

Dynamical erosion of asteroid groups in the region of the Pallas family

V. Carruba,^{*} J. F. Machuca and H. P. Gasparino

UNESP, Univ. Estadual Paulista, Grupo de dinâmica Orbital e Planetologia, Guaratinguetá, SP 12516-410, Brazil

Accepted 2010 November 16. Received 2010 October 27; in original form 2010 September 15

ABSTRACT

In a previous paper, the current state of knowledge on the region of the Pallas dynamical family was revised. Here the dynamical evolution and possible origin of dynamical groups in the region are investigated. First, we study the case of asteroids at high eccentricity ($e > 0.31$). These objects are unstable because of encounters with Mars on time-scales of up to 340 Myr. Local background asteroids are currently the major source of high-eccentricity objects, but Barcelona family members will become the dominant source in about 250 Myr.

Next, attention is focused on the lack of chaotic dynamics near the ν_6 secular resonance border in the region. Contrary to the case of the Phocaea family region, very limited chaotic behaviour was observed for real and fictitious particles in the central main belt near the ν_6 resonance. Using analytical and numerical tools, we find that the limited amplitude of the inclination region near the ν_6 resonance in the Pallas family region for which close encounters with Mars are possible explains the lack of chaotic behaviour found in a previous paper by Carruba.

Finally, we investigate the long-term stability of the minor families and clumps identified in the previous paper, when non-gravitational effects are considered. We find that none of the minor clumps obtained by Carruba is currently interacting with non-linear secular resonances in the region. The classical clumps around (40134) 1998 QO53, (75938) 2000 CO80, (33969) 2000 NM13, (208080) 1999 VV180 and (70280) 1999 RA111 have large detectability times and could be considered reasonable candidates for groups originating from collisional events. We confirm the presence of the (4203) Brucato family observable in the space of proper frequencies ($n, g, g + s$) that has the largest detectability time of all groups in the region.

Key words: celestial mechanics – minor planets, asteroids: general.

1 INTRODUCTION

In Carruba (2010b), the current status of our knowledge regarding the Pallas family region was revised. In that paper, amongst other things, families and clumps in the space of proper elements and frequencies (Carruba & Michtchenko 2007) were obtained and the current knowledge of asteroid taxonomy was revised, along with the albedo and absolute magnitude distribution of objects in the area. Furthermore, the dynamics of asteroids were studied using dynamical maps and chaos indicators. Several interesting results, such as the identification of the frequency family around (4203) Brucato, of the new family of (1222) Tina and of several clumps were obtained in that paper.

Here we try to investigate the questions left unanswered by that work. Asteroids at high eccentricity, the orbits of which are characterized by very low values of Lyapunov time, may experience encounters with Mars that can quickly destabilize their orbits. Here

we study the long-term stability of these high-eccentricity objects and identify the orbital regions that may replenish this population.

Another question left unanswered by Carruba (2010b) concerns the lack of chaotic behaviour near the ν_6 secular resonance border, especially when the Pallas family region is compared with the much more chaotic Phocaea area (Carruba 2009b). Using analytical (Yoshikawa 1987) and numerical (Carruba et al. 2007) tools, here, we study the effect of the proximity of the ν_6 secular resonance on asteroid eccentricity and find a simple analytical criterion to identify the asteroids most likely to show chaotic behaviour.

Finally, we investigated the long-term stability of the minor families and clumps identified in the previous paper, when non-gravitational effects are considered, and studied the influence (or lack of) that non-linear secular resonances may have on asteroids in the region.

The layout of this paper is as follows. In Section 2, we review the preserving effect of the Lidov–Kozai resonance on highly inclined objects when planetary effects are considered. In Section 3, we investigate the mechanisms that could create the currently observed high-eccentricity population. In Section 4, we study the dynamics of orbits near the ν_6 resonance centre and why so little chaotic

^{*}E-mail: vcarruba@feg.unesp.br

behaviour was observed in the region in Carruba (2010b). In Section 5, we investigate the long-term stability of minor families and clumps in the region. Finally, in Section 6, we present our conclusions.

2 MARS-CROSSER ASTEROIDS IN THE REGION: NUMERICAL SIMULATIONS

As seen in Carruba (2010b), asteroids in the region of the Pallas dynamical family at high eccentricities are characterized by interactions with Mars. In order to understand the long-term stability of asteroids in the region, we turn our attention to the results of numerical simulations. We start by performing short-term simulations on selected asteroids in the region.

2.1 Short-term numerical simulations

In Carruba (2010a), we investigated the long-term stability of high-eccentricity asteroids in the region of the Phocaea family and saw that objects with $e > 0.31$ were unstable because of encounters with Mars on time-scales of up to 270 Myr. Here we start by repeating the same analysis. There are 105 asteroids in the region of the Pallas family (25 numbered ones) that have eccentricities larger than 0.31. Two of these objects belong to the Barcelona frequency family (Carruba 2010b) and a member of the Pallas frequency family with an eccentricity of 0.3077 is in the proximity of this region, but, overall, the great majority of high-eccentricity asteroids in the region do not belong to families.

For asteroids at high inclination, it is possible to use the Lidov–Kozai Hamiltonian approximation (Carruba 2010a). The conserved integral of the Lidov–Kozai Hamiltonian is (Thomas & Morbidelli 1996; Gronchi & Milani 1999; Carruba 2010a):

$$H = \sqrt{a(1-e^2)} \cos i, \quad (1)$$

where a , e and i are the semimajor axis, eccentricity and inclination (evaluated with respect to the invariable plane of the Solar system), respectively. At a given semimajor axis, the conservation of H implies that there are maximum allowed values of eccentricity and inclination (see Carruba 2010a). When the eccentricity is large enough, the nodal distance at ascending node

$$d_{\text{nod}}^+ = \frac{a(1-e^2)}{1+e \cos \omega} - a' \quad (2)$$

and that at descending node

$$d_{\text{nod}}^- = \frac{a(1-e^2)}{1-e \cos \omega} - a' \quad (3)$$

between the ellipse of the asteroid orbit and the circular orbit (with radius a') of some perturbing planets can become zero. In this case, a node crossing is said to occur and a collision between the asteroid and the planet is possible. At $\omega = \pm 90^\circ$, $\cos \omega = 0$ and therefore the denominators in equations (2) and (3) become equal to 1. As a consequence, for the values of ω for which the eccentricity reaches its maximum value, the right-hand side in equations (2) and (3) reduces to its minimum value $a(1-e^2) - a'$. The fact that for the maximal value of the eccentricity the nodal distance is not minimal is called the ‘Lidov–Kozai protection mechanism’. For $\omega = 0^\circ$, equation (2) reduces to the first-order criterion $q = a'$, with q the asteroid pericentre distance, $q = a(1-e)$ (note that for $\omega = 0^\circ$ the value of the eccentricity is the minimum one).

In this section, we want to know what other information can be obtained by studying Lidov–Kozai conserved quantities in the region of the Pallas dynamical family. For this purpose, we focus our attention on the conserved values of the H integral.

Fig. 1 (panel A) shows the current distribution of proper inclination and eccentricities for asteroids in the region with $e > 0.31$. Superimposed, we plotted (e, I) values of the H integral in the range of H values for asteroids in the region. The minimum value of H is 0.8679, the maximum value is 1.4955 and the median value is 1.3152.

Fig. 1 (panel B) shows Mars node crossing lines for asteroids with semimajor axes equal to the median value of semimajor axes for $e > 0.31$ asteroids in the region ($a = 2.635$ au), obtained with equations (2) and (3), assuming Mars on a circular orbit of radius a' . As can be seen in the figure, with the exception of three asteroids, all objects in the region have nodal distances smaller than the value needed to collide with Mars. However, the long-term stability of these objects and the ability of the Lidov–Kozai mechanism to protect them from close encounters with Mars needs further research.

To start investigating the orbital behaviour of these objects, we integrated the 105 asteroids with SWIFT-SKEEL, the symplectic integrator of Levison & Duncan (2000) that is able to model close encounters between a massive planet and a massless particle, over 20 Myr and under the gravitational influence of all planets from

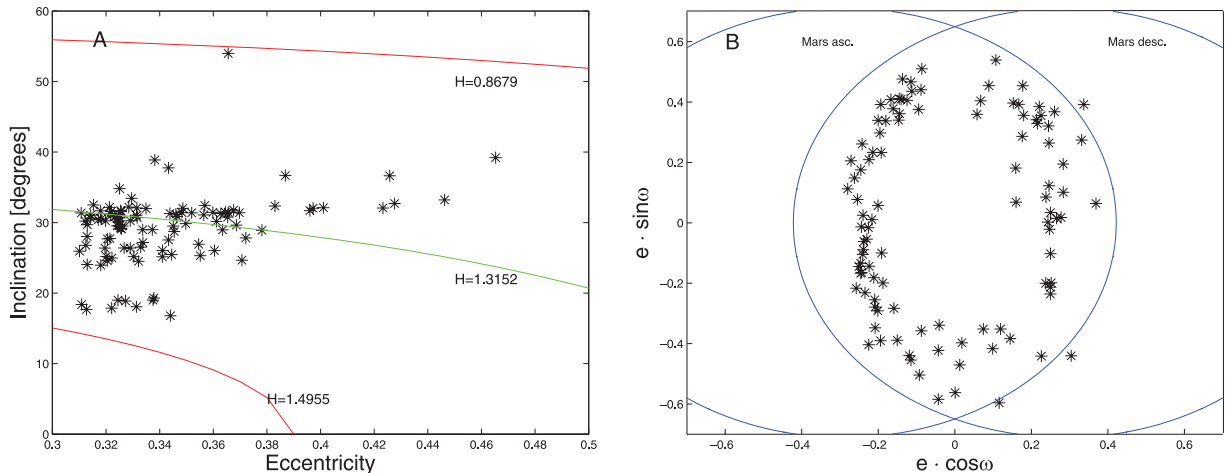


Figure 1. Panel A: (e, I) values of the H integrals for asteroids in the region of the Pallas family. Panel B: Mars node crossing lines for asteroids in the same region.

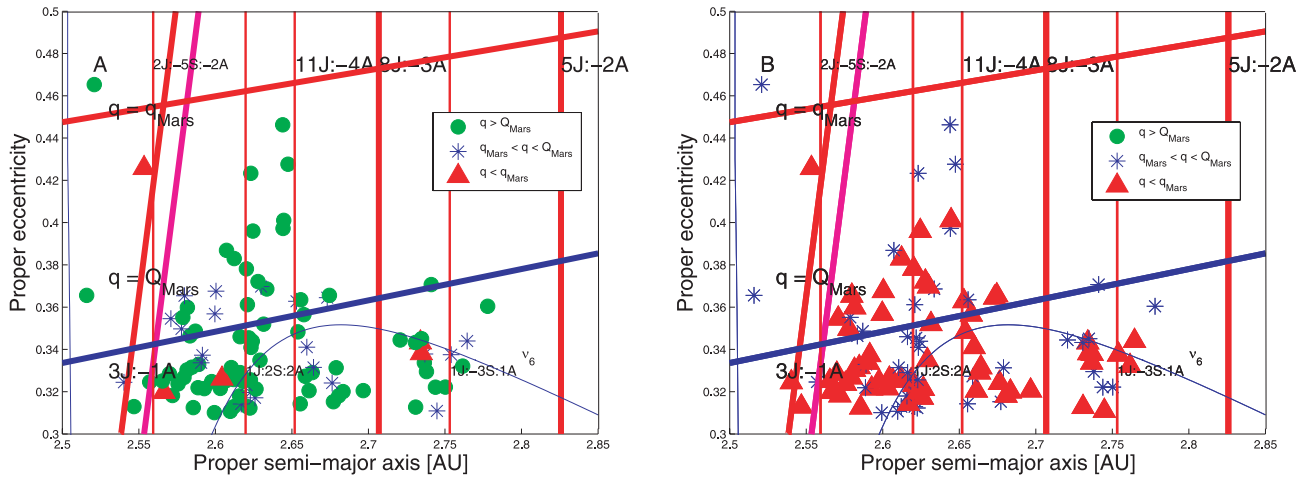


Figure 2. Panel A: an (a, e) projection of asteroids in the region of the Pallas family, for $e > 0.30$, with average values of pericentres. Panel B: the minimal values of pericentres of the same asteroids at the end of the 20-Myr integration.

Venus to Neptune (Mercury was accounted for as a barycentric correction in the initial condition of the system).

Fig. 2 (panel A) displays a proper (a, e) projection of the final fate of asteroids in the region. The red triangles are associated with asteroids whose mean pericentre was less than the pericentre of Mars, the blue asterisks are associated with asteroids whose mean pericentre was between the values of the pericentre and apocentre of Mars, and the green circles are associated with asteroids that had mean pericentres above the apocentre of Mars. The vertical red lines display the location of mean-motion resonances, the magenta lines display the location of the chaotic layer near the $3J: -1A$ mean-motion resonance as defined in Guillens, Vieira Martins & Gomes (2002) and the blue lines are associated with the secular resonances in the area. Inclined lines show the location of orbits with $q = Q_{\text{Mars}}$ (in blue) and $q = q_{\text{Mars}}$ (in red). Fig. 2 (panel B) shows the same, but for the minimum values of the pericentre that was reached by the particles during the simulation.

As can be seen in the figure, several asteroids near the v_6 separatrix attain values of the mean pericentre low enough to allow them to interact with Mars. All asteroids in the region with $e > 0.31$ had values of minimum pericentres during the integration, low enough to plunge into the orbit of Mars. To further investigate the stability of orbits in the region, we created a grid of test particles with initial osculating eccentricity between 0.20 and 0.40, in the range of the semimajor axis between those of the $3J: -1A$ and the $5J: -2A$ mean-motion resonances, and with the values of inclination determined by the constancy of the H integral. In particular, we used the median value of H for asteroids in the region ($H = 1.3152$). The initial values of Ω , ω and M were those at J2000 of 1999 VF87, an asteroid with the median H value. There was a total of 2601 particles in this simulation, integrated with SWIFT-SKEEL over 20 Myr.

Fig. 3 (panel A) shows an averaged (a, e) projection for test particles in the region of the Pallas family, for $e > 0.30$, with minimum values of pericentres. Here we used averaged elements rather than the proper ones, because many particles were lost before a time-span long enough to obtain reliable proper elements. The colour code for the test particles and the other symbols is the same as in Fig. 2 (panel B). Fig. 3 (panel B) does the same, but for the maximum survival times (see the figure legend for the description of the meaning of the particle colour codes).

As can be seen in the figures, essentially, all asteroids above the v_6 resonance had values of pericentres lower than Mars' apocentre and could therefore experience martian close encounters. Yet, only particles with $q < q_{\text{Mars}}$ were lost before the end of the integration (Fig. 3, panel B). This is further confirmed by the density maps of q_{min} (panel C) and t_{max} (panel D) values. Following the approach of Carruba & Michtchenko (2009), in these figures, we show the mean values of q_{max} and t_{max} in a grid covering the intervals (2.50–2.85) in a (we took 20 equally spaced intervals) and (0.3–0.5) in e (also 20 equally spaced intervals). Superimposed to the density maps, there is the orbital location of real asteroids in the region (black asterisks). The density map scale of the q_{min} values was scaled up to the value of Mars' apocentre.

During the length of the integration (20 Myr), some particles in the region with $q < Q_{\text{Mars}}$ achieved larger values of maximal eccentricity, but they were not lost. To further investigate the long-term stability of asteroids in the region, we performed simulations on longer time-scales for asteroids in these regions. The setup of the simulation and the results will be discussed in the next section.

2.2 Long-term numerical simulations

Fig. 2 (panel A) displays an (a, e) projection of the 105 asteroids in the region of the Pallas family. Of the objects with $e > 0.31$, (3712) Kraft is the only one for which a spectral classification is available (it is an S-type object in the Pallas family area). SDSS-MOC3 data are available for (123237) (2000 UH58) and suggest that this object, also in the Pallas family region, belongs to the C-complex. No information is available on the albedos of these 105 objects.

As can be seen in Fig. 2 (panel A), there are essentially seven mechanisms that can increase the asteroid eccentricity from the values observed in the region of the Pallas family to Mars-crossing values: interaction with the $3J: -1A$, $11J: -4A$, $8J: -3A$ and $5J: -2A$ two-body mean-motion resonances, and interaction with the $2J: -5S: 2A$, $1J: -2S: -2A$ and $1J: -3S: 1A$ three-body resonances. Of these mechanisms, the interaction of particles with the $3J: -1A$ mean-motion resonance is a proven very effective mechanism in raising the eccentricity of test particles to Mars-crossing levels. Secondary, the interplay of the Yarkovsky effect with higher order two- and three-body mean-motion resonances as a mechanism to

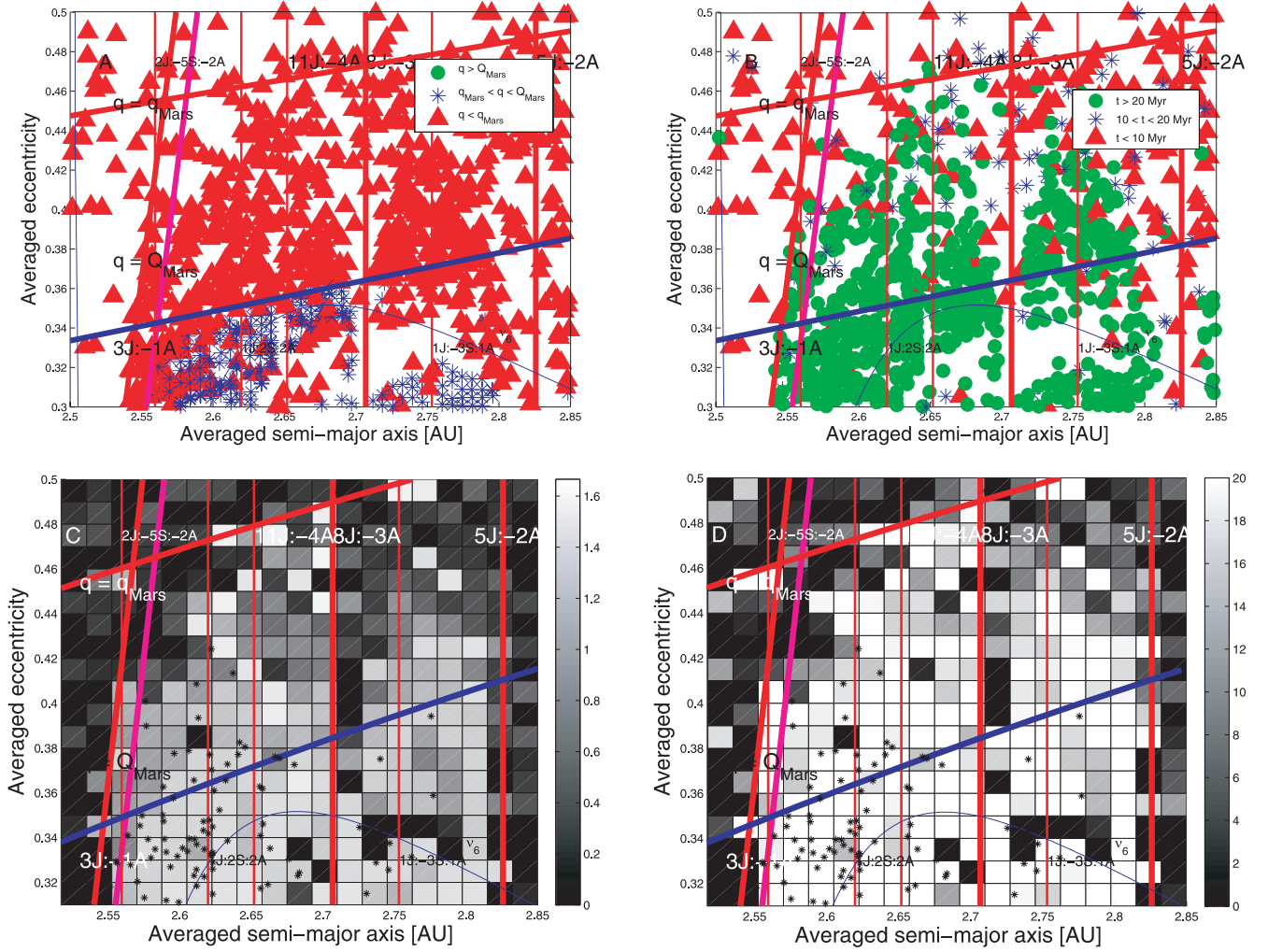


Figure 3. Panel A: an (a, e) projection of test particles in the region of the Pallas family, for $e > 0.30$, with minimum values of pericentres. Panel B: the maximum survival time of the same test particles at the end of the 20-Myr integration. Panels C and D display the position of real asteroids in the region, superimposed with density maps of minimum pericentre (in au) and survival time (in Myr).

increase asteroid eccentricities should also be investigated. More information on these mechanisms will be given in Section 3.

To further investigate the long-term stability of these 105 asteroids, we performed the following numerical experiments. First, we integrated the 105 asteroids with SWIFT-SKEEL over 300 Myr under the gravitational influence of all planets from Venus to Neptune (Mercury was accounted for as a barycentric correction to the Sun initial conditions). Following the approach of Nesvorný et al. (2008), we define a region of interest, which is between the $3J: -1A$ and the $5J: -2A$ mean-motion resonances, with $e > 0.31$ and $q < Q_{\text{Mars}}$. The last condition was set so as to investigate the asteroids with large eccentricity that could potentially be destabilized by close encounters with Mars. We then checked how many objects remained in the area as a function of time during the numerical integration.

Fig. 4 (panel A) shows this number as a function of time (blue-dashed line) and the number of objects initially in the area of interest that remained in the region as a function of time (black line). We should emphasize that the parameter that gives information on the stability of objects is the one related to the number of objects *initially* inside the area, that is, the black line. The fact that other objects originally not in the area may be temporarily displaced inside the

region of interest is per se not an indication of the stability of the initial asteroid population.

As can be seen in the figure, the number of objects with $q < Q_{\text{Mars}}$ rapidly drops and goes to zero after 140 Myr, if we consider the initial population. As a second numerical experiment, we integrated the same particles with SWIFT_CE, an integrator developed in Carruba et al. (2007) that simultaneously models the effect of close encounters between a massive planet and a massless particle, and the diurnal and seasonal versions of the Yarkovsky effect. We used typical values of Yarkovsky parameters for S-type asteroids (thermal conductivity $K = 0.001 \text{ W m}^{-1} \text{ K}^{-1}$, thermal capacity $C = 680 \text{ J kg}^{-1} \text{ K}^{-1}$, surface density of 1500 kg m^{-3} , density of 2500 kg m^{-3} , bond albedo of 0.1, Carruba et al. 2003). The asteroid radius was computed using equation (1) in Carruba et al. (2003) and the average value of the geometric albedo of Phocaea members, $p_V = (0.24 \pm 0.12)$ (Carruba 2009b), and we gave to one set of objects an inclination of the spin axis of 90° , while to the second was assigned an obliquity of -90° . No re-orientations were considered, so that the drift caused by the Yarkovsky effect was the maximum possible. We integrated our test particles over 400 Myr.

Fig. 4 (panel B) displays the number of objects currently (blue-dashed line) and since the integration begun (black line) in the

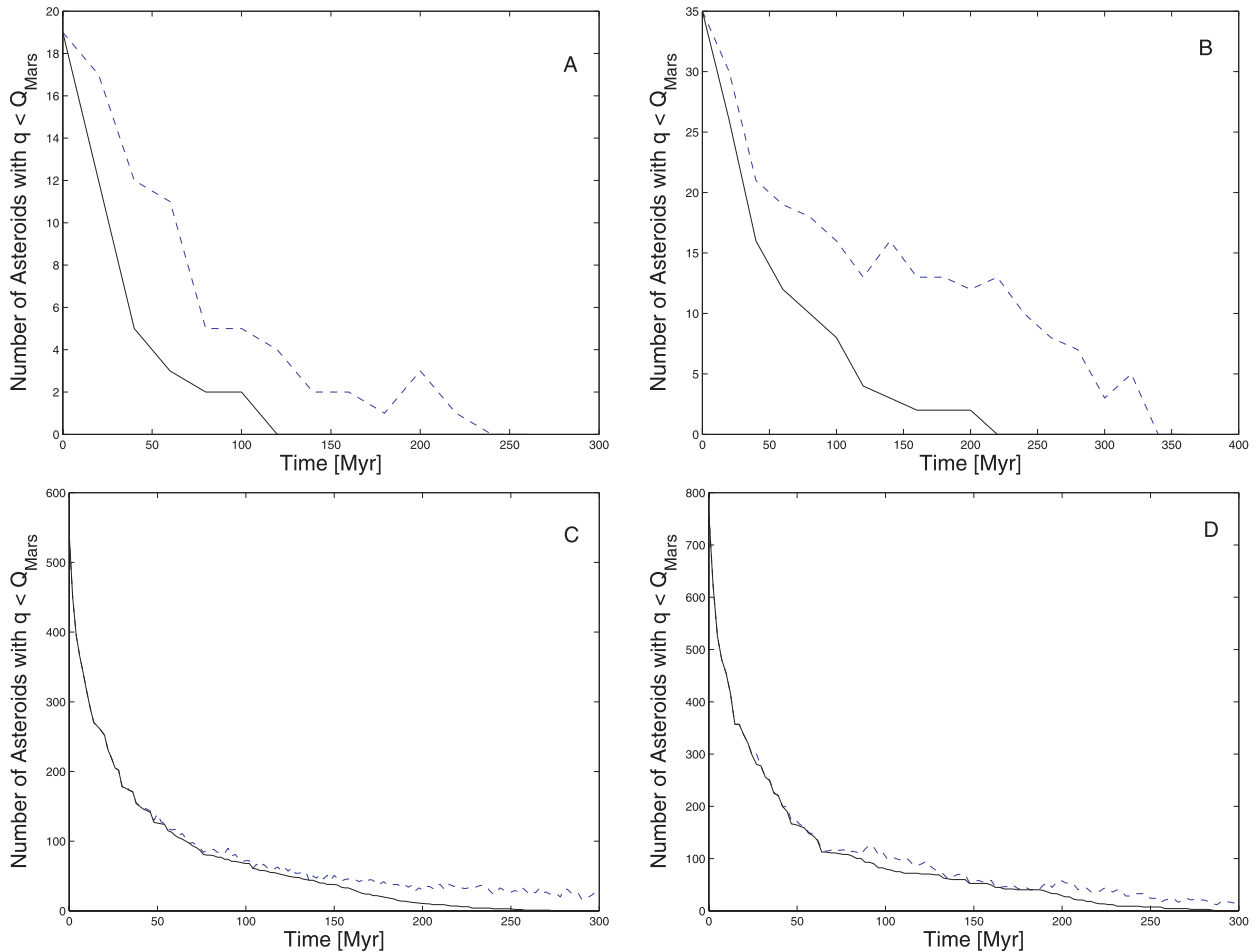


Figure 4. The number of objects (blue-dashed line) and of asteroids initially inside the $q < Q_{\text{Mars}}$ region defined in the text (black line), as a function of time. See text for a description of the different panels.

area of interest during the length of the simulation. As for the case without non-gravitational effects, the number of objects in the area still drops, but on a somewhat longer time-scale. The last particle originally in the area of interest was removed after 220 Myr and all particles left the region after 340 Myr. As observed for the high-eccentricity, Mars-crossing region in the Phocaea family area (Carruba 2010a), test particles in the simulation with non-gravitational effects tend to stick longer than the same particles in the conservative integration. This is due to the interplay of the Yarkovsky effect with some of the local non-linear secular resonances, such as the $2\nu_6 - \nu_5 + \nu_{16}$ resonance in the Pallas region, that forced a few particles to stick to the resonances (and to the high-eccentricity area for longer times). Overall, this phenomenon only delays the inevitable loss of the original high-eccentricity, $q < Q_{\text{Mars}}$ population, which is lost on time-scales of at most 220 Myr.

One possible objection to these data is the low number of objects currently present at high eccentricity. To obtain a statistically more robust estimate on the stability time, we integrated the same grid of 2600 high-eccentricity particles defined in Section 2.1 with SWIFT-SKEEL and SWIFT_CE over 300 Myr. Fig. 4 (panels C and D) shows the number of objects (blue-dashed line) and of asteroids originally inside the $q < Q_{\text{Mars}}$ region (black line) as a function of time.

As can be seen in the figure, the time-scales for survival of the high-eccentricity, $q < Q_{\text{Mars}}$ population are of 260 Myr for the

particles in the SWIFT-SKEEL simulation and of 280 Myr for the particles in the SWIFT_CE simulation, in agreement with what found in the previous run with real asteroids. In the next section, we will investigate the mechanisms that can create the observed high-eccentricity population.

3 CREATING THE HIGH-ECCENTRICITY ASTEROID POPULATION

Now that we have estimated the lifetime of high-eccentricity objects, one natural question that may arise is how to create and sustain such an unstable population of asteroids. Three natural possible sources of high-eccentricity objects are the local background asteroids and the two families in the region with largest eccentricities: the Barcelona and Pallas families.¹ In principle, the source of these asteroids could be distinguished based on the taxonomy: C-type asteroids would be more likely to come from the Pallas family and S-type asteroids should be associated with the Barcelona one. Unfortunately, taxonomical information is available for only

¹ The 208080 clump in the region of the Barcelona family is composed of low-eccentricity objects and could not possibly furnish members to the high-eccentricity region in short time-scales.

one high-eccentricity object, (3712) Kraft, which is an S-type asteroid. SDSS-MOC3 data are also only available for just one object, (123237) (2000 UH58), also belonging to the S-complex. At this stage of our knowledge, it seems therefore that only dynamics can provide some clues about the possible origin of these high-eccentricity objects.

To investigate the possible mechanisms to replenish the high-eccentricity region, as defined in Section 2.2, we simulated the evolution under the Yarkovsky effect of two sets of clones of local background asteroids [asteroids with $\sin i > 0.5$ that do not belong to the Barcelona, Pallas or 208080 dynamical groups], the Barcelona and Pallas families, one with a spin axis obliquity of 90° and the other with a spin axis obliquity of -90° , so as to maximize the speed of migration towards larger and smaller semimajor axes. The periods were obtained assuming that the rotation frequency was inversely proportional to the radius and that a 1-km asteroid had a rotation period of 5 h (Farinella, Vokrouhlický & Hartmann 1998). We used the set of Yarkovsky parameters typical of S-type objects for the Barcelona family members (thermal conductivity of $0.001 \text{ W m}^{-1} \text{ K}^{-1}$, thermal capacity of $680 \text{ J kg}^{-1} \text{ K}^{-1}$, density of 2500 kg m^{-3} , surface density of 1500 kg m^{-3} , bond albedo of 0.1 and thermal emissivity of 0.95, Carruba et al. 2003) and typical of C-type objects for the Pallas family members (same as for S-type objects, but with a density of 1500 kg m^{-3}), and integrated the members of both classical and frequency families over 300 Myr. The radius of each test particle was estimated using the median value of the geometric albedo for the family as found in Carruba (2010a) ($p_V = 0.2416$ for the Barcelona family, $p_V = 0.1569$ for the Pallas one and $p_V = 0.2000$ for local background objects) and the absolute magnitude of each object, using the formula

$$D = \frac{D_0}{\sqrt{p_V}} 10^{-0.2H}. \quad (4)$$

Multi-opposition objects for which no absolute magnitude data were available were given a radius of 1000 m. At each time-step, we checked the number of objects that reached the high-eccentricity region, as defined in Section 2.2.

Fig. 5 shows the number of high-eccentricity objects that originated as a member of the Barcelona family (blue line), as a member

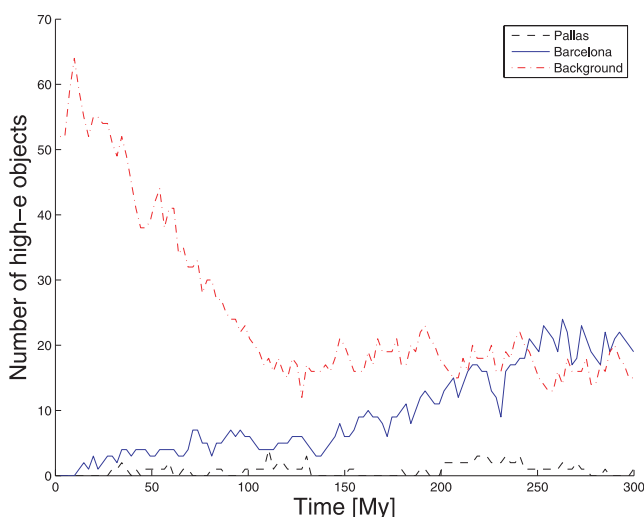


Figure 5. The number of high-eccentricity objects that originated as a member of the Barcelona family (blue line), as a member of the Pallas family (black dashed line) or as background object (red dashed line), as a function of time.

of the Pallas family (black dashed line) or as a background object (red dashed line), as a function of time, over the length of the integration (300 Myr). Among the mechanisms that can increase the eccentricity of asteroids, we found that powerful mean-motion resonances with Jupiter, such as the $3J$: $-1A$, $5J$: $-2A$ and $8J$: $-3A$ resonances, and the ν_6 secular resonance effectively increased the eccentricity of test particles on short time-scales, but also tended to create a population of objects on short time-scales (2 Myr or less). Three-body mean-motion resonances, such as the $1J$: $2S$: $2A$ and $1J$: $-3S$: $1A$ resonances, increased the asteroid eccentricities on longer time-scales, but also produced a longer lived high-eccentricity population.

As can be seen in the figure, the current high-eccentricity population is dominated by background objects, with a secondary contribution from the Barcelona family and a very minor influx of Pallas objects. According to our simulations, Barcelona family members will become the dominant source of high-eccentricity asteroids 250 Myr into the future. The fact that background objects dominate the current high-eccentricity population in the region implies that, contrary to the case of the Phocaea family region (Carruba 2010a), it is not possible to set lower limits on the age of either the Barcelona or the Pallas family based on the time-scales needed to family members to reach the high-eccentricity region.

4 THE (ABSENCE OF) CHAOTIC DYNAMICS NEAR THE ν_6 SECULAR RESONANCE SEPARATRIX

One of the questions left unanswered in Carruba (2010b) was the absence of significant chaotic behaviour near the ν_6 secular resonance separatrix. In this section, we will investigate this problem with analytical and numerical tools.

4.1 Analytical model of the ν_6 resonance

Since the late 1980s, it is known that asteroids in the proximity of the ν_6 secular resonance have their eccentricity raised to planetary crossing values and are so destabilized on short time-scales (see, amongst others, Yoshikawa 1987). To more quantitatively estimate the effect of the proximity of the ν_6 secular resonance, here, we briefly revise the non-linear model for the ν_6 resonance of Yoshikawa (1987). In his model, Yoshikawa kept terms up to third degree in eccentricity and inclination for the secular part of the disturbing function and, under the hypothesis of proximity to the ν_6 resonance that $l_6 = \varpi - \varpi_6^*$ moves much slower than $l_5 = \varpi - \varpi_5^*$, where $\varpi_i^* = v_i t + \beta_i$, v_i and β_i being known constants (Bretagnon 1974), obtained the following expression for the Hamiltonian of the problem:

$$F_{\nu_6} = \frac{1}{2}(b - \nu_6)e^2 + \frac{1}{4}ce^4 - d_6e \cos(\varpi - \varpi_6^*), \quad (5)$$

where $\nu_6 = 26.217 \text{ arcsec yr}^{-1}$ is the value of the precession frequency of the pericentre of Saturn and b , c and d_6 are coefficients whose values can be found in Yoshikawa (1987) and Carruba (2010a) (b roughly corresponds to the g pericentre precession frequency).

In this paper, we are interested in investigating the dynamics of objects near the ν_6 resonance separatrix. Following Yoshikawa (1987), we introduce the k_6 parameter:

$$k_6 = b - \nu_6. \quad (6)$$

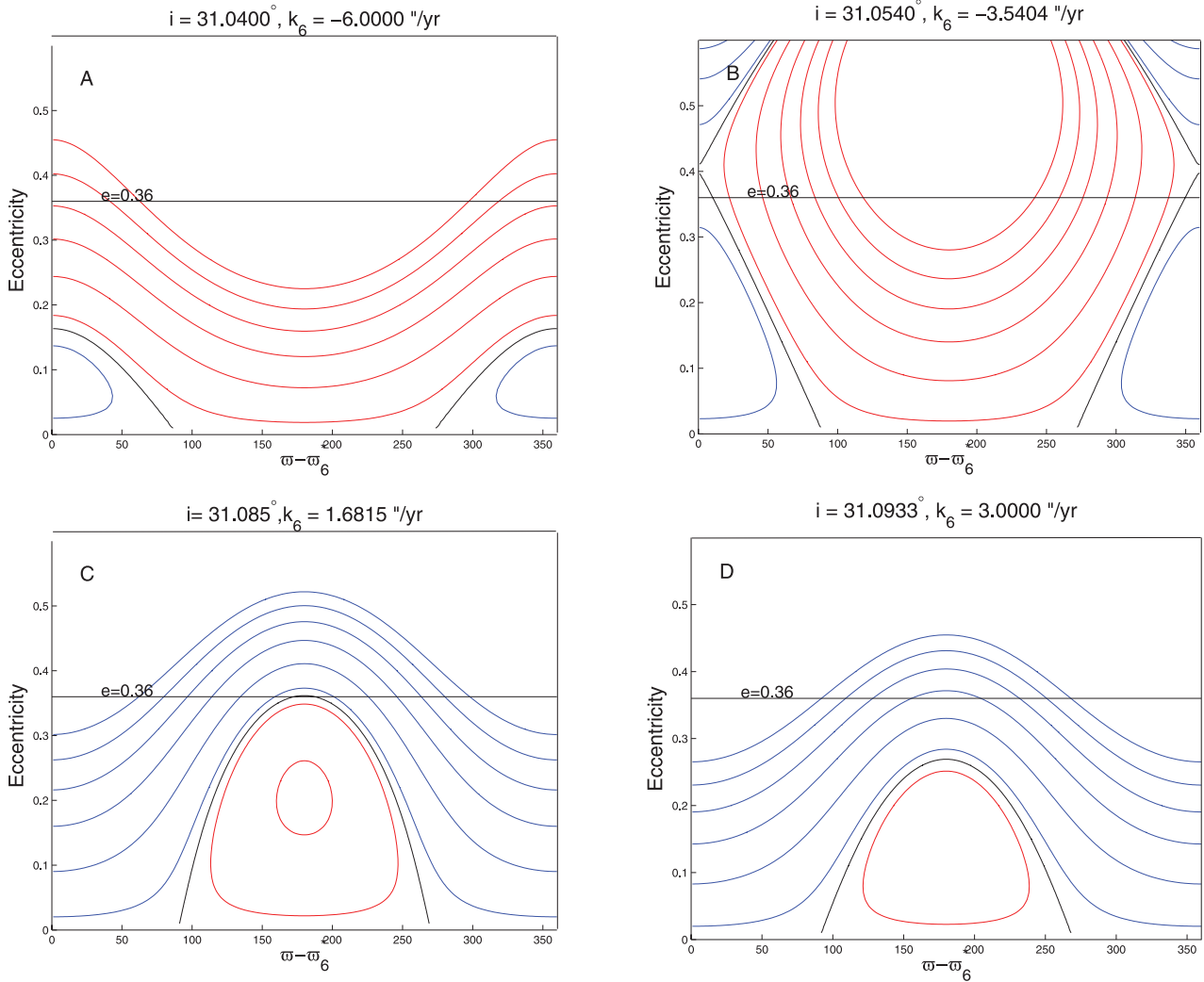


Figure 6. Diagrams of equi-Hamiltonian curves for $a = 2.6$ au and four values of inclinations and of the corresponding k_6 parameters.

For the $e > 0.31$ asteroids in the region of the Pallas family, values of k_6 in the range -120.5 to 10.3 arcsec yr^{-1} are observed. Only five asteroids have values of $|k_6| < 6$ arcsec yr^{-1} and are therefore close enough to the ν_6 resonance separatrix so that the approximations of the Yoshikawa model (i.e. we are in the proximity of the ν_6 resonance, so that the $l_6 = \omega - \omega_6$ variable moves much slower than the $l_5 = \omega - \omega_5$ one, Yoshikawa 1987) hold.

Following the approach of Yoshikawa (1987), we computed equi-Hamiltonian curves for two values of the semimajor axis [$a = 2.6$ au in the Barcelona area (Fig. 6) and $a = 2.8$ au in the Pallas region (Fig. 7)] and four different values of the inclination and of the corresponding value of k_6 .

Fig. 6 (panel A) displays equi-Hamiltonian curves for $k_6 = -6.000$ arcsec yr^{-1} in the $(e, \omega - \omega_6^*)$ plane.² Since the Yoshikawa model is not accurate for eccentricities larger than 0.6, the plots display values of e in the range (0,0.6) only. The black line displays the level $F_{\nu_6} = 0$, red lines represent negative values of F_{ν_6} and blue lines represent positive values. The horizontal line shows the $e = 0.360$ level that corresponds to the eccentricity that an aster-

oid at $a = 2.6$ au should have in order to have its pericentre equal to the apocentre of Mars. For $k_6 = -6.000$ arcsec yr^{-1} , there is a libration island at $\omega - \omega_6^* = 0^\circ$, but overall this does not perturb significantly the circulating red orbits. Objects with eccentricities at $\omega - \omega_6^* = 0^\circ$ less than 0.360 are not pushed into a region of Martian close encounters and should present high values of Lyapunov times. For $k_6 = -3.5404$ arcsec yr^{-1} (Fig. 6, panel B), circulating orbits of negative energies are now trapped in a libration island, while the former librating orbits of positive energies now become circulating ones. All asteroids in the libration island and several of the asteroids on circulation orbits are eventually forced to reach Mars-crossing eccentricities, and should be expected to present low Lyapunov times.

For $k_6 = 1.6815$ arcsec yr^{-1} (Fig. 6, panel C), the curve of zero Hamiltonian has a maximum at $\omega - \omega_6^* = 90^\circ$ for $e = 0.360$. As a consequence, all circulating curves are forced to reach Mars-crossing values of eccentricities and should be rather chaotic. Finally, for $k_6 = 3.0000$ arcsec yr^{-1} (Fig. 6, panel D), most of the low-eccentricity circulating orbits are unperturbed even at $\omega - \omega_6^* = 90^\circ$ and should have rather high Lyapunov times.

Overall, circulating orbits at $a = 2.6$ au in the k_6 range -3.5404 to 1.6815 arcsec yr^{-1} are forced to reach Mars-crossing values of eccentricity and should be strongly perturbed by that planet. One

² See Yoshikawa (1987) for the definition of ω_6^* .

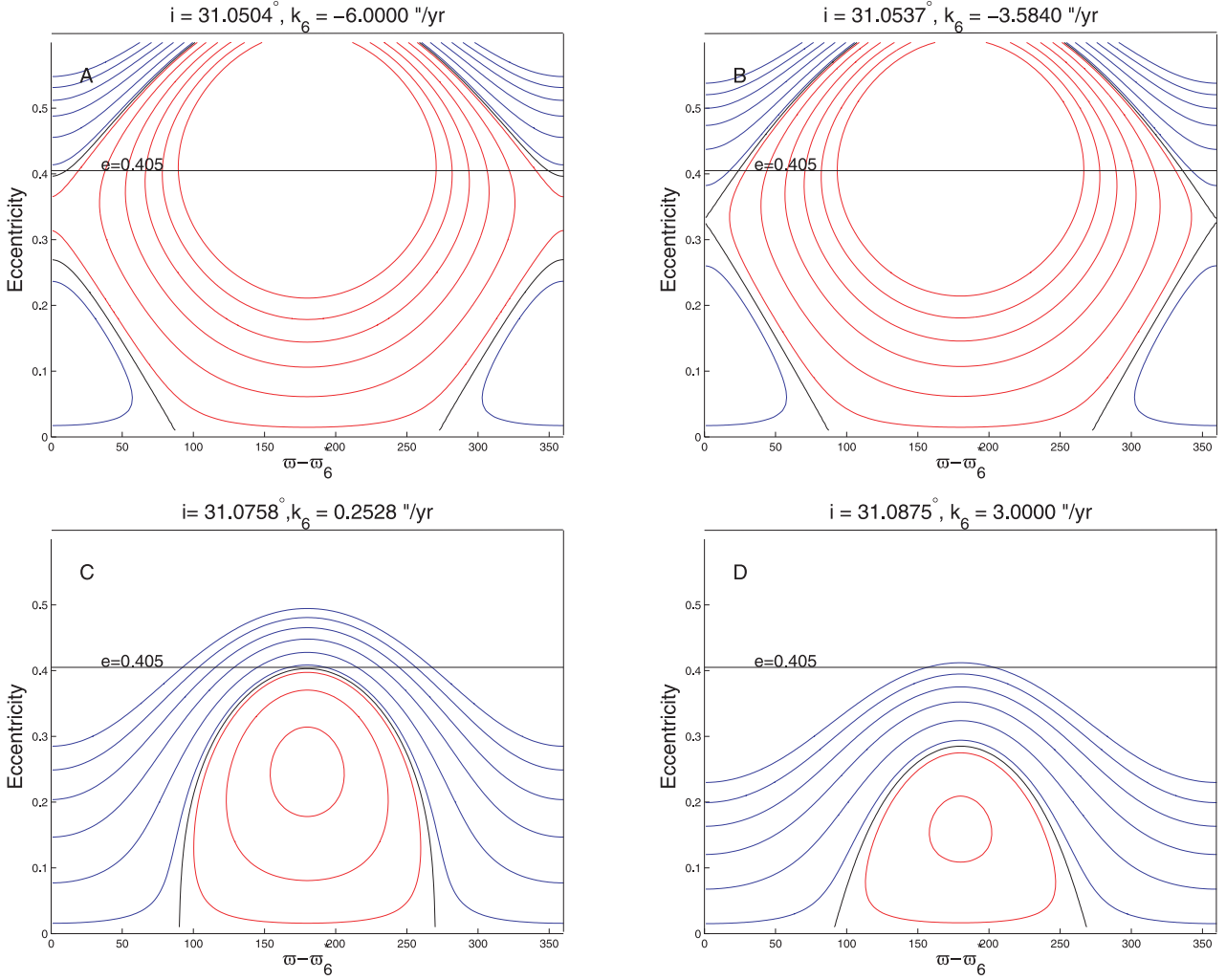


Figure 7. Diagrams of equi-Hamiltonian curves for $a = 2.8$ au and four values of inclinations and of the corresponding k_6 parameters.

may be expecting that chaotic orbits of low Lyapunov times could be found in this range of k_6 values. One may notice that the amplitude in inclination of this chaotic layer is quite limited: only 0.0313° . This, in principle, may be explaining the lack of significant chaotic dynamics found in Carruba (2010b). We will further investigate this hypothesis in Section 4.2.

The topology of the ν_6 resonance is different at larger semimajor axes, as is the minimum value of the eccentricity needed so that $q = Q_{\text{Mars}}$ (0.405 at $a = 2.8$ au, in the Pallas region). Fig. 7 (panel A) displays equi-Hamiltonian curves at $a = 2.8$ au, for $k_6 = -6.0000$ arcsec yr $^{-1}$. For this value of k_6 , there are two islands of libration for positive energy orbits at $w - w_6 = 0^\circ$ and some of the circulating negative energy orbits are not pushed into the Mars-crossing region. Libration, positive energy orbits become circulating for $k_6 = -3.5840$ arcsec yr $^{-1}$ (Fig. 7, panel B) and the libration island of negative energy orbits has a maximum at $e = 0.405$ for $k_6 = 0.2528$ arcsec yr $^{-1}$ (Fig. 7, panel C). Fig. 7 (panel D) shows relatively unperturbed orbits at $k_6 = 3.0000$ arcsec yr $^{-1}$. In this case, the chaotic layer in the k_6 range -3.5840 to 0.2528 arcsec yr $^{-1}$ is even more limited in inclination than for $a = 2.6$ au, with a width of just 0.0221° . We will further investigate the implication of this fact for chaotic orbits in the next section.

4.2 Numerical results

In Carruba (2010b), very little chaotic behaviour was found for orbits near the ν_6 resonance separatrix in the region of the Pallas family, contrary to what observed for orbits near the ν_6 resonance in the Phocaea region. The causes of the absence of significant chaotic behaviour were left as an unanswered question. In the previous section, we saw that chaotic behaviour should be expected, according to the Yoshikawa (1987) model of the ν_6 resonance, for values of k_6 in the range -3.5404 to 1.6815 arcsec yr $^{-1}$ at 2.6 au and in the range -3.5840 to 0.2528 arcsec yr $^{-1}$ at 2.8 au. In both cases, the amplitude in inclination of the chaotic layer was of only 0.0313° at 2.6 au and less at 2.8 au.

To further confirm that the chaotic behaviour of small-inclination asteroids is indeed caused by the ν_6 secular resonance, we computed values of k_6 for the test particles that we used for estimates of Lyapunov times in the $(a, \sin i)$ plane in Carruba (2010b). Fig. 8 (panel A) displays a blow-up of the low-inclination region of small Lyapunov times found in Carruba (2009b). Orbits with Lyapunov times of less than 20 000 yr are displayed as yellow dots. Fig. 8 (panel B) shows the orbital location of $-3.5404 < k_6 < 1.6815$ arcsec yr $^{-1}$ asteroids in the space of averaged $(a, \sin i)$ (red

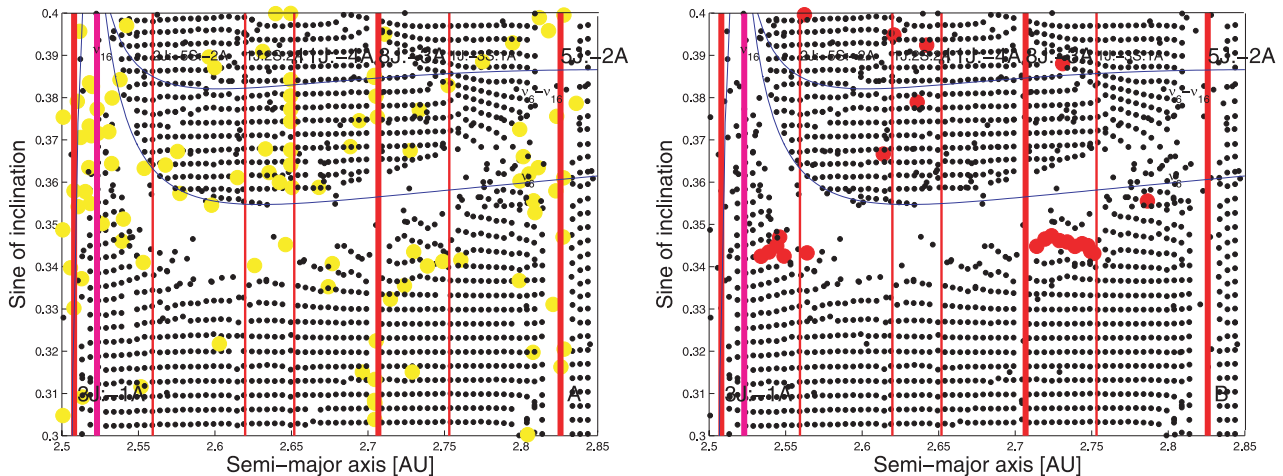


Figure 8. Panel A: Lyapunov times projected in the space of averaged (a , $\sin i$) elements. Yellow circles display Lyapunov times smaller than 20 000 yr, while black dots are associated with times larger than 20 000 yr. Panel B: k_6 values in the same range of averaged (a , $\sin i$) elements. Red full dots display values of k_6 in the range $-3.5404 < k_6 < 1.6815$ arcsec yr $^{-1}$.

full dots). Here, we are using the wider 2.6-au criterion in order to maximize the amplitude of the chaotic layer in inclination.

As can be seen in the figure, there is a very good agreement between Lyapunov times near the ν_6 separatrix and results of the Yoshikawa model: only very few particles have values of k_6 in the right range and several of these were so much destabilized by Martian close encounters that they were lost during the length of the integration. The result is that only a few chaotic particles were visible and those match well the predictions of the Yoshikawa model. This result, in our opinion, should explain the absence of the chaotic behaviour near the ν_6 secular resonance separatrix found in Carruba (2010b).

5 LONG-TERM STABILITY OF MINOR FAMILIES AND CLUMPS IN THE REGION

In Carruba (2010b), several small dynamical groups were identified in the region of the Pallas dynamical family. Many of these groups had a limited number of members, sometimes just large enough for the group to be considered a clump. A question left unanswered by the previous work was about the statistical significance of these groups. Were these clusters created by real collisions or were they just random association of bodies that happened to be in nearby orbits for a limited period of time? In order to estimate the statistical significance and the time-scales over which these clusters are still bound, we devised the following numerical experiment: we created two sets of clones of members of the clusters and integrated them with SWIFT-RMVSy.f, the symplectic integrator of Brož (1999), which simulates the diurnal and seasonal versions of the Yarkovsky effect. Using typical values of the Yarkovsky parameters (Carruba et al. 2003), we gave to one set of objects an inclination of the spin axis of 90° , while to the second was assigned an obliquity of -90° . No re-orientations were considered, so that the drift caused by the Yarkovsky effect was the maximum possible.

We integrated the clones of members of the classical and frequency clusters over 200 Myr in the future and 200 Myr in the

past,³ and obtained synthetic proper elements according to the definition of Knežević & Milani (2000) for the clones every 2.4576 Myr. We then re-obtained families and clumps for the set of synthetic proper elements of the clones at each time-step, using the barycentre of the clusters (Carruba 2009b, equation 7) as the first body for the family. As soon as the cluster (obtained for the values of the velocity cut-off of 160 m s^{-1} and of the frequency cut-off of $0.605 \text{ arcsec yr}^{-1}$, Carruba 2009b) reached the minimum number of objects for being considered a clump, the cluster was considered dispersed and a minimum limit for the dispersion time was found.

We will start by discussing the results for the classical families and clumps in the next section.

5.1 Classical groups

In Carruba (2010b), eight minor clumps were identified in the space of proper elements at a velocity cut-off of 122 m s^{-1} : a clump around (208080) 1999 VV180 in the Barcelona area, five clumps around (4203) Brucato, (18511) 1996 SH4, (36240) 1999 VN44, (70280) 1999 RA111 and (75938) 2000 CO80 in the Olympia region, a clump around (33969) 2000 NM13 in the Hansa area and a clump around (40134) 1998 QO53 in the region of the Gallia family. For simplicity, in this section and in the next section, we will refer to the dynamical groups by their first body number only, omitting the name.

Fig. 9 displays the number of members of the integrated (4203) clump (panel A) and of the (40134) clump (panel B), as a function of time. The horizontal black line shows the limit ($N_{\min} = 12$) for a group to be recognized as a clump, while the horizontal red line in panel B shows the limit for a group to be classified as a family ($N_{\text{fam}} = 30$), at a velocity cut-off of 122 m s^{-1} . The vertical line separates the backward and forward integrations. The higher the time

³ Concerning the integration in the past, we should caution the reader that integrations with the Yarkovsky effect are not conservative and therefore technically speaking not time-reversible. Backward integrations are, however, interesting from a statistical point of view, since they allow to increase our sample of integrated objects.

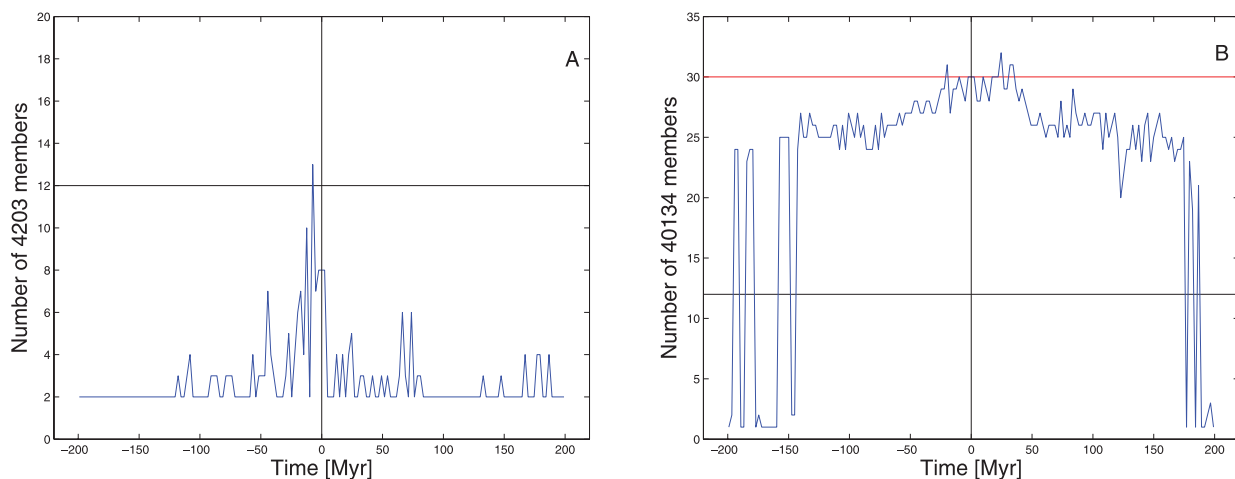


Figure 9. The number of members of the integrated (4203) clump (panel A) and of the (40134) clump (panel B), as a function of time.

Table 1. Detectability times over 400 Myr for classical groups in the Pallas region.

Name	$T(N > 12)$ (Myr)
(208080)	14.75
(4203)	1.25
(18511)	7.38
(36240)	2.46
(70280)	12.30
(75938)	51.61
(33969)	15.98
(40134)	173.26

the integrated clumps have more than $N_{\min}$ members (hereinafter detectability time), the more likely that they are real collisional groups rather than statistical flukes. The (4203) and (40134) groups have the lowest and highest detectability times with $N > N_{\min}$, respectively, taken as the average of the detectability times over the length of the forward and backward integrations. Detectability times for all the classical clumps in the region are given in Table 1.

None of the members of the clumps in the area seems to be currently interacting with non-linear secular resonances (the Hansa families interact with the $\nu_6 - \nu_{16}$ resonance and the Pallas ones with the $2\nu_6 - \nu_5 + \nu_{16}$ resonance). Of all the clumps, those around (40134), (75938), (33969), (208080) and (70280) have the largest detectability time and should be considered reasonable candidates for groups originating from collisional events. The three other clumps have very short detectability times and may be considered statistical flukes. The fact the they are englobed by larger groups in the $(n, g, g + s)$ frequency domain [(4203), (18511) and (70280) are all part of the frequency (4203) family (Brucato family), while (75938) and (36240) form the larger frequency clump around (36240)] seems to confirm this analysis. More details on this subject will be given in Section 5.2.

5.2 Frequency groups

In the $(n, g, g + s)$ domain, two frequency clumps were identified in Carruba (2010b): one around (36240) 1999 VN44 in the Olympia area and the other around (82426) 2001 NB20 in the Hansa area. The group around (4203) Brucato in the Olympia area was a family

in the frequency domain. These groups were identified with a cut-off of $0.605 \text{ arcsec yr}^{-1}$. At this cut-off, a group should have at least 22 members to be identified as a clump and 55 members to be recognized as a family.

The clump in the Hansa area around (82426) quickly dispersed after 1.25 Myr and should not be considered statistically significant. More interesting was the case of the Olympia groups. The (36240) frequency clump that englobes the (75938) and (36240) classical groups was observable as a family for a large interval of time [but not at $T = 0$ Myr, see Fig. 10 (panel A)] and only dispersed for $T = 150$ Myr. It is observable in the past for times up to -180 Myr and it has a detectability time, $N > N_{\min}$ (see Section 5.1), of 149.92 Myr. It should therefore be considered as a statistically robust group.

The (4203) frequency family that englobes the (4203), (18511) and (70280) classical clumps, finally disperses for $T = \pm 330$ Myr (Fig. 10, panel B), and it has a detectability time, over 200 Myr, of 129.03 Myr. It is the largest dynamical group observed in the Olympia region and it should be considered a reliable dynamical family. Results are summarized in Table 2.

6 CONCLUSIONS

In this work, we studied the dynamical evolution of asteroids in the region of the Pallas dynamical family. Amongst other things:

(i) We investigated the long-term stability of asteroids at high eccentricity ($e > 0.31$). These objects are unstable because of encounters with Mars on time-scales of up to 340 Myr. Local background asteroids are currently the major source of high-eccentricity objects, but Barcelona family members will become the dominant source in about 250 Myr;

(ii) We studied the problem of asteroids in the proximity of the ν_6 secular resonance separatrix with analytical (Yoshikawa 1987) and numerical tools. Asteroids with $-3.5404 < k_6 < 1.6815 \text{ arcsec yr}^{-1}$, where $k_6 = b - \nu_6$, b being a coefficient that roughly corresponds to the pericentre precession frequency g ,⁴ are forced to reach values of pericentres low enough to allow them to experience deep close encounters with Mars. The fact that this chaotic layer has a very limited amplitude in inclination (only 0.0313) explains why very

⁴ See Yoshikawa (1987) and Carruba (2010) for the exact definition of b .

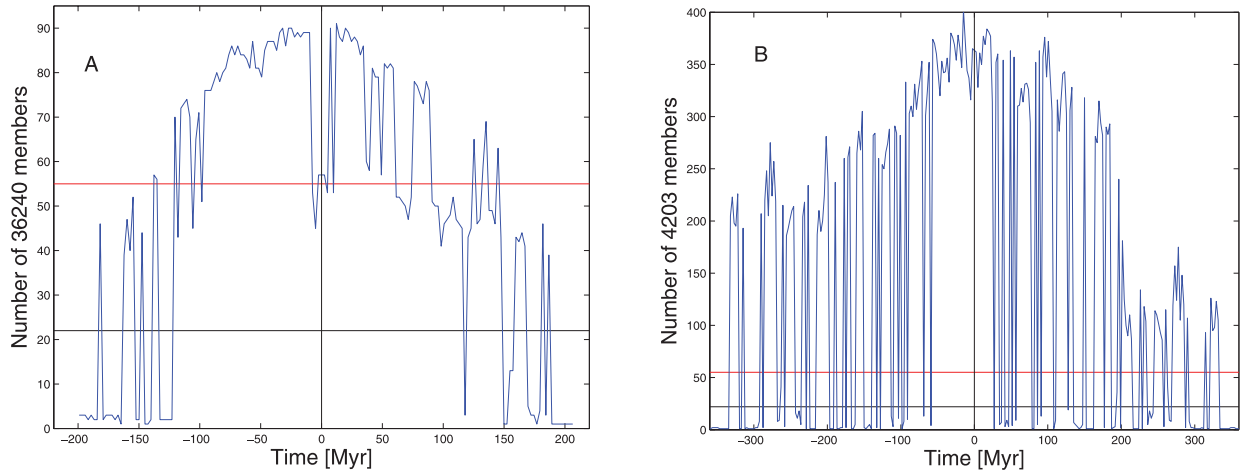


Figure 10. The number of members of the integrated (36240) clump (panel A) and of the (4203) frequency family (panel B), as a function of time.

Table 2. Detectability times over 400 Myr for frequency groups in the Pallas region.

Name	$T(N > 22)$ (Myr)
(4203)	129.03
(36240)	149.92
(82426)	1.25

limited chaotic behaviour was observed near the ν_6 separatrix in Carruba (2010b);

(iii) We investigated the dynamical evolution when the Yarkovsky effect is considered of minor clumps and families identified in Carruba (2010b) in the domains of proper elements and frequencies. We find that none of the minor clumps obtained in Carruba (2010b) is currently interacting with non-linear secular resonances in the region. The classical clumps around (40134) 1998 QO53, (75938) 2000 CO80, (33969) 2000 NM13, (208080) 1999 VV180 and (70280) 1999 RA111 have large detectability times and could be considered reasonable candidates for groups originating from collisional events. We confirm the presence of the (4203) Brucato family observable in the space of proper frequencies ($n, g, g + s$) that has the largest detectability time of all groups in the region.

We believe that this work should have answered some of the questions raised by Carruba (2010b), but as often in science, many other questions are left unanswered. For instance, what is the long-term effect that the $2\nu_6 - \nu_5 + \nu_{16}$ secular resonance and several quasi-resonances with Jupiter and Ceres have on the Pallas family members? Also, is the Brucato frequency family a real collisional group or, as suggested by the presence of asteroids belonging to both the C- and X-type complexes, just an agglomeration of nearby objects? Our current analysis suggests that the family is dynamically stable, so are some of the members of the Brucato family that belong

to the C- or X- complexes interlopers? No information is currently available to draw a final conclusion on these issues that remain, in our opinion, interesting subjects for future work.

ACKNOWLEDGMENTS

We are grateful to the reviewer of this paper, Ricardo Gil-Hutton, for his careful revision and comments that significantly improved the quality of this paper. We would like to thank the São Paulo State Science Foundation (FAPESP) that supported this work via the grant 06/50005-5 and the Brazilian National Research Council (CNPq, grants 302183/2008-6 and 473345/2009-9).

REFERENCES

- Bretagnon P., 1974, *A&A*, 30, 141
- Brož M., 1999, Thesis, Charles Univ., Prague, Czech Republic
- Carruba V., 2009b, *MNRAS*, 398, 1512
- Carruba V., 2010a, *MNRAS*, 403, 1834
- Carruba V., 2010b, *MNRAS*, 408, 580
- Carruba V., Michtchenko T., 2007, *A&A*, 475, 1145
- Carruba V., Michtchenko T., 2009, *A&A*, 493, 267
- Carruba V., Burns J. A., Bottke W., Nesvorný D., 2003, *Icarus*, 162, 308
- Carruba V., Roig F., Michtchenko T., Ferraz-Mello S., Nesvorný D., 2007, *A&A*, 465, 315
- Farinella P., Vokrouhlický D., Hartmann W. K., 1998, *Icarus*, 132, 378
- Gronchi G. F., Milani A., 1999, *A&A*, 341, 928
- Guillens S. A., Vieira Martins R., Gomes R. S., 2002, *AJ*, 124, 2322
- Knežević Z., Milani A., 2000, *Celest. Mech. Dyn. Astron.*, 78, 17
- Levison H. F., Duncan M. J., 2000, *AJ*, 120, 2117
- Nesvorný D., Roig F., Gladman B., Lazzaro D., Carruba V., Moth-Diniz T., 2008, *Icarus*, 183, 85
- Thomas F., Morbidelli A., 1996, *Celest. Mech.*, 64, 209
- Yoshikawa M., 1987, *Celest. Mech.*, 40, 233

This paper has been typeset from a $\text{\LaTeX}$ file prepared by the author.