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Tension force in nanosatellite tethers systems

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Abstract. The use of innovative technologies in space missions has evolved considerably in the last decade. The use of large cables in space structures to connect spacecrafts and satellites with the goal to minimize cost of missions created a new field to be explored. A brief explanation will be considered about papers related to the equilibrium and stability of the movement of space systems connected by cables, known as Tether Systems. It will be presented the mathematical formulation for the system formed by two point masses connected by a tether in the central force field, in a Keplerian movement. The Lagrangian formulation was used to describe the rotational movement of the dumbbell-like system. Results of system behavior, tension and kinetic energy will be presented for two different situations, considering equal masses and different masses.

1. Introduction

Tether systems consists by rigid objects connected each other by flexibles cables and with high length, known as tethers [1]. It has a great potential for transporting cargo into space without a propellant consumption. Among its several applications in space missions we can mention the concept of space elevator and the creation of artificial gravity.

The concept of Space Tether was created by the Russian scientist Tsiolkovskii in 1895, who idealized the creation of artificial gravity in a spacecraft connected to a counterweight by rotating around the mass center of the system. In the year of 1966, this idea was used during the Gemini-11 space mission. Tsiolkovskii was also one of the first to imagine a “tower” which the base is located on the Earth’s surface and its top would be in a geostationary orbit [2]. It was based on this idea that the concept of a space elevator was created.

Beletsky and Levin [2], discuss the concept of Space Tether with their applications and describe the dynamics of these systems from analytical formulations. Besides being presented the restrictions of mass and tensions, along with a table of all possible materials to be used in the tether manufacturing. Systems with massless tether, massive tether and pendular motions are analyzed searching for stability and equilibrium solutions in a Newtonian Field. In some cases, there is a simplification of the problem in order to obtain partial solutions, such as a system without disturbances, stationary motions and smalls curvatures along the tether.

Aslanov et al. in one of his papers [3] conducts a study of tether dynamics after a rupture of a space elevator due to a space debris collision. It was considered that the rupture occurred in a geostationary orbit, taking into account the action of gravitational, inertial and aerodynamic forces. He discovered that some tether fragments have reached the Earth’s surfaces with a



considerable velocity. In another paper, Aslanov et al. [4], discuss about a chaotic motion of one system formed by a low thrust tug-debris, caused by the orbital eccentricity effects and the magnitude of the thrust. The system was considered as two-point masses connected by a massless tether. A general equation for the motion was obtained in order to analyze the effects caused by the eccentricity and the disturbance conducted by the movement outside the reference plane.

Burov and Toger [5] approaches a system composed by a pendulum suspended by a fixed tether in a rigid body in a Newtonian central field of forces. The pendulum consists of a point mass attached to a massless and non-deformable tether in a circular orbit. The equilibrium conditions were defined and the stability solutions for the system was analyzed by Routh's method. Burov et al. [6] used the same previously formulation for analyze the dynamics and stability of a spacecraft connected by a tether on a surface of an asteroid, which has a constant angular velocity. The system is modeled like a pendulum and the asteroid is considered a sphere. The results are obtained for two different situations, when the first one for a known position of the anchoring, and the second for a given spacecraft position.

Ferreira et al. [7] performs a study of a dynamics of system formed by a double asteroid which a tether will be anchored in one of it. In order to obtain the equilibrium points and to analyze the system stability, it was necessary to know the shape, dimensions and mass distribution of the asteroid. Firstly, the bodies were treated as spheres and a partial result was obtained. At that time, irregularities were added making the body more realistic. The asteroids Ostro and Atiope were analyzed, both are double and synchronous asteroids. It was obtained that for irregular asteroids, which the point of anchor was in the plane formed by the primary bodies, no solutions were found outside this plane. Ferreira et al. [8] discuss a "capture portal" capable of alter the movement of a spacecraft, performing a "slingshot" maneuver without fuel consumption. This maneuver consists of a tether fixed to the surface of a body while the other end has a net attached, which will be used by the vehicle to perform the capture or escape maneuver. The Restricted Three-Body Problem was used for the formulation, also a massless and inextensible tether were considered. Equilibrium points on the surface of the body to connect the tether were found. The use of this maneuver causes a greater gain (or loss) of energy in relation to assisted gravitational maneuvers.

Santos et al. [9] analyzed the equilibrium conditions for a dumbbell-like system, formed by two point masses connected by a massless tether with variable length. The planar elliptic orbit was described from the position of the center of mass of the system, and the motion equations were obtained through the Lagrangian formulation. Control laws for angles of rotation around center of mass were considered, it was used the Floquet Theory to obtain the stability conditions. The result showed that small perturbations do not affect the system behavior, also was found stability regions for several values of eccentricity for the case of uniform rotations. Santos et al. [10] approaches a dumbbell-like system, this time considering a three-dimensional movement, outside the reference plane. Two cases were analyzed, tether length was dependent of eccentricity, true anomaly and initial length; and the case of constant length.

The main objective of this work is to simulate two configurations in which two point masses are connected by a massless tether. The tension force, kinetic energy and total energy of the system will be presented as a result.

2. Mathematical Modeling

It was analyzed a dumbbell-like system formed by two point masses (m_1 and m_2), connected each other by a massless cable with variable length, the model is shown in figure 1. The system movement is described from the position of the center of mass which moves in a Keplerian orbit around a primary body along the XY plane.

The position of the center of mass (C) which coordinates are given by (x_0, y_0) , can be written

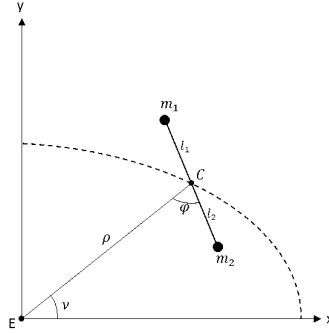


Figure 1: Tether dumbbell-like system model

as:

$$\begin{cases} x_0 = \rho \cos(\nu) \\ y_0 = \rho \sin(\nu) \\ \rho = \frac{p}{1 + e \cos(\nu)} \end{cases} \quad (1)$$

$$\begin{cases} x_1 = x_0 + l_1 \cos(\nu + \varphi) \\ y_1 = y_0 + l_1 \sin(\nu + \varphi) \\ x_2 = x_0 - l_2 \cos(\nu + \varphi) \\ y_2 = y_0 - l_2 \sin(\nu + \varphi) \end{cases} \quad (2)$$

Which ρ the distance of C from the primary body E , the angle ν is the true anomaly, the focal parameter is denoted by p and the orbit eccentricity (e). The coordinates of each of the masses m_1 and m_2 are given by,

Where φ is the angle formed between the tether and ρ , l_1 and l_2 are the distances of m_1 and m_2 to the center of mass, respectively.

Lagrangian Formulation was used, equations of motion that relate the Potential and Kinetic energies and their generalized coordinates, since it is not necessary to know all the forces acting in the system. Its general equation is given by:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = Q_j \quad (3)$$

Which L is the Lagrangian given by $L = T - V$, where T is the kinetic energy and V is the potential energy of the system, q_j is the generalized coordinate, and Q_j is the generalized force acting on the system.

For the modeling of this system, it was defined that it is conservative, where there are no perturbative forces, $Q_j = 0$, and the generalized coordinates used were φ and l (the tether length), obtaining the equations:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\varphi}} \right) - \frac{\partial L}{\partial \varphi} = 0 \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{l}} \right) - \frac{\partial L}{\partial l} = 0 \quad (4)$$

For this work it was chosen to use the first of the above equations, where we initially define conditions for φ in order to obtain the behavior of l .

The kinetic and potential energies of the system are given by

$$T = \frac{1}{2} m \left(\frac{p^2 (1 + e^2 + 2e \cos \nu) \dot{\nu}^2}{(1 + e \cos \nu)^4} - (-1 + \mu) \mu (l^2 + l^2 (\dot{\nu} + \dot{\varphi})^2) \right) \quad (5)$$

$$V = -\frac{m \mu_0 (1 + e \cos \nu)}{p} + \frac{m (-1 + \mu) \mu \mu_0 (1 + e \cos \nu)^3 (1 + 3 \cos 2\varphi) l^2}{4 p^3} \quad (6)$$

Where T is the kinetic energy and V is the potential energy. $m = m_1 + m_2$, $\mu = \frac{m_1}{m}$ and $\mu_0 = GM$. G is the gravitational constant and M is the mass of the primary body. p is the focal parameter, e is the orbital eccentricity and l is the tether length.

Realizing the simplifications and developing the equations shown above, making it the function of the true anomaly ν , independent of time, we obtain the equation below, which establishes a relation between the angle φ and the length l of the tether. Which $(.)' = \frac{d}{d\nu}$.

$$(1 + e \cos \nu) \varphi'' + 2 \left[\frac{l'}{l} (1 + e \cos \nu) - e \sin \nu \right] (1 + \varphi') + 3 \cos \varphi \sin \varphi = 0 \quad (7)$$

Adopting a uniform rotation of the system around the center of mass, a control law for the system is given by,

$$\varphi = \omega \nu + \varphi_0 \quad (8)$$

Where ω is the angular velocity, ν is the true anomaly, φ_0 is the initial condition and $\varphi' = \omega$. Rewriting equation (7) as a function of $\frac{l'}{l}$ and replacing equation (8), we have the variation of the length of the tether along its orbit as a function of the true anomaly.

$$\frac{l'(\nu)}{l(\nu)} = \frac{e \sin \nu}{1 + e \cos \nu} - \frac{3 \sin(2(\omega \nu + \varphi_0))}{4(1 + \omega)(1 + e \cos \nu)} \quad (9)$$

The tension force in the tether is given by the equation below [11],

$$\vec{\tau} = m_1 \left(\vec{r}_1 + \frac{\mu_0 \vec{r}_1}{r_1^3} \right) = m_2 \left(\vec{r}_2 + \frac{\mu_0 \vec{r}_2}{r_2^3} \right) \quad (10)$$

Where $\vec{r}_1 = (x_1, y_1)$ and $\vec{r}_2 = (x_2, y_2)$.

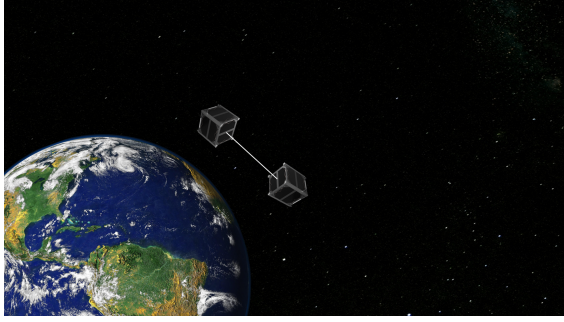
3. Results

The simulations comprehend two configurations in which two point masses are connected by a tether. In the first one is analyzed the behavior of a system formed for two equal masses, while in the second one is considered two bodies with a great difference between their masses., as shown in Figure 2 . Both systems are in low orbit but the atmosphere drag is not considered. The rotational movement, the tension force, kinetic energies and the total energy of the systems are analyzed as a function of true anomaly.

Nanosatellites may have some different configurations, the simplest is the 1u, which represent a small cube of approximately 10 cm edges, volume of 1 liter and mass up to 1.33 kg. The other configurations are given by the union of nanosatellites 1u, as is the case of 3u, formed by three 1u configuration.

3.1. First Case: Equal Masses

Two equal masses connected by a tether are considered ($m_1 = m_2$). To exemplify this case, two nanosatellites 1u connected by a massless tether are used. In this simulation the following initial conditions are considered $m_1 = m_2 = 1.33 \text{ kg}$, $l(0) = 10 \text{ km}$ and $p = 7200 \text{ km}$. The results are shown in Figure 3.

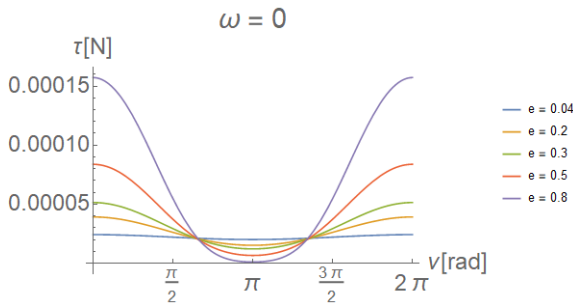
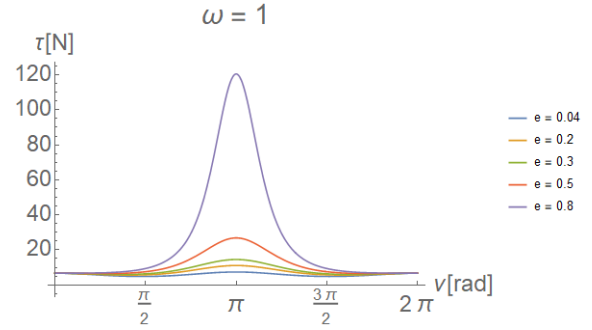
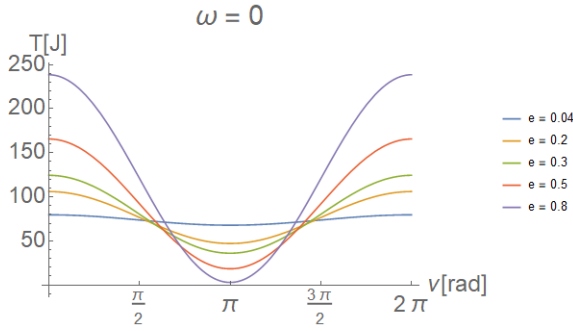
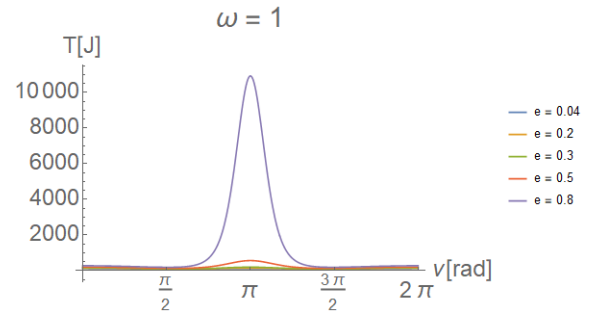
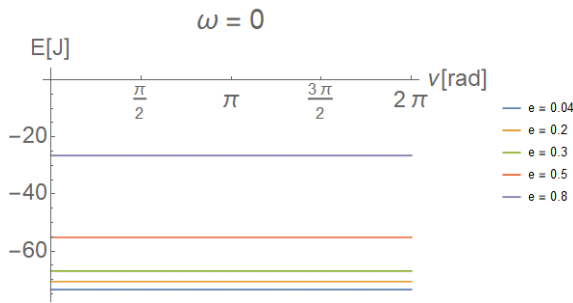
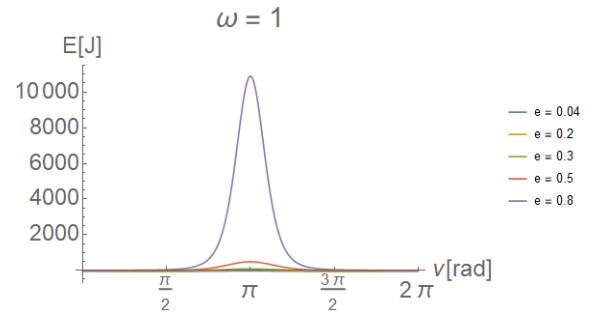


(a) Two nanosatellites connected by a cable



(b) Main satellite and nanosatellite connected by a cable

Figure 2: Artistic conception of the suggested system.

(a) Tension force for $\omega = 0 \text{ rad/s}$ (b) Tension force for $\omega = 1 \text{ rad/s}$ (c) Kinetic Energy for $\omega = 0 \text{ rad/s}$ (d) Kinetic Energy for $\omega = 1 \text{ rad/s}$ (e) Total Energy for $\omega = 0 \text{ rad/s}$ (f) Total Energy for $\omega = 1 \text{ rad/s}$ Figure 3: Results of the first case analyzed: $m_1 = m_2 = 1.33 \text{ kg}$, $l(0) = 10 \text{ km}$, $p = 7200 \text{ km}$

3.2. Second Case: Different Masses

Two masses, with a great different between it, connected by a tether are considered ($m_1 \gg m_2$). To exemplify this case, a main satellite, illustrated by the Sino-Brazilian satellite CBERS-2, and a 1u nanossatélite are connected by a tether of negligible mass. In this simulation the following initial conditions are considered, $m_1 = 1450 \text{ kg}$, $m_2 = 1.33 \text{ kg}$, $l(0) = 10 \text{ km}$ and $p = 7200 \text{ km}$. The results are shown in Figure 4.

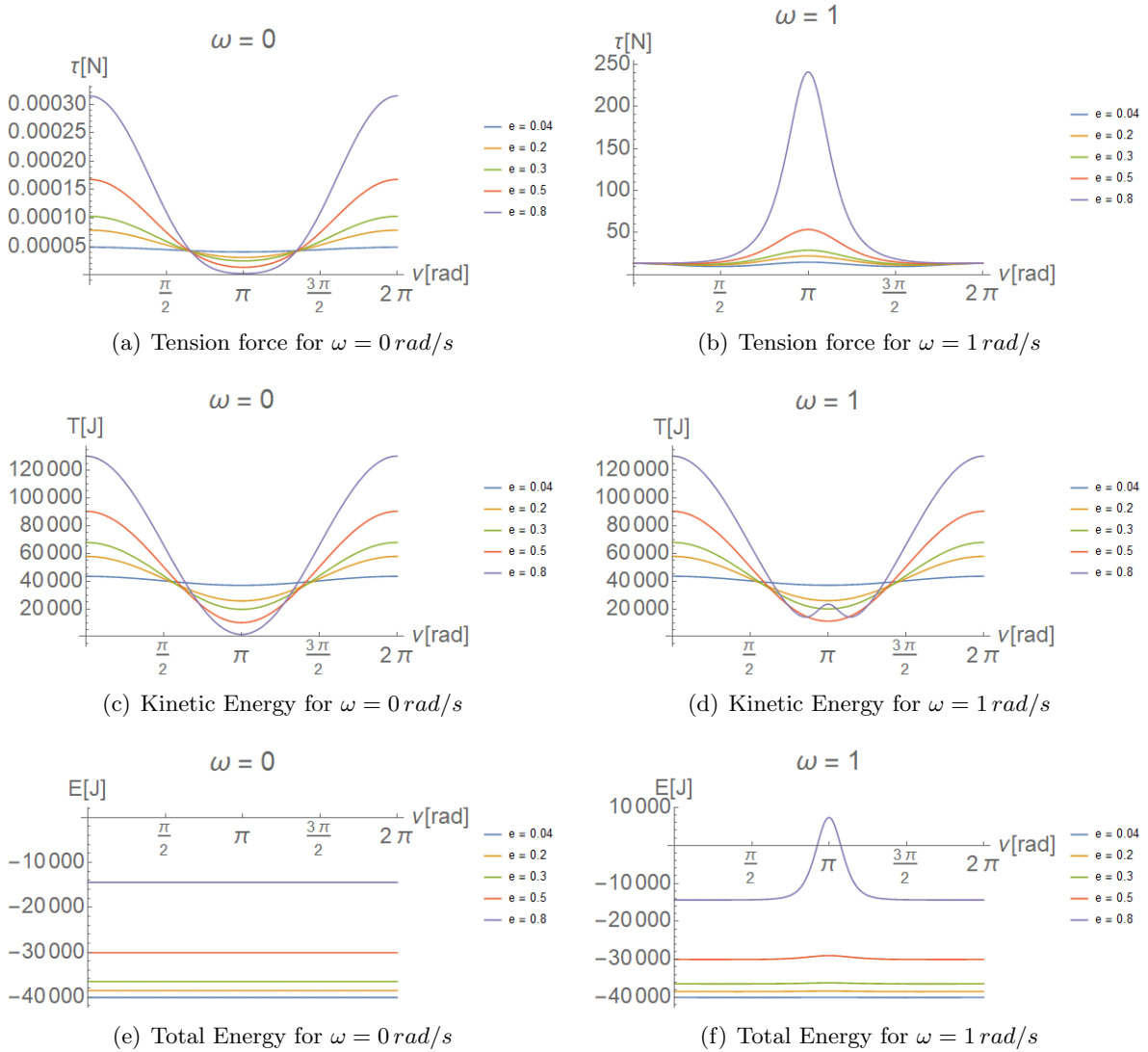


Figure 4: Results of the second case analyzed: $m_1 = 1450 \text{ kg}$, $m_2 = 1.33 \text{ kg}$, $l(0) = 10 \text{ km}$, $p = 7200 \text{ km}$

4. Conclusion

The same initial conditions are considered for both simulations in order to compare its results. The tension τ is seen that although the difference of the total masses of each system is large, $m_{T1} = 2.66 \text{ kg}$ and $m_{T2} = 1451.33 \text{ kg}$, the tension forces in the tether are of comparable magnitudes. Therefore, the distribution of masses in the system is a more influential factor for the intensity of the tension when compared to the total mass of the system. It was also

noted that for higher values of eccentricity and rotational velocity higher is the force in the tether. When $\nu = \pi$ the higher variations of the energies occurs because the center of mass of the system lies in the periapsis of the orbit, where it is the point of maximum kinetic energy.

In the control law for the rotation of the system, considered in this work, an active control can be implemented, so the attitude of tether can be defined, like a pointing towards the Earth, where $\omega = 0$ and varying φ_0 as a function of the orbital movement.

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