UNIVERSIDADE ESTADUAL PAULISTA

PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA

ÁREA DE ENSINO E APRENDIZAGEM DA
MATEMÁTICA E SEUS FUNDAMENTOS FILOSÓFICO-CIENTÍFICOS

EMPOWERING ADDITIONAL LANGUAGE LEARNERS AND THEIR TEACHERS IN 21ST-CENTURY MATH CLASSES THROUGH TECHNOLOGY AND AI

Jinos Nejad Mikailli

INSTITUTO DE GEOCIÊNCIAS E CIÊNCIAS EXATAS

Rio Claro – SP
2025

UNIVERSIDADE ESTADUAL PAULISTA
"Júlio de Mesquita Filho"
Instituto de Geociências e Ciências Exatas
Campus de Rio Claro

Jinos Nejad Mikailli

EMPOWERING ADDITIONAL LANGUAGE LEARNERS AND THEIR TEACHERS IN 21ST-CENTURY MATH CLASSES THROUGH TECHNOLOGY AND AI

Dissertação apresentada ao Programa de Pós-Graduação em Educação Matemá-tica do Instituto de Geociências e Ciências Exatas da Universidade Estadual Paulista "Júlio de Mesquita Filho", campus de Rio Claro, como requisito para obtenção do grau de Mestre em Educação Matemática.

Orientador: Prof. Dr. Marcus Vinicius Maltempi

Rio Claro (SP)
2025

N417e

NejadMikailli, Jinos

Empowering additional language learners and their teachers in 21st-century math classes through technology and AI / Jinos NejadMikailli. -- Rio Claro, 2025

105 p.

Dissertação (mestrado) - Universidade Estadual Paulista (UNESP), Instituto de Geociências e Ciências Exatas, Rio Claro

Orientadora: Marcus Vinicius Maltempi

1. Matemática do século XXI. 2. Proficiência linguística. 3. Educação matemática. 4. Desafios culturais. 5. Integração de AI. I. Título.

Sistema de geração automática de fichas catalográficas da Unesp. Dados fornecidos pelo autor(a).

IMPACTO POTENCIAL DESTA PESQUISA

Esta pesquisa está diretamente relacionada ao 4º objetivo da Agenda 2030 das Nações Unidas para o Desenvolvimento Sustentável, que destaca a educação de qualidade inclusiva e equitativa para todos. Seu objetivo é melhorar os resultados de aprendizagem, integrando IA e tecnologia em um ambiente multilíngue e multicultural.

POTENTIAL IMPACT OF THIS RESEARCH

This research is directly related to the 4th goal of the United Nations 2030 Agenda for Sustainable Development, highlighting inclusive and equitable quality education for all. It aims to enhance learning outcomes by integrating AI and technology in a multilingual and multicultural setting.

تاثیر بالقوه این پژوهش

این پژوهش مستقیما با چهارمین برنامه توسعه پایدار 2030 سازمان ملل متحد که بر اهمیت آموزش فراگیر، برابر و با کیفیت تاکید دارد، مرتبط است. هدف اصلی این تحقیق بهبود نتایج یادگیری از طریق ادغام هوش مصنوعی و تکنولوژی در محیطهای چند زبانه با تنوع فرهنگی است.

UNIVERSIDADE ESTADUAL PAULISTA
"Júlio de Mesquita Filho"
Instituto de Geociências e Ciências Exatas
Campus de Rio Claro

Jinos Nejad Mikailli

EMPOWERING ADDITIONAL LANGUAGE LEARNERS AND THEIR TEACHERS IN 21ST-CENTURY MATH CLASSES THROUGH TECHNOLOGY AND AI

Dissertação apresentada ao Programa de Pós-Graduação em Educação Matemática do Instituto de Geociências e Ciências Exatas da Universidade Estadual Paulista "Júlio de Mesquita Filho", campus de Rio Claro, como requisito para obtenção do grau de Mestre em Educação Matemática.

Orientador: Prof. Dr. Marcus Vinicius Maltempi

Comissão Examinadora

Prof. Dr. Marcus Vinicius Maltempi – Orientador
IGCE/Unesp/Rio Claro (SP)

Prof. Dr. Marcelo Almeida Bairral
Universidade Federal Rural do Rio de Janeiro (UFRRJ)

Prof. Dr. Roger Miarka
IGCE/Unesp/Rio Claro (SP)

Conceito: Aprovado.

Rio Claro (SP), 17 de Março de 2025.

With deep respect and gratitude, I dedicate this research to the remarkable professors at BIHE University whose wisdom, courage and support have not only shaped my academic journey but also transformed my life.

Acknowledgment

I would like to thank Prof. Dr. Farid Nourani whose support and dedication made my admission to this program possible. I am truly grateful for your invaluable effort in handling all the necessary paperwork, your guidance through the process, your kindness, and your generosity. I would like to thank Prof. Dr. Rosana Miskulin who readily accepted to be my advisor and offered her support despite facing health challenges. I am deeply thankful to my advisor, Prof. Dr. Marcus Vinicius Maltempi, for kindly stepping in with no hesitation and guiding me with his dedication. I am grateful for your tremendous mentorship in conducting my research. Your guidance made a significant difference in my academic journey. I would like to thank all the faculty members of the Graduate Program in Mathematics Education (PPGEM) for this opportunity, in particular, Prof. Roger Miarka and Prof. Heloisa da Silva. Thank you for the constant dedication and effort you put into making things easier for everyone. Thank you Prof. Marcello Bairral and Prof. Roger Miarka, my evaluation committee, for your precious help. I thank Prof. Jessica Shumway, Dr. Danyal Farsani, and Dr. L.C. who participated in this study. Without your passionate participation, this study would have been impossible.

Agradeço aos membros do meu grupo de pesquisa, DIEEM, e aos amigos da Unesp. Ter vocês ao meu lado tornou essa jornada mais leve e especial. Sou imensamente grata pela amizade de cada um de vocês.

Finally, yet importantly, a special thanks goes to my family, my amazing mum, and my big sister Rojan. You are the whole world to me. Thank you for being there and providing unconditional support through this journey. Thank you for your encouragement, patience, and belief in me. Your words, “No storm lasts forever. Keep moving forward.” kept me going through every challenge. Thank you Amoo joon and the whole family for your support during my initial arrive. Your help means more than words can express.

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

This study was financed in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPQ).

همه بار یک داریم و برگ یک شاخسار.

(بهاءالله)

Resumo

Com o dramático aumento do número de refugiados na última década, a prática de uma educação matemática de qualidade enfrenta inúmeros desafios em ambientes multilíngues e multiculturais. Esta pesquisa investiga o potencial das tecnologias e da inteligência artificial (IA) para a formação de aprendizes em aulas de matemática ministradas em outro idioma, particularmente à luz da Agenda 2030 das Nações Unidas para o Desenvolvimento Sustentável, que visa garantir uma educação de qualidade, inclusiva e equitativa para todos. A mudança de abordagens tradicionais para abordagens de matemática do século XXI tornou a proficiência linguística mais crucial do que nunca para o desenvolvimento das habilidades 4C em sala de aula. Esta pesquisa explora os desafios enfrentados por professores e seus alunos que estão aprendendo matemática em uma língua adicional ao se envolverem com habilidades do século XXI. Além disso, visa evidenciar como a IA e as tecnologias digitais podem favorecer o processo de adaptação a novas culturas educacionais, auxiliando estudantes de diferentes origens a superar barreiras linguísticas e prosperar em ambientes acadêmicos desconhecidos. A pesquisa mostra que as tecnologias digitais e a IA podem fornecer acesso 24 horas por dia, 7 dias por semana, a materiais em vários idiomas, oferecer ambientes de aprendizagem personalizados para atender às necessidades únicas dos alunos, fornecer feedback em tempo real para avaliações automatizadas para grandes grupos de alunos, promover colaboração e interação entre todos os participantes e abrir caminho para o desenvolvimento de habilidades de resolução de problemas e pensamento crítico. Com o aumento da interação entre alunos usando tecnologias educacionais e IA, a expectativa é que nenhum aluno fique para trás e todos possam aprender sem o medo de serem julgados.

Palavras-chaves: Matemática do século XXI, educação matemática, proficiência linguística, desafios culturais, integração de IA

Abstract

With the dramatic increase in the number of refugees over the past decade, implementing a 21st-century education has presented numerous challenges for teaching and learning math in multilingual and multicultural environments. This research investigates the potential of technology and artificial intelligence (AI) to empower additional language learners in 21st-century math classes particularly in light of United Nations 2030 Agenda for Sustainable Development, which aims to guarantee inclusive and equitable quality education for all learners. The shift from traditional to 21st-century math approaches has made language proficiency more crucial than ever in applying 4C skills effectively in the classroom. This research seeks to explore the challenges teachers and their students who are learning math in an additional language face while engaging with 21st-century skills. It also aims to demonstrate how AI and digital technologies can facilitate the process of adjusting to new educational cultures, helping students from diverse backgrounds overcome language barriers and thrive in unfamiliar academic environment. Digital tools and AI can provide 24/7 access to materials in various languages, offer personalized learning environments to meet students' unique needs, deliver real-time feedback for automated assessments to large group of students, promote collaboration and interaction among all participants and paves the way to gain problem-solving and critical thinking skills. With an increase in interaction between students using educational technologies and AI, no learner is left behind and all can learn without the fear of being judged.

Keywords: 21st century, math-education, language proficiency, cultural challenges, AI-integration

چکیده مقاله

با افزایش چشمگیر پناهجوبان در دهه ی گذشته، تدریس و همچنین یادگیری ریاضی در چهارچوب قرن 21 در محیطی چند زبانه و چند ملیتی با چالش های بسیاری روبرو شده است. این پژوهش به دنبال بررسی پتانسیل های فناوری تکنولوژی اموزشی و هوش مصنوعی در توانمند ساختن دانشجویان غیر بومی در کلاسهای ریاضی قرن 21 میباشد. این مطالعه در چهارچوب طرح توسعه پایدار (2030) سازمان ملل متحد، که بر آموزش فراگیر و با کیفیت برای همه دانش آموزان و دانشجویان تاکید دارد، انجام شده است. تغییر رویکرد از تدریس ریاضی به روش سنتی به روشهای نوین قرن 21، نقش زبان را در به کارگیری موثر توانایی های 4C (تفکر انتقادی، خلاقیت، همکاری و تعامل) در کلاس پررنگتر کرده است. از این رو این مقاله تلاش دارد تا چالش های دانش آموزانی که به زبانی به غیر از زبان مادری خود ریاضی را با اهداف قرن 21 فرا میگیرند و همچنین معلمهایشان زا بیابد و به علاوه نشان دهد چگونه تکنولوژی اموزشی و هوش مصنوعی میتواند پروسه تطبیق فرد با فرهنگ آموزشی جدید را تسهیل ببخشد تا دانش آموزانی با فرهنگ ها و زبانهای مختلف به موانع گفتاری خود فایق آیند و در محیط آموزشی نا آشنا پیشرفت کنند. فناوری دیجیتال با فراهم کردن دسترسی 24 ساعته به منابع آموزشی در زبانهای مختلف، برآورده کردن نیازهای فردی دانش آموزان در محیط یادگیری شخصی سازی شده، ارایه بازخورد آنی برای ارزیابی خوکار به گروه بزرگی از دانش آموزان، مسیر را برای حل مسایل از طریق تفکر انتقادی با به کار گیری تعامل و همکاری مهیا می سازد.با افزایش تعامل بین دانش آموزان از طریق فناوری آموزشی و هوش مصنوعی، هیچ دانش آموزی از فرایند آموزش حذف نمیشود و همه بدون ترس از قضاوت شدن میتوانند به یادگیری بپردازند.

کلید واژه: آموزش قرن بیست و یک، آموزش ریاضی، مهارت زبانی، چالشهای فرهنگی، هوش مصنوعی

Summary

1 Introduction

The context of my research interest emerged at a young age from my personal experiences as an Iranian Baha'i[1]. Having been born and raised in Iran, I have witnessed severe educational discrimination which has impacted the access of Baha'is, the largest non-Muslim religious minority, to education at various levels. Few years ago, Baha'i children even faced obstacles in attending schools for their beliefs but their resilience and effort have improved their access to primary and secondary education over time. However, they still do not have the right to attend colleges and universities. Since 1979, belonging only to a religion that is recognized by the Iranian government, including Islam, Christianity, Judaism or Zoroastrianism, has been the main qualification for education in Iran. Therefore, after taking the entrance exam of the university, many Baha'is have been told that their applications are incomplete and this has led to their exclusion from the university.

In response to this disparity, the Baha'i community of Iran has established the Baha'i Institute for Higher Education (BIHE) with the help of over 1000 volunteer professors from many different universities in the world to provide quality education for their youth. However, 17 Baha'i educators in Iran were arrested and sentenced to prison in 2011 to stop their efforts to deliver higher education for Baha'is in Iran. While BIHE has continued offering Bachelor's and Master's degrees, our lack of valid higher education certificates has impacted our access to professional opportunities in Iran. As a result, this inequity has left many members of this religious community with no alternative but to make the difficult decision to leave their country in search of a better future for their children.

According to Kirmayer et al. (2011), the displacement has been just the beginning of going through bigger problems for these families, especially for those with kids. They are likely to experience the trauma of migration and many other mental problems. Many kids in these families have stated that they suffered from mental health issues, depression, and anxiety as they entered schools where everything was taught in another language. Many of them have been disappointed by their inability to participate in class activities, express their knowledge effectively, and show excellent work in the exams. Additionally, the differences in

[1] Baha'i faith was founded in the mid-19th century by Bahaullah, which in Arabic means "Glory of God." Baha'u'llah, regarded by Baha'is as the last Messenger of God, proclaimed the message that humanity is one, all people are created by the same God and the purpose of religion is to foster unity and peace.

curriculum and education standards between countries have left gaps in children's knowledge and have caused a lack of interest in school. Also, failure to fit in with their peers in class and collaborate and interact with other participants has led to feelings of isolation and exclusion.

Immigration can be challenging for everyone, especially children. According to the Canadian Collaboration for Immigrant and Refugee Health (CCIRH), "children and adolescents must learn a new language, renegotiate their cultural identity, and deal with social isolation, racism, prejudice and discrimination". Bustamante (2017) also introduces some of the main sources of mental health problems among refugees as feelings of "not belonging to a single place", "weak social conditions, linguistic and cultural changes, loneliness" and "failure of one's migration project." Therefore, the stress of dislocation among children can be triggered by the need to adapt to a new education system, new language, and cultural identity.

Sadly, ongoing conflicts and human rights crises have spread all around the world and are not confined to Iran (#OurStoryIsOne[2]). The United Nations High Commissioner for Refugees (UNHCR) has affirmed a surge in the number of refugees at the end of 2023 to approximately 117.3 million people worldwide for "persecution conflict, violence, human rights violations, and events seriously disturbing the public order." With the very recent increase in the global conflicts in Palestine, Ukraine, Russia, Afghanistan, and Syria, UNHCR has estimated that this number will reach to 120 million at the beginning of 2024 and 40 percent of this number has been estimated to be children below the age of 18.

This rings an alarm bell to education authorities to be aware of potential challenges in providing education for this large number of language novices. United Nations set the *2030 Agenda for Sustainable Development* consisting of 17 goals in 2015. The fourth objective of this plan is a global education goal "to ensure inclusive and equitable quality education and promote lifelong learning opportunities for all." This strategic plan has aimed to guarantee that all individuals have access to inclusive and continuous learning throughout their lives to "acquire the knowledge and skills needed to exploit opportunities and to participate fully in society" regardless of their "sex, age, race, ethnicity, and persons with disabilities, migrants, indigenous peoples, children and youth, especially those in vulnerable situations." (UN Dept. of Economic and Social Affairs, 2015)

---

[2] The hashtag #OurStoryIsOne is used to convey the idea of shared human experience and unity and in the context of the Baha'i Faith, it honors the memory of 40 Baha'i women in Iran who were executed for their beliefs. This campaign reflects the shared narrative of courage, resilience, and unity in the face of oppression, reinforcing the universal truth that inequality knows no borders.

Also, many educational organizations, like the National Council of Teachers of Mathematics, have put increasing emphasis on equity as an essential element for education and mathematics learning and believe that “excellence in mathematics education requires equity, high expectations, and strong support for all students” (NCTM, 2000, p. 12). It also claims that “all students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding” (NCTM, 2000, p. 50). Therefore, providing every single student with equal chance, resources and support to succeed has become a necessity in the field of education.

In March 2023, I started my master’s at Unesp University in Portuguese with very little language proficiency. What I knew back then was that this academic journey wouldn’t be easy for me. My experience of pursuing a master’s degree in an additional language equipped me with the skills, perspectives, and empathy needed to address the complexities of language proficiency in education and helped me gain a deep understanding for students in similar situations. I have reflected this interdisciplinary approach in my research where I integrate insights from education, technology and cultural studies to develop solutions for equitable learning environments. This invaluable two-year journey made me immerse myself in a new language and culture to enhance my cultural sensitivity and awareness and understand the language and cultural contexts influencing learning. My master’s degree made me bridge gaps between different fields and linguistic barriers.

This two-year master’s degree with little language proficiency contributes to my research to a great extent. Most of the courses I took, required us to read articles and evaluate their argument. In other words, we were asked to apply critical thinking while reading the articles. We were also expected to use our creativity to find a new way to explain parts of concepts to the class. Additionally, we were given opportunities to collaborate with peers, exchange ideas, and communicate our insights through group work and discussions.

From the beginning, my participation in English often made the class less engaging for my classmates, as many did not understand English. At the same time, being exposed to a lot of Portuguese was exhausting, as I found it hard to understand many things. It was also difficult to ask my questions and clarify my doubts as I was unable to communicate with my classmates. Additionally, the students’ culture of participation in the class was interesting and challenging to me as class participation in my home country is structured differently. The students’ interaction with the professors and their discussion method was new to me. In the first sessions of the semester, I just observed students comfortably engaging in lively debates. Due to my cultural background, I was waiting for the class to become quiet before attempting to participate, even after raising my hand. This is something that sometimes made me feel isolated. Additionally, simultaneous conversations in the class made my comprehension difficult due to the language barrier. Little by little, the professors and my classmates learned about my attitude and invited me to take the chance to ask questions or leave comments. This experience proved the interplay between language and culture in educational settings to me.

In the same class, we were assigned projects where we had to use our creativity and technological tools that we were comfortable with to teach part of the curriculum in groups of two or three. A group benefited from the interactive whiteboard, called Jamboard, to facilitate collaboration. This increased my confidence to participate in discussions, using translation tools, while staying anonymous. This was how I gained a sense of belonging to the class. The other group used Nearpod, an application that shows PowerPoint presentations ensuring that all students progress together. This was also helpful as it let me know where we were and my language barrier wouldn't make me feel lost. Other than that, group activities and required communications brought us together and built a strong bond between us as we were fighting for a common objective.

In another class, exit assignments were used as a fair way to assess students' understanding. The flexibility allowed me to express my understanding in a way that was more comfortable for me and the impact of the language barrier was minimized. Additionally, the teacher implemented a flipped environment to enhance students' engagement and comprehension. We were supposed to read a chapter from the book in Portuguese or English to be prepared for our discussion in class. What happened was that all the students had an equal chance to gain information at their own pace outside of class.

However, my graduate studies were in a different area. When I took a course from the pure mathematics group, I needed to learn so many mathematics registers that I hadn't heard of before, neither in Persian nor in English. What helped me a lot was watching many videos on YouTube that went beyond simple definitions of math registers. Examples and step-by-step problem-solving strategies in the videos could make the complex ideas more accessible to me.

I began my master's studies with an interest in addressing some of the challenges around multilingual math class in 21st century and how technology and AI can empower both teachers and students through this journey. As a graduate with a bachelor's degree in both computer science and English language studies, I would like to bring a unique perspective to my work while recalling some of the hands-on experiences I had in my master's degree.

The research topic was supposed to be how second language learners can benefit from

AI and technologies in 21^{st}-century math classes. However, I made little changes to it after doing some research. According to Skovsmose (2020), teaching and learning are inseparable and they are “features of the same integral process” (p. 239). This is why I decided to analyze how a non-native environment can affect both teaching and learning. In addition, I found that the official language in many countries can differ from the language or dialect which is used in homes. Therefore, the language used at school in host countries might be children’s third language. Therefore, I revised the research title to *Empowering Additional Language Learners and Their Teachers in 21^{st}-Century Math Classes through Technology and AI*. By additional language learners, I mean any student whose language differs from the language of instruction in class. Therefore, the research does not only address refugee students and immigrants but also those students who have to use another language at school for any reason. For this reason, I aimed to address the commonalities shared among these groups to ensure that the findings and solutions are inclusive and applicable across diverse contexts.

As my research addresses a broad concept, I have divided it into three main chapters each of which answers one of the following objectives of the research:

1. What is 21^{st}-century math class and what skills do students need to develop within it?

2. What challenges do additional language learners and their teachers encounter in math classes due to language barriers and cultural differences?

3. How can technology and AI assist additional language learners and their teachers in overcoming those challenges in 21^{st}-century math classes?

The first chapter will set a foundation by elaborating on the skills students need to gain in math classes in today’s world. The chapter starts with demonstrating the failure of traditional math teaching and learning, which emphasized rote math learning or math learning without understanding. Teachers used to make their utmost effort to teach formulas and make students apply them in math tasks. Over time, the traditional approach has been criticized for the same reason. As Boaler (2016) confirms, it became clear that students were unable to apply their learnings in real-life scenarios and this would cause disengagement for learners. Little by little, many reforms reshaped mathematic learning to a more meaningful learning environment, which could develop students’ academic performance (Polman, 2021). With the emergence of Generation Z as a consequence of rapid advancement in technology, many organizations including NCTM, PISA, and P21 took steps toward defining successful math learning and preparing students for a complex world. They also attempted to align math learning with learners’ needs. Gen Z, with an interest in interaction and collaboration through digital technologies, desired skills that could help them with their careers (Mohr, 2017). Roll (2016) believes that “the dynamic nature of job requirements encourages schools to develop curricula that focus on knowledge application, collaboration, and self-regulated learning skills” and “Knowledge is becoming a verb (something we do) rather than a noun (something we possess)” (p. 591). Therefore, skills can refer to different abilities that learners need to gain to apply knowledge effectively. Now that the education content has shifted to a more student-centered class where math problems are understood, analyzed, and evaluated well, creativity is applied to create new ideas, and findings are shared through communication and collaboration. Therefore, the main objectives of 21st-century math class have a focus on the development of four skills including critical thinking, creativity, collaboration and communication. These skills which help students beyond school, provide them the chance to explore, examine, and reflect on their findings using collaboration and communication. For this, a comfortable environment should be provided for the students that lets them take risks and not be afraid of making mistakes.

The second chapter aims to provide the content by analyzing the challenges that additional language learners and their teachers might face in a 21st-century math class. Back to the problem I mentioned at the beginning of this part, a surge in the number of refugees has increased the number of non-native students at schools in recent years. Despite the benefits it can bring to the class, it can also be very challenging both for students and teachers. As mathematics is not a language-free content, students’ language barrier can be a big concern. Additionally, considering the objectives of the 21st-century math class and a shift from individual activities in teacher-centered classes to more social activities in students-centered classes, students need to be able to communicate the language. To achieve this goal, students should learn many homonyms, math notations, and registers as well as obtain a strong language ability to gain a full understanding of math tasks, instructions, feedback, and problems. They are also supposed to communicate mathematically to engage in math discussions and explain their reasoning.

The idea that I appreciated very much was the idea of the interrelation of language and culture and that culture also plays an important role in learning math. Many researchers admit the interconnectedness of education, language, and culture. Therefore, they believe that to have equity in education, students’ cultural context should be considered in a multicultural environment. This cultural context might refer to either contextual differences, including student cultural values, gender norms, and cultural representations, or educational culture which refers to pedagogical differences, curriculum design, and learning styles in different cultures. This raises a red flag to be aware of the role of culture to avoid a cultural bias and also to pre-teach cultural aspects before going through tasks.

The third section will show practical solutions by demonstrating how technology and AI can help teachers and additional language learners in 21st-century math classes overcome their challenges. According to Collier (1987), “young children with little or no formal schooling in their L1 require approximately 5-7 years to reach the level of a native speaker” (p. 619). If they acquire the additional language before entering school in host countries, it definitely makes them fall behind others. Therefore, AI-driven tools and digital platforms can provide language support, personalized learning experiences, and other assistance to bridge language barriers and make math more accessible to students with diverse linguistic and cultural backgrounds by facilitating communication, collaboration, and engagement and providing an inclusive learning environment for them. This work is not without its limitations. Therefore, this chapter concludes by discussing the prerequisites for effectively integrating technology and AI in the classroom and the potential challenges it may have for both teachers and students.

To analyze this study, I considered interviewing immigrant children to learn more about their academic challenges. However, I'm concerned that they might only admit difficulties they have been through, without providing further insights. I also considered working with students in bilingual schools. However, I found that the methodology that is applied in these schools does not align with the objectives of my research. Therefore, this research is theoretical in nature and I will make inferences from the data collected from bibliographical studies and will provide insights from educators who navigate these challenges daily. All three chapters of this research end with the insights gathered from interviewees (Bridging theory and practice), which tend to elaborate on how theoretical recommendations align or diverge from real-world application. For this purpose, I had the opportunity to interview three professors whose diverse backgrounds, research interests, professional roles, and firsthand experience with additional language learners bring valuable perspectives to the study of math education for multilingual students. It is important to note that not all of them will address every research goal, as their input will be limited to the areas they are most familiar with.

The study gathered insights from Professor Jessica Shumway, an associate professor of mathematics education at Utah State University. Dr. Shumway’s research is about improving elementary mathematics education. She has also been investigating instructional practices and emerging learning technologies that foster mathematical thinking in preschool and elementary school. She also has extensive experience in working with EFL students (English as a Foreign Language) as well as elementary students for years. Her extensive background made me realize that she can provide valuable insights for my interview, particularly in areas related to language learning challenges, strategies for younger learners, and the integration of technology in education. She was a Fulbright scholar at Universidade Estadual Paulista (Unesp) in Rio Claro in 2023 and I first met her when I took a technology course instructed by her at Unesp.

Dr. Danyal Farsani is an associate professor in the department of teacher education in the Norwegian University of Science and Technology (NTNU). He completed his Ph.D. in Education at the University of Birmingham followed by two postdoctoral research positions. His research explores the role of both verbal and non-verbal language across cultures and how language is utilized as a resource in teaching and learning across diverse settings and tasks. Dr. Farsani brings unique insights from his personal experience as an Iranian refugee who completed high school in England in an additional language, enhancing the relevance and depth of his work on multilingual and multicultural education. Taking a course called "Culture and Cognition" with him and participating in his seminars convinced me that his insights could significantly contribute to my research, particularly with the second research aim.

Dr. L.C., who holds a Ph.D. from the department of Teaching and Learning at Stockholm University, specializes in math education with a particular focus on multilingual students. Her research addresses the unique challenges and opportunities within math learning environments for students who speak multiple languages, contributing valuable insights to the field of inclusive education. Dr. L.C. has been working in a school with many students from different nationalities, including Bangladesh, Morocco, Iraq, Turkey, Somalia, Egypt, Colombia, Russia, Poland, Greece, Ukraine, Lebanon, and Kurdistan in Stockholm. I was honored to have her at my research presentation, where she learned about my work and shared her observations on the challenges refugee students face at her school. During this discussion, I realized I could benefit from her valuable experience in my research.

For the interviews that I had with these professors, I benefited from the necessary steps that Creswell (2016) introduces in conducting qualitative interviews, including deciding on the type of sampling and open-ended, general research questions that are focused on understanding one's central phenomenon, determining the type of interview and using recording procedures. Following these steps, one-by-one interviews, each taking about 40-60 min, were conducted as conversations rather than strict Q&A sessions via videoconference in

March 2024. This type of interview is also known as semi-structured interviews, in-depth interviews, or flexibly structured interviews, allowing participants to express their thoughts more freely and enabling a co-construction of meaning between the interviewer and interviewee (Bogdan, 1997). Interviewees were briefly informed about the research aims and theories found in the literature and studies and were asked to reflect on how the theories align or differ from their real-world experiences. A list of open-ended questions was also provided to lead the interview. The flexible nature of open-ended questions enables the participants to respond based on their perspective, rather than being guided by predetermined questions (Bogdan, 1997). The interviewees were asked whether the education system is truly adapting to 21^{st}-century practice in real-world application and what advantages and disadvantages this change can bring to the education system. They were also asked about the role of language and culture in education in a multilingual setting and the potential advantages of having students from different cultures together in a 21^{st}-century class. In addition, they were asked about the strategies that are used in a multicultural and multilingual math class to make learning happen and what differences technology and AI have made in math classes.

This research will mainly address 6th-9th grade non-native math students. The main reason is that the students have already completed elementary school and have gotten used to the educational culture in their own country. In addition, at this level, students encounter more complicated math registers which require better language proficiency. Therefore, these grades can be the beginning of a big challenge for math students in a non-native classroom.

2. The 21st-Century Math Approach

According to Bishop (1996), in the 19th century mathematics used to be considered as "training of the thinking muscle" or "thinking power gymnastics" as it could foster intellectual discipline and sharpen the mind (p. 31). Little by little, mathematics gained importance for its uses and ability to change modern society. It was believed that mathematics had helped the non-mathematician to apply changes in modern society with deeper insight and greater efficiency.

Mathematics got fame as a subject that had an important role in the economic development of a country and it started being taught to all technicians and skilled workers. With the passage of time, the perspective of math education was broadened and after World War II "new math curriculum" was developed and understanding became the predominant goal of math teaching. As a consequence, in the 20th century, the focus of math shifted to "mathematical processes (such as exploration, investigation, conjecturing, problem posing/formulation/solving, representing, proving, modelling) and not only products (concepts, results, methods, skills)." (Bishop, 1996, p. 41)

At the end of the 20th century and the beginning of the 21st, the importance of learning mathematics started to shift to practical applications of mathematics. Findell (2001) claims that youth are being raised in an environment rich in mathematics. The tools and systems which are used in households, educational institutions, and professional settings are based on mathematical principles. Many educational pathways and careers need advanced mathematical skills. Engineers need math to calculate structural loads and design circuits, and accountants require math to prepare financial statements and perform cost analysis. Economists demand math to model economic theories and architects need math to calculate dimensions and design structures. Market researchers need math to conduct surveys and analyze consumer data and physicists need math to conduct experiments and analyze physical properties. Therefore, basic mathematic proficiency has become essential for any engagement in society.

Learning math is an essential life skill that opens doors to economic opportunities and promotes critical thinking by fostering problem-solving skills, developing financial literacy, facilitating data analysis, driving technological innovation, and practicing critical thinking abilities. According to Schoenfeld (2016), "learning math is empowering" (p. 2). Schoenfeld (2016) aims to say that mathematically proficient students have quantitative literacy, which enables them to interpret great amounts of data and make proper judgments. They apply math from simple tasks like reasoning to complex activities such as budget projections and computer modeling. With the help of math, students become flexible thinkers, proficient at solving new problems with various techniques and perspectives, and analytical in both their thinking and evaluating others' arguments.

Noreen (2019) emphasizes the significance of mathematics and claims that while math is thought to be challenging, it fosters logical thinking and enables precise clarification of thoughts and decisions. “In this changing world, those who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. Mathematical competence opens doors to productive futures and a lack of mathematical competence keeps those doors closed. All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding” NCTM (2000, p. 50). Therefore, let us consider mathematics not only as a means for academic success but also for personal and professional development.

The important role of math in personal and professional achievements led to a re-evaluation of how it is being taught. As a result, the emergence of the Internet and technologies and the arrival of native digitals (Gen Z) have been transforming the focus of mathematic teaching from a traditional teaching approach with a focus on procedural fluency to a more dynamic approach with a focus on understanding of mathematics and problem-solving skills that prepare students for their future careers in the 21st century. This section aims to elaborate on this shift and explore the key concepts of 21st-century skills in math education.

2.1 Traditional Approach to Mathematics Teaching and Learning

Traditional math teaching and learning has been the primary method which has been used worldwide for years and has shaped the experience of many students so far. This approach is based on a “teacher-directed environment” and rote memorization, with a focus on procedural fluency and “little significance to the real world” (Stephan, 2020, p. 594).

Ardeleanu (2019), perfectly portrays a traditional math class as where students are typically found seated neatly, allowing everyone a clear view of the teacher at the front of the class. Having the primary role, the teacher introduces a new concept and ensures if students can recall previously learned material. Visual aids, such as black or whiteboards, are commonly used to illustrate examples and explain the concepts. There is an active interaction between the teacher and students, especially to ask students’ opinions and engage them in problem-solving activities to encourage their thought process. Following the introduction of the new concept, students are allowed to apply their newfound knowledge through practice exercises. According to Noreen (2019), in a classroom where a traditional approach is used, the majority of learners are often limited only to “duplicate” what is presented on the board. Consequently, they may find it hard to effectively analyze data and engage in critical evaluation. Noreen continues that students may not even get the chance to put what they learn into practice. In this model, the teacher directs the learning process by delivering lectures, providing instructions, and leading discussions while students are passive receivers of knowledge.

There are many similar ideas about traditional math approaches. According to Ardeleanu (2019), traditional math teaching often relies on passive methods, where students solve problems individually and the teacher offers explanations if needed. There is more interaction between students and teachers rather than among students. This interaction normally happens when students ask the instructors for help or when they respond to the questions. Findell (2001) states that the teacher evaluates students’ responses by categorizing them as “right” or “wrong” before proceeding to the next question. This cycle of questioning-responding continues, often at a rapid pace, until the material has been completely reviewed. According to Findell (2001), new concepts are introduced by the instructor through either verbal explanation or demonstration. At the end of the lesson, the class tackles more complex problems together. Not surprisingly, when the teacher leads the modeling, students are not allowed space for creativity or critical thinking.

Stephan (2020) also describes “Teacher-centered” classrooms as where the teacher prioritizes proficiency in content and fundamental skills and often delivers knowledge through lectures and drilling. When instructors try to deliver the information rather than asking the students to construct their knowledge or explore things, students become receptors rather than producers. Such restrictions on thinking ability can make students lose their interest in learning. In Hattie's (2008) description, traditional math teaching is a method that requires the students to memorize algorithms and follow problem-solving steps without necessarily understanding the concepts. That could be why Boaler (2015) highlighted math as what was believed to be “all about getting correct answers, not interpretation or meaning” (p. 4). Does this simplistic view limit our understanding of how mathematical thinking and problem-solving truly evolve?

Wu (1996) claims that the traditional approach has been criticized for focusing too much on memorization and routine procedures. Boaler (2016) believes that this can lead to disengagement and may slow down critical thinking. It continues that its main reason is that there aren't many opportunities for students to connect mathematical ideas and apply them to diverse situations. In addition, passive teaching methods, like traditional lectures, often don't allow much time for student discussions with the instructor or collaboration among students. These “non-interactive” methods continue into higher education and make it difficult for students to fully engage with mathematics (Abdulwahed, 2012).

Findell (2001) declares that in the traditional approach to math teaching and learning, skills become broken parts of knowledge as they are gained without understanding. Hiebert (2006) claims that “a mathematical idea or procedure or fact is understood if it is a part of an internal network” (p. 67). Hilbert believes that this network could be students’ prior knowledge. Polman (2021) also asserts that an important element of “meaningful learning” is an emphasis on “the cognitive processes by which students incorporate new mathematics knowledge into their existing knowledge structures” (p. 471). Therefore, learning new topics becomes more difficult for learners in traditional math learning as there is no connection to previously learned concepts and skills to relate to the new material.

Findell (2001) continues that another consequence of learning without understanding is the separation between school-based learning and real-world application. Boaler (2016) also admits a difference between the skills taught in academic settings and those demanded in real-world scenarios. What happens is that children develop one set of problem-solving methods to use outside of school, without recognizing the relationship between that and what they learned at school. Boaler (2016) believes that there is a need to align math education with more essential skills for success in diverse contexts such as critical thinking, problem-solving, and adaptability skills.

2.2 A Shift to Modern Approaches to Math Teaching in 21st Century

Considering the importance of math and the failure of traditional math education in equipping students with essential skills, there have been efforts at improving math education since the 20th century. The development in math education started with NCTM’s statement in its *Agenda for Action* that “problem solving must be the focus of school mathematics” and that mathematics must equip “individuals” with the skills to address the unique challenges they will encounter in their careers (NCTM, 1980, p. 1). Ellis (2005) also claims that a lot of research was conducted for the necessity of “more skill building and less use of calculators” and many called for “cooperative learning activities” aimed at fostering students' comprehension of fundamental concepts (p. 7).

According to Findell (2001), several changes occurred in the definition of successful mathematics learning, which were influenced by changes in society and educational reforms.

In the beginning, proficiency in math operation was the primary measure of success in mathematics education from pre-kindergarten to eighth grade. Findell (2001) approaches the reform movements of the 1980s and 1990s as the ones which redirected the focus of math learning to developing "mathematical power”, reasoning, and problem-solving (p. 55). These reforms also focused on connecting mathematical concepts and communicating mathematical ideas effectively. According to Polman (2021), these reforms introduced new strategies to improve students' motivation as well as their academic performance. It continues that these strategies involve creating meaningful learning environments based on students' interests and needs. These environments, which develop a deeper understanding of the value of learning, facilitate better application of knowledge. Polman (2021) suggests four elements as researchers’ findings for meaningful learning which are “activating prior knowledge, connecting to students’ personal worlds and showing the value beyond school” and “working collaboratively” (p. 474).

Regarding this idea, Papert (1993) introduces two concepts in *The Childrens’ Machine* book, which are constructionism and instructionism. In instructionism, classes are more teacher-centered as instructors have a central role. They are supposed to give lectures and do drills to transfer knowledge to the students. In return, students have passive roles. They need to memorize the information they receive and they hardly ever explore or construct knowledge in the class. On the contrary, in constructionism, educational technologies are used to involve students in projects that they can create and explore. A lot of collaboration and interaction happens among students therefore students are active participants. They learn best when they construct knowledge and understanding through hands-on experience.

The Children’s Machine highlights the importance of allowing children to explore and construct knowledge in a self-directed manner with the support of technology. Papert’s vision suggests a shift from instructionism to constructionism, recognizing the opportunity to benefit from computers for meaningful education experience. Papert's concept of constructionism closely aligns with what Gen Z (new generation of education) seeks in education.

2.3 Generation Z (New Math Audiences)

Engelbrecht (2020) introduces Generation Z (Gen Z) as those who were born between 1994 -2010. Being born in different “socio-economic conditions” or better to say “in the world of Internet-connected technology”, Generation Z have different preferences and expectations as learners (Szymkowiak, 2021, p. 2, 3). Gen Z is recognized for their proficiency and interest in digital technologies. Not surprisingly, traditional classrooms, where teachers have a central role, don't interest this population. Gen Z refuses to be only passive learners in a classroom. Instead, they actively seek opportunities to engage with course material and participate in the learning process (Puangpunsi, 2021). They prefer a “learner-centered self-paced learning environment” where they can contribute, and collaborate, rather than merely listening passively. (Szymkowiak, 2021, p. 4)

Engelbrecht (2020) and Puangpunsi (2021) claim that in the age of technology, students’ preferences have changed and we need to respond to their requests. “Today’s students prefer seeking their own information rather than being presented with it, they prefer on-demand access to knowledge, disseminated over the internet, and absorb knowledge rapidly across different channels” (Engelbrecht, 2020, p. 826). Engelbrecht (2020) describes the new generation with different characteristics, however, he believes that they have certain things in common. They prefer multitasking and acquiring information very fast “using visuals, sounds, and color from multiple multimedia sources in a novel or useful way” (p. 826). As they have been using the Internet and social media, they enjoy learning through interaction with others. Additionally, Gen Z has the desire for “an education that prepares them for a meaningful career” (Mohr, 2017, p. 6). Therefore, instructors should “equip students with the skills and attitudes that will enable them to function in work-related roles in the future” and adjust assignments and course contents in a way that they help students with their life beyond school (Mohr, 2017, p. 6). One of the skills that can benefit Generation Z could be critical thinking. According to Ku (2009), argues that critical thinking, which has become an important objective of modern education, is not essential for students’ better performance at school but is vital for their future workplace. As they gain skills in reasoning, judging information, and evaluating evidence, critical thinking becomes crucial.

Mohr (2017) describes Gen Z “somewhat conflicted” with “paradoxical insights” (p. 87). While they “see themselves as problem-solvers, who prefer to work alone,” (Mohr, p. 88) “they like to be active and collaborate using the latest technology and visualization opportunities” (Engelbrecht, 2020, p. 826). Therefore, “the way of teaching should be completely adapted to meet these challenges and to respond to the new requests” (Engelbrecht, 2020, P. 826). For this purpose, using flipped classroom or jigsaw formats is suggested as students should seek information online before the class to share with a group for a bigger project later through interactions in class.

Smit (2016) also believes that rapid advancement in technology and globalization in the 21st century has been changing the education content. As a result, Education authorities and instructors have been looking for ways to equip students with the essential skills to succeed in this digital age. Engelbrecht (2020) claims that Social media has been playing an important role in education and when media changes, “knowledge acquiring process” can change (p. 826). According to Engelbrecht (2020), the education process has shifted from “push” in the traditional approach in which teachers have the role of know-of-all to a student-centered “pull approach” where students can learn through interaction, collaboration, and communication with others. This shift aligns with the constructionism approach suggested by Papert.

In the context of mathematics education suggested for Gen Z, the focus is on shifting from only coming up with the correct answer for math problems to the development of 21st-century skills, such as critical thinking, problem-solving, creativity, and collaboration. Students should also be given “opportunities to search, explore, make conjectures, hypothesize, examine, refute, adapt strategies, devise plans, conclude, reason and justify the conclusions and reflect on them, monitor, and experience the processes that mathematicians have gone through” and should be “encouraged to take risks, make mistakes and have chance to investigate alternative pathways” (Nadjafikhah, 2012, p. 289). Additionally, they should be provided with the opportunities to “learn how to think, inquire, question, solve problem and apply these skills in new situations with new information” (Puangpunsi, 2021, p. 2).

Bishop (1996), Mohr (2017), and Engelbrecht (2020) believe the main reason for recent changes in mathematics education is to design a student-centered class that meets the needs of Gen Z as its primary recipients. However, this is not a simple task as mathematics is taught to different types of personalities and characters each year. Therefore, Mohr (2017) suggests that the education authorities should give up on many certain priorities and tailor educational objectives to the characteristics, preferences, and needs of the current student population to enhance engagement and effectiveness. “Educators realize that we need to rethink the entire model of education and redesign it so that it is more student-centered. This means adopting new technologies, but it also means giving up on certain attitudes about what constitutes educational success. These new technologies also seriously influence the nature of mathematics, e.g., application of procedures is becoming less important and new ways of validation (and practicing in general) in mathematics are being developed” (Engelbrecht, 2020, p. 825). Engelbrecht (2020) adds that this shift requires teachers to collaborate to design a learning environment based on the recent changes in students’ preferences. However, this might not be easy for solitary teachers.

2.4 Framework/ Key Organizations Driving the Reform of 21st-Century Math Education

As a response to the failure of traditional education systems in adequately equipping students with the skills needed to succeed in the modern world, many organizations contributed to the reform of math education. Among these organization, NCTM, PISA and P21 took remarkable actions. However, as Isaac Newton says for every action there is an equal opposite action and that is true about math reform in 20th century which faced counter-reform (Kilpatrick, 2009). Kilpatrick explains the concept of "math wars" referring to the conflict between the supporter of math reform and the opponents during mid 1950s to mid 1970s. This is the moment when NCTM started its effort to redefine school math skills.

The National Council of Teachers of Mathematics (NCTM) was founded in 1920 "as a response to educational reformers" to improve mathematics education in the United States. (McLeod, 1996, p.30) After the publication of *Agenda for Action*, NCTM published the "Curriculum and Evaluation Standards for School Mathematics" to provide a guideline for school math programs and promote a shift from memorization of formulas to meaningful learning in 1989 (Kilpartrik, 2009). In 2000, NCTM updated its previous work and offered more recommendations in "Principle and Standards for School Mathematics" both focusing on "mathematical problem solving, communication, reasoning, and connections" (McLeod, 1996, p. 29).

According to NCTM, learning math should be connected to real-life situations and students should be provided with opportunities to use mathematical concepts in meaningful ways with an emphasis on mathematical thinking and reasoning (McLeod, 1996). "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000, p. 20). NCTM claims that "teachers' actions are what encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions." (NCTM, 2000, p. 18) Therefore, teachers attended in collaboratives to question what, how, and why they teach specific content in the class. Collaborative activities helped teachers to share their new ideas and self-reflections to go through a big change process with the support of one another. (McLeod, 1996).

PISA, Programme for International Student Assessment, was first implemented with the assistance of the Organization for Economic Co-operation and Development (OECD). The main purpose of PISA is to evaluate education systems to see what competencies students have gained and can apply in real-world contexts. In other words, PISA has been trying to answer the following questions: “Are students well prepared to meet the challenges of the future? Are they able to analyze, reason, and communicate their ideas effectively? Do they have the capacity to continue learning throughout life?” (OECD, 2010, p.1)

The PISA test is conducted every three years among 15-year-old students worldwide. Schools are randomly chosen in participating countries and 4500-10000 students take the test in two sessions, each lasting about two hours. OECD analyzes the data from the test to generate insights into the effectiveness of education systems in the world. These assessments have widely affected educational reforms in mathematics by highlighting strengths and weaknesses in many education systems globally. Countries have used PISA results to revise their math curricula, focusing more on problem solving, real-world application, and reasoning. PISA has always encouraged continuous assessment and feedback, and the use of technology in math education. There are three main assessment areas in PISA, including, reading literacy, mathematical literacy, and scientific literacy.

The PISA framework has always been updated and the most recent framework for mathematics assessment in 2022 demonstrates a rapidly changing world due to the presence of “new technologies and trends in which citizens are creative and engaged, making judgments for themselves and the society in which they live” (OECD, 2023, p. 22). Pisa’s test on mathematical Literacy is defined to assess students’ ability of “evaluating situations, selecting strategies, drawing logical conclusions, developing and describing solutions, and recognizing how those solutions can be applied” as well as problem-solving (OECD, 2023, p. 14). Pisa has emphasized critical thinking and problem solving and the meaning of mathematical literacy in the 21st Century which includes “mathematical reasoning” and “some aspects of computational thinking” (OECD, 2023, p. 19). PISA defines the meaning of mathematical literacy as “that every student should learn to think mathematically, using mathematical reasoning (both deductive and inductive) in conjunction with a small set of fundamental mathematical concepts that support this reasoning” (OECD, 2023, p. 20). PISA believes that “Mathematical reasoning involves evaluating situations, selecting strategies, drawing logical conclusions, developing and describing solutions, and recognizing how those solutions can be applied” (OECD, 2023, p. 14). PISA introduces eight 21st-century skills in the PISA 2022 assessment framework, including critical thinking, creativity, research and inquiry, communication, and reflection.

P21 or the Partnership for 21st-Century Learning was established in 2002 through collaboration among education, business, and government leaders. The main purpose of P21 is to integrate 21st-century skills into education to better prepare students for modern challenges. Their framework highlights that "students must also learn the essential skills for success in today's world, such as critical thinking, problem solving, communication, and collaboration" (PARTNERSHIP FOR 21^{st}-CENTURY LEARNING, 2019, p. 2). In math education, this means that students should be motivated to apply the formulas they learn to solve real-world problems, analyze data critically, and collaborate effectively with their peers. It is believed that these key skills can help students survive in today's global economy. In terms of learning and innovation skills, P21 focuses on 4Cs, including creativity, critical thinking, communication, and collaboration, which are believed to equip students for more complex life and work environments.

2.5 21st-Century Math Classes

A critical component of the 21^{st}-century approach is the emphasis on 4Cs-Creativity and Innovation, Critical Thinking and Problem Solving, Communication, and Collaboration-which are essential for the modern era. The first element is Creativity which Henriksen (2016) defines as what "lies in the ability to create ideas or works that are novel, high in quality, and task appropriate. This suggests that creative work is dependent on context, because it is assigned value in relation to the domain it is created within" (p. 29). According to Trilling (2012), students need to "create new and worthwhile ideas", "Elaborate, refine, analyze and evaluate their own ideas in order to improve and maximize creative efforts" and "be open and responsive to new and diverse perspectives." Therefore, students are always expected to listen to others' reasoning and feedback, and apply different strategies to evaluate their novel ideas. In addition, they should "view failure as an opportunity to learn" (p. 59).

The second element is Critical Thinking and Problem Solving. According to Peter (2012), the importance of critical thinking lies in the fact that it fosters dealing with "social, scientific and practical problems" (Shakirova, 2007), and also it encourages the learner to focus on the process of learning instead of facts (Supena, 2021). Zakaria (2009) believes that according to the Curriculum Development Centre (2003), an important objective in teaching mathematics is to develop students' mathematical problem-solving skills. It continues that in math education, a key goal is teaching students how to solve problems, as problems are a natural part of life. Kriegler (2008) defines a problem as a situation in which an individual must find solutions that are not immediately obvious or in other words, he introduces problem-solving as what requires having the mathematical tools to figure out what to do when one does not know what to do. It also involves the student's willingness to accept challenges. Accepting a challenge here means student is willing to find appropriate methods to solve a problem. For this purpose, there is an emphasis on asking "critical questioning techniques to engage students actively in the learning process." These questions might include:

1) What do you think about this?

2) Why do you think that?

3) What is your knowledge based upon?

4) What does it imply and presuppose?

5) How are you viewing it?

6) Should it be viewed differently? (p. 3)

Peter (2012) believes that asking these questions makes students "think about thinking" (p. 3). They also need to recall and use their prior knowledge and experience. Above all, they have the opportunity to apply self-correction by looking at their understanding again and see if they have missed anything or if they could make any changes to their perceptions. This could happen through collaboration with peers/group mates.

Aizikovitsh (2105) and Peter (2012) claim that teachers should be aware that knowledge and thinking should be taught together. Aizikovitsh (2015) believes that "I see, I think, and I wonder" should be added to the routines of students to practice "careful observation, thoughtful interpretation, and imaginative speculation" (p. 456). However, it's not easy to engage the students who are used to being passive learners in an education system which focuses on test scores into applying critical thinking. Lai (2011) suggests that teachers can model critical thinking and make the reasoning clear to the students. Applying critical thinking in real-world problems becomes easier for students when the teachers think aloud and students observe them providing evidence to support their arguments.

According to Lai (2011), critical thinking can be taught but it requires teachers to provide the students with many opportunities from an early age in their education life to apply it in a variety of contexts.

> Those who have the ability to hear, do not always actively listen. Similarly, those who have the ability to know, do not always critically think. The premise that critical thinking is to knowing as listening is to hearing implies that critical thinking is a learned skill that must be developed, practiced, and continually integrated into the curriculum to engage students in active learning. (Peter, 2012, p.1).

Therefore, we cannot expect students to apply critical thinking without being taught.

In 21st-century math classes, students are suggested to solve real-world problems which is also called project-based learning. Stehle (2019) believes that in real-world problems, "students work to identify the problem, propose a solution for a specific client, test the solution, and share their ideas" as there is no existing solution for them. It continues that this is the time that "students develop knowledge in a meaningful way", "gain experience defending their choices through evidence and effective communication skills" and "learn from failures" (p. 3). This includes applying critical thinking in solving the problem which Supena (2021) introduces as "curious thinking about the available information to reach a deep understanding" (p. 875).

Based on Pólya (2004), critical thinking and problem-solving could happen through four main steps, including understanding the problem, devising a plan, carrying out the plan, and looking back and further developing the topic. In the first step students should fully understand the problem by answering the questions like "What is the unknown? What are the data? What is the condition?" (p.23). In the second step, students should apply their creativity and knowledge of the problem to choose an appropriate strategy to solve the problem. In the third step, students implement the strategy from the previous step and execute the calculations. Students might need to share their thoughts through collaboration and communication to reach the best result. The last step is the time to apply critical thinking, reconsider, and re-examine what one has done. Students are always encouraged to collaborate and communicate with one another to see if they can improve their solutions.

Communication and collaboration are the other elements of 4C, which require students to negotiate their thoughts, be able to work in a team, and be flexible to opposing ideas. Zakaria (2009) believes that students with different potentials have different attitudes toward problem-solving. In his studies, he addresses different research that has been done on the role of students' attitudes on problem-solving. He assumes that each of these attitudes can be considered as a capability. Smith (1992) also demonstrates that students from different backgrounds, learning styles, and experiences have different perspectives (this will be discussed further in the following section) Therefore, educators should not look for a specific approach that fits all students but instead provide them opportunities for them to share their thoughts.

Singley (2000) believes that collaboration lets learners find that a single viewpoint may not consider different aspects of a complex subject. For this purpose, learners need to effectively listen to their peers and also present their mathematical insights and defend their strategies with their peers during problem-solving. Stehle (2019) believes that students get to a higher level of solution through collaboration and communication because they "respond to feedback and questions to create solutions that better fit the problem" (p. 4). NCTM (2000) also introduces communication as one of the essential skills in math education. Cooke (2005) asserts that many children enter school with little communication skills and they need to be supported to be able to communicate mathematically. It continues that elicitation techniques might encourage them to participate more in communication as they can share their understanding of the world.

The connection between the 21st-century skills demonstrates that they are interconnected. According to Stehle (2019), “by building one skill, others are reinforced” (p. 5). Lai (2011) claims that critical thinking relates to creativity. Both require “good purposeful thinking” and good thinking needs one to “be aware, strategic, and critical about the quality of intellectual product” (p. 22). Lai (2011) also believes that critical thinking can be perceived through collaboration and cooperation. In other words, there should be an interaction among students to develop critical thinking skills and one student should respond constructively to others in a discussion to prove the credibility of their reasoning. Supena (2021) admits Piaget’s theory of constructivist learning (1986) in which students are encouraged to construct their knowledge and findings (applying creativity) through collaboration and interaction. In addition, it confirms that critical thinking helps students to apply what they have just learned to practical, real-life problems which require creativity. Moreover, collaborative tasks often involve problem-solving and decision-making. They also require communication to express ideas effectively. Therefore, mastering competency in one of the 4C skills—whether it is communication, collaboration, critical thinking, or creativity—leads to the enhancement of others.

2.6 4C Learning Model

This section will demonstrate the practical application of the 4C model in a math class, exploring how each element-class arrangement, effective elicitation technique, engaging activities, comprehensive assessment, mistake handling, and timely feedback-contribute to a dynamic and effective learning environment in 21st-century math class.

Supena (2021) claims that in the 21st century, classes should be viewed as “a learning community” (p. 877). According to Ardeleanu (2019), a big change is noticeable in 21st-century classrooms compared to traditional environments. Instead of rows of desks, one can notice desks arranged in groups to facilitate interaction during the learning process. Many small work teams of five to six students are formed including both high and low-achieving students. Consequently, students have the opportunity to apply collaboration during the class and enjoy a sense of teamwork. In student-centered classes, students learn to rely on one another for support when needed. Students have different sources to learn from, including books, lectures by their peers, and the communication between them (Supena, 2021).

According to Cooke (2005), teachers can promote communication by encouraging the students with a “powerful message” of “say whatever you want to” (p. 369). This can be accompanied by a smile, a compliment and a dialogue conveying that every single person is important in the class. Therefore, it’s the teacher who should apply different strategies to create a comfortable and inviting environment in which students feel free to “generate verbal participation” (p. 369). De Arauja (2018) believes that “Eliciting discourse” can motivate learners to participate in class interactions. Another advantage of elicitation is that mathematics educators can learn “what a student knows, can do, and is disposed to do” before knowing “how and what it means to help students become better problem solvers” (Chapman, 2015, p. 31). According to Peter, nobody expects students to know how to think critically. This is a skill that needs to be learned and developed. Therefore, teachers can model a critical thinking process by asking questions that do not have one certain correct answer (De Arauja, 2018).

Peter (2012) suggests four types of activities to promote critical thinking in the class. First, students should be asked “ill-structured problems” which require “reflective judgment.” It can be a comparison of two items that need logical reasoning with no right or wrong answers. Secondly, students ought to be asked to think about their thinking or evaluate their thinking by asking about the reason behind their thoughts or solutions. Then, students should be given feedback. Finally, students should improve their thinking with the help of feedback they have received or reflect on their thinking and reasoning. Internalizing the culture of critical thinking and problem-solving requires a lot of effort from both the teacher and students. Lai induces motivation as one of the prerequisites of critical thinking. Students need to be given a challenging task to apply high-level thinking skills. This can make them more motivated than the time they are given a simple task which can be solved through the application of formulas.

Think-Pair-Share is a collaborative learning model that was introduced by Razak (2016). Through Think-Pair-Share students can work individually and also have the chance to collaborate and apply creativity in problem-solving. One of the advantages of using this model is that students with different mathematical proficiency can work together and assist one another. The other advantage of this strategy is that students can apply critical thinking and problem-solving through communication and teamwork. This model includes three main steps. In the first step, students take their time to think about a specific question or problem. Then, they are paired to discuss their thoughts and exchange ideas with their peers. In the last step, students share their findings and solutions with the group. Razak (2016) believes that in the last two steps, students can learn from one another and explore many possible solutions.

Supena (2021) suggests a lot of group activities in 21st-century math classes as there are more opportunities for learners from different academic capabilities to interact and discuss with one another and learn from each other. Smith (1992) introduces various forms of collaborative learning activities in the class which can vary from short lectures or discussions to long-term research projects. However, it believes that this is the teacher who decides how to engage students based on their interests. Other than think-Pair-Share, Lai (2011) suggests that students should do interviews, gallery walks, jigsawing, and other collaborative activities that require active participation.

Engelbrecht (2020) has reported a study conducted by Voigt, Fredriksen, and Rasmussen on flipped classrooms that address “the efficacy of using a flipped classroom approach on student outcomes” (p. 833). They suggest providing the students the chance to watch some lectures or videos at home “to think critically about mathematical problems individually before engaging with the task in a collective environment” and then participate in collaborative activities in the class (p. 833). This type of activity increases the time spent on other tasks and practices in the class with “the possibility of integrating new knowledge with existing beliefs and real-time feedback” (p. 9).

In 21st-century math classes, many other student-centered activities can be done which leads to meaningful learning. Polya (1962) highlights the growing popularity of Project-Based Learning (PBL) in which learners are involved in an active learning process. Learners gain knowledge and skills through complex, authentic tasks which enables them to apply mathematics to problems that occur in the real world with multiple solutions. Barron (2008) also suggests using Problem-Based Learning in which there is a focus on reasoning and problem-solving. “Students work in small groups to investigate meaningful problems, identify what they need to learn in order to solve a problem, and generate strategies for solution” (p. 5). Lastly, students are supposed to evaluate and revise their solutions to practice critical thinking in pairs or groups and see if they can make any changes to improve their solutions.

Kokotsaki (2016) claims that both Problem Based and Project Based Problem Learning demand students to gain a shared goal through collaboration. Erdogan (2015) defines features of PBL as:

> (a) authentic assessment and content, (b) challenging projects with complex tasks, (c) decision making and problem solving, (d) explicit

> objectives with individual and collective learning, (e) realistic products to real-world problems, (f) student directed and teacher facilitated and (g) time limited. (p. 32)

Therefore, students should be involved in challenging tasks followed by an authentic assessment. Learners can also benefit from technologies (technology-enhanced PBL) to access online resources or document the process of problems-solving and share it in a digital format (Kokotsaki, 2016). Kokotsaki (2016) asserts that PBL can end in a formative evaluation or assessment with authentic tasks to measure a child's performance with a self or peer reflection. Self-assessment skills can help students to manage their learning (Kokotsaki, 2016; Mergendoller, 2006). Students need to assess their work, time management, collaboration with others, and competencies.

Black (1998) criticizes classroom evaluation for

> concentrating on recall of isolated details, usually items of knowledge which pupils soon forget. Teachers do not generally review the assessment questions that they use and do not discuss them critically with peers, so there is little reflection on what is being assessed. The grading function is over-emphasized and the learning function underemphasized. (p. 12)

Ku (2009) claims that educators should pay more attention to assessing students' ability to think and applying what they have learned to practice instead of just a memory call (Snyder, 2008; Polya, 1962).

> Teachers should adopt different assessment methods, such as exercises that allow students to self-construct answers, assignments that facilitate the practice of strategic use of thinking skills in everyday contexts, and when adopting multiple-choice exercises, follow-up questions should be given to probe students' underlying reasoning. (Ku, 2009, p. 75).

Therefore, a possible alternative for assessment could be a mixture of multiple questions with open-ended tests where learners can analyze and debate. According to Lai (2011), authentic contexts and real-world problems with many possible solutions should be used in assessments. This is what makes students go beyond what they have learned to analyze mathematical tasks from different views as a single response format of assessments may not demonstrate a learner's critical thinking ability (Ku, 2009). In addition, mathematical tasks with multiple correct answers encourage students to participate more without the fear of making mistakes.

21st-century math class aims to provide a comfortable environment for students to take risks and not be afraid of making mistakes. Risk takers credit learning for grades, therefore they welcome challenging problems as it requires more understanding of the information without concerns for mistakes.

There has been a shift in mistake handling in recent years. In traditional math classes, there was little chance for students to explore. Mistakes often demonstrated failures or a lack of understanding. Therefore, they were corrected on the spot by teachers to avoid fossilization. However, modern math educators and researchers have been perceiving mistakes as valuable learning opportunities and a natural part of learning. This shift in mistake handling and opportunities for collaborative learning has provided the students the chance to listen to one another and apply self or peer correction (Kokotsaki, 2016).

In recent years, there has been much attention to praise students for their efforts, and willingness to learn from their mistakes. Heinze (2007) believes that “mistakes are the best teachers”. Grégoire (2016) asserts that “students need to explore, with the possibility of going in the wrong direction and making mistakes. They should have the possibility of finding various correct answers and/or various ways to arrive at the correct solution” (p. 9). According to Grégoire (2016), it develops a culture among students to see mistakes as opportunities to learn and grow.

Heinze (2007) has conducted research focusing on the effects of an in-service teacher training program on students' performance in geometry, specifically in the context of mistake-handling activities in the mathematics classroom. The researchers implemented a training program for mathematics teachers to enhance their ability to handle mistakes effectively while teaching geometry. In this research, students’ geometry proficiency was tested before (pre-test) and after (post-test) the in-service teacher training. The activities were designed in a way that it could encourage students to learn from their errors and it helped students to develop better mathematical understanding and problem-solving skills. The research results may show that students who benefited from the mistake-handling service showed a better performance. By creating an environment where mistakes are seen as learning opportunities, teachers can provide constructive feedback that leads to further development and growth.

Shute (2008) introduces formative feedback as “information communicated to the learner that is intended to modify his or her thinking or behavior for the purpose of improving learning” (p. 154). It adds that formative feedback can be very helpful as it removes uncertainty and fills a gap between what students have understood and what they need to understand (Hattie, 2008; Black, 1998; Ennis, 1993). However, Shute (2008) believes that not all feedbacks are powerful facilitators of learning. It asserts that feedback is effective if it provides the students with information on how to correct their answers, in addition to marking them as correct or incorrect. Black (1998) and Shute (2008) agree with Hattie that effective feedback should answer three main questions, “where am I going? (what are the goals), How am I going, (what progress is being made toward the goal?) and where to next? (what activities need to be undertaken to make better progress)" (p. 177). Additionally, there are a variety of types of feedback depending on who provides it, be it teacher, peers, or textbooks, how and when they are given. Hattie (2008) adds that there are two types of feedbacks regarding the time it is provided, immediate and delayed. It claims that immediate error correction should be given during task acquisition and delayed feedback can be provided for more complicated tasks. It is worth mentioning that feedback provision is not limited to assessments. Individualized feedback for students' responses through critical thinking and problem solving helps them "assess their thinking" (Snyder, 2008, p. 5). Therefore, teachers should be concerned about providing clear, and meaningful, and comprehensible feedback and take the responsibility to decide on when, how, and what type of feedback to provide depending on the tasks.

As we consider the framework of 21st-century skills, it is valuable to reflect on broader influences shaping modern education. Firstly, many organizations, including P21 and PISA, have contributed to the big reform in math education. However, it is essential to remember that the intentions behind their efforts are complex and not always fully transparent. Therefore, with a critical eye, we might recognize their contribution as a tool with both the potential to enhance global education and what might not align with the best interests of all students. Secondly, while 21st-century skills are recommended to prepare students for the future, their real-world implementation might face limitations like curriculum restrictions, limited resources, or level of support.

2.7 Bridging Theory and Practice

In response to a shift from the traditional approach to 21st-century math teaching, Shumway talks about William Brownell's 1947 article "Meaning in Arithmetic" with doctoral students at Utah University, emphasizing meaning and not procedures at school. She adds that the concept of the 'thinking classroom', introduced by Canadian researcher Peter Liljedahl, which has been very successful in Canada, has recently gained significant attention in America, specifically among math educators in elementary and secondary schools. Liljedahl's work has been appealing to teachers and is "about getting up from the desk, standing up with each other, writing on walls and thinking through problems together." His method resonates with the growing push for innovative, student-centered teaching practices that move away from traditional, lecture-based instruction. This can engage students deeply with math concepts, develop problem-solving skills, and build confidence in a collaborative setting. However, she believes that “it takes time for the reform of mathematics to change.”

In addition, Shumway reminds us the fact that “while teachers are so enthusiastic to apply techniques to encourage creativity, collaboration, problem-solving, they need support to know how to do that so as not to let go of class management.” This can indicate that implementation of the 21st-century math approach is more easily proposed than executed because many factors matter here, including class size and the curriculum that should be covered.

In terms of Cs, Shumway has noticed a move toward collaboration in education and there are more courses available that highlight 21st-century skills. “In the US, there is a focus on career readiness and there has been an emphasis on STEM than has been traditionally.” This aligns with the idea of Mohr (2017) and could be the first steps toward preparing students with the skills they need for their future.

L.C. claims that in the school where she works, teachers tend to invite students to work in pairs and groups to advocate collaboration. “There has been an emphasis on giving rapid feedback to students, highlighting common mistakes, letting them collaborate to identify the mistakes they made in these questions and determine the correct solution.” For example, they analyze math tasks that were answered incorrectly in tests to see if the students did not understand that specific question fully (due to lack of language proficiency) or if it was for the context. Even though it is time-consuming, she invites the class to get the conclusion together. This can provide the class with an opportunity for communication, collaboration, and critical thinking.

All in all, Shumway’s reference to William Brownell’s article demonstrates the importance of developing a deep understanding of math rather than rote memorization. However, she emphasizes the support the teachers need to balance skills like creativity, collaboration, and time management, considering different factors, like class size, curriculum demands, and teacher preparedness. Therefore, the application of modern math education requires strategic planning and resources. While L.C.’s experiences admit the complexity of the application of modern math education, they prove the feasibility of the integration of 4C skills in real-world classes by creating the chance for students to learn from their mistakes and engage in discussions.

3. Challenges for Additional Language Learners and Their Teachers in 21st-Century Math Classes

As discussed in the previous section, there has been a revolution in the 21st-century education system, including big changes in the objectives and content of education, particularly in math classes. The model of 4C math learning demands a lot of discussions in the class to empower students with critical and creative thinking. This learning environment encourages the students to learn from textbooks and other materials as well as communication and interactions with their peers. Moschkovich (2002) describes this change in math teaching and learning as “a shift from a focus on primarily silent and individual activities to more verbal and social” which has made a group of bilingual students encounter new challenges in math class. It continues that this problem gets more serious as students are supposed to go beyond learning vocabularies or fostering their comprehension skills and they should be able to “communicate mathematically, both orally and in writing, and participate in mathematical practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments” (p. 190). What is critical in this environment is to make sure that students can interpret and analyze all aspects of math problems and justify their responses to ensure their understanding of math tasks.

Many multicultural classrooms have been formed around the world as a consequence of an increase in the rate of emigration. Although it can bring a lot of benefits to the class, like learning about different cultures and ideas, a significant concern is the language barrier. It is interesting to know it’s not only refugees or immigrants who might need to continue their education in another language. This is true in many developing countries in which the official language of instruction is different from the first language of learners.

According to Jones (1982), for some political reasons, the language of instruction in Papua New Guinea is English. Outside school, there is no communication in English and kids start school with no or very little knowledge of English. Sadly, there is no language support for these students in their first language when they are in doubt. The math curriculum in Papua New Guinea is similar to what is taught in developed countries. According to Jones, math students don’t perform as well as their counterparts in the countries where math is taught in students’ first language. Clearly, since math is taught, understood, and discussed through language, non-native students or additional language learners can face significant challenges in their math education.

Barton (2007) tells a similar story about Maori children in New Zealand. Maori is a language which was brought to New Zealand 1000 years ago. Maori children were alienated from math and science for years as they had many challenges to “accurately explain the meaning and function of many mathematical terms and concepts" (p. 3). The Maori language was adapted to the teaching of math to some extent by producing vocabulary and grammar. However, after the colonization of New Zealand by English speakers, the education system shifted to English. The author explains one of their linguistic challenges as "in English there are verbal forms for the number 1 to 4: I can single someone out. I can double my debt" (p. 4). This is what does not exist in the Maori language and "it is often difficult to write in one language the equivalent of what is being said in another" (p. 7). Barton (2007) highlights the importance of considering the link between language and mathematics as speakers of different languages may have different mathematical conceptions.

According to Adoniou (2014), many researchers refer to mathematics as a universal language meaning that anyone with math understanding can solve math problems no matter in which language they speak. Take a math problem in Spanish as an example, describing a situation in which a person had a number of an object the day before and he sold a few today and the question is asking how many of this item is left if he buys a few more the day after. Is it possible for a person with no Spanish language background to understand the problem and solve it?

Ji-Yeong (2020) claims that "there is no language-free content; language use always presents some content, and most representations of content require some language use, even with multimodal resources for meaning-making" (as cited in Nasem, 2018, p. 58). In addition, considering the objectives of 21st-century education, can we expect children to be engaged in creative problem-solving, critical thinking, and collaboration without proper knowledge of the language taught and used in the class? "From the foregoing, it can be seen that language is critical to many of the processes of learning and instruction and it confers many benefits in terms of enabling us to articulate, objectify and discuss the problems that the field of mathematics presents" (MULWA, 2015, p. 28).

Adoniou (2014) also believes that most mathematical tasks are not "language free" and this is what many students find challenging. One of the biggest challenges they meet is that "they are learning the English language at the same time as they are learning mathematics through that language" (p. 3). It continues that despite acknowledging the importance of language proficiency in mathematics education, the language demand of mathematics textbooks and curricula is not apparent to educators. This is why many math teachers are not aware of the complexity of math learning by language learners.

Speaking of language proficiency might make you think of only word problems to understand classroom instruction and math problems. However, Moschkovich (2002) believes that the language barrier in math class goes beyond solving word problems. According to Moschkovich (2002), bilinguals acquire language through these three steps: acquiring vocabulary, constructing meanings, and participating in discourses. It explains the main linguistic problems that non-native students struggle with in math classes as follows:

> In many mathematics classrooms today, students are not grappling primarily with acquiring technical vocabulary, developing comprehension skills to read and understand mathematics textbooks, or solving traditional word problems. In many classrooms, teachers are incorporating many forms of mathematical communication, and students are expected to participate in classroom mathematical practices that go beyond solving computation or word problems on a worksheet students are expected to participate in a variety of oral and written practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments. Reading textbooks and solving traditional word problems are thus no longer the best examples of how language and learning mathematics intersect. (p. 192-3)

As previously stated, in the 21st-century math class, learners need to apply critical thinking by analyzing and evaluating the information they are given and the mathematical concepts they have learned to solve complex problems. Therefore, they need to have a good understanding of the language to see what their teachers mean while giving instruction. That means they should comprehend and interpret word problems and follow complex instructions. They also need to share their novel ideas with their classmates through collaboration and communication. According to Moschkovich (2002), "students are now expected to communicate mathematically, both orally and in writing, and participate in mathematical practices, such as explaining solution processes, describing conjectures, proving conclusions, and presenting arguments" (p. 190). Therefore, they need to be able to communicate in the language.

It is also essential for them to understand their peers and the solutions they find, make some comments on them, and help one another to develop their ideas. These tasks require a strong command of the language used in instruction which makes language proficiency an essential skill for success in math. Freeman (2012) claims that teaching math concepts without providing language support is a false opportunity for non-native speakers to learn math. In other words, we cannot expect equal achievement from the ones who are not equally facilitated.

In a multilingual environment in which students speak plenty of languages not everyone feels comfortable participating. They might not feel a sense of belonging to the class or they might fall behind the schedule as they cannot follow their teachers or even their peers. In addition to the language barrier, the influence of cultural differences can affect the way students interpret mathematical concepts. Yoon (2008) believes that “the teachers’ frustration was largely caused by their focus on the students’ linguistic needs only. The issue of the students’ cultural and social needs was not taken into consideration for their learning” (p. 497). Learners of different cultures perceive the knowledge differently and show various attitudes toward different activities in the class. For example, students’ participation in the class or their openness to receiving feedback or being corrected might vary from one culture to another. This can be a frustrating process both for teachers and students.

This section of the study will elaborate more on the importance of these two factors, language proficiency, and culture, in learning 21st-century math skills among non-native students and will bring first-hand experiences from the teachers involved.

3.1 Case Studies of Non-Native Performance in Math Classes

Andoniou (2014) claims that up to 15 percent of language learners cannot perform well in math classes for their language barriers. Neville-Barton (2005) has conducted five pieces of research carried out by a team of practitioners from different institutions. The main purpose of the research is to find the impact of language on mathematics learning among English as an Additional Language (EAL) students. It also aims to produce practical recommendations for math educators of EAL students. The project also seeks to design language support programs and develop teacher-researchers.

The first study, called Auckland Girls’ Grammar School, was conducted on 40 Chinese Mandarin-speaking students of years 12 and 13. Bilingual tests in two languages (English and Mandarin) were done. The findings indicate that 15% of students faced significant challenges when tested in a second language. The second study, called Wellington Girls’ College, was undertaken on 13 Chinese Mandarin-speaking students of years 12 and 13. Similar bilingual tests were done as the previous one but this time with a shorter interval. The outcomes highlighted that 12% of students revealed major language-related issues including particular misunderstandings and lack of comprehension. The third study research, called Macleans College Study, was done on two groups of year 13 students, a group of Chinese-speaking students (considered as EAL) and a group of native English speakers. The study displays that native students performed better in mathematics and EAL students mostly struggled with prepositions, word order, and interpreting context. The fourth research, called Tanagra College study, was carried out on students of two Year 12 mathematics classes. They received questionnaires in English with a translation to their first Language. The study showed that language barriers impacted the student's results greatly. The biggest challenge faced in this study was the combination of syntax and technical vocabulary in a second language. The last study, called The University of Auckland, was executed on two groups of third-year undergraduate students of the University of Auckland from both groups of EAL and native English speakers. The students were tested on some mathematical items and were supposed to provide a self-perception of their understanding of the course they had passed. The outcome proves that the performance of those with lower English proficiency was unexpectedly poor and surprisingly they were not even aware of their difficulties with mathematical understanding.

Neville-Barton (2005) summarizes the result of these 5 studies as follows:

> All five studies offered quantitative or qualitative evidence that EAL students suffer a disadvantage in mathematics learning due to language difficulties. The extent of this disadvantage was measured as 12 percent and 15 percent in two of the studies…Three of the studies revealed that both general and technical English were factors in the problems experienced by EAL students. The Macleans College study, with students of uniformly higher mathematical proficiency, indicated that general English was a bigger factor. The Auckland Girls' Grammar School study, with students of varied mathematical proficiency, reported that the mathematical discourse was a bigger factor. The indication is that the type of language causing difficulty is related to the mathematical proficiency of the student. What specific language features cause difficulty? The features varied across the studies, and appear to depend on the mathematical level as well as the home language and English language proficiency levels. Each study reports particular items. Vocabulary on its own is not the big issue that was anticipated. However it was a component of the difficulty experienced with understanding mathematical discourse as a whole. (p. 13)

The case studies above revealed that EAL students performed worse compared to their native English-speaking peers for their language barriers. They all faced difficulties either with mathematical vocabulary, terminology, and syntax, or mathematical discourse. Therefore, there is no question that language proficiency is crucial for success in math.

3.2 Challenges in Terms of Language Proficiency

Schleppegrell (2007) claims that "students come to school with an everyday language with which they have constructed their knowledge of the world, the school can build on that knowledge and language and move students toward new and more scientific and technical understandings by being aware of the linguistic challenges that accompany the conceptual challenges of learning" (Schleppegrell, 2007, p. 140). Therefore, knowing the language of a discipline is an essential part of learning that new discipline.

According to Adoniou (2014), in all areas, the essential concept for reading comprehension is learning vocabulary. “Vocabulary is key to comprehension and as we progress through the years of schooling, the vocabulary of mathematics becomes increasingly idiosyncratic to the learning area, and as the concepts become more abstract so does the vocabulary” (p. 8). Firstly, students need to learn words but it is more complex than it sounds. There are plenty of words in everyday language that are used differently in mathematics or might have different meanings in context. For example, the meaning of the word ‘meet’ in “parallel lines never meet” is different from its meaning in everyday English (Moschkovich, 2002, p. 199). In the example of “find the value of x”, students do not need to look for anything but to do the algorithm. Moreover, as Moschkovich mentions, “any number” might mean “all numbers” in mathematics (p. 194). Gee (1999) also discusses the complexity of comprehending the meaning of words as follows:

> A situated view of the meaning of words means that the meanings of words are not stable and general. Rather words have multiple and ever changing meanings created for and adapted to specific contexts of use. At the same time, the meanings of words are integrally linked to social and cultural groups in ways that transcends individuals. (p. 40)

Another key challenge that impacts students’ math learning in an additional language in terms of learning words is working with math registers. Halliday (1978) defines a register as

> a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to the “mathematics register,” in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195)

Registers can include vocabularies, symbols, or expressions that are unique in mathematical communication. For example, the word function which means the purpose in “what is the function of this tool?” means differently in math as a relation of a set of inputs which provides a set of outputs representing as (f(x)). The word ‘Integral’ in French everyday conversation like “C’est une partie integrale de l’equipe.” means entire or whole, however in mathematics, it refers to integration in calculus. The word ‘Tabe’ تابع in Persian means dependent or following as in “او تابع قوانین است.” However, in mathematics it means a set or collection of distinct objects. The word ‘Nesbat’ نسبت, which means relationship, in mathematics means ratio. Therefore, students should learn mathematics registers to competently understand and use math language.

In addition to mathematics registers, there might be words that do not exist in students' first language or have different mathematical meanings. In other words, it gets even more complicated when additional language learners do not have any idea of the math registers in their own language. Ji-Yeong (2020) believes that

> When EBs (Emergent Bilinguals) learn a new mathematical concept that they have not learned in their home countries, language then becomes a huge barrier because it is through language that the meaning of the new concept is delivered. If a verbal explanation is the only instructional tool, EBs will not understand the new concept, and they will soon fall behind in their math class unless their parents hire a bilingual tutor for them. (p. 21)

An ELL (English Language Learner) student wrote about his earlier years of classroom-based, teacher-led math experiences: "I'm still guessing what 'perform the operation' means when the teacher is erasing the board. So, I just stop listening and figure I can't do math" (Tran, 2005, as cited in Freeman, 2012, p. 3). Therefore, language incompetence can affect the pace of the class and make the class fall behind schedule.

Other than math terminologies used in the field of mathematics, there are some math notation or a format for conveying mathematical ideas that might vary from one country to another. Lopez (2008) has provided a comparison of mathematical notation in the U. S. and Latin American countries. For example, the calendar date is shown as 'May 15, 2007' or '5/15/2007' in the US while in Latin American countries it is written as '15 May 2007' or '15/5/2007'. In Mexico negative numbers can be written as -5 or the negative sign can appear on top of the number (p. 2,3). Additionally, in some countries symbols representing multiplication and division can be shown differently. In German mathematical notation, a dot (.) is used for multiplication while in Iran the traditional multiplication sign (X) is used in math textbooks. This adds to the challenges that non-native speakers have regarding learning vocabulary and symbols.

Schleppegrell (2007) argues that "each subject area has its own ways of using language to construct knowledge, and students need to be able to use language effectively to participate in those ways of knowing" (p. 140). Linguistic demands in math class extend far beyond learning vocabulary and require high abilities to construct and interpret sentences and texts. Considering 21st-century skills, mathematical problems provide many real-world examples and necessitate that students read math problems and understand them thoroughly. In many fields of studies or even language proficiency tests, readers can apply reading techniques to understand the message of the text without knowing the meaning of all the words. They can skip the new words or guess the meaning of the words they don't know in context to get a good approximation of what the word might mean or decipher the text's central theme. However, in mathematics, the meaning of the sentence or problem can change if the reader does not understand all of the words.

On the other hand, excessive focus on vocabulary learning among non-native students might cause confusion and errors of interpretation (Moschkovich, 2002). Jones (1982) claims that "More" and "Less" are two of the most commonly used words in the early years of schools that can be challenging to language learners when they are used in more linguistically complex forms. Jones offers a set of questions from native and non-native English math learners in his study. It believes that both first-language learners and second-language learners could provide the correct answers to simple questions like "Which is more, 10 or 13?" and "Which is less, 7 or 9?" However, in some problems, like "What number is 1 more than 5? What number is 2 less than 5?" students need to go beyond their vocabulary knowledge and determine the meaning by the context in which the terms are used. Jones asserts that in the research study conducted for the second set of questions, "the second-language learners lagged well behind the first-language learners in achieving mastery of the meaning of 'more' and 'less' in a given context" (p. 276). For the question 'What number is 1 more than 5?' many language learners mistakenly did the comparison again without understanding the question and wrote 5 as their answer. Schleppegrell (2007) believes that the reason for this failure is that "students who described a relation between numbers in an informal, unclear or immature way were unable to relate it to a mathematical operation" (As cited in MacGregor, 2002, p. 1).

In another math problem, students are told that Alice had 20 dollars and won 10 more. How much money does Alice have? To a language learner the word 'more' can imply addition operation. That is why in the following math problem the language learner is very likely to make a mistake. The following problem says that Alice has 10 dollars and wants to get a book which costs 15 dollars. How much more does Alice need? The language learner needs a good comprehension of the instruction to understand that subtraction operation is required here. However, the language learner is likely to make a mistake, especially if they are asked these two questions consecutively. The first question is simple and makes the student lower their guard and make them overconfident. Consequently, the language learner does not assess the complexity of the question.

Ellerton (1996) suggests Newman Procedure for students without language proficiency to go through these sequences. According to Ellerton (1996), students need to "read the questions, comprehend what they have read, carry out a mental transformation from the words of the question to the selection of an appropriate mathematical strategy, apply the Process Skills demanded by the selected strategy and encode the answer in an acceptable written form" (p.16). This procedure which aligns with critical thinking and problem-solving strategies in the 21st century seems to be a big challenge for language learners.

The other challenge that non-native students face in 21st-century math classes is an unequal chance to participate in collaborations and communications due to the language barrier. Students are expected to use the language to engage in math discussions, explain their reasoning, demonstrate the steps they took to arrive at their conclusions, and defend their approaches with peers and groups through collaboration and communication. All these activities require strong language skills which can add significant pressure to non-native language speakers (Moschkovich, 2002). Learners sometimes find it so hard to express their ideas and make themselves understood. They might get to the right answer but feel unable to explain their reasoning (Schleppegrell, 2007).

Jourdain (2016) believes that students must understand mathematical concepts and prove that they can link mathematical knowledge with their everyday lives and communicate their knowledge to others. However, native and non-native students do not always have equal chance to participate in the class.

> With regard to power relations in the classroom, both native and international students seemed to acknowledge the fact that power is unevenly distributed among graduate students depending on the nativeness of English. That is, native students who need much less effort in formulating and articulating utterances in English may gain better positions in classroom communities. (Yoon, 2013, p. 67)

In an interview that Yoon (2013) conducted with international students, one of them asserts that

> For us, we are more like audience, not the, how to say, the same level participant as them. So we listen to their lecture, listen to what they speak, but actually, even when we talk, we show some comments. So I think they're more like the speakers, and we are more like the listeners. (p. 68)

Yoon (2013) continues that those participants who have better language proficiency might feel superior to those with limited language skills.

Furthermore, the language factor in math assessment, especially verbal format, is undeniable. Jones (1982) asserts that "although community school children in Papua New Guinea performed relatively well on word-free computational questions, they performed poorly on any question couched in verbal terms in English" (p. 270). According to Abedi (2001), children perform 10-30% worse in verbal assessment compared to numeric format. It continues that the scores of English language learner students are even lower in these assessments. This proves the important role of language proficiency in solving math problems. Abedi's (2001) research shows that non-native math students scored significantly lower, on average, than their native-speaking counterparts.

Interestingly, in the second part of this study, a similar test was revised to reduce the complexity of linguistic features. The findings indicate that simplicity in linguistic structures in math word problems can result in improvement in students' comprehension and their scores on tests. Abedi (2001) claims that

> Data from this study were consistent with previous research suggesting that unfamiliar or infrequent vocabulary and passive voice constructions may affect comprehension for certain groups of students and that average and low-achieving students may be at a relatively greater disadvantage in answering mathematics items with complex language. (p. 14)

Students' decisions depend on their knowledge and understanding. The distinction is important because it shifts the focus from blaming the students for being wrong to identifying a need to develop their language gap. According to Alrø (1996), "the students might not be mistaken if they do not understand a certain problem" (p. 7). The answer that students provide is based on what they understand. When language learners cannot perform well in their math tests due to their language barriers, their Knowledge of math is unvalued. This can serve as a reminder for educators to consider various educational tools and technologies to address gaps in students understanding.

Black (2009) and McFadzien (2015) highlight the importance of feedback provision in the 21st century as what moves learners forward by demonstrating their level of performance and their desired level. McFadzien (2015) defines effective feedback with the definition of the Ministry of Education of New Zeeland as "specific, descriptive, student initiated (combined with self/ peer assessment), time appropriate, suited to needs of individuals, provides strategies for improvement, allows time for action, happens as a conversation and communication surrounding the adequacy of the feedback provided" (p. 16). Feedback can happen through a conversation or interaction between student and teacher. Black (2009) describes interactive feedback as a question or a task from the teacher and a response from the students which ends in an evaluation by the teacher. McFadzien (2015) urges the issue of language in feedback provision, especially for students as receivers of feedback. Therefore, considering the importance of providing feedback in the 21st century, there is a concern for language learners to avoid misinterpretation of feedback they are provided with.

All in all, language deficiency can be demonstrated in different situations like misunderstanding instructions or comprehension of questions and tasks, expressing math ideas regarding the objectives of 21st-century math class, assessments or feedback. Considering the importance of language proficiency in math class, I would like to highlight the role of teachers as the ones who can benefit from technologies, AI, visual aids, or anything that helps them to mediate language complexity for language learners in math class. Class activities should also undergo reflection to ensure that all students are provided with clear instruction and feedback, encouraged to participate in interactions, and given additional resources regardless of their deficiencies.

3.3 Challenges in Terms of Cultural Differences

Ji-Yeong (2020) believes that language is not the only factor that can impact math learning among non-native speakers or what he calls emergent bilinguals (EB). It believes that EBs bring diverse cultural norms and values to the class that might affect their perceptions of mathematical concepts. De Araujo (2018) asserts that there is a deep connection between language and culture. Therefore, other than students' language proficiency, researchers have been wondering if addressing students' cultural resources can support their mathematics learning. Cummins (2000) and Nasir (2008) also claim that in the context of education, language and culture are interconnected and that culture plays a significant role in math learning. They believe that culture, values, and norms influence how students perceive mathematical concepts, engage in activities, and approach problem-solving (Wong, 2004).

Andrews (2010) introduces culture as "a collective manifestation of psychological conditioning" and what "can be construed as a societal analogy to individual memory." It continues that "culture includes those beliefs, artifacts, and practices that history has shown to be effective for the maintenance of a society and its future generations" (p. 4). This means that culture is deeply tied to identity and influencing a person's culture might be challenging. Therefore, Cummins (2000) and Nasir (2008) suggest that to have educational equity, education systems need to integrate students' cultural context into the curriculum to make learning more relevant and engaging for them and consider cultural relevance to ensure that students from diverse backgrounds have meaningful learning experiences.

In addition, the OECD (2015) highlights the importance of immigrants' culture and the education that they received in their countries and addresses them as important elements that can strongly affect the performance of students. It believes that with an increase in the number and diversity of immigrants, host communities might need to change "to develop the skills needed to adapt to new concepts of identity, culture and citizenship" (p. 21). The main reason to highlight cultural differences is to ensure that multicultural classes are free from cultural bias, and that any potentially relevant cultural aspects are addressed *before* introducing math problems. Additionally, teachers can benefit from the presence of learners from different countries to provide more real-world math tasks that reflect the identities of the students to engage and motivate all (Yoon, 2008; Wong, 2004; De Araujo, 2018).

Here, I would like to demonstrate how cultural factors can challenge mathematical learning process. These cultural differences can be categorized into two main groups: cultural contextual differences and differences within the education system or educational culture.

3.4 Cultural Contextual Differences and Their Manifestations in Mathematics Education

One of the biggest challenges that students have in a multicultural setting is Contextual Relevance, which is the fact that they cannot relate to the context of the problem. Consider a math problem asking students from various geographical backgrounds about the cost of skiing. Students from countries like Saudi Arabia, Sri Lanka, Ghana, and Bangladesh might not have any experience with snow and the leisure activity of skiing. Students struggle not with the math but with understanding the context of the problem. De Araujo (2018) indicates many studies confirming that "in tasks with familiar contexts, students were able to undertake actions that promoted meaning making while they solved them, rather than simply relying on procedures." It also highlights the "importance of allowing ELs (English Learners) to use their cultural and linguistic resources while they engage with mathematics" (p. 900).

Here is a list of examples of contextual relevance that illustrate the diverse challenges in multicultural classrooms.

Measurement Units: The usage of measurement systems varies in the world. Many countries use the metric system (meters and centimeters) while some countries, including the United States and Myanmar, use the imperial system (like inches and feet). This global division might cuase confusions for students of different parts of the world.

Currency Differences: Another element that can distract learners in multicultural settings is the different values and symbols of currencies. A question that includes the price of a book in euros (with the symbol) might seem totally unfamiliar to many students from the East. They also might not have any idea about the power of the currency or the exchange rate.

Cultural occasions: Special occasions vary in different countries. Students' lack of cultural knowledge might make it hard for them to understand problem scenarios. For example, Students are given the price of different decorative items and are asked to find the total cost of house decoration with Rangoli patterns during the Diwali festival.

Geographical content: A problem asking about the distance between the Empire State Building and Central Park might be difficult for students from anywhere other than the United States. They might find it hard to visualize the problem and understand the context. In addition, problems involving the snowfall measurement on New Year's Day in America which falls on the first of January sound unfamiliar to Iranian Students who celebrate New Year's Day on the 20th of April in Spring or those from tropical countries. Calculating the height of trees in the Amazon forest might seem unbelievable to a student from Saudi Arabia, too.

Cultural Values: Children are raised with different cultural values in different countries. For example, traditional views about gambling in many countries in Asia are different from Europe or Western countries. Questions focusing on probability using the names of some specific gambling games might sound challenging to Asian students or those belonging to certain religions. In addition, in some Asian countries, like Iran, there is a tradition of standing on ceremony which involves politeness. It includes offering, refusing, and inviting to maintain social harmony. It also involves showing extreme politeness and humility. Therefore, in this culture, it sounds polite to refuse what you are offered (be it food or anything) many times to sound polite. In math problems including probability, this cultural norm can affect students' responses. For example, in a math problem saying that 2 friends are served with 3 cookies, students are asked to calculate the probability that one takes two cookies. Unintentionally, the culture of refusing offers and acting politely influences their answers, and the probability that one person takes two cookies decreases.

Gender Norms: There are certain activities or sports that are reserved for a certain gender in some countries. For example, the idea of a female wrestler may not align with cultural norms in some Arab Countries. A math problem introduces a female wrestler who practices 5 days a week, each day about 2 hours. It asks the students to find out how many hours she trains a week. This question might lead to confusion for many students with traditional cultures.

Representations and names: There are also some names of fruits, food, religious books, and even names of people that students might not have ever heard of. This can also be something that they cannot relate to. Jourdain (2016) has brought an interesting example from Brown, Cady, and Taylor (2009) of a Mexican girl in an American classroom. She was dealing with a problem including the phrase ‘Hanging Streamers’ for a party. As streamers have never been used in parties in her culture, she recalled her knowledge of “knowing that a stream was a moving body of water” (p. 9). This is how she mistakenly related streamer to stream and tried to figure out how to measure stream.

Math Gestures: Gestures have been used as a communication tool to support the understanding of students for decades. Farsani (2015) has analyzed gestures in the context of a bilingual math classroom. The result shows that using gestures in math class goes beyond mere pointing and it complements speech mainly in bilingual context. However, it formulates a question at the end of the research asking to what extent gesturing can convey the message in a math class. Students from different cultures might perceive gestures depending on their cultural background. Kita (2009) claims that different languages “have different lexical and syntactic resources to express spatial information. This linguistic difference is reflected in how gestures express spatial information” (Kita, 2009, 145). According to Núñez (2006), gestures are culturally influenced so their meanings depend on social and cultural factors. According to what was mentioned before about the differences in mathematical notions, there is no surprise that gestures do not mean the same to everyone in multicultural settings. Other than that, some gestures are not appropriate in many cultures and are considered taboo or offensive.

3.5 Educational Practices and Pedagogical Differences

The impacts of culture on the education system can be analyzed from different aspects, including teaching methods, policy and curriculum design, learning styles, and learning preferences.

Stigler (1998) compares teaching with family dinner when everyone comes to the table to eat from containers of food brought to the table. Adults help children to eat and there is an open conversation among all participants. He believes that family dinner which is a cultural activity is “learned implicitly, through observation and participation—not by deliberate study” (p. 1). He continues that teaching is different from learning a skill like working with computers and it is more like a cultural activity. He adds that “although most people have not studied to be teachers, most people have been students. People within a culture share a mental picture of what teaching is like. We call this mental picture a script" (p. 2). This script can tell participants of the class what to expect next.

It is true that things have changed in teaching in recent years and many teachers, particularly in developed countries, need to have teaching certificates to find teaching jobs but in many disadvantaged countries, competence in math is enough to have a teaching career. The other thing is that even educated teachers are affected by what they observe in classrooms in their academic journey as students or what they monitor in their peer observations. "Teaching is a complex system created by the interactions of the teacher, the students, the curriculum, the local setting, and other factors that influence what happens in the classroom" (p. 2). Therefore, educational pedagogy, assessment and techniques, policy and curriculum design in each culture vary from others, and these are what need consideration in a multicultural setting.

TIMSS, Trends in International Mathematics and Science Study, provided a report in 2015 describing a comprehensive comparison of math teaching approach and student performance among 57 countries. TIMSS includes 800 assessment items, the majority of which assess students' applying and reasoning math in 4th and 8th grade. TIMSS aims to help countries make improvements in teaching and learning in mathematics and science. The data collected by TIMSS (2015) reveals that many East Asian classrooms demonstrated the application of teacher-centered approaches with emphasis on memorization while countries like the US and Finland showed more student-centered methods encouraging critical thinking and problem-solving. Countries like Singapore had frequent formative assessments, which allowed constant feedback during the semester, while many countries focused on summative exams which provided end-of-term exams with limited feedback. Many East Asian cultures valued education highly with strong parental support while this was less observed in Western counterparts. Unfortunately, TIMSS does not study the education system in many disadvantaged countries with a high rate of refugees but it proves that different education system values different factors in math teaching which calls for attention in multilingual classrooms.

Teaching Methods: Continuing with different pedagogies in different cultures and countries, Leung (2001) pictures two contrasting images of math classes in Eastern and Western countries. It is interesting to know that the source of the differences lies in the culture of these countries. Leung asserts that in East Asia, students have outperformed learners from Western countries in the last decades. It describes math classes in Asian countries as where there is no or little group work or activities. Chinese classes are more teacher-centered and everything, including the decision making process, class activities, and flow of information, is controlled by the teacher who is expected to be an “expert”. Wong (2004) quotes students’ ideas about education in China as, “you just sit there listen to what teachers said” and “The tutors spoon feeding you, not so much on discussion” which is just the opposite of the idea of 21st-century method (p. 158). A Malaysian student says that “as long as you study and memorise what you learn you can do well” and another student claims that “before I came here...teacher will tell you everything and then you just read, memorize, and then go to the exam, that is all. Most of the students do not need to express our own opinion” (p. 159). The class is based on rote memorization with many mathematical practices and repetitions that do not necessarily lead to understanding. It is also filled with many competitive exams that don’t let the students enjoy their learning. Leung (2001) adds that Chinese parents keep telling their children from an early age that success lies in hard work and pleasure contradicts the process of learning and success in studies. Therefore, children always have this mentality that they should apply perseverance in their studies.

Leung (2001) believes that many East Asian countries have one thing in common and that is the fact that they “share much of the Confucian values in education” (p. 36). Andrews (2010) and Wong (2004) also admit the impacts of Socratic and Confucian philosophies on Chinese educational traditions. According to Leung, Chinese educators provide “examination oriented” classes and believe that the pressure of these exams will lead to an “extrinsic motivation” in students to study more and attempt to learn math (p. 42). In China, exams are what differentiate successful students from less successful ones. Since these examinations and grades affect their future positions, students become more aware of the importance of achieving academic success through extrinsic motivation.

On the other hand, Leung (2001) claims that, unlike Asian countries, Westerners stress “the process of doing mathematics rather than learning the mathematics content itself. Classroom activities such as problem solving and investigations, instead of lecturing and drill and practice, are highly promoted” (p. 38). They also advocate meaningful learning. Students need to understand new information before committing it to their memories. In the Western math approach, learners are the center of the class. Therefore, the class should be learner-centered, and “learning should be a pleasurable experience for the child” (p. 41). The pleasure of learning math creates intrinsic motivation for learners to engage deeply with the subject and persist through challenges. Moreover, the teacher has the role of a facilitator who helps the students to learn to learn.

Stigler (1998) also has conducted studies comparing teaching practices in the US, Japan, and Germany. Stigler observes that in Japan students are “interested in exploring mathematics by developing new methods for solving problems” (p. 2). Japanese teachers believe that students can best learn through critical thinking, collaboration, and sharing the pros and cons of different methods. Japanese teachers are lenient with frustration and confusion that happens to students in different stages of learning. They believe that students need to “explore and invent, to make mistakes, to reflect, and to receive the needed information at the appropriate time” to apply problem-solving fully (p. 3). Additionally, one of the essential parts of the learning process is believed to be mistakes.

According to Stigler (1998), Japanese teachers usually start the class with a story or by asking a challenging problem and always let the students take their time to understand the task and work on a solution. Then, teachers do the monitoring to find the best moment to encourage the students to collaborate. Problem-solving usually leads to a class discussion and highlights important features of students’ methods. Japanese teachers promote discussions, appreciate individual differences, and “view differences as resources in the mathematics class” as different types of learners can bring different ideas, methods, and solutions (p. 4). Stigler continues that American teachers don’t welcome differences among students in math class and consider it as an “obstacle to effective teaching” (p. 4). They believe that dealing with different students requires providing them with different instructions which is not easy in a large class. So, while American and German students use the teacher's provided solutions as examples, Japanese students apply creativity to what they have learned so far to find a solution by themselves. This is how Japanese classes go on like a “tightly connected, coherent story” (p. 4). Stigler claims that Japanese and German math learners are provided with more advanced mathematics. While Japanese teachers promote thinking and understanding, German and American teachers focus on skills.

Problem-solving strategies: Non-native students might struggle with methods that do not align with their cultural background. This can have many negative consequences for learners. Ji-Yeong (2020) brings an interesting example of what he witnessed in her class.

> A few years ago, one high school EB student, let’s call her Sunni, explained what happened in her math class. She had learned how to solve quadratic equations by factoring in her home country, and the method was different from the common method used in the U.S. When she had a test on this concept, she used the method she learned in her home country. After receiving her test back, Sunni was shocked to discover that, her math teacher marked her answers wrong although the answers were correct. Even after Sunni explained how she solved the problems (she could not explain

> herself very well in English), the math teacher insisted she must use the method taught in the class. (p. 13)

These differences in problem-solving methods get worse when they cannot be communicated. Learners sometimes do not have the chance to explain their thinking process due to their language barriers. This might lead to devaluing their math knowledge as in Sunni's case.

Policy and Curriculum design: Andrews (2010) claims that "the curriculum reflects both a way of life, including the shared values and meanings common to members of the group and the practices by which meaning is constructed and shared within the group" (p. 7). It continues that "a loosely structured curriculum reflects societal norms," like those that promote "each day is taken as it comes, but also a culture in which dissent and deviation are tolerated and people are willing to take risks" (p. 6). For example, Andrews (2010) claims that in Belgium, there is a tightly structured curriculum due to their culture which values rules, laws, and structured situations.

Different goals and priorities in different countries have made teachers plan their lessons differently. Alrø (1996) addresses a study by Marilyn Nickson concluding that the role of learners in class is a reflection of the view of their teachers about Mathematics. Andrews (2010) asserts that values in educational systems are affected by the culture and teachers are the ones who reflect these values in their strategies. This is what can distinguish teachers in different countries. He brings an example from English teachers that

> felt themselves encouraged to cover the walls of their classrooms with examples of students' work or posters highlighting in colorful ways various mathematical facts and procedures, while such practices were alien to Hungarian teachers who tended to work within classrooms with, essentially, bare walls. (p. 12)

Andrews believes that teachers' routines and class policies are widely accepted in their culture.

Additionally, education systems in different countries offer varying math content and different assessment systems in each academic year. Wong (2004) quotes from Asian students "Yes, this is what most of the students do. It is very exam based. They only look for the information that they get then can pass. It is very exam based, they only teach you to pass the exam. Probably also the students want it that way." and "Before I came here...teacher will tell you everything and then you just read, memorize, and then go to the exam, that is all. Most of the students do not need to express our own opinion" (p. 159).

Additionally, Hiebert (2006) highlights the importance of prior knowledge in current learning. It claims that "students interpreted and respond to new situations in terms of what they know" and adds that "prior knowledge that is well-understood influences learning differently than prior knowledge that is less understood" (p. 80). If the students have not fully understood the foundational knowledge in their countries, this gap in understanding can create significant challenges in their educational journey.

Learning style: Joy (2009) conducted a study with participants from different countries all around the world to find if culture has an impact on learning styles. The result of the study suggests many different dimensions of culture that can shape students' learning styles. This can illustrate why students from different cultures show different learning preferences and performances in class. Some of these cultural dimensions are explained below.

Joy defines a society high in group collectivism as one where there is an emphasis on group harmony, group activities and cooperation, and developing constant reflections. However, in individual collectivism, the focus is more on individual achievement and personal goals. Therefore, people work individually and competitively and their priority is to outperform their individuals. The learners from this cultural setting might not feel free to get involved in group activities and collaborations. One of the differences in learning style between these two cultures is as Hattie (2007) claims that learners from collectivist cultures like Chinese students prefer "indirect and implicit feedback, more group focused feedback, and no self-level feedback" while those from individualist cultures prefer "more direct feedback particularly related to effort, were more likely to use direct inquiry to seek feedback, and preferred more individual focused self-related feedback" (p. 100).

Joy introduces another important cultural factor uncertainty avoidance. In this cultural setting, teachers are considered the ultimate authority who always provide clear guidance, rules, and laws to avoid ambiguity. Unknown experiences are not welcome and there is no tendency to take risks. Innovative approaches to teaching and creativity in learning are not tolerated. Therefore, one raised in this culture might not be open to applying creativity in learning. Moreover, mistakes are not tolerated. However, in cultures with low uncertainty, the society or system favors open-ended learning, like applying risks, creative thinking, and exploring new ideas, group activities, and collaborative learning. Above all, mistakes are considered part of learning.

The other cultural factor introduced by Joy (2009) is assertiveness. In cultures with high assertiveness, members are more likely to communicate and express one's views openly. However, individual success is given priority, so members live in a competitive environment. To achieve this goal, everyone is allowed to provide direct feedback and constructive criticism. Unlike high-assertive cultures, a lot of collaboration, group activities, respect for other members, gentle feedback, and encouragement are seen in low-assertive cultures.

The other influential cultural dimension is power distance. Societies high on power distance "value social hierarchies" (Joy, 2009, p. 76). Individuals are not allowed to do whatever they want. Members of the society are always involved in showing appropriate behavior to members of the higher hierarchy. Learners in this education system might not feel free to participate actively, or ask many questions for fear of interrupting others or offending the teacher. Teachers have the authority in the class and students normally follow him blindly. They might even hesitate to apply critical thinking and creativity to find a new solution to the problems.

The other cultural factor is gender inequality, which can lead to a feminine or masculine society. Psaki (2018) has done a research study to examine gender disparities in education across 43 countries. The study focuses on the rate of enrollment, progression, and completion among different genders in low-income countries. The results show that traditional gender norms impact educational attainment greatly and there are many countries that still face obstacles to removing gender inequality in education. It claims that primary education enrolment for both genders has progressed but disparities remain in secondary and higher education. It asserts that "beyond the magnitude of gender gaps in school enrollment and attainment, many argue that continued investment in girls' education are warranted because girls face numerous additional challenges in receiving a quality education" (p. 119). This can increase the gap in girls' education which will have negative consequences for their education life. In addition, in many traditional settings, there is a wrong stereotype that girls are considered inferior gender and they are not allowed to participate in society. According to Andrews (2010), in masculine cultures, there is a large distinction between what men and women are supposed to do. Other than having limited access to education, girls might be discouraged from involvement in activities or speaking up in classroom discussions. Even in the host countries these learners might have fear of being judged for their viewpoints and hesitate to participate in collaborations and communications. Joy (2009) introduces Gender Egalitarianism as the level of human equality and justice in society while accepting their differences. According to Joy (2009), in societies with higher egalitarianism, women and men are more likely to be similarly educated and employed in the same occupations. On the contrary, participants of lower gender egalitarianism may feel less confident in participating in class discussions and group work. Therefore, the conflicts they may find in the host country can be stressful and distracting for many, leading them to experience anxiety and show reduced performance.

Cultural differences do impact the way students communicate and interact with one another and their learning styles. It is worth bringing an example of Taarof, which is one of the very common traditional etiquettes in my country. Taarof is an example of a cultural element associated with low assertiveness. It encompasses another cultural element in Joy (2009), too. Taarof, which is collectivist in nature, involves behavior that shows respect and consideration for others. It often prioritizes a group's needs over individual desires and can suppress individual creativity. In 21st-century education that values collaboration and group work and direct and honest communication is key, Taarof often involves indirectness out of politeness which avoids clear and direct communication. For the same reason, constructive feedback which is very important in 21st-century education is often moderated. Taarof also shows the culture of power distance as it emphasizes respect for social hierarchies, authorities, and elders. Younger individuals are always expected to give up their turns to elders and those with higher status. Therefore, they might not even have the chance to participate in discussions and collaborations equally as other participants in the class. Wong (2004) brings another example from Chinese culture where teachers are considered an authority. It asserts that Chinese students are believed to be very quiet in class and less likely to question and challenge the teachers. Wong (2004) also quotes a student saying that "not so many of students giving answers and ideas. I think Asian students are this kind of culture, they expect the tutors to spoon-feed them" (p. 163). Therefore, students are more like receivers of information and hardly ever share their ideas unless they are asked.

Therefore, the cultural dimensions which are mentioned above can sometimes conflict with the principles of 21st-century education. The one from a culture with high assertiveness who values competition and directness might find it hard to participate in 21st-century education which requires effective communication, involving active listening and collaboration. The one with a high in-group collectivism culture, which promotes group harmony, might struggle with the idea of individual creativity in the 21st century. The one with an individualistic culture, who gives priority to individual achievement and personal goals, could encounter difficulties with 21st-century education which promotes collaboration and valuing group success. Learners with high power distance cultures that accept hierarchical order without much questioning can have a hard time applying critical thinking, offering challenging ideas, and questioning one another. While 21st-century math education highlights collaboration and communication among all and promotes contribution to learning, one from a masculine society values competition. This makes it hard for female participants in the class to take part in communications. In a society with high uncertainty avoidance, one prefers predictable situations over uncertain situations so as not to make mistakes. Therefore, one prefers not to get involved with questioning, thinking critically, or creativity which requires risk-taking.

As Yoon (2008) says, teacher's approaches to "building a trusting relationship and celebrating ELLs' differences" can affect the way they meet "the students' cultural, social, and academic needs" (p. 505). Students need to know that their lives and cultures are valued after they move to a new culture. Yoon (2008) explains his observation of a class run by Mrs. Young as a teacher who embraced different cultures in the class to involve more students. She asked students questions like "How do you celebrate that?" "How do people feel about this?" and "What are your traditions?" Yoon claims a Korean student got so excited about having the chance to explain his culture to the class. This provided the Korean student to find his voice in the class, have a sense of belonging to the class, and start to participate in the activities. Wong (2004) also indicates the satisfaction of a Malaysian student in Australia as "when we discuss topic on cultural, they will ask, what do you think? They ask about Malaysia. They try to get me more involved in tutorial or lecture" (p. 164).

Wong (2004) claims that students are flexible in their learning practices. He explains his experience of being brought up in a traditional Chinese family with the role of 'passive recipient' in education. He claims that when he was exposed to another method of learning in a different culture, he managed to adapt himself to the new approach. It also brings a study that was conducted on a group of Asian international college students most of which were with teacher-centered educational backgrounds. These students were offered different types of teacher/student-centered activities. The results show that most of the students preferred more open and global types of learning experiences (student-centered learning) despite their educational background. Wong (2004) quotes a student saying that "Because I like this style, so I can adjust to it. Yes, I like to learn myself" (p. 164). That means that students can adapt themselves to new learning approaches.

However, Wong (2004) admits that cultural resources should be considered before implementing educational theories because it is hard to use a student-centered approach for the one who is used to having the role of a passive recipient. Wong brings an example from an Asian student who has shifted from a teacher-centered education system to a student-centered environment in Australia and lacks the experience in note-taking during lectures.

> The hardest for me is to take down lecture notes, because I never had to do that before. Most of the time teachers would give out notes,

> coming here certain lectures we have to write our own notes, which is pretty hard because we are not used to it. By the time you finished writing, the lecturers are way out and you may miss part of it. That was hardest for me. (p. 161)

Another student says that

> and also group work. I realise that here they emphasis a lot of group assignment. While in KL (Kuala Lumpur), mostly is individual. So is very personal workload. Whereas here, they make you work in groups. So far I found it not as effective way for me. For me the group members do affect teamwork. I find it so far not a helpful way of learning. Probably the background that I come from is very personal. (p. 162)

Therefore, education authorities should know about different information perceptions among learners. Wong (2004) brings another example proving the importance of better understanding between students and teachers as:

> Last semester, I felt really lost, this semester the Japanese student and I tried to talk to the lecturers how we feel, and everything we are not happy. They become realised, it is not their fault, because they don't understand. We let them know it is not their faults. It is our problems. So they can understand now. (p.164)

There is no question that teachers play a critical role in bridging cultural gaps and eliminating it as a barrier. However, adapting to a class that promotes equality might need additional support. Educators should bridge these cultural gaps by promoting an environment that values different perspectives and encourages the development of 4C while respecting cultural backgrounds.

3.6 Bridging theory and practice

Farsani introduces language as an important factor in education. He believes "when there is no common language, there is no common understanding." He brings many examples of language barriers among additional language learners in math class. For example, the word 'mean' usually implies 'meaning' while in math 4+8 means 12. The word 'leave' means 'departing' while in math 17-3 leaves 14. He claims that many additional language learners tend to locate x instead of solving the problem to calculate x when they are asked to 'Find x'.

Farsani brings more examples of language barriers from his experience of working with Kurdish students. He observed a case involving Kurdish students where a point (.) was used instead of (x) demonstrating the multiplication symbol. He claims that "many Kurdish students were wondering what this symbol (x) was doing in their math books in English." In addition, he asserts that language learners assume that the numbers grow bigger when they are raised to power. Therefore, 3^{-2} could be a mismatch for them. It reminds us of what Jones (1982) claimed about the words “More” and “Less” and that the second-language learners lagged well behind the first-language learners. Farsani has witnessed many Haitian students (French speakers) in Chile having difficulties in understanding math due to their language barrier. The interesting thing to him was that the teacher of the class kept telling him that ‘time is healing’ indicating that the students will understand the math points through the time. However, it wasn’t easy for the students to deal with it.

L.C. has been teaching math to 7 to 12-year-old students from more than 12 different nationalities, where only the Swedish language is used. She claims that “many of them were born in Sweden and learn Swedish in kindergarten. However, having been raised in non-Swedish families, where the first language is used at home, they might face language challenges when they enter school. They sometimes struggle with understanding academic content because they are still developing proficiency in the school’s language. This sometimes makes it harder for them to understand lessons and participate in discussions.”

According to L.C., in Sweden, students can benefit from a teacher who speaks their language an hour a week. Therefore, teachers who speak the language of children are hired to re-cover part of the content that was covered in the class earlier. However, these classes are available only in a few languages, like Turkish, Somalian, and Lebanese.

L.C. admits the importance of language proficiency in math class and claims that “when math is taught to additional language learners, they sometimes have problems with reading, especially technical words and those words with different meanings in math” (ambiguous terms or polysemous terms). These terms can create confusion when used in different contexts because their meanings depend on the subject matter. For example, when students are asked to look at the table (on the board), they mistakenly look at their study table. She adds other examples, like “ring”, “group”, “field”, “graph”, “function” and “matrix”. L.C. claims that in Swedish, there are many of these words that can be mistaken by students, especially by additional language learners.

L.C. normally pre-teaches the challenging words to the class and boards them to set clear tasks and let them know what exactly they are supposed to do. However, she sometimes notices some additional language learners with little confidence in responding to math questions. “They are given whiteboards to write their answers instead of answering them orally.” She explains the challenge as “in this situation, other students nag about why it takes him long to respond or when I listen to one, the rest start talking together. I am so pressured to tell them that they need to wait. Therefore, part of what I teach in the class is that we need to be patient and support one another. And that we are not all the same."

Shumway brings her experience of working in a school full of immigrants from all over the world and highlights the beautiful asset-based approach. In the asset-based approach, the focus is on recognizing the strengths, resources and potentials within individuals instead of highlighting problems, and deficits. The asset-based approach emphasizes what is working well now and how the existing situations can be used for the growth of that specific field. She claims that "now with the emergence of Dual Immersion Language programs, there is more cultural awareness. The students know what it feels like to learn a new language. Therefore, they understand one another better."

Farsani suggests using visual symbolism, colors, body language, and gestures in class so that students will not rely much on the language. However, he believes that "language is not the only problem that additional language learners and their teachers should deal with." He believes that "we should learn the language with the culture." He brings many examples of cultural contextual differences. He claims that "there are many short, dark-skinned people in the eastern areas of Chile and when you take the subway to Western parts, you find tall, light-skinned people. Therefore, the word tall in Chile might mean differently according to the area they belong to." What if the students are asked to consider a tall man? They can have different ideas about what 'tall' really means.

Another example he provides about the interconnectedness of language and culture is the electricity outage. This word might mean differently to one from Sweden, Australia, Iran, and Africa. In Sweden, electricity outage happens rarely as Sweden depends much on renewable energy sources. Therefore, people normally have no idea of electricity cuts. In Australia, electricity outages are generally rare and often happen due to severe weather events and normally there is a backup plan for essential needs. Iranians have experienced more frequent electricity outages in recent years due to high demand in the summer which might take hours. In some areas in Africa, they do not even have any idea about the electricity, let alone the electricity outage. What if the students are asked about the average duration of electricity cuts? Therefore, Farsani believes that cultural experience in education should be considered.

Farsani also presents a case study he observed among Chilean families where the firstborn son is called after his father while the second might have a unique name. In a conditional probability, where a couple (Carlos and Maria) with two children is introduced, students are asked to find the probability that the first child of this couple is a girl while knowing that the second child is a boy called Daniel. Since the second child is a boy, the boy-girl and girl-girl outcome is excluded and only boy-boy and girl-boy are left. Non-Chilean students might say that the probability is 50% based on purely mathematical logic. However, to a student raised with Chilean culture, the firstborn boy must be Carlos, a boy, and the child cannot be a girl. Therefore, a Chilean might say that the probability of 'first child is a girl' is 0 based on cultural naming tradition.

Shumway also addresses many cultural differences explained by Joy (2009) asserting that "teachers should be aware of cultural differences, gender differences, and power dynamics and they should make sure that all voices are heard." This shows how critical classroom management is. She adds that "everything comes from social emotional well-being that teachers having the skills set to know how to facilitate difficult discussions and how to set expectations for group work. While being in a multilingual class, teachers should make sure they have some visuals to guarantee that all students can participate in what is being done in class. She admits that it takes some effort on the teacher side to build that awareness and believes that teachers should have choices on how to engage different types of students in class activities. However, it is not easy."

L.C. shares one of her secrets of success in a multicultural and multilingual setting as using culturally responsive practices. She tries to use math problems with some cultural points. This is what motivates the students to share some information about their country of origin. She believes that when students are asked to share something from their culture with their friends, it can help them feel a greater sense of belonging in the class, be more open to communicate with peers, and participate in class activities.

Considering cultural differences, Farsani has also worked with many students who are not open to participating in group activities. He believes that teachers need to be aware of different strategies to handle this big difference between students' tendencies and class objectives.

All in all, the insights reveal that teaching math in multilingual settings is not only about overcoming language barriers but also understanding and addressing the cultural issues that influence learning. Cultural differences as well as language barriers can complicate understanding of math and can affect how students interpret concepts. Therefore, teachers need to be equipped to adapt their instructions, learning environment and strategies to manage diverse classrooms and involve different types of learners. The experiences of these three professors highlight that using visual aids and culturally responsive practices can help students feel supported in class. In addition, some programs, like Dual Immersion Language that value cultural awareness, have increased mutual understanding among students which can empower them to succeed.

4. Leveraging Technology and AI to Address Language and Cultural Challenges in 21^{st}-Century Math Education

According to Engelbrecht (2020), technology has helped a lot to change mathematics education to a more student-centered model since the 1970s. Social media, in particular, has transformed education from a ‘push’ to a student-centered ‘pull’ approach in which students have an essential role. Engelbrecht (2020) elaborates that over the last decades, teacher-driven classes, where one-way transmission of knowledge from teacher to students happens (pushing knowledge), have been replaced by student-driven approach, where students take control of their learning process through collaboration and cooperation (pull process). In a pull process, students have the chance to select, discover, design, and develop the information they want to learn. In this process, students may have the role of critical thinkers and learning can happen through discussions and collaboration. This change does not only mean benefiting from technologies but also signifies a change in the concept of educational success. Therefore, technology has been useful for rethinking education. Through this change, there has been less focus on the “application of procedures” in mathematics and more focus on “new ways of practicing mathematics” (Engelbrecht, p. 825).

Additionally, Engelbrecht(2020) believes that the emergence of a new generation that is interested in multitasking in education, “collaborat(ing) using the latest technology” and requiring “information quickly using visuals, sounds, and color from multiple multimedia sources in a novel or useful way” along with the formation of digital educational tools has challenged “the efficacy of current teacher practices and traditional classrooms” (p. 826). Marshall McLuhan (1967) also highlights the role of media and believes that it is media that shape how we perceive and interact with the world. Therefore, now that the nature of media has shifted, the entire process of acquiring and developing knowledge may also transform.

As discussed in the chapter “The 21st-Century Math Approach”, the 21st-century approach and its emphasis on 4Cs-Creativity and Innovation, Critical Thinking and Problem Solving, Communication, and Collaboration- is the consequence of the emergence of Gen Z and the shift in the nature of media and technologies. Engelbrecht (2020) quotes the idea of Vygotsky’s theory, highlighting the critical role of communication and interaction as a “primary role in the construction of learning” and a way to gain “higher levels of understanding and achievement” (p. 835). However, with a sudden increase in the number of refugees and additional language learners in educational systems all around the world and the importance of communication and interaction in learning, the challenges and opportunities posed by these language novices have become a concern to educational authorities.

As discussed in the chapter “Challenges for Additional Language Learners and Their Teachers in 21st-Century Math Classes,” many researchers admit that additional language learners do not perform well in math class and their performance is far below their native-speaking peers (Cummins, 2000). Additionally, as Gómez-Rey (2016) claims environments where multiple cultures exist have been expanding. While it can bring a lot of advantages, it might cause many misunderstandings and difficulties in communication in an educational context. Gomez highlights the key role of ‘Culture’ in “social behavior, communication, cognitive processes, and pedagogical technologies” and believes that both instructional designers and practitioners should take this element seriously in learning environments (p. 225).

Designers should respect certain viewpoints of international students and the teachers should consider the differences among international students and their needs and try to deliver culturally relevant courses. Klotz (2006) interprets Culturally Relevant Education as an educational system “that honors, respects, and values diversity in theory and in practice where teaching and learning are made relevant and meaningful to students of various cultures” (p. 11). According to NCTM (2000), “teachers need help to understand the strengths and needs of students who come from diverse linguistic and cultural backgrounds, who have specific disabilities, or who possess a special talent and interest in mathematics” to design programs and courses according to that (p. 14). It adds that “all students should have access to an excellent and equitable mathematics program that provides solid support for their learning and is responsive to their prior knowledge, intellectual strengths, and personal interests” (p. 13).

To achieve this goal, Engelbrecht (2020) claims that digital technologies can provide innovative methods for designing, implementing and, evaluating professional development. Therefore, digital technology might be a solution to two main problems in education at this moment which are “What we teach, and how we teach it” (Holmes, 2019, p. 2). Holmes explains ‘What’ as what is worth teaching/learning in an age of AI and ‘How’ as how AI and educational technologies can enhance and transform education. It can also be translated as how school content can be taught effectively to remove the gap between native and non-native learners.

According to Walter (2024), AI not only serves as a means of delivering content but also as a tool to develop questioning, analyzing, and thinking deeply about the information the learners receive and to foster critical thinking. Walter adds that giant databases in AI systems can provide complex problems that promote meaningful learning which requires relating their new learning experience to what they have already learned. AI can provide personalized learning experiences that adapt to the individual learning styles and abilities of students. Online discussions can provide this opportunity for all participants to generate different views “in the course of social interaction” (Engelbrecht, 2020, p. 830).

All in all, this chapter will discuss the ways AI and technology (How) can benefit additional language learners in gaining 21st-century skills in math class (What). This section will also show the collaborative role of teachers, technology and AI in transforming education from a traditional passive form to an engaging 21st-century intelligent learning. Then, this education system can be where everyone regardless of their learning style, language, culture, and educational background can benefit from.

4.1 Artificial Intelligence (Technologies) in Education (AIED)

According to the Merriam-Webster dictionary, technology is defined as “the practical application of knowledge, especially in a particular area.” Januszewski (2008) claims that according to the Association for Educational Communications and Technology (AECT) “educational technology is a complex and integrated process of people, procedures, ideas, devices, and organization for analyzing problems and devising, implementing, evaluating, and managing solutions to those problems, involved in all aspects of human learning” (p. 270). Therefore, what educational technology tends to do is to facilitate learning by using technological resources. The integration of technology in education is the fusion of technology resources, like “computers and specialized software, network-based communication systems, and other equipment and infrastructure” in daily routines of educational environments (Schmitt, 2002, p.75). Through this routine process of supporting school objectives, one can manage to apply collaborative work and communication and benefit from digital and network-based access to data (Schmitt, 2002).

Additionally, UNESCO (2019b) defines artificial intelligence as “machines capable of imitating certain functionalities of human intelligence, including such features as perception, learning, reasoning, problem-solving, language interaction, and even producing creative work” (p. 24). Baker (2019) believes that AI consists of “computers which perform cognitive tasks, usually associated with human minds, particularly learning and problem-solving” (p. 10). Therefore, AI is not a single technology but instead includes a range of technologies and methods, such as algorithms, neural networks, machine learning, natural language processing, and data mining.

Baker (2019) defines AI applications as what can be used for learning and teaching in two categories of learner-facing AIED and teacher-facing AIED. It claims that the definition of AIED goes beyond the application of AI technologies, such as intelligent tutoring systems, chatbots, robots, and automated assessment, with the potential to enhance learning, teaching, and assessment by offering learners more personalized and adaptive learning and providing immediate feedback. Instead, it is a move away from a 'one-size-fits-all' approach to learning, enabling students to learn at their own pace or tailor learning materials to their own interests. Such tools are used by learners in the classroom, particularly in large mixed-ability classes, where teaching through a single human offering direct instruction may be more difficult. They are also used to carry out homework, or to facilitate flipped classroom, where students familiarize themselves with new concepts via the intelligent tutoring system outside the classroom, with classroom time used to develop understanding of those concepts (p. 11). Holmes (2019) adds that:

> AIED includes everything from AI-driven, step by-step personalized instructional and dialogue systems, through AI-supported exploratory learning, the analysis of student writing, intelligent agents in game-based environments, and student-support chatbots, to AI-facilitated student/tutor matching that puts students firmly in control of their own learning. It also includes students interacting one-to-one with computers, whole-school approaches, students using mobile phones outside the classroom, and much more besides.

Additionally, according to Baker (2019), the teacher-facing system can help teachers benefit from AI to "reduce their workload, gain insights about students and innovate in their classroom" (p. 12).

4.2 How AIED and Technology Can Support Additional Language Learners in Math Classes

According to Chiu (2023), there should be a paradigm shift in delivering education in the AI era to provide more dynamic, interactive, and student-centered learning environments. To support 4C skills in a multilingual environment, a range of educational technologies and artificial intelligence (AI) based tools have been enhancing students' learning experience in the 21st-century math classroom. The emergence of AI has been a quick game-changer in the field of education. It has led to a new era in learning and teaching methodologies by offering a personalized learning platform and supporting a diverse range of educational needs and learner types.

AI-driven tools in this section are divided into six categories, though they can also be categorized in other ways.

AI approved language support tools

The first category of AI-driven tools is AI-approved language support tools which can assist math learners, particularly with their language barrier, by providing real-time translation and facilitating communication. AI-approved language support tools can form the foundation of other AIEDs that will be explained in this section. These language support tools, such as translation tools, speech recognition and text-to-speech tools complement and enhance accessibility and understanding for diverse learners. Tapalova (2022) introduces AI tools that simulate human listening (machine translation, speech recognition), speech (speech synthesis, human-computer dialogue), and observation (computer vision, recognition images, text recognition) that can help those who struggle with local language.

AI-powered translation tools, such as Chatgpt, provide multiple support for multilingual students, such as revising the text and checking the grammar, offering suggestions for clearer or more accurate wording, translating and paraphrasing, summarizing, and simplifying unfamiliar math concepts. Therefore, learners can enter the instructions, explanations, or their reasoning for math tasks in any language in the form of text, audio, or visuals and ask Chatgpt or other AI-driven translation tools to apply any of the commands above in a few seconds ensuring that the respond is clear, correct and easy to understand.

Additionally, Son (2023) introduces Automatic speech recognition (ASR) as “a technology that uses AI and machine learning techniques to understand and produce spoken and written text.” This technology uses “voice recognition and speech-to-text”, like “intelligent personal assistants (IPAs), automatic transcribers, and note taking apps” (p. 8). This type of technology can increase interactions in an anxiety-free environment. Son (2023) believes that an animated chatting system, like Alexa and Google Assistant, can increase students’ willingness to interact with one another. It also adds that these technologies can personalize learning content for students by adopting the pace and content based on the needs of individuals in different languages.

Another example of AI-powered language support tool could be Otter.ai which transcribes the whole class or part of discussions and makes it easier for additional language learners to follow the class or discussions. There is also a chance for everyone to apply real-time translation. In addition, plugins for summarizing YouTube or providing subtitles offer real-time captions for everyone. Furthermore, Immersive Reader by Microsoft can read texts and break down complex sentences, translate, and provide visual aids. According to Liu

(2023), “language inclusive interventions could play a role in improving equity and access to information for all citizens, not only those in higher education” (p. 26).

Adaptive learning system

The impacts of culture on the education system, pedagogical differences, and various types of policies and curriculum designs were analyzed in detail in the previous chapter. It was discussed that different goals and priorities in different countries can lead to different forms of participation and various outcomes in math classes. In addition, different countries might prioritize certain mathematical skills based on cultural, societal, or economic needs. This can cause gaps in understanding due to differences in educational culture and prior knowledge among learners of different countries. Tapalova (2022) claims that “learning is a natural human ability that starts in personal experience, cognitive awareness, personal biases, opinions, cultural background and environment” (p. 642). Therefore, traditional teaching where everyone is encouraged to follow similar learning procedures without similar “knowledge, preferences, learning goals and needs” might not lead to success (p. 642).

Adaptive learning systems aim to remove the gap between multicultural learners by substituting a one-size-fit-all traditional class to a personalized learning environment that tailors education based on students’ pace, learning styles, needs, “age, attainment level, prior knowledge and personal relevance” by monitoring their progress, recording their performance, and finding their weaknesses through real-time feedback (Major, 2021, p. 1937). It also makes sure that each student “receive the right level of challenge and support” (Onesi-Ozigagun et al., 2024, p. 591). Major (2021) claims that this learning system can be adjusted according to learners' preferences and cultural context and can also support “self-led learning.” It highlights the fact that personalized learning does not mean ‘individual ‘or ‘individualized’ learning. Instead, students can experience “personalized learning while working individually, in small groups or in the whole class” (p. 1938).

According to Holmes (2019), personalized learning platforms can record a profile for each student by collecting data based on their learning/problem-solving styles, the time they spend on tasks, and their types of mistakes. For the students to become better problem solvers, this system applies elicitation to find what they already know and understand their prior knowledge. This was highlighted earlier by Chapman (2015) as one of the components of 21st-century math class. Onesi-Ozigagun et al. (2024) state that “if a student is struggling with a particular concept, the system can provide additional practice exercises or resources to help them improve” and “if a student is excelling in a certain area, the system can offer more advanced materials to keep them engaged and challenged" (p. 591). The advantage of this system is that it keeps all the students engaged, regardless of their educational background.

Holmes (2019) adds that personalized learning systems can provide personalized feedback that suits their individual needs which can encourage the students to think critically about their strategies of solving problems. This tool can also promote deeper reasoning, and encourage creative problem-solving by providing different types of tasks and collaborative learning based on adaptive grouping. Holmes (2019) believes that teachers used to personalize their teaching to some extent by walking around the class, providing extra support for those who found mathematics tasks too challenging, and offering more challenging tasks for those who found the tasks easy to deal with. Personalized learning platforms can increase the quality of education by decreasing the negative effects of teacher to student ratio (Major, 2021) regardless the number of students.

Intelligent tutoring systems (ITS) have been designed to provide personalized and interactive instruction by detecting students' errors and evaluating their abilities without the involvement of human teachers (Son, 2023). Zawacki-Richter (2019) believes that ITS has great potential to provide one-to-one personal tutoring where individual learning paths are structured based on students' models. ITS also has the capability to provide cultural information tailored to each student's background and offer immediate, personalized feedback.

As discussed in the chapter "*The 21st-Century Math Approach"*, the learning process should provide feedback to achieve educational goals and this feedback is essential both for teachers and students to improve their work (Jurs, 2020). There is no question that good feedback can cause thinking. As Black (1998) concluded, "the provision of challenging assignments and extensive feedback lead to greater student engagement and higher achievement" (p. 23). Holmes (2019) also highlights the importance of feedback provision to enhance learning outcomes and admits that this is almost impossible "when tests are marked by hand." Real-time feedback systems in personalized math education can provide immediate feedback to students as they can engage with specific math problems that have procedures to solve. This process may develop critical thinking as students analyze their performance and make real-time adjustments to achieve better outcomes. Interestingly, students are given the opportunity to discuss the feedback they receive with their peers, allowing for deeper reflection and a more interactive learning experience which can be regarded as a form of critical thinking.

According to Chiu (2023), one of the uses of AIED has been to provide real-time feedback by “analyzing their work and learning process.” (p. 8) ALEKS is an example of the combination of personalized or adaptive learning systems and real-time feedback provision tools. ALEKS provides students with a new page and information according to their correct/incorrect answers on the previous page. Students always receive feedback by choosing incorrect answers to understand the reason. The system can also provide additional corrective information before they proceed. In other words, adaptive tasks are assigned to each student based on their individual competence (Holmes, 2019) that provide targeted lessons based on their learning needs. Finally, advanced language processing (ALP) which understands and processes many languages can be employed to provide feedback in different languages.

In the previous chapter, low assertiveness, gender inequality, and individual collectivism were introduced as characteristics of culture that could limit the participation of students. What can be concluded from the explanations above is that the customized and self-paced model for individuals that is designed in personalized learning systems can bridge cultural differences by providing a safe, non-judgmental environment for everyone to achieve equitable outcomes.

To conclude, personalized learning systems might seem contradictory to 4C objectives at first as they appear to be learning systems designed for individuals, and this might raise the question of how they can promote collaboration. The key point is that it is not just about the tool. It is often the teachers’ role that brings out the 4C through intentional integration into pedagogy, setting clear objectives and applying blended approaches. As mentioned earlier, students from different countries experience educational gaps for several reasons, like different curricula and teaching styles or interrupted schooling. This can act as a barrier to effective teamwork. With the help of personalized learning systems, while students work on content suitable for their individual levels, teachers can encourage teamwork by asking them to participate in team challenges where they should complete levels and earn badges.

In addition, as previously stated, personalized learning can identify the strengths, weaknesses, and leaning styles of every single student. This can help teachers intelligently group students for collaborative tasks based on these findings. For example, two students, one strong in conceptual understanding and the other strong in practical application can be paired to work on different parts of a joint project while following personalized paths.

Interactive math learning systems

Human–machine conversation is another way that AI allows students to have conversations with machines about their learning. According to Son (2023), “A chatbot, also known as a bot, chatterbot, dialogue system, conversational agent, virtual assistant, or virtual agent, is a software application that interacts with users via chat and stimulates human conversations by asking and answering questions via text or audio” (p. 10).

Son (2023) believes that virtual agents can provide a comfortable environment for those who do not feel like taking part in collaboration and communication in class due to their cultural background, language barrier, or any other reasons. Interaction with virtual agents can help them gain more confidence to actively participate in class activities and discussions. In addition, Walkington (2024) claims that students can benefit from the outputs of math problems provided by GPT 3.5 to “generate ideas for how to approach the problem, to compare their reasoning with GPT-3.5” and to “evaluate and correct GPT solutions” (p. 3). This is how students can apply critical thinking by reasoning behind the solutions for math problems. Son (2023) adds that teachers can also recognize students’ mistakes or weaknesses if chatbots provide teachers with a record of conversations between the machine and the learners.

Additionally, Son (2023) believes that chatbots have unlimited patience, time, and capability to respond an unlimited number of requests, unlike human resources. Knowing this fact can help learners with language barriers to ask questions more focused and without hesitation, anxiety and, the fear of being judged. Chen in her article called *AI will transform teaching and learning. Let’s get it right* introduces “fear of judgment from peers” as one of the main reasons which holds students back from fully engaging in many contexts. Additional language learners might “believe themselves to be behind”, therefore, they are not very likely to engage in the class. Technology helps the ones who ask questions in the class stay anonymous. That is why many teachers believe that AI and EdTech tools help encourage class participation by insisting that “there is no such thing as a stupid question”. This can also help those from high-assertiveness cultures. The other thing is that while additional language learners benefit from chatbots in their math tasks, they can also improve their speaking performance through communication with bot in multiple languages.

Other than virtual tutors and chatbots, interactive simulations and games can facilitate interactive math learning. Virtual manipulative (VM) is an example of an interactive simulation system that can revolutionize education by engaging all types of learners and making complex math concepts available to all. VM is “an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, and presents opportunities for constructing mathematical knowledge" (Moyer-Packenham, 2016, p. 3). Teachers can integrate the use of visual manipulative tools into their instruction to teach new concepts and provide students with opportunities to engage in interactive activities during guided and/or independent practice (Shin, 2017). Educational games, like Prodigy, that integrate math problems into fun experiences, can promote critical thinking and collaboration. According to Walkington (2024), they encourage students to solve problems creatively by offering different scenarios that require creative solutions. This approach may keep the students engaged and motivated to learn while bridging language gaps. It also promotes a sense of teamwork, as students learn to rely on one another for support until task completion without noticing language barriers.

Visual models in GeoGebra and Socratic have the potential to convey the meaning of the relationship of fractions, including addition, multiplication, and division of fractions, in an interactive way through diagrams, fraction bars, and area models and are easy to access through their mobile or computers. Virtual manipulatives can also be integrated into games maybe in the form of virtual-manipulatives-based quizzes to check understanding of a particular concept (Shin, 2017) while promoting equity by providing a language-independent pathway to understanding. It is worth mentioning that students need teachers' support to find what VMs represent and how to use them. (Moyer-Packenham, 2013).

Teachers can provide instructions in Conceptua Math or Fun Fraction, two apps with the ability to read aloud the fractions, or through games using Illuminations (http://illuminations.nctm.org/Games-Puzzles.aspx). VMs can be connected to interactive whiteboards to provide this opportunity for students to share their work and how they use the visual model tools.

The integration of digital interactive whiteboards, like Jamboard, with translation tools, can provide the students the chance to collaborate problem-solving real-time in multiple languages. According to Google Workspace, teachers can provide access to a shared board through a link and multiple users can write, draw, add sticky notes, upload images, and create diagrams with changes visible to everyone in real time.

Real-time assessment and feedback tools

Two of the essential components of learning and teaching are assessment and feedback provision. Black (1998) claims that assessments are vital to improve students learning and feedback on teachers' success. This is how teachers and students can benefit from assessments in education and might be the reason why Holmes (2019) introduces assessments as what “define(s) the education we get” (p. 6). In addition, Chassignol (2018) claims that “assessment refers to the wide variety of methods or tools that educators use to evaluate, measure, and document the academic readiness, learning progress, skill acquisition, or educational needs of students” (p. 18). Therefore, proper techniques should be applied to make this essential part of education lead to progress in teaching and learning.

AI grading tools have made it possible to provide immediate insights into student performance and identify misunderstandings or challenges as they occur in classes with large groups of students with different backgrounds. In addition, students’ outcomes and a view of their progress can be presented in a user-friendly format through tools for predictive analytics.

According to Chiu (2023), AI-enhanced grading systems can provide automatic marking for mathematics and provides “more accurate, fast, and secure grading in tests and examinations than teachers” (p. 9). Fu (2020) introduces automatic scoring applications (ASA) as a similar AI-driven tool that can grade learners and assess their written and multiple-choice questions in real time. Fu (2020) continues that some students hesitate to ask their teachers many questions and show their doubts to other students, especially those with the language barrier.

AI tools have provided real-time scores which “make trial-and-error learning more effective by providing real-time scores” and “immediate feedback” to demonstrate students’ strengths and areas for improvement without delay (p. 2). Additionally, According to Chiu (2023), the interaction between AI tools and learners has helped “low-achieving students feel more confident and useful and less embarrassed” and has made the class more engaging (p. 10). It adds that students consider AI-driven assessment tools more objective and are happier with the scores they receive from these tools. This is what can create more motivation and engagement among learners. Gradescope, MyMathLab, and ALEKS are some of the most famous AI grading tools.

Along with AI grading tools, analytical platforms are used to track students’ understanding. Chiu (2023) claims that analytical platforms are designed to predict students’ performance “by assessing the extent and quality of their participation in learning activities, such as discussion forums” (p. 9). This tool can have the ability to “predict the probability of a student failing an assignment or dropping out of a course with high levels of accuracy” (Zawacki-Richter, 2019, p. 4). This can be very practical for those students with any kind of limitations, like language barriers or gaps in educational background. However, Chiu (2023) believes that selecting data for prediction is still challenging.

Culturally responsive AI and language diversity tools

In the previous chapter, culture was introduced as a factor that can impact learning styles. Some learners are more open to communicating and collaborating in class while others prioritize individual success. Many do not feel like applying critical thinking and creativity as they value the teacher's authority in the class and many do not feel like participating in activities due to gender inequality in their home country. AI-driven tools have provided the opportunity to bridge language and cultures. One of the AI-driven tools that can accommodate cultural diversity is LMS.

Learning management systems (LMS), like Google Workspace and Microsoft Teams, provide the chance for the students to recognize and minimize their weaknesses through interactions and let students work together on math projects and communicate effectively. Furqon (2023) introduces the LMS as a pedagogical instrument that can offer "a range of functions, including the presentation of learning materials, sharing and discussion of presented materials, course management, completion of coursework, attendance of exams, provision of feedback on homework and exams, organization of learning materials, and maintenance of records pertaining to students, teachers, and systems" (p. 1083). Then, according to that learning process using LMS, teachers, and students have their way of building interactions with each other, collaborative responses, and collaboration.

LMS also allows teachers and students to form fun new ways of communication through the use of updated learning media, which are proven to increase motivation and learning achievement. Then the LMS makes students and teachers flexible to investigate, explore, and receive information related to assignments and learning activities at certain times with the same rights and speed for each student (Furqon, 2023). It is worth mentioning that videos, reading, or other materials can be stored on the site and assigned to be read at home and classroom time can be used for active learning, like discussions and hands-on activities. This learning strategy is known as flipped classroom which can help additional language learners personalize their pace and provide this chance for their teachers to use class time to address individual challenges instead of giving lectures.

The LMS platform has a forum section where students can ask questions and make mistakes in a safe setting. In asynchronous online discussions, additional language learners can benefit from extra time to understand the questions, as they can use translation tools. They can receive direct and immediate feedback, too. In forums, students are allowed to share their diverse strategies and techniques for solving mathematical problems. They also have the chance to apply peer feedback by commenting on each other's work. This is how the idea of think-pair-share discussed earlier by Razak (2016) can be implemented.

AI for Content Design and Customization

Chatbots powered by AI can be valuable tools for teachers in designing lessons. Walkington (2024) asserts that teachers can ask chatbots, like ChatGPT, to customize their lesson plans with culturally relevant examples or in other words to suggest math problems based on real-world scenarios from a specific region or culture to meet their students' needs. Regarding the view of Supena (2021), claiming that classes should be learning communities, one of the advantages that using culturally relevant examples can bring to the class is that it fosters a classroom environment that values diversity over standardization and provides the opportunity for the teachers to celebrate cultural differences in math practices.

In the previous chapter, many examples of cultural and contextual relevance were provided, including cultural values, currency differences, and geographical content. Teachers can ask chatbots to apply math in meaningful ways, like suggesting a math problem involving currency conversion for students to learn about international trade. This can foster both engagement and critical thinking among multicultural students. Walkington (2024) asserts that students themselves can write their personalized problems based on their interests with the help of GPT-4. It believes that this can help students feel more valued and empowered.

In the previous chapter, we observed a class run by Mrs. Young (Yoon, 2008) where the use of cultural examples led to student satisfaction. Walkington (2024) also quotes a teacher saying that "if it's (math problem) talking about a place, thing, or situation that they're actually familiar with, that they've actually had hands-on experience with, or have seen with their own eyes then, of course, it's gonna be a little bit easier for them to comprehend the problem" (p. 5). Chatbots can provide real-world math problems rooted in different cultural contexts or reflect local traditions to make lessons more relatable for students from diverse backgrounds and let them learn about other cultures through math problems, as well. Integration of chatbots with interactive software can allow students from different cultures to share some information about their culture with their classmates in any language and gain more sense of belonging to the class (Walkington, 2024).

Additionally, it was mentioned in the previous chapter that gestures are a powerful communicative tool in teaching mainly to clarify concepts. However, they might mean differently across cultures which can lead to misunderstandings or might not be understood well. Incorporating virtual representations can substitute potentially problematic gestures. AI tools have the potential to help teachers customize visuals as well as explanations to accompany math tasks for diverse audiences while ensuring culturally sensitive content (Walkington, 2024). The author believes that explanations generated by GPT-4 are clearer and more detailed compared to those offered by teachers.

4.3 AIED and Objectives of 21st-Century

Sajja (2024) admits the effectiveness of traditional teaching methods in some cases, though criticizes it for not providing personalized support, instant feedback, critical thinking, and analytical skills. It emphasizes that students need support to apply creativity and critical analysis. This has resulted from using AI, with the potential to revolutionize education in this term. According to Furqon (2023), technology development, the availability of information and the consequent interactions among the participants resulting from technology tools have changed the acquisition of knowledge for the better because “teachers and students can access knowledge as a whole due to the large availability of information and various types of interactions that can be built between students, teachers, and with learning materials” (p. 1082).

Howard (2015) claims that access to educational tools has provided the opportunity for students to shift from the role of passive receivers of information to active creators of knowledge which can help to make the vision of a student-centered classroom a reality. Ramaila (2022) declares that “the use of digital technologies drives innovation and fosters creativity” and makes “learning more contextualized, social, reflective and active” (p. 9,10). It also highlights the role of the teacher in selecting appropriate tools and their “beliefs, knowledge and goals” (p. 10). It claims that it is teachers’ beliefs that affect prioritizing the goals of learning and how to use tools in the process of learning.

Zawacki-Richter (2019) wraps up the role of AI in education as what can support collaborative learning by “supporting adaptive group formation based on learner models, by facilitating online group interaction or by summarizing discussions that can be used by a human tutor to guide students towards the aims and objectives of a course” (p. 4). Through collaboration and communication, students learn to hear different ideas and work together to choose the best solutions.

Zawacki-Richter (2019) adds that the just-in-time feedback and assessment that AI provides, can be “an ongoing analysis of student achievement” instead of a traditional form of “stop-and-test” (p. 4). Holmes (2019) adds that the feedback that these tools provide does not only “lead him(student) to the right answer, clearly do more than test him; they also teach him" (p. 23). As was mentioned in the chapter "The 21st-Century Math Approach", students used to learn to solve math problems in one certain way to come up with the correct answer. However, with the help of AIED, students can think of different ways of solving math problems to get many possible correct answers. Real-time feedback not only correct students' mistakes but also helps them reflect on their way of thinking. Holmes (2019) also claims that AI-driven tools can make sure that "no learning opportunities were missed" in the class (p. 23).

4.4 How Teachers Can Benefit from Technology and AI Tools (Enhancing Teachers' Abilities to Teach)

"AI technologies have been applied not only to support teaching but also to support the professional development of teachers" (Chiu, 2023, p. 9). Apart from AI tools introduced in the previous part, the following explanations elaborate on how technology can benefit, particularly teachers, to maximize teaching in a non-native environment and facilitate effective communication with additional language learners in math class.

As mentioned before, Baker (2019) introduces one perspective of educational AI tools as "teacher-facing systems" that are designed to reduce teachers' workload by providing automated assessment, feedback, and plagiarism detection (Zawacki-Richter, 2019). Therefore, AIED can offer opportunities for teachers to save time through automating tasks, such as automated assessment, feedback, and plagiarism detection, and demonstrate students' progress. Therefore, Tapalova (2022) believes that the role of educators changes to "coordinator" or even "mentor" instead of being responsible for making decisions for different aspects of the learning process under the traditional approach. Roll (2016) also describes this shift as "from the sage on the stage, teachers become the guide on the side" (p. 592). This means that teachers are not considered as know-it-all but the ones who are "tasked with supporting their learners in seeking, finding, and integrating information, and becoming independent collaborative thinkers" (p. 592)

"AI can automate routine tasks such as grading and lesson planning, freeing up teachers' time to focus on more meaningful interactions with students" (Onesi-Ozigagun et al., 2024, p. 591). Feedback provision and automated grading tools can relieve the pressure on teachers to spend more time providing additional support to additional language learners. In addition, the report received from ITS or analytical platforms can offer an understanding of students' academic growth for teachers and administrators to provide guidance where necessary (Zawacki-Richter, 2019) and track students' progress to make informal decisions to improve teaching strategies. Educational technology tools have proven what was mentioned earlier by Skovsmose (2020) about the intertwined nature of learning and teaching as they provide tools that enhance teaching effectiveness while simultaneously enriching the learning experience for students.

AI technologies also provide teachers with some suggestions based on analysis of real data in class to reflect on their teaching practices. According to Onesi-Ozigagun et al. (2024), "AI-powered analytics tools can analyze vast amounts of data to identify patterns and trends, allowing teachers to make informed decisions about their instructional strategies" (p.591). Chiu (2023) believes that these comments sound less offensive to teachers. Major (2021) adds that these suggestions can reduce educational gaps "through adaptive remedial instruction and performing routine tasks to free up teachers to spend more time on aspects of education where they have comparative advantages over technology" (p. 1939). It concludes that these potential benefits can align with the 4th goal of the UN's Sustainable Development to ensure inclusive and equitable quality education for all.

Chiu (2023) also introduces some classroom management applications, such as tools for "assigning, and distributing learning materials and assignments" or the ones suggesting "adaptive teaching content and teaching methods" to design a course with the potential to improve teacher's performance in class (p. 9).

Additionally, collaboration tools can significantly provide a platform for teachers for communication, resource sharing, peer learning, and exchanging ideas. Platforms, like Google Docs, can provide the chance for teachers to work together on the same lesson plan or presentation in real-time. This can make co-teaching and planning easier for those who feel shy or less extroverted to collaborate with their colleagues.

Furthermore, it is not surprising that when home and schools come together as a team, students normally show better attendance and behavior and get better grades. According to Paxton (2011), parental engagement has a key role in effective education for students with refugee backgrounds. However, according to the Victorian Refugee Status Report, there are multiple barriers to refugee parents participating in their children's education, one of which is language. Communication platforms foster refugee parents' involvement by providing the chance for them to use translation tools.

4.5 Actionable Insight

According to Holmes (2019), artificial intelligence has quietly been employed in the classroom whether the educational community welcomes it or not. Holmes (2019) believes that it has been many years that many companies, like Amazon, Google, and Facebook, have invested millions of dollars in this field for the benefits they expect it to bring to the classroom. However, despite its many promises, the integration of AI in education comes with plenty of challenges that must be addressed to fully realize its benefits. These challenges can be categorized as follows:

One of the biggest challenges in the process of integration of AI in education is teachers' professional development, such as lack of training, resistance to change, and ongoing support. Chiu (2023) and Ramaila (2022) believe that many educational professionals "lack an understanding of how the technologies operate" and many have reported that they don't feel like using AIED to support their classroom as they believe using AIED reduces their ability to manage classroom activities effectively (Chiu, 2023, p. 9). Zhao (2003) introduces the unreliability of technology and its possibility to break down at any time as a concern to many teachers. It believes that this can be a reason why they prefer not to use it in class. According to Howard (2015) and Chiu (2023), some teachers feel less confident while facing technological errors.

The unfamiliarity with using technologies and their value can confuse teachers and make them feel like they are forced to use AI in class and this can lead to their resistance to using them (Zhao, 2003). Furthermore, Zhao (2003) and Mayes (2015) claim that AI can bring so many risks in higher education including the fear of teacher replacement by automated intelligent tutors for budget cuts.

Additionally, those with less 'openness to change' are less likely to apply AIED in the classroom as they doubt the role of AI in enhancing student learning (Howard, 2015). Mayes (2015) claims that integration of technology in education can change their role from deliverers of content to facilitators and mentors which need training and professional development efforts.

Shin (2017) complains that many teachers have not been trained to use AI and need to receive instructions through video tutorials to see how to implement them in learning. However, it admits the insufficiency of video tutorials in using AIED, like virtual manipulatives, accurately. Therefore, an educational technologist should support the new interaction between teachers and students that technology can bring to the class, ETI (Educational Technology Integration) competency preparation, staff collaboration efforts, and time management (Mayes, 2015). Additionally, educational technologists can get educators to adopt the new learning management system and instructional designers can conduct needs assessments to find the requirements for a stronger policy for encouraging the implication of technology.

The second challenge is the ethical and privacy concerns, such as data security, bias in AI algorithms, and student autonomy. Zhao (2003) and Mayes (2015) are concerned about large amount of confidential information that AI requires about students and faculties and believe that it can "raise serious issues of privacy and data protection." Zawacki-Richter (2019) also agrees with the importance of the concept of privacy in the development of personalized learning. It claims that information like students' learning style and capabilities is private and many don't want others know anything about it. Mayes (2015) warns about many types of cyber-attacks, such as denial of service (DoS) attacks, "Click jacking", "Spear phishing", "Cross-site scripting" (targeted information gathering) as a big threat to the integrity of databases. Therefore, sensitive personal information should be stored securely and accessed only by authorized personnel so as not to put students at risk of data misuse or identity exposure. In addition, Mayes (2015) suggests that children should be watched while using the internet, their access to harmful pages should be restricted and they should be taught about the culture of proper use of the internet to find the information they need. For this purpose, schools need to put policies and everyone should be obliged to follow them.

The third challenge is student support, like emotional support and technical guidance and training. Chiu (2023) claims that many students feel anxious and less confident while using AI in learning. Shin (2017) believes that students' abilities to use technologies like virtual manipulatives should be evaluated to maximize the benefits of technology and connect it to mathematics. During this process, "students should be encouraged to verbalize their mathematical thinking and justify their representations of problem situations.

The fourth challenge is technological infrastructure, such as access to resources and the digital divide. While technology and AI aim to provide equity in education, Ramaila (2022) claims that digital tools are unavailable in many schools due to "poor infrastructure or lack of finances". Johnson (2016) also believes that effective technology integration requires broad accessibility of educational technologies. Otherwise, it makes it hard for the educational community to provide ETI.

The other challenge is school culture and adaptation to AI. Howard (2015) introduces school culture as a key factor that can affect teachers' decisions to integrate technology in class. It continues that schools should promote risk-taking by encouraging a culture of change. By culture of change, Howard means that "teachers can work towards change and feel they are able to experiment with new tools, approaches, and teaching strategies without punishment or fears of negative impact on students' learning."

For this purpose, Howard suggests that "school leadership should identify and limit perceived threats to teaching and learning where possible. They can do this by involving teachers in the decision-making processes about technology-related change and the school's vision for integration, discussing how it is expected to be used in the classroom, and provide appropriate technological and pedagogical support." Howard continues that teachers need to have an opportunity to learn about tools and collaborate with their colleagues which can reduce the fear of failure and punishment develop "a shared vision of integration" of AIED (p. 12).

The last challenge is the high cost and sustainability of AI systems. According to Zawacki-Richter (2019), one of the concerns of developing AI-driven tools in education is the costs and time involved. This is what many government-funded schools cannot afford. Major (2021) also doubts that this big change in education such as using personalized technology can be cost-effective. If not, this also can contradict equity in education.

4.6 Bridging Theory and Practice

Farsani defines technology as "anything that eases your job." He claims that it shouldn't necessarily cost a fortune. He adds that "being innovative can also be a technology. For example, teachers use gestures to facilitate learning. This can be considered as technology." About the role of technology, he asserts that "technology is not designed to add variety but it does and anything that helps to have less boring class can be beneficial."

Shumway claims that "school teachers are more up to using technology than universities are keeping up with." As a mum of two kids in 4^{th} and 6^{th} grade, she has observed that technology has been used for memorization instead of conceptual understanding. She claims that it is not better but at least they are using technology and getting familiar with it.

L.C. admits that schools are more eager to use technology than before. She says that in Sweden, students have access to laptops and iPads, which are kept at school. The teacher receives a pack of iPads from the authorities for a specific duration of time. For example, sometimes students are asked to take pics of geometrical representations/ forms in the class. Then, they are paired to collaborate on their findings. In addition, multiple-choice tests are given using laptops. Their responses are corrected using three colors, green, red, and yellow (representing correct, wrong, and partially correct). "Students are given real-time feedback. Common mistakes are found based on the highest number of red and yellow responses. Students are sometimes asked which mistakes they would like to start with. They normally go for yellow first as they feel less stressed." She believes that technology lets students feel better by knowing that some sections are difficult for many. In addition, as discussed earlier, technology has not only helped them provide real-time feedback but also has allowed them to discuss it in groups for deeper reflection.

Shumway recalls her experience of being in Brazil as a language learner in an academic environment, relying a lot on translation tools. When she was at a lecture, translation tools helped her to follow along with the speech and the available visual dynamics helped her know what the content was about. That's why she calls technologies and AI tools "very critical pieces." She claims that "AI and translation tools can make teachers sure that all group members are involved in collaborations and discussions." She adds that through AI and technology, there are so many different things that students can do together to support each other while working on projects.

Shumway has sent her two kids to language immersion schools where students are taught in two languages. Typically, one language is their native and the other is a second language. This could be like learning math and science in one language and other subjects in a second language. The goal of this program is to develop bilingualism and cultural understanding among students. Shumway says that when she sent her kids to language immersion school, "the teacher said that the first month of school would be very tiring for the students. They will have more frustration and tears in the first month but we know how to support them to learn to communicate in the second language." Shumway claims that this was so heartwarming to her as a mum and helped her to trust the teachers. She highlights the role of teacher-parent meeting tools that can update both teachers and parents about what is going on in the class and the challenges they have. In Shumway's case, both teacher and parent speak the same language. As discussed earlier, undoubtedly if they didn't know a similar language, a teacher-parent meeting tool, using translation tools, would have been essential.

Shumway has just started observing classes at language immersion schools, noticing that teachers are more focused on visuals, vocabulary development, and using gestures to let students know what is going on even if they don't have much language knowledge. She recalls the days she worked as an EFL teacher working with language learners and claims that things are different from when she was a teacher. "Language immersion schools tend to use more PowerPoint slides, visual dynamics, and the use of video and she has noticed how the teachers are using technology and videos, music and gestures, and the combination of multimedia resources to help students understand the content and also to learn basic language skills through repetition." Moyer-Packenham (2013) also admits that virtual manipulatives can be used to support a variety of teaching and learning styles.

All three interviewees agree that using AI and technology in education is not without challenges. Shumway believes that "teachers are more focused on specific technologies that they are being encouraged to use at school. But they need to figure out how to evaluate technology whether as a tool to encourage collaboration or using technology in traditional math teaching." According to Nietschke (2023, p. 8), "the application of digital technology varies by community and socioeconomic level, by teacher willingness and preparedness, by education level, and by country income." Therefore, as discussed earlier, teachers' preparedness is a concern. Shumway adds that teachers are not equipped with those infrastructural approaches but we need to think big as things change rapidly.

L.C. asserts that it takes both teachers and students time to get used to using technology in class. She continues that the class sometimes gets noisy during the process of log-in (while using iPads). The teachers are called many times to tackle the Internet problem, so time management becomes an issue. Therefore, support for technical issues is essential. In addition, while using iPads is mandatory in her school, there are so many teachers who prefer not to.

In terms of challenges in using technology in education, Farsani mentions two main points. He highlights the importance of incorporating culture in the use of technology. He claims that it does not matter how many visuals and gestures are provided to the students when they are not paying attention or they don't know the purpose of that. He adds that sometimes new technology can limit old technologies or kill students' creativity. Therefore, more research should be done on using technology in education settings.

Shumway agrees with Farsani saying that there isn't much we know about using technology in education. She adds that there is a strict policy in the US regarding Internet use and technology for the same reason. This has held them back from using technology, especially in classroom settings. This is exactly what Nietschke (2023, p.11) says: "Research on technology in education is as complex as technology itself." Therefore, "while there is much general research on education technology, the amount of research into specific applications and contexts is insufficient, making it difficult to prove that a particular technology enhances a particular kind of learning."

The perspectives of all highlight both the potential and challenges of technology and AI and call for further research into how technology can best be integrated into educational settings. All interviewees agree that teachers need to be equipped with technology tools to facilitate learning in multilingual and multicultural settings and provide inclusive learning environments for all. However, it is noted that teachers should learn strategies to evaluate their effectiveness in class and meet challenges like time management and technical problems.

4.7 Last Word

Technology and artificial intelligence are not infallible. Therefore, it is essential to consider their potential negative aspects alongside their benefits. AI tools can sometimes misinterpret complex problems, especially when they involve ambiguous phrasing. Walkington (2024) claims that "32% of the hints generated by GPT were initially disqualified for inaccuracy" (p. 5). It also introduces GPT-4 as a safe tool for middle school students with only a 4% error rate as it has faced difficulties solving advanced problems at graduate levels. AI tools also often use standardized formats and symbols, like '÷' vs ':' which might sound challenging to students from diverse educational systems. In addition, some AI tools focus on providing quick answers rather than step-by-step explanations or might favor certain problem-solving methods and "provide biased results and misleading information" (Resnick, 2024, p. 7). Resnick (2024) also criticizes AI tools for not being able to "cultivate a caring community the way a human teacher can" to let the students feel understood and supported (p. 6). Furthermore, relying much on AI for answers can interfere with developing critical thinking skills and potentially undermine students' understanding of math reasoning. Walkington (2024) also believes that much interaction with chatbots can lower students' confidence in problem-solving as they get too reliant on chatbots' assistance.

Therefore, "while AI can assist in delivering content and assessing learning, it is crucial to balance its use with activities that promote creativity and critical thinking" (Onesi-Ozigagun et al., 2024, p. 599). Accordingly, technology alone cannot lead to a development in learning and human resources are necessary to push the borders as there are some skills that AI might not excel by itself, "such as creative and social intelligence, reasoning skills, critical thinking and interpersonal skills" (Liu, 2023, p. 45).

"Teachers will therefore remain at the frontline of education; it is misinformed to say that AI can replace teachers" (Pedro, 2019, p. 29). "It is the change in processes of teaching using digital technologies that results in gains in learning" (Howard, 2015, p. 2). Teachers should remain central to education for several important reasons. Firstly, Shin (2017) and Ramaila (2022) believe that it is teachers' responsibility to adapt and implement a proper app or technology for each task, suggest using a rubric for that and "decide how and when it would be appropriate to use AI-enabled tools" (Pedro, 2019, 29). According to Chassignol (2018), each student has his learning curve. Therefore, a good teacher is essential to deliver information in a way that suits each student and consider his behavioral or psychological problems if he has any. For this purpose, Howard suggests that teachers should take risks in designing different tasks using technology tools to explore which tool can support learning the most and increase interaction in class. Secondly, Walkington (2024) claims that if students are guided well by their teachers, wrong answers provided by AI tools can be an opportunity for students to improve their learning. Lastly, Chassignol (2018) adds that AI is not equipped to handle overly involved or under-involved parents. Therefore, there should be an instructor to connect parents with the class.

Howard (2015) defines "Culture of Change" as "teachers can work towards change and feel they are able to experiment with new tools, approaches and teaching strategies without punishment or fears of negative impact on students' learning" (p. 12). It suggests that schools should involve teachers "in the decision making processes about technology-related change and the school's vision for integration, discuss how it is expected to be used in the classroom and provide appropriate technological and pedagogical support" to minimize unexpected challenges to teaching and learning (p. 12). With this aim, teachers need to spend time learning about new tools and how to plan and develop curriculum using these tools.

According to Onesi-Ozigagun et al. (2024), while "AI has the potential to enhance learning experiences, it is not a replacement for teachers. Teachers play a crucial role in guiding and supporting students" (p. 597). They need to monitor and address any mistakes or misleading outputs generated by AI tools. They should also encourage students to question AI output instead of accepting them blindly. Students could be asked to explain why a particular AI-suggested solution is correct or incorrect in groups to practice reasoning, critical thinking, and collaborative learning. It is teachers who need to add a personal touch by addressing learning habits. What matters in this process of change are school culture, confidence in using technology, and beliefs about technology in teaching. Not surprisingly, teachers feel less confident without support from school and this might result in having problems, like not understanding or panicking while receiving errors of tools, and also not having the "openness to change" (p. 12). AI and technology, when used thoughtfully, can act as a second teacher alongside human educators. According to Pedro (2019) AI is reshaping teaching methodologies by making "a dual-teacher model entailing a teacher and a virtual teaching assistant, which can take over the teacher's routine task, frees up teachers' time, enabling them to focus on student guidance and one-to-one communication" (p. 13).

5. Conclusion

This research was undertaken with the aim of drawing the attention of people miles away to the growing inequalities of the world that are not a priority yet, which are the challenges faced by additional language learners in 21st-century math classes. It also focused on the opportunities technology and AI offer both students and teachers to bridge linguistic, cultural, and pedagogical gaps.

This study has organized its objectives into three chapters. Chapter 2 answered the first research question, which is to define 21st-century math class and the skills students need to develop within it. It explored the inefficacy of traditional rote-based learning with a focus on coming up with the correct answer and defined an evolution to 21st-century skills in math education as a response to the demands of this era. With the emergence of Gen Z, the new audience of math education that is interested in collaboration using technology, and the development of technology in the 21st century, education has undergone a fundamental shift to learning 4C skills, including critical thinking and problem-solving, creativity, collaboration, and communication, that prepares students for their future. The main objective of learning these skills in 21st-century math classes is to teach the students how to solve problems as problems are inseparable parts of life. Therefore, 4C skills provide them the chance to understand, analyze, and evaluate math problems well, apply creativity to create new ideas and reflect on their findings using collaboration and communication. The application of 4C skills requires a lot of interaction among students to allow them to construct their knowledge in pairs and groups and discuss the credibility of their reasoning in real-world math tasks. Many group activities, like Think-Pair-Share, are suggested to provide this chance for students of different math proficiency and different academic capabilities to work together, help one another, and learn from each other.

Chapter 3 discussed the linguistic structure of mathematics and explores the interconnectedness of language, culture, and math education to answer the second research question, which is to explain the challenges additional language learners and their teachers encounter in achieving 21st-century math skills due to language barriers and cultural differences. Considering the objectives of math classes in the 21st century, one of the main requirements is language proficiency. Understanding math requires language proficiency and linguistic barriers can affect logical reasoning and problem comprehension. According to the case studies addressed in the research, a high percentage of students revealed language-related issues, like lack of comprehension and an inability to provide self-perception of their understanding. There are moments when the students get to the right answer but are unable to explain their reasoning. It is also important to note that linguistic demands go beyond learning vocabulary and need a high ability to construct and interpret sentences.

This chapter also highlighted how different cultures can approach math differently and how values and norms influence how students perceive mathematical concepts, engage in activities, and approach problem-solving. Those female students from a masculine society are less likely to participate in group activities as they have never had their voice in the class and those from societies with high individual collectivism are less into collaboration. Many examples were also brought about how culture can affect the way students solve math tasks. Cultural differences do impact the way students communicate and interact with one another and their learning styles and eliminating deeply rooted cultural habits in students requires significant effort.

Chapter 4 answered the third research question, which is exploring how AI and technology can benefit additional language learners as well as their teachers in overcoming the challenges in multilingual and multicultural settings. This chapter includes the key findings of the research achieved through a review of existing literature, interviews with experienced professors, and an analysis of available technology tools. The technologies that were introduced in this chapter go beyond providing translation. Instead, they create an environment that considers linguistic barriers and cultural diversities where all students are valued.

Personalized learning offered through adaptive learning systems and intelligent tutoring systems can remove the gaps in understanding due to differences in prior knowledge among learners of different countries and adjusts the difficulty of math tasks based on the performance of each student. Real-time assessment and feedback tools can provide various forms of tests and immediate feedback to evaluate students' learning progress. Culturally responsive systems can be used to design culturally relevant real-world math tasks that can engage all students in meaningful problem-solving. Interactive math learning systems can also encourage students to actively participate in class activities and discussions, using visual tools, virtual manipulatives, and chatbots. Integration of translation tools with these AIEDs can remove language barriers for all. Automated tools introduced above can also benefit teachers by freeing up their time and allowing them to come together as a team and exchange ideas.

However, the integration of technology and AI will not be without its challenges. The findings of this research revealed many challenges, including teacher unpreparedness or unwillingness to integrate AI and technology in their classes, inequity in access to resources, and ethical considerations, such as privacy and bias. Overcoming these challenges requires systematic support and collaboration among educators, policymakers, and technologists.

AI and technology tools in this research are believed to have the potential to help instructors as thinking partners in designing a curriculum that fulfills the objectives of 21st-century math classes and meets the needs of math learners in multilingual and multicultural settings. The next steps in this research will involve collaborating with educational institutions to pilot these technologies in real-world classrooms, collecting feedback from both teachers and students to optimize the findings above, and seeing whether the findings of this study are universally applicable.

References

ABEDI, J.; LORD, C. The language factor in mathematics tests.

Applied Measurement in Education, v. 14, n. 3, p. 219-234, 2001.

ABDULWAHED, M.; JAWORSKI, B.; CRAWFORD, A. Innovative approaches to teaching mathematics in higher education: a review and critique. 2012.

ADONIOU, M.; YI, Q. Language, mathematics and English language learners. Australian Mathematics Teacher, The, v. 70, n. 3, p. 3-13, 2014.

AIZIKOVITSH-UDI, E.; CHENG, D. Developing critical thinking skills from dispositions to abilities: mathematics education from early childhood to high school. Creative Education, v. 6, n. 4, p. 455-462, 2015.

ALRØ, H.; SKOVSMOSE, O. On the right track. For the Learning of Mathematics, v. 16, n. 1, p. 2-22, 1996.

An Encyclopedia Britannica Company. Technology. Merriam-Webster. Available at: http://www.merriam-webster.com/dictionary/technology. Accessed on: 2 October 2024.

ANDREWS, P. The importance of acknowledging the cultural dimension in mathematics teaching and learning research. Acta Didactica Napocensia, v. 3, n. 2, p. 3-16, 2010.

ARDELEANU, R. Traditional and modern teaching methods in mathematics. Journal of Innovation in Psychology, Education and Didactics, v. 23, n. 2, p. 133-140, 2019.

BAKER, T.; SMITH, L.; ANISSA, N. Educ-AI-tion rebooted? Exploring the future of artificial intelligence in schools and colleges. Nesta Foundation, 2019. Available at: https://media.nesta.org.uk/documents/Future_of_AI_and_education_v5_WEB.pdf.

BARTON, B. The language of mathematics: telling mathematical tales. v. 44. New York: Springer Science & Business Media, 2007.

BARRON, B.; DARLING-HAMMOND, L. Teaching for Meaningful Learning: A Review of

Research on Inquiry-Based and Cooperative Learning. Book Excerpt. George Lucas Educational Foundation, 2008.

BISHOP, A. J. (Ed.). International handbook of mathematics education. v. 4, n. 1. Springer Science & Business Media, 1996.

BLACK, P. Inside the black box: Raising standards through classroom assessment. School of Education, King’s College, 1998.

BLACK, P.; WILIAM, D. Assessment and classroom learning. Assessment in Education: Principles, Policy & Practice, v. 5, n. 1, p. 7-74, 1998.

BLACK, P.; WILIAM, D. Developing the theory of formative assessment. Educational Assessment, Evaluation and Accountability (formerly: Journal of Personnel Evaluation in Education), v. 21, p. 5-31, 2009.

BOALER, J. Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching. San Francisco, CA: Jossey-Bass, 2016.

BOALER, J.; STAPLES, M. Creating mathematical futures through an equitable teaching approach: The case of Railside School. Teachers College Record, v. 110, n. 3, p. 608-645, 2008.

BOALER, J.; WILLIAMS, C.; CONFER, A. Fluency without fear: Research evidence on the best ways to learn math facts. Reflections, v. 40, n. 2, p. 7-12, 2015.

BOGDAN, R.; BIKLEN, S. K. Qualitative research for education. 368. ed. Boston: Allyn & Bacon, 1997.

BUSTAMANTE, L. H.; CERQUEIRA, R. O.; LECLERC, E.; BRIETZKE, E. Stress, trauma, and posttraumatic stress disorder in migrants: A comprehensive review. Revista Brasileira de Psiquiatria, v. 40, n. 2, p. 220-225, 2017.

CHAPMAN, O. Mathematics teachers’ knowledge for teaching problem solving. LUMAT: International Journal on Math, Science and Technology Education, v. 3, n. 1, p. 19–36, 2015. Disponível em: https://doi.org/10.31129/lumat.v3i1.1049.

CHASSIGNOL, M.; KHOROSHAVIN, A.; KLIMOVA, A.; BILYATDINOVA, A. Artificial Intelligence trends in education: a narrative overview. Procedia Computer Science, v. 136, p. 16-24, 2018.

CHEN, C. AI will transform teaching and learning. Let's get it right. Stanford University AI+ Education Summit, 7 June 2023. Available at: https://acceleratelearning.stanford.edu/story/ai-will-transform-teaching-and-learning-lets-get-it-right/.

CHIU, T. K.; XIA, Q.; ZHOU, X.; CHAI, C. S.; CHENG, M. Systematic literature review on opportunities, challenges, and future research recommendations of artificial intelligence in education. Computers and Education: Artificial Intelligence, v. 4, p. 100118, 2023.

COLLIER, V. P. Age and rate of acquisition of second language for academic purposes. TESOL Quarterly, v. 21, n. 4, p. 617-641, 1987.

COOKE, B. D.; BUCHHOLZ, D. Mathematical communication in the classroom: A teacher makes a difference. Early Childhood Education Journal, v. 32, p. 365-369, 2005.

CRESWELL, J. W.; POTH, C. N. Qualitative inquiry and research design: choosing among five approaches. Thousand Oaks: Sage, 2016.

CUMMINS, J. Language, power, and pedagogy: Bilingual children in the crossfire. v. 23. Multilingual Matters, 2000.

DE ARAUJO, Z.; ROBERTS, S. A.; WILLEY, C.; ZAHNER, W. English learners in K–12 mathematics education: A review of the literature. Review of Educational Research, v. 88, n. 6, p. 879-919, 2018.

ELLERTON, N. F.; CLARKSON, P. C. Language factors in mathematics teaching and learning. In: International Handbook of Mathematics Education: Part 1, p. 987-1033, 1996.

ELLIS, M. W.; BERRY III, R. Q. The paradigm shift in mathematics education: Explanations and implications of reforming conceptions of teaching and learning. The Mathematics Educator, v. 15, n. 1, 2005.

ENGELBRECHT, J.; LLINARES, S.; BORBA, M. C. Transformation of the mathematics classroom with the internet. ZDM, v. 52, n. 5, p. 825-841, 2020.

ENNIS, R. H. Critical thinking assessment. Theory into Practice, v. 32, n. 3, p. 179-186, 1993.

ERDOGAN, N.; BOZEMAN, T. D. Models of project-based learning for the 21st century. In: A practice-based model of STEM teaching. Brill, 2015. p. 31-42.

FARSANI, D. Deictic gestures as amplifiers in conveying aspects of mathematics register. In: CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education. February 2015, p. 1382-1386.

FREEMAN, B. Using digital technologies to redress inequities for English language learners in the English-speaking mathematics classroom. Computers & Education, v. 59, n. 1, p. 50-62, 2012.

FINDELL, B.; SWAFFORD, J.; KILPATRICK, J. (Eds.). Adding it up: Helping children learn mathematics. National Academies Press, 2001.

FU, S.; GU, H.; YANG, B. The affordances of AI-enabled automatic scoring applications on learners' continuous learning intention: An empirical study in China. British Journal of Educational Technology, v. 51, n. 5, p. 1674-1692, 2020.

FURQON, M.; SINAGA, P.; LILIASARI, L.; RIZA, L. S. The Impact of Learning Management System (LMS) Usage on Students. TEM Journal, v. 12, n. 2, 2023.

GÓMEZ-REY, P.; BARBERA, E.; FERNÁNDEZ-NAVARRO, F. The impact of cultural dimensions on online learning. Journal of Educational Technology & Society, v. 19, n. 4, p. 225-238, 2016.

GRÉGOIRE, J. Understanding creativity in mathematics for improving mathematical education. Journal of Cognitive Education and Psychology, v. 15, n. 1, p. 24-36, 2016.

HALLIDAY, M. A. K. Language as social semiotic. London, England: Edward Arnold, 1978. p. 195.

HATTIE, J.; TIMPERLEY, H. The power of feedback. Review of Educational Research, v. 77, n. 1, p. 81-112, 2007.

HATTIE, J. Visible learning: A synthesis of over 800 meta-analyses relating to achievement. Routledge, 2008.

HEINZE, A.; REISS, K. Mistake-handling activities in the mathematics classroom: Effects of an in-service teacher training on students' performance in geometry. In: Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education. Seoul: PME, 2007. v. 3, p. 9-16.

HENRIKSEN, D.; MISHRA, P.; FISSER, P. Infusing creativity and technology in 21st-century education: A systemic view for change. Journal of Educational Technology & Society, v. 19, n. 3, p. 27-37, 2016.

HIEBERT, J.; CARPENTER, T. Learning and teaching. In: Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics, p. 65, 2006.

HOFSTEDE, G. Cultural differences in teaching and learning. International Journal of Intercultural Relations, v. 10, n. 3, p. 301-320, 1986.

Holmes, W.; Bialik, M.; Fadel, C. Artificial intelligence in education promises and implications for teaching and learning. Center for Curriculum Redesign, 2019.

HOWARD, S. K.; MOZEJKO, A. Teachers: technology, change and resistance. In: Teaching and digital technologies: Big issues and critical questions, v. 2, n. 1, p. 307-317, 2015.

I, Ji-Yeong; MARTINEZ, R. Teaching math for emergent bilinguals: Building on culture, language, and identity. Iowa State University Digital Press, 2020. p. 12-15.

JANUSZEWSKI, A.; PERSICHITTE, K. A. A history of the AECT's definitions of educational technology. In: Educational Technology. Routledge, 2013. p. 259-282.

JONES, P. L. Learning mathematics in a second language: A problem with more and less. Educational Studies in Mathematics, v. 13, n. 3, p. 269-287, 1982.

JOURDAIN, L.; SHARMA, S. V. Language challenges in mathematics education for English language learners: A literature review. Waikato Journal of Education, v. 21, n. 2, 2016.

JOY, S.; KOLB, D. A. Are there cultural differences in learning style? International Journal of Intercultural Relations, v. 33, n. 1, p. 69-85, 2009.

JURS, P.; ŠPEHTE, E. The value and topicality of feedback in improving the learning process. Society. Integration. Education. Proceedings of the International Scientific Conference, v. 3, p. 244, 20 May 2020. Available at: https://doi.org/10.17770/sie2020vol3.5014. Accessed on: 9 April 2023.

KILPATRICK, J. The mathematics teacher and curriculum change, 2009.

KITA, S. Cross-cultural variation of speech-accompanying gesture: A review. Language and Cognitive Processes, v. 24, n. 2, p. 145-167, 2009.

KIRMAYER, L. J.; NARASIAH, L.; MUNOZ, M.; RASHID, M.; RYDER, A. G.; GUZDER, J.; … POTTIE, K. Common mental health problems in immigrants and refugees: General approach in primary care. CMAJ, v. 183, n. 12, p. E959-67, 2011. DOI: 10.1503/cmaj.090292.

Klotz, M. B. Culturally competent schools: Guidelines for secondary school principals. Principal Leadership, 2006. Available at: http://www.nasponline.org/resources/principals/Culturally%20Competent%20Schools%20NASSP.pdf.

KOKOTSAKI, D.; MENZIES, V.; WIGGINS, A. Project-based learning: A review of the literature. Improving Schools, v. 19, n. 3, p. 267-277, 2016.

KRIEGLER, S. Just what is algebraic thinking. Retrieved September 10, 2008.

KU, K. Y. Assessing students’ critical thinking performance: Urging for measurements using multi-response format. Thinking Skills and Creativity, v. 4, n. 1, p. 70-76, 2009.

LAI, E. R. Critical thinking: A literature review. Pearson's Research Reports, v. 6, n. 1, p. 40-

41, 2011.

LEUNG, F. K. In search of an East Asian identity in mathematics education. Educational Studies in Mathematics, v. 47, p. 35-51, 2001.

Liu, B. L., Morales, D., Roser-Chinchilla, J., Sabzalieva, E., Valentini, A., Vieira do Nascimento, D., & Yerovi, C. (2023). Harnessing the era of artificial intelligence in higher education: a primer for higher education stakeholders.

LOPEZ, N. R. Mathematical notation comparisons between US and Latin American countries, 2008.

Major, L.; Francis, G. A.; Tsapali, M. The effectiveness of technology-supported personalised learning in low-and middle-income countries: A meta-analysis. British Journal of Educational Technology, v. 52, n. 5, p. 1935-1964, 2021.

MCFADZIEN, N. Why is effective feedback so critical in teaching and learning? 2015.

MCLEOD, D. B.; STAKE, R. E.; SCHAPPELLE, B. P.; MELLISSINOS, M.; GIERL, M. J. Setting the Standards: NCTM's role in the reform of mathematics education. Bold Ventures: Case Studies of US Innovations in Mathematics Education, p. 13-132, 1996.

MCLUHAN, M. The medium is the massage. A Bentham Book, 1967.

MERGENDOLLER, J. R.; MARKHAM, T.; RAVITZ, J.; LARMER, J. Scaffolding project-based learning: Tools, tactics, and technology to facilitate instruction and management. Buck Institute for Education Novato, California, USA, 2006.

MOHR, K. A.; MOHR, E. S. Understanding Generation Z students to promote a contemporary learning environment. Journal on Empowering Teaching Excellence, v. 1, n. 1, p. 9, 2017.

MOSCHKOVICH, J. A situated and sociocultural perspective on bilingual mathematics learners. Mathematical Thinking and Learning, v. 4, n. 2-3, p. 189-212, 2002.

MOYER-PACKENHAM, P. S.; WESTENSKOW, A. Effects of virtual manipulatives on student achievement and mathematics learning. International Journal of Virtual and Personal Learning Environments, v. 4, n. 3, p. 35–50, 2013. doi:10.4018/jvple.2013070103.

MOYER-PACKENHAM, P. S.; BOLYARD, J. J. Revisiting the definition of a virtual manipulative. International Perspectives on Teaching and Learning Mathematics with Virtual Manipulatives, p. 3-23, 2016.

MULWA, E. C. Difficulties Encountered by Students in the Learning and Usage of Mathematical Terminology: A Critical Literature Review. Journal of Education and Practice, v. 6, n. 13, p. 27-37, 2015.

NADJAFIKHAH, M.; YAFTIAN, N.; BAKHSHALIZADEH, S. Mathematical creativity: Some definitions and characteristics. Procedia-Social and Behavioral Sciences, v. 31, p. 285–291, 2012.

NASIR, N. I. S.; HAND, V.; TAYLOR, E. V. Culture and mathematics in school: Boundaries between "cultural" and "domain" knowledge in the mathematics classroom and beyond. Review of Research in Education, v. 32, n. 1, p. 187-240, 2008.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS. Principles and Standards for School Mathematics. Reston, VA: NCTM, 2000. Disponível em: https://www.nctm.org/flipbooks/standards/agendaforaction/html5/index.html

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS. Statement of beliefs.

NCTM, 2024. Available at: https://www.nctm.org/About/At-a-Glance/Statement-of-Beliefs/. Accessed on: 15 June 2024.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (NCTM). An Agenda for Action: Recommendations for School Mathematics of the 1980s. Retrieved June 28, 2024, from https://www.nctm.org/Standards-and-Positions/More-NCTM-Standards/An-Agenda-for-Action-(1980s)/

NEVILLE-BARTON, P.; BARTON, B. The relationship between English language and mathematics learning for non-native speakers, 2005.

NIETSCHKE, Y.; DABROWSKI, A. Technology in education-a tool on whose terms?. 2023.

NOREEN, R.; RANA, A. M. K. Activity-Based Teaching versus Traditional Method of Teaching in Mathematics at Elementary Level. Bulletin of Education and Research, v. 41, n. 2, p. 145-159, 2019.

NÚÑEZ, R.; SWEETSER, E. With the future behind them: Convergent evidence from Aymara language and gesture in the crosslinguistic comparison of spatial construal of time. Cognitive Science, v. 30, p. 401-450, 2006.

OECD. PISA 2009 Results: What Students Know and Can Do: Student Performance in Reading, Mathematics, and Science (Vol. I). OECD Publishing, 2010. https://doi.org/10.1787/9789264083943-en

OECD. OECD Reviews of Migrant Education: Immigrant Students at School Easing the Journey Towards Integration. OECD Publishing, 2015.

Onesi-Ozigagun, O., Ololade, Y. J., Eyo-Udo, N. L., & Ogundipe, D. O. (2024). Revolutionizing education through AI: a comprehensive review of enhancing learning experiences. International Journal of Applied Research in Social Sciences, 6(4), 589-607.

ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT (OECD). PISA 2018 Results (Volume I): What Students Know and Can Do. Available at: https://www.oecd-ilibrary.org/docserver/5f07c754-en.pdf, 2019.

ORGANISATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT (OECD). PISA 2022 Assessment and Analytical Framework. OECD Publishing, 2023.

PAPERT, S. The children's machine. Technology Review-Manchester NH, v. 96, p. 28-28, 1993.

PAXTON, G.; SMITH, N.; WIN, A. K.; MULHOLLAND, N.; HOOD, S. Refugee Status Report: A Report on How Refugee Children and Young People in Victoria Are Faring. Melbourne: Department of Education and Early Childhood Development (DEECD), 2011. Available at: https://search.informit.org/doi/abs/10.3316/aeipt.190061. Accessed on: 20 April 2023.

Pedro, F., Subosa, M., Rivas, A., & Valverde, P. (2019). Artificial intelligence in education: Challenges and opportunities for sustainable development.

PETER, E. E. Critical thinking: Essence for teaching mathematics and mathematics problem solving skills. African Journal of Mathematics and Computer Science Research, v. 5, n. 3, p. 39-43, 2012.

POLMAN, J.; HORNSTRA, L.; VOLMAN, M. The meaning of meaningful learning in mathematics in upper-primary education. Learning Environments Research, v. 24, n. 3, p. 469-486, 2021.

PÓLYA, G. How to solve it: A new aspect of mathematical method. Princeton: Princeton University Press, 2004.

POLYA, G. Mathematical Discovery: On Understanding, Learning, and Teaching Problem Solving. John Wiley & Sons, 1962.

PSAKI, S. R.; McCARTHY, K. J.; MENSCH, B. S. Measuring gender equality in education: Lessons from trends in 43 countries. Population & Development Review, v. 44, n. 1, 2018.

PUANGPUNSI, N. Learners' Perception towards Project-Based Learning in Encouraging English Skills Performance and 21st Century Skills. Thaitesol Journal, v. 34, n. 1, p. 1-24, 2021.

RAZAK, F. The effect of cooperative learning on mathematics learning outcomes viewed from students' learning motivation. JRAMathEdu (Journal of Research and Advances in Mathematics Education, v. 1, n. 1, p. 49-55, 2016.

Resnick, M. (2024). Generative AI and creative learning: Concerns, opportunities, and choices.

Roll, I., & Wylie, R. (2016). Evolution and revolution in artificial intelligence in education. International journal of artificial intelligence in education, 26, 582-599.

Sajja, R.; Sermet, Y.; Cikmaz, M.; Cwiertny, D.; Demir, I. Artificial intelligence-enabled intelligent assistant for personalized and adaptive learning in higher education. Information, v. 15, n. 10, p. 596, 2024.

SCHLEPPEGRELL, M. J. The linguistic challenges of mathematics teaching and learning: A research review. Reading & Writing Quarterly, v. 23, n. 2, p. 139-159, 2007.

SCHMITT, C. Technology in schools: Suggestions, tools, and guidelines for assessing technology in elementary and secondary education. US Department of Education, Office of Educational Research and Improvement, National Center for Education Statistics, 2002.

SCHOENFELD, A. H. Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). Journal of Education, v. 196, n. 2, p. 1-38, 2016.

SHIN, M.; BRYANT, D. P.; BRYANT, B. R.; MCKENNA, J. W.; HOU, F.; OK, M. W. Virtual manipulatives: Tools for teaching mathematics to students with learning disabilities. Intervention in School and Clinic, v. 52, n. 3, p. 148-153, 2017.

SHUTE, V. J. Focus on formative feedback. Review of Educational Research, v. 78, n. 1, p. 153-189, 2008.

SINGLEY, M. K.; SINGH, M.; FAIRWEATHER, P.; FARRELL, R.; SWERLING, S. Algebra jam: Supporting teamwork and managing roles in a collaborative learning environment. In Proceedings of the 2000 ACM Conference on Computer Supported Cooperative Work, p. 145-154, 2000.

SKOVSMOSE, O. Critical mathematics education. Springer International Publishing, 2020. p. 154-159.

SON, J. B.; RUŽIĆ, N. K.; PHILPOTT, A. Artificial intelligence technologies and applications for language learning and teaching. Journal of China Computer-Assisted Language Learning, 2023.SMIT, L. S. A better understanding of 21st century skills in mathematics education and a view of these skills in current practice. Master's Thesis, 2016.

SMITH, B. L.; MACGREGOR, J. T. What is collaborative learning, 1992.

STEPHAN, M. Teacher-centered teaching in mathematics education. Encyclopedia of

Mathematics Education, p. 836-840, 2020.

STIGLER, J. W.; HIEBERT, J. Teaching is a cultural activity. American Educator, v. 22, n. 4, p. 4-11, 1998.

SZYMKOWIAK, A.; MELOVIĆ, B.; DABIĆ, M.; JEGANATHAN, K.; KUNDI, G. S. Information technology and Gen Z: The role of teachers, the internet, and technology in the education of young people. Technology in Society, v. 65, 101565, 2021.

SUPENA, I.; DARMUKI, A.; HARIYADI, A. The Influence of 4C (Constructive, Critical, Creativity, Collaborative) Learning Model on Students' Learning Outcomes. International Journal of Instruction, v. 14, n. 3, p. 873-892, 2021.

STEHLE, S. M.; PETERS-BURTON, E. E. Developing student 21st Century skills in selected exemplary inclusive STEM high schools. International Journal of STEM Education, v. 6, p. 1-15, 2019.

SNYDER, L. G.; SNYDER, M. J. Teaching critical thinking and problem-solving skills. The Journal of Research in Business Education, v. 50, n. 2, p. 90, 2008.

TAPALOVA, O.; ZHIYENBAYEVA, N. Artificial intelligence in education: AIEd for personalised learning pathways. Electronic Journal of e-Learning, 20(5), 639-653, 2022.

THE UNITED NATIONS HIGH COMMISSIONER FOR REFUGEES (UNHCR). Global Trends: Forced Displacement in 2023. Available at: https://www.unhcr.org/global-trends-report-2023.

TRILLING, B.; FADEL, C. 21st Century Skills: Learning for Life in Our Times. John Wiley & Sons, 2012.

WALTER, Y. Embracing the future of Artificial Intelligence in the classroom: the relevance of AI literacy, prompt engineering, and critical thinking in modern education. International Journal of Educational Technology in Higher Education, v. 21, n. 1, p. 15, 2024.

WONG, J. K. K. Are the Learning Styles of Asian International Students Culturally or Contextually Based? International Education Journal, v. 4, n. 4, p. 154-166, 2004.

WU, H. The mathematician and the mathematics education reform. Notices of the AMS, v. 43, n. 12, p. 1531-1537, 1996.

YOON, B. Uninvited guests: The influence of teachers’ roles and pedagogies on the positioning of English language learners in the regular classroom. American Educational Research Journal, v. 45, n. 2, p. 495-522, 2008.

YOON, H. J. Challenging the “non-native English speaker" identity in US higher education: A case of international graduate students. Working Papers in Educational Linguistics (WPEL), v. 28, n. 2, p. 55-75, 2013.

UNITED NATIONS DEPARTMENT OF ECONOMIC AND SOCIAL AFFAIRS. The 2030 Agenda for Sustainable Development, the 17 Goals. Available at: https://sdgs.un.org/goals#history. Accessed on: 15 June 2023.

ZAKARIA, E.; YUSOFF, N. Attitudes and problem-solving skills in algebra among Malaysian matriculation college students. European Journal of Social Sciences, v. 8, n. 2, p. 232-245, 2009.

ZAWACKI-RICHTER, O.; MARÍN, V. I.; BOND, M.; GOUVERNEUR, F. Systematic review of research on artificial intelligence applications in higher education–where are the educators? International Journal of Educational Technology in Higher Education, v. 16, n. 1,

p. 1-27, 2019.

ZHANG, L.; BASHAM, J. D.; YANG, S. Understanding the implementation of personalized learning: A research synthesis. Educational Research Review, v. 31, 100339, 2020. Disponível em: https://doi.org/10.1016/j.edurev.2020.100339

UNESCO. (2019b). Steering AI and Advanced ICTs for Knowledge Societies: Human Rights implications - A ROAM Perspective. Paris: UNESCO.

WALKINGTON, C. The Implications of Generative Artificial Intelligence for Mathematics Education. 2024. DOI: 10.13140/RG.2.2.11192.15367.