla\cdot(W^\alpha\nabla W) = W^{\alpha+1}(\alpha W^{-2}\nabla W\cdot\nabla W + W^{-1}\nabla^2 W). \tag{9}$$

Combining Eqs. (7)–(9) will allow us to write

$$\nabla\cdot(W^\alpha\nabla W) = \frac{W^{\alpha+1}}{p(p+1)}[\alpha(W^{-2}\bar{R} - R^\mu_\mu) + (p-\alpha)(\tilde{R} - R^a_a)]. \tag{10}$$

As an *ansatz* for the stress-energy tensor, we can write

$$T_{AB} = -\Lambda G_{AB} - \sum_i T^{(i)}_q P[G_{AB}]^{(i)}_q \Delta^{(D-q-1)}(r-r_i) + \tau_{AB}, \tag{11}$$

where Λ is the bulk cosmological constant, $T^{(i)}_q$ is the ith q-brane tension, $\Delta^{(D-q-1)}(r-r_i)$ is the covariant combination of delta functions which localizes the brane, $P[G_{AB}]^{(i)}_q$ is the pullback of the spacetime metric to the world volume of the q-brane (or the induced metric on the brane), and any other matter contribution is represented by τ_{AB}. Following this *ansatz* one obtains

$$T^\mu_\mu = -(p+1)\Lambda + \tau^\mu_\mu - \sum_i T^i_q\Delta^{(D-q-1)}(r-r_i)(p+1) \tag{12}$$

and

$$T^a_a = -(D-p-1)\Lambda + \tau^a_a - \sum_i T^i_q \Delta^{(D-q-1)}(r-r_i)(q-p). \tag{13}$$

Inserting Eqs. (2) and (3) into Eq. (10), with T^μ_μ and T^a_a as defined in Eqs. (12) and (13), respectively, it is straightforward to demonstrate—following the previous *ansatz* (12) and (13) and performing an integration in the compact internal space as well—that

$$\oint W^{\alpha+1}\Big\{\alpha\bar{R}W^{-2} + (p-\alpha)\tilde{R} - \Big[\frac{p+1}{D-2}[(p-2\alpha)(D-p-1)+2\alpha] + \frac{(D-p-1)p(2\alpha-p+1)}{D-2}\Big]\Lambda$$
$$- 8\pi G_D\Big[\sum_i\Big[\frac{p+1}{D-2}((p-2\alpha)(D-p-1)+2\alpha) + (q-p)\frac{p(2\alpha-p+1)}{D-2}\Big]T^{(i)}_q\Delta^{(D-q-1)(r-r_i)}$$
$$- \frac{(p-2\alpha)(D-p-1)+2\alpha}{D-2}\tau^\mu_\mu - \frac{p(2\alpha-p+1)}{D-2}\tau^a_a\Big]\Big\} = 0. \tag{14}$$

We shall refer to Eq. (14) as the one parameter family of consistency conditions for braneworld scenarios. If one considers, as an example, the case $D=5$, $p=3$, with $\alpha=-1$, setting $\tau^a_a = \tau^\mu_\mu = 0$, the equation above reduces simply to

$$-\bar{R}\oint W^{-2} = 32\pi G_5\sum_i T^{(i)}_3, \tag{15}$$

since $\tilde{R}=0$ for a single internal direction.

In trying to reproduce our Universe, it is acceptable to set $\bar{R}=0$, and Eq. (15) reduces to

$$32\pi G_5\sum_i T^{(i)}_3 = 0, \tag{16}$$

making the presence of a negative brane tension necessary in the Randall-Sundrum setup.

After this short review, let us start the analysis of the $f(R)$ case itself by considering the nonlinear (in the Ricci scalar) bulk action in D dimensions given by

$$S_{\rm bulk} = -\frac{1}{8\pi G_D}\int d^Dx\sqrt{-G}[f(R) + \mathcal{L}_m], \tag{17}$$

where $\mathcal{L}_m$ stands for the matter (and brane) Lagrangian. The variation of the above action with respect to the bulk metric gives the following field equation:

$$F(R)R_{AB} - \tfrac{1}{2}G_{AB}f(R) + G_{AB}\Box F(R) - \nabla_A\nabla_B F(R) = 8\pi G_D T_{AB}, \tag{18}$$

which may be recast in the more familiar form

$$R_{AB} - \frac{1}{2}RG_{AB} = \frac{1}{F(R)}\Big[8\pi G_D T_{AB} - \Big(\frac{1}{2}RF(R) - \frac{1}{2}f(R) + \Box F(R)\Big)\times G_{AB} + \nabla_A\nabla_B F(R)\Big], \tag{19}$$

with $F(R) = df(R)/dR$. In order to implement the bulk $f(R)$ gravity in the consistency conditions program, we call attention to the fact that Eq. (10) is obtained without any reference to the field equation in question, being instead purely geometric. Therefore, it is a suitable starting point for our generalization. From Eq. (19) it is easy to see that the Ricci scalar is given by

$$R = \frac{2}{(2-D)F(R)}\times\Big[8\pi G_D T - \frac{D}{2}RF(R) + \frac{D}{2}f(R) + (1-D)\Box F(R)\Big]. \tag{20}$$

Note that, obviously, it is possible to isolate the Ricci scalar, obtaining $R = [8\pi G_D T + (D/2)f(R) + (1-D)\Box F(R)]/F(R)$. However, we shall develop our presentation with Eq. (20) for the sake of clarity, since in this way the usual general relativity limit [$f(R)=R$ and $F(R)=1$] is easily obtained. From Eq. (20) the contact with general relativity is, indeed, more explicit.

Substituting Eq. (20) into (19) we have

$$R_{AB} = \frac{1}{F(R)}\Big[8\pi G_D\Big(T_{AB} - \frac{G_{AB}}{D-2}T\Big) + \nabla_A\nabla_B F(R) + \frac{1}{D-2}G_{AB}[RF(R) - f(R) + \Box F(R)]\Big], \tag{21}$$

whose limit to general relativity gives only the first term, as expected. The partial traces of the above equation, following the same standard previous notation, are given by

$$R^\mu_\mu = \frac{1}{(D-2)F(R)}[8\pi G_D[(D-p-3)T^\mu_\mu - (p+1)T^m_m] + (D+p-1)W^{-2}\nabla^\mu\nabla_\mu F(R) + (p+1)[RF(R) - f(R) + \nabla^m\nabla_m F(R)]] \tag{22}$$

and

$$R_m^m = \frac{1}{(D-2)F(R)}[8\pi G_D[(p-1)T_m^m - (D-p-1)T_\mu^\mu] + (2D-p-3)\nabla^m\nabla_m F(R) + (D-p-1)[RF(R) - f(R) + W^{-2}\nabla^\mu\nabla_\mu F(R)]]. \tag{23}$$

Now it is possible to apply Eq. (10) to the $f(R)$ bulk gravity case. Hence with Eqs. (22) and (23) it reads

$$\begin{aligned}\nabla\cdot(W^\alpha\nabla W) = \frac{W^{\alpha+1}}{p(p+1)(D-2)F(R)}&[(D-2)F(R)[\alpha\bar{R}W^{-2} + (p-\alpha)\tilde{R}] \\ &+ 8\pi G_D T_\mu^\mu[(p-\alpha)(D-p-1) - \alpha(D-p-3)] + 8\pi G_D T_m^m[\alpha(p+1) - (p-\alpha)(p-1)] \\ &- [RF(R) - f(R)][\alpha(p+1) + (D-p-1)(p-\alpha)] - W^{-2}\nabla_\mu\nabla^\mu F(R)[(D-p-1) + \alpha(D+p-1)] \\ &- \nabla^m\nabla_m F(R)[\alpha(p+1) + (p-\alpha)(2D-p-3)]].\end{aligned} \tag{24}$$

In an internal compact space the integral of the equation above vanishes, since $\oint\nabla\cdot(W^\alpha\nabla W) = 0$. In order to complete the analysis we shall use the comprehensive stress-tensor ansatz defined by Eq. (11) with the partial traces given by Eqs. (12) and (13). Substituting Eqs. (12) and (13) into Eq. (24) we arrive, after some algebra, at the generalization of Eq. (14) for the $f(R)$ bulk gravity case:

$$\begin{aligned}\oint\frac{W^{\alpha+1}}{F(R)}\Big\{&[(D-2)F(R)[\alpha\bar{R}W^{-2} + (p-\alpha)\tilde{R}]] - [RF(R) - f(R) + 2\Lambda][(D-p-1)(p-\alpha) + \alpha(p+1)] \\ &- 8\pi G_D\{(p+1)[(D-p-1)(p-2\alpha) + 2\alpha] + p(1+2\alpha-p)(q-p)\}\sum_i T_q^i\Delta^{(D-q-1)}(r-r_i) \\ &+ 8\pi G_D[(D-p-1)(p-2\alpha) + 2\alpha]\tau_\mu^\mu + 8\pi G_D p(1+2\alpha-p)\tau_m^m \\ &- W^{-2}\nabla_\mu\nabla^\mu F(R)[(D-1)(\alpha+1) + p(\alpha-1)] - \nabla_m\nabla^m F(R)[\alpha(p+1) + (p-\alpha)(2D-p-3)]\Big\} = 0.\end{aligned} \tag{25}$$

In order to make our point clearer, we shall particularize the analysis to the five-dimensional bulk case. Therefore, we take $D = 5$, $p = q = 3$, and for simplicity, we set $\tau_\mu^\mu = \tau_m^m = 0$, disregarding any extra bulk matter contribution. Note that for this case $\tilde{R} = 0$, since there is only one extra (transverse) dimension. With such particularizations Eq. (25) reads

$$\begin{aligned}\oint\frac{W^{\alpha+1}}{F(R)}\Big\{&\alpha F(R)\bar{R}W^{-2} - (\alpha+1)[RF(R) - f(R) + 2\Lambda] - 32\pi G_5\sum_i T_3^i\delta(r-r_i) \\ &- \frac{W^{-2}}{3}(1+7\alpha)\nabla_\mu\nabla^\mu F(R) - 4\nabla^m\nabla_m F(R)\Big\} = 0.\end{aligned} \tag{26}$$

In trying to describe our Universe, it is conceivable to set $\bar{R}$, which vanishes with an accuracy of $10^{-120}M_{\text{Planck}}^2$. Besides, from Eqs. (3) and (4) it is easy to see that the bulk scalar of curvature for the case in question is given only in terms of the warp factor by

$$R = R_\mu^\mu + R_m^m = -\frac{4}{W}\nabla^2 W - \frac{1}{W^4}\nabla^2 W^4, \tag{27}$$

where $\nabla^2 = \frac{1}{2g_{rr}}(\partial_r g_{rr})g^{rr}\partial_r + (\partial_r g^{rr})\partial_r + g^{rr}\partial_r^2$. For the simplest case of stabilized distance between the branes (g_{rr} constant)—as in the RSI model—we have simply $\nabla^2 = \partial_r^2$, where $g_{rr} = 1$ for simplicity. In view of Eq. (27), and taking into account that the warp factor depends only on the extra dimension, we have

$$\begin{aligned}\oint\frac{W^{\alpha+1}}{F(W)}\Big\{&\frac{(\alpha+1)}{4}\Big[2\Lambda - f(W) - F(W) \\ &\times\Big(\frac{4}{W}\nabla^2 W + \frac{1}{W^4}\nabla^2 W^4\Big)\Big] \\ &+ 8\pi G_5\sum_i T_3^i\delta(r-r_i) + \nabla^2 F(W)\Big\} = 0.\end{aligned} \tag{28}$$

It is important to stress that for $f(R) = R = -\frac{4}{W}\nabla^2 W - \frac{1}{W^4}\nabla^2 W^4$ the usual sum rules for general relativity are recovered.

The most direct way to obtain a necessary condition concerning the brane tension sign is by choosing $\alpha = -1$ [11,12], since it eliminates the overall warp factor and the first term of Eq. (28). Hence, in this particularly interesting case we have

$$\oint \frac{1}{F(W)}\left\{8\pi G_5 \sum_i T_3^i \delta(r - r_i) + \nabla^2 F(W)\right\} = 0, \quad (29)$$

which leads to the following necessary condition:

$$8\pi G_5 \sum_i \frac{T_3^i}{F_i(W)} + \oint \frac{\nabla^2 F(W)}{F(W)} = 0, \quad (30)$$

where $F_i(W) = F(W(r = r_i))$. Note that, according to Eq. (30), a negative brane tension in the compactification scheme is not necessary, provided that[1] $\oint \frac{\nabla^2 F(W)}{F(W)} < 0$. We remark, in passing, that the claim above is not applicable in the pathological case given by

$$\nabla^2 F(W) + \frac{(\alpha+1)}{4}\bigg[2\Lambda - f(W) - F(W)\left(\frac{4}{W}\nabla^2 W + \frac{1}{W^4}\nabla^2 W^4\right)\bigg] = 0, \quad (31)$$

which, certainly, is not the most general case.

As it is possible to evade the necessity of a negative brane tension by means of a condition which depends on the warp factor, it would be interesting to explore a little further the analysis by studying the possibility of warp factors being suitable for the hierarchy problem solution and satisfying $\oint \frac{\nabla^2 F(W)}{F(W)} < 0$ at the same time.

In five dimensions it is easy to see that the bulk scalar of curvature is given by

$$R = -\frac{12(W'(r))^2 + 8W(r)W''(r)}{(W(r))^2}, \quad (32)$$

where a prime means a derivative with respect to r. Obviously, any warp factor leading to a constant R—even solving the hierarchy problem—does not satisfy the constraint given by Eq. (30), and shall not be used to set a compactification scheme without a negative brane tension.

There are several warp factor profiles which may be used in order to approach the hierarchy problem. To fix ideas, let us work with the simplest choice $W(r) = e^{-ar^n}$, with $a(>0)$ and n being constants.[2] From Eq. (32) we have

$$R = -4nar^{n-2}[5nar^n - 2(n-1)]; \quad (33)$$

[1]Note that the warp factor must be of class C^4 at least.

[2]We shall not be concerned with the absolute value of r in this simple example.

hence models with $n = 1$ should be excluded. Following this reasoning, it is not difficult to test a bulk $f(R)$ gravity braneworld model as a candidate to approach the hierarchy problem without a negative brane tension. In our simple example outlined above, if $n \neq 1$ then, in principle, it would be possible to use such a warp factor to attack both problems. Of course, to complete the analysis one should fix the specific model by setting the functional form of $f(R)$.

III. CONCLUDING REMARKS

The generalization of the braneworld consistency conditions to $f(R)$ bulk gravity was considered in order to derive criteria on the possible warp factors which could solve the hierarchy problem, leading, at the same time, to a consistent compactification framework. By applying this program it is possible to verify, among several braneworld scenarios, what type of models may be phenomenologically useful.

By inspecting Eq. (30) it is simple to see that any braneworld model whose warp factor leads to a constant bulk scalar of curvature necessarily requires at least one negative brane tension. For some typical warp factors (suitable for the hierarchy problem solution) the constraint (30) cannot be satisfied.

On the other hand, if the model does not concern the hierarchy problem—leaving it for supersymmetric extensions of the standard model, for instance—then Eq. (30) is the relevant constraint. A nonconstant R (with respect to the extra transverse dimension) may lead to a scenario without a negative brane tension. The presence of $F(R)$ terms in the consistency conditions acts in order to relax the conditions to be satisfied, opening the possibility for a well-defined model. It is certainly a gain when compared with the usual general relativity based braneworld models.

It is important to remark that in the bulk $f(R)$ gravity context, other types of warp factors may be useful to approach the hierarchy problem and to evade the necessity of a negative brane tension in the compactification scheme. In this approach the $F(R)$ [and, accordingly, the $F(W)$] term just depends on the extra transverse dimension, in such a way that any experimental contradiction with local tests of gravity on the brane is not expected. However, in order to give a cogent argument, one should project the $f(R)$ bulk geometrical quantities on the brane, via e.g. the Gauss-Codazzi formalism with appropriate junction conditions, and calculate the relevant post Newtonian parameters. On the other hand, keeping in mind the conformal equivalence between $f(R)$ and scalar tensor theories, one may speculate that, indeed, there is no violation of any experimental bound; i.e., Eq. (30) is in fact well defined from the phenomenological point of view.

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