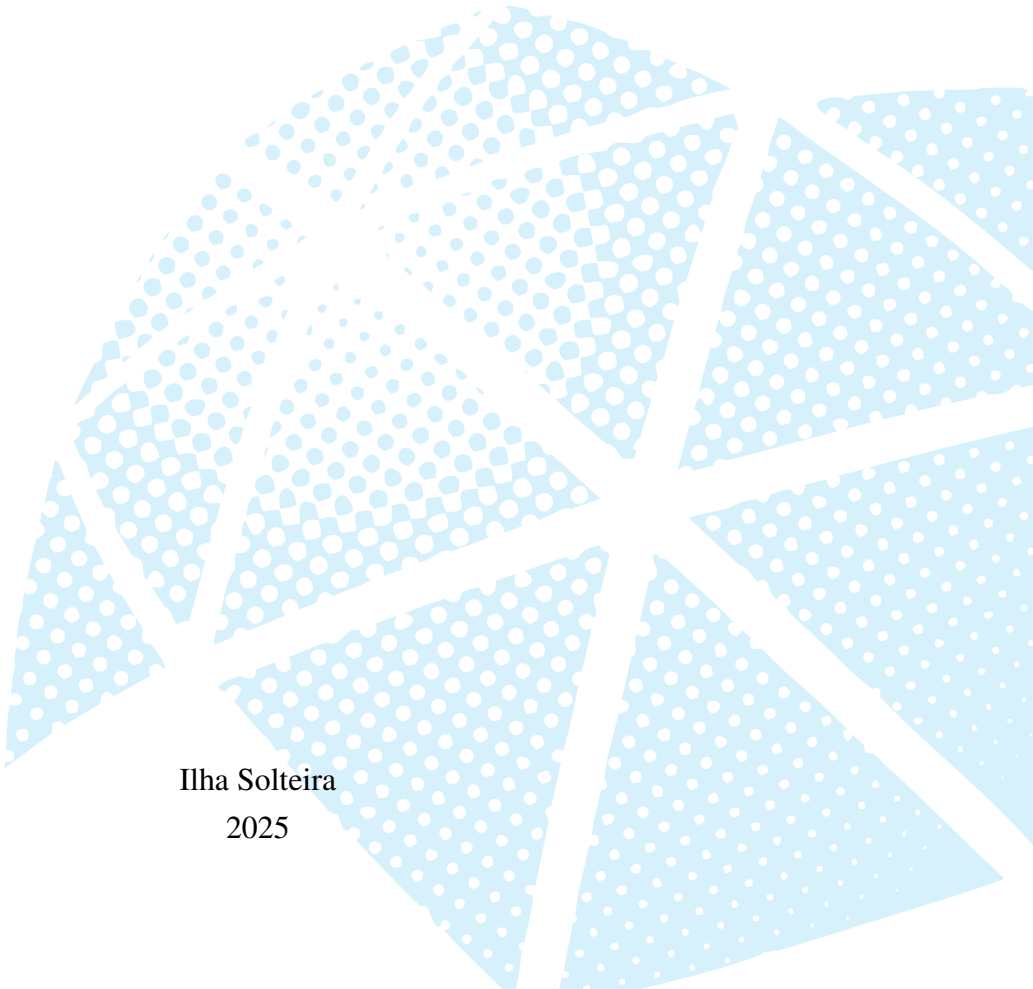


**UNIVERSIDADE ESTADUAL PAULISTA “JÚLIO DE MESQUITA FILHO”
CAMPUS DE ILHA SOLTEIRA**

Lorena Lopes Dias

**Temperature Effects in EMI-based SHM: Theoretical and Experimental
Approaches**

Ilha Solteira
2025



Lorena Lopes Dias

Temperature Effects in EMI-based SHM: Theoretical and Experimental Approaches

Thesis presented to São Paulo State University (UNESP), School of Engineering of Ilha Solteira, as part of the requirements for obtaining the PhD degree in Mechanical Engineering.

Knowledge Area: Solid mechanics

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Ilha Solteira, 24 de julho de 2025

*To my parents,
for their constant love and support that strengthened me to persevere.*

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*"As long as you're learning, you're
not failing."*

Bob Ross

ABSTRACT

Structural Health Monitoring (SHM) is dedicated to the continuous assessment of structural conditions, aiming to detect damage at early stages and, consequently, ensure the functionality and safety of different structures. Among the various methodologies employed in SHM, the Electromechanical Impedance (EMI) technique stands out for its sensitivity to incipient damage. Important findings have been achieved from experimental data, establishing this approach as the predominant one in the literature. However, theoretical analyses have also driven substantial advancements, demonstrating great potential. Despite this, the practical application of the EMI technique faces challenges due to its sensitivity to external conditions, such as temperature. Thermal variations induce changes in EMI signals, which can be mistakenly interpreted as damage, compromising the accuracy of structural diagnostics. To address this challenge, temperature compensation methods have been developed to minimize its influence on EMI signatures. In this context, the present thesis investigates the temperature effects on the EMI technique through two complementary approaches: theoretical and experimental. An analytical EMI model is developed to examine an Euler-Bernoulli beam with symmetrically coupled piezoelectric transducers. Three different dynamics are evaluated, i.e., longitudinal, flexural, and a combination of both. The study investigates both the influence of temperature variations and the presence of symmetric and asymmetric damages. The results exhibit strong similarity with experimental, particularly in peaks position. In addition, a temperature compensation technique is proposed through the incorporation of a new step involving the selection of the frequency range for computing damage detection indices, aiming to improve the detection accuracy. The methodology is demonstrated in an aluminum beam subjected to three different structural conditions: baseline (undamaged) and two types of damage. The obtained results confirm the effectiveness of the proposed approach in reducing false alarms during damage detection via EMI.

Keywords: structural health monitoring; electromechanical impedance; temperature; analytical model; experimental tests.

RESUMO

O Monitoramento da Integridade Estrutural (SHM, do inglês *Structural Health Monitoring*) dedica-se a análise contínua das condições estruturais, visando detectar danos em estágios iniciais e, conseqüentemente, garantir a funcionalidade e a segurança de diferentes estruturas. Dentre as diversas metodologias empregadas em SHM, a técnica da Impedância Eletromecânica (EMI, do inglês *Electromechanical Impedance*) destaca-se pela sua sensibilidade a danos incipientes. Importantes descobertas têm sido alcançadas a partir de dados experimentais, configurando-se como a abordagem predominante na literatura. Entretanto, análises teóricas também têm impulsionado avanços significativos, mostrando-se promissoras. Apesar disso, a aplicação prática da técnica de EMI apresenta desafios relacionados à sua sensibilidade a condições externas, como a temperatura. Variações térmicas induzem alterações nos sinais de EMI, que podem ser erroneamente interpretadas como danos, comprometendo a precisão do diagnóstico estrutural. Diante desse desafio, têm sido desenvolvidos métodos de compensação da temperatura, visando minimizar sua influência nas curvas de EMI. Neste contexto, a presente tese investiga os efeitos da temperatura na técnica de EMI considerando análises teóricas e experimentais. Para isso, implementa-se um modelo analítico de EMI para investigar uma viga de Euler-Bernoulli com transdutores piezoelétricos simetricamente acoplados. Com essa configuração são analisadas as dinâmicas longitudinal, flexural e a combinação de ambas. O estudo avalia tanto a influência das variações térmicas quanto a presença de danos simétricos e assimétricos. Os resultados apresentam uma importante similaridade com dados experimentais, especialmente na localização dos picos. Também, propõe-se uma técnica de compensação de temperatura mediante a incorporação de uma etapa envolvendo a seleção da faixa de frequência para a estimativa dos índices de detecção de danos, visando melhorar a detecção. A metodologia é demonstrada considerando uma viga de alumínio com três condições estruturais distintas: sem e com dois tipos de danos. Os resultados obtidos corroboram a efetividade da abordagem proposta em reduzir falsos alarmes durante a detecção de danos via EMI.

Palavras-chave: monitoramento da integridade estrutural; impedância eletromecânica; temperatura; modelo analítico; ensaios experimentais.

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LIST OF ABBREVIATIONS AND ACRONYMS

AA	Aluminum Association
ADM	Aluminum Design Manual
AE	Acoustic Emission
ANN	Artificial Neural Network
CCDM	Correlation Coefficient Deviation Metric
CNC	Computer Numerically Controlled
CNN	Convolutional Neural Network
CVM	Comparative Vacuum Monitoring
DAQ	Data Acquisition
DI	Damage Index
EC	External Condition
EFS	Effective Frequency Shift
EMI	Electromechanical Impedance
FBG	Fiber Bragg Grating
FR	Frequency Range
GW	Guided Wave
IoT	Internet of Things
LS-HSS	Large-Scale Hydro-Steel Structure
LW	Lamb Waves
ML	Machine Learning
NDE	Non-destructive Evaluation
PCA	Principal Component Analysis
PZT	<i>Lead Zirconate Titanate</i>
RBF	Radial Basis Function

RMSD	Root Mean Square Deviation
SDOF	Single-degree-of-freedom
SHM	Structural Health Monitoring
UND	Undefined
WSN	Wireless Sensor Network

LIST OF SYMBOLS

Superscripts

$()^*$	Denotes a complex number
$()'$	Indicates the first derivative with respect to x
$()_{\sim}$	Represents temperature dependence
$()_{\dot{}}$	Denotes the first derivative with respect to time t
$()_{\ddot{}}$	Indicates the second derivative with respect to time t
$()''$	Represents the second derivative with respect to x
$()^T$	Indicates the transpose of a matrix
$()_{\bar{}}$	Denotes the mean value

Subscripts

$()_L$	Represents the longitudinal dynamics
$()_b$	Indicates the beam
$()_p$	Denotes the PZT
$()_F$	Represents the flexural dynamics
$()_C$	Represents the coupled dynamics
$()_d$	Denotes the damage
$()_{ad}$	Represents the asymmetric damage
$()_U$	Indicates the reference value
$()_R$	Indicates the unknown value
$()_{b_1}$	Denotes the undamaged beam region located before the damage
$()_{b_2}$	Denotes the undamaged beam region located after the damage

Matrices and Vectors

$\mathbf{M}$	Total mass matrix [kg]
$\mathbf{K}$	Total stiffness matrix [N/m]
$\mathbf{M}_b$	Mass matrix of the beam [kg]
$\mathbf{M}_p$	Mass matrix of the PZT [kg]
$\mathbf{K}_b$	Stiffness matrix of the beam [N/m]
$\mathbf{K}_p$	Stiffness matrix of the PZT [N/m]
$\mathbf{F}_L$	Force vector generated by the PZTs in longitudinal dynamics [J/V]
$\mathbf{g}$	System's generalized coordinates [m]
Φ	Eigenvector matrix for zero electric field
$\mathbf{C}_{gg}$	Damping matrix [Ns/m]

Ψ	Modal shape matrix
$\mathbf{N}_F$	Moment vector generated by the PZTs in flexural dynamics [J/V]
$\mathbf{K}_A$	Longitudinal-flexural coupling matrix [N/m]
α	Damping coefficient matrix for α values [s^{-1}]
β	Damping coefficient matrix for β values [s]
$\mathbf{I}$	Identity matrix
$\mathbf{0}$	Matrix of zeros
$\mathbf{M}_{A_d(\cdot)}$	Asymmetric damage-induced coupling mass matrix [kg]
$\mathbf{M}_{F_d(\cdot)}$	Asymmetric damage-induced flexural mass matrix [kg]
$\mathbf{K}_{A_d(\cdot)}$	Asymmetric damage-induced coupling stiffness matrix [N/m]
$\mathbf{K}_{F_d(\cdot)}$	Asymmetric damage-induced flexural stiffness matrix [N/m]
$\mathbf{Dif}$	Mean differences vector [Ω]

Electrical Properties

ϵ_{33}^T	Dielectric constant of the PZT in the 3-3 direction (z) under a constant voltage [F/m]
E_3	Electric field [V/m]
D_3	Electric displacement [C/m^2]
V_3	Electric voltage [V]
Q	Electric charge [C]
I	Electric current [A]
δ	Dielectric loss
C_p	PZT capacitance [F]

Greek Letters

ϵ	Mechanical strain
σ	Mechanical stress [Pa]
π_b	Strain energy of the beam [J]
ρ	Material density [kg/m^3]
π_p	Strain energy of the PZT [J]
$\pi(\cdot)$	Total strain energy in the dynamics () [J]
ψ	Modal shape function
α	Damping coefficient [s^{-1}]
β	Damping coefficient [s]
ψ_n	Natural modal shape function

Latin Letters

Y_{em}	Electromechanical admittance [Ω^{-1}]
Z_{em}	Electromechanical impedance [Ω]
L	Length [m]
b	Width [m]
h	Thickness [m]
Y_{11}^E	Elastic modulus of the PZT in the 1-1 direction (x) under a constant electric field [Pa]
j	Pure imaginary number
Z	Structure's mechanical impedance [kg/s]
Z_A	PZT mechanical impedance [kg/s]
u	Displacement in the x -direction [m]
v	Displacement in the y -direction [m]
w	Displacement in the z -direction [m]
E_b	Young's modulus of the beam [Pa]
V_b	Beam's volume [m ³]
A	Cross-sectional area [m ²]
T_b	Kinetic energy of the beam [J]
S_{11}^E	Mechanical compliance of the PZT [Pa ⁻¹]
T_p	Kinetic energy of the PZT [J]
V_p	PZT volume [m ³]
x_1	Distance from the first PZT's end to the origin [m]
x_2	Distance from the second PZT's end to the origin [m]
$T_{()}$	Total kinetic energy in the dynamics () [J]
$n_{()}$	Number of modes in the dynamics ()
q	Modal coordinate [m]
m_b	Element of the beam mass matrix [kg]
m_p	Element of the PZT mass matrix [kg]
m_b	Element of the beam stiffness matrix [N/m]
m_p	Element of the PZT stiffness matrix [N/m]
I_b	Beam's area moment of inertia [m ⁴]
k_a	Element of the longitudinal-flexural coupling matrix [N/m]
k_n	Natural wavenumber [m ⁻¹]
k	Wavenumber [m ⁻¹]
$Z_{em_{exp}}$	Experimental electromechanical impedance [Ω]
$Z_{em_{theo}}$	Theoretical electromechanical impedance [Ω]
f_{min}	Minimum frequency [Hz]
f_{max}	Maximum frequency [Hz]
N_{pk}	Number of peaks

f_{pk_i}	Frequency corresponding to the i -th peak [Hz]
x_d	Initial position of the damage [m]
h_d	Reduced damage thickness [m]
I_d	Damage area moment of inertia [m ⁴]
$m_{ad(\cdot)}$	Element of the asymmetric damage coupling mass matrix [kg]
d_{ad}	Distance between the neutral axis of the damaged section and those of the adjacent undamaged regions [m]
$m_{d(\cdot)F}$	Element of the asymmetric damage-induced flexural mass matrix [kg]
I_{ad}	Asymmetric damage moment of inertia [m ⁴]
$k_{ad(\cdot)}$	Element of the asymmetric damage-induced coupling stiffness matrix [N/m]
$k_{d(\cdot)F}$	Element of the asymmetric damage-induced flexural stiffness matrix [N/m]
L_{pd}	Distance between the PZT and the damage [m]
C_C	Cross-correlation coefficient
Z_{emR}	Reference electromechanical impedance [Ω]
Z_{emU}	Unknown electromechanical impedance [Ω]
$Z_{emheating}$	Electromechanical impedance measured during the heating process of the structure [Ω]
$Z_{emcooling}$	Electromechanical impedance measured during the cooling process of the structure [Ω]
N_r	Number of frequency points
$Z_{emUmeasured}$	Measured unknown electromechanical impedance [Ω]
T_R	Reference temperature [$^{\circ}$ C]
T_U	Unknown temperature [$^{\circ}$ C]
f_{lim}	Limiting frequency [Hz]
f_{cp}	Clipped frequency [Hz]
f_{or}	Original frequency [Hz]
N_{cp}	Number of frequency points of the clipped frequency
N_{or}	Number of frequency points of the original frequency
ΔZ_{em}	Amplitude of the electromechanical impedance signal [Ω]
n_f	Number of sub-intervals
f_{sub}	Frequency sub interval [Hz]
N_{dif}	Number of frequency points of the frequency sub interval
$\overline{Dif}$	Average value of the mean differences vector [Ω]
σ_{Dif}	Standard deviation of the mean differences vector [Ω]
V_d	Volume of the damaged region [m ³]
u_{ad}	Longitudinal displacement generate by the asymmetric damage [m]
θ	Angular displacement

Special Notations

$\otimes$	Kronicher product
F	Function to determine the damping coefficients $[\Omega]$
$max()$	Maximum value
F_{sub}	Sub-intervals function $[\Omega]$
V_a	Sum of the real EMI changes squared $[\Omega^2]$
$Re()$	Indicates the real part
δ^S	Average difference between reference and measured impedance curves $[\Omega]$
S	Parameter for determining the number of points to be removed
$min()$	Minimum value

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1 INTRODUCTION

Structural Health Monitoring (SHM) is an engineering field dedicated to the continuous evaluation of structural integrity and the early detection of damage, focus on enhancing safety and operational efficiency by mitigating the risk of severe functional and structural failures. This field has gained increasing prominence from the scientific and engineering communities due to the growing demand for safer structures and cost-effective maintenance strategies (Burgos *et al.*, 2020; Mishra; Lourenço; Ramana, 2022). In this context, the Electromechanical Impedance (EMI) technique has emerged as a promising SHM approach due to its high sensitivity to incipient damage and low-cost implementation (Tenreiro; Lopes; Silva, 2022; Parida; Moharana, 2023). However, despite this method's proven effectiveness across various engineering applications, its practical implementation continues to face significant challenges, especially due to its inherent sensitivity to environmental changes, such as the temperature. Thermal variations affect the properties of both transducer and monitored structure, resulting in variations in the EMI signatures that can be mistakenly interpreted as structural damage. Hence, the development of effective temperature compensation strategies is essential for practical SHM applications to mitigate the adverse effects of temperature variations and ensure the reliability of the monitoring process (Koo *et al.*, 2009; Giancesini *et al.*, 2021).

In literature, the majority of studies on EMI rely on experimental data, which may limit the physical understanding of the EMI technique, particularly to situations beyond those established for each particular test condition. However, the engineering community has been increasingly relying on theoretical models due to advancements in computational resources and the need for performance optimization. This approach enables the simulation and prediction of EMI across different scenarios, offering insights that complement experimental findings and contribute to the optimization of project design (Giurgiutiu; Zagrai, 2000; Davis; Eisenhardt; Bingham, 2006).

In this context, the present thesis investigates the temperature effects on the EMI technique through theoretical and experimental approaches. In the first one, an analytical EMI model is developed to examine an Euler-Bernoulli beam with coupled PZTs. The theoretical formulations have been improved to incorporate symmetrically coupled PZTs, enabling the analysis of EMI signals considering different dynamics, i.e, longitudinal and flexural separately and coupled to each other. The study evaluates the influence of temperature variations within a range of 24°C to 65°C, along with the effects of symmetric and asymmetric damage in different configurations. The results provide valuable insights into the influence of temperature variations and structural damage on EMI signals, contributing to the advancement of more robust and reliable monitoring techniques. The experimental results exhibit strong similarity, particularly in terms of the peaks positions. In addition, a temperature compensation technique is proposed to select the frequency range to compute

damage detection indices, aiming to improve detection accuracy. The effectiveness of this methodology is demonstrated in an aluminum beam subjected to three different structural conditions: baseline (undamaged) and two types of damage. The results are investigated for four different frequency ranges. The findings demonstrate the effectiveness of the proposed approach in reducing false alarms in damage detection via EMI.

1.1. OBJECTIVES

The main objective of the thesis is to develop a theoretical model to compute the EMI and a technique to compensate the temperature variations in beam-like structures. The following specific objectives have been delineated:

- To formulate an analytical EMI model that comprehensively considers the longitudinal and flexural dynamics;
- To extend the modeling to represent both symmetric and asymmetric damages on EMI signatures;
- To design a strategy to reduce false alarms in damage detection by applying a temperature compensation technique.

1.2. CONTRIBUTIONS

The main contribution of this thesis is to demonstrate how the longitudinal and flexural dynamics are coupled to establish the EMI in beam-like structures.

Additional contributions were achieved as described below:

- An extensive and thorough bibliographic review on the EMI technique, focused on its modeling - (article I.c);
- An investigation of the effects of temperature variations on the EMI curves - (articles II.f and II.d);
- An enhanced and more accurate theoretical modeling of the EMI based on flexural dynamics - (article II.d);
- An adapted model to include symmetric damage - (articles II.b and II.a);
- Experimental test campaigns to demonstrate the proposed modeling for longitudinal and coupled dynamic responses in undamaged and symmetric damage configurations - (article I.b);
- A methodology to enable the appropriate application of Rayleigh damping across broad frequency ranges - (article II.c);
- A literature review on compensation techniques, coupled with a thorough assessment of their effectiveness - (article II.e);
- A specific compensation technique and formulation of a novel strategy to reduce false

alarms in damage detection - (article I.a).

1.2.1 Publications

I. *Publications and submissions to journals*

I.a - DIAS, L. L.; LOPES, K. W.; BUENO, D. D.; GONSALEZ-BUENO, C. G. An enhanced approach for damage detection using the electromechanical impedance with temperature effects compensation. **Journal of the Brazilian Society of Mechanical Sciences and Engineering**, v. 45, n. 228, 2023. DOI: 10.1007/s40430-023-04127-8.

I.b - DIAS, L. L.; GONSALEZ-BUENO, C. G.; BUENO, D. D. SHM-based Electro-mechanical Impedance: on the coupling of longitudinal and flexural dynamics. **Structural Health Monitoring**.

Awaiting review

I.c - DIAS, L. L.; GONSALEZ-BUENO, C. G.; BUENO, D. D. Electromechanical Impedance for SHM: A State-of-the-Art Review of Modeling and Applications. **Journal of Intelligent Material Systems and Structures**.

Awaiting review

II. *Publications in conferences*

II.a - DIAS, L. L.; GONSALEZ-BUENO, C. G.; BUENO, D. D. Electromechanical Impedance in Periodic Structures: Analytical Modeling and Experimental Results. **International Conference on Structural Engineering Dynamics (ICEDyn)**, 2025.

II.b - DIAS, L. L.; GONSALEZ-BUENO, C. G.; BUENO, D. D. Influence of Damage and Structural Periodicity on the Electromechanical Impedance for SHM. **XX International Symposium on Dynamic Problems of Mechanics (DINAME)**, 2025. DOI: 10.26678/ABCM.DINAME2025.DIN2025-0173.

II.c - DIAS, L. L.; BUENO, D. D.; AYALA, P. C.; GONSALEZ-BUENO, C. G. Análise do Amortecimento de Rayleigh Aplicado ao Modelo de uma Viga com PZT Acoplado. **XII Congresso Nacional de Engenharia Mecânica (CONEM)**, 2024. DOI: 10.26678/ABCM.CONEM2024.CON24-1629.

II.d - DIAS, L. L.; BUENO, D. D.; GONSALEZ-BUENO, C. G. SHM based on the Electromechanical Impedance Technique with Temperature Variations: Theoretical and Experimental Approach. **27th International Congress of Mechanical Engineering (COBEM)**, 2023. DOI: 10.26678/ABCM.COBEM2023.COB2023-1251.

II.e - DIAS, L. L.; GONSALEZ-BUENO, C. G.; LOPES, K. W.; BUENO, D. D. Temperature Compensation Strategies for SHM based on Electromechanical Impedance: A Comparative Study. **8th International Symposium on Solid Mechanics (MECSOL)**, 2022. DOI: 10.26678/ABCM.MECSOL2022.MSL22-0196.

II.f - DIAS, L. L.; GONSALEZ-BUENO, C. G.; BUENO, D. D. Study of the Temperature Effect on the Electromechanical Impedance Technique Applied to Structural Health Monitoring of a Beam. **26th International Congress of Mechanical Engineering (COBEM)**, 2021. DOI: 10.26678/ABCM.COBEM2021.COB2021-1649.

1.3. THESIS OUTLINE

This section outlines the structure of this thesis, providing a brief overview of the content addressed in each chapter. The organization is structured to progress from theoretical investigations to experimental analysis and, ultimately, the final conclusions. The following chapters are organized as follows:

- **Chapter 1** provides a brief introduction and justification for this thesis, outlining its objectives and achieved contributions, along with the respective publications;
- **Chapter 2** presents a comprehensive literature review, addressing the central themes of this thesis: SHM, EMI, its modeling formulations, temperature effects and the importance of the frequency range selection;
- **Chapter 3** provides a detailed explanation of the modeling approach for the three dynamics investigated – longitudinal, flexural, and the coupled one –, alongside the formulations associated with the presence of both symmetric and asymmetric damages. It also discusses the modal shape functions, the employed damping model and the temperature effects on the properties of the beam and the PZTs.
- **Chapter 4** presents the theoretical results obtained when considering the effects of temperature variations and different damage characteristics – namely, depth, length, and position – on the EMI response.
- **Chapter 5** shows the experimental investigations, outlining the methodology for data acquisition, as well as the application of the temperature compensation technique and the step-by-step process of the novel approach developed to reduce false alarms in damage detection. It also includes the experimental validations of the theoretical modeling.
- **Chapter 6** presents the final remarks of this thesis, detailing the main conclusions and suggesting potential future studies to advance and complement this research;
- **Appendix A** discusses the applicability of the Rayleigh damping model across broad

frequency ranges and the adjustment of its coefficients;

- **Appendix B** provides the mathematical formulations derived to obtain the equations associated with the presence of both symmetrically and asymmetrically damaged structural sections;
- **Appendix C** presents the behavior of the electromechanical impedance curves when considering the proposed temperature compensation technique.

2 LITERATURE REVIEW

This chapter provides a comprehensive review of the main topics covered in the present thesis. An overview of the SHM field is presented in Section 2.1, followed by a detailed discussion on the EMI technique in Section 2.2. In addition, the adoption of the theoretical modeling to simulate EMI responses is examined in Section 2.3, and the effects of temperature variations and their compensations techniques are addressed in Section 2.4. Finally, Section 2.5 discusses the importance of selecting an appropriate frequency range for reliable damage detection.

2.1. STRUCTURAL HEALTH MONITORING

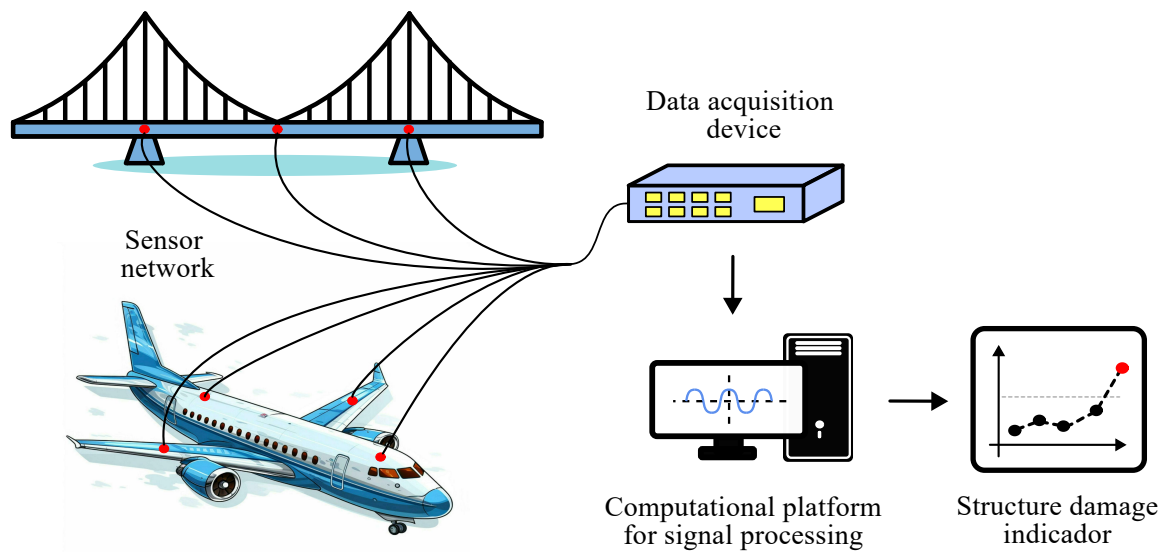
The inevitable aging and degradation of structures due to operational environments and prolonged service life, along with the growing need to ensure high performance under demanding operating conditions, have driven the development of advanced methodologies for continuous and real-time structural evaluation. This field, known as Structural Health Monitoring (SHM), focuses on the periodic assessment of structural conditions to detect damage at early stages and, ultimately, reduce the likelihood of severe structural failures (Jiao *et al.*, 2020; Fan; Li; Hao, 2021). As a result, SHM contributes to enhanced safety and operational efficiency, while also decreasing downtime associated with frequent inspections, thus directly benefiting engineers, maintenance teams and end users (Chang, 1999).

Although the SHM approach shares similarities with non-destructive evaluation (NDE) methods, it distinguishes itself by providing real-time and automated inspections through the integration of sensor networks, data acquisition devices, computational platforms and signal processing algorithms, thereby facilitating structural assessment with minimal manual intervention and greater efficiency (Giurgiutiu, 2007; Boller; Chang; Fujino, 2009). Thus, driven by both economic and safety perspectives, as well as significant advancements in acquisition and computational technologies, this field has experienced substantial growth in the last decades, both in academic research and practical applications across different sectors such as civil, mechanical and aerospace engineering (Cortez; Vieira Filho; Baptista, 2012; Sepehry; Bakhtiari-Nejad; Shamshirsaz, 2014).

The basic premise of SHM consists of employing sensors to periodically extract damage-sensitive data from the monitored structures, which are subsequently analyzed to assess their integrity. Over time, a continuously updated diagnosis is obtained regarding the structure's ability to perform its function safely (Worden; Dulieu-Barton, 2004; Inman *et al.*, 2005). Through this evaluation process, SHM enables both the early detection of damage and the tracking of its progression over time, enhancing safety, reducing maintenance costs, and extending the service life of structures.

Figure 1 presents a schematic representation of different SHM applications – in civil and aerospace engineering –, as well as the main stages that typically comprise an SHM system. The monitoring process begins by integrating sensors along the structure, forming a sensor network, from which damage-sensitive information are collected via a data acquisition device. These data are subsequently processed and interpreted using computational platforms and signal processing algorithms, ultimately yielding a diagnosis of the structural condition, i.e., the presence or absence of damage.

Figure 1 – Different applications (civil and aerospace engineering) and stages of a typical SHM system.

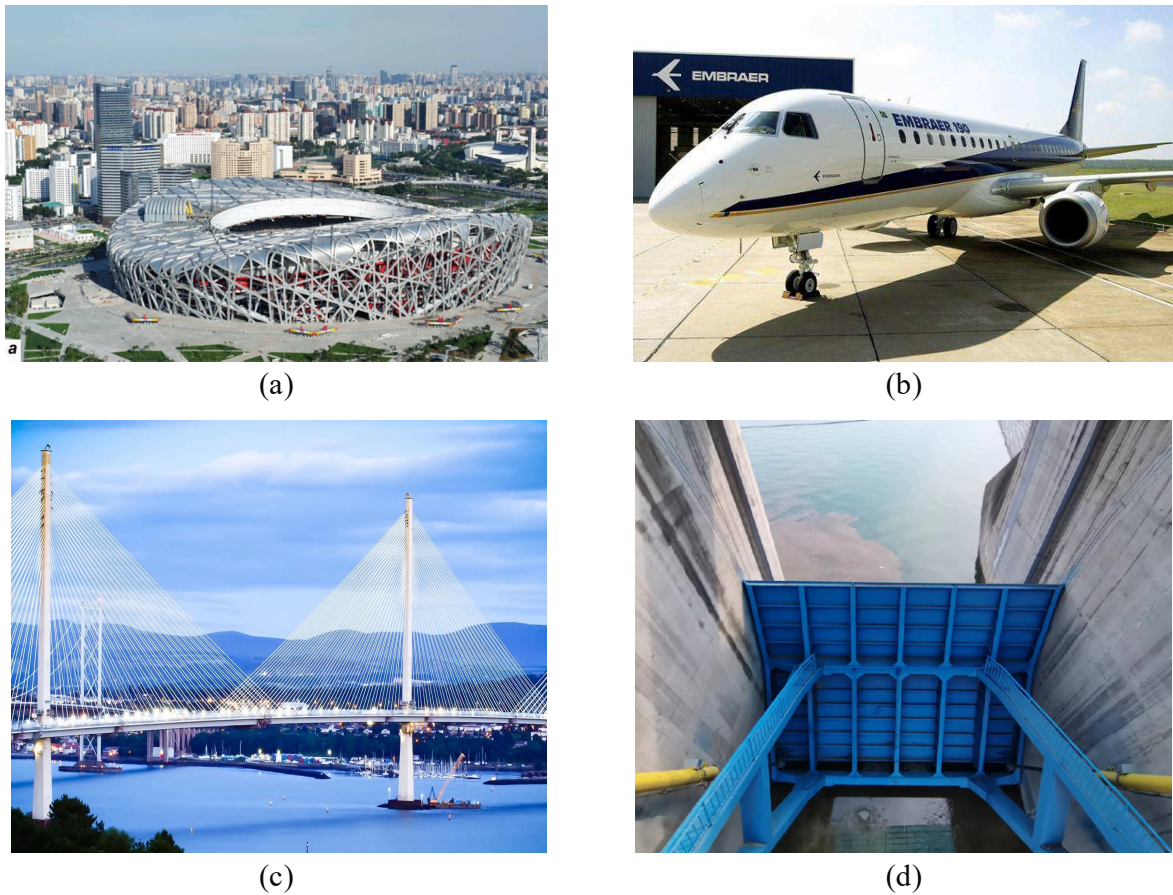


Source: Adapted from Ahani, Yang and Ghannad (2025).

Several practical applications demonstrate the versatility and effectiveness of SHM methodologies across different engineering scenarios. A notable example is the National Stadium of China, commonly known as the “Bird’s Nest”, a large-scale building constructed for the 2008 Beijing Olympic Games, with a capacity for more than 90.000 occupants (see Fig. 2a). A multitype wireless sensor network (WSN) was implemented to assess its structural health, comprising 290 sensors designed to monitor stress, displacement, acceleration, wind, and temperature (Shen *et al.*, 2013). In the aerospace sector, Embraer conducted evaluations of SHM technologies on its flight test aircraft Embraer-190 (see Fig. 2b). Two methods – Lamb Waves (LW) and Comparative Vacuum Monitoring (CVM) – were initially tested under ground conditions to investigate their capabilities of withstanding the real aircraft environmental and operational conditions (Rulli; Dotta; Silva, 2013). Another prominent example is the Queensferry Crossing bridge (see Fig. 2c), opened in August 2017, in Scotland. About 2000 sensors was installed to monitor parameters such as wind, weather, temperature, corrosion, motion and strains in real-time, allowing operators to respond quickly to extreme events, target inspections and carry out preventive interventions to avoid potential failures (Arup, 2025). A recent study, conducted in 2024, investigated the feasibility of

implementing an SHM system on the large-scale hydro-steel structure (LS-HSS), Luhun Reservoir spillway in China, by monitoring its radial gates (see Fig. 2d). The proposed SHM framework integrates finite element modeling, multi-sensor data fusion, and Internet of Things (IoT). The system demonstrated stable and reliable performance for real-time anomaly detection and decision-making (Li *et al.*, 2024a).

Figure 2 – Examples of practical applications of SHM systems in: (a) the “Bird’s Nest” stadium, (b) the aircraft Embraer-190, (c) the Queensferry Crossing bridge and (d) the radial gates of the Luhun Reservoir spillway.



Source: (a) Shen *et al.* (2013), (b) Rulli, Dotta and Silva (2013), (c) Arup (2025) and (d) Li *et al.* (2024a).

Different techniques or technologies can be employed for the SHM process, such as:

- **Acoustic Emission (AE)**, which captures the waves generated by the emergence of damage (Han *et al.*, 2015; Tziavos *et al.*, 2020; Tonelli *et al.*, 2020);
- **Fiber Bragg Grating (FBG)**, which constitutes a versatile and robust sensing technology capable of measuring a wide variety of parameters (Yeager *et al.*, 2017; Zhang; Wu; Cheng, 2020; Braunfelds *et al.*, 2022);
- **Guided Wave (GW)**, which relies on the propagation of waves over large distances to assess damage (Salas; Cesnik, 2009; Moll *et al.*, 2010; Memmolo; Maio; Ricci, 2022);
- **Comparative Vacuum Monitoring (CVM)**, which detects damage through variations in vacuum levels caused by the formation of leakage paths associated with cracks (Sba-

rufatti; Manes; Giglio, 2014; Rulli *et al.*, 2017; Blond *et al.*, 2023).

- **Electromechanical Impedance (EMI)**, which is based on the coupled electromechanical response to detect damage (Ai; Luo; Zhu, 2017; Divsholi; Yang, 2012; Naoum *et al.*, 2023).

Among these approaches, the EMI technique has attracted considerable attention due to its ability to detect incipient damage through non-intrusive piezoelectric transducers, as well as its potential for a relatively low-cost implementation (Wang; Li; Zhu, 2016).

2.2. ELECTROMECHANICAL IMPEDANCE

The Electromechanical Impedance (EMI) technique is a promising SHM approach, demonstrating significant potential for effective structural assessment (Han *et al.*, 2020; Soman *et al.*, 2020; Ali *et al.*, 2021). First proposed by Liang, Sun and Rogers (1994), this technique employs piezoelectric transducers, especially *Lead Zirconate Titanate* (PZT) ceramics, to act as actuators and sensors. This dual functionality arises from their ability to convert electrical signals into mechanical strain and vice-versa, enabling the PZTs to both excite the structure and measure its response. These transducers, renowned for their lightweight, robust and cost-effective nature, enable the detection of incipient damage through high-frequency interrogation of the local mechanical response (Yang; Lim; Soh, 2008a; Tenreiro; Lopes; Silva, 2022). This enhanced sensitivity stems from the fact that, at higher frequencies, the wavelengths becomes shorter, which increases the system's sensitivity to subtle structural changes, typically associated with incipient damage – and this is one of the major advantages of the EMI technique over other SHM approaches (Gresil *et al.*, 2012; Gonsalez, 2012). These characteristics make EMI an appealing and economically viable solution for a wide range of applications, particularly those where early-stage damage detection is critical to structural safety and performance.

The fundamental principle of the EMI technique lies in the electromechanical coupling between the electrical impedance of the PZT and the mechanical impedance of the structure. If a PZT is bonded to a structure, it induces mechanical vibrations that interact with the surrounding material. Therefore, due to the electromechanical coupling, any change in the structure's mechanical impedance – such as that caused by damage – are sensed by the PZT, which in turn alters its electrical impedance. Thus, structural health is assessed by comparing the EMI signal of the undamaged state – known as baseline – with another one acquired from a monitored condition (i.e., an unknown state). Deviations between these signals – such as changes in terms of peaks positions and amplitude levels – reveal possible modifications in the structural impedance and serve as indicators of a damage presence (Sun *et al.*, 1995a; Rabelo *et al.*, 2017; Meher; Sunny, 2022).

Since the introduction of the EMI technique by Liang, Sun and Rogers (1994), several researchers – such as Sun *et al.* (1995a), Zhou, Liang and Rogers (1996), Giurgiutiu,

Reynolds and Rogers (1999), Park, Cudney and Inman (2000b), among others – have investigated its feasibility and effectiveness, contributing significantly to its widespread application across various scenarios. For instance, Park, Cudney and Inman (2001) explore the EMI capability of detecting structural damage in pipes connected by bolted joints, particularly in scenarios requiring rapid condition assessment, such as post-earthquake evaluations. Their experimental results demonstrate the method's effectiveness in identifying imminent damage both under routine operational conditions and in the aftermath of seismic events.

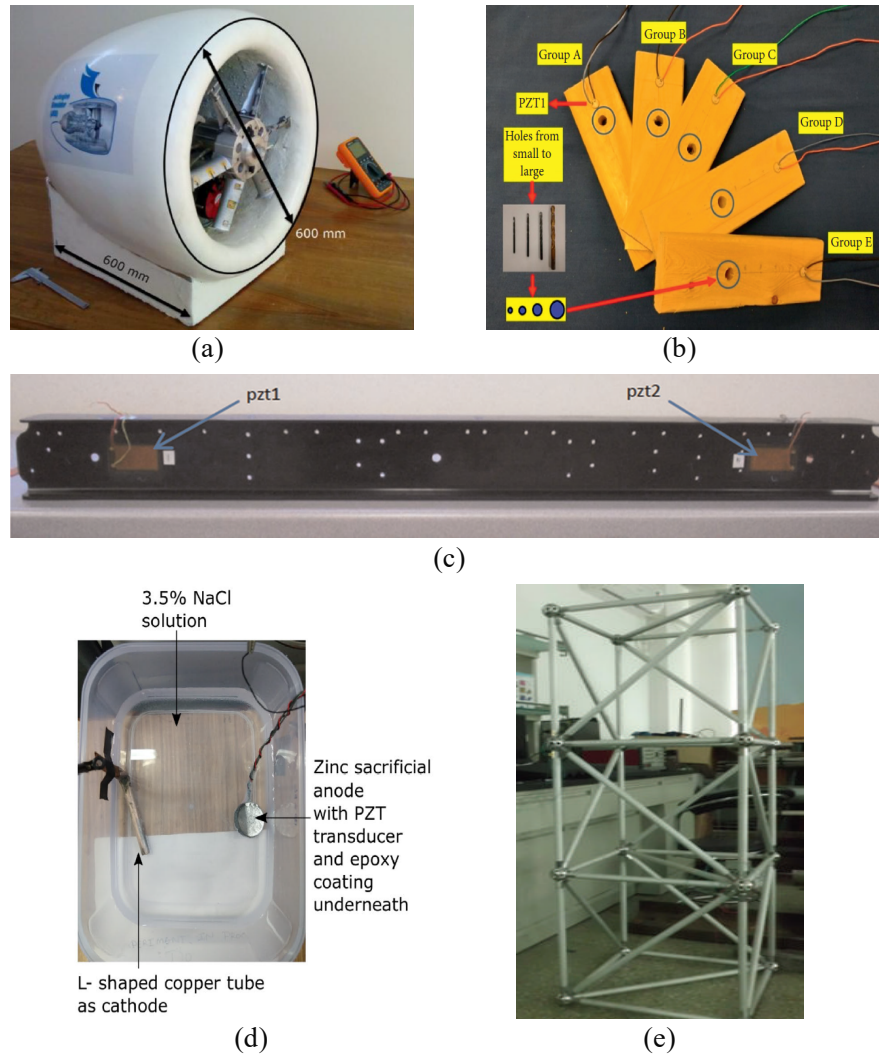
Hoshyarmanesh and Abbasi (2018) implement EMI-based monitoring on rotating aerospace components, such as turbomachinery blades, and analyze the effects of damage formation and rotational speed on EMI signals, showcasing the technique's potential for industrial applications under dynamic operational conditions. Likewise, Li *et al.* (2024b) investigate aircraft structural damages by verifying the feasibility of a novel portable wireless impedance analyzer on a wing of an unmanned aerial vehicle. Han *et al.* (2020) prove the EMI effectiveness and sensitivity to damage in timber specimens, particularly within the frequency range of 150 *kHz* to 300 *kHz*. Ali *et al.* (2021) validate the method's high sensitivity in detecting sub-millimetric cracks in welded steel joints of offshore jacket platforms under cyclic load.

Tamhane *et al.* (2021) employ the EMI technique to monitor the corrosion progression of zinc sacrificial anodes, aiming to identify early signs of protective cover deterioration and to provide advance warning for preventive maintenance of critical infrastructures, such as the marine ones. The authors also highlight the potential of the method for real-time monitoring of electrode degradation in batteries and fuel cells. The EMI technique has also been tested on composite materials, as demonstrated by Tawie *et al.* (2019), who investigate the method's capabilities and performance in glass fiber plates by exploring different PZT attachment methods for the detection of delamination and crack, and by Cherrier *et al.* (2013), who generate a damage localization map for composite structures based on indicators computed from EMI spectrums and inverse distance weighting interpolation.

More recently, artificial intelligence tools have been integrated with EMI data to improve damage quantification across different structural applications, as exemplified by the study of He *et al.* (2014), who combine EMI and a backpropagation neural network to quantify the health state of truss structures; by Perera *et al.* (2021), who propose an innovative approach that integrates EMI and machine learning methodologies to evaluate reinforced concrete beams under sustained loads and varying temperatures; and by Jiang, Zhang and Zhang (2021), who employ different artificial intelligences – principal component analysis, K-means clustering and artificial neural network – to accurately distinguish and identify structural damage, simulated by added mass on an aluminum plate, from four kinds of sensor damages, namely, pseudosoldering, debonding, wear, and breakage. Figure 3 present some examples of EMI applications.

Despite the proven versatility of the EMI technique to a wide range of structural

Figure 3 – Examples of research applications of the EMI technique in: (a) turbomachine, (b) timber specimens, (c) composite stiffener of an aircraft door in the shape of an I-beam, (d) sacrificial anode, and (e) truss structure.



Source: (a) Hoshyarmanesh and Abbasi (2018), (b) Han *et al.* (2015), (c) Cherrier *et al.* (2013), (d) Tamhane *et al.* (2021) and He *et al.* (2014).

and material applications, review articles emphasize the importance of addressing different technical aspects and pursuing further investigations to improve its reliability and extend its practical application. Park *et al.* (2003) present the first review on the EMI technique, highlighting its advantages over other NDE approaches, most notably its high sensitivity to incipient damage and automated operation. Annamdas and Soh (2010) discuss key developments in EMI-based SHM and provided insights into future research directions and necessary enhancements, such as the inclusion of wireless technology, baseline-free methods, temperature compensation techniques, and integration with other NDE approaches. Similarly, Na and Baek (2018) conduct a comprehensive literature review, emphasizing unresolved challenges in the application of EMI – such as limited sensing range, the need for appropriate frequency band selection, sensitivity to environmental factors (e.g., temperature variations),

among other problems —, which must be addressed to enable reliable damage detection. More recently, Cao *et al.* (2023) review the use of the EMI technique for damage identification from a numerical modeling perspective. They underscore that significant challenges still hinder practical applications and highlight the need for ongoing research to improve the accuracy and convergence of numerical models, aiming to bridge the gap between research and practice.

Therefore, despite its numerous advantages, practical problems have limited the implementation of the EMI technique in real structures, with temperature effects cited as one of the most critical and challenging issues (Baptista *et al.*, 2014). This limitation have motivated the development of compensation techniques and the incorporation of theoretical modeling capable of predicting and interpreting the EMI signatures under varying environmental conditions.

2.3. EMI MODELING

Applications involving EMI commonly focus on evaluating experimental signals (Giurgiutiu; Zagrai, 2000; Yan *et al.*, 2007c). These studies contribute to a more comprehensive understanding of the method's practical performance by providing direct insights into the system behavior (Habib *et al.*, 2024). However, the engineering community has increasingly adopted the use of theoretical models, mainly due to the advancement of computational resources (Bishnu; Alnouri; Al-Mohannadi, 2023; Zhang *et al.*, 2024). This approach enables detailed EMI simulations across various scenarios, allowing informed decision-making, design optimization and enhanced result interpretation by providing a controlled environment for exploring complex interactions that may be difficult to isolate and/or collect experimentally (Davis; Eisenhardt; Bingham, 2006). Its predictive abilities makes it possible, for instance, to assess the PZTs sensitivity to diverse system configurations, thereby supporting the determination of optimal transducer placement and sizing, the selection of an appropriate frequency range for damage detection, the enhancement of the sensor/structure adhesive interface, among other design considerations, in a more timely manner, contributing to the development of more precise and reliable monitoring systems (Giurgiutiu, 2010). Therefore, there is an important effort to broaden the understanding of the EMI technique through numerical-computational simulations, so that response trends and parameter dependencies can be explicitly analyzed.

In this context, the modeling of EMI has been explored through a variety of approaches, including the Finite Element Method (Yang; Lim; Soh, 2008b; Antunes *et al.*, 2019), Spectral Element formulations (Ritdumrongkul *et al.*, 2004; Wang; Song; Zhu, 2015), boundary-element-based methodologies (Zou; Aliabadi, 2015; Zou; Aliabadi, 2017) and analytical models (Zagrai; Giurgiutiu, 2001; Wang *et al.*, 2024). The latter one, in particular, enables the derivation of exact solutions, typically for relatively simplified problems,

expressed directly in terms of the governing variables, which facilitates the analysis of the parameters that influence the EMI response (Lee, 2009).

Liang, Sun and Rogers (1994) propose the first analytical EMI model of a single-degree-of-freedom (SDOF) structure coupled with a piezoelectric transducer. Subsequently, Zhou, Liang and Rogers (1996) extend this one-dimensional (1D) model to a two-dimensional (2D) representation, expanding its applicability to more complex structural behaviors. Giurgiutiu and Zagrai (2000) derive the dynamics of a free piezoelectric sensor under different boundary conditions and extend their formulation to include the PZT attached to a structural substrate. The structural dynamics are incorporated through the pointwise dynamic stiffness that the structure transmits to the sensor. The model accounts for both longitudinal and flexural vibration modes. Bhalla and Soh (2004) are one of the first authors to address the influence of bonding layers between the PZTs and the host structure. They investigate its mechanism of force transfer and present a step-by-step derivation to integrate the shear lag effect into EMI modeling, in both 1D and 2D configurations. Their findings underscore that, if not carefully controlled during the installation process, the bonding layer can substantially modify the EMI signals. Annamdas and Soh (2006) employ the average sum impedance concept to model the coupled dynamics of embedded PZT-structure systems, considering vibrations in the length and thickness directions of the PZT patch.

Yan *et al.* (2007a), Yan *et al.* (2007b), Yan *et al.* (2007c) combine the EMI technique with a reverberation matrix method to develop analytical models that quantitatively correlate stiffness loss – model as Young’s modulus reductions – with high-frequency EMI signatures in bars, beams, and framed beam structures. Yang and Miao (2008), Yang and Miao (2010) develop 1D and 2D EMI models for beams and plates, considering the influence of external excitation. Their analysis reveals that its effects become non-negligible when the excitation frequency and PZT’s actuation are comparable, highlighting the necessity to mitigate the interference caused by external conditions. Kuang, Li and Chen (2008) model a cracked curved beam by segmenting it into elastic and piezoelectric bimorph components, introducing a linear spring to represent the crack. They observe that the frequency shift amplitude increases as the crack approaches the anti-node of the corresponding radial displacement mode, whereas it decreases when the crack is located near a node point.

Sepehry, Shamshirsaz and Bastani (2010), Sepehry, Bakhtiari-Nejad and Shamshirsaz (2014) detail different formulations, for beams and plates, showing interesting correlation between theoretical results and experimental tests, considering the influence of ambient temperature. Yan, Wang and Chen (2011) introduce an analytical model of a laminated beam containing a delamination represented by a Kelvin-type viscoelastic interface. The authors employ an improved version of the reverberation-ray matrix method to compute the EMI, deriving a recursive formulation that improves numerical stability, particularly at high frequencies. Lim and Soh (2012) propose an analytical model to investigate the effects of varying axial loadings and boundary conditions on the admittance signatures acquired from

a PZT bonded on a beam. Both extensional and transverse vibrations modes are considered. Their results reveal that the peaks associated with transverse modes shift with increasing axial force, whereas those corresponding to axial modes remain unaffected. Buethe, Eckstein and Fritzen (2013) model the EMI behavior under varying temperature conditions and propose a physics-based temperature compensation strategy that is capable of distinguishing sensor faults from temperature effects, reducing the need for extensive baseline measurements.

Wang *et al.* (2018) introduce a novel EMI model for quantitative monitoring of bolted looseness under varying pre-load conditions through the integration of fractal contact theory. Li *et al.* (2020) model the EMI of an circular piezoelectric-metal transducer for corrosion monitoring applications, based on electroelastic and Kirchhoff plate theories. Their results demonstrate that in the flexural dynamics the peak frequencies decrease linearly with increasing thickness loss. Hamzeloo, Barzegar and Mohsenzadeh (2020) develop a theoretical model of L-shaped beams by computing the dynamic stiffness to obtain the EMI at each point of the structure, showing good agreement with the curves and damage indices obtained from experimental data. Zhang *et al.* (2020a) employ piezo-elasticity theory to relate the EMI response of embedded PZTs to the evolving soil's mechanical properties during freeze–thaw process. Based on the proposed model, the authors observe that it is possible to predict the soil's strength from the EMI signatures. Similarly, Zhang *et al.* (2020b) propose a simplified 1D model of a PZT embedded in cementitious material to evaluate the applicability of using EMI-based to monitor its stiffness degradation under uniaxial compression loadings, concluding that the elastic modulus of hardened cement exerts a greater influence on the EMI response than the applied loads.

Beyond these contributions summarized herein, other researchers have also proposed additional EMI analytical models. A concise review of the articles that employ this modeling strategy – 45 in total – is presented in Tabs. 1 to 3, with the references provided in chronological order, from the earliest to the most recent work. The tables compile key information from each reference, including the material, structure, frequency range (FR) and any external conditions (EC) investigated. The abbreviation UND (undefined) is used to indicate cases where the information was not specified by the authors.

Due to the adoption of an analytical approach, most of these cited studies investigate beams (54.1%) or plates (19.6%), with more than half of the reviewed works proposing one-dimensional (1D) models (66%). However, note that the adoption of simplified model does not inherently compromise its appropriate representation of the real system, once this feature depends on whether its assumptions and simplifications are capable to successfully capture the characteristics relevant to each particular analysis. In fact, models of different dimensions have shown satisfactory agreement with experimental, numerical or analytical results, with the majority of the listed articles (86.7%) conducting at least one form of these validations for their proposed models. For three-dimensional (3D) dynamic models, researchers commonly

Table 1 – Analytical EMI models (part I). FR defines the frequency range, EC the external conditions and UND corresponds to an undefined information.

Ref.	Material	Structure	Damage	FR	EC
Liang, Sun and Rogers (1994), Liang, Sun and Rogers (1996)	Aluminum	Beam (1D)	-	0 - 3 kHz	-
Zhou, Liang and Rogers (1996)	Aluminum	Plate, shell (2D) and beam (1D)	-	0.05 - 0.7 kHz	-
Giurgiutiu and Zagrai (2000)	Steel	Beam (1D)	-	0 - 30 kHz	-
Zagrai and Giurgiutiu (2001)	Aluminum	Circular plate (2D)	-	0.5 - 40 kHz	-
Giurgiutiu and Zagrai (2002)	Steel	Beam (1D)	-	1 - 30 kHz	-
Bhalla and Soh (2004)	Aluminum	Block (1D and 2D)	-	0 - 250 kHz	-
Bois and Hochard (2004), Bois, Herzog and Hochard (2007)	Composite	Beam (1D)	Delamination	100 Hz - 30 kHz	-
Giurgiutiu and Zagrai (2005)	Aluminum	Circular plate (2D)	-	0.5 - 40 kHz	-
Kuang, Li and Chen (2006)	Aluminum and steel	Beam (1D)	Crack	2.9 - 3.5 kHz	-
Yan <i>et al.</i> (2006)	Steel	Beam (1D)	Local stiffness reduction	0 - 300 kHz	-
Yan <i>et al.</i> (2007a)	UND	Bar (1D)	Local stiffness reduction	60 - 200 kHz	-
Yan <i>et al.</i> (2007b)	Composite	Beam (1D)	Local stiffness reduction	0 - 250 kHz	-
Yan <i>et al.</i> (2007c)	UND	Framed beam (1D)	Local stiffness reduction	0.5 kHz - 115 kHz	-
Yan <i>et al.</i> (2008)	UND	Beam (1D)	Crack	0.68 - 120 kHz	-

Source: Elaborated by the author.

Table 2 – Analytical EMI models (part II). FR defines the frequency range, EC the external conditions and UND corresponds to an undefined information.

Ref.	Material	Structure	Damage	FR	EC
Kuang, Li and Chen (2008)	Aluminum	Circular beam (1D)	Crack	0 - 1.4 kHz	-
Yang and Miao (2008)	Aluminum	Beam (1D)	-	5 - 50kHz	Vibration
Dugnani (2009)	Aluminum	Hexagonal washer and plate (2D)	-	0 - 20kHz (washer) and 0 - 600 kHz (plate)	-
Yan, Cai and Chen (2009)	Composite	Beam (2D)	Interfacial imperfections and PZT debonding	11.35 - 19.86 kHz	-
Sepehry, Shamshirsaz and Bastani (2010)	Aluminum	Beam (1D)	-	16 - 34 kHz	Temperature
Yang and Miao (2010)	Aluminum	Plate (2D)	-	5 - 50 kHz	Vibration
Yan, Cai and Chen (2011)	Composite	Beam (1D)	Crack	0 - 20 kHz	-
Yan, Wang and Chen (2011)	Composite	Beam (1D)	Delamination	0 - 200 kHz	-
Yan <i>et al.</i> (2011)	Titanium and ceramic	Functionally graded beam (1D)	Crack	0 - 50 kHz	-
Zhang <i>et al.</i> (2011)	UND	Beam (1D)	Crack	40 - 200 kHz	-
Bhalla and Moharana (2012)	Aluminum	Block (1D and 2D)	-	0 - 250 kHz	-
Lim and Soh (2012)	Aluminum	Beam (1D)	-	10 - 50 kHz	Axial load
Giurgiutiu <i>et al.</i> (2012)	Steel	Beam (1D)	-	10 - 100 KHz	-
Buethe, Eckstein and Fritzen (2013)	Aluminum	Coupons (2D)	Degradation and sensor breakage	200 - 800 kHz	Temperature
Song, Lim and Sohn (2013)	Steel	Beam (1D)	-	0 - 25 kHz	-

Source: Elaborated by the author.

Table 3 – Analytical EMI models (part III). FR defines the frequency range, EC the external conditions and UND corresponds to an undefined information.

Ref.	Material	Structure	Damage	FR	EC
Sepehry, Bakhtiari-Nejad and Shamshirsaz (2014)	Aluminum	Plate (2D)	-	5 - 180 kHz	Temperature
Moharana and Bhalla (2014)	Aluminum	Block (2D)	-	0 - 250 kHz	-
Islam and Huang (2014)	Aluminum	Beam (1D)	-	0 - 500 kHz	-
Kamas, Giurgiutiu and Lin (2015)	Aluminum	Rectangular and circular plate (1D)	-	0 - 10 MHz	-
Dugnani (2016)	Aluminum	Plate (1D)	-	150 - 200 kHz	-
Zhu and Scalea (2016)	Aluminum and steel	Bar (1D)	-	50 - 205 kHz	Load
Dugnani and Chang (2017)	Hysol EA 9696	Adhesive joints (2D)	-	0 - 1 MHz	-
Wang <i>et al.</i> (2018)	Steel	Bolted joint (2D)	-	100 - 400 kHz	Preload
Zhu, Wang and Qing (2019)	Aluminum	Plate (2D)	Hole	2.4 - 3.0 MHz	-
Li <i>et al.</i> (2020)	Steel	Plate (2D)	Corrosion	1 - 200 kHz	-
Zhang <i>et al.</i> (2020a)	Soil	Cylinder (1D)	-	0 - 4 MHz	Temperature
Zhang <i>et al.</i> (2020b)	Cement	Block (1D)	-	0 - 4 MHz	Load
Hamzeloo, Barzegar and Mohsenzadeh (2020)	Aluminum	L-shaped beam (2D)	Crack	26 - 40 kHz	-
Wang <i>et al.</i> (2024)	UND	UND (1D)	-	0.5 - 1.5 MHz	Static stress

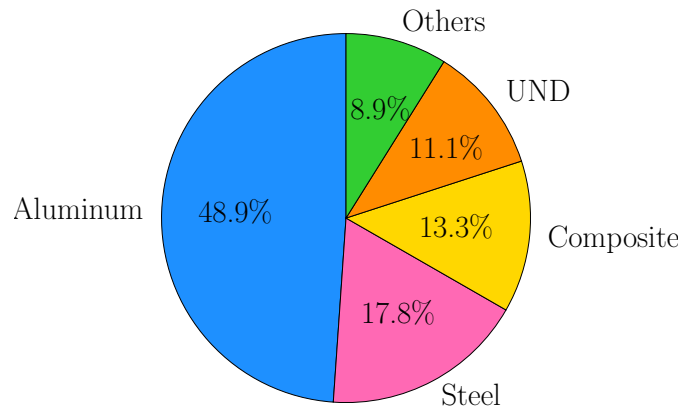
Source: Elaborated by the author.

resort to semi-analytical models, which complement the analytical formulations by incorporating either the Finite Element or the Ritz methods (Annamdas; Soh, 2007a; Annamdas; Soh, 2007b; Madhav; Soh, 2007; Annamdas; Soh, 2008; Wang; Song; Zhu, 2014; Wang; Li; Zhu, 2016).

Figure 4 shows the distribution of the reviewed articles according to the structural

material considered in their investigations. The more typical materials are aluminum and steel, representing a total of approximately 66.7%. These materials are widely used in engineering structures, and they are assumed to be commonly homogeneous, which simplifies the theoretical modeling, in comparison with heterogeneous materials such as cement or composite.

Figure 4 – Types of materials considered for different studies using analytical modeling.



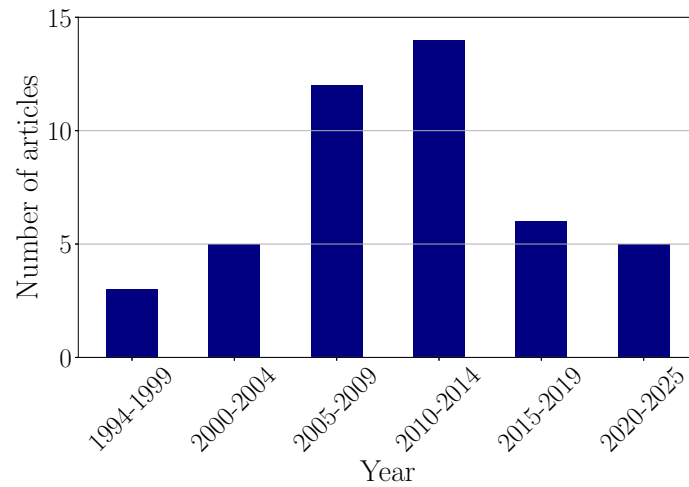
Source: Elaborated by the author.

In terms of external conditions, just 11 (24.4%) of those works consider them, and the most common are temperature, load, vibration and stress. On the other hand, 18 (40%) studies include damages, encompassing crack, hole, corrosion, stiffness reduction, debonding, degradation and/or breakage of the PZTs, besides delamination in composite materials. Additionally, just one article (i.e., 2.2%) combines both aspects, which suggests a potential for further research exploring these topics, once they are relevant for the effective application of the EMI technique in real-world structures. A broader investigation encompassing both external conditions and damage can assist engineers in distinguishing between their effects and, consequently, in ensuring more accurate damage detection.

Figure 5 presents the number of articles published in five-year intervals. The highest number of publications occurred during period 2010-2014, followed by a decreasing trend. This decrease can be associated with the growing adoption of numerical approaches, including machine learning (ML), driven by advances in computational capabilities (Pereira *et al.*, 2024).

According to the literature, analytical modeling has been widely employed to investigate EMI across diverse scenarios; nonetheless, relatively few studies analyze the presence of structural damage and external conditions simultaneously. This gap highlights the potential for further developments involving theoretical aspects.

Figure 5 – Number of articles employing analytical modeling published in five-year intervals.



Source: Elaborated by the author.

2.4. TEMPERATURE EFFECTS

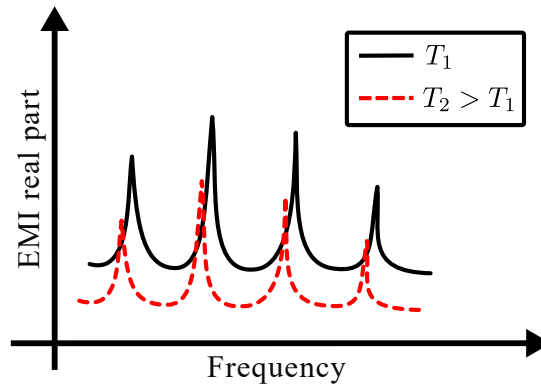
In practical applications the structures are continuously exposed to different environmental conditions, which can adversely impact the effectiveness of EMI-based SHM. Among these external influences, temperature stands out as one of the most critical effects due to its pronounced potential to change the dynamic behavior of the monitored structure (Abbas *et al.*, 2021; Li *et al.*, 2021; Du *et al.*, 2023).

This sensitivity to the temperature arises from the fundamental principle of the EMI technique, which relies on the electromechanical coupling between the PZT and the host structure, making the resulting EMI response inherently dependent on their properties. As such, temperature-induced changes in the geometric and physical properties of these materials can modify the EMI curves. This characteristic is widely investigated in the literature, with different researchers identifying key factors that influence the signal variations. Sepehry, Shamshirsaz and Bastani (2010) develop an analytical model to analyze the EMI response under varying temperature conditions by incorporating temperature-dependent material properties for both the aluminum beam and the PZT. Their work reveals that a decrease in EMI amplitude is primarily attributed to changes in the PZT properties with increasing temperature, whereas the leftward frequency shift results from temperature-induced variations in the structure's properties. Xu *et al.* (2016) independently corroborate these findings through a spectral element model, reaching the same conclusions regarding EMI behavior. In an experimental study, Yang, Lim and Soh (2008a) highlight that the bonding layer also influence the EMI shifts. As temperature increases, the stiffness of the bonding layer decreases, which impairs the stress transfer between the PZT and the host structure, ultimately contributing to a leftward shift in the EMI peaks. The authors further emphasize that thicker bonding layers

tend to exacerbate temperature-induced deviations. In the numerical counterpart of their study, Yang, Lim and Soh (2008b) confirm these findings through finite element simulations.

Abbas *et al.* (2021) further reinforce the significant impact of temperature variations on EMI signals, emphasizing the high sensitivity of the PZT dielectric constant (ϵ_{33}^T) to thermal fluctuations. Their experimental work, spanning from 25°C to 160°C, highlight how temperature-induced variations can undermine the accuracy and reliability of EMI-based damage detection. Other researchers have also investigated the EMI technique in the context of temperature variations, employing both experimental and theoretical approaches (Baptista; Vieira Filho; Inman, 2012; Sepehry; Bakhtiari-Nejad; Shamshirsaz, 2014; Na; Lee, 2016; Zhang; Zhou; Li, 2019; Li *et al.*, 2021; Dahl *et al.*, 2024). These findings collectively demonstrate that the temperature significantly affects both the amplitude and frequency of EMI curves. Consequently, impedance signatures collected at different temperatures may differ considerably, even when the structural condition remains unchanged. Such variations can be mistakenly interpreted as damage, compromising the accuracy of the diagnostic process. Figure 6 illustrates these typical changes in the EMI real part, which are noted when comparing the curves obtained at two different temperatures T_2 and T_1 .

Figure 6 – Schematic representation of two curves of EMI real part at two different temperatures, $T_2 > T_1$, showing the downward and leftward shifts induced by the temperature increased.



Source: Elaborated by the author.

Due to the significant impact of temperature on damage detection via EMI, different researchers have developed strategies known as temperature compensation techniques. They are focused on mitigating the effect of the changes on the damage detection. Sun *et al.* (1995b) propose a compensation method based on the cross-correlation between the reference and the unknown impedance signals to mitigate the horizontal shifts caused by temperature variations. Krishnamurthy, Lalande and Rogers (1996) introduce a linear frequency-independent temperature coefficient to vertically compensate the shifts on the PZT transducer in a narrow frequency and temperature range. However, their approach do not mitigate the temperature effects on the structure, i.e., the horizontal shifts. To overcome both vertical and horizontal displacements, Park *et al.* (1999) employ a modified damage metric to compensate them.

Koo *et al.* (2009) introduce the Effective Frequency Shift (EFS) method, which compensates for temperature variations by shifting the unknown EMI signature to maximize its cross-correlation with the baseline signal. Expanding on this approach, Baptista *et al.* (2014) study the influence of temperature on the EMI technique and perform horizontal compensation by maximizing the correlation coefficient. Their findings highlight that frequency shifts tend to increase with frequency, suggesting that the EFS approach performs more effectively over narrow frequency bands, where the displacements have approximately the same value.

Wandowski, Malinowski and Ostachowicz (2017) divide the entire frequency range into smaller sub-bands and apply a different EFS to each band to address this limitation. They also perform vertical compensation via whole-signal normalization using the root mean square value. Rabelo *et al.* (2017) propose a hybrid optimization method associated with different damage metrics to compensate the horizontal and vertical shifts. The authors use more than one baseline signal to achieve more accurate compensation across broader temperature ranges. Antunes *et al.* (2019) employ a linear interpolation technique to eliminate temperature effects, which can be mainly applied to structures where both amplitude and frequency shifts exhibit linear dependence on temperature and frequency. On the other hand, Giancesini *et al.* (2021) propose an analytical polynomial-based temperature compensation method capable of addressing nonlinear dependencies on both temperature and frequency.

In addition to the previously mentioned approaches, others authors have explored alternative strategies for temperature compensation in EMI-based SHM. Park, Shin and Yun (2009) and Grisso and Inman (2010) develop compensation techniques focused on sensor self-diagnosis by investigating the imaginary part of the electromechanical admittance. Lim *et al.* (2011) and Huynh, Dang and Kim (2018) employ a Principal Component Analysis (PCA) to improve damage detectability under varying temperature conditions. Sepehry, Shamshirsaz and Abdollahi (2011) and Huynh and Kim (2018) propose temperature compensation methods based on Radial Basis Function (RBF) for damage detection in different structures. More recently, Du *et al.* (2023) introduce a multitask Convolutional Neural Network (CNN) that integrates two subnetworks: one dedicated to temperature compensation and another to lightweight damage identification of bolt loosening.

Despite the extensive research on temperature compensation techniques, most studies focus primarily on applying these methods to baseline signals, i.e., undamaged structural conditions, without evaluating their impact on damage detectability when also implemented to damaged states.

2.5. FREQUENCY RANGE SELECTION

The frequency range selected in the EMI technique has a direct impact on the sensitivity and accuracy of damage detection. This is because the effect of a given damage on EMI curves may vary across the frequency spectrum, with certain regions exhibiting more

pronounced signal changes than others. Likewise, the number of peaks within the investigated frequency range – which are associated with the structure’s natural frequencies and can therefore provide a more robust set of information about its dynamic behavior – also differ across frequency bands (Park *et al.*, 2003; Singh; Fakhri; Malinowski, 2023). Therefore, choosing an appropriate range is essential to enhance detection performance while minimizing unnecessary data collection, computational effort and the risk of misdiagnosis.

However, the best frequency range is not universal – it depends on different factors, including the characteristics of the piezoelectric transducer, the host structure and the type of damage (Baptista; Vieira Filho, 2010; Annamdas; Radhika, 2013; Na; Baek, 2018). Generally, this selection is performed through trial-and-error procedures, by identifying frequency bands that exhibit greater damage sensitivity. Nonetheless, some authors have proposed alternative methods for this selection. Peairs, Tarazaga and Inman (2007) compute a damage index for each sub-band of the overall frequency range, followed by the application of statistical hypothesis testing to identify the ones more sensitive to damage detection. Annamdas and Rizzo (2009) evaluate the sensitivity of narrow frequency ranges by computing two different damage metrics, aiming to identify the most responsive range within the broader frequency spectrum. In contrast to previous methods that rely on experimental data, Baptista and Vieira Filho (2010) use a modified equivalent electromechanical circuit to analyze the sensitivity of the PZT transducer in terms of the frequency and the mechanical impedance of the host structure. The theoretical model suggests that there are frequencies where sensitivity is optimal and others where it is minimal. Min, Park and Yun (2010) propose an innovative technique for autonomous selection of damage-sensitive frequency ranges for the EMI technique using Artificial Neural Networks (ANNs). The ANN is trained using the cross-correlation coefficient values computed for each sub-band from the original wide frequency range. Singh and Malinowski (2022) propose a standard deviation-based approach, computed from the differences between healthy and damaged states, for selecting the effective frequency ranges. Singh, Fakhri and Malinowski (2023) compute the standard deviation of the rescaled impedance data to identify a single maximum, from which a threshold is defined to select the appropriate frequency range.

Despite the critical role of the selected frequency range in the effectiveness of the EMI technique, relatively few studies have investigated this subject, with the majority of works still manually seeking the most sensitive frequency range.

2.6. CONCLUSIONS

This chapter provided a comprehensive review of the key concepts addressed in the present thesis. The fundamental principles of SHM and EMI were discussed, followed by an overview of the theoretical modeling approaches used to simulate the impedance response. Special attention was given to the influence of temperature, which remains a critical factor for

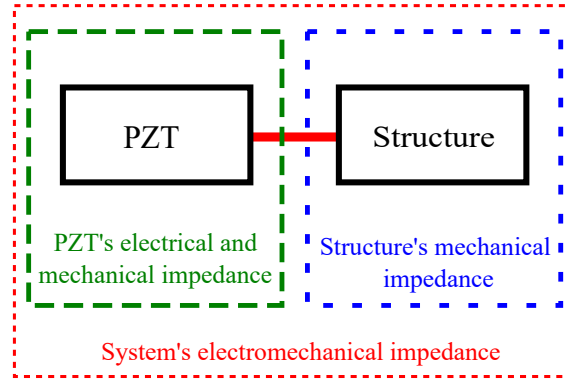
the broader implementation of the EMI technique in real structures. Moreover, the selection of the frequency range was highlighted as a decisive factor in the performance of EMI-based SHM.

Altogether, the literature reveals that although the EMI technique has shown great potential for effective damage detection, there are still challenges related to environmental influences, modeling complexity and frequency optimization. These aspects are essential for the greater understanding and applicability of the EMI technique in practical SHM applications.

3 THEORETICAL MODELING

The Electromechanical Impedance technique commonly uses one or more PZTs to monitor the structures. These devices undergo mechanical deformation when they are exposed to an electric field; thus, operating as actuators. Conversely, when subjected to mechanical stress, they generate electrical charges, acting as sensors. This dual role enables PZTs to dynamically couple the mechanical impedance of the structure with their electrical impedance, so that any changes in the structural dynamics are captured through variations in the PZT response signals (Na; Baek, 2018; Cao *et al.*, 2023). This connection is visually represented in Fig. 7 and formally expressed in Eq. (3.1) (Liang; Sun; Rogers, 1994), where the electromechanical admittance, Y_{em} , is the inverse of the EMI, Z_{em} .

Figure 7 – Schematic representation of a generic system's electromechanical impedance due to a PZT coupled to a mechanical structure.



Source: Elaborated by the author.

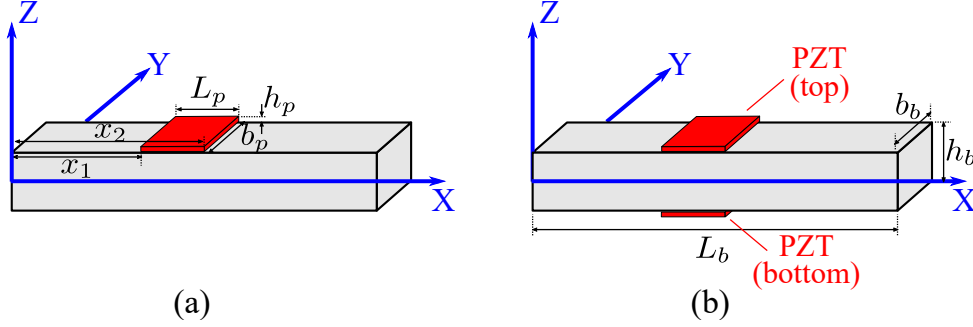
$$Y_{em}(\omega) = j\omega \frac{b_p L_p}{h_p} \left[\epsilon_{33}^{T*} - \frac{Z(\omega)}{Z_A(\omega) + Z(\omega)} d_{31}^2 Y_{11}^{E*} \right], \quad (3.1)$$

where L_p , b_p and h_p are the length, width and thickness of the PZT, respectively, ϵ_{33}^{T*} is the complex dielectric constant of the PZT in the 3-3 direction (z) under a constant voltage, d_{31} is the piezoelectric constant, Y_{11}^{E*} is the complex elastic modulus of the PZT in the 1-1 direction (x) under a constant electric field, $j = \sqrt{-1}$ is the pure imaginary number, and Z and Z_A are the mechanical impedances of the structure and PZT, respectively.

This theoretical model proposed by Liang and his colleagues is considered by Sepehry, Shamshirsaz and Bastani (2010) to introduced the EMI for the flexural dynamics of beams. Their modeling considers a PZT asymmetrically placed on the structure, in relation to its longitudinal direction. A schematic drawing is shown in Fig. 8(a). However, in practical application, this type of PZT coupling excites both flexural and longitudinal dynamics. On the other hand, if two PZTs are symmetrically placed on the beam as shown in Fig. 8(b), they can be conveniently used to excite only the longitudinal or flexural dynamics. Therefore, this thesis presents the theoretical developments and practical aspects concerning both the

adoption of a symmetric piezoelectric configuration (Fig. 8b), in which longitudinal and flexural dynamics are treated separately, as well as the coupling between these two dynamics when using a single PZT (Fig. 8a).

Figure 8 – Schematic drawing of a beam with PZT coupled in (a) asymmetric and (b) symmetric configurations.



Source: Elaborated by the author.

The EMI modeling is formulated based on the principle of energy conservation. By defining the kinetic and strain energies of each system component (beam and PZTs), the equation of motion can be established. The mechanical analysis can be combined with the electrical response of the PZTs to determine the system's EMI. These formulations for each dynamics - longitudinal, flexural and the coupled one - are described in Sections 3.1 to 3.3, respectively. Details regarding the modal shape functions appear in Section 3.4, whereas the adjustment of EMI peaks is discussed in Section 3.5. Formulations for symmetric and asymmetric damage are introduced in Section 3.6, followed by an analysis of temperature effects on the material properties of the beam and PZTs presented in Section 3.7.

3.1. EMI FOR LONGITUDINAL DYNAMICS

Consider the axial displacement $u(x, t) \equiv u_L(x, t)$ in the x -direction (see Fig. 8), where the subscript L indicates the longitudinal dynamics. The displacements in the y - and z -directions are respectively $v_L(x, t) = 0$ and $w_L(x, t) = 0$. Then, the mechanical strains are given by

$$\begin{aligned} \epsilon_{xx_L} &= u'_L(x, t); \\ \epsilon_{xy_L} &= \epsilon_{yy_L} = \epsilon_{zz_L} = \epsilon_{yz_L} = \epsilon_{zx_L} = 0, \end{aligned} \quad (3.2)$$

where $(\)'$ indicates the first derivative with respect to x .

Based on the stress-strain relation $\sigma_{xx} = \tilde{E}_b \epsilon_{xx}$, the longitudinal strain energy of the beam π_{b_L} is $\pi_{b_L} = \frac{1}{2} \int_{V_b} \sigma_{xx_L} \epsilon_{xx_L} dV_b$, such that

$$\pi_{b_L} = \frac{1}{2} \int_0^{L_b} \tilde{E}_b A_b u'_L(x, t)^2 dx, \quad (3.3)$$

where V_b is the volume of the beam, L_b is its length, $\tilde{E}_b$ is the Young's modulus and A_b is cross-sectional area. The subscript b denotes the beam, while the superscript $(\tilde{\cdot})$ indicates temperature dependence. On the other hand, the kinetic energy of the beam T_{b_L} is given by

$$T_{b_L} = \frac{1}{2} \int_0^{L_b} \tilde{\rho}_b A_b \dot{u}_L(x, t)^2 dx, \quad (3.4)$$

where $\tilde{\rho}_b$ is the temperature-dependent material density and $(\dot{\cdot})$ denotes the first derivative with respect to time t .

Assuming a beam-shaped PZT, the well-known constitutive equations of this device are given by

$$\begin{cases} \epsilon_1 = S_{11}^E \sigma_1 + \tilde{d}_{31} E_3 \\ D_3 = \tilde{d}_{31} \sigma_1 + \tilde{\epsilon}_{33}^T E_3 \end{cases}, \quad (3.5)$$

where ϵ_1 is the mechanical strain, σ_1 is the stress, E_3 is the electric field, D_3 is the electric displacement and S_{11}^E is the mechanical compliance. For simplicity, Eq. (3.5) can be rewritten by $\sigma_1 = c\epsilon_1 - dE_3$ and $D_3 = d\epsilon_1 + e_p E_3$, where $c = \frac{1}{S_{11}^E}$, $d = \frac{\tilde{d}_{31}}{S_{11}^E}$ and $e_p = \tilde{\epsilon}_{33}^T - \frac{\tilde{d}_{31}^2}{S_{11}^E}$.

The kinetic energy T_{p_L} and strain energy π_{p_L} of the beam-type PZT are computed by

$$T_{p_L} = \frac{1}{2} \int_{x_1}^{x_2} \rho_p A_p \dot{u}_L(x, t)^2 dx, \quad (3.6)$$

$$\pi_{p_L} = \frac{1}{2} \int_{V_p} (\sigma_{1L} \epsilon_{1L} - D_3 E_3) dV_p, \quad (3.7)$$

where ρ_p is the material density of the PZT, A_p is its cross-sectional area, V_p is the volume, and x_1 and x_2 correspond to the coordinates where the PZT is bonded to the beam, such that $(x_2 - x_1) = L_p$ (see Fig. 8a). The subscript p indicates the PZT. The second term of Eq. (3.7) corresponds to the electric potential energy stored within a capacitor. It is a typical approximation once the PZT behaves equivalent to a capacitive element (Liang; Sun; Rogers, 1996; Giurgiutiu; Zagrai, 2000; Baptista; Vieira Filho, 2010).

Using the above equations and substituting $\epsilon_{1L} = u_L'(x, t)$, Eq. (3.7) is rewritten as

$$\pi_{p_L} = \frac{1}{2} \int_{x_1}^{x_2} (I_{1L} u_L'(x, t)^2 - I_{2L} E_3 u_L'(x, t) - I_3 E_3^2) dx, \quad (3.8)$$

where $I_{1L} = c b_p h_p$, $I_{2L} = 2d b_p h_p$ and $I_3 = e_p b_p h_p$.

The total kinetic T_L and strain π_L energies of the system are respectively given by $T_L = T_{b_L} + T_{p_L}$ and $\pi_L = \pi_{b_L} + \pi_{p_L}$. The Rayleigh-Ritz method is employed to discretize the resulting system, such that

$$u_L(x, t) = \sum_{i=1}^{n_L} \psi_{i_L}(x) q_{i_L}(t), \quad (3.9)$$

where n_L is the number of longitudinal modes, $\psi_{i_L}(x)$ represents the i -th longitudinal modal shape function, which fulfills the specified boundary conditions, and $q_{i_L}(t)$ is the associated modal coordinate. Then, the resulting energies are given by

$$T_L = \sum_{i=1}^{n_L} \sum_{j=1}^{n_L} \frac{1}{2} \int_0^{L_b} \tilde{\rho}_b A_b \psi_{i_L}(x) \psi_{j_L}(x) dx \dot{q}_{i_L}(t) \dot{q}_{j_L}(t) + \sum_{i=1}^{n_L} \sum_{j=1}^{n_L} \frac{1}{2} \int_{x_1}^{x_2} \rho_p A_p \psi_{i_L}(x) \psi_{j_L}(x) dx \dot{q}_{i_L}(t) \dot{q}_{j_L}(t), \quad (3.10)$$

$$\begin{aligned} \pi_L = & \sum_{i=1}^{n_L} \sum_{j=1}^{n_L} \frac{1}{2} \int_0^{L_b} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx q_{i_L}(t) q_{j_L}(t) \\ & + \sum_{i=1}^{n_L} \sum_{j=1}^{n_L} \frac{1}{2} \int_{x_1}^{x_2} I_{i_L} \psi'_{i_L}(x) \psi'_{j_L}(x) dx q_{i_L}(t) q_{j_L}(t) \\ & - \sum_{i=1}^{n_L} \frac{1}{2} \int_{x_1}^{x_2} I_{2_L} E_3 \psi'_{i_L}(x) dx q_{i_L}(t) - \frac{1}{2} I_3 E_3^2 L_p. \end{aligned} \quad (3.11)$$

The Lagrange equation (Eq. 3.12) is applied to the case without external forces to obtain the system's motion equation, given by Eq. (3.14).

$$\frac{d}{dt} \left(\frac{\partial T_L}{\partial \dot{r}_i} \right) + \frac{\partial \pi_L}{\partial r_i} = 0. \quad (3.12)$$

Setting $r_i = q_{i_L}$, the following equation is obtained

$$\begin{aligned} & \sum_{j=1}^{n_L} \int_0^{L_b} \tilde{\rho}_b A_b \psi_{i_L}(x) \psi_{j_L}(x) dx \ddot{q}_{j_L}(t) \\ & + \sum_{j=1}^{n_L} \int_{x_1}^{x_2} \rho_p A_p \psi_{i_L}(x) \psi_{j_L}(x) dx \ddot{q}_{j_L}(t) \\ & + \sum_{j=1}^{n_L} \int_0^{L_b} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx q_{j_L}(t) \\ & + \sum_{j=1}^{n_L} \int_{x_1}^{x_2} I_{1_L} \psi'_{i_L}(x) \psi'_{j_L}(x) dx q_{j_L}(t) \\ & - \int_{x_1}^{x_2} I_{2_L} E_3 \psi'_{i_L}(x) dx = 0, \end{aligned} \quad (3.13)$$

where $\ddot{(\cdot)}$ indicates the second time derivative.

Equation (3.13) can be rewritten as

$$\mathbf{M}_L \ddot{\mathbf{q}}_L(t) + \mathbf{K}_L \mathbf{q}_L(t) = \mathbf{F}_L E_3, \quad (3.14)$$

where $\mathbf{M}_L$ and $\mathbf{K}_L$ represent the total mass and stiffness matrices, respectively, obtained by summing the individual contributions from the beam and PZT components, such that

$$\mathbf{M}_L = \mathbf{M}_{b_L} + \mathbf{M}_{p_L}, \quad (3.15)$$

$$\mathbf{K}_L = \mathbf{K}_{b_L} + \mathbf{K}_{p_L}, \quad (3.16)$$

in which $\mathbf{M}_{b_L}$ and $\mathbf{M}_{p_L}$ are the mass matrices of the beam and the PZT, respectively; and, similarly, $\mathbf{K}_{b_L}$ and $\mathbf{K}_{p_L}$ are their respective stiffness matrices. The analytical expressions used to calculate the elements corresponding to the i -th row and j -th column of the mass and stiffness matrices are presented in Eqs. (3.17) and (3.18), respectively.

$$\begin{aligned} m_{b_L}^{(i,j)} &= \int_0^{L_b} \tilde{\rho}_b A_b \psi_{i_L}(x) \psi_{j_L}(x) dx; \\ m_{p_L}^{(i,j)} &= 2 \int_{x_1}^{x_2} \rho_p A_p \psi_{i_L}(x) \psi_{j_L}(x) dx. \end{aligned} \quad (3.17)$$

$$\begin{aligned} k_{b_L}^{(i,j)} &= \int_0^{L_b} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx; \\ k_{p_L}^{(i,j)} &= 2 \int_{x_1}^{x_2} I_{1_L} \psi'_{i_L}(x) \psi'_{j_L}(x) dx. \end{aligned} \quad (3.18)$$

Note that if the PZT symmetric configuration is adopted (i.e., two PZTs symmetrically placed on the beam), the mass and stiffness matrices pertaining to the structure remain unchanged. However, the PZT-related matrices, along with the vector of forces generated by these devices (see Eq. 3.19), are changed, leading to a duplication of the original values of these tensors. This adjustment reflects the combined contribution of each PZT to the system's dynamic response.

The i -th element of the force vector $\mathbf{F}_L$ generated by the PZTs is given by

$$F_L^{(i)} = 2 \int_{x_1}^{x_2} I_{2_L} \psi'_{i_L}(x) dx. \quad (3.19)$$

The vector $\mathbf{q}_L(t) = [q_{1_L} \ q_{2_L} \ \dots \ q_{n_L}(t)]^T$ is rewritten as the product of the system's generalized coordinates $\mathbf{g}_L(t)$ and the eigenvector matrix Φ_L for zero electric field, such that Eq. (3.14) becomes

$$\mathbf{M}_{gg_L} \ddot{\mathbf{g}}_L(t) + \mathbf{K}_{gg_L} \mathbf{g}_L(t) = \Phi_L^T \mathbf{F}_L E_3, \quad (3.20)$$

where $\mathbf{M}_{gg_L} = \Phi_L^T \mathbf{M}_L \Phi_L$ and $\mathbf{K}_{gg_L} = \Phi_L^T \mathbf{K}_L \Phi_L$.

The structural damping matrix $\mathbf{C}_{gg_L}$ is introduced in Eq. (3.20) by using the Rayleigh proportional damping as:

$$\mathbf{C}_{gg_L} = \alpha_L \mathbf{M}_{gg_L} + \beta_L \mathbf{K}_{gg_L}, \quad (3.21)$$

where α_L and β_L are coefficients determined as introduced in the Section 3.5.

The electric field E_3 is assumed to be uniform over the PZT and expressed in terms of the electric voltage V_3 , such that $E_3 = V_3/h_p$. Thus,

$$\mathbf{M}_{gg_L} \ddot{\mathbf{g}}_L(t) + \mathbf{C}_{gg_L} \dot{\mathbf{g}}_L(t) + \mathbf{K}_{gg_L} \mathbf{g}_L(t) = \mathbf{F}_{g_L} V_3, \quad (3.22)$$

where $\mathbf{F}_{g_L} = \Phi_L^T \mathbf{F}_L / h_p$. By setting a harmonic voltage $V_3 = V_0 e^{j\omega t}$ and $\mathbf{g}_L(t) = g_{0_L} e^{j\omega t}$ into Eq. (3.22), it is obtained that

$$\mathbf{g}_{0_L} = \left(-\mathbf{M}_{gg_L} \omega^2 + \mathbf{C}_{gg_L} j\omega + \mathbf{K}_{gg_L} \right)^{-1} \mathbf{F}_{g_L} V_0. \quad (3.23)$$

Thus, the modal function vector can be computed as:

$$\mathbf{q}_{0_L} = \Phi_L \left(-\mathbf{M}_{gg_L} \omega^2 + \mathbf{C}_{gg_L} j\omega + \mathbf{K}_{gg_L} \right)^{-1} \mathbf{F}_{g_L} V_0. \quad (3.24)$$

The electrical response for the symmetrically placed PZTs (named herein symmetric configuration) is obtained from the electric charge $Q(t)$, which is determined from the electric displacement D_3 . Since this configuration involves two capacitors-like transducers (i.e., the PZTs) connected in parallel, the resulting electric charge is doubled. Then, $Q(t) = 2 \int_{A_p} D_3 dA_p$, such that

$$Q(t) = 2 \int_{x_1}^{x_2} \int_0^{b_p} (d\epsilon_{1L} + e_p E_3) dx dy. \quad (3.25)$$

Replacing $\epsilon_{1L} = \epsilon_{xx_L} = u'_L(x, t)$:

$$Q(t) = 2 \left[\int_{x_1}^{x_2} db_p u'_L(x, t) dx + e_p E_3 b_p L_p \right]. \quad (3.26)$$

Applying the Rayleigh-Ritz method, and substituting the result from Eq. (3.24), it is obtained this following equation

$$Q(t) = 2 \left[db_p \left(\int_{x_1}^{x_2} \Psi'_L(x) dx \right) \mathbf{q}_L(t) + e_p E_3 b_p L_p \right], \quad (3.27)$$

$$Q(t) = 2V_0 e^{j\omega t} \left[db_p \Psi_L(x) \Big|_{x_1}^{x_2} \Phi_L \left(-\mathbf{M}_{gg_L} \omega^2 + \mathbf{C}_{gg_L} j\omega + \mathbf{K}_{gg_L} \right)^{-1} \mathbf{F}_{g_L} + \frac{e_p b_p L_p}{h_p} \right], \quad (3.28)$$

where $\Psi_L(x) = [\psi_{1_L}(x) \ \psi_{2_L}(x) \ \dots \ \psi_{n_L}(x)]$ is the longitudinal modal shape matrix, obtained for n_L modes.

Differentiating $Q(t)$ with respect to time, it is obtained the electric current $I(t) = j\omega Q(t)$. Thus, the electromechanical impedance Z_{em_L} for the system's longitudinal dynamics is given by

$$Z_{em_L}(\omega) = \frac{V_3(t)}{I(t)} = \left\{ 2j\omega \left[db_p \Psi_L(x) \Big|_{x_1}^{x_2} \Phi_L \left(-\mathbf{M}_{gg_L} \omega^2 + \mathbf{C}_{gg_L} j\omega + \mathbf{K}_{gg_L} \right)^{-1} \mathbf{F}_{g_L} + \frac{e_p b_p L_p}{h_p} \right] \right\}^{-1}. \quad (3.29)$$

3.2. EMI FOR FLEXURAL DYNAMICS

The EMI for a beam flexural dynamics is proposed by Sepehry, Shamshirsaz and Bastani (2010), considering the Euler-Bernoulli beam theory. The displacements and mechanical strains for flexural dynamics are equivalent to $u_F(x, t) = -zw'(x, t)$, $v_F(x, t) = 0$

and $w_F(x, t) = w(x, t)$. In addition,

$$\begin{aligned}\epsilon_{xx_F} &= -zw_F''(x, t); \\ \epsilon_{xy_F} &= \epsilon_{yy_F} = \epsilon_{zz_F} = \epsilon_{yz_F} = \epsilon_{xz_F} = 0,\end{aligned}\tag{3.30}$$

where the subscript F denotes the flexural dynamics and $()''$ indicates the second spatial derivative. The strain π_{b_F} and kinetic T_{p_F} energies of the beam are given by

$$\pi_{b_F} = \frac{1}{2} \int_0^{L_b} \tilde{E}_b I_b w_F''(x, t)^2 dx,\tag{3.31}$$

$$T_{b_F} = \frac{1}{2} \int_0^{L_b} \tilde{\rho}_b A_b \dot{w}_F(x, t)^2 dx,\tag{3.32}$$

where I_b is the beam's area moment of inertia.

The kinetic T_{p_L} and strain π_{p_L} energies of the PZT considering the flexural dynamics are given by

$$T_{p_F} = \frac{1}{2} \int_{x_1}^{x_2} \rho_p A_p \dot{w}_F(x, t)^2 dx,\tag{3.33}$$

$$\pi_{p_F} = \frac{1}{2} \int_{x_1}^{x_2} (I_{1_F} w_F''(x, t)^2 + I_{2_F} E_3 w_F''(x, t) - I_3 E_3^2) dx,\tag{3.34}$$

where $I_{1_F} = cb_p \frac{z^3}{3} \Big|_{h_p}^{h_b/2+h_p}$ and $I_{2_F} = db_p z^2 \Big|_{h_p}^{h_b/2+h_p}$, in which b_b is width of the beam.

Similarly to longitudinal dynamics, the kinetic T_F and strain π_F energies of the system are obtained by summing the energies of the individual components, i.e., $T_F = T_{b_F} + T_{p_F}$ and $\pi_F = \pi_{b_F} + \pi_{p_F}$. The Rayleigh-Ritz method is once more used to discretize the system, such that $w_F(x, t) = \sum_{i=1}^{n_F} \psi_{i_F}(x) q_{i_F}(t)$, where n_F is the number of flexural modes, and $\psi_{i_F}(x)$ and $q_{i_F}(t)$ denote the i -th flexural mode shape and modal coordinate, respectively. Lagrange's equation is applied to the case without external forces, deriving the system's dynamics in the context of flexural motion, as presented in the equations below.

$$\begin{aligned}& \sum_{j=1}^{n_F} \int_0^{L_b} \tilde{\rho}_b A_b \psi_{i_F}(x) \psi_{j_F}(x) dx (t) \ddot{q}_{j_F}(t) \\ & + \sum_{j=1}^{n_F} \int_{x_1}^{x_2} \rho_p A_p \psi_{i_F}(x) \psi_{j_F}(x) dx \ddot{q}_{j_F}(t) \\ & + \sum_{j=1}^{n_F} \int_0^{L_b} \tilde{E}_b I_b \psi_{i_F}''(x) \psi_{j_F}''(x) dx q_{j_F}(t) \\ & + \sum_{j=1}^{n_F} \int_{x_1}^{x_2} I_{1_F} \psi_{i_F}''(x) \psi_{j_F}''(x) dx q_{j_F}(t) \\ & + \int_{x_1}^{x_2} I_{2_F} E_3 \psi_{i_F}''(x) dx = 0,\end{aligned}\tag{3.35}$$

or, alternatively, in matrix form,

$$\mathbf{M}_F \ddot{\mathbf{q}}_F(t) + \mathbf{K}_F \mathbf{q}_F(t) = \mathbf{N}_F E_3. \quad (3.36)$$

The elements of the mass and stiffness matrices for both beam and PZT, along with the moment vector $\mathbf{N}_F$ generated by the PZTs, are given by

$$\begin{aligned} m_{b_F}^{(i,j)} &= \int_0^{L_b} \tilde{\rho}_b A_b \psi_{i_F}(x) \psi_{j_F}(x) dx; \\ m_{p_F}^{(i,j)} &= 2 \int_{x_1}^{x_2} \rho_p A_p \psi_{i_F}(x) \psi_{j_F}(x) dx, \end{aligned} \quad (3.37)$$

$$\begin{aligned} k_{b_F}^{(i,j)} &= \int_0^{L_b} \tilde{E}_b I_b \psi_{i_F}''(x) \psi_{j_F}''(x) dx; \\ k_{p_F}^{(i,j)} &= 2 \int_{x_1}^{x_2} I_{1_F} \psi_{i_F}''(x) \psi_{j_F}''(x) dx, \end{aligned} \quad (3.38)$$

$$N_F^{(i)} = -2 \int_{x_1}^{x_2} I_{2_F} \psi_{i_F}''(x) dx. \quad (3.39)$$

Following the procedures outlined in the longitudinal dynamics, the modal function vector for flexural motion is derived as

$$\mathbf{q}_{0_F} = \Phi_F \left(-\mathbf{M}_{gg_F} \omega^2 + \mathbf{C}_{gg_F} j\omega + \mathbf{K}_{gg_F} \right)^{-1} \mathbf{N}_{g_F} V_0, \quad (3.40)$$

where $\mathbf{M}_{gg_F} = \Phi_F^T \mathbf{M}_F \Phi_F$, $\mathbf{K}_{gg_F} = \Phi_F^T \mathbf{K}_F \Phi_F$, $\mathbf{C}_{gg_F} = \alpha_F \mathbf{M}_{gg_F} + \beta_F \mathbf{K}_{gg_F}$ and $\mathbf{N}_{g_F} = \Phi_F^T \mathbf{N}_F / h_p$.

Then, the electric charge $Q(t)$ of the PZTs in this case is equivalent to

$$Q(t) = 2 \int_{x_1}^{x_2} \int_0^{b_p} (d\epsilon_{1_F} + e_p E_3) dx dy. \quad (3.41)$$

Defining $\epsilon_{1_F} = \epsilon_{xx_F} = -zw_F''(x, t)$ and $z = h_b/2$, it is obtained these following equations

$$Q(t) = 2 \left[-db_p \left(\frac{h_b}{2} \right) \Psi_F'(x) \Big|_{x_1}^{x_2} \mathbf{q}_F(t) + e_p E_3 b_p L_p \right]. \quad (3.42)$$

$$Q(t) = 2V_0 e^{j\omega t} \left[-db_p \left(\frac{h_b}{2} \right) \Psi_F'(x) \Big|_{x_1}^{x_2} \Phi_F \left(-\mathbf{M}_{gg_F} \omega^2 + \mathbf{C}_{gg_F} j\omega + \mathbf{K}_{gg_F} \right)^{-1} \mathbf{N}_{g_F} + \frac{e_p b_p L_p}{h_p} \right], \quad (3.43)$$

where $\Psi_F(x) = [\psi_{1_F}(x) \ \psi_{2_F}(x) \ \dots \ \psi_{n_F}(x)]$ is the flexural modal shape matrix, obtained for n_F modes. Then, the electromechanical impedance Z_{em_F} for the system's flexural dynamics is given by

$$Z_{em_F}(\omega) = \left\{ 2j\omega \left[-db_p \left(\frac{h_b}{2} \right) \Psi_F'(x) \Big|_{x_1}^{x_2} \Phi_F \left(-\mathbf{M}_{gg_F} \omega^2 + \mathbf{C}_{gg_F} j\omega + \mathbf{K}_{gg_F} \right)^{-1} \mathbf{N}_{g_F} + \frac{e_p b_p L_p}{h_p} \right] \right\}^{-1}. \quad (3.44)$$

3.3. COUPLED DYNAMICS - ASYMMETRIC CONFIGURATION

Consider a single transducer placed on the beam, as shown in Fig. 8(a). It is an asymmetric configuration, in terms of the PZT position in relation to the beam longitudinal direction. In practice, with this arrangement, the system exhibits a coupled dynamics, i.e., both longitudinal and flexural. Then, the displacements are given by

$$\begin{aligned} u_C(x, t) &= u_L(x, t) - zw'_F(x, t); \\ v_C(x, t) &= 0; \quad w_C(x, t) = w_F(x, t), \end{aligned} \quad (3.45)$$

where the subscript C denotes the coupled dynamics. The mechanical strains are defined as

$$\begin{aligned} \epsilon_{xx_C} &= u'_L(x, t) - zw''_F(x, t); \\ \epsilon_{xy_C} &= \epsilon_{yy_C} = \epsilon_{zz_C} = \epsilon_{yz_C} = \epsilon_{xz_C} = 0. \end{aligned} \quad (3.46)$$

So, the kinetic T_{b_C} and strain π_{b_C} energies of the beam are equivalent to

$$\begin{aligned} T_{b_C} &= \frac{1}{2} \int_0^{L_b} \tilde{\rho}_b A_b [\dot{u}_L(x, t)^2 + \dot{w}_F(x, t)^2] dx \\ &= T_{b_L} + T_{b_F}, \end{aligned} \quad (3.47)$$

$$\begin{aligned} \pi_{b_C} &= \frac{1}{2} \int_{V_b} \tilde{E}_b \epsilon_{xx_C}^2 dV_b = \frac{1}{2} \int_{V_b} \tilde{E}_b (u'_L - zw''_F)^2 dV_b \\ &= \frac{1}{2} \int_0^{L_b} [\tilde{E}_b A_b u'_L(x, t)^2 + \tilde{E}_b I_b w''_F(x, t)^2] dx \\ &= \pi_{b_L} + \pi_{b_F}. \end{aligned} \quad (3.48)$$

Considering $\epsilon_{1_C} = \epsilon_{xx_C}$, the kinetic T_{p_C} and strain π_{p_C} energies of the PZT for the coupled dynamics are given by

$$\begin{aligned} T_{p_C} &= \frac{1}{2} \int_{x_1}^{x_2} \rho_p A_p [\dot{u}_L(x, t)^2 + \dot{w}_F(x, t)^2] dx \\ &= T_{p_L} + T_{p_F}, \end{aligned} \quad (3.49)$$

$$\pi_{p_C} = \frac{1}{2} \int_{x_1}^{x_2} \left(I_{1_F} w''_F{}^2 + I_{1_L} u'_L{}^2 - I_4 w''_F u'_L + I_{2_F} E_3 w'' - I_{2_L} E_3 u'_L - I_3 E_3 \right) dx, \quad (3.50)$$

where $I_4 = cb_p z^2 \Big|_{h_p}^{h_b/2+h_p}$. The total kinetic and strain energies of the system are obtained by summing the corresponding energies of each component, such that $T_C = T_{b_C} + T_{p_C}$ and $\pi_C = \pi_{b_C} + \pi_{p_C}$. Employing the discretizations approach, and applying Lagrange's equation (Eq. 3.12), with $r_i = q_{i_L}$ and $r_i = q_{i_F}$, the equations of motion for the coupled dynamics are given by

$$\begin{aligned} \mathbf{M}_L \ddot{\mathbf{q}}_L + \mathbf{K}_L \mathbf{q}_L - \mathbf{K}_A \mathbf{q}_F &= \mathbf{F}_L E_3; \\ \mathbf{M}_F \ddot{\mathbf{q}}_F + \mathbf{K}_F \mathbf{q}_F - \mathbf{K}_A \mathbf{q}_L &= \mathbf{N}_F E_3. \end{aligned} \quad (3.51)$$

This equation above can be rearranged in matrix notation, and the following equation is obtained

$$\begin{bmatrix} \mathbf{M}_L & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_F \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_L \\ \ddot{\mathbf{q}}_F \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_L & -\mathbf{K}_A \\ -\mathbf{K}_A^T & \mathbf{K}_F \end{bmatrix} \begin{Bmatrix} \mathbf{q}_L \\ \mathbf{q}_F \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_L \\ \mathbf{N}_F \end{Bmatrix} E_3, \quad (3.52)$$

or, alternatively, in a compact form, $\mathbf{M}_C \ddot{\mathbf{q}}_C(t) + \mathbf{K}_C \mathbf{q}_C(t) = \mathbf{F}_C E_3$. $\mathbf{K}_A$ is the longitudinal-flexural coupling matrix, and each element $k_a^{(i,j)}$ in its i -th row and j -th column is given by this following equation

$$k_a^{(i,j)} = \int_{x_1}^{x_2} I_4 \psi'_{i_L}(x) \psi''_{j_F}(x) dx. \quad (3.53)$$

Note that the total mass and stiffness for the asymmetric configuration are determined by the individual contributions of each dynamics, resulting in a combination of the matrices corresponding to each type of motion. However, due to the use of a single PZT, the tensors – mass, stiffness and loads generated by the PZT – that describe the properties of this component are no longer duplicated, as presented in the previous sections corresponding to the longitudinal and flexural dynamics.

By following the procedures presented for the longitudinal dynamics, the modal function vector for this coupled dynamics is given by

$$\mathbf{q}_{0_C} = \Phi_C \left(-\mathbf{M}_{gg_C} \omega^2 + \mathbf{C}_{gg_C} j\omega + \mathbf{K}_{gg_C} \right)^{-1} \mathbf{F}_{g_C} V_0, \quad (3.54)$$

where $\mathbf{M}_{gg_C} = \Phi_C^T \mathbf{M}_C \Phi_C$, $\mathbf{K}_{gg_C} = \Phi_C^T \mathbf{K}_C \Phi_C$ and $\mathbf{F}_{g_C} = \Phi_C^T \mathbf{F}_C / h_p$. The structural damping matrix $\mathbf{C}_{gg_C}$ is given by

$$\mathbf{C}_{gg_C} = \alpha \otimes \mathbf{I} \begin{bmatrix} \mathbf{M}_{gg_L} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{gg_F} \end{bmatrix} + \beta \otimes \mathbf{I} \begin{bmatrix} \mathbf{K}_{gg_L} & -\mathbf{K}_A \\ -\mathbf{K}_A^T & \mathbf{K}_{gg_F} \end{bmatrix}, \quad (3.55)$$

where the superscript $()^T$ denotes the transpose of a matrix, α and β are the following 2×2 matrices, $\otimes$ denotes the Kronicher product and $\mathbf{I}$ is a $(n_L + n_F)$ identity matrix defined below, in which $\mathbf{0}_{()}$ are matrices of zeros.

$$\alpha = \begin{bmatrix} \alpha_L & 0 \\ 0 & \alpha_F \end{bmatrix}, \quad (3.56)$$

$$\beta = \begin{bmatrix} \beta_L & 0 \\ 0 & \beta_F \end{bmatrix}, \quad (3.57)$$

$$\mathbf{I} = \begin{bmatrix} \mathbf{I}_{n_L} & \mathbf{0}_{n_L \times n_F} \\ \mathbf{0}_{n_F \times n_L} & \mathbf{I}_{n_F} \end{bmatrix}. \quad (3.58)$$

The electric charge $Q(t)$ for this PZT asymmetric configuration is given by $Q(t) = \int_{A_p} D_3 dA_p$, such that

$$Q(t) = \int_{x_1}^{x_2} db_p \Psi_C(x) \mathbf{q}_C(t) + e_p E_3 b_p L_p, \quad (3.59)$$

where

$$\Psi_C(x) = \begin{Bmatrix} \Psi_L(x) \\ -\frac{h_b}{2}\Psi'_F(x) \end{Bmatrix}. \quad (3.60)$$

Thus, the electromechanical impedance Z_{em_C} for the system's coupled dynamics is given by

$$Z_{em_C}(\omega) = \left\{ j\omega \left[db_p \Psi_C(x) \Big|_{x_1}^{x_2} \Phi_C \left(-\mathbf{M}_{gg_C} \omega^2 + \mathbf{C}_{gg_C} j\omega + \mathbf{K}_{gg_C} \right)^{-1} \mathbf{F}_{g_C} + \frac{e_p b_p L_p}{h_p} \right] \right\}^{-1}. \quad (3.61)$$

3.4. MODAL SHAPE FUNCTIONS

The investigations are focused on an Euler-Bernoulli beam under the free-free boundary condition. The expressions for calculating the longitudinal and flexural modal shape functions are presented below, and they are respectively given by Giurgiutiu (2007) and Gonçalves, Peplow and Brennan (2018).

- Longitudinal Dynamics

$$\psi_{n_L}(x) = \cos(k_{n_L}x), \quad (3.62)$$

where

$$k_{n_L} = \frac{n}{L_b} \left(\pi - \frac{\pi}{4n} \right), \quad (3.63)$$

in which k_{n_L} is the longitudinal wavenumber related to the natural frequencies of the beam. A phase shift of $\frac{\pi}{4L_b}$ is applied (see Eq. 3.63) to achieve better correlation with experimental data for both longitudinal and coupled dynamics.

- Flexural Dynamics

$$\begin{aligned} \psi_{n_F}(x) = & \left[\frac{-e^{(k_{n_F}(x-2L_b))} + [\cos(k_{n_F}L_b) - \sin(k_{n_F}L_b)]e^{(k_{n_F}(x-L_b))}}{1 - e^{(-2k_{n_F}L_b)} - 2\sin(k_{n_F}L_b)e^{(-k_{n_F}L_b)}} \right] \\ & + \frac{1}{2}e^{(-k_{n_F}x)}(1 + \eta_n) + \cos(k_{n_F}x) - \eta_n \sin(k_{n_F}x), \end{aligned} \quad (3.64)$$

where

$$\eta_n = \frac{1 + e^{(-2k_{n_F}L_b)} - 2e^{(-k_{n_F}L_b)} \cos(k_{n_F}L_b)}{1 - e^{(-2k_{n_F}L_b)} - 2e^{(-k_{n_F}L_b)} \sin(k_{n_F}L_b)}, \quad (3.65)$$

Table 4 – Flexural wavenumbers k_{n_F} .

n	$k_{n_F} L_b$
1	4.73004
2	7.85320
3	10.9956
4	13.3518
5	16.4934
$n > 5$	$\frac{(2n + 1)\pi}{2}$

Source: Elaborated by the author.

in which k_{n_F} represents the flexural wavenumber of the beam's natural frequencies, calculated as shown in Tab. 4.

These expressions for flexural dynamics are applicable when the Euler-Bernoulli beam theory is valid, that is, when the wavelength of a flexural wave is approximately ten times greater than the beam thickness (Gebhart, 1969; Gonçalves; Peplow; Brennan, 2018). Consequently, these expressions remain valid over a wide frequency range, including high frequencies, as long as the assumptions of the Euler-Bernoulli theory are satisfied, regardless of the number of modes required. This is particularly advantageous, as the EMI technique operates at high frequencies. For the beam analyzed in this thesis, the Euler-Bernoulli model is valid up to 30 kHz.

By examining the expressions for the natural wavenumbers $k_{n_{()}}$ in Eq. (3.63) and Tab. 4, alongside the definitions of the longitudinal and flexural wavenumbers, given by $k_L = \omega (\rho_b/E_b)^{1/2}$ and $k_F = (\rho_b A_b/E_b I_b)^{1/4} \omega^{1/2}$, respectively, one can determine the number of modes required to accurately represent the entire frequency range of interest. The theoretical investigations conducted in this thesis required 14 and 35 structural modes to represent the longitudinal and flexural dynamics, respectively, considering a maximum frequency of 30 kHz.

3.5. ADJUSTMENT OF EMI PEAKS: THEORETICAL VS. EXPERIMENTAL RESULTS

The adjustment of EMI peaks between theoretical and experimental results can be done by computing the $\alpha_{()}$ and $\beta_{()}$ coefficients for both longitudinal and flexural dynamics. This strategy involves to determine the values of these coefficients to obtain a good fit between the theoretical prediction and experimental data. The proposed approach involves the Nelder-Mead numerical method to locate the minimum of an objective function F . It is defined as the absolute difference between the corresponding electromechanical impedance at the highest experimentally and theoretically measured peak, obtained at a reference temperature

(24°C, in this case), such as represented in Eq. (3.66).

$$F = |\max[Z_{em_{\text{exp}}}(\omega_i)] - \max[Z_{em_{\text{theo}}}(\omega_i)]|, \quad (3.66)$$

where $i = f_{\min}, f_{\min} + 1, \dots, f_{\max}$, $\max[\]$ denotes the maximum value, $Z_{em_{\text{exp}}}$ and $Z_{em_{\text{theo}}}$ represent the experimental and theoretical electromechanical impedance, respectively, and $f_{\min}$ and $f_{\max}$ correspond to the minimum and maximum frequencies for a range of interest.

The approach presented above is not necessarily recommended for applications involving a wide frequency range, as it hinders precise representation of the structure's damping and may result in underdamping or overdamping states. However, it is easily adapted to compute frequency-dependent coefficients. The wide frequency range can be divided by a desired number of sub-intervals, and F is replaced by F_{sub} , such as defined in Eq. (3.67). Polynomial functions are subsequently fitted to these values to describe the frequency-dependent coefficients, enabling their application across broader operational ranges. The computed coefficients are then used herein to adjust the EMI peaks for the coupled dynamics (see Eq. 3.55).

$$F_{\text{sub}} = \max \left[Z_{em_{\text{exp}}} \left(f_{\text{pk}_{i+1}} \right) - Z_{em_{\text{exp}}} \left(f_{\text{pk}_i} \right) \right], \quad (3.67)$$

where $i = 1, 2, \dots, (N_{\text{pk}} - 1)$, N_{pk} is the number of peaks within the desired frequency range and f_{pk_i} refers to the frequency corresponding to the i -th peak. See complementary information in Appendix A.

3.6. STRUCTURAL DAMAGE

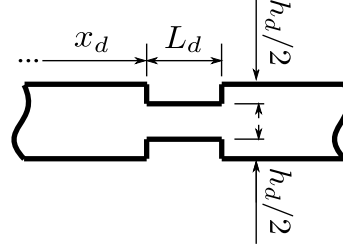
Structural damage is represented by a localized reduction in the beam's cross-sectional area. Two damage configurations are considered: symmetric and asymmetric. This first one considers a reduction equally distributed relative to the neutral axis of the beam, whereas in the latter, the reduction is asymmetric. These configurations are illustrated in Figs. 9 and 10, respectively. The damaged regions are characterized by a horizontal length L_d and a local thickness reduction $0 < h_d < h_b$, where h_b is the beam nominal thickness. The physical properties of the damaged sections, such as Young's modulus and material density, remain identical to those of the undamaged regions. These structural modifications do not affect the PZT-related tensors; however, the beam's mass and stiffness matrices are adjusted according to the equations detailed in the following sections, specific to each damage configuration. For further details regarding the derivation of these equations, see Appendix B.

3.6.1 Symmetric Damage

A locally symmetric reduction of the beam cross-section area has been used by different authors to represent a type of damage (Shen; Pierre, 1990; Lopes; Gonzalez-Bueno; Bueno, 2022; Lopes *et al.*, 2024). Although simpler and less common in practical

applications, Lopes *et al.* (2022) demonstrate that the dynamics of a small asymmetric damage can be predicted by analyzing the symmetric case, mainly in terms of frequencies at which it can be detected from the system's response. The presence of this type of damage changes the beam's matrices as follows:

Figure 9 – Schematic illustration of a symmetric cross-sectional area reduction h_d .



Source: Elaborated by the author.

$$m_{b(\cdot)}^{(i,j)} = \int_0^{x_d} \tilde{\rho}_b A_b \psi_{i(\cdot)}(x) \psi_{j(\cdot)}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{\rho}_b A_d \psi_{i(\cdot)}(x) \psi_{j(\cdot)}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{\rho}_b A_b \psi_{i(\cdot)}(x) \psi_{j(\cdot)}(x) dx, \quad (3.68)$$

$$k_{b_L}^{(i,j)} = \int_0^{x_d} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{E}_b A_d \psi'_{i_L}(x) \psi'_{j_L}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx;$$

$$k_{b_F}^{(i,j)} = \int_0^{x_d} \tilde{E}_b I_b \psi''_{i_F}(x) \psi''_{j_F}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{E}_b I_d \psi''_{i_F}(x) \psi''_{j_F}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{E}_b I_b \psi''_{i_F}(x) \psi''_{j_F}(x) dx, \quad (3.69)$$

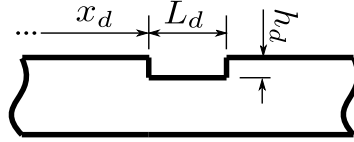
where x_d represents the initial position of the damage, L_d is its length, h_d is its thickness reduction, A_d is the respective cross-sectional area, defined as $A_d = b_b(h_b - h_d)$, and I_d is the area moment of inertia of the damaged region, calculated as: $I_d = \frac{b(h_b - h_d)^3}{12}$. The subscript d indicates the damage and $(\cdot)$ indicates the corresponding dynamics, i.e., longitudinal, L , or flexural, F .

3.6.2 Asymmetric Damage

A typical damage is characterized by asymmetric changes in the structure's geometric properties (Chen; Byüköztürk, 2017; Lopes *et al.*, 2024). Damage of this nature causes scattering of both longitudinal and flexural waves, which occurs due to mode conversion at damage sites, even when only one type of wave is initially present. Therefore, the asymmetry in a damaged section leads to coupling between longitudinal and flexural wave motion, as a consequence of the distance between the neutral axis of the damaged section and the adjacent regions (Alleyne; Cawley, 1992; Gonzalez-Bueno, 2019). Due to these distinctive

characteristics and their greater prevalence in practical applications, this asymmetric reduction of the beam's cross-section area has been employed by different authors to represent a structural damage (Chen; Byüköztürk, 2017; Gonzalez-Bueno, 2019; Lopes *et al.*, 2022; Vandecruys *et al.*, 2023). This damage configuration changes the motion equations governing the longitudinal and flexural dynamics of the system as follows:

Figure 10 – Schematic illustration of an asymmetric cross-sectional area reduction h_d .



Source: Elaborated by the author.

- Longitudinal Dynamics

$$\begin{bmatrix} \mathbf{M}_L & (\mathbf{M}_{A_d} + \mathbf{M}_{A_{d_2}}) \\ (\mathbf{M}_{A_d} + \mathbf{M}_{A_{d_2}})^T & (\mathbf{M}_F + \mathbf{M}_{F_d} + \mathbf{M}_{F_{d_2}} + \mathbf{M}_{F_{d_3}}) \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_L \\ \ddot{\mathbf{q}}_F \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_L & (\mathbf{K}_{A_d} + \mathbf{K}_{A_{d_2}}) \\ (\mathbf{K}_{A_d} + \mathbf{K}_{A_{d_2}})^T & (\mathbf{K}_F + \mathbf{K}_{F_d} + \mathbf{K}_{F_{d_2}}) \end{bmatrix} \begin{Bmatrix} \mathbf{q}_L \\ \mathbf{q}_F \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_L \\ \mathbf{0} \end{Bmatrix} E_3. \quad (3.70)$$

- Flexural Dynamics

$$\begin{bmatrix} \mathbf{M}_L & (\mathbf{M}_{A_d} + \mathbf{M}_{A_{d_2}}) \\ (\mathbf{M}_{A_d} + \mathbf{M}_{A_{d_2}})^T & (\mathbf{M}_F + \mathbf{M}_{F_d} + \mathbf{M}_{F_{d_2}} + \mathbf{M}_{F_{d_3}}) \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_L \\ \ddot{\mathbf{q}}_F \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_L & (\mathbf{K}_{A_d} + \mathbf{K}_{A_{d_2}}) \\ (\mathbf{K}_{A_d} + \mathbf{K}_{A_{d_2}})^T & (\mathbf{K}_F + \mathbf{K}_{F_d} + \mathbf{K}_{F_{d_2}}) \end{bmatrix} \begin{Bmatrix} \mathbf{q}_L \\ \mathbf{q}_F \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{N}_F \end{Bmatrix} E_3, \quad (3.71)$$

where $\mathbf{M}_L$ and $\mathbf{M}_F$ are the total mass matrices for the longitudinal and flexural dynamics, respectively, obtained by summing the mass matrices of the beam, $\mathbf{M}_{b(\cdot)}$, and the PZTs, $\mathbf{M}_{p(\cdot)}$. The latter is defined by Eqs. (3.17) and (3.37) for the longitudinal and flexural dynamics, respectively, while the elements of the beam-related matrices are given by

$$m_{b(\cdot)}^{(i,j)} = \int_0^{x_d} \tilde{\rho}_b A_b \psi_{i(\cdot)}(x) \psi_{j(\cdot)}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{\rho}_b A_d \psi_{i(\cdot)}(x) \psi_{j(\cdot)}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{\rho}_b A_b \psi_{i(\cdot)}(x) \psi_{j(\cdot)}(x) dx. \quad (3.72)$$

The coupling mass matrices $\mathbf{M}_{A_d}$ and $\mathbf{M}_{A_{d_2}}$, arising from the presence of the asymmetric damage, are defined as

$$m_{a_d}^{(i,j)} = \int_{x_d}^{x_d+L_d} 2\tilde{\rho}_b A_d d_{ad} \psi_{i_L}(x) \psi'_{j_F}(x) dx; \quad (3.73)$$

$$m_{a_{d_2}}^{(i,j)} = - \int_{x_d}^{x_d+L_d} \tilde{\rho}_b \gamma_{ad} \psi_{i_L}(x) \psi'_{j_F}(x) dx,$$

where $d_{ad} = h_d/2$ is the distance between the neutral axis of the damaged section and those of the adjacent undamaged regions and $\gamma_{ad} = b_b(h_d^2 - h_b h_d)$. The subscript ad indicates the asymmetric damage.

The matrices $\mathbf{M}_{F_d}$, $\mathbf{M}_{F_{d_2}}$, and $\mathbf{M}_{F_{d_3}}$ represent additional flexural mass effects induced by the damaged region, and their elements are defined as follows:

$$\begin{aligned}
 m_{d_F}^{(i,j)} &= \int_0^{x_d} \tilde{\rho}_b I_b \psi'_{i_F}(x) \psi'_{j_F}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{\rho}_b I_{ad} \psi'_{i_F}(x) \psi'_{j_F}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{\rho}_b I_b \psi'_{i_F}(x) \psi'_{j_F}(x) dx; \\
 m_{d_{2F}}^{(i,j)} &= \int_{x_d}^{x_d+L_d} \tilde{\rho}_b A_d d_{ad}^2 \psi'_{i_F}(x) \psi'_{j_F}(x) dx; \\
 m_{d_{3F}}^{(i,j)} &= - \int_{x_d}^{x_d+L_d} \tilde{\rho}_b \gamma_{ad} d_{ad} \psi'_{i_F}(x) \psi'_{j_F}(x) dx,
 \end{aligned} \tag{3.74}$$

where $I_{ad} = \frac{b_b(h_b - h_d)^3}{12} + b_b \left(\frac{h_d}{2} \right)^2 (h_b - h_d)$ is the second moment of area of the asymmetric damage.

Similarly, $\mathbf{K}_L$ and $\mathbf{K}_F$ denote the total stiffness matrices for the longitudinal and flexural dynamics resulting from the sum of the beam, $\mathbf{K}_{b(\cdot)}$, and PZTs, $\mathbf{K}_{p(\cdot)}$, stiffness matrices. The latter is defined by Eqs. (3.18) and (3.38) for the longitudinal and flexural dynamics, respectively, and the former is given by

$$\begin{aligned}
 k_{b_L}^{(i,j)} &= \int_0^{x_d} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{E}_b A_d \psi'_{i_L}(x) \psi'_{j_L}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx; \\
 k_{b_F}^{(i,j)} &= \int_0^{x_d} \tilde{E}_b I_b \psi''_{i_F}(x) \psi''_{j_F}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{E}_b I_{ad} \psi''_{i_F}(x) \psi''_{j_F}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{E}_b I_b \psi''_{i_F}(x) \psi''_{j_F}(x) dx.
 \end{aligned} \tag{3.75}$$

The coupling stiffness matrices $\mathbf{K}_{A_d}$ and $\mathbf{K}_{A_{d_2}}$ are given by

$$k_{a_d}^{(i,j)} = \int_{x_d}^{x_d+L_d} 2\tilde{E}_b A_d d_{ad} \psi'_{i_L}(x) \psi''_{j_F}(x) dx; \tag{3.76}$$

$$k_{a_{d_2}}^{(i,j)} = - \int_{x_d}^{x_d+L_d} \tilde{E}_b \gamma_{ad} \psi'_{i_L}(x) \psi''_{j_F}(x) dx,$$

and the stiffness matrices $\mathbf{K}_{F_d}$ and $\mathbf{K}_{F_{d_2}}$ for additional flexural behavior induced by the asymmetric damage are given by

$$k_{d_F}^{(i,j)} = \int_{x_d}^{x_d+L_d} \tilde{E}_b A_d d_{ad}^2 \psi''_{i_F}(x) \psi''_{j_F}(x) dx; \tag{3.77}$$

$$k_{d_{2F}}^{(i,j)} = - \int_{x_d}^{x_d+L_d} \tilde{E}_b \gamma_{ad} d_{ad} \psi''_{i_F}(x) \psi''_{j_F}(x) dx.$$

Note that Eqs. (3.70) and (3.71) exhibit a similar structure, differing only in the right-hand side of the equations, where the loading induced by the PZTs are introduced. These correspond exclusively to the axial forces, $\mathbf{F}_L$, in the longitudinal dynamics and to the bending moments, $\mathbf{N}_F$, in the flexural one. In the case of the coupled dynamics, both types of loading — the longitudinal forces $\mathbf{F}_L$ and the bending moments $\mathbf{N}_F$ — are simultaneously present. Additionally, it becomes necessary to include the contribution of the coupling matrix arising from the asymmetric piezoelectric configuration, $\mathbf{K}_A$, defined in Eq. (3.53).

3.7. EFFECTS DUE TO TEMPERATURE VARIATIONS

The EMI technique is a temperature-sensitive approach because the environmental temperature variations affect the properties of both the piezoelectric transducer and the monitored structure. If the SHM process compares two EMI curves — one related to a baseline/undamaged condition and the other to an unknown structural state —, and they are obtained at different temperatures, the analysis can indicate an incorrect diagnoses in terms of damage presence. Nevertheless, different works demonstrate that theoretical predictions can be sufficiently accurate by adjusting the structure and PZT physical properties to reflect temperature variations (Raghavan; Cesnik, 2008; Yang; Lim; Soh, 2008b; Dodson; Inman, 2013). In this sense, the proposed theoretical approach is evaluated across a temperature range from 24°C to 65°C.

The PSI-5H4E piezoelectric transducer, manufactured by *Piezo Systems*[®], is used to monitor the structure integrity. Information regarding its properties is obtained from the technical document provided by the piezoelectric ceramic manufacturer. These properties are investigated at five different temperatures: 24, 35, 45, 55 and 65°C. Table 5 presents the values of the dielectric constant $\tilde{\epsilon}_{33}^T$ and the piezoelectric constant $\tilde{d}_{31}$ in terms of the temperature, along with the corresponding equations that describe these variations.

Table 5 – Piezoelectric transducer PSI-5H4E properties in terms of the temperature.

Temperature [°C]	Property	
	$\tilde{d}_{31}$ [10 ⁻¹² m/V]	$\tilde{\epsilon}_{33}^T$ [10 ⁻⁸ F/m]
24	-327.18	3.248
35	-339.79	3.520
45	-351.25	3.824
55	-362.71	4.182
65	-374.17	4.594

$$\tilde{d}_{31}(^{\circ}\text{C}) = -1.1459 \times T - 299.6837$$

$$\tilde{\epsilon}_{33}^T(^{\circ}\text{C}) = (0.304 \times T^2 + 10.0408 \times T + 3.2537 \times 10^3) 8.85 \times 10^{-12}$$

Source: Elaborated by the author.

A 6063-T5 aluminum alloy beam is investigated. The Aluminum Design Manual

(ADM) (Aluminum Association - AA, 2010) is utilized to extract the Young's modulus values at different temperatures and to formulate an equation to describe these variations, whereas the reference source for the material density is Meseguer, Martínez-Conesa and Portolés (2019). The values of the Young's modulus $\tilde{E}_b$ and material density $\tilde{\rho}_b$ in terms of the temperature are presented in Tab. 6, along with the equations describing these variations.

Table 6 – 6063-T5 aluminum alloy beam properties in terms of the temperature.

Temperature [°C]	Property	
	$\tilde{E}_b$ [GPa]	$\tilde{\rho}_b$ [kg/m ³]
24	69	2680
35	68.601	2678
45	68.238	2676
55	67.875	2674
65	67.511	2672
$\tilde{E}_b(^{\circ}\text{C}) = -3.6316 \times 10^7 \times T + 6.9872 \times 10^{10}$ $\tilde{\rho}_b(^{\circ}\text{C}) = -0.2 \times T + 2.6850 \times 10^3$		

Source: Elaborated by the author.

3.8. CONCLUSIONS

This chapter provided a comprehensive formulation of the proposed analytical model for the Electromechanical Impedance technique, covering longitudinal, flexural, and coupled dynamics. The derivations presented serve as the foundation for all theoretical investigations conducted in this thesis.

A key contribution is the introduction of a novel longitudinal-flexural coupling matrix, which enhances the model's capability to represent asymmetric piezoelectric configurations more accurately. Additionally, the employed modal shape functions were detailed, and a methodology for peak adjustment was proposed through a comparison between theoretical and experimental results.

Furthermore, the presence of structural damage — both symmetric and asymmetric — was modeled, offering insight into how it modifies the system's equations. Finally, the effects of temperature variations were presented to discuss the EMI sensitivity to environmental conditions.

4 RESULTS AND DISCUSSION

This chapter presents the results obtained through the theoretical model proposed in this thesis. The investigations focuses on assessing the effects of temperature variations (Section 4.1) and different damage configurations (Section 4.2) on the EMI response, in order to understand how these factors alter the EMI curves. The theoretical investigations are performed using a in-house Python[®]-based algorithm. The EMI curves are evaluated within the frequency range of 5 kHz to 30 kHz, as the Euler–Bernoulli model restricts the analysis to a maximum frequency of 30 kHz (see Section 3.4). The formulation consider $n_L = 14$ longitudinal modes and $n_F = 35$ flexural modes.

4.1. EFFECTS OF TEMPERATURE VARIATIONS

The temperature effects on EMI curves were investigated in three different cases. In the first one, the temperature's influence was considered exclusively on the piezoelectric transducer (Section 4.1.1). In the second case, the temperature investigations were restricted to the structure (Section 4.1.2). Lastly, in the third case, temperature variations affected both the PZT and the structure simultaneously (Section 4.1.3). In all cases, temperature-induced variations were incorporated into the model by modifying the physical properties of the system components, according to Tabs. 5 and 6. Five temperatures were analyzed: 24°C, 35°C, 45°C, 55°C and 65°C. The properties of the PZT and the beam under investigation are presented in Tabs 7 and 8, respectively, for the temperature of 24°C. Additionally, it was employed the complex dielectric constant, given by $\tilde{\epsilon}_{33}^{T*} = \tilde{\epsilon}_{33}^T(1 - j\delta)$, where δ is the dielectric loss, to account for the electrical losses of the PZT and, consequently, achieve a more accurate representation of a real system. The analysis was restricted to the coupled dynamics.

Table 7 – Physical and geometric properties of the PZT PSI-5H4E at 24°C.

Property	Symbol	Value
Length	L_p	12 mm
Width	b_p	12 mm
Thickness	h_p	0.2667 mm
Compliance	S_{11}^E	$1.6129 \times 10^{-11} \text{ Pa}^{-1}$
Density	ρ_p	7800 kg/m ³
Dielectric loss	δ	0.02
Distance to 1st PZT's end	x_1	81 mm
Distance to 2nd PZT's end	x_2	$x_1 + L_p = 93 \text{ mm}$

Source: Elaborated by the author.

Table 8 – Geometric properties of the 6063-T5 aluminum alloy beam.

Property	Symbol	Value
Length	L_b	499 mm
Width	b_b	12 mm
Thickness	h_b	3 mm

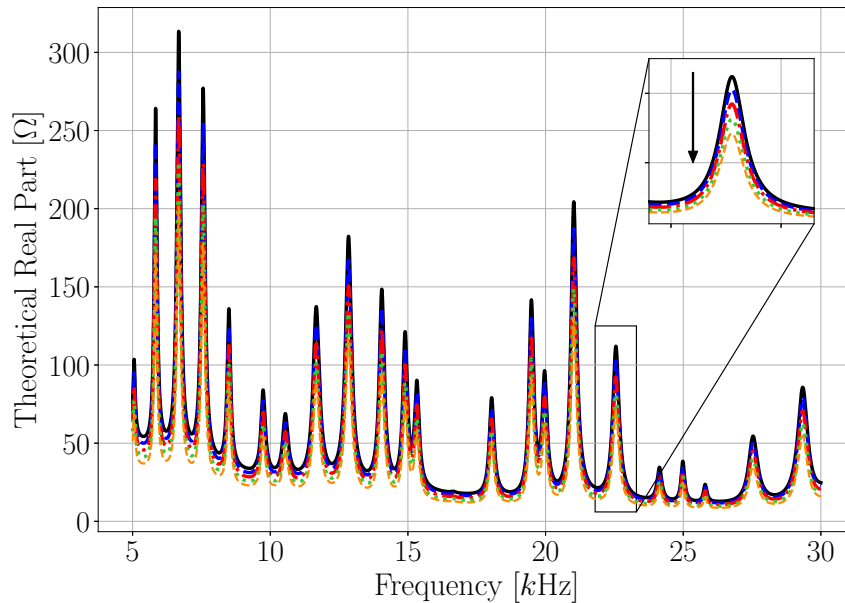
Source: Elaborated by the author.

4.1.1 First Case - PZT

Figure 11 shows the real part of the theoretical EMI curves obtained when only the properties of the piezoelectric transducer vary with ambient temperature. As temperature increases, changes in the dielectric $\tilde{\varepsilon}_{33}^{T*}$ and piezoelectric $\tilde{d}_{31}$ constants cause a decrease in the EMI amplitudes, affecting both the peak values and the overall average level.

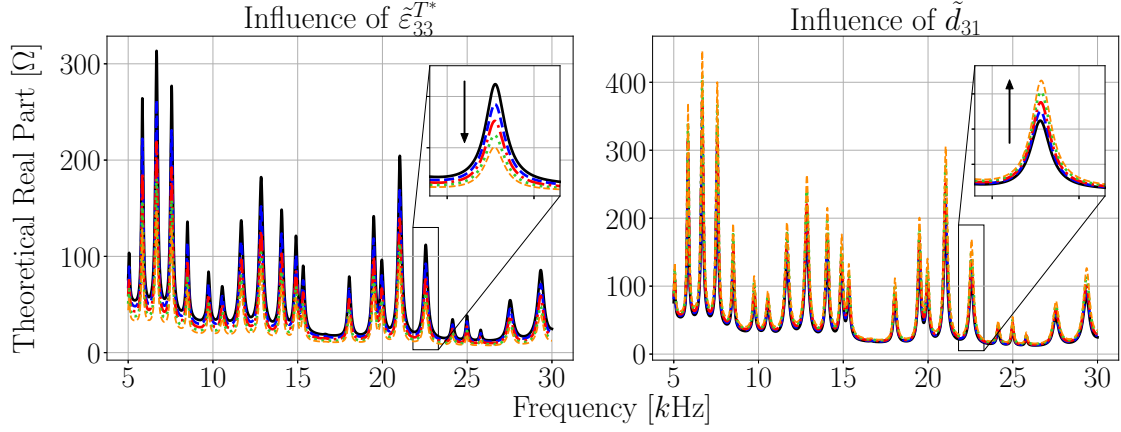
To further investigate these effects, Fig. 12 presents the real part of the theoretical EMI curves obtained by varying $\tilde{\varepsilon}_{33}^{T*}$ (left) and $\tilde{d}_{31}$ (right) separately, while keeping the other constant at the reference temperature (24°C). Changes in the dielectric constant lead to a reduction in peak amplitudes, along with a downward vertical displacement in the average level of the EMI signals, as temperature increases. In contrast, variations in the piezoelectric constant exhibit the opposite behavior. Nevertheless, the overall response is dominated by downward vertical displacements (see Fig. 11), mirroring the effect of the dielectric constant

Figure 11 – Theoretical EMI real part with temperature-induced variations in the piezoelectric properties ($\tilde{\varepsilon}_{33}^{T*}$ and $\tilde{d}_{31}$) at: 24°C (solid black line), 35°C (dashed blue line), 45°C (dashed-dotted red line), 55°C (dotted green line) and 65°C (thinner dashed orange line).



Source: Elaborated by the author.

Figure 12 – Theoretical EMI real part with temperature-induced variations in $\tilde{\varepsilon}_{33}^{T*}$ (left) and $\tilde{d}_{31}$ (right) separately at: 24°C (solid black line), 35°C (dashed blue line), 45°C (dashed-dotted red line), 55°C (dotted green line) and 65°C (thinner dashed orange line).



Source: Elaborated by the author.

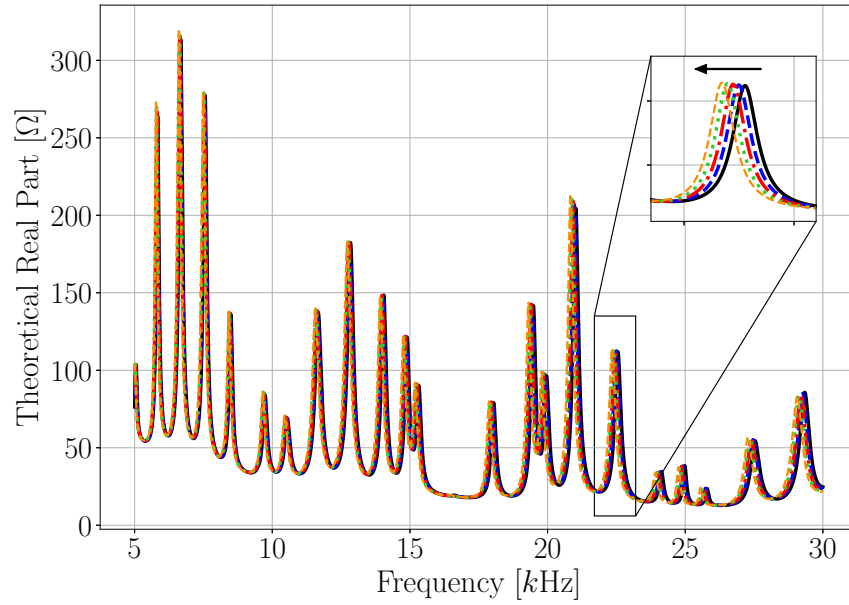
alone, which is likely due to the greater magnitude of this parameter.

4.1.2 Second Case - Structure

Figure 13 shows the real part of the theoretical EMI curves obtained by varying only the structural properties: Young's modulus, $\tilde{E}_b$, and material density, $\tilde{\rho}_b$. Variations in these parameters induce a leftward horizontal displacement in the EMI curves, resulting in lower peak frequencies as temperature increases.

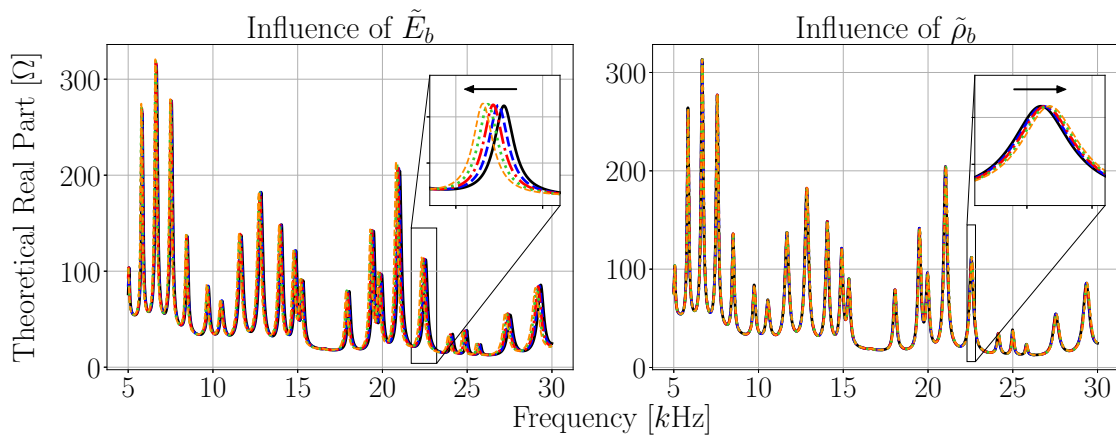
This shift behavior is mainly attributed to the loss of structural stiffness. As the temperature rises, there is a reduction in stiffness – evidenced by the decrease in Young's modulus (see Tab. 6) –, which, in turn, leads to lower peak frequencies. This statement is further verified by Fig. 14, which presents the real part of the theoretical EMI curves obtained by varying the parameters $\tilde{E}_b$ (left) and $\tilde{\rho}_b$ (right) independently. It is evident that variations in Young's modulus alone replicate the overall behavior observed in Fig. 13, while changes in material density produce an opposite effect.

Figure 13 – Theoretical EMI real part with temperature-induced variations in the structure properties ($\tilde{E}_b$ and $\tilde{\rho}_b$) at: 24°C (solid black line), 35°C (dashed blue line), 45°C (dashed-dotted red line), 55°C (dotted green line) and 65°C (thinner dashed orange line).



Source: Elaborated by the author.

Figure 14 – Theoretical EMI real part with temperature-induced variations in $\tilde{E}_b$ (left) and $\tilde{\rho}_b$ (right) separately at: 24°C (solid black line), 35°C (dashed blue line), 45°C (dashed-dotted red line), 55°C (dotted green line) and 65°C (thinner dashed orange line).

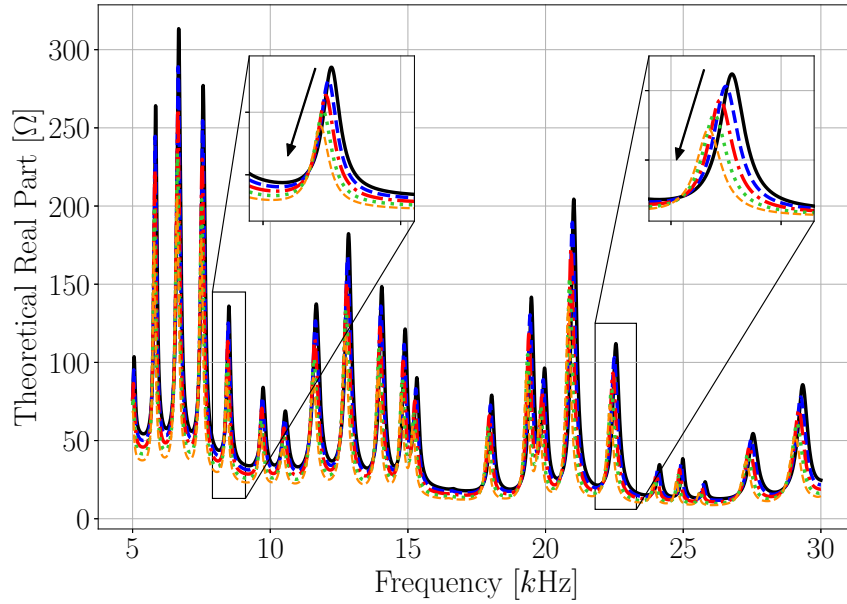


Source: Elaborated by the author.

4.1.3 Third Case - Coupled System

Figure 15 shows the real part of the theoretical EMI curves obtained by simultaneously varying all properties of both the beam and the PZT. It is evident that, for the coupled system, an increase in temperature simultaneously reduces both frequency and amplitude of the EMI signatures. This trend reflects a combination of the behaviors observed in the previous two cases. The downward vertical displacement in peak amplitude and average level is associated with the PZT influence, while the leftward horizontal displacement is characteristic of the structural variations. Furthermore, these horizontal shifts becomes progressively more pronounced as frequency increases, as evidenced by the comparison between the two zoomed-in plots in Fig. 15, which highlight low- and high-frequency regions.

Figure 15 – Theoretical EMI real part with temperature-induced variations in piezoelectric ($\tilde{\varepsilon}_{33}^{T*}$ and $\tilde{d}_{31}$) and structural ($\tilde{E}_b$ and $\tilde{\rho}_b$) properties at: 24°C (solid black line), 35°C (dashed blue line), 45°C (dashed-dotted red line), 55°C (dotted green line) and 65°C (thinner dashed orange line).



Source: Elaborated by the author.

4.2. DAMAGE-INDUCED MODIFICATIONS

Table 9 shows the damage configurations explored, which are categorized into three case studies: (i) variation in damage depth, (ii) variation in damage length, and (iii) variation in damage location. In each case, a single parameter is changed and the other two remain constant, allowing an isolated assessment of the influence of each damage characteristic on the EMI response. The parameters are: L_d denotes the damage length; h_d represents the

reduced thickness, expressed as the percentage reduction of the nominal thickness h_b ; and L_{pd} corresponds to the distance between the closest edges of the PZTs and the damaged region.

Table 9 – Damage configurations defined by the combination of three different parameters.

Case Study	Varied Parameter	Values	Constant Parameters
(i) Depth variation	h_d	5%, 15%, 25%	$L_d = 25$ mm, $L_{pd} = 20$ cm
(ii) Length variation	L_d	0.1 mm, 5 mm, 25 mm	$h_d = 15\%$, $L_{pd} = 20$ cm
(iii) Position variation	L_{pd}	5 cm, 20 cm, 35 cm	$h_d = 15\%$, $L_d = 25$ mm

Source: Elaborated by the author.

The impact of different damages on the EMI signatures is commonly quantified using metric indices, which represent the signal variations through a single scalar value. Among the available metrics, the Root Mean Square Deviation (RMSD) and the Correlation Coefficient Deviation Metric (CCDM) are used in the present thesis due to their simplicity, effectiveness and widespread adoption (Giurgiutiu, 2007). These damage indices (DI) compare two electromechanical impedance signals. One of them is obtained for the undamaged structure (reference value) and the second one corresponds to the unknown structural condition. The RMSD index is more sensitive to variations in the magnitude of EMI curves, whereas the CCDM index is more affected by frequency shifts (Baptista *et al.*, 2014). The formulas to compute them are presented in Eqs. (4.1) and (4.2), respectively (Baptista *et al.*, 2014).

$$RMSD = \sum_{i=1}^{N_f} \sqrt{\frac{[Z_{em_U}(\omega_i) - Z_{em_R}(\omega_i)]^2}{[Z_{em_R}(\omega_i)]^2}}, \quad (4.1)$$

$$CCDM = 1 - C_C;$$

$$C_C = \frac{\sum_{i=1}^{N_f} [Z_{em_R}(\omega_i) - \bar{Z}_{em_R}][Z_{em_U}(\omega_i) - \bar{Z}_{em_U}]}{\sqrt{\sum_{i=1}^{N_f} [Z_{em_R}(\omega_i) - \bar{Z}_{em_R}]^2} \sqrt{\sum_{i=1}^N [Z_{em_U}(\omega_i) - \bar{Z}_{em_U}]^2}}, \quad (4.2)$$

where C_C is the cross-correlation coefficient, Z_{em_R} and Z_{em_U} are the reference and unknown electromechanical impedance signals, respectively, and $\bar{Z}_{em_R}$ and $\bar{Z}_{em_U}$ correspond to their respective mean signatures. The superscript $(\bar{})$ indicates the mean value, and the subscripts $()_R$ and $()_U$ refer to the reference and unknown conditions, respectively.

The system under study comprises a 6063-T5 aluminum alloy beam instrumented with two PSI-5H4E piezoelectric transducers symmetrically bonded to its surface. The

Table 10 – Physical and geometric properties of the 6063-T5 aluminum alloy beam at 24°C.

Property	Symbol	Value
Length	L_b	500 mm
Width	b_b	12 mm
Thickness	h_b	3 mm
Young's modulus	E_b	69 GPa
Density	ρ_b	2680 kg/m ³

Source: Elaborated by the author.

Table 11 – Physical and geometric properties of the PZTs PSI-5H4E at 24°C.

Property	Symbol	Value
Length	L_p	12 mm
Width	b_p	12 mm
Thickness	h_p	0.2667 mm
Compliance	S_{11}^E	1.6129×10^{-11} Pa ⁻¹
Density	ρ_p	7800 kg/m ³
Piezoelectric constant	d_{31}	-327.18×10^{-12} m/V
Dielectric constant	ε_{33}^T	3.248×10^{-8} F/m
Dielectric loss	δ	0.02
Distance to 1st PZT's end	x_1	80 mm
Distance to 2nd PZT's end	x_2	$x_1 + L_p = 92$ mm

Source: Elaborated by the author.

properties and dimensions of the beam and the PZTs at 24°C are presented in Tabs. 10 and 11, respectively.

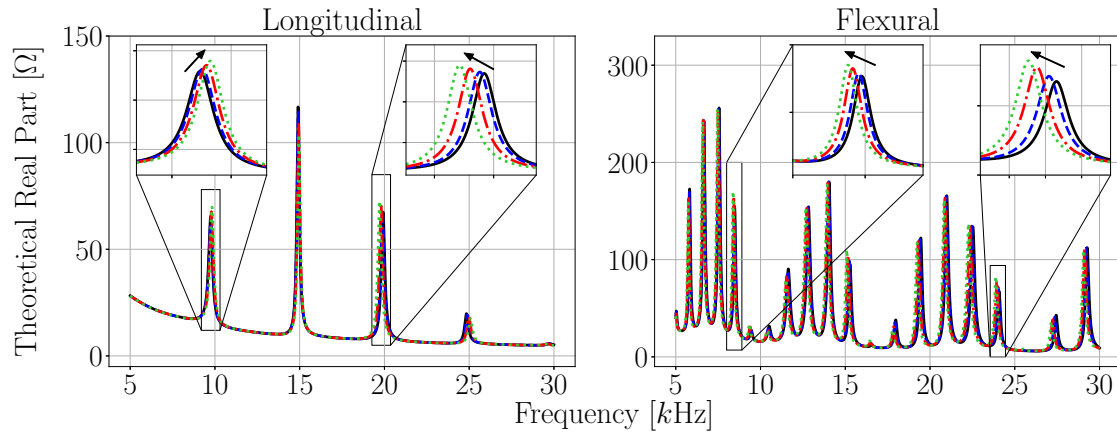
The EMI curves obtained from the three case studies defined in Tab. 9, along with their corresponding damage indices are presented in the following sections exclusively for the structural condition involving a symmetric reduction in the beam's cross-sectional area. The dynamic behavior is examined separately in the longitudinal and flexural dynamics, as the coupled one represent a combination of these two dynamics. Therefore, insights drawn from the uncoupled responses are easier to understand and can be extended to characterize the coupled dynamics. All investigations are carried out at a temperature of 24°C.

4.2.1 (i) Damage Depth Variation

Figure 16 presents the real part of the theoretical EMI curves obtained in case study (i) for the longitudinal (left) and flexural (right) dynamics. The figure shows the EMI signals corresponding to the baseline (undamaged) condition and to the three distinct damage depths, whose values are specified in Tab. 9. The corresponding damage indices for the longitudinal and flexural dynamics are displayed in Figs. 17 and 18, respectively.

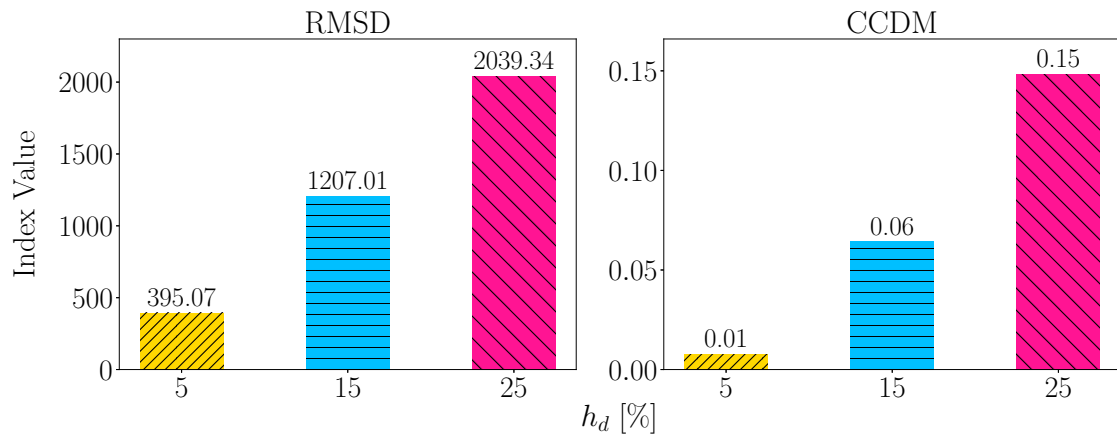
It is observed that, for both dynamics, the EMI curves progressively deviate from the baseline condition as the damage depth increases. This trend is confirmed by the metric

Figure 16 – Theoretical EMI real part for longitudinal (left) and flexural (right) dynamics under baseline condition (solid black line) and damage depths h_d of 5% (dashed blue line), 15% (dashed-dotted red line) and 25% (dotted green line).



Source: Elaborated by the author.

Figure 17 – RMSD (left) and CCDM (right) indices for longitudinal dynamics at the three distinct damage depths h_d .

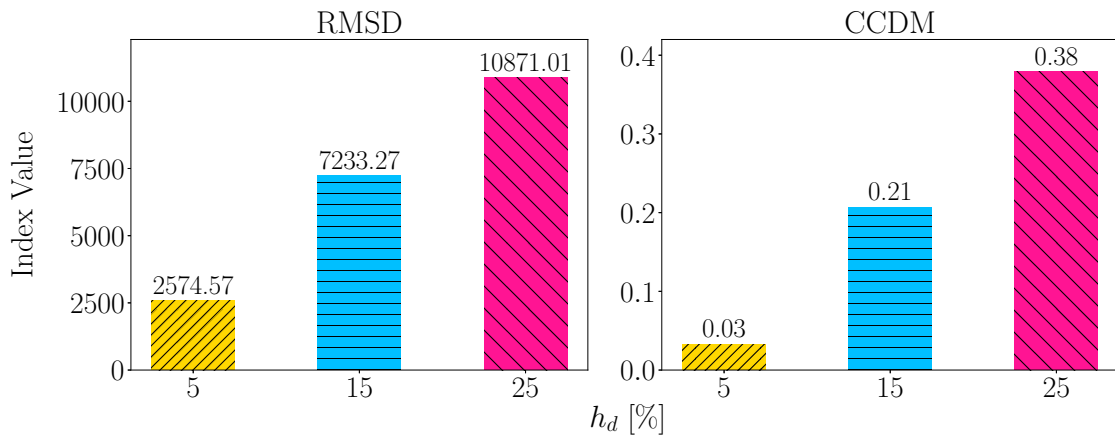


Source: Elaborated by the author.

values shown in Figs. 17 and 18, where both RMSD and CCDM indices increase with damage intensity. In the case of flexural dynamics, the curves exhibit a leftward shift with increasing damage, whereas the longitudinal dynamics show an opposite behavior, with the curves shifting to the right, except for the third peak, which moved to the left. Moreover, at higher frequencies, the deviation of the damaged curves from the baseline signal becomes more pronounced, as the excitation wavelengths shorten and, consequently, the system becomes more sensitive to local and incipient damage (Park *et al.*, 2003; Annamdas; Soh, 2010).

Despite both dynamics may detect and quantify the increase in damage depth, flexural dynamics proved to be more sensitive, since it presented higher values for both RMSD and CCDM indices. This greater sensitivity can be attributed to two main factors: first, the amplitude of the flexural curves, more than twice that of the longitudinal ones, may increase

Figure 18 – RMSD (left) and CCDM (right) indices for flexural dynamics at the three distinct damage depths h_d .



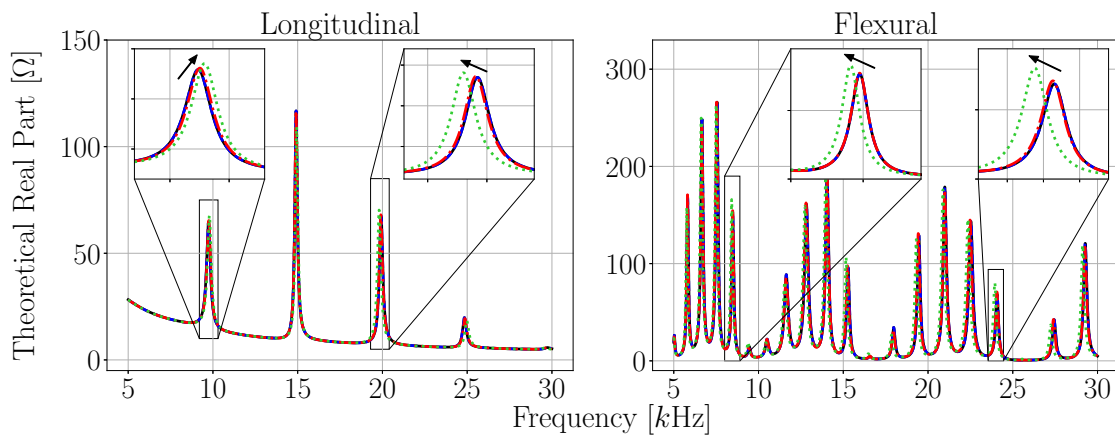
Source: Elaborated by the author.

the index values; and second one, the greater density of peaks. The presence of damage alters the mechanical impedance of the structure, changing the position and/or intensity of the EMI peaks. Therefore, the analysis of these changes is essential for detecting damage. Consequently, a greater number of peaks in a given frequency range provides a more robust set of information about the structural state, increasing the reliability of damage detection.

4.2.2 (ii) Damage Length Variation

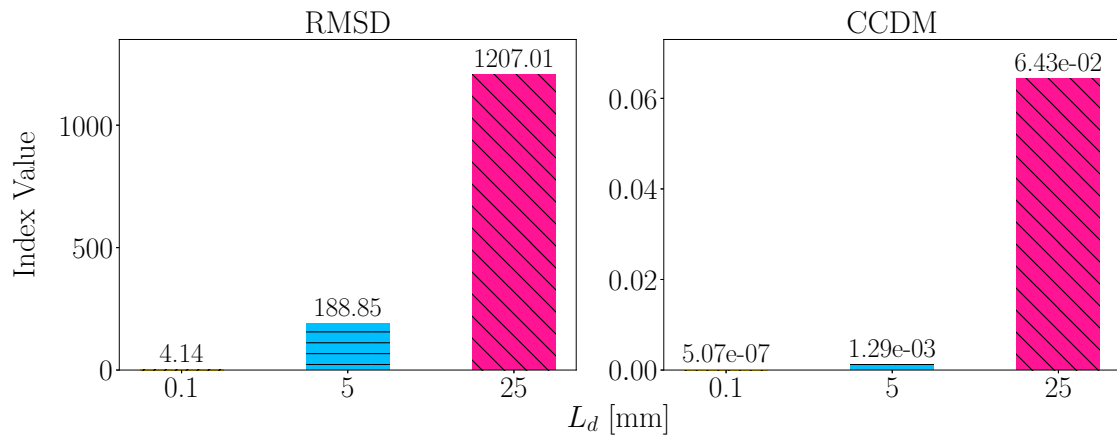
The real part of the theoretical EMI curves obtained from case study (ii) is shown in Fig. 19 to the longitudinal and flexural dynamics. Each subplot includes the baseline state and the three different damage lengths defined in Tab 9. The associated damage indices are

Figure 19 – Theoretical EMI real part for longitudinal (left) and flexural (right) dynamics under baseline condition (solid black line) and damage lengths L_d of 0.1 mm (dashed blue line), 5 mm (dashed-dotted red line) and 25 mm (dotted green line).



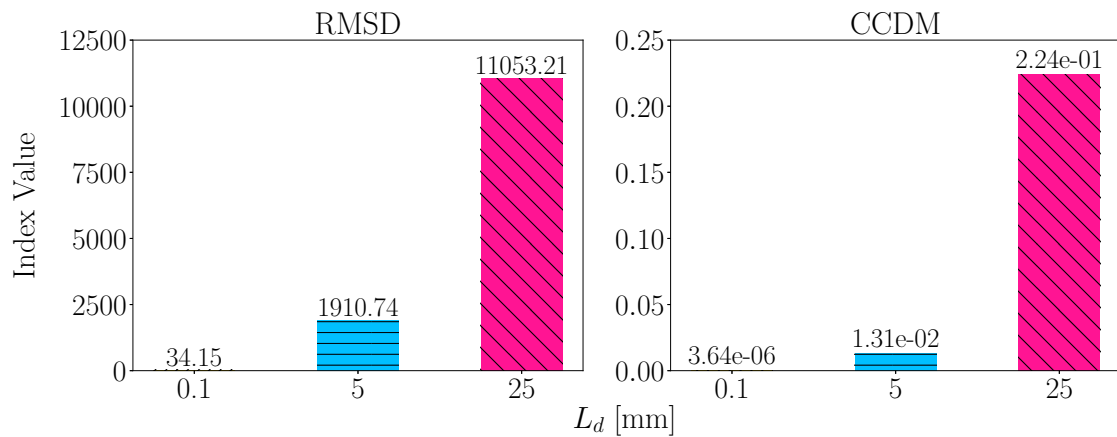
Source: Elaborated by the author.

Figure 20 – RMSD (left) and CCDM (right) indices for longitudinal dynamics at the three distinct damage lengths L_d .



Source: Elaborated by the author.

Figure 21 – RMSD (left) and CCDM (right) indices for flexural dynamics at the three distinct damage lengths L_d .



Source: Elaborated by the author.

illustrated in Figs. 20 and 21 for the longitudinal and flexural dynamics, respectively.

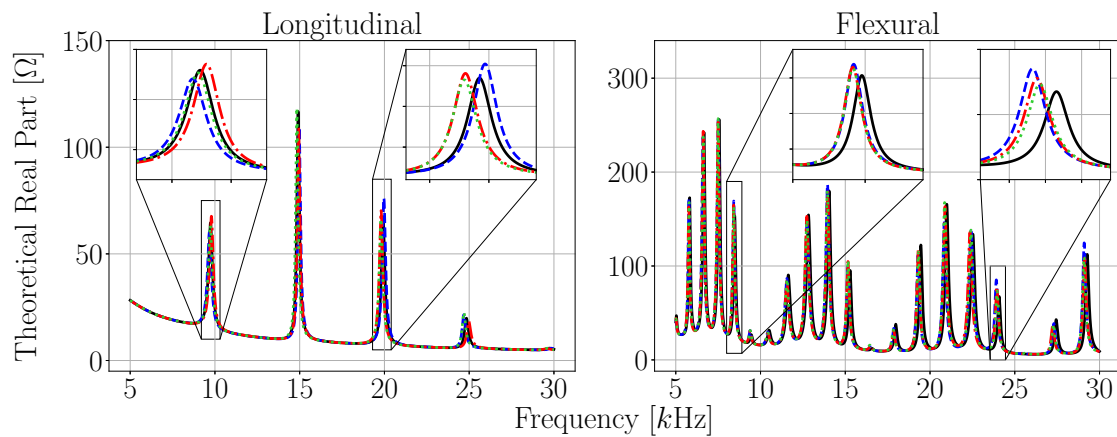
According to Fig. 19, the damaged EMI curves exhibit an increasingly pronounced deviation from the baseline curve as the damage length increases, for both longitudinal and flexural dynamics. This trend is quantitatively confirmed by the progressive rise in the RMSD and CCDM indices shown in Figs. 20 and 21. Particularly, the smallest damage length analyzed – 0.1 mm, representing a crack – produces negligible changes in the EMI signatures, indicating that damage of this scale remains largely undetectable within the considered frequency range. This characteristic is further evidenced by its small damage indices shown in Figs. 20 and 21. Similarly, the 5 mm damage length results in only minor deviations in the EMI curves, which become slightly more noticeable at higher frequencies. In contrast, the damage length of 25 mm induces greater changes in the EMI signals, as reflected by its higher indices values. These observations are consistent across both dynamics.

The general behavior of the EMI curves with respect to increasing damage length mirrors that observed in case study (i): the flexural curves shift leftward as the damage increases, and the longitudinal curves shift to the right, except for the third peak, which moves to the left. Moreover, this pattern becomes more pronounced as the damage length and the frequency increase, and the flexural dynamics continue to demonstrate greater sensitivity to damage in both indices.

4.2.3 (iii) Damage Position Variation

Figure 22 shows the real part of the theoretical EMI curves for the longitudinal (left) and flexural (right) dynamics considering case study (iii). This figure presents the baseline EMI signal along with the curves associated with the three distinct damage positions defined in Tab. 9. The associated damage indices are illustrated in Figs. 23 and 24 for the longitudinal and flexural dynamics, respectively.

Figure 22 – Theoretical EMI real part for longitudinal (left) and flexural (right) dynamics under baseline condition (solid black line) and damage positions L_{pd} of 5 cm (dashed blue line), 20 cm (dashed-dotted red line) and 35 cm (dotted green line).

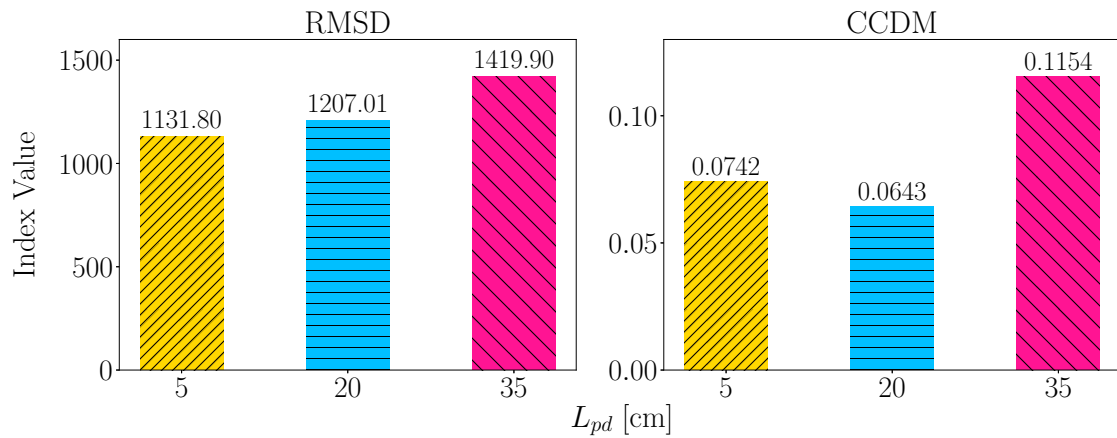


Source: Elaborated by the author.

The sensing area of a PZT depends on various factors, including its own properties, the characteristics of the structure and the frequency range under investigation. In the EMI technique, the PZT sensitivity generally decreases as its distance from the damage increases. In other words, variations in EMI signals induced by damage become less pronounced as the distance between the PZT and the damage region grows (Giurgiutiu; Zagari, 2005; Divsholi; Yang, 2012). According to Inman *et al.* (2005), this detection area can extend up to 2 meters in simple metal beams. Considering that this beam is 50 cm long, it is expected that the PZTs are capable to detect damage located anywhere along the structure length.

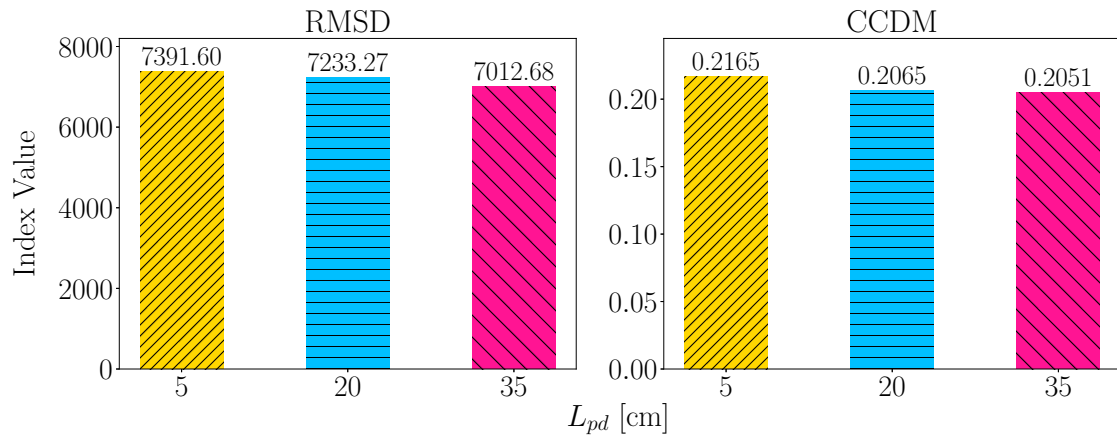
This above statement is supported by Fig. 22, which shows that, regardless of the damage position, there are noticeable discrepancies between the damaged and baseline EMI

Figure 23 – RMSD (left) and CCDM (right) indices for longitudinal dynamics at the three distinct damage positions L_{pd} .



Source: Elaborated by the author.

Figure 24 – RMSD (left) and CCDM (right) indices for flexural dynamics at the three distinct damage positions L_{pd} .



Source: Elaborated by the author.

curves for both dynamics, even at lower frequencies. However, the behavior of the curves appeared more random compared to the previous case studies, with the location of the EMI peaks do not following a consistent pattern as the damage moved along the beam, i.e., with increasing L_{pd} . This effect is observed in both flexural and longitudinal dynamics.

Despite of this behavior shown in Fig. 24, both RMSD and CCDM indices computed for the flexural EMI curves exhibit a decreasing trend as the damage is placed farther from the PZTs. Nonetheless, the variation of the index values is relatively low, probably due to the small length of the beam, which restricted the spatial variation of L_{pd} . On the other hand, for the longitudinal dynamics, no clear correlation is observed between damage location and the index values. The reasons for this lack of linearity remain unclear and may involve several contributing factors, such as the short length of the structure, the frequency range analyzed, the nature of the investigated damage and the considered metrics. Furthermore, it

is not yet possible to determine whether this behavior is inherent to longitudinal dynamics or merely a peculiarity of the system investigated in this thesis. Further research involving different structural geometries, damage positions, frequency ranges and damage metrics is therefore necessary to better understand this index behavior and assess the generality of this observation.

4.3. CONCLUSIONS

This chapter examined the effects of both temperature variations and symmetric damage configurations through the application of the proposed theoretical model. Modifications in the physical properties of both the beam and the PZT were considered to simulate environmental temperature changes, whereas parametric variations in damage depth, length, and position were independently investigated to evaluate their influence on the EMI signals of an Euler-Bernoulli beam with symmetrically coupled PZTs.

Temperature variations confined to the piezoelectric transducer resulted in vertical displacements of the EMI curves. In contrast, temperature changes restricted to the structure predominantly led to horizontal shifts, characterized by lower peak frequencies as the temperature increased. Finally, a combined response, featuring both vertical and horizontal displacements, was observed when both the PZT and the structure were simultaneously affected by temperature variations.

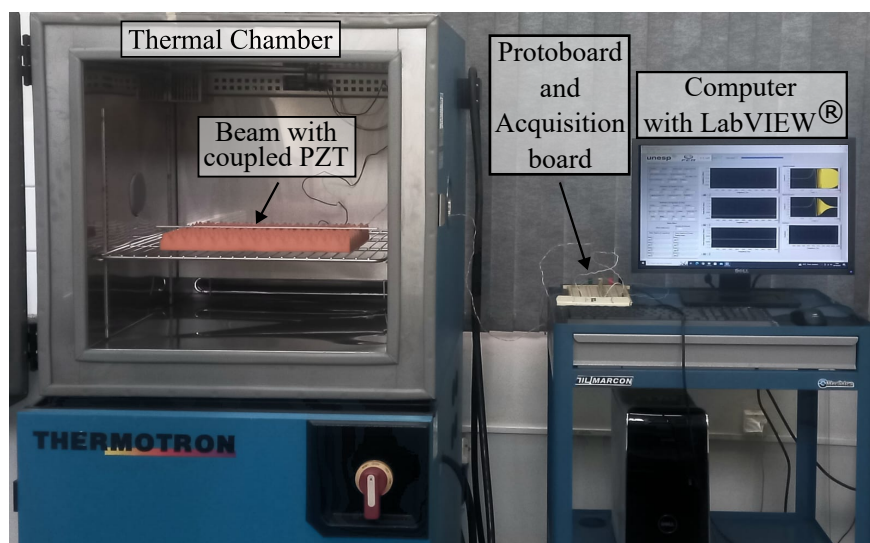
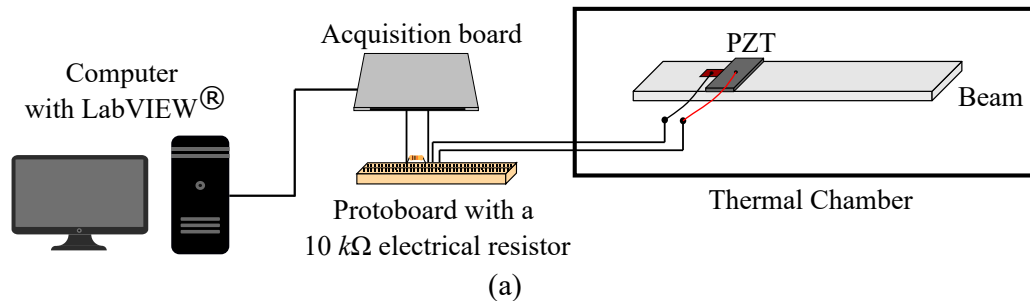
Regarding damage effects, increases in depth and length consistently led to greater deviations in the EMI signatures, for both longitudinal and flexural dynamics. Moreover, for the flexural case, the influence of damage decreased as its distance from the PZTs increased, whereas the longitudinal dynamics did not exhibit a clear linear correlation with damage position.

Overall, the results demonstrate how variations in the physical properties of both the PZT and the structure induce different shift behaviors in the EMI signatures, highlighting the critical impact of temperature on the reliability of SHM evaluations. Additionally, the findings elucidate how distinct damage parameters influence the EMI response differently for longitudinal and flexural dynamics, with the latter demonstrating greater sensitivity to damage detection within the investigated system. Furthermore, the results highlight the capability of the theoretical model to simulate diverse thermal and damage scenarios, as well as the sensitivity of the EMI technique to various forms of structural degradation, thereby reaffirming its potential as a reliable SHM tool.

5 EXPERIMENTAL RESULTS

This chapter presents a description of the experimental tests conducted to collect electromechanical impedance signals, along with the proposed false alarm reduction strategy and the validation of the theoretical model. The experiments were performed on a 6063-T5 aluminum alloy beam with PSI-5H4E piezoelectric transducers, from Piezo Systems®, attached to its surface. Figure 25 shows both a schematic illustration and a photograph of the experimental setup. It was used a National Instruments® board, model NI-USB 6211 (16 bits); a computer equipped with the impedance analyzer software developed by Baptista and Vieira Filho (2009) in LabVIEW® environment; and a protoboard containing a resistance of $10\text{ k}\Omega$, which acts as an auxiliary electrical circuit, whose function is to limit the voltage in the PZT, thus avoiding potential damage to this device. The data acquisition (DAQ) board is responsible for capturing the response signal and, simultaneously, transmitting the excitation signal to the PZT. This signal, in turn, is generated by the impedance analyzer, which also calculates the EMI based on the response signal captured by the DAQ. The aluminum beam was placed inside a thermal chamber, model Thermotron® S-Series, to control the environmental

Figure 25 – (a) Schematic illustration and (b) photograph of the experimental setup employed to measure the EMI signals.



Source: Elaborated by the author.

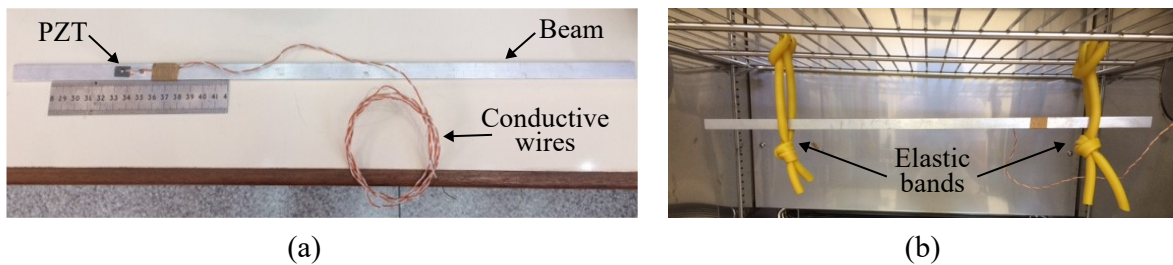
temperature. The sampling rate used was 250 kS/s.

Two distinct datasets signals were used, and the first one was available in the research group (FAPESP, Grant number 2016/12241-0), which was obtained using a PZT asymmetrically coupled to the structure (see Fig. 8a). These signals served two primary purposes: implementing a temperature compensation technique described in the literature and developing the novel false alarm reduction strategy, as detailed in Section 5.1. The second dataset was collected by this author, and involves PZTs symmetrically coupled to the beam (see Fig. 8b). It is used to validate the analytical model under different dynamic and structural conditions, as presented in Section 5.2.

5.1. TEMPERATURE COMPENSATION AND FALSE ALARM REDUCTION TECHNIQUES

Figure 26(a) shows the aluminum beam with an asymmetrically coupled PZT, and Fig. 26(b) shows the beam suspended by elastic bands, simulating a free-free boundary condition, inside the thermal chamber. Geometric characteristics for the beam and the PZT are presented in Tab. 12.

Figure 26 – (a) Aluminum beam with an asymmetrically coupled PZT and (b) suspended by elastic bands inside the thermal chamber.



Source: Adapted from Lopes (2018).

Table 12 – Geometric characteristics of the beam and the PZT.

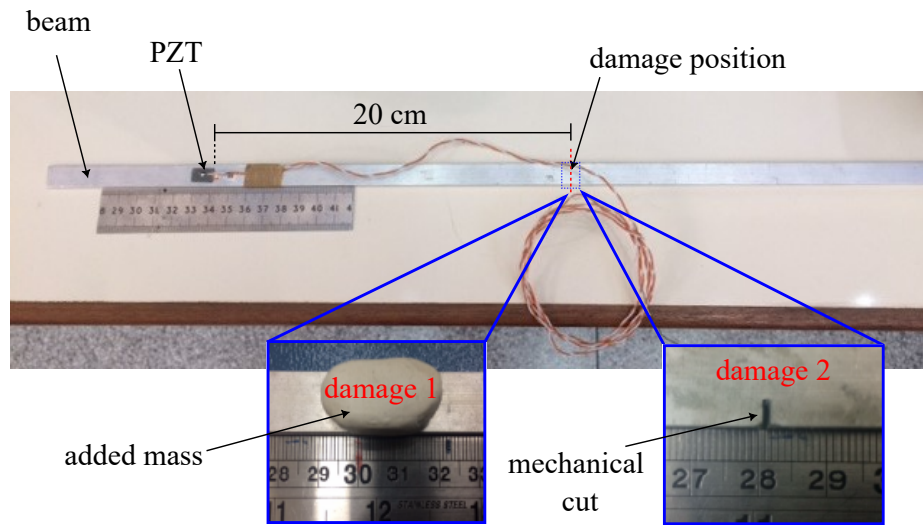
Property	Beam [mm]	PZT [mm]
Length	500	10
Width	13	5
Thickness	3	0.2667

Source: Lopes (2018).

The EMI signals were measured across a temperature range from -10°C to 80°C , which corresponds to the operating conditions of various structural applications. An incremental step of 2°C and 5°C was considered for the undamaged and damaged conditions, respectively. A sinusoidal frequency sweep (*chirp*) with an amplitude of $\pm 5\text{ V}$ in a frequency range of 0 to 125 kHz, with increments of 1 Hz, was used to excite the structure through the PZT. Two

types of damage were considered (both 20 cm from the PZT). The first one is the addition of a 4.89 g durepoxy mass (damage 1) and the second one is a 4 mm introduced mechanical cut (damage 2), as shown in Fig. 27. The signals were collected during both the heating, $Z_{em_{heating}}$, and cooling, $Z_{em_{cooling}}$, processes of the structure to account for hysteresis effects. However, the employed signal corresponds to the mean of these EMI measurements, computed as $\frac{Z_{em_{cooling}} + Z_{em_{heating}}}{2}$. In total, 92 signals were collected for the undamaged structure, and 38 signals were measured for each damage condition.

Figure 27 – Beam-like structure with a PZT transducer and both types of damages: added mass (damage 1) and mechanical cut (damage 2).



Source: Adapted from Lopes (2018).

The seven selected temperatures for the undamaged and damaged signals are shown in Tab. 13, with the reference signature (at 24°C) highlighted by a red rectangle. The definition of this reference EMI is important for the future application of the temperature compensation and false alarm reduction techniques discussed. Four different frequency ranges are evaluated, as shown in Tab. 14, where f_{min} and f_{max} are the minimum and maximum frequencies for each frequency range (FR). Note that the lower frequency remained constant in all investigated cases, as the EMI technique operates at high frequencies, typically exceeding 30 kHz (Park *et al.*, 2003; Yang; Divsholi, 2010; Ai *et al.*, 2014).

Figures 28 to 30 show the real part of the EMI signal curves for both damaged and undamaged conditions at the selected temperatures on the frequency range number 4, i.e., with 45 kHz of bandwidth. Only the real part of the EMI is analyzed, because it has been reported to exhibit greater sensitivity to damage, in comparison with the total electromechanical impedance signal (Park *et al.*, 2003; Yang; Hu; Lu, 2008; Wang *et al.*, 2018). The reference EMI curve is also shown to allow one the comparison among the different signatures. As temperature increases, the curves progressively deviate to the left-hand side, exhibiting lower peak frequencies. This behavior is primarily governed by the reduction in structural stiffness (see Tab. 6). Furthermore, the temperature elevation induces a downward vertical

Table 13 – Selected temperatures for the undamaged and damaged signals, with the reference signature highlighted by a red rectangle.

Temperature	Undamaged	Damage 1	Damage 2
1	-10°C	-10°C	-10°C
2	4°C	5°C	5°C
3	24°C	25°C	25°C
4	36°C	35°C	35°C
5	52°C	50°C	50°C
6	64°C	65°C	65°C
7	80°C	80°C	80°C

Source: Elaborated by the author.

Table 14 – Different frequency ranges (FR) investigated and each corresponding bandwidth.

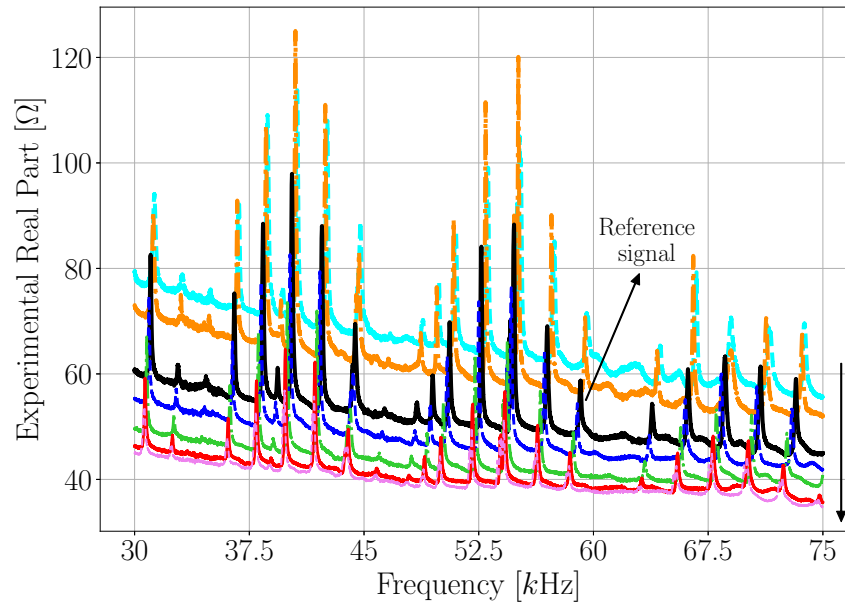
Number	f_{min} [kHz]	f_{max} [kHz]	Bandwidth [kHz]
1	30	45	15
2	30	60	30
3	30	65	35
4	30	75	45

Source: Elaborated by the author.

displacement, attributed to the rise in the PZT dielectric constant $\tilde{\epsilon}_{33}^T$ (see Tab. 5). The piezoelectric transducer ideally behaves as a capacitor, and a temperature rise leads to an increase in its capacitance, $C_p = \tilde{\epsilon}_{33}^T \frac{b_p L_p}{h_p}$ (Baptista, 2010), and, consequently, a decrease in the impedance. This relationship is evident in Eq. (3.1), which associates the PZT capacitance with the first term of the electromechanical admittance, $Y_{em} = 1/Z_{em}$. Additionally, a rise in environmental temperature also induces peak smoothing. These behaviors have also been verified by other authors, whether through experimental data (Krishnamurthy; Lalande; Rogers, 1996; Baptista *et al.*, 2014), dynamic models (Xu *et al.*, 2016), or a combination of both (Sepehry; Shamshirsaz; Bastani, 2010), and are likewise observed in Section 4.1 of the present thesis.

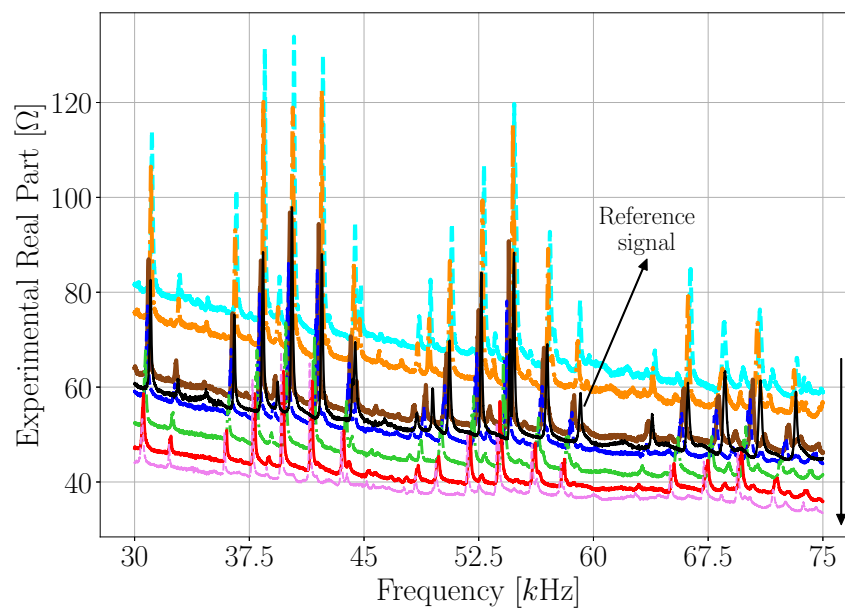
Figures 28 to 30 show that the temperature induces significant changes in the EMI signals, even in the absence of structural damage. These variations can be mistakenly interpreted as damage and, consequently, lead to false alarms in terms of damage detection, i.e., indicating damage where it does not exist. This reinforces the great importance of compensation techniques to reduce temperature effects on damage detection. Then, the strategy proposed by Park *et al.* (1999) was specifically investigated because the author's previous studies has demonstrated its high efficiency to compensate temperature effects and its potential to detect damage (Dias, 2021; Dias *et al.*, 2022).

Figure 28 – EMI real part for the undamaged condition at the selected temperatures: -10°C (thicker dashed cyan line), 4°C (thicker dashed-dotted orange line), 24°C (reference: thicker solid black line), 36°C (dashed blue line), 52°C (dashed-dotted green line), 64°C (solid red line) and 80°C (thinner dashed violet line).



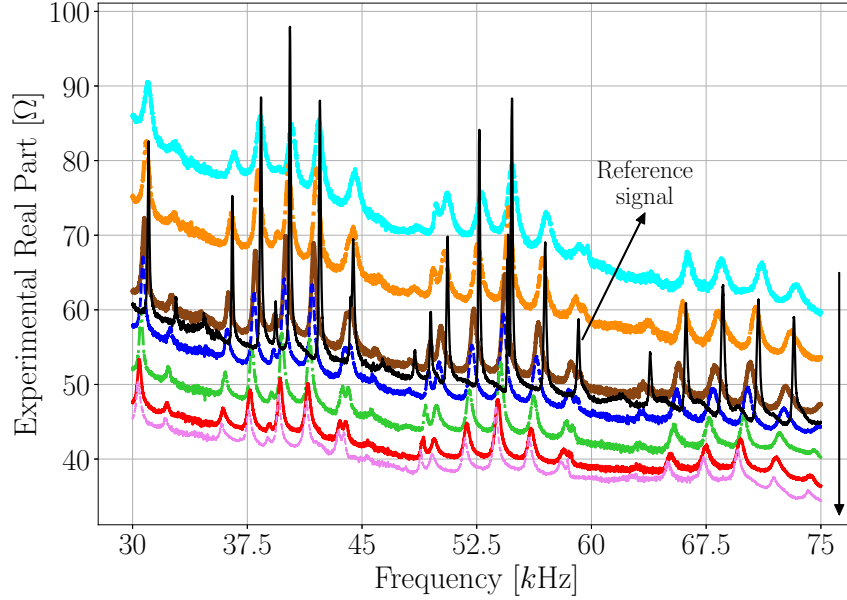
Source: Elaborated by the author.

Figure 30 – EMI real part for the damage condition 2 at the selected temperatures: -10°C (thicker dashed cyan line), 5°C (thicker dashed-dotted orange line), 25°C (thicker solid brown line), 35°C (dashed blue line), 50°C (dashed-dotted green line), 65°C (solid red line) and 80°C (thinner dashed violet line). Reference signal represented by the thinner solid black line.



Source: Elaborated by the author.

Figure 29 – EMI real part for the damage condition 1 at the selected temperatures: -10°C (thicker dashed cyan line), 5°C (thicker dashed-dotted orange line), 25°C (thicker solid brown line), 35°C (dashed blue line), 50°C (dashed-dotted green line), 65°C (solid red line) and 80°C (thinner dashed violet line). Reference signal represented by the thinner solid black line.



Source: Elaborated by the author.

5.1.1 Park *et al.* (1999) Compensation Technique

Park *et al.* (1999) present the technique based on an empirical approach to reduce the temperature effects on EMI-based SHM methods. The compensation procedure relies on minimizing V_a , which represents the sum of the real EMI changes squared and it is defined by Eq. (5.1). In this technique, rather than assuming the summation indices i and k to be equal, the authors exploit the fact that EMI variations are dominated by horizontal peak shifts and vary k independently.

$$V_a = \sum_{i,k=1}^{N_f} [Re(Z_{em_R}(\omega_i)) - Re(Z_{em_U}(\omega_k))]^2, \quad (5.1)$$

where N_f denotes the number of frequency points, $Re(Z_{em_R}(\omega_i))$ represents the real part of the reference electromechanical impedance (Z_{em_R}) at i -th frequency and $Re(Z_{em_U}(\omega_i))$ corresponds to the real part of the electromechanical impedance (Z_{em_U}) for the unknown structural condition at k -th frequency (lagged with respect to i). It can be derived from the unknown measured signal using the following relation:

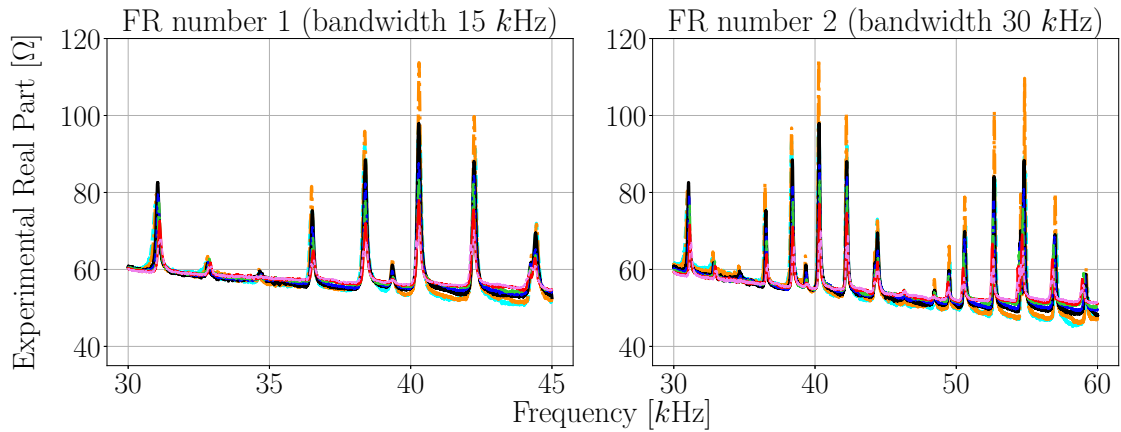
$$Re(Z_{em_U}(\omega_k)) = Re(Z_{em_U}(\omega_k)_{\text{measured}}) + \delta^S, \quad (5.2)$$

where δ^S is defined as the average difference between reference and measured impedance curves. The unknown EMI, $Re(Z_{em_U})$, is vertically compensated, i.e., shifted closer to the

reference curve than the the measured one, $Z_{em_U}(\omega_k)_{\text{measured}}$, due to the addition of δ^S . The values of (k,i) and δ^S can be found through numerical iteration by shifting the EMI curves to the right side when considering temperatures above the reference temperature, or to the left side, for temperatures below the reference temperature, until the minimum value of V_a is achieved. For a better understanding of this procedure, the behavior of the electromechanical impedance curves during the application of the Park *et al.* (1999) technique is presented in Appendix C.

Figures 31 to 36 show the compensated curves obtained by applying the Park *et al.* (1999) technique to the EMI signals depicted in Figs. 28 to 30. The compensated results present, respectively, the EMI curves for the undamaged, damage 1 (added mass), and damage 2 (mechanical cut) conditions across the four investigated frequency ranges (see Tab. 14). It is observed that as the bandwidth increases, the effectiveness of the temperature compensation technique decreases, revealing differences among the curves, especially in the higher-frequency regions, for all signals conditions (undamaged, and damage 1 and 2). These findings are presented in detail in Fig. 37.

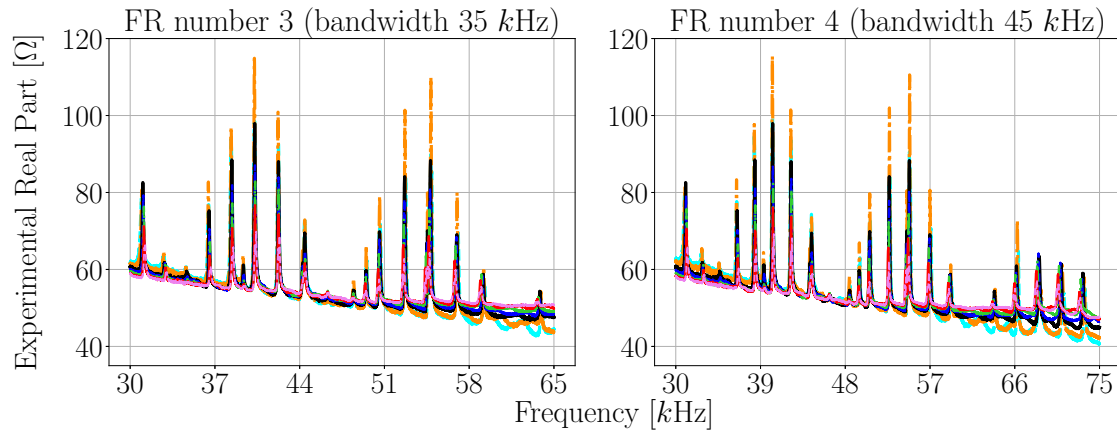
Figure 31 – EMI real part for the undamaged condition after applying the Park *et al.* (1999) technique in FR numbers 1 (left) and 2 (right) at the temperatures: -10°C (thicker dashed cyan line), 4°C (thicker dashed-dotted orange line), 24°C (reference: thicker solid black line), 36°C (dashed blue line), 52°C (dashed-dotted green line), 64°C (solid red line) and 80°C (thinner dashed violet line).



Source: Elaborated by the author.

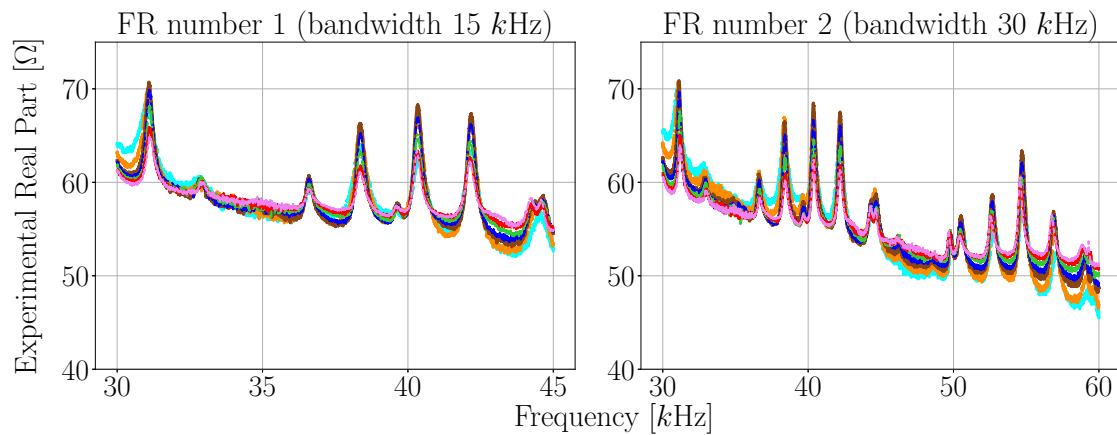
The aforementioned behavior is verified by the different values in the displacements of EMI peaks along the frequency spectrum, with greater shifts observed at higher frequencies and smaller ones at lower frequencies (see Section 4.1). Consequently, a more accurate alignment along the horizontal axis (i.e., in the frequency) occurs for peaks whose frequency shifts are closer to each other. On the other hand, the effectiveness of the temperature compensation approach along the vertical axis (i.e., in the magnitude) decreases as the investigated frequency range (i.e., the bandwidth) increases. This behavior occurs mainly due the addition of a constant value, δ^S , across the entire analyzed frequency range. These

Figure 32 – EMI real part for the undamaged condition after applying the Park *et al.* (1999) technique in FR numbers 3 (left) and 4 (right) at the temperatures: -10°C (thicker dashed cyan line), 4°C (thicker dashed-dotted orange line), 24°C (reference: thicker solid black line), 36°C (dashed blue line), 52°C (dashed-dotted green line), 64°C (solid red line) and 80°C (thinner dashed violet line).



Source: Elaborated by the author.

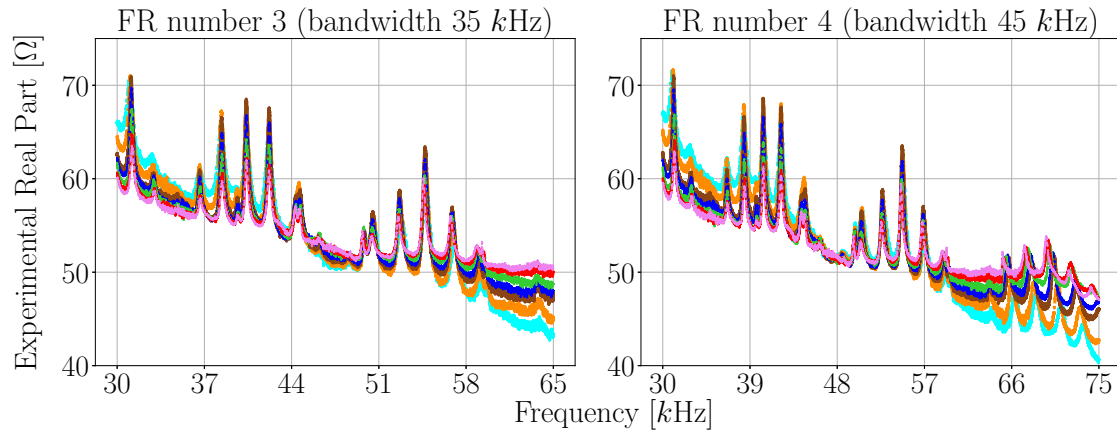
Figure 33 – EMI real part for the damage condition 1 after applying the Park *et al.* (1999) technique in FR numbers 1 (left) and 2 (right) at the temperatures: -10°C (thicker dashed cyan line), 5°C (thicker dashed-dotted orange line), 25°C (thicker solid brown line), 35°C (dashed blue line), 50°C (dashed-dotted green line), 65°C (solid red line) and 80°C (thinner dashed violet line).



Source: Elaborated by the author.

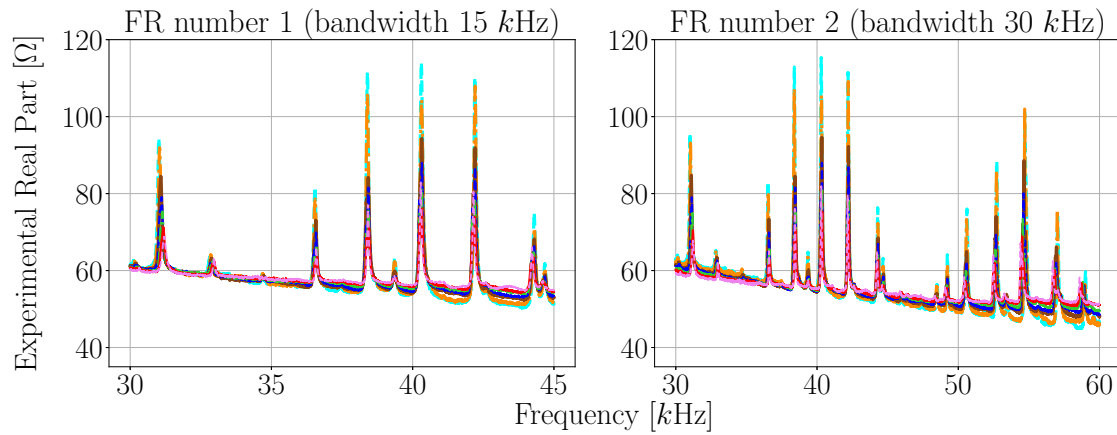
characteristics are observed for all the three structural conditions: undamaged, and damage 1 and 2.

Figure 34 – EMI real part for the damage condition 1 after applying the Park *et al.* (1999) technique in FR numbers 3 (left) and 4 (right) at the temperatures: -10°C (thicker dashed cyan line), 5°C (thicker dashed-dotted orange line), 25°C (thicker solid brown line), 35°C (dashed blue line), 50°C (dashed-dotted green line), 65°C (solid red line) and 80°C (thinner dashed violet line).



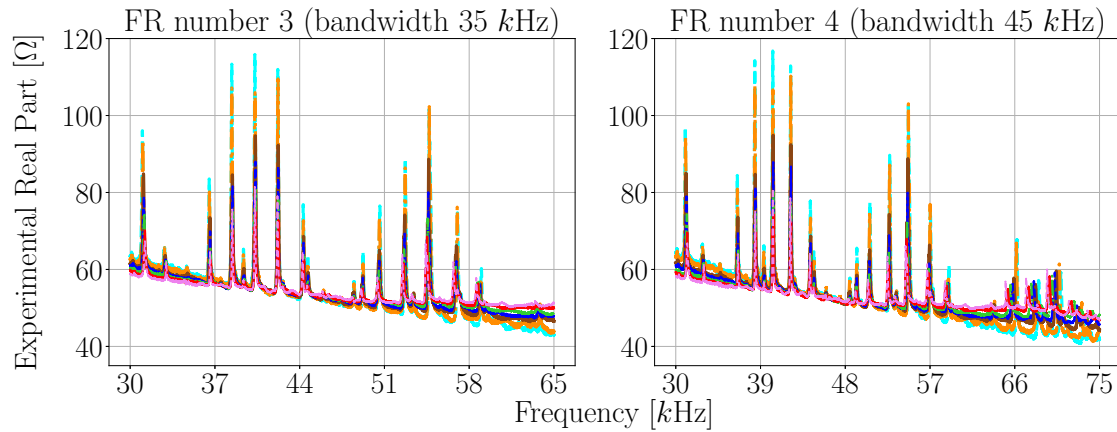
Source: Elaborated by the author.

Figure 35 – EMI real part for the damage condition 2 after applying the Park *et al.* (1999) technique in FR numbers 1 (left) and 2 (right) at the temperatures: -10°C (thicker dashed cyan line), 5°C (thicker dashed-dotted orange line), 25°C (thicker solid brown line), 35°C (dashed blue line), 50°C (dashed-dotted green line), 65°C (solid red line) and 80°C (thinner dashed violet line).



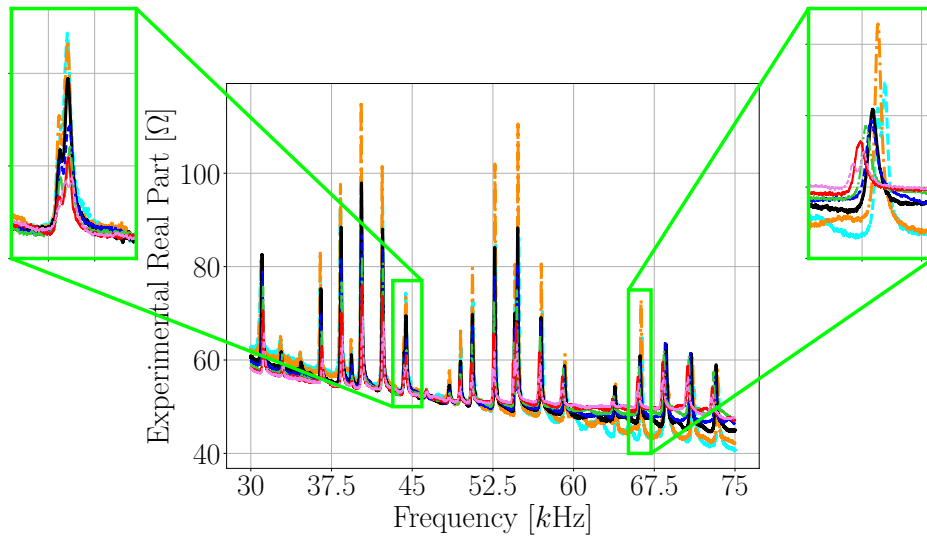
Source: Elaborated by the author.

Figure 36 – EMI real part for the damage condition 2 after applying the Park *et al.* (1999) technique in FR numbers 3 (left) and 4 (right) at the temperatures: -10°C (thicker dashed cyan line), 5°C (thicker dashed-dotted orange line), 25°C (thicker solid brown line), 35°C (dashed blue line), 50°C (dashed-dotted green line), 65°C (solid red line) and 80°C (thinner dashed violet line).



Source: Elaborated by the author.

Figure 37 – A zoomed view in two frequencies regions of the undamaged EMI curve shown in Fig. 32 (right-hand side).



Source: Elaborated by the author.

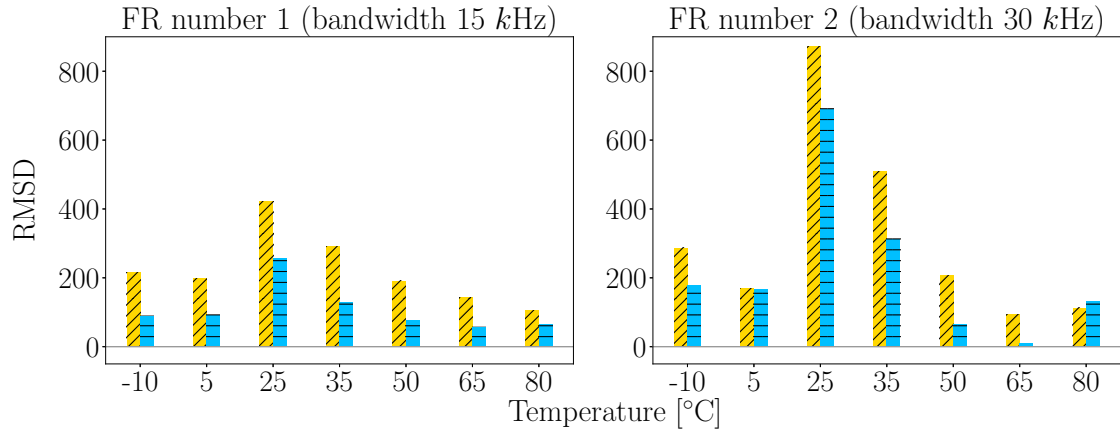
5.1.1.1 Quantitative Analysis of the Park *et al.* (1999) Technique

Evaluating the differences between two EMI signals (corresponding to the baseline and the unknown structural condition) is essential for monitoring and detecting structural damage. Therefore, a quantitative assessment of the EMI curves was conducted to evaluate the variations induced by the presence of damage to further enhance the analyses (Tseng; Naidu, 2002; Gulizzi *et al.*, 2015). In the present thesis, such investigation was performed

using the RMSD and CCDM indices, which are computed according to Eqs. (4.1) and (4.2), respectively.

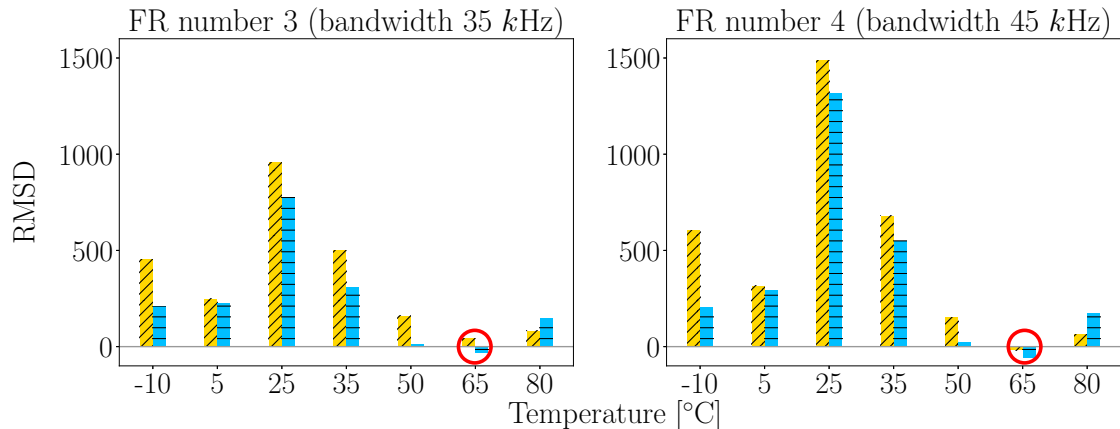
The results of the quantitative analysis of the Park *et al.* (1999) technique are presented in Figs. 38 and 41. These figures show the indices of differences, DI_{diff} , for the RMSD (in Figs. 38 and 39) and CCDM (in Figs. 40 and 41) metrics, calculated as the difference between the indices of the damaged and undamaged conditions obtained at the same temperature T_i , i.e., $DI_{\text{diff}}(T_i) = DI_{\text{damaged}}(T_i) - DI_{\text{undamaged}}(T_i)$, where i indicates the selected temperature

Figure 38 – Difference in RMSD values computed for the signals in the damaged and undamaged conditions, compensated by Park *et al.* (1999) technique, for the FR numbers 1 (left) and 2 (right). Yellow hatched bar (diagonal pattern) depicts damage 1, blue hatched bar (horizontal pattern) represents damage 2, and red circle highlights the false negatives.



Source: Elaborated by the author.

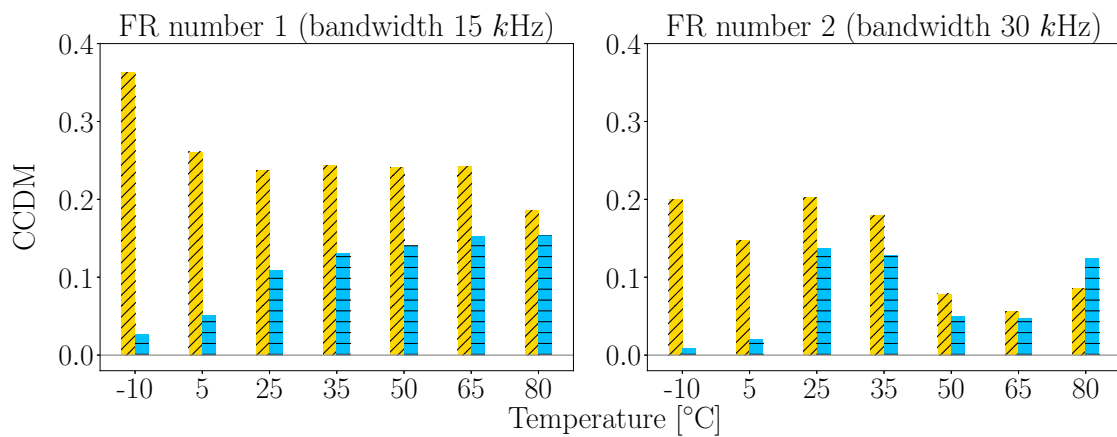
Figure 39 – Difference in RMSD values computed for the signals in the damaged and undamaged conditions, compensated by Park *et al.* (1999) technique, for the FR numbers 3 (left) and 4 (right). Yellow hatched bar (diagonal pattern) depicts damage 1, blue hatched bar (horizontal pattern) represents damage 2, and red circle highlights the false negatives.



Source: Elaborated by the author.

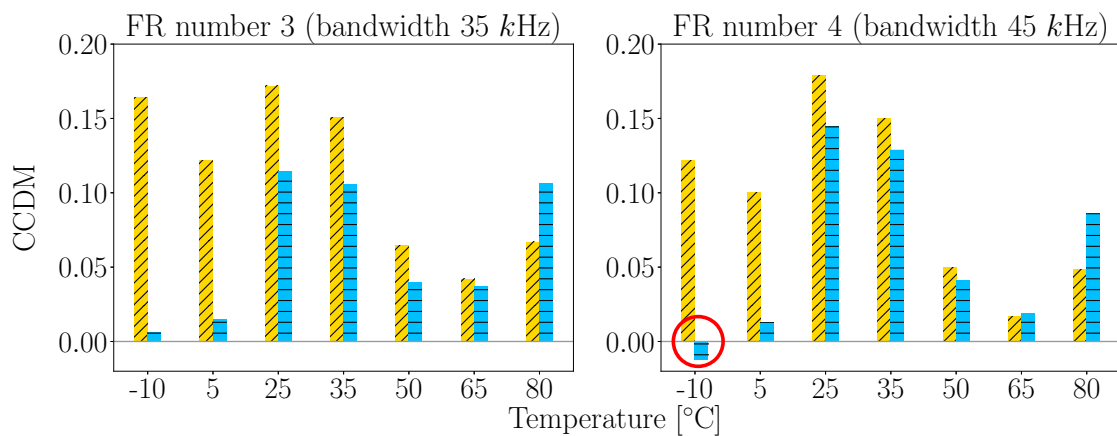
(see Tab. 13). The results are computed for both damage types across the four frequency bandwidths (see Tab. 14). Note that damage is detected at a given temperature if these indices of differences yield positive values. On the other hand, an index equal to zero indicates the absence of damage, while negative values represent a false negative, meaning the damage was not detected due to the influence of temperature.

Figure 40 – Difference in CCDM values computed for the signals in the damaged and un-damaged conditions, compensated by Park *et al.* (1999) technique, for the FR numbers 1 (left) and 2 (right). Yellow hatched bar (diagonal pattern) depicts damage 1, blue hatched bar (horizontal pattern) represents damage 2, and red circle highlights the false negatives.



Source: Elaborated by the author.

Figure 41 – Difference in CCDM values computed for the signals in the damaged and un-damaged conditions, compensated by Park *et al.* (1999) technique, for the FR numbers 3 (left) and 4 (right). Yellow hatched bar (diagonal pattern) depicts damage 1, blue hatched bar (horizontal pattern) represents damage 2, and red circle highlights the false negatives.



Source: Elaborated by the author.

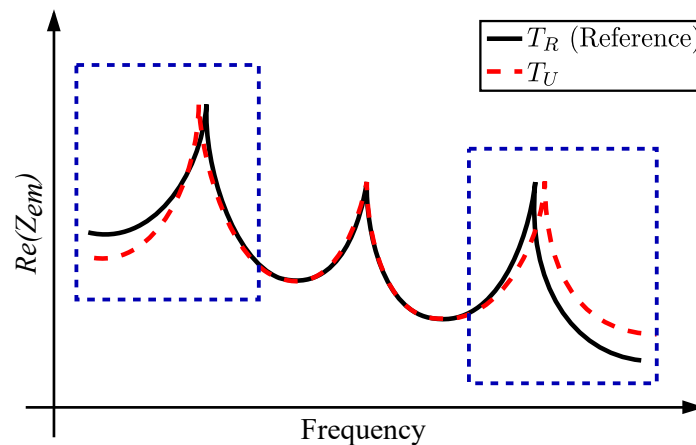
The outcomes for both indices – RMSD and CCDM – reveal that the Park *et al.* (1999) technique allows one to detect both damage types in different temperatures. However,

in particular, at 65°C, the RMSD-based index (Fig. 39) fails to detect both damage types using the frequency range number 4. It is also unable to detect damage type 2 using the frequency range number 3. Similarly, the CCDM-based index fails at -10°C for the frequency range number 4, specifically for damage 2 (Fig. 41). These cases with false negatives in terms of damage detection and their corresponding temperatures are highlighted by red circles in both Figs. 39 and 41. These findings demonstrate that the frequency range considered to compute damage detection indices affects the accuracy of the SHM process when applying the Park *et al.* (1999) technique. Furthermore, the results suggest that larger frequency bandwidths tend to produce less accurate detections, increasing the likelihood of false negatives in identifying damage presence. Hence, this next section introduces a new approach to overcome this practical limitation.

5.1.2 The Proposed Approach

The enhanced technique is developed to eliminate false negatives, and it introduces a strategy to remove regions of the compensated EMI curves obtained at a temperature T_U that present important discrepancies in their real part when compared with the reference signal measured at a temperature T_R . These differences of shape are illustrated in the schematic drawing shown in Fig. 42, in which two rectangles are used to highlight the frequency ranges with the higher differences.

Figure 42 – Representation of typical discrepancies between the compensated (Z_{em_U}) and reference (Z_{em_R}) EMI curves.



Source: Elaborated by the author.

The approach considers parts of the clipped EMI curve measured in a different temperature T_U from the reference one T_R , and employs the undamaged signals for its execution. The procedure is implemented and executed computationally as follows:

- **Step 1:** consider the frequency ranges and temperatures to be used for the application of this new approach. It is recommended to assess a broader frequency range, or even

the entire measured spectrum, rather than a smaller frequency range.

- **Step 2:** identify the peaks in the reference signal. The procedure is done by applying the *prominence* parameter. The function takes the signal and finds all local maxima by simple comparison of neighboring values. The prominence of a peak measures how much it stands out from the surrounding signal and is defined as the vertical distance between the peak and its lowest contour line. In practice, the analyst needs to observe the original curve and defines a minimum value to establish this contour line. This procedure is employed to avoid detecting small peaks associated with the noise interference in the measured signals. A greater prominence corresponds to a higher EMI peak.
- **Step 3:** remove the frequency ranges containing the identified peaks to eliminate their influence on the EMI curves. Figure 43(a) illustrates the real part of a generic EMI signal and Fig. 43(b) show it after removing the frequency ranges associated with the peaks. Note that there are limiting frequencies f_{lim_i} , $i = 1, 2, \dots, 2N_{pk} + 2$ for a given number of peaks N_{pk} . The new clipped frequency range f_{cp} , considered to evaluate the EMI curve in the proposed damage detection process, is defined by:

$$f_{cp} = [f_{lim_a}, f_{lim_b}], \begin{cases} a = 1, 3, \dots, N_{pk} + 1 \\ b = 2, 4, \dots, N_{pk} + 2 \end{cases}. \quad (5.3)$$

N_r points to the right and left-hand sides of each peak are taken from the original frequency f_{or} , obtaining the clipped frequency f_{cp} . The corresponding number of points are given by:

$$N_{cp} = N_{or} - 2N_r \times N_{pk}. \quad (5.4)$$

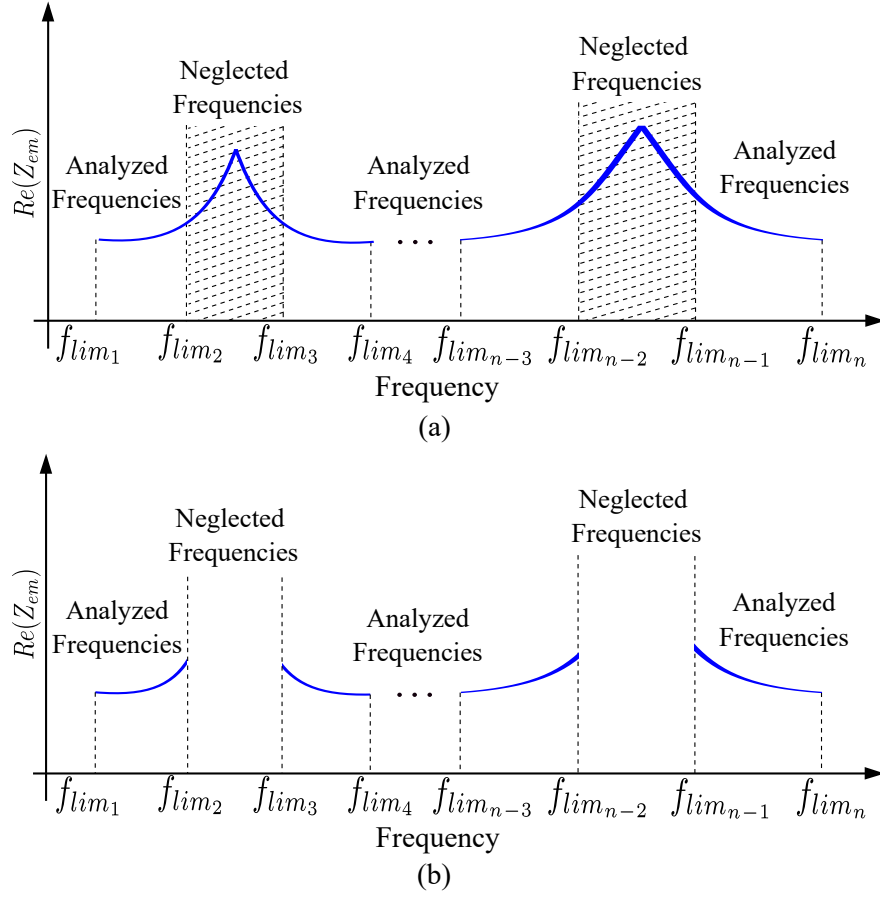
N_r is obtained by selecting the smallest number of points for which the introduced parameter S reaches its approximately stable value, which usually corresponds to a numerical variation smaller than 0.001. S is defined by the division between the amplitude of the real part of the clipped signal $\Delta Z_{em_{cp}}$ and the amplitude of the real part of the original signal $\Delta Z_{em_{or}}$, as shown in Eq. (5.5). The values of S are computed for different N_r until it assumes its stable value. Figure 44 shows the behavior of S when varying the number of points removed from the curve, and highlights the chosen N_r by the biggest red circle.

$$S = \frac{\Delta Re(Z_{em_{cp}})}{\Delta Re(Z_{em_{or}})}, \quad (5.5)$$

where

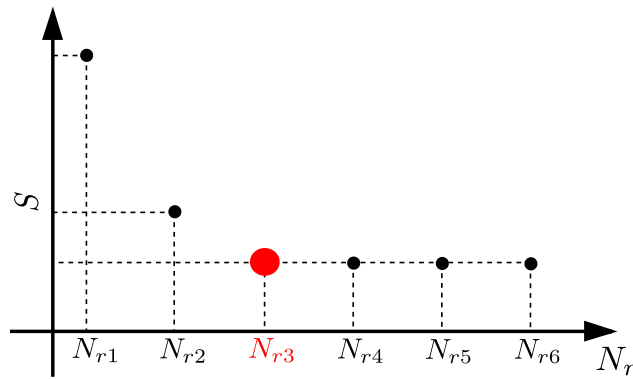
$$\Delta Z_{em_{()}} = \max(Z_{em_{()}}) - \min(Z_{em_{()}}). \quad (5.6)$$

Figure 43 – Signal (a) before and (b) after removing the frequencies associated with the peaks, where $n = 2N_{pk} + 2$.



Source: Elaborated by the author.

Figure 44 – S -parameter as a function of N_r and the number of points chosen in red (N_{r3}).



Source: Elaborated by the author.

- **Step 4:** find the absolute differences between the means of the unknown signals and the reference curve. The frequency f_{cp} is divided into n_f frequency intervals or sub-bands, which are employed to analyze the absolute difference between the mean value of the reference signal and the mean value of the unknown signal in that sub-band. These

frequency intervals, f_{sub} , is defined as:

$$f_{sub} = [f_{cp1+a*N_{dif}}, f_{cpN_{dif}+a*N_{dif}}], \quad (5.7)$$

$$a = 0, 1, 2, \dots, n_f.$$

In the present thesis, it is used $n_f = 21$ intervals. Note that the standard deviation (Eq. 5.8) can be computed by considering N_{dif} in the denominator. However, if $n_f \leq 20$, the common statistical approach suggests using $(N_{dif} - 1)$ instead. In practice, several computational tests indicate that $n_f > 20$ provides accurate results for this proposed approach. Then, Eq. (5.8) is employed to calculate the standard deviation of the mean differences vector, **Dif**, where $\overline{Dif}$ is its average value.

$$\sigma_{Dif} = \sqrt{\frac{\sum_{i=1}^{21} (\mathbf{Dif}_i - \overline{Dif})^2}{N_{dif}}}, \quad (5.8)$$

where

$$N_{dif} = \text{int} \left(\frac{N_{cp}}{n_f - 1} \right). \quad (5.9)$$

The width of the sub-bands is determined and a temperature of the electromechanical impedance signal is selected to perform the difference analysis. Then, the vector **Dif** of the mean differences (absolute value) is computed for each sub-band as shown in Eq. (5.10).

$$\mathbf{Dif} = \left\{ \left| \bar{Z}_{em_{R_1}} - \bar{Z}_{em_{U_1}} \right| \left| \bar{Z}_{em_{R_2}} - \bar{Z}_{em_{U_2}} \right| \dots \left| \bar{Z}_{em_{R_{n_f}}} - \bar{Z}_{em_{U_{n_f}}} \right| \right\}, \quad (5.10)$$

where

$$\bar{Z}_{em_{()}_y} = \frac{\sum_{i=f_{subx}} Z_{em_{()}}(\omega_i)}{N_{dif}}, \quad x, y = 1, 2, \dots, n_f - 1, \quad (5.11)$$

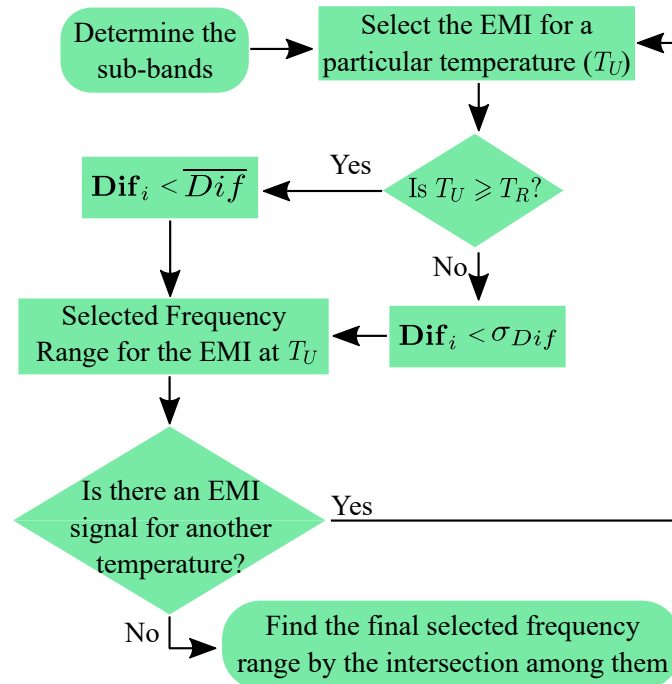
$$\bar{Z}_{em_{()}_n} = \frac{\sum_{i=f_{subn_f}} Z_{em_{()}}(\omega_i)}{N_{cp} - (n_f - 1)N_{dif}}, \quad (5.12)$$

in which Eq. (5.12) uses the remaining points of the clipped signal, calculated as: $N_{cp} - (n_f - 1)N_{dif}$, to determine the average electromechanical impedance.

The proposed procedure requires to define whether the chosen temperature is higher or lower than the reference temperature, since the curves of the vector of differences presented relatively different behaviors as a function of this condition. In the case of temperatures below the reference temperature, the selected frequency range is determined by selecting the largest frequency interval whose **Dif** is less than σ_{Dif} . On the other hand, for temperatures above the reference temperature, the selected frequency range is determined by selecting the largest frequency interval whose **Dif** is less than its average, $\overline{Dif}$. This process is repeated for all temperatures chosen in Step 1. Lastly,

the intersection of all the selected ranges obtained for each temperature is determined, thus obtaining the final selected frequency range. The step-by-step of this stage is introduced in the flowchart shown in Fig. 45.

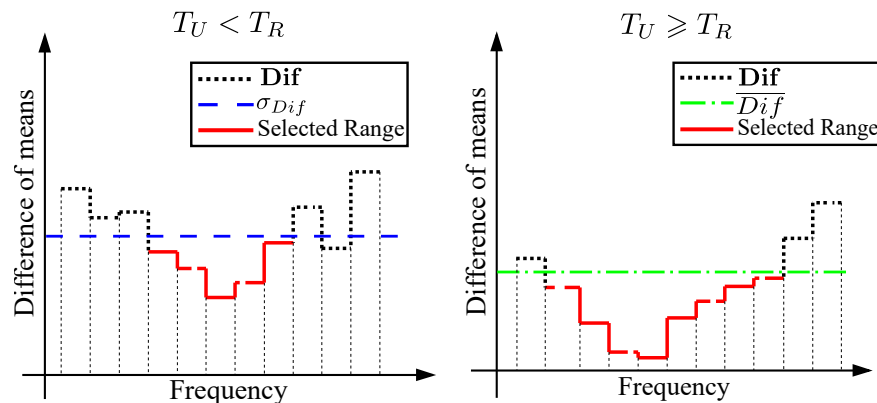
Figure 45 – Flowchart showing the sequential procedures involved in Step 4.



Source: Elaborated by the author.

Figure 46 contains the curves of the mean differences obtained for an arbitrary signal at a different temperature T_U , both above and below the reference temperature T_R . The thick red line indicates the selected frequency range, since it corresponds to the largest part whose **Dif** is less than σ_{Dif} , for $T_U < T_R$, or $\overline{Dif}$, for $T_D \geq T_R$. Note that there are some blank spaces, which correspond to the location of the removed peaks.

Figure 46 – Mean differences curves of an illustrative and arbitrary signal for scenarios in which T_U is lower (left) or higher (right) than T_R .

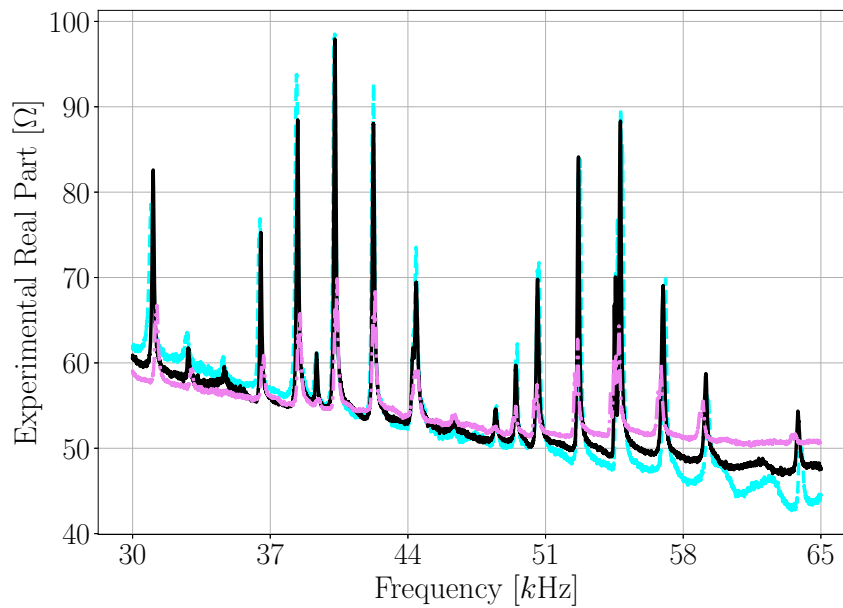


Source: Elaborated by the author.

5.1.2.1 Analysis of The New Approach

This study analyzes the EMI signals shown in Fig. 47 to demonstrate the proposed approach, which contains the reference curve (solid black line), acquired at 24°C, along with two additional signatures measured from the undamaged state at -10°C (dashed cyan line) and 80°C (dashed-dotted violet line). These signals are obtained after applying the Park *et al.* (1999) technique. For illustrative purposes, and without loss of generality, the procedure is specifically implemented on frequency range number 3, corresponding to a 35 kHz bandwidth. The results of applying the proposed approach (see Section 5.1.2) to these EMI signals are presented below.

Figure 47 – EMI real for the undamaged condition obtained after applying the Park *et al.* (1999) technique in the frequency range number 3 at temperatures of -10°C (dashed cyan line), 24°C (reference: solid black line), and 80°C (dashed-dotted violet line).

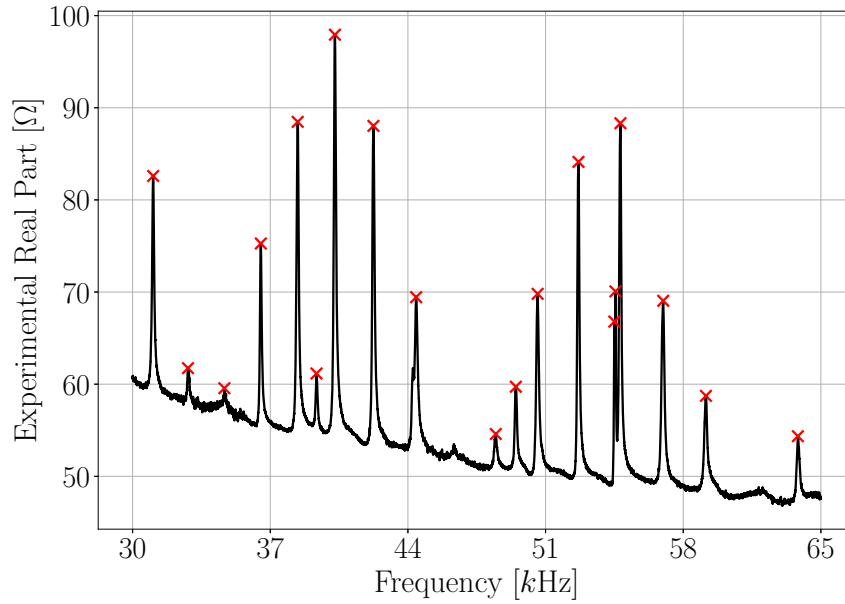


Source: Elaborated by the author.

According to this proposed approach, an important step consists in identifying the peak values of the real part of the reference EMI. Figure 48 shows the curve for the reference temperature (24°C), and the peaks are highlighted by red symbols ($\times$). Note that even small peaks are identified by employing the *prominence* equal to 2, and their associated frequencies are accurately determined based on their corresponding position in the frequency vector.

Subsequently, the S -parameter values need to be determined. The methodology is applied exclusively to frequency range number 3; however, Tab. 15 presents the S values computed for all investigated frequency ranges. Note that it is approximately a constant value (around 10^3) for the frequency ranges 2, 3 and 4. However, its variation is more significant for the frequency range number 1. This behavior suggests that the approach can not be

Figure 48 – Reference signal (real part of the EMI) with the identified peaks highlighted by red $\times$ symbols.



Source: Elaborated by the author.

successfully applied when using this specific frequency range. Therefore, the evaluation of S -parameter is an important part of this proposed strategy to select the frequency range. Based on the $S \times N_r$ data presented in Tab. 15, the parameter S achieves a numerically stable value when considering $N_r \approx 300$ points for FRs numbers 2, 3 and 4.

Table 15 – S and N_r values for the four frequency ranges (FR).

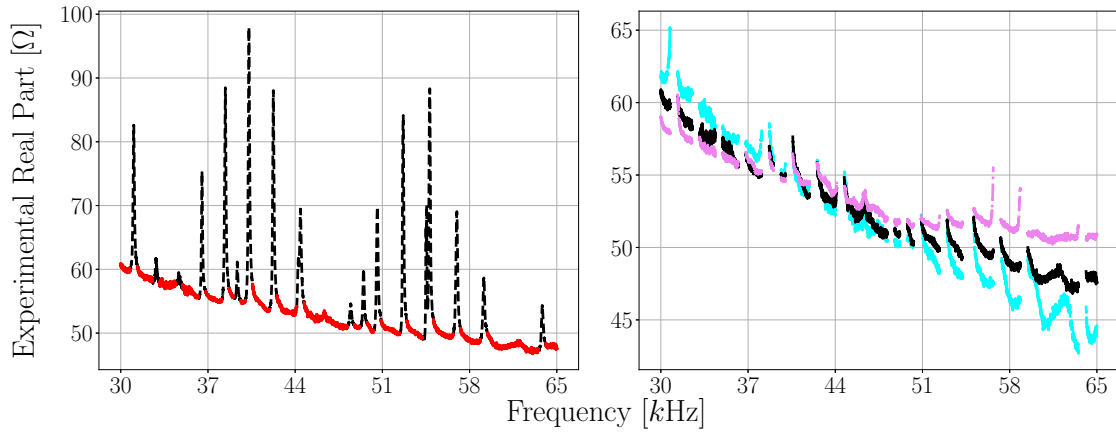
FR N_r	S			
	1	2	3	4
100	0.3368	0.3989	0.4133	0.4376
200	0.1933	0.2688	0.2863	0.3159
300	0.1812	0.2578	0.2756	0.3056
400	0.1812	0.2578	0.2756	0.3056
500	0.1812	0.2578	0.2756	0.3054
600	0.1761	0.2578	0.2756	0.3054
700	0.1740	0.2578	0.2756	0.3054

Source: Elaborated by the author.

Figure 49 shows the signal parts retained after removing the influence of the peak values, i.e., the clipped curves, and a noticeable reduction of their impact can be observed. The vector of the means differences are obtained by considering N_{dif} equal to 1224 points. Figure 50 presents this vector **Dif** for temperatures of -10°C (left) and 80°C (right), and one can

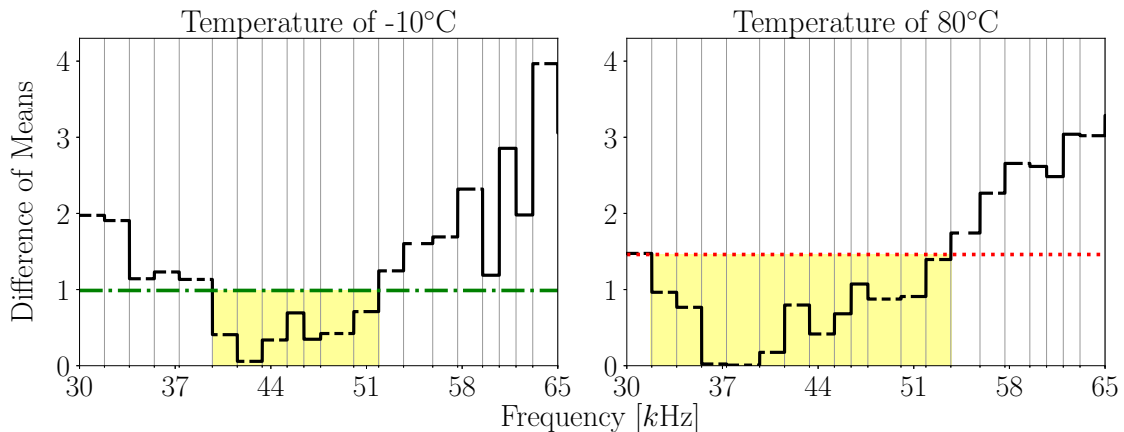
notice distinct behaviors when comparing them with each other. These variations suggest that different strategies need to be applied to select the frequency range depending on whether the unknown temperature is above or below the reference temperature. Therefore, the employed procedure considers that when $T_U < T_R$, the selected frequency range corresponds to the interval where the values of **Dif** are smaller than its standard deviation σ_{Dif} . Conversely, if $T_U \geq T_R$, the selected frequency range is defined by the region where **Dif** values remain below its mean $\overline{Dif}$. Using this selection strategy, it is possible to evaluate more accurately the EMI signals measured at different temperatures.

Figure 49 – Clipped EMI curves. Left-hand side: original (dashed black line) and clipped (solid red line) curves at the reference temperature. Right-hand side: clipped curves at the three different temperatures: -10°C (dashed cyan line), 24°C (reference: solid black line), and 80°C (dashed-dotted violet line).



Source: Elaborated by the author.

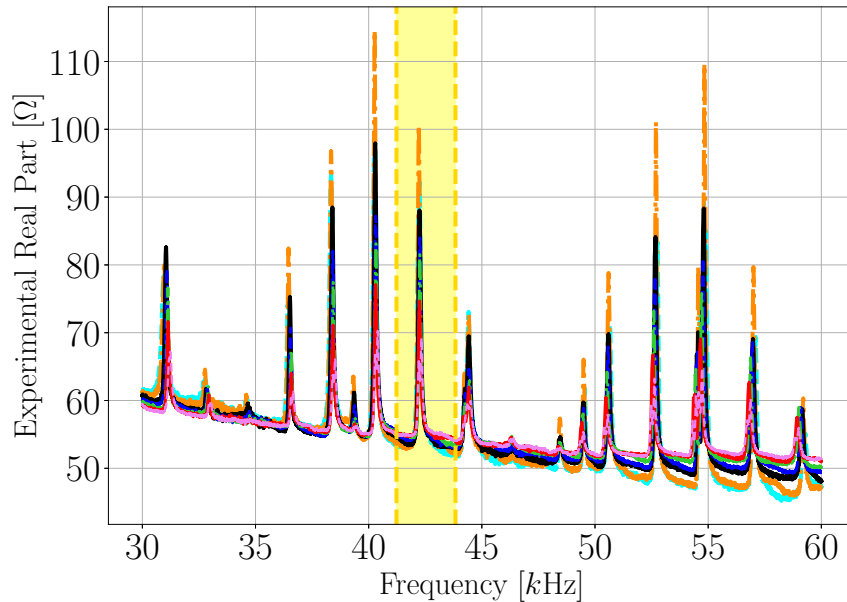
Figure 50 – Curves of the absolute differences of means for the temperatures of -10°C (left) and 80°C (right). The solid black line represents **Dif**, the dashed-dotted green line corresponds to σ_{Dif} , and the dotted red line indicates $\overline{Dif}$. The yellow region represents the selected frequency range, which is 39.72 to 51.89 kHz for $T_U = -10^\circ\text{C}$ and 31.82 to 53.71 kHz for $T_U = 80^\circ\text{C}$.



Source: Elaborated by the author.

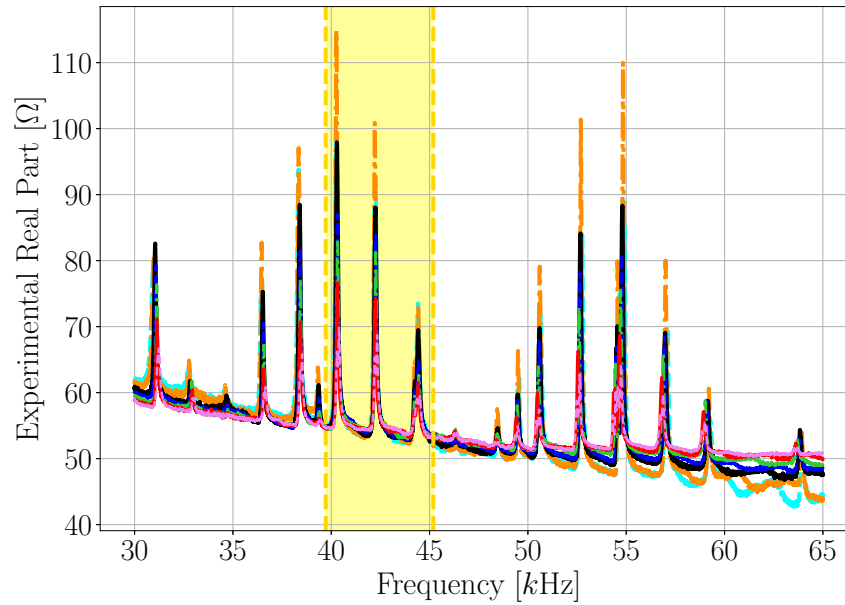
The strategy employed to select the frequency range demonstrated above is also applied for the undamaged system at the remaining temperatures (4°C, 36°C, 52°C and 64°C) to determine the final selected frequency range (see Step 4 in Section 5.1.2). Figures 51 to 53 present this final range, delimited by the vertical yellow lines, for FRs numbers 2, 3, and 4, respectively. The results for frequency range number 1 (bandwidth 15 kHz) are not displayed, as the proposed approach cannot be applied to this FR since its S -parameter has not stabilized (see Tab. 15). By analyzing Figs 51 to 53, it is observed that the new frequency ranges are located in the regions where the EMI curves exhibit the smallest discrepancies. This behavior confirms that the proposed approach successfully removes the sections of the compensated EMI curves that show significant deviations from the reference signal. Furthermore, to assess whether the false negatives have been eliminated, Figs. 54 to 55 present the re-computed RMSD and CCDM-based indices, considering the new selected frequency ranges for the originals FRs numbers 2, 3 and 4, respectively. The findings confirm that both damage types are successfully detected at all temperatures, meaning that the new approach has effectively eliminated the false negatives.

Figure 51 – EMI real part for the undamaged condition (Fig. 31, right) in FR number 2, with the final selected frequency range, from 41.23 to 43.84 kHz, delimited by vertical yellow lines.



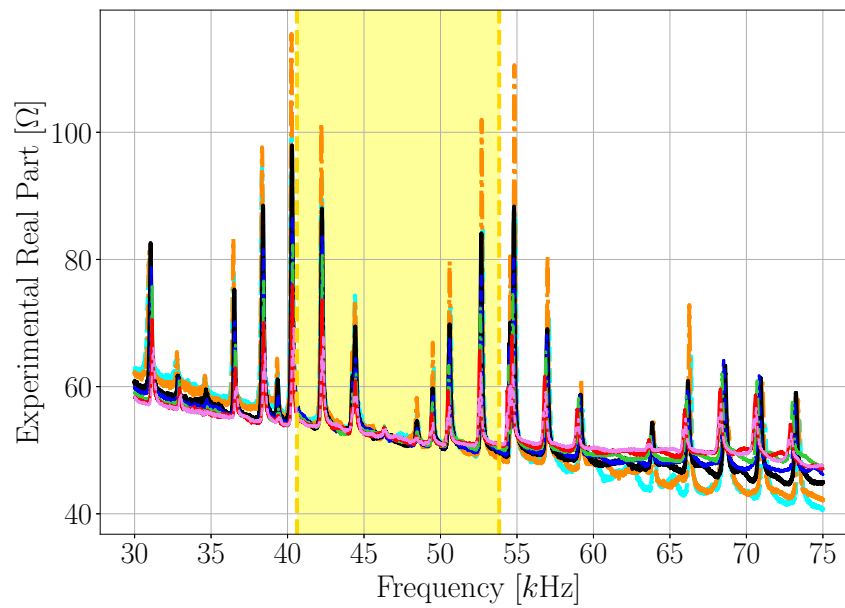
Source: Elaborated by the author.

Figure 52 – EMI real part for the undamaged condition (Fig. 32, left) in FR number 3, with the final selected frequency range, from 39.72 to 45.19 kHz , delimited by vertical yellow lines.



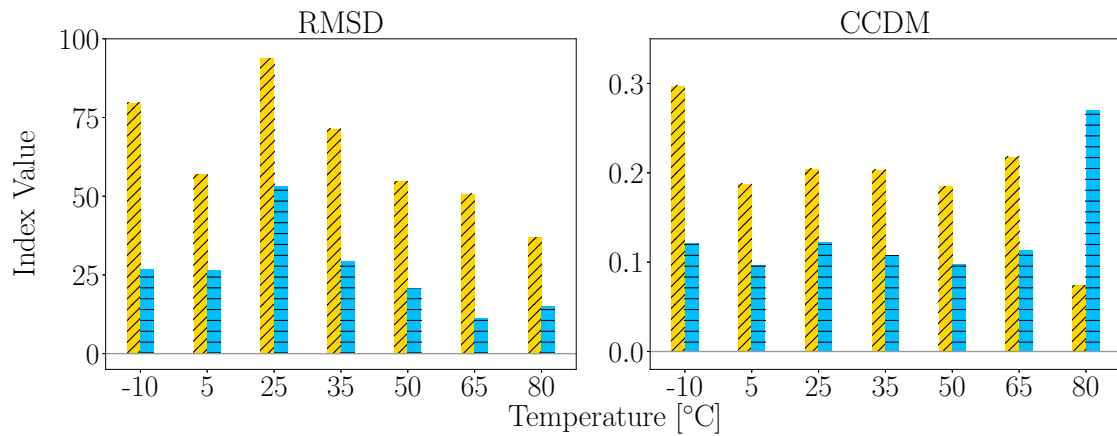
Source: Elaborated by the author.

Figure 53 – EMI real part for the undamaged condition (Fig. 32, right) in FR number 4, with the final selected frequency range, from 40.62 to 53.84 kHz , delimited by vertical yellow lines.



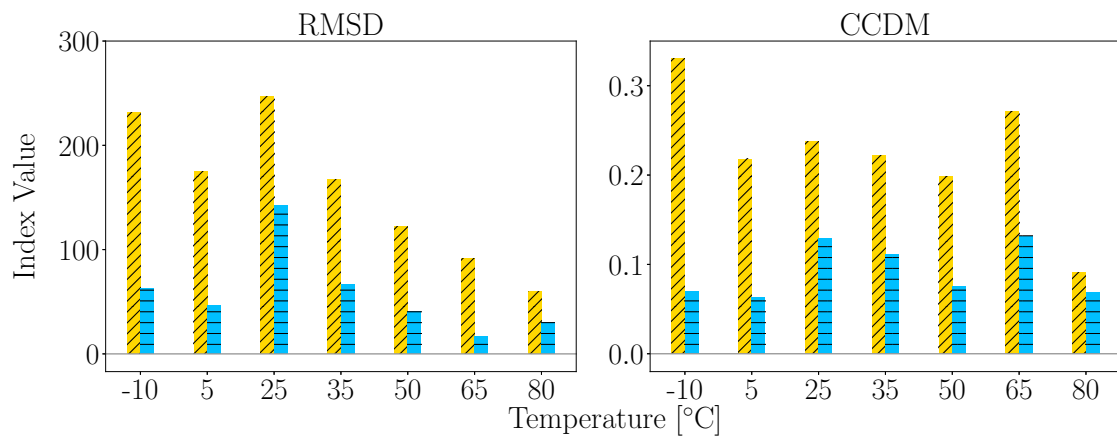
Source: Elaborated by the author.

Figure 54 – Difference in RMSD (left) and CCDM (right) values computed for the signals in the damaged and undamaged conditions after applying the new approach for the FR number 2. Yellow hatched bar (diagonal pattern) depicts damage 1 and blue hatched bar (horizontal pattern) represents damage 2.



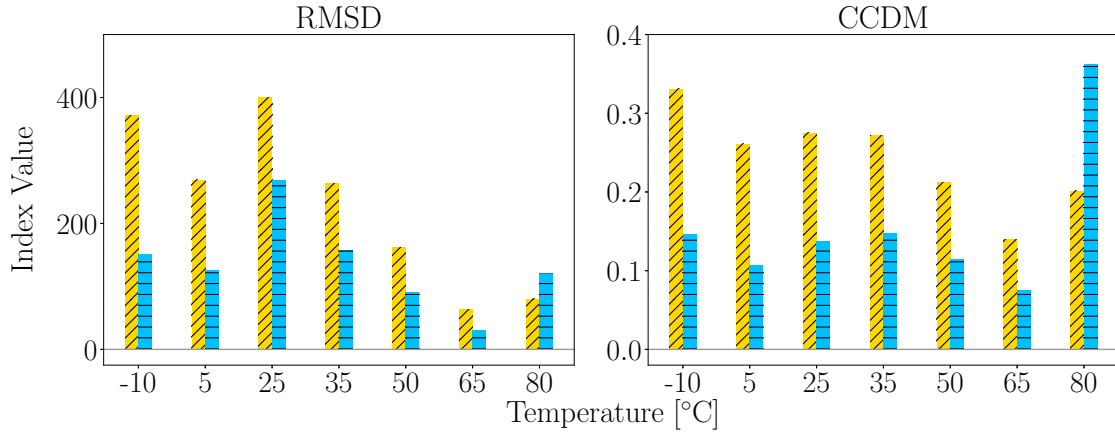
Source: Elaborated by the author.

Figure 55 – Difference in RMSD (left) and CCDM (right) values computed for the signals in the damaged and undamaged conditions after applying the new approach for the FR number 3. Yellow hatched bar (diagonal pattern) depicts damage 1 and blue hatched bar (horizontal pattern) represents damage 2.



Source: Elaborated by the author.

Figure 56 – Difference in RMSD (left) and CCDM (right) values computed for the signals in the damaged and undamaged conditions after applying the new approach for the FR number 4. Yellow hatched bar (diagonal pattern) depicts damage 1 and blue hatched bar (horizontal pattern) represents damage 2.



Source: Elaborated by the author.

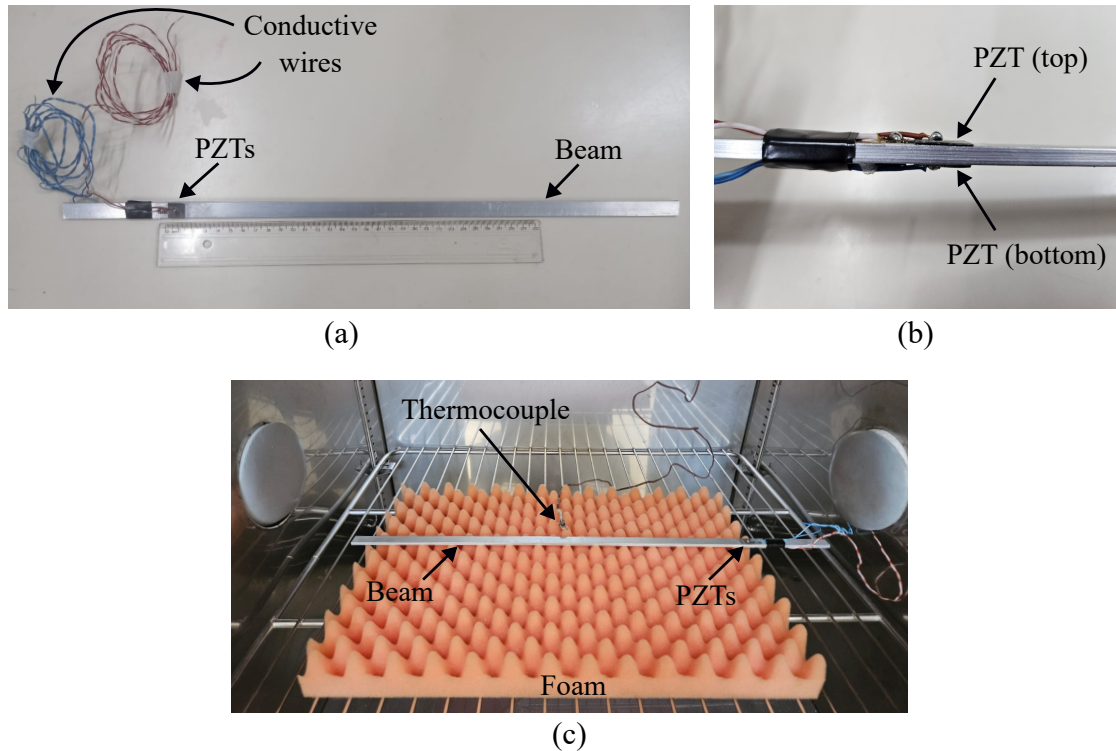
5.2. ANALYTICAL MODEL VALIDATION

The second dataset of EMI measurements was collected by this author employing a distinct piezoelectric configuration to the previous dataset. In this arrangement, the PZTs were symmetrically coupled to the aluminum beam in relation to its longitudinal direction, as shown in Fig. 57(a)-(b). Different structural conditions were considered for the three dynamics: longitudinal, flexural and the coupled one. The instrumented beam was placed on a foam layer to replicate a free-free boundary condition, as presented in Fig. 57(c). The properties of the piezoelectric transducer and the beam at 24°C are detailed in Tabs. 7 and 8, respectively.

The arrangement of the connection between the piezoelectric transducers plays an important role in ensuring the proper excitation of the desired dynamics. By applying in-phase electrical potentials to the PZTs, axial forces are induced, thereby generating longitudinal waves. In contrast, out-of-phase potentials produce bending moments, leading to the excitation of flexural waves (Brennan, 1994; Tseng; Wang, 2005a). Lastly, to obtain the EMI signals associated with the coupled dynamics, only one of the PZTs is employed, thus simulating an asymmetric piezoelectric configuration.

After measuring the nominal (baseline) condition at each temperature, two damage configurations – symmetric and asymmetric – were introduced to the beam independently to each other, i.e., each damage type was applied to a separate beam specimen, ensuring that only one form of damage was present per structure. Both damage types were 25 mm in length and were positioned 94 mm from the PZTs ($x_d = 187$ cm). The damages were machined using a Emco Emcotronic® Computer Numerically Controlled (CNC) lathe, model F1. For the

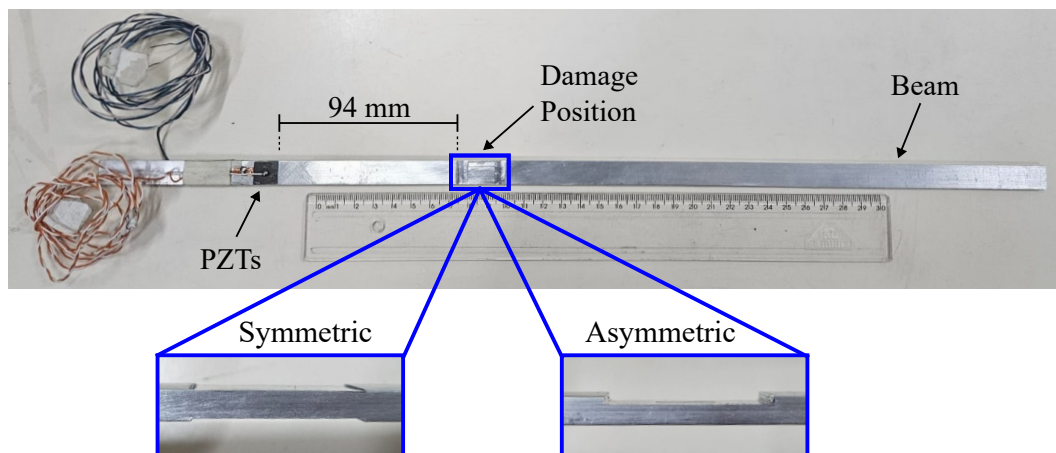
Figure 57 – (a) Aluminum beam with (b) symmetrically coupled PZTs (c) placed on a foam layer inside the thermal chamber.



Source: Elaborated by the author.

symmetric damage cases, two identical thickness reductions were introduced on the upper and lower surfaces of the beam, resulting in total cross-sectional area reductions of 3%, 6%, and 15% relative to the beam's original thickness, as shown in Fig. 58. Conversely, the asymmetric damage involved material removal from only one surface of the beam, with cross-sectional area reductions of 5%, 10%, and 20%, as shown in Fig. 58.

Figure 58 – Beam-like structure with symmetrically coupled PZTs and two types of damage: symmetric and asymmetric.



Source: Elaborated by the author.

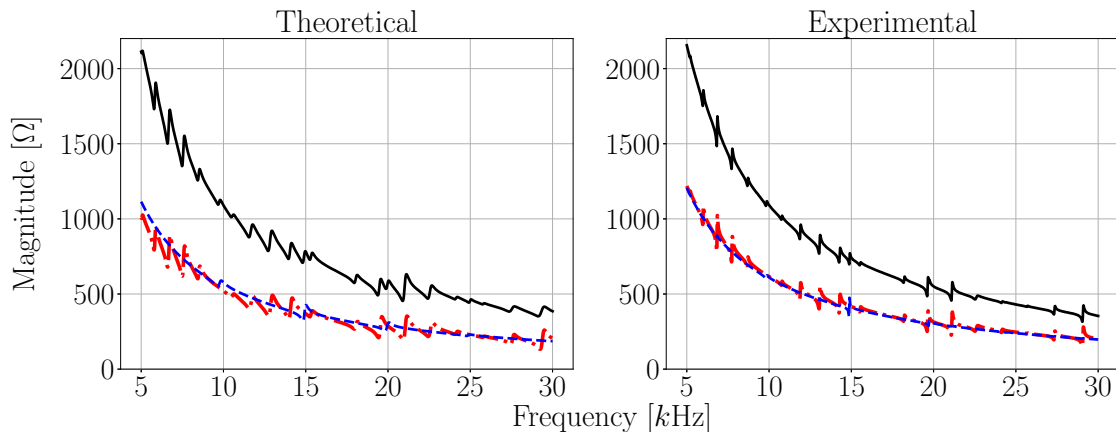
The EMI signals were measured in a temperature range from 24°C to 65°C. An incre-

mental step of 10°C and 20°C was considered for the undamaged and damaged conditions, respectively. A sinusoidal frequency sweep (*chirp*) with an amplitude of ± 1 V in a frequency range from 1 Hz to 100 kHz, with increments of 1 Hz, was used to excite the structure through the PZTs. Ten sets of signals were collected for each temperature to verify the reliability of the results, exclusively during the heating process of the beam, and 30 minutes after achieving the steady state for each temperature.

The theoretical simulations were performed separately for the longitudinal, flexural and coupled dynamics. All theoretical results were obtained using an in-house Python[®]-based algorithm. The EMI curves were analyzed across a frequency range from 5 kHz to 30 kHz, as this represents the upper limit within which Euler-Bernoulli beam theory remains valid (see Section 3.4). A total of $n_L = 14$ and $n_F = 35$ modes were considered for the longitudinal and flexural dynamics, respectively.

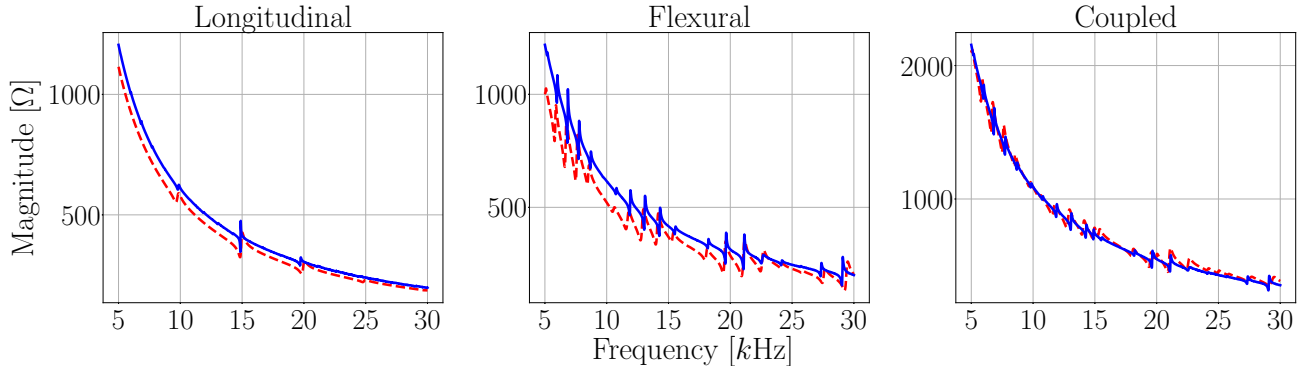
The analytical model is mainly validated by analyzing the real part of the EMI curves, as it exhibits greater sensitivity to structural damage. Nevertheless, as supplementary material for the reader, Figs. 59 to 62 present the results for the magnitude (in Fig. 59 and 60) and the imaginary part (in Fig. 61 and 62) at the reference temperature of 24°C. These figures provide both an overall (in Fig. 59 and 61) and a curve-by-curve (in Fig. 60 and 62) comparison between the theoretical and experimental EMI signals for the three dynamics. Note that there is a good agreement between the two types of curves, with the signatures exhibiting consistent behavioral patterns across all investigated dynamics, thereby demonstrating the accuracy of the proposed approach. However, upon examining Figs. 60 and 62, it becomes evident that the most pronounced discrepancies between the theoretical and experimental signals occur in the longitudinal and flexural dynamics. According to Bhalla and Soh (2004), an increase in the thickness of the adhesive layer leads to a downward shift in the imaginary part of the EMI curve and, consequently, an increase in its magnitude. Since the proposed model neglects the

Figure 59 – EMI magnitude at 24°C: theoretical (left) and experimental (right) results for longitudinal (dashed blue line), flexural (dashed-dotted red line) and coupled (solid black line) dynamics.



Source: Elaborated by the author.

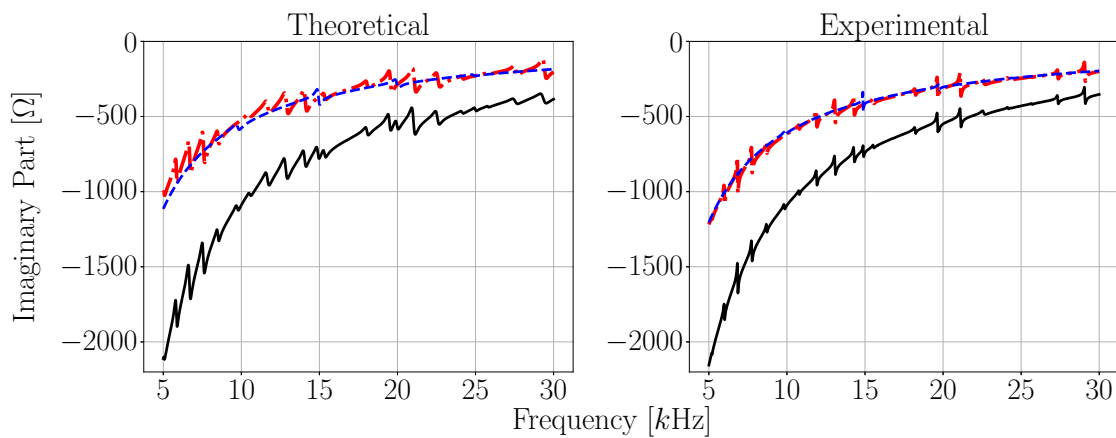
Figure 60 – EMI magnitude of the theoretical and experimental EMI curves for the three dynamics at 24°C. Red dashed line represents the theoretical curve and blue solid line corresponds to the experimental curve.



Source: Elaborated by the author.

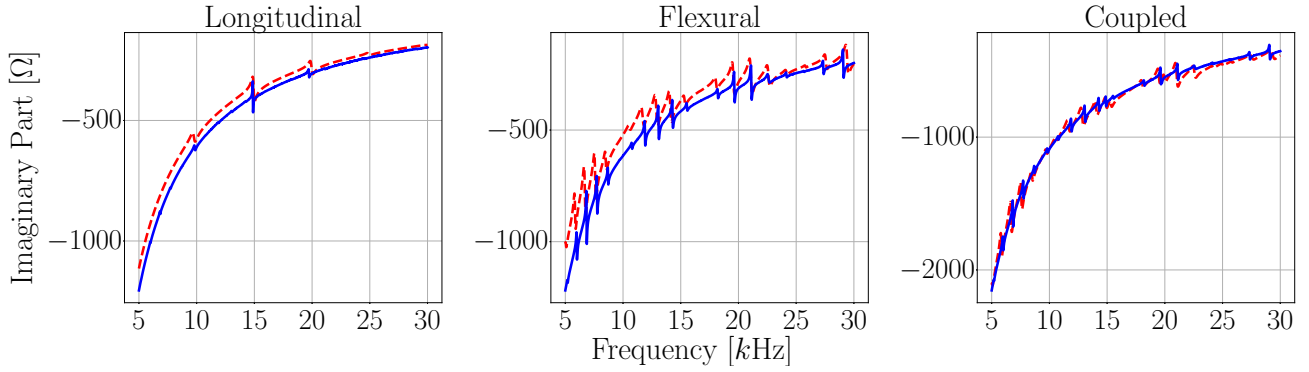
presence of this adhesive layer, its greater influence in the longitudinal and flexural dynamics – resulting from the use of two PZTs in the experimental setup – may have contributed to the more pronounced discrepancies observed, when compared to the coupled dynamics, which employs only a single PZT. Nevertheless, a closer examination of Figs. 60 and 62 reveals that the number of peaks in the experimental and theoretical data is identical for both the magnitude and the imaginary part, with only minor discrepancies in terms of damping influence. Note that all theoretical results presented herein incorporate the adjusted damping coefficients, $\alpha_{()}$ and $\beta_{()}$, which demonstrates the feasibility and sufficient accuracy of this methodology.

Figure 61 – EMI imaginary part at 24°C: theoretical (left) and experimental (right) results for longitudinal (dashed blue line), flexural (dashed-dotted red line) and coupled (solid black line) dynamics.



Source: Elaborated by the author.

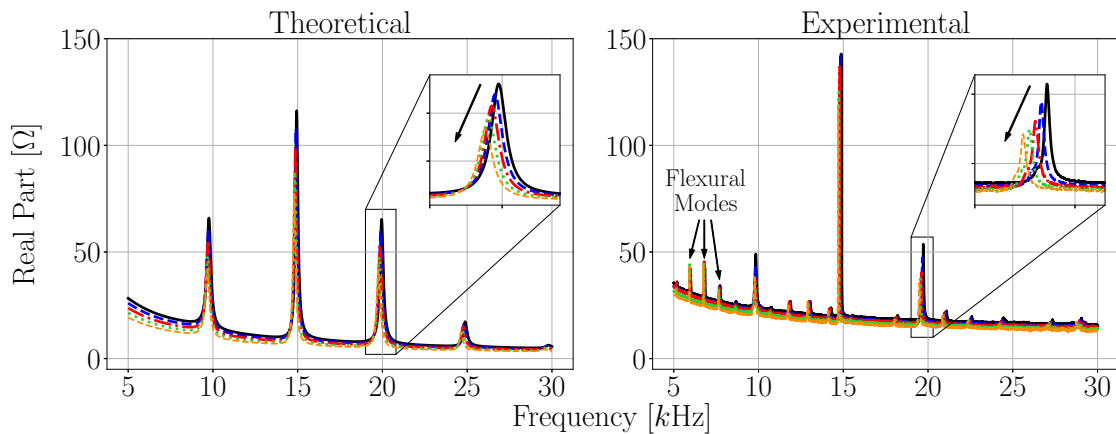
Figure 62 – EMI imaginary part of the theoretical and experimental EMI curves for the three dynamics at 24°C. Red dashed line represents the theoretical curve and blue solid line corresponds to the experimental curve.



Source: Elaborated by the author.

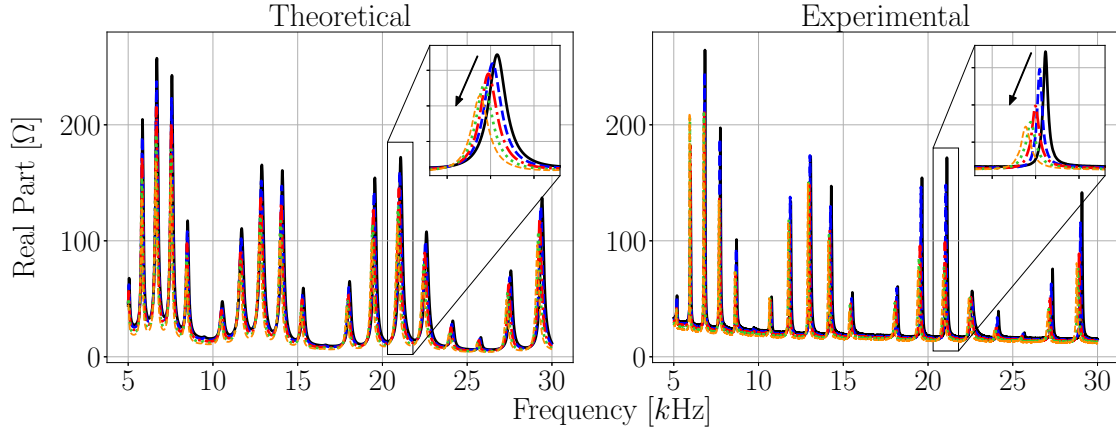
Figures 63 to 65 show the theoretical (right) and experimental (left) EMI real parts at the five investigated temperatures – 24°C, 35°C, 45°C, 55°C, and 65°C –, respectively to the longitudinal, flexural and coupled dynamics. These graphics demonstrate that an increase in temperature result in a decrease in both peak amplitude and frequency, a trend consistently observed at all investigated temperatures. It is a general behavior previously noted in literature (Yang; Lim; Soh, 2008a; Baptista *et al.*, 2014), and these results exhibit it for each dynamics (longitudinal, flexural and coupled). These shift patterns are consistently observed in both the theoretical and experimental curves, highlighting the model's effectiveness in predicting the electromechanical impedance behavior under temperature environmental variations.

Figure 63 – Theoretical (left) and experimental (right) EMI real part for the longitudinal dynamics: 24°C (solid black line), 35°C (dashed blue line), 45°C (dashed-dotted red line), 55°C (dotted green line) and 65°C (thinner dashed orange line).



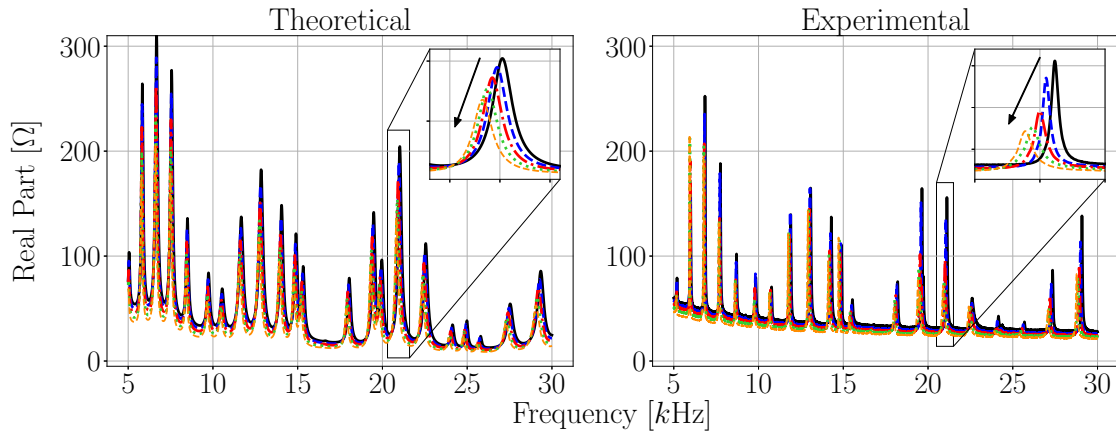
Source: Elaborated by the author.

Figure 64 – Theoretical (left) and experimental (right) EMI real part for the flexural dynamics: 24°C (solid black line), 35°C (dashed blue line), 45°C (dashed-dotted red line), 55°C (dotted green line) and 65°C (thinner dashed orange line).



Source: Elaborated by the author.

Figure 65 – Theoretical (left) and experimental (right) EMI real part for the coupled dynamics: 24°C (solid black line), 35°C (dashed blue line), 45°C (dashed-dotted red line), 55°C (dotted green line) and 65°C (thinner dashed orange line).



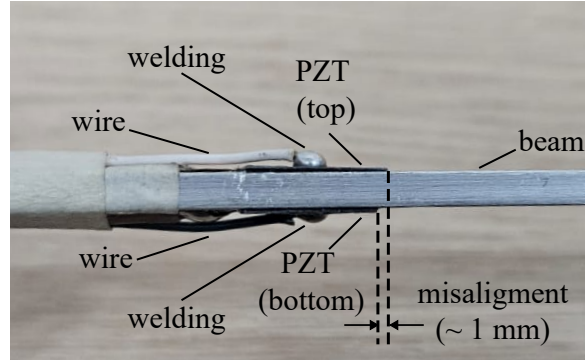
Source: Elaborated by the author.

The experimental data show that a slight misalignment of approximately 1 mm between the PZTs (see Fig. 66) is sufficient to provide the presence of some flexural dynamics when exciting the structure to obtain the longitudinal one. Note in Fig. 63 that the experimental signals of the longitudinal curve, specially from 5 kHz to 15 kHz, exhibit low-amplitude peaks corresponding to flexural modes, highlighting the influence of the PZTs misalignment. This is an involuntary misalignment resulting from the PZT's bonding process. Consequently, ensuring this symmetric configuration of the PZTs is crucial for the accurate analyses of the longitudinal and flexural dynamics separately.

Additionally, note that, in Figs. 63 and 64, the number of modes considered for the theoretical model ($n_L = 14$ and $n_F = 35$) does not necessarily correspond to the number of

modes (i.e. peaks) observed in the experimental data. In practice, numerical solutions indicate that an accurate theoretical prediction requires a higher number of modes in comparison with the experimental results, and this is a good topic for further investigations.

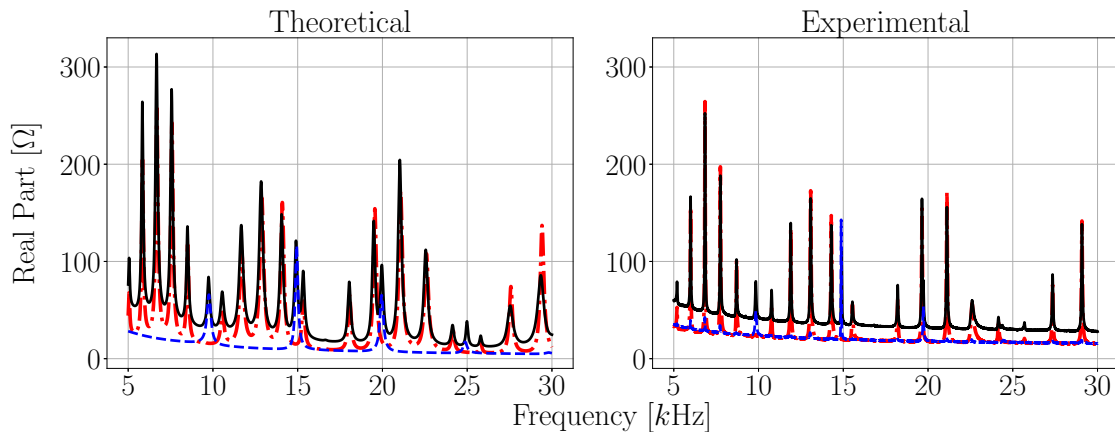
Figure 66 – Misalignment of the symmetric PZTs in relation to the beam longitudinal direction.



Source: Elaborated by the author.

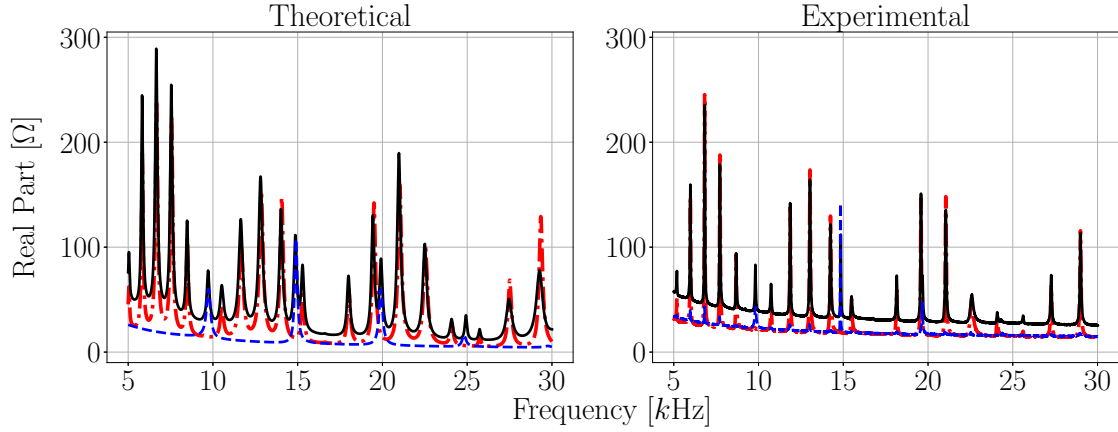
Figures 67 to 71 present the theoretical (left) and experimental (right) EMI real part for the three dynamics at 24°C, 35°C, 45°C, 55°C and 65°C, respectively. It is observed that both the theoretical and the experimental curves exhibit similar behavioral trends for the three dynamics analyzed – longitudinal, flexural and coupled –, highlighting the model's effectiveness in predicting the EMI response. An examination of both results reveals that the average amplitude levels of the longitudinal and flexural dynamics are lower than those associated with the coupled one. This difference arises from the electrical configuration of the system: the piezoelectric transducers can be represented as capacitors. Consequently, the parallel connection of the PZTs in the longitudinal and flexural dynamics results in a higher equivalent capacitance, which, in turn, leads to a reduction in the average levels of the EMI

Figure 67 – EMI real part at 24°C: theoretical (left) and experimental (right) results for longitudinal (dashed blue line), flexural (dashed-dotted red line) and coupled (solid black line) dynamics.



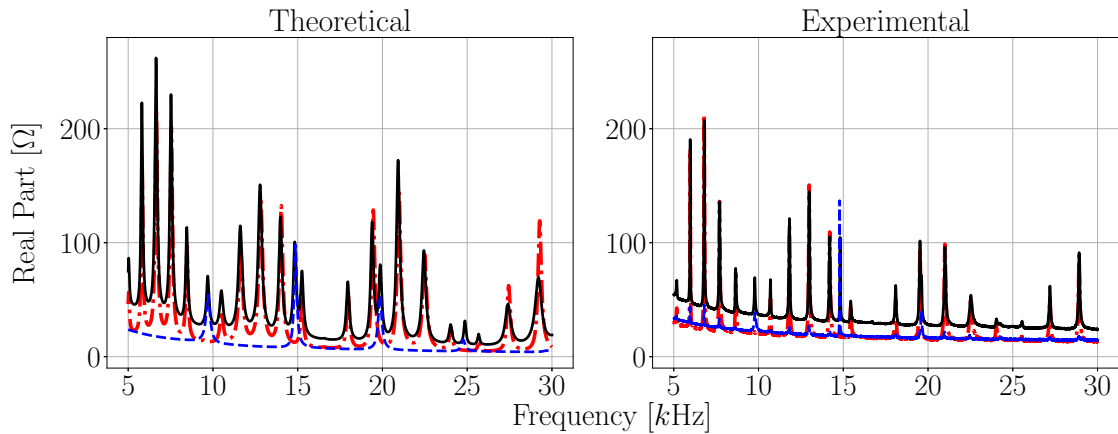
Source: Elaborated by the author.

Figure 68 – EMI real part at 35°C: theoretical (left) and experimental (right) results for longitudinal (dashed blue line), flexural (dashed-dotted red line) and coupled (solid black line) dynamics.



Source: Elaborated by the author.

Figure 69 – EMI real part at 45°C: theoretical (left) and experimental (right) results for longitudinal (dashed blue line), flexural (dashed-dotted red line) and coupled (solid black line) dynamics.

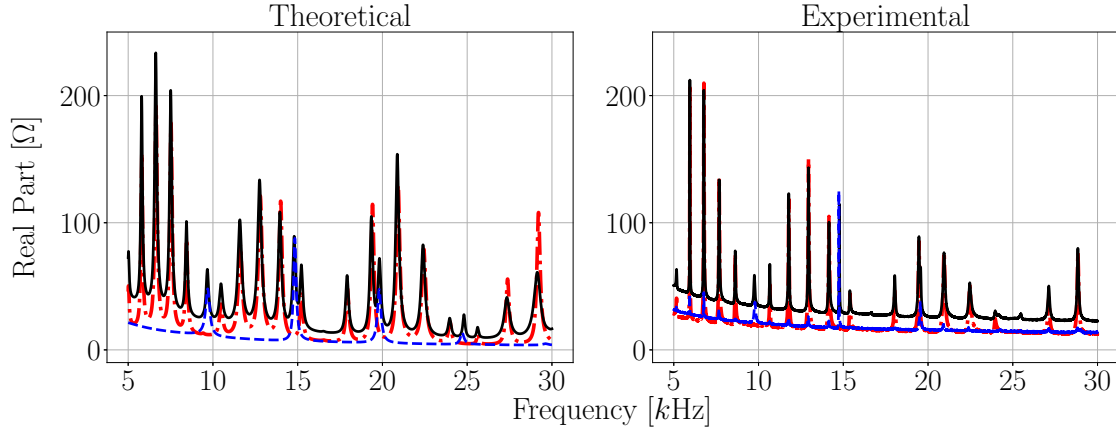


Source: Elaborated by the author.

curves. Additionally, it is observed that the number of modes (or peaks) in the beam excited by a single PZT (i.e., the coupled dynamics) is equal to the sum of the flexural and longitudinal modes. This occurrence is explained by the nature of the asymmetric piezoelectric excitation, which simultaneously induces both flexural and axial deformations in the structure. Conversely, both amplitude and number of peaks in the EMI curves corresponding to the longitudinal dynamics are, in general, lower than those observed for the flexural one. This difference is due to the greater beam's axial stiffness, which relatively hinders the excitation of longitudinal modes and, consequently, results in fewer and lower EMI peaks in this dynamics.

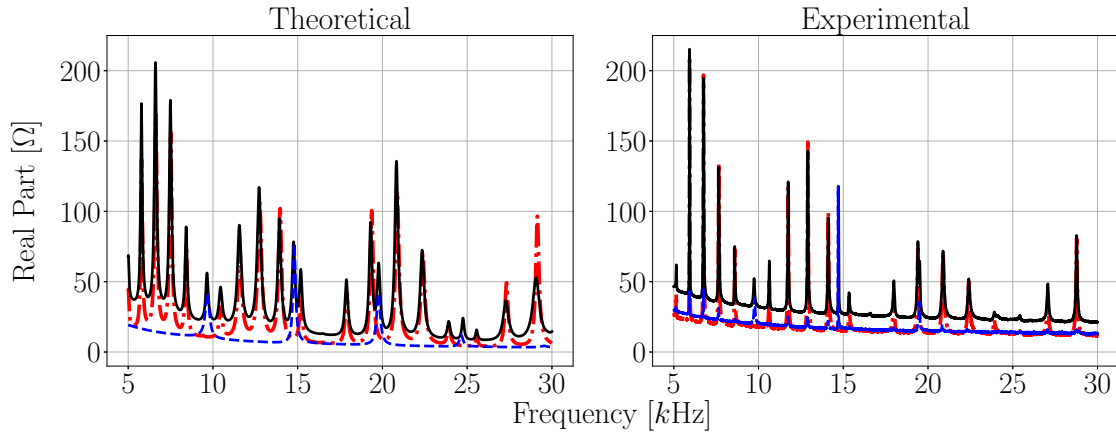
Figures 72 to 76 show a more detailed comparison between the theoretical and experimental results, for longitudinal (left), flexural (middle) and coupled (right) dynamics at all investigated temperatures (24°C, 35°C, 45°C, 55°C, and 65°C). The main differences

Figure 70 – EMI real part at 55°C: theoretical (left) and experimental (right) results for longitudinal (dashed blue line), flexural (dashed-dotted red line) and coupled (solid black line) dynamics.



Source: Elaborated by the author.

Figure 71 – EMI real part at 65°C: theoretical (left) and experimental (right) results for longitudinal (dashed blue line), flexural (dashed-dotted red line) and coupled (solid black line) dynamics.



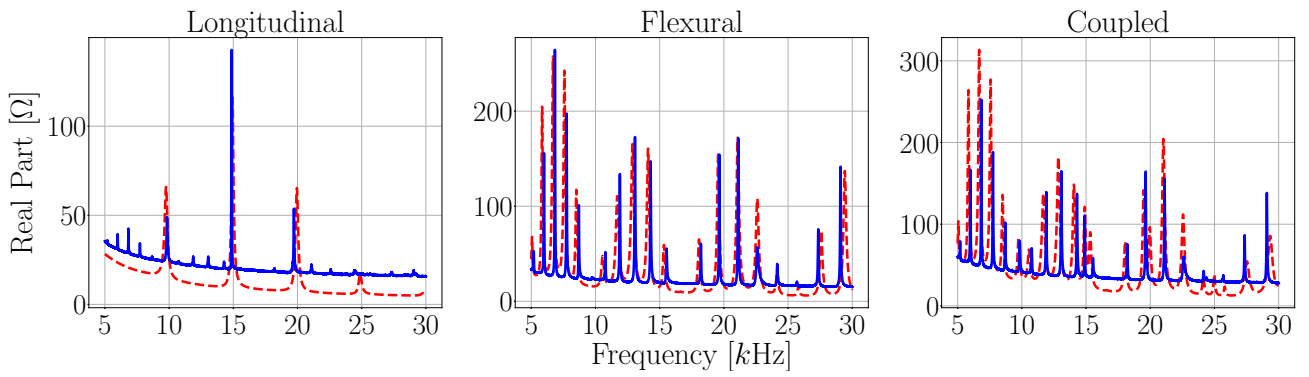
Source: Elaborated by the author.

between the two types of signals involve the average levels of the curves, the amplitudes and frequency locations of some peaks, as well as the overall peak widths. In particular, the latter discrepancy is attributed to the higher damping required in the analytical modeling in comparison with the experimental results, which reflects in broader spectral widths in the theoretical frequency responses.

The results also demonstrate that the EMI curves shift upward from the zero vertical axis and display a characteristic slope, observed in both experimental and theoretical curves. The real part of the EMI signal reflects contributions from both electrical and mechanical properties. The electrical component is primarily associated with resistive effects and inherent electrical losses, while the mechanical component is mainly influenced by the damping characteristics of the monitored structure. From a theoretical standpoint, the model incorpo-

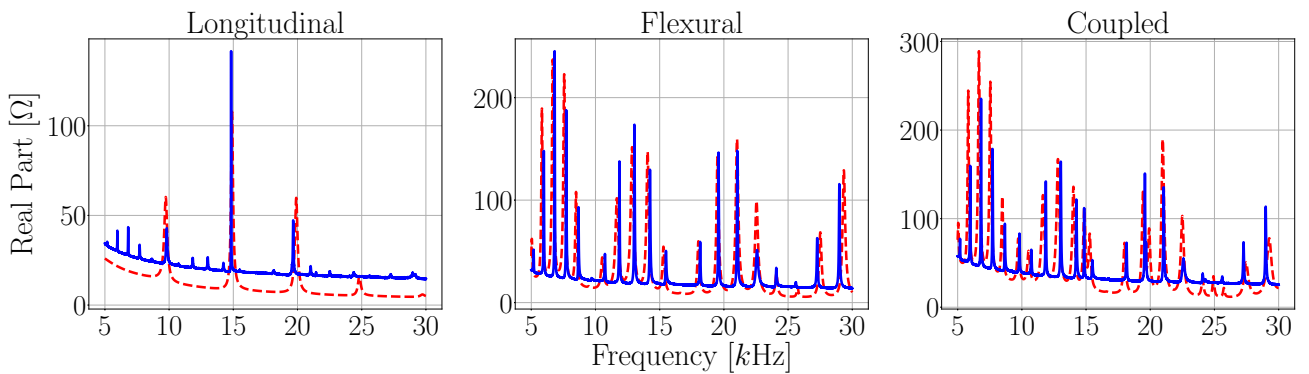
rates electrical losses through a complex dielectric constant expressed as $\tilde{\varepsilon}_{33}^T(1 - i\delta)$, where δ denotes the dielectric loss factor, specified in Tab. 7. Consequently, the observed discrepancies between the average levels of the theoretical and experimental curves — with the latter presenting higher amplitudes — can be attributed to additional resistive effects. These arise from the electrical resistance of the conducting cables, the data acquisition system, and other non-modeled losses within the PZTs themselves, all of which contribute to an increase in the measured impedance during experimental testing.

Figure 72 – EMI real part of the theoretical and experimental EMI curves for the three dynamics at 24°C. Red dashed line represents the theoretical curve and blue solid line corresponds to the experimental curve.



Source: Elaborated by the author.

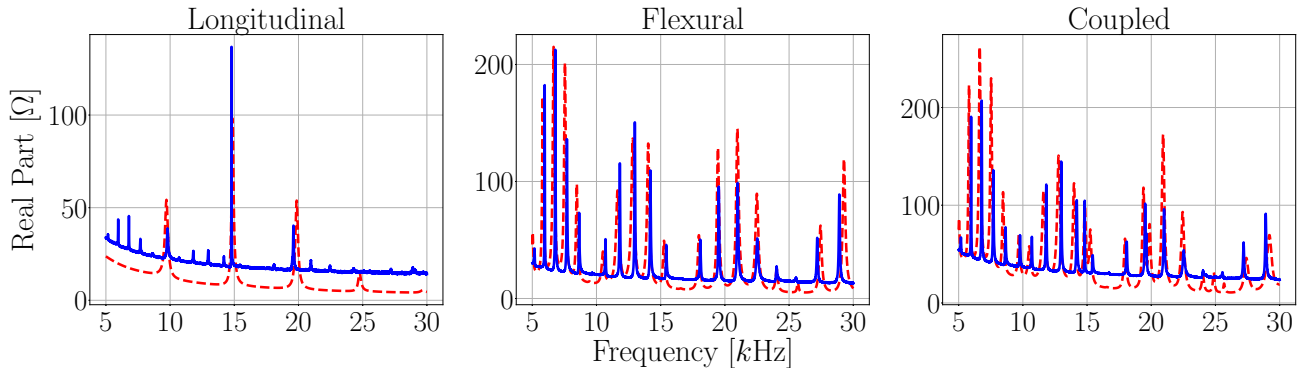
Figure 73 – EMI real part of the theoretical and experimental EMI curves for the three dynamics at 35°C. Red dashed line represents the theoretical curve and blue solid line corresponds to the experimental curve.



Source: Elaborated by the author.

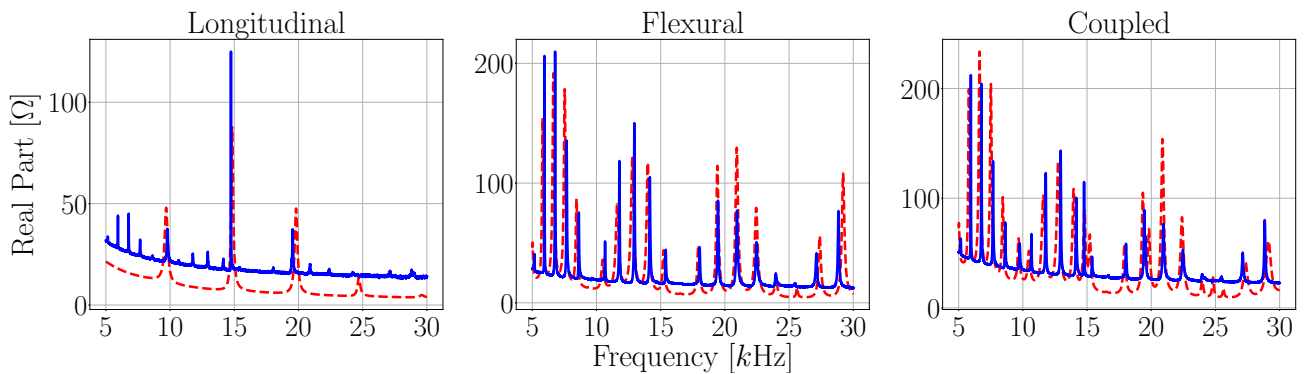
The direct comparisons between the EMI real parts of theoretical and experimental signals reveal that the theoretical predictions closely replicate the experimental behavior, particularly with respect to peak locations. Across the five investigated temperatures and a frequency range from 5 kHz to 30 kHz, the maximum percentage error in terms of peak frequency is 2.09% for longitudinal dynamics, 2.78% for flexural dynamics, and 2.82% for

Figure 74 – EMI real part of the theoretical and experimental EMI curves for the three dynamics at 45°C. Red dashed line represents the theoretical curve and blue solid line corresponds to the experimental curve.



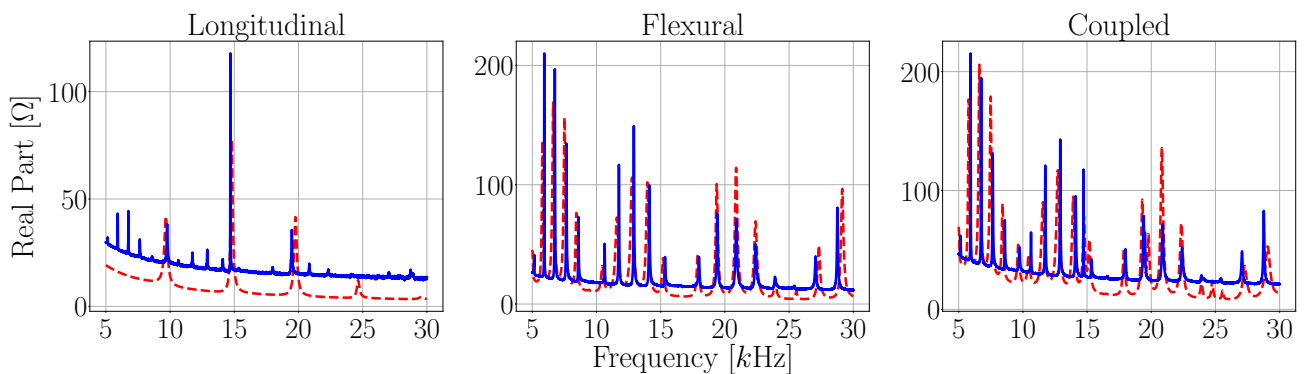
Source: Elaborated by the author.

Figure 75 – EMI real part of the theoretical and experimental EMI curves for the three dynamics at 55°C. Red dashed line represents the theoretical curve and blue solid line corresponds to the experimental curve.



Source: Elaborated by the author.

Figure 76 – EMI real part of the theoretical and experimental EMI curves for the three dynamics at 65°C. Red dashed line represents the theoretical curve and blue solid line corresponds to the experimental curve.



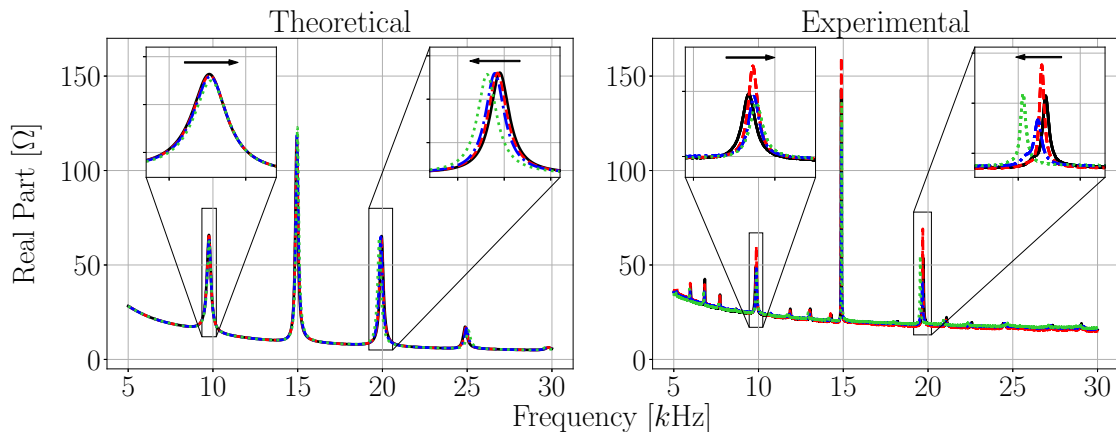
Source: Elaborated by the author.

coupled dynamics. These results highlight the model's ability to accurately replicate the resonant characteristics of the system under varying temperature conditions.

5.2.1 Symmetric Cross-Sectional Area Reduction

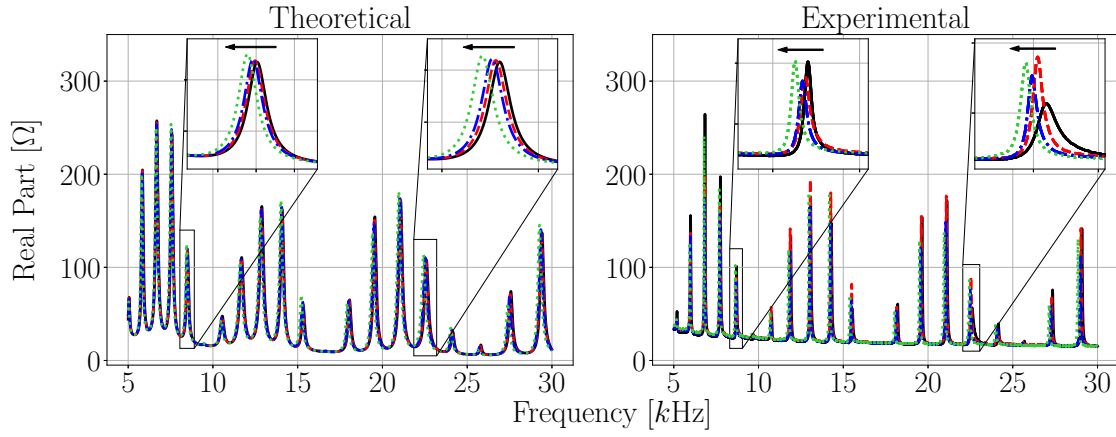
A local and symmetric cross-sectional area reduction is considered to represent a damaged region of the beam. Figures 77 to 79 present the EMI real parts for this damage configuration corresponding, respectively, to the longitudinal, flexural and coupled dynamics, obtained at 24°C. Theoretical (left) and experimental (right) curves are shown for both nominal (i.e., baseline - solid black line) and damaged conditions; the latter represented by symmetric thickness reductions of 3% (dashed red line), 6% (dashed-dotted blue line), and 15% (dotted green line). The theoretical signals corresponding to the damaged condition exhibit horizontal displacements patterns relative to the nominal condition, consistent with those observed in the experimental curves. In the flexural dynamics, all peaks of the EMI curves – both theoretical and experimental – shifted to the left-hand side, indicating an effect analogous to a local reduction in structural stiffness. On the other hand, in the longitudinal dynamics, the third peak shifts to the left-hand side, whereas the other ones shift to the right-hand side. These behaviors are observed in both theoretical and experimental data. Lastly, in the coupled dynamics, each peak can be associated with one type of motion, either flexural or longitudinal. Therefore, the displacement behavior observed in the coupled dynamics mirrors the patterns recorded in each dynamics separately. Additionally, the results also show that the relative displacement between the damaged and nominal EMI curves becomes progressively more pronounced as the frequency increases, in both theoretical and experimental results. This phenomenon corroborates previous works which stated that damage detection exhibits

Figure 77 – EMI real part at 24°C for the longitudinal dynamics: theoretical (left) and experimental (right) results for both baseline/nominal (solid back line) and symmetric damaged conditions, with thickness reductions of 3% (dashed red line), 6% (dashed-dotted blue line) and 15% (dotted green line).



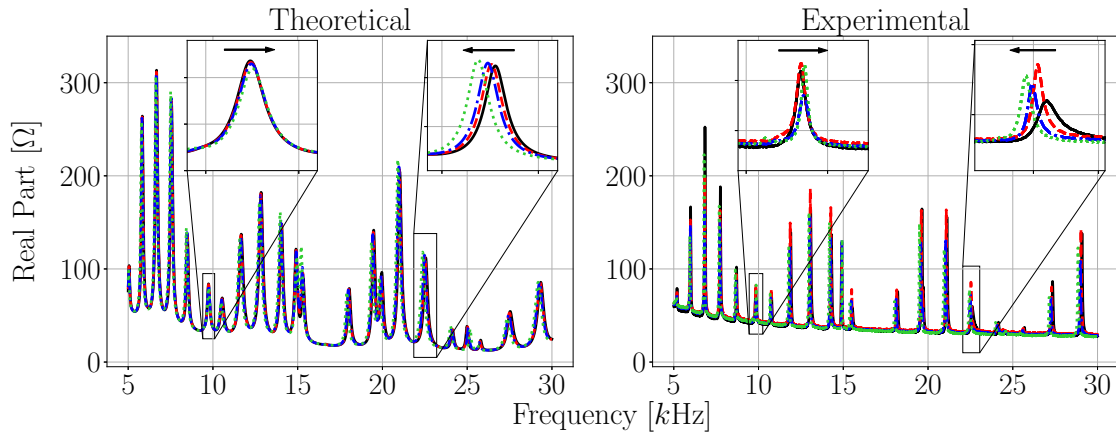
Source: Elaborated by the author.

Figure 78 – EMI real part at 24°C for the flexural dynamics: theoretical (left) and experimental (right) results for both baseline/nominal (solid black line) and symmetric damaged conditions, with thickness reductions of 3% (dashed red line), 6% (dashed-dotted blue line) and 15% (dotted green line).



Source: Elaborated by the author.

Figure 79 – EMI real part at 24°C for the coupled dynamics: theoretical (left) and experimental (right) results for both baseline/nominal (solid black line) and symmetric damaged conditions, with thickness reductions of 3% (dashed red line), 6% (dashed-dotted blue line) and 15% (dotted green line).



Source: Elaborated by the author.

greater sensitivity at higher frequencies (Park; Cudney; Inman, 2000a; Tseng; Wang, 2005b; Zhang *et al.*, 2011). Similar conclusions are achieved when evaluating the EMI signals for both damaged and undamaged conditions at the other two environmental temperatures (45°C and 65 °C).

The presented results demonstrate greater peak amplitudes variations in the experimental data, when comparing the baseline and damage conditions, than those observed in the theoretical model. The reason for this occurrence is that both damaged and baseline theoretical EMI consider the same set of $\alpha_{()}$ and $\beta_{()}$ coefficients, i.e., these coefficients were obtained to compute the nominal curve, and also used to compute the EMI corresponding

to the damaged conditions. Furthermore, while the Euler-Bernoulli hypothesis – which requires a length-to-thickness ratio greater than 20 (Musgraves; Hu; Calvez, 2019) – is satisfied for the full beam, it is not strictly valid within the damaged region. As a result, the experimental measurements may contain additional effects that were not considered in the analytical modeling. Nevertheless, both types of signals exhibit remarkable similarity, particularly in the peaks locations, with percentage of errors below 2.86% for all investigated cases.

5.2.2 Asymmetric Cross-Sectional Area Reduction

This section presents the validation of the analytical model for a beam with a local and asymmetric cross-sectional area reduction, representing a damage condition that promotes mode conversion between longitudinal and flexural motions. Unlike the previous cases – undamaged and symmetric damaged beams – the analytical and experimental results of this structural condition exhibited some discrepancies. Therefore, the experimental and theoretical results are presented separately, followed by a comparative discussion.

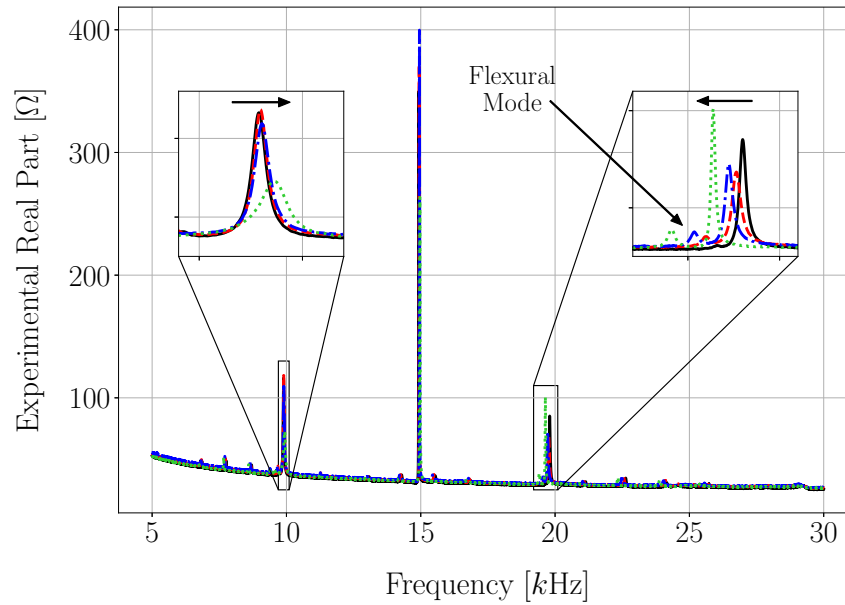
5.2.2.1 Experimental Asymmetric Results

Figures 80 to 82 present the experimental results of the EMI real parts for the asymmetric damage configuration corresponding, respectively, to the longitudinal, flexural and coupled dynamics, obtained at 24°C. The signals are shown for both nominal (i.e., baseline - solid black line) and damaged conditions; the latter represented by asymmetric thickness reductions of 5% (dashed red line), 10% (dashed-dotted blue line), and 20% (dotted green line).

Figures 80 and 81 highlight the appearance of some EMI peaks that differ from those typically expected for the respective dynamics under investigation. Specifically, longitudinal modes become evident within the flexural response, and conversely, flexural modes emerge within the longitudinal dynamics. This behavior is attributed to the mode conversion effects induced by the asymmetric reduction in the beam's cross-sectional area, which introduces coupling between the two vibration modes. However, it is important to emphasize that the PZTs misalignment (see Fig. 66), along with the short energy duration of the excitation signal – a 1-second chirp – may have hindered the proper excitation and visualization of additional peaks arising from the asymmetric damage, particularly in the longitudinal dynamics, which, even in the undamaged condition, already exhibited some flexural modes. Nevertheless, it is observed that as the damage severity increases, the amplitude of the newly emerged peaks also rises, suggesting that greater geometric asymmetry enhances the degree of mode conversion.

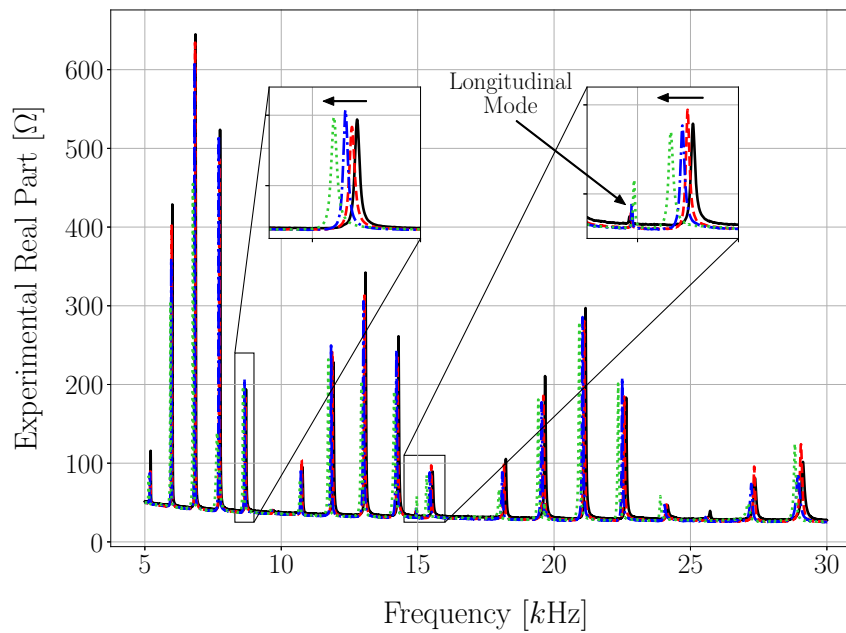
All peaks in the EMI flexural curves shifted to the left-hand side as the damage severity increased, whereas, in the longitudinal dynamics, the first and second peaks shifted

Figure 80 – Experimental EMI real part at 24°C for the longitudinal dynamics: baseline/nominal (solid black line) and asymmetric damaged conditions, with thickness reductions of 5% (dashed red line), 10% (dashed-dotted blue line) and 20% (dotted green line).



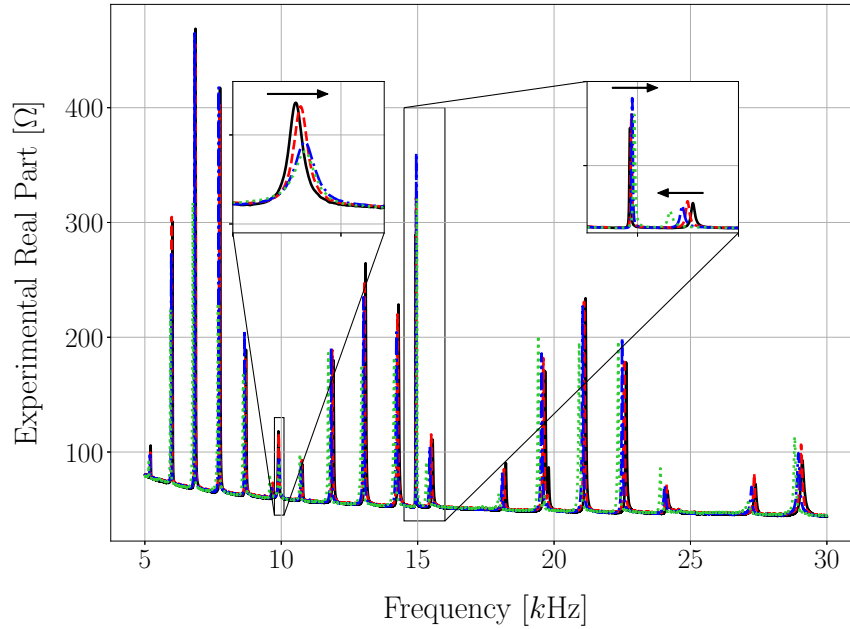
Source: Elaborated by the author.

Figure 81 – Experimental EMI real part at 24°C for the flexural dynamics: baseline/nominal (solid black line) and asymmetric damaged conditions, with thickness reductions of 5% (dashed red line), 10% (dashed-dotted blue line) and 20% (dotted green line).



Source: Elaborated by the author.

Figure 82 – Experimental EMI real part at 24°C for the coupled dynamics: baseline/nominal (solid black line) and asymmetric damaged conditions, with thickness reductions of 5% (dashed red line), 10% (dashed-dotted blue line) and 20% (dotted green line).



Source: Elaborated by the author.

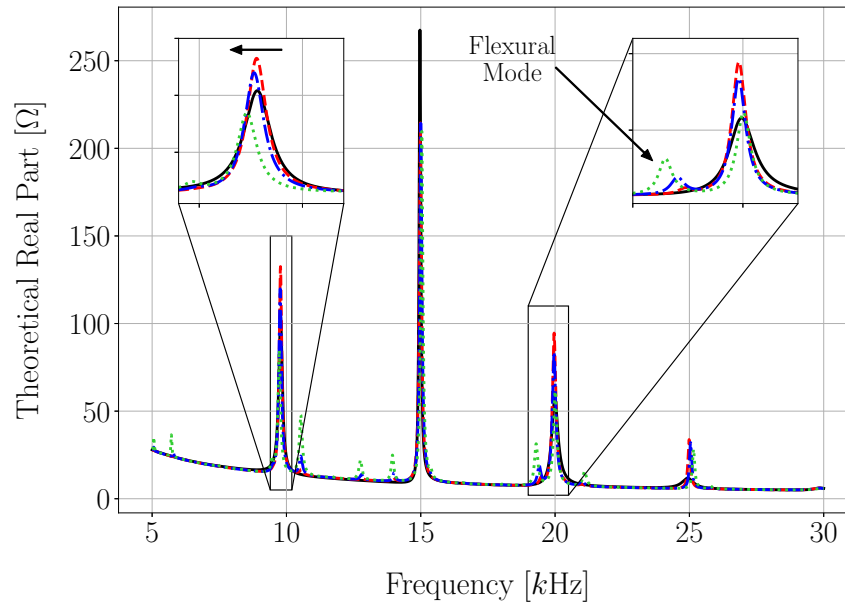
to the right-hand side, whereas the third peak shifted to the left-hand side. The coupled dynamics, on the other hand, presented peak shift behaviors that reflected a combination of the patterns observed in the individual longitudinal and flexural responses. Additionally, the results also indicate that the relative displacement between the damaged and baseline EMI curves becomes progressively more pronounced as the frequency increases, in all three dynamics.

5.2.2.2 Theoretical Asymmetric Results

Figures 83 to 85 present the theoretical results of the EMI real parts for the asymmetric damage configuration corresponding, respectively, to the longitudinal, flexural and coupled dynamics, obtained at 24°C. The signals are shown for both nominal (i.e., baseline - solid black line) and damaged conditions; the latter represented by asymmetric thickness reductions of 5% (dashed red line), 10% (dashed-dotted blue line), and 20% (dotted green line).

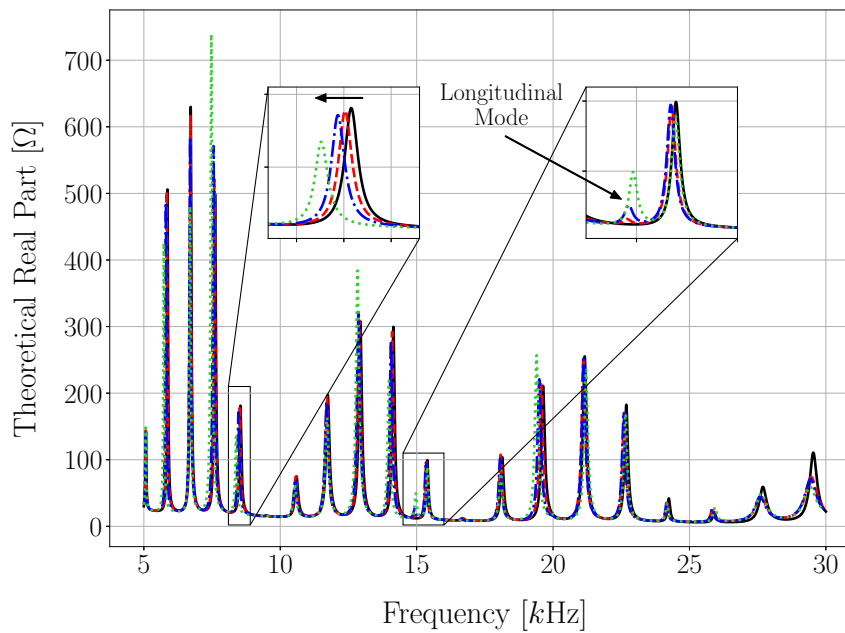
Figures 83 and 84 show that the dynamics under investigation exhibit new peaks that originate from a different dynamics, which confirms the coupling introduced into the model through the mass and stiffness matrices modified by the asymmetric damage. Moreover, the amplitudes of these newly emerged peaks increased with the severity of the damage, reinforcing the sensitivity of the model to this type of structural modification. In the case of the 20% cross-sectional area reduction, certain peaks shifted in the opposite direction to

Figure 83 – Theoretical EMI real part at 24°C for the longitudinal dynamics: baseline/nominal (solid back line) and asymmetric damaged conditions, with thickness reductions of 5% (dashed red line), 10% (dashed-dotted blue line) and 20% (dotted green line).



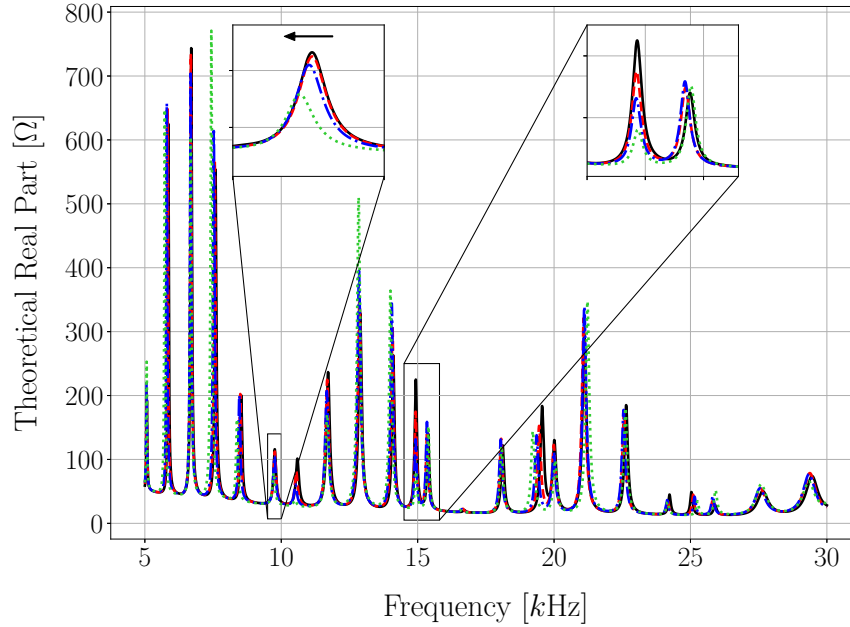
Source: Elaborated by the author.

Figure 84 – Theoretical EMI real part at 24°C for the flexural dynamics: baseline/nominal (solid back line) and asymmetric damaged conditions, with thickness reductions of 5% (dashed red line), 10% (dashed-dotted blue line) and 20% (dotted green line).



Source: Elaborated by the author.

Figure 85 – Theoretical EMI real part at 24°C for the coupled dynamics: baseline/nominal (solid back line) and asymmetric damaged conditions, with thickness reductions of 5% (dashed red line), 10% (dashed-dotted blue line) and 20% (dotted green line).



Source: Elaborated by the author.

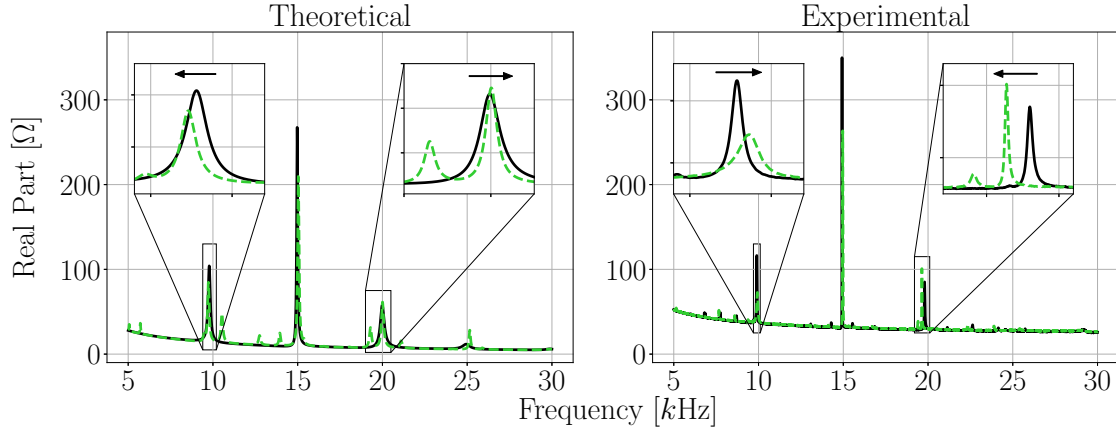
the other damage levels. This behavior suggests that, unlike the undamaged and symmetric damage scenarios (both experimental and theoretical), peak displacements does not follow a consistent pattern as damage intensifies.

Nevertheless, some trends can still be identified. In the longitudinal dynamics, the first peak shifted to the left-hand side as damage severity increased, whereas the second and fourth peaks shifted to the right-hand side. In contrast, the third peak displayed a different behavior (see Fig. 83). In the flexural dynamics, most peaks shifted to the left-hand side as the damage depth increased; however, some exceptions were observed, such as the 11th peak, located at approximately 15.3 kHz and highlighted in Fig. 84, and the 14th peak, around 21.2 kHz, which shifted to the right-hand side. As a consequence, these displacements differences were also observed in the coupled EMI signals. Furthermore, no clear relationship was observed between the increase in frequency and the magnitude of the relative displacement between the baseline and damaged EMI curves, and it is a good topic for further investigations in future works.

5.2.2.3 Discussion and Comparison

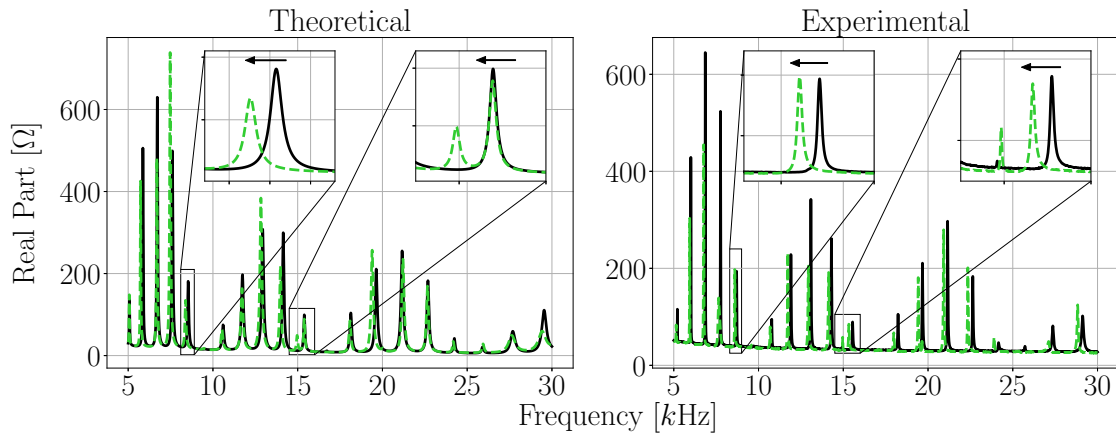
Figures 86 and 87 show a comparison between the theoretical and experimental results for the longitudinal and flexural dynamics, respectively, considering the baseline condition (solid black line) and the 20% asymmetric thickness reduction (dashed green line).

Figure 86 – EMI real part at 24°C for the longitudinal dynamics: theoretical (left) and experimental (right) results for both the baseline/nominal condition (solid back line) and the 20% asymmetric damage (dashed green line).



Source: Elaborated by the author.

Figure 87 – EMI real part at 24°C for the flexural dynamics: theoretical (left) and experimental (right) results for both the baseline/nominal condition (solid back line) and the 20% asymmetric damage (dashed green line).



Source: Elaborated by the author.

Based on Figs. 80 to 85, as well as the more direct comparisons provided in Figs. 86 and 87, the theoretical and experimental longitudinal peaks shifted in opposite directions, except for the second peak. On the other hand, in the flexural dynamics, all experimental peaks exhibited a leftward shift, with the magnitude of this displacement increasing with frequency. This trend, however, was not reflected in the theoretical curves, where several peaks presented minimal movement, especially at higher frequencies.

Therefore, these discrepancies between the theoretical and experimental results indicate that further investigations are necessary to more accurately capture the dynamic behavior induced by the asymmetric damage, and it is an interesting topic for future works.

5.3. CONCLUSIONS

This chapter presented the experimental investigations on electromechanical impedance behavior under varying structural, environmental and piezoelectric configurations. The experimental setups were described, including the instrumentation, data acquisition procedures and test conditions.

The effectiveness of the Park *et al.* (1999) temperature compensation technique was investigated, leading to the development of a new approach to reduce false alarm in damage detection. The efficiency of the proposed method was quantitatively verified, demonstrating its potential to enhance the reliability of damage detection by mitigating temperature-induced false alarms in the EMI technique.

The analytical model was validated through experimental data acquired from a beam instrumented with symmetrically attached PZTs to investigate three dynamics: longitudinal, flexural and the coupled one. Two types of damage configurations – symmetric and asymmetric cross-sectional area reductions – were considered for different reduction levels and environmental temperatures. The comparison between theoretical and experimental results revealed a strong agreement between the undamaged and symmetric damaged conditions at different temperatures, particularly with respect to the location of EMI peaks, thereby verifying the model's ability to accurately predict the system's EMI behavior under these distinct structural and temperature conditions. However, differences were observed for the asymmetric damage when comparing experimental and theoretical EMI curves.

6 FINAL REMARKS

This thesis presented a theoretical and experimental study of the Electromechanical Impedance technique, with particular emphasis on its sensitivity to temperature variations to contribute to improvement of the understanding of EMI-based SHM. The investigations were conducted on an aluminum beam instrumented with piezoelectric transducers arranged in both symmetric and asymmetric configurations.

A comprehensive literature review addressing the main topics relevant to this thesis was presented, with the objective of introducing the theoretical foundations and contextualizing the research. A theoretical model was developed considering an Euler-Bernoulli beam instrumented with PZTs coupled in both symmetric and asymmetric configurations. With this, the results demonstrate how the EMI is affected by the longitudinal, flexural and coupled dynamics on beam-like structures. Furthermore, the formulations accounts for symmetric and asymmetric damages, in addition to a derived coupling matrix that characterizes the interaction between longitudinal and flexural modes, thereby enhancing the representativeness of the theoretical EMI signals.

This theoretical model was employed to evaluate different symmetric damage configurations. The results showed the influence of damage depth, length, and position on the EMI signature, revealing distinct patterns for the longitudinal and flexural dynamics. In general, increases in damage depth and length led to more pronounced discrepancies between the baseline and damaged EMI curves for both dynamics. On the other hand, damages located farther from the PZTs resulted in reduced variations in the flexural response, whereas a different behavior with respect to damage position was observed in the longitudinal dynamics. These findings also highlighted the greater detection capability of the flexural dynamics for the investigated system, as well as the potential and sensitivity of the EMI-based SHM to small geometric changes.

Experimental investigations were carried out to assess the practical behavior of the EMI technique under varying structural and external conditions, as well as to demonstrate the theoretical modeling and the proposed false alarm reduction strategy. The evaluation of the Park *et al.* (1999) temperature compensation technique led to the development of an enhanced methodology for reducing false negatives, whose effectiveness was quantitatively demonstrated, underscoring its potential to improve the reliability of damage detection via EMI. In addition, comparative analyses between the analytical predictions and the experimental measurements — performed on a beam instrumented with symmetrically bonded PZTs — confirmed the model's capability to reproduce the EMI response under different structural (undamaged and symmetric damaged) and external conditions (i.e., temperatures from 24°C to 65°C), specially regarding the peak frequencies, for the three dynamics, i.e., longitudinal, flexural and the coupled one. However, such agreement was not observed for the asymmetric damage, which presented different peak shifting behaviors for the theoretical and

experimental results, highlighting the need for further investigations. These contributions strengthen the reliability of the EMI technique for structural damage detection, and expand its applicability and study to environments subject to temperature variations.

In conclusion, the results obtained from this thesis provide relevant information into the physics of EMI-based SHM under varying temperature and damage, specially regarding to different dynamics. Then, they contribute to a more comprehensive understanding of this structural health monitoring technique. Furthermore, the integration of theoretical modeling and experimental investigations reinforces the results of this study and also highlights the potential of hybrid approaches for understanding the complex interactions in EMI responses. This combination can provide a basis for the development of more effective and robust monitoring procedures, broadening the applicability of the EMI technique to practical applications.

6.1. SUGGESTIONS FOR FUTURE WORKS

This section presents suggestions for future works, with the aim of furthering advancing the research:

- To improve and correct the modeling of the asymmetric damage to better capture the dynamic coupling effects and frequency shift patterns induced by this damage configuration;
- To explore the modeling of the asymmetric damage in a series of case studies (varying the damage depth, length, and position) to investigate its influence on the longitudinal and flexural EMI curves;
- To adapt the proposed model to incorporate the Timoshenko beam theory, thereby enabling higher operational frequency ranges and the understanding of the effects of shear waves on the EMI signatures;
- To refine the model to consider different damage configurations, such as irregular geometries, gradual material degradation due to corrosion or erosion, among other examples;
- To conduct additional experimental investigations to assess the applicability and effectiveness of the proposed false alarm mitigation strategy across different structures and/or damage configurations;
- To investigate the correlation between damage location and metric indices in longitudinal dynamics, with additional structural geometries, damage positions, frequency ranges and damage metrics;
- To determine which modes are most sensitive to damage and to obtain a more comprehensive understanding of their influence on the EMI curves.

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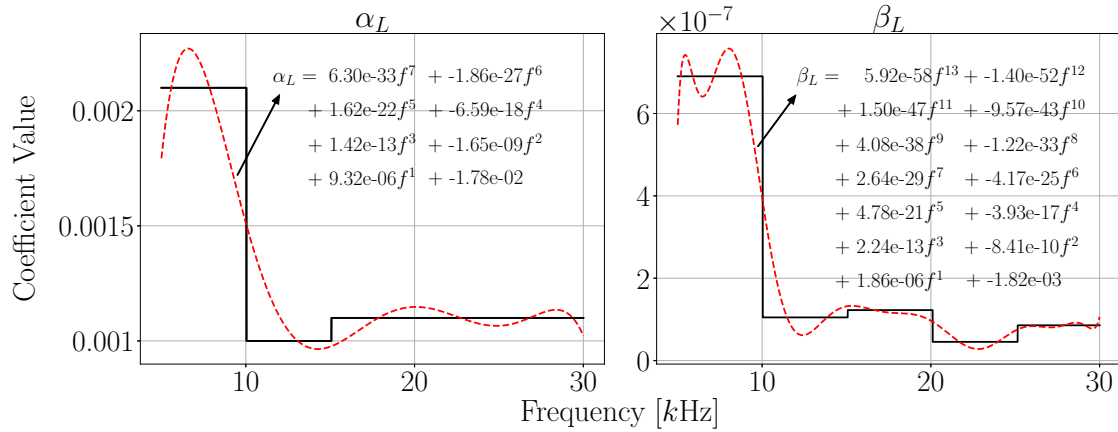
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APPENDIX A – ADJUSTMENT OF DAMPING COEFFICIENTS

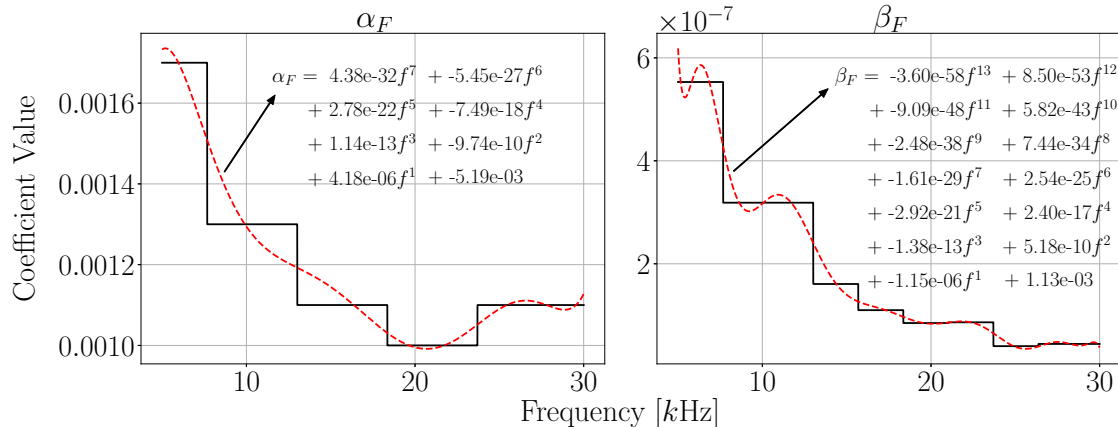
The damping coefficients $\alpha_{()}$ and $\beta_{()}$ for both longitudinal and flexural dynamics can be determined by using two different approaches. The first assumes these coefficients remain constant across the entire frequency range (Eq. 3.66), while the second employs frequency-dependent values (Eq. 3.67). In the latter approach, the coefficients are constant just on specific frequency sub-intervals, where best-fit polynomial functions are derived for each dynamics, as shown in Figs. 88 and 89. These figures illustrate the damping coefficient values corresponding to each F_{sub} (solid black line), along with their polynomial approximations (dashed red line) for longitudinal and flexural dynamics. The respective polynomial equations are also provided. The frequency interval widths for longitudinal and flexural dynamics are 5035 Hz and 2671 Hz, respectively.

Figure 88 – Damping coefficient values for each F_{sub} (solid black line) and the corresponding polynomial function (dashed red line) in longitudinal dynamics.



Source: Elaborated by the author.

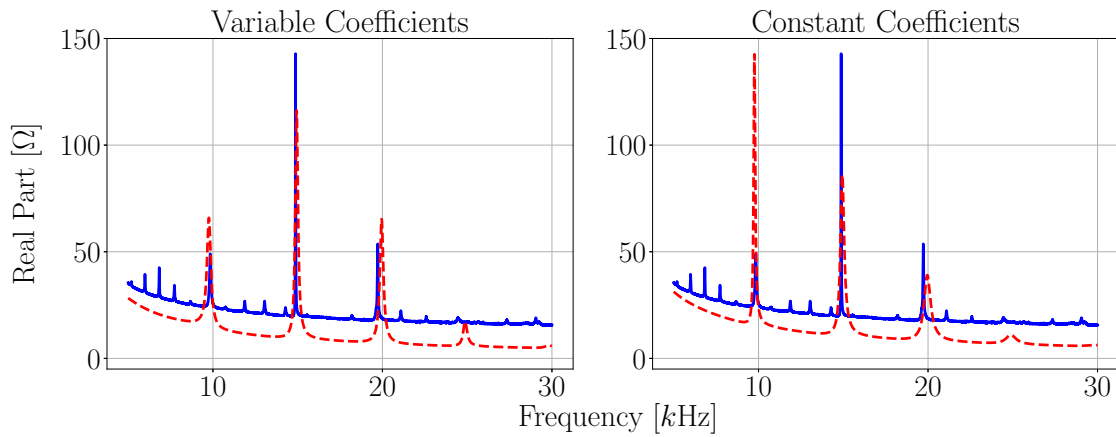
Figure 89 – Damping coefficient values for each F_{sub} (solid black line) and the corresponding polynomial function (dashed red line) in flexural dynamics.



Source: Elaborated by the author.

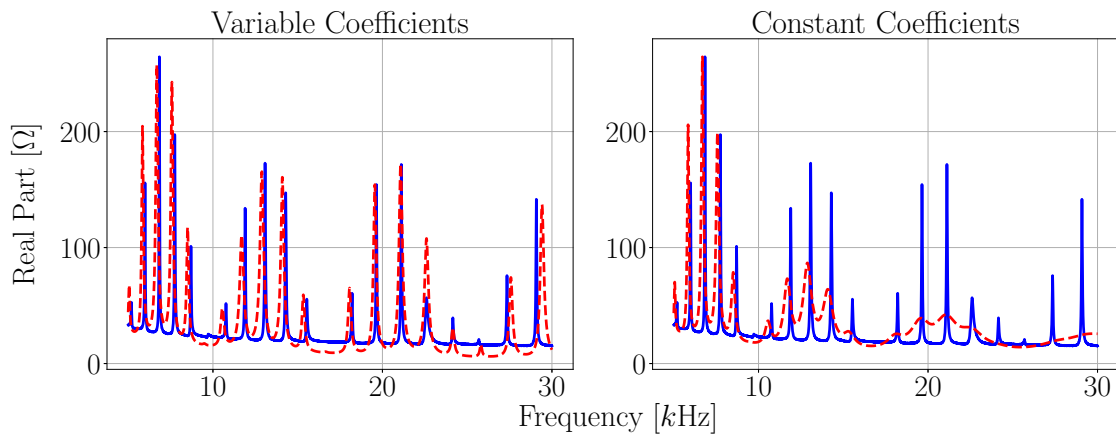
Conversely, adopting constant coefficients results in the following values: $\alpha_L = 0.0012$, $\beta_L = 1.8337 \times 10^{-7}$, $\alpha_F = 0.0017$ and $\beta_F = 5.5303 \times 10^{-7}$. Figures 90 and 91 present the real part of experimental (solid blue line) and theoretical (dashed red line) EMI signals for both longitudinal and flexural dynamics at 24°C, respectively. Each figure shows the theoretical results obtained by using the frequency-dependent (left) and the constant (right) coefficients. The findings indicate that variable coefficients enhance the accuracy of theoretical predictions, in terms of EMI peaks, by improving their agreement with the experimental peak amplitudes. In contrast, constant coefficients can lead the system to underdamped or overdamped states, resulting in poorer concordance with experimental data over large frequency ranges. The best strategy to be adopt depends on each specific problem.

Figure 90 – Real part of the theoretical (dashed red line) and experimental (solid blue line) EMI for longitudinal dynamics at 24°C, considering variable and constant damping coefficients.



Source: Elaborated by the author.

Figure 91 – Real part of the theoretical (dashed red line) and experimental (solid blue line) EMI for flexural dynamics at 24°C, considering variable (left) and constant (right) damping coefficients.



Source: Elaborated by the author.

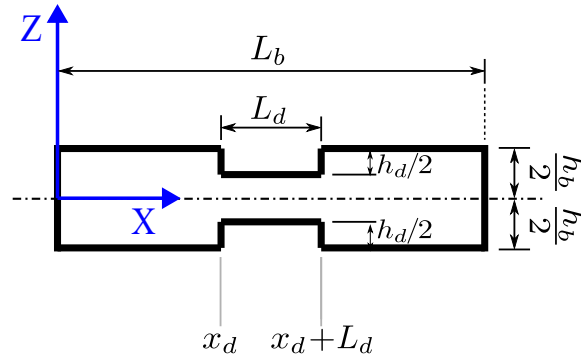
APPENDIX B – DERIVATION OF THE DAMAGE EQUATIONS

This appendix presents the mathematical derivations of the equations that characterize the effects of both symmetric (Eqs. 3.68 and 3.69) and asymmetric (Eqs. 3.70 and 3.71) structural damage within the EMI analytical model. These expressions serve as the basis for obtaining the beam's mass and stiffness matrices, and subsequently, the system's motion equation, which ultimately enables the characterization of the system's EMI under damaged conditions. The formulations are derived for both longitudinal and flexural dynamics.

B.1. SYMMETRIC DAMAGE FORMULATIONS

Figure 92 shows a schematic illustration of a beam with a symmetrically damaged region of length L_d , located between x_d and $x_d + L_d$. This damage is characterized by a thickness reduction of $h_d/2$ on both the upper and lower surfaces of the nominal beam, resulting in a total reduction of h_d . The damaged region presents the same width as the undamaged beam, represented by b_b . Therefore, the damage's cross-sectional area is given by $A_d = b_b(h_b - h_d)$. The coordinate system and axis orientation are also illustrated in the figure (in blue).

Figure 92 – Schematic illustration of a beam with a symmetrically damaged region located between x_d and $x_d + L_d$.



Source: Elaborated by the author.

The step-by-step derivation of the strain $\pi_{d(\cdot)}$ and kinetic $T_{d(\cdot)}$ energies for both the longitudinal and flexural dynamics, specifically for the symmetrically damaged region limited between x_d and $x_d + L_d$, are defined as follows:

- Longitudinal Dynamics

Based on the stress-strain relation $\sigma_{xx} = \tilde{E}_b \epsilon_{xx}$ and the longitudinal strain formulation provided in Eq. (3.2), the strain energy of the damaged region for longitudinal dynamics

is computed as:

$$\begin{aligned}
\pi_{d_L} &= \frac{1}{2} \int_{V_d} \sigma_{xx_L} \epsilon_{xx_L} dV_d \\
&= \frac{1}{2} \int_{V_d} \tilde{E}_b (\epsilon_{xx_L})^2 dV_d \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \int_{-(\frac{h_b}{2}-\frac{h_d}{2})}^{(\frac{h_b}{2}-\frac{h_d}{2})} \int_0^{b_b} \tilde{E}_b u'_L(x, t)^2 dx dz dy \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \int_{-(\frac{h_b}{2}-\frac{h_d}{2})}^{(\frac{h_b}{2}-\frac{h_d}{2})} \tilde{E}_b u'_L(x, t)^2 y \Big|_0^{b_b} dx dz \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{E}_b b_b u'_L(x, t)^2 z \Big|_{-(\frac{h_b}{2}-\frac{h_d}{2})}^{(\frac{h_b}{2}-\frac{h_d}{2})} dx \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{E}_b b_b (h_b - h_d) u'_L(x, t)^2 dx \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{E}_b A_d u'_L(x, t)^2 dx,
\end{aligned} \tag{B.1}$$

where V_d is the volume of the damaged section. In turn, the kinetic energy of the damaged region for longitudinal dynamics, T_{d_L} , is given by

$$T_{d_L} = \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{\rho}_b A_d \dot{u}_L(x, t)^2 dx. \tag{B.2}$$

- Flexural Dynamics

According to the definition of the flexural strain provided in Eq. (3.30), the strain energy of the symmetrically damaged region for flexural dynamics is expressed as:

$$\begin{aligned}
\pi_{d_F} &= \frac{1}{2} \int_{V_d} \sigma_{xx_F} \epsilon_{xx_F} dV_d \\
&= \frac{1}{2} \int_{V_d} \tilde{E}_b (\epsilon_{xx_F})^2 dV_d \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \int_{-(\frac{h_b}{2}-\frac{h_d}{2})}^{(\frac{h_b}{2}-\frac{h_d}{2})} \int_0^{b_b} \tilde{E}_b [-zw''_F(x, t)]^2 dx dz dy \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \int_{-(\frac{h_b}{2}-\frac{h_d}{2})}^{(\frac{h_b}{2}-\frac{h_d}{2})} \tilde{E}_b z^2 w''_F(x, t)^2 y \Big|_0^{b_b} dx dz \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{E}_b b_b w''_F(x, t)^2 \frac{z^3}{3} \Big|_{-(\frac{h_b}{2}-\frac{h_d}{2})}^{(\frac{h_b}{2}-\frac{h_d}{2})} dx \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{E}_b b_b \frac{(h_b - h_d)^3}{12} w''_F(x, t)^2 dx \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{E}_b I_d w''_F(x, t)^2 dx,
\end{aligned} \tag{B.3}$$

where $I_d = \frac{b(h_b - h_d)^3}{12}$ is the area moment of inertia of the damaged region. On the other hand, its kinetic energy is given by

$$T_{d_F} = \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{\rho}_b A_d \dot{w}_F(x, t)^2 dx. \tag{B.4}$$

The energies of the nominal regions located before (from 0 e x_d) and after (from $x_d + L_d$ e L_b) the damage are defined according to Eqs. (3.3) and (3.4) for longitudinal dynamics, and Eqs. (3.31) and (3.32) for flexural dynamics; requiring only an adjustment of the integration limits to reflect their respective positions along the damaged beam. Consequently, the total energy of the beam depicted in Fig. 92 is obtained by summing the energy contributions of each segment – namely, the region preceding the damage, the damaged region itself, and the region following the damage –, as detailed below.

$$\begin{aligned}\pi_{b_L} &= \pi_{b_{1L}} + \pi_{d_L} + \pi_{b_{2L}} \\ &= \frac{1}{2} \int_0^{x_d} \tilde{E}_b A_b u_L'(x, t)^2 dx + \frac{1}{2} \int_{x_d}^{x_d + L_d} \tilde{E}_b A_d (u_L')^2 dx + \frac{1}{2} \int_{x_d + L_d}^{L_b} \tilde{E}_b A_b u_L'(x, t)^2 dx,\end{aligned}\quad (\text{B.5})$$

$$\begin{aligned}T_{b_L} &= T_{b_{1L}} + T_{d_L} + T_{b_{2L}} \\ &= \frac{1}{2} \int_0^{x_d} \tilde{\rho}_b A_b \dot{u}_L(x, t)^2 dx + \frac{1}{2} \int_{x_d}^{x_d + L_d} \tilde{\rho}_b A_d \dot{u}_L(x, t)^2 dx + \frac{1}{2} \int_{x_d + L_d}^{L_b} \tilde{\rho}_b A_b \dot{u}_L(x, t)^2 dx,\end{aligned}\quad (\text{B.6})$$

$$\begin{aligned}\pi_{b_F} &= \pi_{b_{1F}} + \pi_{d_F} + \pi_{b_{2F}} \\ &= \frac{1}{2} \int_0^{x_d} \tilde{E}_b I_b w_F''(x, t)^2 dx + \frac{1}{2} \int_{x_d}^{x_d + L_d} \tilde{E}_b I_d (w_F'')^2 dx + \frac{1}{2} \int_{x_d + L_d}^{L_b} \tilde{E}_b I_b w_F''(x, t)^2 dx,\end{aligned}\quad (\text{B.7})$$

$$\begin{aligned}T_{b_F} &= T_{b_{1F}} + T_{d_F} + T_{b_{2F}} \\ &= \frac{1}{2} \int_0^{x_d} \tilde{\rho}_b A_b \dot{w}_F(x, t)^2 dx + \frac{1}{2} \int_{x_d}^{x_d + L_d} \tilde{\rho}_b A_d \dot{w}_F(x, t)^2 dx + \frac{1}{2} \int_{x_d + L_d}^{L_b} \tilde{\rho}_b A_b \dot{w}_F(x, t)^2 dx,\end{aligned}\quad (\text{B.8})$$

where the subscripts b_1 and b_2 indicate the nominal beam regions located before and after the damage section, respectively. Then, by applying the Rayleigh–Ritz method in conjunction with the Lagrange equation, the expressions to calculate the elements of the mass and stiffness matrices of the damaged beam are given by:

$$m_{b_{()}}^{(i,j)} = \int_0^{x_d} \tilde{\rho}_b A_b \psi_{i_{()}}(x) \psi_{j_{()}}(x) dx + \int_{x_d}^{x_d + L_d} \tilde{\rho}_b A_d \psi_{i_{()}}(x) \psi_{j_{()}}(x) dx + \int_{x_d + L_d}^{L_b} \tilde{\rho}_b A_b \psi_{i_{()}}(x) \psi_{j_{()}}(x) dx,\quad (\text{B.9})$$

$$k_{b_L}^{(i,j)} = \int_0^{x_d} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx + \int_{x_d}^{x_d + L_d} \tilde{E}_b A_d \psi'_{i_L}(x) \psi'_{j_L}(x) dx + \int_{x_d + L_d}^{L_b} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx;$$

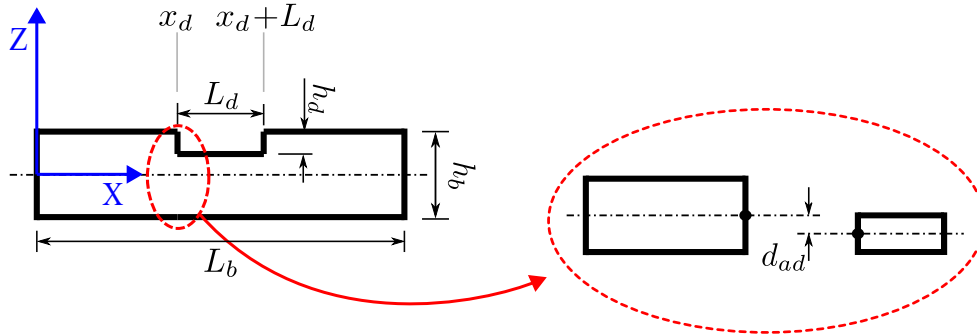
$$k_{b_F}^{(i,j)} = \int_0^{x_d} \tilde{E}_b I_b \psi''_{i_F}(x) \psi''_{j_F}(x) dx + \int_{x_d}^{x_d + L_d} \tilde{E}_b I_d \psi''_{i_F}(x) \psi''_{j_F}(x) dx + \int_{x_d + L_d}^{L_b} \tilde{E}_b I_b \psi''_{i_F}(x) \psi''_{j_F}(x) dx,\quad (\text{B.10})$$

It is observed that the kinetic energy expressions for both the longitudinal and flexural dynamics presented in Eqs. (B.6) and (B.8), respectively, share the same structural form, differing only in terms of the specific motion characteristics, such as velocity or mode shape. Therefore, they are computed into a single generalized expression, as presented in Eq. (B.9).

B.2. ASYMMETRIC DAMAGE FORMULATIONS

Figure 93 shows a schematic illustration of a beam with an asymmetrically damaged region of length L_d , located between x_d and $x_d + L_d$. This structural modification induces the coupling between the longitudinal and flexural dynamics due to the distance d_{ad} between the neutral axis of the damaged section and those of the adjacent undamaged regions, as shown in Fig. 93. This geometric distance is computed as $d_{ad} = h_d/2$. Such offset generates a longitudinal displacement $u_{ad} = d_{ad}\theta$, where θ is the angular displacement, expressed as $\theta = w'_F(x, t)$ for an Euler–Bernoulli beam.

Figure 93 – Schematic illustration of a beam with an asymmetrically damaged region located between x_d and $x_d + L_d$, with close-up view of the junction between the damaged and undamaged regions.



Source: Elaborated by the author.

The step-by-step derivation of the strain π_{ad} and kinetic T_{ad} energies for the beam with an asymmetrically damaged region, located between x_d and $x_d + L_d$, are presented below. In this formulation, both longitudinal and flexural dynamics follow similar procedures, differing only in the type of loading induced by the PZT actuators. Accordingly, in both dynamics, the displacements in the x , y , and z directions, denoted respectively as $u(x, t)$, $v(x, t)$, and $w(x, t)$, are given by:

$$\begin{aligned} u(x, t) &= u_L(x, t) - zw'_F(x, t) + d_{ad}w'_F(x, t); \\ v(x, t) &= 0; \\ w(x, t) &= w_F(x, t), \end{aligned} \tag{B.11}$$

where u_L represents the longitudinal displacement, associated with the axial dynamics, w_F denotes the transverse displacement linked to the flexural dynamics, and $d_{ad}w'_F(x, t)$

corresponds to the displacement generated by the presence of the asymmetric damage. The subscript ad refer to this type of damage. Thus, the mechanical strain is defined as: $\epsilon_{xx_{ad}} = u'_L(x, t) - zw''_F(x, t) + d_{ad}w''_F(x, t)$, and, consequently, the strain energy of the beam with an asymmetric damage, π_{ad} , is given by:

$$\begin{aligned}\pi_{ad} &= \frac{1}{2} \int_{V_b} \tilde{E}_b \epsilon_{xx_{ad}}^2 dV_b \\ &= \frac{1}{2} \int_{V_b} \tilde{E}_b [u'_L(x, t) - zw''_F(x, t) + d_{ad}w''_F(x, t)]^2 dV_b \\ &= \frac{1}{2} \int_{V_b} \tilde{E}_b [(u'_L)^2 + z^2(w''_F)^2 + d_{ad}^2(w''_F)^2 + 2d_{ad}u'_L w''_F - 2zu'_L w''_F - 2zd_{ad}(w''_F)^2] dV_b.\end{aligned}\quad (B.12)$$

The first two terms of Eq. (B.12) correspond to the strain energies of the longitudinal and flexural dynamics defined in Eqs. (3.3) and (3.31), respectively. Consequently, their total energy expressions are obtained by summing the energy contributions of each segment – namely, the region preceding the damage, the damaged region itself, and the region following the damage –, as detailed below.

$$\begin{aligned}& \frac{1}{2} \int_{V_b} \tilde{E}_b [(u'_L)^2 + z^2(w''_F)^2] dV_b \\ &= \frac{1}{2} \int_0^{x_d} \tilde{E}_b A_b (u'_L)^2 dx + \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{E}_b A_d (u'_L)^2 dx + \frac{1}{2} \int_{x_d+L_d}^{L_b} \tilde{E}_b A_b (u'_L)^2 dx \\ &+ \frac{1}{2} \int_0^{x_d} \tilde{E}_b I_b (w''_F)^2 dx + \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{E}_b I_{ad} (w''_F)^2 dx + \frac{1}{2} \int_{x_d+L_d}^{L_b} \tilde{E}_b I_b (w''_F)^2 dx,\end{aligned}\quad (B.13)$$

where $I_{ad} = \frac{b_b(h_b - h_d)^3}{12} + b_b \left(\frac{h_d}{2} \right)^2 (h_b - h_d)$ is the second moment of area of the asymmetric damage. On the other hand, the last four terms in Eq. (B.12) are related to phenomena arising from the presence of this damage type. As such, these terms are nonzero exclusively over its extent, that is, in $x \in [x_d, x_d + L_d]$. Therefore:

$$\begin{aligned}& \frac{1}{2} \int_{V_d} \tilde{E}_b [d_{ad}^2(w''_F)^2 + 2d_{ad}u'_L w''_F - 2zu'_L w''_F - 2zd_{ad}(w''_F)^2] dV_d \\ &= \frac{1}{2} \int_{x_d}^{x_d+L_d} \int_{-\frac{h_b}{2}}^{\frac{h_b}{2}-h_d} \int_0^{b_b} \tilde{E}_b [d_{ad}^2(w''_F)^2 + 2d_{ad}u'_L w''_F - 2zu'_L w''_F - 2zd_{ad}(w''_F)^2] dx dz dy \\ &= \frac{1}{2} \int_{x_d}^{x_d+L_d} \int_{-\frac{h_b}{2}}^{\frac{h_b}{2}-h_d} \tilde{E}_b [d_{ad}^2(w''_F)^2 + 2d_{ad}u'_L w''_F - 2zu'_L w''_F - 2zd_{ad}(w''_F)^2] y \Big|_0^{b_b} dx dz \\ &= \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{E}_b b_b \left[zd_{ad}^2(w''_F)^2 + 2zd_{ad}u'_L w''_F - 2\frac{z^2}{2}u'_L w''_F - 2\frac{z^2}{2}d_{ad}(w''_F)^2 \Big|_{-\frac{h_b}{2}}^{\frac{h_b}{2}-h_d} \right] dx \\ &= \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{E}_b A_d d_{ad}^2(w''_F)^2 + 2\tilde{E}_b A_d d_{ad}u'_L w''_F - \tilde{E}_b \gamma_{ad} u'_L w''_F - \tilde{E}_b \gamma_{ad} d_{ad}(w''_F)^2 dx,\end{aligned}\quad (B.14)$$

where $\gamma_{ad} = b_b(h_d^2 - h_b h_d)$. Thus, the total strain energy of the beam with asymmetric damage is obtained by summing the energy expressions provided in Eqs. (B.13) and (B.14).

Accordingly, the kinetic energy T_{ad} is defined as:

$$T_{ad} = \frac{1}{2} \int_{V_b} \tilde{\rho}_b \dot{u}(x, t)^2 dV_b + \frac{1}{2} \int_{V_b} \tilde{\rho}_b \dot{w}(x, t)^2 dV_b, \quad (\text{B.15})$$

where the second term corresponds to the kinetic energy of the flexural dynamics defined in Eq. (3.32) and its total value is obtained by summing the energy contributions of each segment – namely, the region preceding the damage, the damaged region itself, and the region following the damage –, as presented below.

$$\begin{aligned} & \frac{1}{2} \int_{V_b} \tilde{\rho}_b \dot{w}(x, t)^2 dV_b \\ &= \frac{1}{2} \int_0^{x_d} \tilde{\rho}_b A_b \dot{w}_F(x, t)^2 dx + \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{\rho}_b A_d \dot{w}_F(x, t)^2 dx + \frac{1}{2} \int_{x_d+L_d}^{L_b} \tilde{\rho}_b A_b \dot{w}_F(x, t)^2 dx. \end{aligned} \quad (\text{B.16})$$

On the other hand, the first term in Eq. (B.15), associated with the longitudinal motion, is defined as:

$$\begin{aligned} & \frac{1}{2} \int_{V_b} \tilde{\rho}_b \dot{u}(x, t)^2 dV_b \\ &= \frac{1}{2} \int_{V_b} \tilde{\rho}_b [\dot{u}_L(x, t) - z \dot{w}'_F(x, t) + d_{ad} \dot{w}'_F(x, t)]^2 dV_b \\ &= \frac{1}{2} \int_{V_b} \tilde{\rho}_b [(\dot{u}_L)^2 + z^2 (\dot{w}'_F)^2 + d_{ad}^2 (\dot{w}'_F)^2 + 2d_{ad} \dot{u}_L \dot{w}'_F - 2z \dot{u}_L \dot{w}'_F - 2z d_{ad} (\dot{w}'_F)^2] dV_b. \end{aligned} \quad (\text{B.17})$$

Among the terms in Eq. (B.17), only the first two admit integration over the entire length of the beam, i.e., from 0 to L_b , and are explicitly defined below. Additionally, it should be noted that the first one of these terms corresponds to the kinetic energy associated with the longitudinal dynamics defined in Eq. (3.4).

$$\begin{aligned} & \frac{1}{2} \int_{V_b} \tilde{\rho}_b [(\dot{u}_L)^2 + z^2 (\dot{w}'_F)^2] dV_b \\ &= \frac{1}{2} \int_0^{x_d} \tilde{\rho}_b A_b (\dot{u}_L)^2 dx + \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{\rho}_b A_d (\dot{u}_L)^2 dx + \frac{1}{2} \int_{x_d+L_d}^{L_b} \tilde{\rho}_b A_b (\dot{u}_L)^2 dx \\ &+ \frac{1}{2} \int_0^{x_d} \tilde{\rho}_b I_b (\dot{w}'_F)^2 dx + \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{\rho}_b I_{ad} (\dot{w}'_F)^2 dx + \frac{1}{2} \int_{x_d+L_d}^{L_b} \tilde{\rho}_b I_b (\dot{w}'_F)^2 dx. \end{aligned} \quad (\text{B.18})$$

The last four terms in Eq. (B.17) have nonzero contributions only within the asymmetric damage region, i.e., in the interval from x_d to $x_d + L_d$:

$$\begin{aligned}
& \frac{1}{2} \int_{V_b} \tilde{\rho}_b \left[d_{ad}^2 (\dot{w}'_F)^2 + 2d_{ad} \dot{u}_L \dot{w}'_F - 2z \dot{u}_L \dot{w}'_F - 2zd_{ad} (\dot{w}'_F)^2 \right] dV_b \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \int_{-\frac{h_b}{2}}^{\frac{h_b}{2}-h_d} \int_0^{b_b} \tilde{\rho}_b \left[d_{ad}^2 (\dot{w}'_F)^2 + 2d_{ad} \dot{u}_L \dot{w}'_F - 2z \dot{u}_L \dot{w}'_F - 2zd_{ad} (\dot{w}'_F)^2 \right] dx dz dy \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \int_{-\frac{h_b}{2}}^{\frac{h_b}{2}-h_d} \tilde{\rho}_b \left[d_{ad}^2 (\dot{w}'_F)^2 + 2d_{ad} \dot{u}_L \dot{w}'_F - 2z \dot{u}_L \dot{w}'_F - 2zd_{ad} (\dot{w}'_F)^2 \right] y \Big|_0^{b_b} dx dz \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{\rho}_b b_b \left[zd_{ad}^2 (\dot{w}'_F)^2 + 2zd_{ad} \dot{u}_L \dot{w}'_F - 2\frac{z^2}{2} \dot{u}_L \dot{w}'_F - 2\frac{z^2}{2} d_{ad} (\dot{w}'_F)^2 \right] \Big|_{-\frac{h_b}{2}}^{\frac{h_b}{2}-h_d} dx \\
&= \frac{1}{2} \int_{x_d}^{x_d+L_d} \tilde{\rho}_b A_d d_{ad}^2 (\dot{w}'_F)^2 + 2\tilde{\rho}_b A_d d_{ad} \dot{u}_L \dot{w}'_F - \tilde{\rho}_b \gamma_{ad} \dot{u}_L \dot{w}'_F - \tilde{\rho}_b \gamma_{ad} d_{ad} (\dot{w}'_F)^2 dx. \quad (B.19)
\end{aligned}$$

Thus, the total kinetic energy of the beam with an asymmetric damage is obtained by summing the energy expressions provided in Eqs. (B.16), (B.18) and (B.19).

Subsequently, the kinetic and strain energies of the PZTs are incorporated, then the Rayleigh-Ritz method is employed to discretize the displacements, followed by the application of the Lagrange equation for $r_i = q_L$ and $r_i = q_F$. This procedure yields the system's motion equations for longitudinal and flexural dynamics presented below.

- Longitudinal Dynamics

$$\begin{aligned}
& \begin{bmatrix} \mathbf{M}_L & (\mathbf{M}_{A_d} + \mathbf{M}_{A_{d_2}}) \\ (\mathbf{M}_{A_d} + \mathbf{M}_{A_{d_2}})^T & (\mathbf{M}_F + \mathbf{M}_{F_d} + \mathbf{M}_{F_{d_2}} + \mathbf{M}_{F_{d_3}}) \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_L \\ \ddot{\mathbf{q}}_F \end{Bmatrix} + \\
& \begin{bmatrix} \mathbf{K}_L & (\mathbf{K}_{A_d} + \mathbf{K}_{A_{d_2}}) \\ (\mathbf{K}_{A_d} + \mathbf{K}_{A_{d_2}})^T & (\mathbf{K}_F + \mathbf{K}_{F_d} + \mathbf{K}_{F_{d_2}}) \end{bmatrix} \begin{Bmatrix} \mathbf{q}_L \\ \mathbf{q}_F \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_L \\ \mathbf{0} \end{Bmatrix} E_3. \quad (B.20)
\end{aligned}$$

- Flexural Dynamics

$$\begin{aligned}
& \begin{bmatrix} \mathbf{M}_L & (\mathbf{M}_{A_d} + \mathbf{M}_{A_{d_2}}) \\ (\mathbf{M}_{A_d} + \mathbf{M}_{A_{d_2}})^T & (\mathbf{M}_F + \mathbf{M}_{F_d} + \mathbf{M}_{F_{d_2}} + \mathbf{M}_{F_{d_3}}) \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_L \\ \ddot{\mathbf{q}}_F \end{Bmatrix} + \\
& \begin{bmatrix} \mathbf{K}_L & (\mathbf{K}_{A_d} + \mathbf{K}_{A_{d_2}}) \\ (\mathbf{K}_{A_d} + \mathbf{K}_{A_{d_2}})^T & (\mathbf{K}_F + \mathbf{K}_{F_d} + \mathbf{K}_{F_{d_2}}) \end{bmatrix} \begin{Bmatrix} \mathbf{q}_L \\ \mathbf{q}_F \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{N}_F \end{Bmatrix} E_3, \quad (B.21)
\end{aligned}$$

where $\mathbf{M}_L$ and $\mathbf{M}_F$ are the total mass matrices for the longitudinal and flexural dynamics, respectively, obtained by summing the mass matrices of the beam, $\mathbf{M}_{b(\cdot)}$, and the PZTs, $\mathbf{M}_{p(\cdot)}$. The latter is defined by Eqs. (3.17) and (3.37) for the longitudinal and flexural dynamics, respectively, while the elements of the beam-related matrices are given by

$$m_{b(\cdot)}^{(i,j)} = \int_0^{x_d} \tilde{\rho}_b A_b \psi_{i(\cdot)}(x) \psi_{j(\cdot)}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{\rho}_b A_d \psi_{i(\cdot)}(x) \psi_{j(\cdot)}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{\rho}_b A_b \psi_{i(\cdot)}(x) \psi_{j(\cdot)}(x) dx. \quad (B.22)$$

The coupling mass matrices $\mathbf{M}_{A_d}$ and $\mathbf{M}_{A_{d_2}}$, arising from the presence of the asymmetric damage, are defined as

$$m_{a_d}^{(i,j)} = \int_{x_d}^{x_d+L_d} 2\tilde{\rho}_b A_d d_{ad} \psi_{i_L}(x) \psi'_{j_F}(x) dx; \quad (\text{B.23})$$

$$m_{a_{d_2}}^{(i,j)} = - \int_{x_d}^{x_d+L_d} \tilde{\rho}_b \gamma_{ad} \psi_{i_L}(x) \psi'_{j_F}(x) dx.$$

The matrices $\mathbf{M}_{F_d}$, $\mathbf{M}_{F_{d_2}}$, and $\mathbf{M}_{F_{d_3}}$ represent additional flexural mass effects induced by the damaged region, and their elements are defined as follows:

$$m_{d_F}^{(i,j)} = \int_0^{x_d} \tilde{\rho}_b I_b \psi'_{i_F}(x) \psi'_{j_F}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{\rho}_b I_{ad} \psi'_{i_F}(x) \psi'_{j_F}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{\rho}_b I_b \psi'_{i_F}(x) \psi'_{j_F}(x) dx;$$

$$m_{d_{2F}}^{(i,j)} = \int_{x_d}^{x_d+L_d} \tilde{\rho}_b A_d d_{ad}^2 \psi'_{i_F}(x) \psi'_{j_F}(x) dx;$$

$$m_{d_{3F}}^{(i,j)} = - \int_{x_d}^{x_d+L_d} \tilde{\rho}_b \gamma_{ad} d_{ad} \psi'_{i_F}(x) \psi'_{j_F}(x) dx. \quad (\text{B.24})$$

Similarly, $\mathbf{K}_L$ and $\mathbf{K}_F$ denote the total stiffness matrices for the longitudinal and flexural dynamics resulting from the sum of the beam, $\mathbf{K}_{b(\cdot)}$, and PZTs, $\mathbf{K}_{p(\cdot)}$, stiffness matrices. The latter is defined by Eqs. (3.18) and (3.38) for the longitudinal and flexural dynamics, respectively, and the former is given by

$$k_{b_L}^{(i,j)} = \int_0^{x_d} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{E}_b A_d \psi'_{i_L}(x) \psi'_{j_L}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{E}_b A_b \psi'_{i_L}(x) \psi'_{j_L}(x) dx;$$

$$k_{b_F}^{(i,j)} = \int_0^{x_d} \tilde{E}_b I_b \psi''_{i_F}(x) \psi''_{j_F}(x) dx + \int_{x_d}^{x_d+L_d} \tilde{E}_b I_{ad} \psi''_{i_F}(x) \psi''_{j_F}(x) dx + \int_{x_d+L_d}^{L_b} \tilde{E}_b I_b \psi''_{i_F}(x) \psi''_{j_F}(x) dx. \quad (\text{B.25})$$

The coupling stiffness matrices $\mathbf{K}_{A_d}$ and $\mathbf{K}_{A_{d_2}}$ are defined as

$$k_{a_d}^{(i,j)} = \int_{x_d}^{x_d+L_d} 2\tilde{E}_b A_d d_{ad} \psi'_{i_L}(x) \psi''_{j_F}(x) dx;$$

$$k_{a_{d_2}}^{(i,j)} = - \int_{x_d}^{x_d+L_d} \tilde{E}_b \gamma_{ad} \psi'_{i_L}(x) \psi''_{j_F}(x) dx. \quad (\text{B.26})$$

Whereas the stiffness matrices, $\mathbf{K}_{F_d}$ and $\mathbf{K}_{F_{d_2}}$, which account for additional flexural

behavior induced by the asymmetric damage, are given by

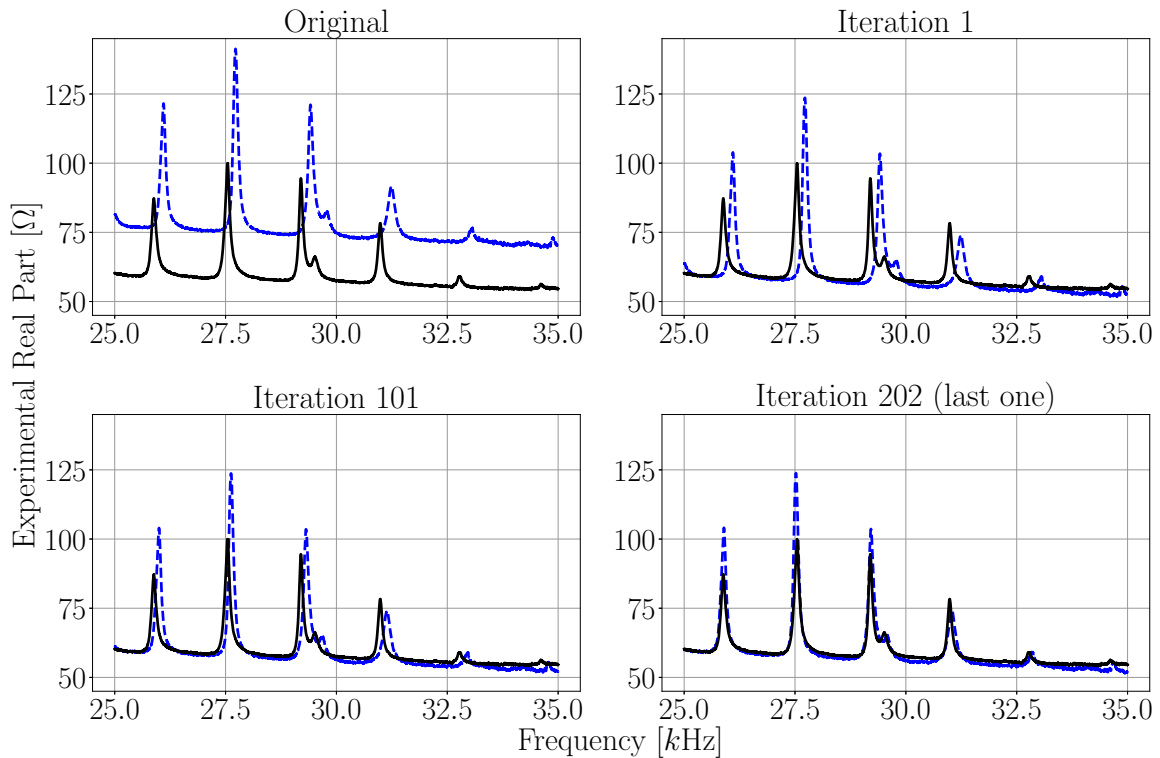
$$k_{d_F}^{(i,j)} = \int_{x_d}^{x_d+L_d} \tilde{E}_b A_d d_{ad}^2 \psi_{i_F}''(x) \psi_{j_F}''(x) dx; \quad (\text{B.27})$$

$$k_{d_{2F}}^{(i,j)} = - \int_{x_d}^{x_d+L_d} \tilde{E}_b \gamma_{ad} d_{ad} \psi_{i_F}''(x) \psi_{j_F}''(x) dx.$$

APPENDIX C – OVERVIEW ON PARK *ET AL.* (1999) COMPENSATION TECHNIQUE

This appendix presents the electromechanical impedance curves obtained during the application of the Park *et al.* (1999) temperature compensation technique. Figure 94 shows the real part of the experimental EMI for the aluminum beam with an asymmetrically coupled PZT in the undamaged condition, measured at two different temperatures (10°C and 30°C) and across four distinct stages of the method (i.e., iterations). The values of V_a and δ^S for each iteration are provided in Tab. 16. The reference signature is the one measured at 30°C. After the first iteration, the vertical discrepancy between the two curves is reduced by summing δ^S to the unknown measured EMI signal (see Eq. 5.2). This parameter presents only minor variations, even after multiple iterations, as indicated in Tab. 16. This occurs because, throughout the iterations of the compensation technique, the vertical discrepancy between the curves remains largely unaffected by the horizontal shifts of $Re(Z_{emU})$. Consequently, δ^S , which is strongly influenced by the average of this difference, also undergoes minimal variation.

Figure 94 – Experimental EMI real part at four distinct iterations of the Park *et al.* (1999) compensation technique for the reference signature at 30°C (solid black line) and the unknown signal at 10°C (dashed blue line).



Source: Elaborated by the author.

In the reference article, Park and his colleagues demonstrate the effectiveness of the

Table 16 – Values of V_a and δ^S in each iteration.

Iteration	V_a [Ω^2]	δ^S [Ω]
0	4242589.902	-17.486
1	970046.711	-17.571
101	593727.990	-17.486
202	143901.751	-17.401

Source: Elaborated by the author.

compensation technique in mitigating the temperature effects through both qualitative (EMI graphs) and quantitative (damage metric charts) analyses, by comparing the uncompensated and compensated results. The investigation considers temperature variations from 25°C to 75°C, with increments of 12.5°C. The application of this technique significantly reduces the temperature influence on the damage metrics, enabling the detection of even incipient damage. The experiments were carried out in different frequency ranges according to the monitored structure: carbon-steel beam (70 kHz to 80 kHz), bolted pipe joint (70 kHz to 80 kHz), gears (190 kHz to 220 kHz), and composite reinforced structure (54 kHz to 63 kHz).