Exploring S-type orbits in the Pluto–Charon binary system

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ABSTRACT

This work generates, through a sample of numerical simulations of the restricted three-body problem, diagrams of semimajor axis and eccentricity which defines stable and unstable zones for particles in S-type orbits around Pluto and Charon. Since we consider initial conditions with $0 \leq e \leq 0.99$, we found several new stable regions. We also identified the nature of each one of these newly found stable regions. They are all associated to families of periodic orbits derived from the planar circular restricted three-body problem. We have shown that a possible eccentricity of the Pluto–Charon system slightly reduces, but does not destroy, any of the stable regions.

Key words: Kuiper belt: general – minor planets, asteroids: general – planets and satellites: dynamical evolution and stability.

1 INTRODUCTION

Pluto and Charon form a binary system with a mass parameter of 0.1165. Along with these massive bodies, two small satellites, named Nix and Hydra, complete this peculiar system. From a dynamical point of view, a binary system presents some differences when compared with other systems. For example, the centre of mass is located outside of the massive bodies and the so-called Lagrangian equilateral points are not stable points (Szebehely 1967; Weaver et al. 2006).

The searching for stable zones can help us to detect additional satellites and a possible ring system, which is one of the goals of the New Horizons mission during its close approach with Pluto. Holman & Wiegert (1999) divided the orbits of a binary system into three classes, following the designations given by Dvorak (1986): in the first one, called P-type orbit, the particle is moving around the centre of mass of the binary system; in the S-type orbit, the particle is around one of the bodies and the third class of orbits are those near the Lagrangian triangular points, $L_4$ and $L_5$. In their work, only the first and the second classes have been considered. They numerically simulated a sample of particles taking into account the elliptical restricted three-body problem for a time-span of $10^4$ orbital periods of the binary. The eccentricity of the binary was taken between 0 and 0.8 and the mass parameter between 0.1 and 0.9. From their results, Holman & Wiegert (1999) derived an empirical expression for the critical semimajor axis which gives the maximum value of the semimajor axis, where the particle will be in a stable zone. However, as they pointed out, the limit between stable and unstable regions is not well defined since it depends on the period of integration.

Stern et al. (1994) have analysed a sample of test particles under the effects of Pluto and Charon. Two types of orbits have been studied: (a) particles in circular S-type orbits around Pluto and (b) particles in P-type orbits around the barycentre of the Pluto–Charon system. They numerically integrated the circular restricted three-body problem, Pluto–Charon particle in S-type orbit, located between Pluto’s Roche limit and Charon’s orbit (0.15$d$–1$d$), where $d$ is the distance between Pluto and Charon. These particles, initially in circular orbits, have inclinations varying from 0° to 180°. Their results show that the stability zone, for particles in prograde orbits and low inclinations, extends up to 0.47$d$. However, for particles in retrograde orbits the stability zone extends up to 0.65$d$. They also found a limit mass for a hypothetical satellite located in the stable zone. A satellite with mass larger than $10^{-4}$ $M_{\text{C-P}}$, where $M_{\text{C-P}}$ is the mass of the binary, is ruled out since it can provoke a large variation in the eccentricity of Charon, which is not observed.

Nagy, Sulli & Erdi (2006) also analysed the phase space of Pluto–Charon system, for particles in P- and S-type orbits, through three different complementary methods. They applied the model of the spatial circular restricted three-body problem. Their results, concerning a sample of particles in S-type orbits around Pluto, show that stable orbits can exist up to 0.5$d$. This value is larger than the value obtained by Holman & Wiegert (1999) for $\mu = 0.1$. They did not find any stable region for particles in S-type prograde orbits around Charon. Particles in S-type retrograde orbits were not analysed in their work.

In the present work, we explore the dynamical behaviour of a sample of test particles, for a time-span of $10^4$ orbital periods of the binary, located between the orbits of Pluto and Charon in order to determine stable regions for different values of the particle’s eccentricity. Holman & Wiegert (1999) and Winter & Vieira Neto (2001) have shown that usually this length of integration time is good enough to identify stable regions (those regions that the particles do not collide with one of the primaries neither escape the system).

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The numerical simulations were performed by using the restricted three-body problem, Pluto–Charon particle. Since Nix and Hydra are too small to add any important gravitational effects on particles located between the orbits of the two massive bodies, their effects have been neglected. We also computed the Poincaré surface of sections in order to verify the location, size and nature of these stable regions. The Poincaré surface of section is an indubitable method to confirm whether a given region is regular or chaotic.

The paper is presented in the following way. In Section 2, we describe the numerical simulations and present the stability diagrams of semimajor axis versus eccentricity. In Section 3, we identify the family of periodic orbits associated to the newly found stable regions through the Poincaré surface of sections. Our results and final conclusions are summarized in the last section.

2 STABILITY DIAGRAMS

The framework of the restricted three-body problem (see for example Murray & Dermott 1999) was used to numerically simulate the behaviour of a sample of test particles under the gravitational effects of Pluto and Charon. The mass ratio of Charon and Pluto was assumed to be \( \mu = 0.1165 \) and the separation between them to be \( d = 19571 \text{ km} \) (Buie et al. 2006). These particles were initially considered in S-type orbits around one of the massive bodies. Those particles which stay around one of these bodies during the time-span of the integration, about \( 10^4 \) orbital periods of the binary \( (T = 65000 \text{ d}) \), are considered to be in stable orbits. The integrator adopted was Radau 15 (Everhard 1985).

In order to obtain the stability diagram, we use a set of about 70 000 initial conditions varied in the following way: (i) the semimajor axis was uniformly distributed, \( \sim 0.012d \) from Pluto and \( \sim 0.006d \) from Charon, with stepsize \( \Delta a = 0.0013d \) and (ii) the eccentricity was varied from 0 to 0.99 with stepsize \( \Delta e = 0.001 \). A collision is computed every time the test particle is \( \sim 0.012d \) from Pluto or \( \sim 0.006d \) from Charon. The particle’s inclination was assumed to be 0° (prograde orbits) and 180° (retrograde orbits).

2.1 Orbits around Pluto

As a result of these numerical simulations (Vieira Neto, Winter & Yokoyama 2006), we obtained two samples of diagrams (Fig. 1) of initial semimajor axis \( (a) \) versus eccentricity \( (e) \) for those particles which remain in S-type orbits around Pluto for \( 10^4 \) orbital periods of the binary. The semimajor axis is given in unit of \( d \). The test particles were divided into two groups: in prograde [Fig. 1(a)] and in retrograde orbits [Fig. 1(b)]. The top half of the diagram \( (e > 0) \) corresponds to conditions at opposition \( (\sigma = 0 \text{ and } \tau = 0, \text{ where } \tau \text{ is the epoch of the pericentre}) \), while the other half \( (e < 0, \text{ this is a symbolical sign}) \) corresponds to conditions at the apocentre at
opposition ($\sigma = \pi$ and $\tau = P/2$, where $P$ is the osculating orbital period).

The limit of the stable region (in white), for those test particles with zero eccentricity [Fig. 1(a) first column], agrees with the result found by Stern et al. (1994) (as shown in their fig. 3) and it is larger than the value (0.42$d$) obtained through the empirical expression given by Holman & Wiegert (1999). Our results also show the existence of stable regions for particles in elliptical orbits. These stable regions can be labelled: region 1 located around 0.6 with eccentricity larger than 0.2 and region 2 between 0.4 and 0.5 with $e < 0.25$.

Stable regions are also present in Fig. 1(b) (first column) for test particles in retrograde orbits around Pluto. Stern et al. (1994) found the limit for a stable region to be $\sim 0.65d$ for particles in circular retrograde orbits. This value is similar to the value (0.67$d$) shown in Fig. 1(b). However, when the eccentricity of the particles is taken to be different from zero, other stable regions are found: region 3 ($0 \leq a < 0.7$ and $e < 0.8$) and region 4 ($e < 0.35$ and $a > 0.7$). It is important to note that there is an unstable region located between 0.5 and 0.6 for eccentricity larger than 0.15.

Tholen et al. (2008) claimed that Charon has an eccentricity equal to 0.0035. We numerically simulated the same initial conditions taking into account the eccentricity of the binary. The results are presented in Fig. 1 (right column). A comparison with Fig. 1 (left column) shows a very slight reduction in the stable regions. Therefore, the circular case gives a very good idea of the stable regions in this system.

### 2.2 Orbits around Charon

Fig. 2 shows the stability diagrams of initial $a$ versus $e$ for particles in orbit around Charon disturbed by the gravitational effects of Pluto. Our results presented in Fig. 2(a) show a stable region (in white) which extends up to 0.13$d$ from Charon for particles with $e < 0.6$. These particles are in prograde orbits.

Our result agrees with the value (0.13$d$) found by Holman & Wiegert (1999) for particles in circular orbits. However, as can be seen in Fig. 2(a), the limit of the stable region can be extended for particles with eccentricity up to 0.6. Our results show one small stable region (region 5), beyond the critical semimajor axis, at $a = 0.18$ and $0.2 < e < 0.6$. Nagy et al. (2006) did not find any stable region beyond the critical semimajor axis for particles in prograde orbits around Charon. The stability diagram (Fig. 2b) also shows two stable regions for particles in retrograde orbits: region 6 ($a < 0.4$) and region 7 ($a > 0.4$).

When the eccentricity of the binary is assumed to be 0.0035, only a negligible decrease in the stable regions is observed, as has been seen for those particles in orbit around Pluto (Fig. 1). Therefore, in the analysis of the periodic orbits (next section) we will assume the eccentricity of the binary to be equal to zero.

### 3 Families of Periodic Orbits

In this section, we introduced the method of the Poincaré surface of section. This is an excellent method to identify whether a trajectory is regular or chaotic. We used this method with two purposes:

(i) to confirm the location and size of the stable regions obtained in Section 2 and
(ii) to identify the nature of these stable regions.

The method of the Poincaré surface of section can be used to determine the location and size of regular and chaotic regions located in phase space of the circular, planar, restricted three-body problem (PCR3BP). In order to determine the orbital elements of the particle at any particular time $t$, it is necessary to know the components of the position $(x, y)$ and the velocity $(\dot{x}, \dot{y})$ of the particle at this time $t$; these will give a point in a four-dimensional phase space. With a fixed value of Jacobi constant $C_J$, the only integral of the motion of the PCR3BP, only three quantities are needed to identify a particle in a three-dimensional surface, for example $x$, $y$, $\dot{x}$. However, if a plane is defined with, for example, $y = 0$, the point in the phase space of two-dimensional is identified at time $t$ with values of $\dot{x}$ and $x$. The ambiguity in the sign of $\dot{y}$ can be removed by considering those points with a fixed sign of $\dot{y}$ (Winter & Murray 1997). In the Poincaré surface of section, a periodic orbit appears as a finite number of points, each one surrounded by islands of quasi-periodic orbits. Chaotic regions appear as irregular distributions of points.

We consider a barycentric coordinate system, rotating with the same angular velocity of the binary. Pluto and Charon are on the $x$-axis, with Pluto at $x < 0$ (Murray & Dermott 1999). The initial conditions $(x, y, \dot{x}, \dot{y})$ of each particle were distributed in the following way: $x$ was taken between the orbits of Pluto and Charon, $y = 0$, $\dot{x} = 0$ and $\dot{y}$ was derived from the Jacobi Constant (Murray 1999).
$C_1 = n^2(x^2 + y^2) + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right) - \dot{x}^2 - \dot{y}^2$,  \hspace{1cm} (1)$

where $n$ is the mean motion of the binary, $\mu_1 = 1 - \mu$, $\mu_2 = \mu$ and $r_1$ and $r_2$ are the distance of the particle from Pluto and Charon, respectively.

About 80 particles, uniformly distributed in the $x$-axis, were numerically integrated in order to result one Poincaré surface of section. A sample of Poincaré surface of section was generated by assuming different values of the Jacobi constant, which ranges between 0.01 and 7.0, with stepsize $\Delta C_J = 0.01$.

The centre of an island in the Poincaré surface of section is associated to one periodic orbit. Following the approach used by Winter (2000), the largest island surrounding this point gives the size of the regular regions. The stability of this kind of periodic orbits can be measured in terms of the maximum amplitude of oscillation about the periodic orbits. For each value of $C_J$, the width of the largest island (values of $x$ at $\dot{x} = 0$) gives the maximum amplitude of oscillation. We identified the islands obtained in the sample of Poincaré surface of section generated in this work. From the values of $C_J$ and $x$, at $\dot{x} = 0$ and $y = 0$, we found the osculating values of the semimajor and eccentricity in order to compare with the stable regions shown in the stability diagrams given in Section 2. The location and size of the stable regions measured from the Poincaré surface of sections are in very good agreement with those stable regions found in the previous section. Therefore, our results confirm that 10$^4$ orbital periods of numerical integration are good enough to identify stable regions in the present dynamical system.

Next, we identify the periodic orbits around Pluto and Charon.

### 3.1 Orbits around Pluto

Fig. 1 shows a stable region (region 1) located at $a \sim 0.6$ with $0.2 < e < 0.6$. Region 1 matches the region obtained from a set of values of $C_J$ ranging between 2.816 and 3.236. Fig. 3(a) presents two Poincaré surface of section for $C_J = 3.154$ and 3.196, where the values of the maximum amplitude of oscillation of the islands located at $x \sim 0.3$ form the stable region 1. The islands at $x > 0.7$ correspond to quasi-periodic orbits around Charon.

After identifying the stable regions in the Poincaré surface of section, we can obtain the trajectory for the stable orbit located at the centre of the island. The initial conditions (position and velocity) were derived from the Poincaré surface of section. Two trajectories are presented in Fig. 3(b). The centre of these islands corresponds to periodic orbits of family ‘BD’ (following the designation given by Broucke 1968).

The same analysis has been carried out for region 2. The values of $C_J$ needed to form this region ranges from 3.466 to 3.706. Fig. 4(a) shows an example of the Poincaré surface of section, and Fig. 4(b)
Figure 4. Region 2: (a) the Poincaré surface of section for $C_J = 3.526$, (b) the trajectory of the stable orbit at $x = 0.320$ and (c) the trajectory of the stable orbit at $x = 0.052$. 

Figure 5. Set of Poincaré surface of section (for regions 3 and 4) for different values of Jacobi constant: (a) $C_J = 0.010$ and $0.20$, (b) $C_J = 0.30$ and $0.40$ and (c) $C_J = 0.50$ and $0.60$. The islands of stability correspond to the stable regions 3 and 4, shown in Fig. 2(a).

the trajectory of the stable orbit located at the centre of the large island ($x = 0.320$). The trajectory of the stable region at the centre of the small island ($x = 0.052$) corresponds to $a < 0.4$ and $e > 0.4$ in the stability diagram [Fig. 1(a)]. The centre of the large islands corresponds to periodic orbits of family ‘BD’ (Broucke 1968).

In order to analyse the stable regions located in Fig. 1(b), we have to generate another set of sample of test particles in retrograde orbits around Pluto. The initial conditions are the same as assumed above, except for the value of $\dot{y}$ which was taken to be negative. The range of values of $C_J$ were ranging between 0.01 and 4.0.

Fig. 5 shows a set for different values of the Jacobi constant. The islands of stability correspond to the stable regions 3 and 4. The centre of the islands corresponds to periodic orbits, whose trajectories are presented in Fig. 6(a). The stable regions are due to the family ‘$A_1$’ of periodic orbits (Broucke 1968). The largest trajectory has $C_J = 0.010$. By increasing the value of $C_J$, the values
Figure 6. Regions 3 and 4: (a) sample of stable orbits for values of Jacobi constant shown in Fig. 5. The size of the trajectories decreases with the increase in the value of Jacobi constant. The largest trajectory has $C_J = 0.010$; (b) stable orbits for $C_J = 0.5$ and 0.6 for a periodic orbit at $x = 0.24$ and 0.15 (Fig. 5), respectively.

of $a$ in the stability diagram decreases. Fig. 6(b) shows the trajectories for the periodic orbits located at $x = 0.24$ and 0.15 in the Poincaré surface of section. The unstable region which separate the two islands can be seen in the stability diagram at $a \sim 0.6$ and $e > 0.15$.

3.2 Orbits around Charon

From Fig. 2(a), we identified a new stable region for prograde orbits (region 5). This region corresponds to islands found in the range of $C_J$ between 3.485 and 3.530. A sample of these Poincaré surface of sections is presented in Fig. 7. The centre of these islands corresponds to periodic orbits of family ‘$H_2$’ (Broucke 1968). This family of periodic orbits was extensively explored in the case of the Earth–Moon system by Winter & Vieira Neto (2002). The evolution of this family of periodic orbits in the Pluto–Charon system is shown in Fig. 8.

In the case of retrograde orbits around Charon [Fig. 2(b)], we identified two stable regions, called regions 6 and 7. The stable regions are well known and were studied in details for the Sun–Neptune system by Winter & Vieira Neto (2001). They showed that these regions are due to the family ‘$f$’ of periodic orbits (Broucke 1968). In the case of Pluto–Charon system, the islands of the stability are found in the Poincaré surface of section for $2.449 \leq C_J \leq 3.157$.

4 FINAL COMMENTS

In this present work, we found several stable regions around Pluto and Charon not identified in previous studies (Stern et al. 1994; Nagy et al. 2006). This analysis has been carried out through a sample of numerical simulations of an ensemble of test particles in orbit around one of the two massive bodies, Pluto or Charon. Our results were presented in stability diagrams of semimajor axis versus eccentricity: given the initial values of the eccentricity and semimajor axis, we can determine whether a particle is in a stable region. These results were confirmed through the Poincaré surface of sections.

Although our results did not cover all the initial conditions space, we have analysed symmetric orbits ($\sigma = 0$ and 180°); the stable regions found in our sample of Poincaré surface of section corresponding to asymmetric orbits are very small. One example of an asymmetric orbit can be seen in the Poincaré surface of section presented in Fig. 5(b) (right column).

The nature of these newly found stable regions were analysed through the Poincaré surface of section. These regions are all
Figure 7. Poincaré surface of section for different values of the Jacobi constant: (a) $C_J = 3.485$ and 3.49, (b) $C_J = 3.50$ and 3.51 and (c) $C_J = 3.52$ and 3.53. The islands of stability correspond to the stable region 6, shown in Fig. 2(a).

We have shown that a possible eccentricity of the Pluto–Charon system (Tholen et al. 2008) slightly reduces, but does not destroy, any of the stable regions. These results can be helpful for the New Horizons mission which will encounter Pluto in 2015 July.

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Figure 8. Periodic orbits with the values of the Jacobi constant given in Fig. 7.

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