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SÃO PAULO STATE UNIVERSITY (UNESP)
"JÚLIO DE MESQUITA FILHO"
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ILHA SOLTEIRA - SP

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$\mathscr{H}_\infty$ AND $\mathscr{H}_2$ GAIN SCHEDULING STATE DERIVATIVE FEEDBACK CONTROL BASED ON LMIs FOR LINEAR PARAMETER-VARYING SYSTEMS

Ilha Solteira
2022

MARCO ANTONIO LEITE BETETO

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Thesis presented to the São Paulo State University (UNESP) - School of Engineering - Campus of Ilha Solteira, in fulfilment of one of the requirements for the degree of Doctor of Science in Electrical Engineering.
Speciality: Automation.

Prof. Dr. Edvaldo Assunção
Advisor

Ilha Solteira
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Câmpus de Ilha Solteira

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TÍTULO DA TESE: Hoo and H2 Gain Scheduling State Derivative Feedback Control Based on LMIs for Linear Parameter-Varying Systems.

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Ilha Solteira, 20 de maio de 2022

I dedicate this work to my parents, Luciana and Claudemir;

To my sister Emanuela;

To my girlfriend Ana Paula;

for all love, support, trust and encouragement at all times.

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”Nothing in this world can take the place of persistence. Talent will not; nothing is more common than unsuccessful men with talent. Genius will not; unrewarded genius is almost a proverb. Education will not; the world is full of educated derelicts. Persistence and determination alone are omnipotent. The slogan Press On! has solved and always will solve the problems of the human race.”

Calvin Coolidge (1872 - 1933)

”The beginning of a habit is like an invisible thread, but every time we repeat the act we strengthen the strand, add to it another filament, until it becomes a great cable and binds us irrevocably thought and act.”

Orison Swett Marden (1848 - 1924)

RESUMO

Neste trabalho, são propostas novas condições para o controle de sistemas lineares dependentes de parâmetros variantes no tempo (do inglês, *Linear Parameter-Varying* - LPV), considerando a realimentação derivativa. A princípio, será abordado o problema $\mathscr{H}_\infty$ para então obter condições para o controlador de realimentação derivativa *gain scheduling* $\mathscr{H}_\infty$. Em seguida, condições para o controlador de realimentação derivativa *gain scheduling* $\mathscr{H}_2$ são obtidas. O projeto dos controladores é baseado em desigualdades matriciais lineares (do inglês, *Linear Matrix Inequalities* - LMIs). É importante ressaltar que será considerada também a $\mathscr{D}$-estabilidade no projeto de controle, como forma de obter bom desempenho com um sinal de controle passível de implementação em um sistema real. Aqui, as condições para a $\mathscr{D}$-estabilidade serão tratadas no sentido de sistemas invariantes no tempo, com valores fixos do parâmetro variante em seu intervalo de variação. Ademais, as condições propostas levam em conta uma função de Lyapunov quadrática comum (do inglês, *Common Quadratic Lyapunov Function* - CQLF) para, em seguida, serem comparadas com as condições propostas considerando uma função de Lyapunov dependente do parâmetro variante (do inglês, *Parameter-Dependent Lyapunov Function* - PDLF). Este trabalho também oferece condições necessárias e suficientes para o controle misto $\mathscr{H}_\infty/\mathscr{H}_2$, ou seja, junta ambos, o problema $\mathscr{H}_\infty$ e o problema $\mathscr{H}_2$. As condições propostas são aplicadas em diversos exemplos para mostrar que utilizando-as é possível diminuir o custo garantido $\mathscr{H}_\infty$ e $\mathscr{H}_2$, ou seja, minimizar o efeito de um possível distúrbio no sistema. Além disso, por meio de um sistema instável, tem-se que com as condições propostas pode-se ao mesmo tempo estabilizar o sistema e minimizar o custo garantido.

Palavras-chave: Desigualdades Lineares Matriciais (LMIs). *Gain Scheduling* (GS). Realimentação Derivativa. Custo Garantido $\mathscr{H}_\infty$. Custo Garantido $\mathscr{H}_2$. Custo Garantido Misto $\mathscr{H}_\infty/\mathscr{H}_2$. $\mathscr{D}$-estabilidade.

ABSTRACT

In this work, new conditions for the control of linear parameter-varying systems (LPV) are proposed, considering the state derivative feedback. At first, the $\mathscr{H}_\infty$ problem will be addressed in order to derive conditions for the $\mathscr{H}_\infty$ gain scheduling state derivative feedback controller. Then, conditions for the $\mathscr{H}_2$ gain scheduling state derivative feedback controller are obtained. The design of the controllers is based on linear matrix inequalities (LMIs). It is important to emphasise that $\mathscr{D}$-stability in the control project will also be considered, as a way to obtain good performance with a control signal that can be implemented in a real system. Here, the conditions for $\mathscr{D}$-stability will be treated in the sense of time-invariant systems, with "frozen" values of the parameter-varying in their range. In addition, the proposed conditions take into account a common quadratic Lyapunov function (CQLF) to then be compared with the proposed conditions considering a parameter-dependent Lyapunov function (PDLF). This work also offers necessary and sufficient conditions for the mixed $\mathscr{H}_\infty/\mathscr{H}_2$ control, i.e., it joins both $\mathscr{H}_\infty$ problem and $\mathscr{H}_2$ problem. The proposed conditions are applied in several examples to show that using them it is possible to decrease the guaranteed cost $\mathscr{H}_\infty$ and $\mathscr{H}_2$, i.e, to minimise the effect of a possible disturbance in the system. In addition, through an unstable system, it is possible to stabilise the system and minimise the guaranteed cost with the proposed conditions.

Keywords: Linear Matrix Inequalities (LMIs). *Gain scheduling* (GS). State Derivative Feedback (SDF). $\mathscr{H}_\infty$ Guaranteed Cost. $\mathscr{H}_2$ Guaranteed Cost. Mixed $\mathscr{H}_\infty/\mathscr{H}_2$ Guaranteed Cost. $\mathscr{D}$-stability.

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LIST OF ABBREVIATIONS AND ACRONYMS

ARE	Algebraic Ricatti Equation
CQLFs	Commom Quadratic Lyapunov Functions
DOF	Dynamical Output Feedback
GS	Gain Scheduling
LMIs	Linear Matrix Inequalities
LPV	Linear Parameter-Varying
LQR	Linear Quadratic
LTI	Linear Time-Invariant
MatLab®	MATrix LABoratory
PDLFs	Parameter-Dependent Lyapunov Functions
PID	Proportional-Integral-Derivative
SDF	State Derivative Feedback
SDOH	Sub-Domain Optimisation Heuristic
SF	State Feedback
T-S	Takagi-Sugeno

LIST OF SYMBOLS

©	Copyright
$diag(.,.,...,.)$	Diagonal matrix with properly dimensions
^{T}	Transpose of a vector or matrix
$^{-T}$	Inverse of a transpose matrix
$*$	Transpose block of a symmetric matrix
0	Null matrix with properly dimensions
I	Identity matrix with properly dimensions
$\wedge_r$	Unitary simplex for $\alpha(t)$
$\wedge_r^d$	Unitary simplex for $\dot{\alpha}(t)$
ρ_l	Upper bound for the derivative of $\alpha(t)$
$M < 0$ $(M \leq 0)$	Negative definite matrices (semi-definite)
$M > 0$ $(M \geq 0)$	Positive definite matrices (semi-definite)
$det(M)$	Determinant of M
$M(\alpha(t))$	Time-varying matrix
$\|\| . \|\|$	Euclidean vector norm
r	Number of polytope vertices
$\mathbb{R}$	Set of real numbers
$\mathbb{R}^{n \times m}$	Set of the real matrices with dimension $n \times m$
γ	$\mathscr{H}_\infty$ guaranteed cost
κ	$\mathscr{H}_2$ guaranteed cost
$\sum_{i=1}^{j}(.)$	A sum from $i = 1$ to j
®	Trademark
$\in$	Belongs to
$\forall$	For all
$\|\|x(t)\|\|_2$	Norm 2 of $x(t) \in \mathbb{R}^n$, given by $\|\|x(t)\|\|_2 = \sqrt{\int_0^\infty x(t)^T x(t) dt}$
$\mathscr{L}_2$	Space of measurable $x(t)$ Lebesgue signals satisfying $\|\|x(t)\|\|_\infty$

SUMMARY

1 INTRODUCTION

In the past few years, the State Derivative Feedback (SDF) has gained close attention in the control literature as we can see in the several papers dealing with it. For instance, in the following problems: vibration control of landing gear components (KWAK; WASHINGTON; YEDAVALLI, 2002), bridge cables (DUAN; NI; KO, 2005), pole placement for linear systems (ABDELAZIZ; VALÁŠEK, 2004), uncertain linear systems (ASSUNÇÃO et al., 2007; ABDELAZIZ, 2009; FARIA et al., 2009), active suspension systems (REITHMEIER; LEITMANN, 2003; SILVA et al., 2013; SEVER et al., 2017; YAZICI; SEVER, 2017), linear quadratic regulator (BETETO et al., 2018; SEVER; YAZICI, 2019), design of SDF control laws in discrete time (ROSSI et al., 2018), adjustment of vehicle's attitude and motion (FALLAH et al., 2012), pitch motion control for a marine vehicle (BASTURK; ROSENTHAL; KRSTIC, 2014), control of discretised systems (LEANDRO; PEREIRA; KIENITZ, 2020), control of boost converters (FAISAL; LATHER, 2020), descriptor systems (DUAN; ZHANG, 2002; CARDIM et al., 2008), singular systems (ZAGHDOUD; SALHI; KSOURI, 2018), Takagi-Sugeno (T-S) *fuzzy* descriptor systems (BARBOSA; SOUZA; PALHARES, 2019; HE et al., 2020), robust PID controllers (VESELỲ; KÖRÖSI, 2019), $\mathscr{H}_\infty$ for linear systems (SEVER; YAZICI, 2017), $\mathscr{H}_\infty$ for uncertain linear systems (YAZICI; SEVER, 2018), $\mathscr{H}_\infty$ T-S *fuzzy* systems (KAEWPRAEK; ASSAWINCHAICHOTE, 2016; HE et al., 2020; RUANGSANG; ASSAWINCHAICHOT, 2019), delayed hybrid descriptor systems (GUANGMING et al., 2019), gain scheduling (LLINS et al., 2017), delayed fractional-order multiagent systems (LIU et al., 2018), partial eigenvalue assignment for linear systems (ARAÚJO, 2019), among others.

The main characteristic of SDF is that the signals of second-derivative are available to feedback owing to the presence of accelerometers as sensors. With this type of sensors, the second-derivative signals represent the acceleration signals. According to Abdelaziz and Valášek (2004), it is possible to obtain the velocity signals by integrating the acceleration signals with good accuracy, but the same does not occurs with the displacement signals. In this way, the SDF is widely applied on vibration suppression control in mechanical systems, hence the fact that the second-derivative signals are acceleration and velocity. It is worth mentioning that when sensors directly measure state-derivatives the State Feedback (SF) does not always solve the problem (SUEUR, 2016), owing to the signal noise when being integrated twice. Therefore, the SDF has the advantage of solving the problem with simplicity, using the signal available for feedback and with lower gains (SUEUR, 2016; TSENG, 2009).

Additionally, owing to their simple structure and low cost, accelerometers have been applied to a large number of engineering problems (SABATO et al., 2016; KASPRZYK et al., 2017; ZHU et al., 2018).

As we can note, the SDF is used to solve a variety of engineering problems. Newly, Yazici and Sever (2018) proposed a robust $\mathscr{H}_\infty$ SDF controller for an active suspension system. In system theory, $\mathscr{H}_\infty$ norm is a very important performance index, which measures the system capacity to reject energy bounded disturbances (MONTAGNER et al., 2005). Additionally, the $\mathscr{H}_\infty$ problem has been addressed by several papers, for instance, Carniato et al. (2018) developed a robust $\mathscr{H}_\infty$ switched static output feedback controller for continuous-time switched linear systems with polytopic uncertainties; Oliveira et al. (2018) introduced a local $\mathscr{H}_\infty$ switched controller design for a class of uncertain nonlinear plants described by T-S *fuzzy* models with unknown membership functions; Rosa, Morais and Oliveira (2018) investigated the problems of stabilisation and mixed $\mathscr{H}_2/\mathscr{H}_\infty$ reduced-order dynamic output-feedback control of discrete-time linear systems. Following the path of SDF and $\mathscr{H}_\infty$ problem, Ren and Zhang (2010) developed a robust $\mathscr{H}_\infty$ control for descriptor systems using a proportional plus derivative state feedback; Kaewpraek and Assawinchaichote (2016) introduced an $\mathscr{H}_\infty$ *fuzzy* SF plus SDF control for photovoltaic systems based on Linear Matrix Inequalities (LMIs); Ruangsang and Assawinchaichot (2019) investigated the problem of a robust $\mathscr{H}_\infty$ SF plus SDF control for a class of uncertain non-linear systems described by a T-S *fuzzy* model.

Highlighting the papers that deal with the the $\mathscr{H}_\infty$-SDF problem, note that they explore only Linear Time-Invariant (LTI) or *fuzzy* systems. Differently from what is presented in (REN; ZHANG, 2010; KAEWPRAEK; ASSAWINCHAICHOTE, 2016; YAZICI; SEVER, 2018; RUANGSANG; ASSAWINCHAICHOT, 2019), this work addresses the $\mathscr{H}_\infty$-SDF problem, but for Linear Parameter-Varying (LPV) systems, using the Gain Scheduling (GS) strategy with LMIs to solve the problem. In control literature, Llins et al. (2017) introduced the GS strategy for LPV systems considering the SDF. Earlier, Apkarian and Gahinet (1995), Montagner et al. (2005) started the study of the $\mathscr{H}_\infty$ problem for LPV systems considering the GS strategy. Now, we intend to derive LMIs conditions for $\mathscr{H}_\infty$-GS-SDF controllers. The main choice of the GS strategy relies on the great interest of the control research community on it. According to Rugh and Shamma (2000), Al-Jiboory and Zhu (2018) it is achievable improve the system performance by means of the GS strategy, accessing the scheduling parameters (in real-time) through measurements or estimations. The surveys Rugh and Shamma (2000), Wei et al. (2014) and the references therein contain a great background on GS strategy.

Regarding the GS strategy to solve the problem of the $\mathscr{H}_\infty$ guaranteed cost for LPV systems, several papers addressed it in specialised literature. For instance, Montagner et al. (2005), Montagner and Peres (2006) developed LMI conditions for the design of $\mathscr{H}_\infty$-GS controllers for LPV systems; Zhou, Zhang and Zheng (2009) addressed the problem of $\mathscr{H}_\infty$-GS filter design for a class of parameter-varying discrete-time systems using LMIs; Caigny et al. (2012) proposed LMIs conditions for GS dynamical output feedback (DOF) controllers and GS-DOF mixed $\mathscr{H}_\infty/\mathscr{H}_2$ controllers for discrete-time LPV systems; Rosa, Morais and Oliveira (2018) investigated the problems of stabilisation and mixed $\mathscr{H}_2/\mathscr{H}_\infty$ reduced-order dynamic output-feedback control of discrete-time linear systems using parameter-dependent LMIs. As we can note, the $\mathscr{H}_\infty$ problem is successfully solved by the GS strategy, hence our motivation to develop the $\mathscr{H}_\infty$-GS-SDF controller.

In addition to the $\mathscr{H}_\infty$, the $\mathscr{H}_2$ control is quite considered in the control literature. For instance, in the following problems: parametric eigenstructure assignment for linear systems (WANG; LIANG; DUAN, 2006), SDF control of overhead crane systems (ALI, 2017), microsatellite attitude control (YANG; SUN, 2002), robust $\mathscr{H}_2$ and $\mathscr{H}_\infty$ filters for uncertain linear systems (LACERDA; OLIVEIRA; PERES, 2011), discrete-time periodic systems (FARGES et al., 2007; PEAUCELLE; EBIHARA; ARZELIER, 2008), active suspension control (AGHAIE; AMIRIFAR, 2007), two-floors building model vibration control (SANTOS et al., 2007), control of parameter dependent systems (OLIVEIRA; SOUZA; TROFINO, 2000). With the $\mathscr{H}_2$ norm it is measured the Root-Mean-Square (RMS), in time domain, value of an impulse response or stationary white noise response (YANG; SUN, 2002). It is worth to mention that we are using LPV systems, which means that the $\mathscr{H}_2$ problem is considered in the parameter-dependent sense as $\mathscr{H}_2$ guaranteed cost. In this work, the definition to this problem is based on the results of (PAGANINI; FERON, 2000; SOUZA; TROFINO; OLIVEIRA, 2003; XIE, 2005), and will be better explored later in the text. Considering the $\mathscr{H}_2$ guaranteed cost for LPV systems, a great number of papers deal with: Xie (2005), Xie (2012) designed new LMIs formulations for the GS control of LPV systems in which the Lyapunov matrix is decoupled from the system matrices; Aouani et al. (2012) developed conditions based on LMIs for the robust stability and the $\mathscr{H}_2$ performance analysis of LPV systems subject to uncertainties and under polytopic structure; Cai et al. (2014) designed sufficient conditions for the $\mathscr{H}_2$-GS-SF and $\mathscr{H}_2$-GS dynamic output feedback controllers for LPV systems; Kang, Lee and Chung (2017) introduced an observer gain scheduling based on $\mathscr{H}_2$ filter for discrete-time LPV systems; Al-Jiboory and Zhu (2018) developed the static output-feedback GS control for LPV systems with scheduling parameters measures affected by uncertainties or noises; Palma, Morais and Oliveira (2020) designed a technique named Sub-Domain Optimisation Heuristic (SDOH) in order to obtain $\mathscr{H}_2$ controllers or filters that treat robust stability independently of performance.

Note that the above papers of the $\mathscr{H}_2$ guaranteed cost for LPV systems do not use the SDF in the problem. In fact, a few papers consider the SDF on the resolution of the $\mathscr{H}_2$ problem. For instance, Zaghdoud, Salhi and Ksouri (2015) proposed a proportional plus derivative feedback controller for continuous and discrete descriptor systems; Zaghdoud, Salhi and Ksouri (2018) developed SDF controllers for LTI descriptor systems considering the $\mathscr{H}_2$ in terms of the Linear Quadratic (LQ) criteria; Ali (2017) derived an $\mathscr{H}_2$ optimal control using the SDF (the $\mathscr{H}_2$ problem is also in terms of the LQ criteria). In this work, unlike what is done in the mentioned papers, we consider the $\mathscr{H}_2$ problem for LPV systems. Then, following this scenario on $\mathscr{H}_\infty$ and $\mathscr{H}_2$ problems, this work has three main objectives, derive LMI conditions for the $\mathscr{H}_\infty$-GS control using SDF, the $\mathscr{H}_\infty$-GS-SDF control; derive LMI conditions for the $\mathscr{H}_2$-GS control using SDF, the $\mathscr{H}_2$-GS-SDF control; and derive LMI conditions for the $\mathscr{H}_2/\mathscr{H}_\infty$-GS control using SDF, the $\mathscr{H}_2/\mathscr{H}_\infty$-GS-SDF control. Note that the controllers are derived in order to reduce the $\mathscr{H}_\infty$ and $\mathscr{H}_2$ guaranteed costs for LPV systems. To the best of the author's knowledge, the conditions for the controllers mentioned above have not been published yet.

Additionally, a region in the left-half plane for pole location is considered. This region may assist us to improve the system performance and/or to reduce the control signal. The chosen region is the $\mathscr{D}$ region, presented in (CHILALI; GAHINET, 1996), where the $\mathscr{H}_\infty$ problem is also considered. It is important to emphasise that the eigenvalue constraints must be understood in the time-invariant sense, i.e., for "frozen" values of the varying-parameter in its range (KAJIWARA; APKARIAN; GAHINET, 1999; PUIG; BOLEA; BLESA, 2012). Furthermore, to derive the LMI conditions a Common Quadratic Lyapunov Function (CQLF) and a Parameter-Dependent Lyapunov Function (PDLF) will be used and compared. The use of a PDLF in the approach of LPV systems seems to lead to less conservative results (WU et al., 1996; OLIVEIRA; GEROMEL, 2005; SATO; PEAUCELLE, 2013; AL-JIBOORY; ZHU, 2018), hence the main motivation to used it.

1.1 CONTRIBUTIONS

The main contributions of this work are:

- Design of an $\mathscr{H}_\infty$-GS-SDF controller for LPV systems;
- Design of an $\mathscr{H}_\infty$-GS-SDF controller for LPV systems considering a PDLF;
- Design of an $\mathscr{H}_2$-GS-SDF controller for LPV systems;
- Design of an $\mathscr{H}_2$-GS-SDF controller for LPV systems considering a PDLF;
- Design of a controller considering the SDF and the $\mathscr{D}$-stability for LPV systems;
- Design of a $\mathscr{D}$-$\mathscr{H}_\infty$-GS-SDF controller for LPV systems;
- Design of a $\mathscr{D}$-$\mathscr{H}_2$-GS-SDF controller for LPV systems;
- Design of an $\mathscr{H}_\infty/\mathscr{H}_2$-GS-SDF controller for LPV systems;

- Design of an $\mathscr{D}$-$\mathscr{H}_\infty/\mathscr{H}_2$-GS-SDF controller for LPV systems;
- Inclusion of a parallel with the robust control.

1.2 STRUCTURE OF THE TEXT

The work is organised as follows:

- Chapter 2 presents some fundamentals concepts and properties that will be used over the text: the LPV system, the SDF for LPV systems, and a characterisation of the dependent parameter (and its derivative) were presented, followed by the introduction of the $\mathscr{H}_\infty$ and $\mathscr{H}_2$ problems in terms of parameter-dependent systems. Also we presented a couple of useful lemmas.
- Chapter 3 presents the conditions for the $\mathscr{H}_\infty$-GS-SDF controllers in terms of LMIs. It is important to mention that the first conditions obtained used a CQLF and, in the subsequent conditions for the $\mathscr{H}_\infty$ problem, a PDLF was used. For illustration, the proposed conditions were applied in four examples. The first example shows that the $\mathscr{H}_\infty$-GS-SDF controllers performed well when applied on an active suspension system. The second example was considered just to indicate that the proposed conditions are able to reduce the $\mathscr{H}_\infty$ guaranteed cost and stabilise an uncertain system. The third example is an analysis of the ρ_l parameter when a PDLF ($P(\alpha(t))$) is considered. With suitable values, it is possible to obtain low $\mathscr{H}_\infty$ guaranteed cost values. The fourth example is concerned with the comparison between the conditions with a CQLF and those with a PDLF, i.e., a feasibility analysis was performed.
- In Chapter 4 we derived GS-SDF controllers considering the $\mathscr{H}_2$ guaranteed cost. To obtain the controllers, the conditions were based on the results of (SOUZA; TROFINO, 2006), and the first conditions take into account a CQLF to later consider a PDLF. To analyse the performance of the proposed conditions, four examples were used. At first, we consider a mass-spring-damper system. Second, an unstable system is used to to show that the proposed $\mathscr{H}_2$-GS-SDF controller is capable of to reduce the $\mathscr{H}_2$ guaranteed cost and stabilise the system. An analysis of the ρ_l parameter is presented in the third example. Finally, a feasibility analysis is showed comparing the conditions with a CQLF with those with a PDLF.
- Chapter 5 presents the conditions for the $\mathscr{D}$-$\mathscr{H}_\infty$-GS-SDF and $\mathscr{D}$-$\mathscr{H}_2$-GS-SDF controllers in terms of LMIs. The chosen $\mathscr{D}$-region was a circular disk in complex plane with center $(-1,0)$, radius $\mathtt{r}$ and decay rate δ, with $q = \delta + \mathtt{r}$. Furthermore, in this work, the eigenvalue constraints must be understood in the time-invariant sense, i.e., for ”frozen”

values of the varying-parameter in its range (KAJIWARA; APKARIAN; GAHINET, 1999; PUIG; BOLEA; BLESA, 2012). The conditions derived were based on a CQLF. For illustration, the proposed conditions were applied in some examples, according to the examples presented in previous sections.

- In Chapter 6 we derived GS-SDF controllers considering the mixed $\mathscr{H}_\infty/\mathscr{H}_2$ problem. The conditions are based on a CQLF. Furthermore, the $\mathscr{D}$-$\mathscr{H}_\infty/\mathscr{H}_2$ controller is also presented. Following the previous chapters, some examples are considered to show that the new conditions performed well.

- Chapter 7 presents some comments about the new conditions proposed in this thesis. Also, it presents a parallel with the robust control, showing some new conditions considering the SDF, the $\mathscr{H}_\infty$ problem, the $\mathscr{H}_2$ problem, the mixed $\mathscr{H}_\infty/\mathscr{H}_2$ problem, and the $\mathscr{D}$-stability.

- Chapter 8 states the conclusions, as well as the related publications and suggestions for future works.

8 CONCLUSIONS

This work proposed methods for the gain scheduling control of linear parameter-varying systems subjects to a disturbance signal. Through the project, the gain scheduling strategy is considered, which has been gaining attention in the control community and, having access to the scheduling parameter in real time, it is possible to improve the performance of the system. Furthermore, the state derivative is used owing to the easy measurement of the second-derivative signals, once that the system has accelerometers as main sensors. In addition, to deal with the disturbance signal, two approaches were considered, the $\mathscr{H}_2$ and the $\mathscr{H}_\infty$ guaranteed costs. With the proposed method, it is achievable to reduce the effects of the disturbance signal in the performance of the systems, improving the system working. Another important fact is that to derive the LMI conditions, the Lemma 2.7 was used. With this lemma it is possible to deal with the cross product between three parameter-dependent variables.

For illustration, some examples have been presented to demonstrate the effectiveness of the proposed methods. Considering the $\mathscr{H}_\infty$ guaranteed cost, the first example consisted of applying the $\mathscr{H}_\infty$-GS-SDF controller to an active suspension system. This system was subject to two disturbance signals, a sinusoidal scan and a square wave. In both cases, the designed controller was able of mitigate the effect of the disturbance signal, ensuring a satisfactory closed-loop performance, increasing the comfort to the driver and minimising the mechanical stress to the suspension system. The second example, an uncertain system was used to show that with the proposed methods it is possible to ensure a low $\mathscr{H}_\infty$ guaranteed cost and stabilise the unstable system. The third and the fourth examples are complementary. They present an analysis of the ρ_l parameter and a feasibility analysis between Theorems 3.1 and 3.2. With these analysis it can be seen that the use of a PDLF is less conservative than the use of a CQLF.

Regarding the $\mathscr{H}_2$-GS-SDF controllers, similar analysis to $\mathscr{H}_\infty$-GS-SDF controller were performed, also considering four examples. The first example considers a mass-spring-damper system subject to two disturbance signals, a sinusoidal scan and a pulse occurring periodically. In both cases the designed controller was able to minimise the effect of the disturbance signal, ensuring a satisfactory closed-loop performance. With the second example, it was shown that the $\mathscr{H}_2$-GS-SDF is capable of to minimise and to stabilise the $\mathscr{H}_2$ guaranteed cost of an unstable system. The third and the fourth examples presented the analysis of the ρ_l parameter and a feasibility analysis for the $\mathscr{H}_2$-GS-SDF.

It is important to highlight that, although the conditions for the $\mathscr{H}_2$-GS-SDF and $\mathscr{H}_\infty$-GS-SDF were based on (YAZICI; SEVER, 2018) and (SOUZA; TROFINO; OLIVEIRA, 2003), respectively, the conditions considering $P(\alpha(t))$ (and the characterisation of $\dot{\alpha}(t)$ was based on the results from (MONTAGNER; PERES, 2006). In this way, the considerations made in (MONTAGNER; PERES, 2006) for ρ_l is valid in this work. Thus, the conditions with $P(\alpha(t))$ (obtained with a PDLF and with a suitable choice of ρ_l) always ensure a lower cost, or at least equal, than the conditions obtained through a CQLF.

Furthermore, in this work was also presented a region in the left-half plane for pole location to improve the system performance and/or to reduce the control signal. The chosen region would be the $\mathscr{D}$ region, presented in (CHILALI; GAHINET, 1996). It is important to emphasise that the eigenvalue constraints must be understood in the time-invariant sense, i.e., for "frozen" values of the parameter in its range (KAJIWARA; APKARIAN; GAHINET, 1999; PUIG; BOLEA; BLESA, 2012). With the $\mathscr{D}$-$\mathscr{H}_2$-GS-SDF and $\mathscr{D}$-$\mathscr{H}_\infty$-GS-SDF controllers, and the properly choice of the parameters $\mathbb{r}$ and δ, it is possible to achieve better transients responses. However, a more detailed analysis of the inclusion of the $\mathscr{D}$-stability would be interesting for LPV systems.

Finally, two topics were still addressed in this thesis. The first deals with the mixed $\mathscr{H}_\infty/\mathscr{H}_2$ control considering SDF, LPV systems and the $\mathscr{D}$-stability. Through the examples we saw that, for this case, if $\mathscr{D}$-stability was not considered, it would not be possible to implement a mixed $\mathscr{H}_\infty/\mathscr{H}_2$ controller, since the controller norm was high. With this, it was also noticed that the sub-optimal guaranteed cost was the best choice, since it was possible to implement the controller.

The second topic deals with the robust control. As the conditions are similar, a parallel was made between the robust control and the GS control. However, this topic only intends to demonstrate that the conditions are similar and that it is possible to obtain conditions for robust control considering the SDF, the $\mathscr{D}$-stability, the $\mathscr{H}_\infty$ problem, the $\mathscr{H}_2$ problem and the mixed $\mathscr{H}_\infty/\mathscr{H}_2$ problem. In addition, it remains as a suggestion for future works to analyse and implement the proposed conditions for the robust case.

8.1 FUTURE RESEARCH SUGGESTIONS

The following suggestions encompass ideas for future works:

- Lemma 2.7 considers the cross product between three variables, and its results can be conservative. In this way, for future works, we intend to study and analyse the triple sum to derive less conservative LMI conditions;
- It would be interesting to make a study of how the frequency interferes in the proposed conditions, since we are considering $P(\alpha(t))$ and the derivative of the parameter-varying appears;

- A more detailed analysis regarding the inclusion of the $\mathscr{D}$-region for LPV systems would be interesting for LPV systems. In addition to this analysis, it would be opportune to study the use of a PDLF to derive the LMI conditions for the $\mathscr{D}$-GS-SDF controllers;

- Derive the conditions for the the mixed $\mathscr{H}_\infty/\mathscr{H}_2$ control considering a PDFL ($P(\alpha(t))$);

- Analyse and compare the conditions for the robust case, considering the SDF, the $\mathscr{D}$-stability, the $\mathscr{H}_\infty$ problem, the $\mathscr{H}_2$ problem and the mixed $\mathscr{H}_\infty/\mathscr{H}_2$ problem.

8.2 PUBLICATIONS

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