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MAURÍCIO ROCHA GONÇALVES

INTEGRATED LOT SIZING AND BLENDING PROBLEMS UNDER DEMAND UNCERTAINTY

São José do Rio Preto
2025

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Tese, apresentada à Universidade Estadual Paulista (UNESP), Instituto de Biociências, Letras e Ciências Exatas, São José do Rio Preto, para obtenção do título de Doutor em Matemática.

Área de Concentração: Matemática Aplicada

Orientador: Profº Dr. Silvio Alexandre de Araujo

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IMPACTO POTENCIAL DESTA PESQUISA

Esta pesquisa contribui para o avanço científico e técnico na área de pesquisa operacional e programação estocástica aplicada ao planejamento da produção sob incerteza na demanda. O potencial social e econômico decorre da aplicação prática em cadeias produtivas que buscam reduzir custos, aumentar a eficiência e mitigar perdas, promovendo maior competitividade industrial. O caráter educacional manifesta-se na apropriação e difusão de conhecimento gerado sobre os modelos matemáticos e métodos de solução inovadores para auxiliar a tomada de decisão em problemas industriais, além da formação de recursos humanos qualificados. A internacionalização é favorecida pela cooperação com grupos de pesquisa estrangeiros. A inserção local, regional e nacional é assegurada pela aderência às demandas do setor produtivo brasileiro, fortalecendo a integração universidade–indústria.

POTENTIAL IMPACT OF THIS RESEARCH

This research contributes to the scientific and technical advancement in the field of operations research and stochastic programming applied to production planning under demand uncertainty. Its social and economic potential arises from the practical application in supply chains that aim to reduce costs, increase efficiency, and mitigate losses, thereby promoting greater industrial competitiveness. The educational dimension is reflected in the appropriation and dissemination of the knowledge generated on innovative mathematical models and solution methods to support decision-making in industrial problems, as well as in the training of qualified human resources. Internationalization is fostered through cooperation with foreign research groups. Local, regional, and national insertion is ensured by addressing the demands of the Brazilian productive sector, strengthening the integration between university and industry.

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"O caminho é mais importante do que a caminhada."
Carlos Drummond de Andrade, em "O avesso das Coisas".

RESUMO

Esta tese aborda variantes de um problema da área de planejamento da produção industrial no qual diferentes componentes são misturados para produzir produtos finais. O planejamento da produção é um tema amplamente estudado na literatura de pesquisa operacional na forma de problemas de dimensionamento de lotes, enquanto o tipo de produção considerado está frequentemente relacionado a problemas de mistura. Além disso, problemas integrados têm recebido crescente atenção na literatura devido à sua relevância em aplicações práticas e ao fato de possibilitarem soluções que otimizam globalmente a cadeia produtiva, especialmente com o objetivo de reduzir o custo total esperado associado. A integração do problema de dimensionamento de lotes e mistura é considerada neste estudo, no qual a tomada de decisão em relação ao plano de produção dos produtos finais e compra dos componentes leva em consideração restrições operacionais, como *setup*, disponibilidade e tempos de entrega, balanço de estoques, requisitos de qualidade, atendimento à demanda, bem como os custos envolvidos. O conhecimento sobre o problema foi estendido principalmente com o estudo da incerteza na demanda por meio de formulações matemáticas e métodos de solução baseados em programação estocástica para apoiar a tomada de decisão relacionada aos planos de produção dos produtos finais e de compra dos componentes. Diferentes esquemas de tomada de decisão no contexto estocástico são propostos nesta tese para orientar o momento em que as decisões sobre os períodos de *setup* para a produção dos produtos finais e para a compra dos componentes, bem como as quantidades correspondentes, devem ser realizadas. A incorporação da incerteza na demanda trouxe desafios adicionais para a resolução das formulações estocásticas, o que motivou o desenvolvimento de métodos de solução fundamentados em programação inteira mista. Dentre esses métodos, foram investigadas abordagens exatas e heurísticas baseadas na utilização do valor esperado da demanda, em conjuntos de cenários de demanda, em aproximação por média amostral, em aproximações sequenciais de subproblemas estocásticos e em técnicas de decomposição. Uma extensa experimentação numérica com instâncias teste foi realizada para avaliar de forma abrangente o desempenho das abordagens propostas. Os resultados considerando diferentes parametrizações nas instâncias e nos métodos de solução evidenciaram diferenças relevantes entre os métodos propostos, destacando principalmente o equilíbrio entre a qualidade da solução, em termos de custo esperado, e o esforço computacional, em termos de tempo de processamento. No geral, além de contribuir com os avanços à literatura acadêmica do problema integrado de dimensionamento de lotes e mistura por meio da proposição de novos modelos, métodos de solução e discussões significativas sobre suas aplicações, esta tese destaca-se por introduzir uma perspectiva inovadora de tomada de decisão sob incerteza na demanda, com potencial de aplicação em cadeias produtivas reais, em particular nos setores agroindustrial, químico, petroquímico e alimentício.

Palavras-Chave: problema de dimensionamento de lotes; problema da mistura; incerteza na demanda; programação estocástica.

ABSTRACT

This thesis addresses variants of a problem in the field of industrial production planning in which different components are blended to produce end products. Production planning is a widely studied topic in the operations research literature as lot sizing problems, while the type of production considered is often related to blending problems. Moreover, integrated problems have received increasing attention in the literature due to their relevance in practical applications and the fact that they enable solutions that globally optimize the supply chain, especially with the goal of reducing the associated expected total cost. The integration of the lot sizing and blending problem is considered in this study, in which decision-making regarding the production plan of the end products and the purchasing of the components takes into account operational constraints such as setup, availability, lead times, inventory balance, quality requirements, demand fulfillment, and the associated costs. Knowledge on the problem was mainly extended by investigating demand uncertainty through the study of mathematical formulations and solution methods based on stochastic programming to support decision-making related to the production and purchasing planning. Different decision-making frameworks in the stochastic context are proposed in this thesis to guide the timing of setup decisions for the production of the end products and for the purchasing of the components, as well as the corresponding quantities. The incorporation of demand uncertainty brought additional challenges to address the stochastic formulations, which motivated the development of solution methods based on mixed-integer programming. Among these methods, both exact and heuristic approaches were developed, based on the use of expected demand, sets of demand scenarios, sample average approximation, sequential stochastic approximations, and decomposition techniques. An extensive numerical experimentation with test instances was carried out to comprehensively assess the performance of the proposed approaches. The results considering different parameterizations of the instances and methods revealed relevant differences among the proposed approaches, in particular highlighting the balance between solution quality, in terms of expected cost, and computational effort, in terms of computational time. Overall, in addition to contributing to the advancement of the academic literature on the integrated lot sizing and blending problem through the introduction of new models, solution methods, and comprehensive discussions of their results, this thesis stands out by introducing an innovative perspective on decision-making under demand uncertainty, with potential applications in real productive chains, particularly in the agro-industrial, chemical, petrochemical, and food sectors.

Keywords: lot sizing problem; blending problem; demand uncertainty; stochastic programming.

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LIST OF ABBREVIATIONS AND ACRONYMS

BOM	Bill-of-Materials
MILP	Mixed-Integer Linear Programming
MCLSB	Multi-item Capacited Lot Sizing and Blending
MV	Mean Value
SAA	Sample Average Approximation
ASAA	Adjustable Sample Average Approximation
MLSBLT	Multi-item Lot Sizing and Blending with Lead Times
STS	Sequential Two-Stage Stochastic
FFS	Fast Forward Selection

CONTENTS

1 INTRODUCTION

Production planning has been extensively studied in the operations research literature as a means of supporting decision-making across a wide range of production chain applications. Within this context, the lot sizing problem is a central topic to address the challenge of determining the optimal setup timing and production quantities for different end products over a planning horizon (Wagner; Whitin, 1958). Mathematical formulations for this problem are primarily developed to capture the trade-offs between setup costs and inventory holding costs while ensuring demand satisfaction. Additionally, extensions of the lot sizing problem further incorporate operational aspects such as capacity restrictions, lead times, and the possibility of unmet demand (Jans; Degraeve, 2008; Brahimi et al., 2017).

The Bill Of Materials (BOM) in manufacturing industries specifies the components that are required to produce the end products. The production planning incorporating the BOM encompasses coordinating the activities of manufacturing plants, where the end products are produced, with those of supply companies, from whom the components are purchased. Thus, the production planning results in a two-level lot sizing problem that is interconnected (Pochet; Wolsey, 2006). One is the production level, where the production timing and quantities for the end products are determined. The other is the purchase level, where the purchase timing and quantities for the components are determined.

The BOM in some industries is rigid, meaning that the components used and their corresponding quantities remain fixed throughout the manufacturing process, whereas other industries allow a certain degree of BOM flexibility the selection of the components and their respective quantities can vary during production (Rutten, 1995). The BOM flexibility typically appears in manufacturing plants that involve the manipulation of powders, liquids, or gases through a mixing process to make the end products, such as in fertilizer production (Ashayeri; Eijs; Nederstigt, 1994), food processing (Akkerman; Meer; Donk, 2010; Kilic et al., 2013), petrochemical industries (Yang; Vayanos; Barton, 2017; Peng; Maggioni; Lisser, 2021; Singh et al., 2000), and coal production (Chen; Duenyas; Jasin, 2022).

The degree of BOM flexibility is typically governed by constraints related to the quality requirements for the end products. In the mentioned applications, the quality requirements that are notably important include the chemical composition in petrochemical industries to control pollutant emissions, the nutritional levels in food processing for dietary requirements, and the calorific value in coal production for energy efficiency. Under quality requirements, the BOM flexibility can take the form of recipe flexibility or quantity flexibility. Recipe flexibility refers to the possibility of choosing among alternative predefined combinations of components that already meet the required quality standards for the end products (Pathumnakul et al., 2011). On the other hand, quantity flexibility allows the proportions of components to vary as long as the overall quality requirements of the end products are satisfied (Kilic et al., 2013).

The blending problem is a possible framework to incorporate the BOM flexibility into the production planning (Williams; Redwood, 1974; Johnson; Montgomery, 1974). The main objective of blending problems is to determine the blending of the components that satisfies the end products' requirements at minimum cost. The blending problem is commonly addressed in the single-period case, but when extended to a multi-period setting, the problem incorporates inventory as a linking element between consecutive time periods, which enables capturing the trade-offs between immediate demand satisfaction and future resource availability (Alfaki, 2012; Pathumnakul et al., 2011; Kilic et al., 2013; Fiorotto; Jans; Araujo, 2021).

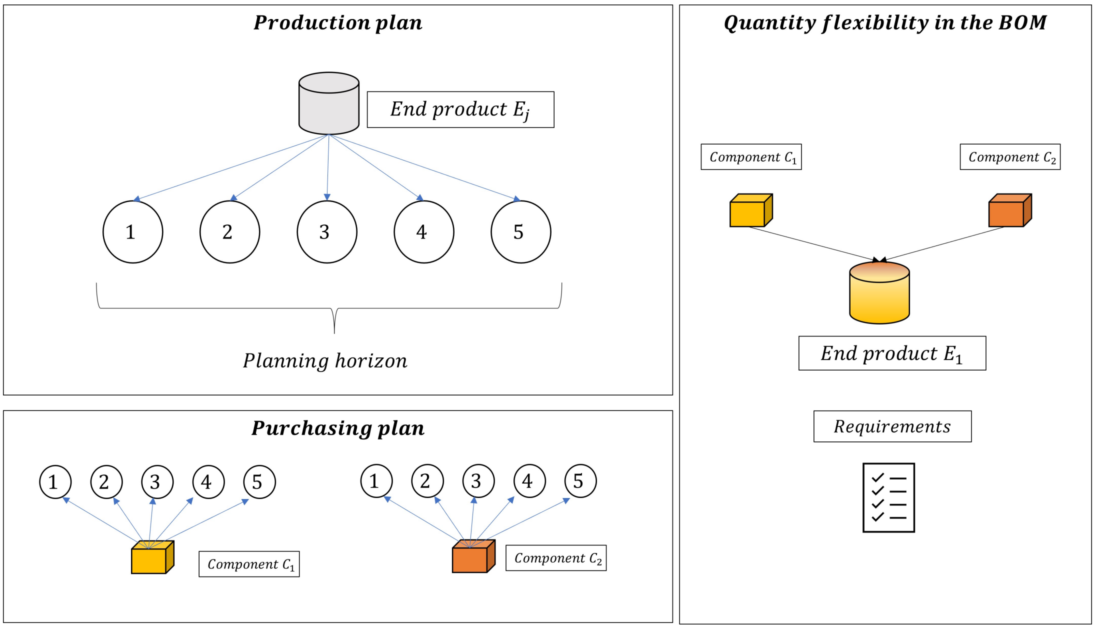
As the demand for the end products can be exploded through the BOM to determine the corresponding demand for the components, there is a hierarchical relationship between the production plan of the end products and the purchasing plan of the components (Rutten, 1993). While this relationship can be coordinated sequentially, the integration of lot sizing decisions accounting for these dependencies has received increasing attention in the literature because of its relevance in enabling solutions that optimize the supply chain in a global manner (Brahimi et al., 2015). Moreover, when quantity flexibility in the BOM is also incorporated into production planning decisions, the integration of the two-level lot sizing and blending problems has been shown to lead to better planning performance (Fiorotto; Jans; Araujo, 2021).

Based on the classification in Fiorotto, Jans & Araujo (2021), we focus on extending the integrated lot sizing and blending problem in this thesis. The lot sizing problem refers to the production plan of the end products and the purchasing plan of the components, while the blending problem provides the framework to incorporate the quantity flexibility assumed in the BOM. Figure 1 illustrates the main elements in the investigated problem: on the left, the production plan involves determining the timing and quantities of the end products across the planning horizon, while the purchasing plan specifies the timing and quantities of the components to be procured. On the right, the blending problem is represented as the mechanism that links components to end products, allowing proportions to vary as long as the requirements related to quality attributes are satisfied.

An important challenge in practice arises due to the uncertainty of the external demand for the end products (Mula et al., 2006). Early studies on the blending problem highlight the critical importance of incorporating demand stochasticity when deciding on the purchase of the components and the production of the end products (Rutten, 1995; Rutten; Bertrand, 1998), as this assumption plays a key role for inventory management and expected total cost minimization. Despite this recognition, the literature on the integrated lot sizing and blending problem has predominantly assumed the demand to be known in advance, thereby leaving a gap in approaches that address demand uncertainty.

The objective of this thesis is to approach the integrated lot sizing and blending problem under demand uncertainty. To this end, *Two-Stage Stochastic Programming* and *Multi-Stage Stochastic Programming* approaches (Birge; Louveaux, 2011; Shapiro; Dentcheva; Ruszczynski, 2009) are investigated to extend existing deterministic formulations for the problem to a stochastic

Figure 1 – Illustration of the elements in the integrated lot sizing and blending problem.



Source: Elaborated by the authors.

setting by partitioning the decision-making framework with respect to the demand realization. In the stochastic setting, the *Static*, *Static-Dynamic*, and *Dynamic* uncertainty strategies originally proposed for stochastic lot sizing problems are applied to orient the timing of the setup and the quantity decisions in the partitioning (Bookbinder; Tan, 1988).

The general contributions of this thesis to the study of the integrated lot sizing and blending problem under demand uncertainty are threefold. First, two-stage and multi-stage stochastic programming formulations are introduced to model alternative decision-making frameworks for the production and the purchasing decisions considering the demand stochasticity. Second, sample approximation, decomposition techniques, and sequential heuristic approaches are proposed as solution methods for these formulations considering a scenario-based representation (Shapiro; Mello, 1998; Mak; Morton; Wood, 1999; Agra; Requejo; Rodrigues, 2018; Escudero et al., 2007; Sereshti; Bodur; Luedtke, 2022). Third, extensive computational experimentation is carried out with test instances to evaluate the trade-offs between the solution quality and the computational effort across the proposed approaches, thereby providing insights regarding the different decision-making frameworks when production and purchasing plans must be made under demand uncertainty.

The next paragraphs provide an overview of the subsequent chapters and the organization of the ideas developed in this thesis.

In Chapter 2, there is a literature review on the blending and stochastic lot sizing problems, including papers with similar applications of two-stage and multi-stage stochastic programming models and methods to deal with demand uncertainty. Based on this review, the specific research gaps to be addressed are identified, and the detailed contributions of the thesis are outlined at the end of the chapter.

Chapter 3 focuses on the integrated lot sizing and blending problem considering a two-stage stochastic programming approach. The problem involves determining the setup periods for purchasing the components and producing the products, along with their respective quantities and blending. Mathematical programming formulations are proposed to minimize the expected total cost over the planning horizon, including inventory, production, purchasing, setup, and lost sales costs. Constraints related to inventory balance, machine capacity, and quality requirements are also taken into account. A novel feature incorporated in this study is the handling of demand uncertainty for the end products through the two-stage stochastic programming formulation. In the two-stage stochastic approach, we assume that the planning prioritizes a stable decision-making approach by employing the static uncertainty strategy. In the first stage, before demand is realized for the entire planning horizon, decisions are made on the setup periods for both production and purchase, and their corresponding quantities. In the second stage, after demand is revealed, the inventory and lost sales decisions are made. The goal in the two-stage stochastic setting is to minimize the expected total cost incurred by the decisions made in both stages. Heuristic, bounding procedure, and decomposition methods based on sample average approximation are proposed, where computational results are reported to evaluate the expected cost and computational time using the two-stage stochastic approach.

Chapter 4 focuses on the integrated lot sizing and blending problem considering a multi-stage stochastic programming approach. The problem involves determining the setup periods for purchasing the components and producing the products, along with their respective quantities and blending. Mathematical programming formulations are proposed to minimize the expected total cost over the planning horizon, including inventory, production, purchasing, setup, and lost sales costs. Constraints related to quality requirements and inventory balance subject to lead times are also taken into account. A novel feature incorporated in this study is the handling of demand uncertainty for the end products through the multi-stage stochastic programming formulation. The demand realization occurs every time period in the stage-wise setting, where different anticipation strategies are proposed for the decisions on the purchase of the components and the production of the products. Based on these strategies, the sequential structure of decisions underlying the alternative settings for the multi-stage stochastic programming problem results in different expected costs. A heuristic method that sequentially yields setup variable values is proposed to define a partial solution to the multi-stage stochastic problem. Computational experiments are carried out with a test instance to assess both the expected cost and the computational effort required by the proposed methods for the multi-stage stochastic approach.

In Chapter 5, the key findings and contributions of this thesis are summarized, and some perspectives for extending the research are indicated.

5 CONCLUSION AND FUTURE RESEARCH

In this thesis, we studied the integrated lot sizing and blending problem under demand uncertainty. The fundamental problem arises in industrial settings that require decisions over a finite planning horizon on the setup periods for purchasing the components and producing the end products, as well as on their respective quantities and blending. This research was motivated by the lack of approaches that incorporate the uncertainty of the demand and its impact on these decisions.

The main objective of this thesis was to develop stochastic programming models and computational methods to support decision-making considering demand stochasticity in the integrated lot sizing and blending problem. We introduced two-stage and multi-stage stochastic formulations to minimize the expected total cost, where alternative decision structures were designed according to uncertainty strategies from the stochastic lot sizing literature. Because solving the stochastic models was computationally challenging, we addressed this issue by developing approximation and heuristic methods within a scenario-based framework.

A set of test instances was designed for the integrated lot sizing and blending problem under demand uncertainty across varied settings, and computational experiments were conducted on these instances to evaluate the proposed modeling and solution approaches. The numerical results provided empirical insights into how computational difficulty and expected cost depend on instance characteristics and method parameterization, thereby clarifying the trade-offs, limitations, and applicability of each approach.

Overall, the contributions of this thesis to the academic literature can serve as a fundamental building block for methodological development, computational analysis, and quantitative evaluation for the integrated lot sizing and blending problem under demand uncertainty. In terms of practical applications, the findings of this thesis enhance the understanding of decision-making in blending problems under demand uncertainty, which are relevant to important segments of process manufacturing, including the food, petrochemical, and chemical industries.

One way to complement the evaluations in this thesis is to conduct additional tests in different settings to provide further insights into the stochastic approaches. In the modeling setting, the formulations could be extended to incorporate grouped capacity constraints for the end products, to treat capacity and lead time constraints simultaneously, and to account for a perishability factor for items in inventory (Tomazella, 2024; Schlenkrich; Cordeau; Parragh, 2025). In the stochastic setting, it would be valuable to consider alternative probability distributions for the demand, such as Poisson, log-normal, or truncated normal, and to model temporal correlation across time periods (Forel, 2021). Additionally, one could adopt decision frames that span more than one time period in the two-stage and multi-stage stochastic approaches (Brandimarte, 2006).

A future investigation using the developed stochastic formulations could quantify the value of key features in the integrated lot sizing and blending problem under demand uncertainty. First, it would be valuable to assess the value of integrating the decisions on the production of the end products and the purchase of the components in the stochastic setting. For example, the stochastic formulations could be decomposed into a production level and a purchase level and solved sequentially, instead of solving the problem as a whole, to examine the impact on the computational effort and solution quality (Fiorotto; Jans; Araujo, 2021). Second, the value of quantity flexibility for producing the end products through blending could be analyzed to study different reactive strategies for handling demand uncertainty. For example, one could model and analyze blending across end products with similar quality requirements as an alternative strategy to mitigate demand uncertainty, rather than considering only inventory and lost sales as the recourse mechanisms (Sereshti; Bodur; Luedtke, 2022).

A direction to improve the methodological framework in this thesis is to test alternative solution methods within the stochastic setting. Progressive Hedging algorithms have been applied to address the scalability of two-stage and multi-stage stochastic problems by decomposing the scenario-based model into independent scenario subproblems within a solution scheme that enforces non-anticipativity with penalties (Schlenkrich; Cordeau; Parragh, 2025). Additionally, Relax-and-Fix strategies have been considered to quickly yield feasible solutions for strong warm starts by partitioning binary variables, relaxing future partitions, solving sequentially, and then fixing decided binary decisions (Beraldi et al., 2006). Furthermore, rolling-horizon and receding-horizon evaluation frameworks that implement a candidate stochastic solution across time periods by incorporating updated demand information have been developed to enable realistic performance assessment under evolving demand uncertainty fashion (Sereshti; Adulyasak; Jans, 2021).

An avenue for future research is to investigate alternative approaches to amend decision-making under demand uncertainty in the integrated lot sizing and blending problem. Robust Optimization is a possible approach that has successfully evolved as a methodology for handling optimization problems with uncertain data (Beyer; Sendhoff, 2007; Li et al., 2009). While stochastic programming incorporates uncertainty via probability distributions and seeks decisions that optimize expected performance over scenarios, robust optimization focuses on decisions that remain feasible across all realizations within a prescribed uncertainty set to provide improved protection against demand variability. In this context, the proposed stochastic programming and the developed robust optimization could be evaluated either as complementary approaches, with stochastic information used to calibrate uncertainty sets, or as alternative approaches that offer different balances between expected performance and robustness under uncertainty (Hu; Hu, 2020).

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