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Searching for orbits around the triple asteroid 2001SN₂₆₃

M P O Cavalca¹, A F B A Prado¹, J K S Formiga² and V M Gomes³

¹National Institute for Space Research - INPE, São José dos Campos (SP), Brazil

²São Paulo State University - UNESP/ICT, São José dos Campos (SP), Brazil

³São Paulo State University - UNESP/FEG, Guaratinguetá (SP), Brazil

E-mail: mpocavalca@hotmail.com

Abstract. The asteroid 2001SN₂₆₃ is one of the possible targets of a proposed mission that would be the first Brazilian exploration in deep space, the Aster Mission. This asteroid is composed by three bodies: Alpha, Beta, and Gamma, in decreasing order of mass. For this study, it is proposed to split this triple system in two double systems: Alpha-Beta-spacecraft and Alpha-Gamma-spacecraft, all of them considered to be points of mass, such that it is possible to use the circular planar restricted three-body problem as the mathematical model. The goal is to find orbits that can be used by a spacecraft to observe the bodies Beta and Gamma. Each orbit can be identified by the initial conditions of the spacecraft with respect to Beta or Gamma: position and velocity. These orbits are classified by the minimum average distance spacecraft-celestial body. The results showed stable orbits around Beta and Gamma, with an average distance below 1.5 km, under the influence of the gravity of Alpha and the solar radiation pressure.

1. Introduction

In recent years, studies and missions related to asteroids have become recurrent and they account for a new space race [1]. Several missions were made or are under preparation to be launched. Several types of bodies have been considered in the literature, including single, double and even triple systems, like the one proposed by the Aster Mission [2-3], where the main target of study is the triple asteroid 2001SN₂₆₃ [4]. The triple system of asteroids 2001SN₂₆₃ is composed by three bodies, denominated by Alpha, Beta and Gamma, in mass descending order. Alpha is the main body, orbited by Gamma, which is located at a short distance from Alpha, below 4 km. Beta also orbits Alpha, but at a larger distance [5-6], above 16 km. The study of this triple system will be performed dividing it into two double systems: Alpha-Beta-spacecraft and Alpha-Gamma-spacecraft. The mathematical model used for the study of the two systems mentioned above is the restricted circular planar three-body problem [7], where it is assumed that Alpha, the most massive body, is orbited by Beta and Gamma, but one at a time, and that there is a spacecraft with negligible mass orbiting the system. It's also included in the mathematical model the solar radiation pressure [8]. This force depends on the area-to-mass ratio (A/m) of the spacecraft and the initial angular position of the Sun (v).

This paper proposes to map orbits [9-10] that can be used by a spacecraft around Beta and Gamma. Each orbit can be identified by the initial conditions of the spacecraft with respect to Beta or Gamma: position and velocity. The orbits will be sorted by the criterion of minimizing the average distance spacecraft-celestial body, also used in [11]. The results presented here are an approximation, since the equations of the planar restricted three-body problem were used to model the problem, which does not



take into account the irregular shapes of the bodies and the perturbation from one smaller body in the orbits around the other one.

2. Metodology

The mathematical model used to describe the motion of the spacecraft is the circular planar restricted three-body problem, which is presented in [7]. The central body, Alpha, is designated by M_1 ; the smaller body, Beta or Gamma, depending on the smaller asteroid considered, is called M_2 ; and the spacecraft, assumed to have a negligible mass, which is designated by M_3 . In each system, M_1 , M_2 , and M_3 move in the same plane; the bodies M_1 and M_2 rotate around their center of mass in circular orbits, and M_3 travels around the bodies M_1 and M_2 influenced by the potential F_G , generated by them. Added to this dynamical model, the solar radiation pressure is added, and it corresponds to the force F_{SRP} exerted by the light of the Sun, as it directly hits the surface of a space vehicle. This force depends on the relationship between the cross-sectional area of the spacecraft and its mass (A/m); of the surface reflectivity coefficient (C_R); of the solar flux (P_S); and the average distance between the system under consideration and the Sun (r_S). When the solar radiation pressure is considered, the value of A/m is assumed to be $0.01 \text{ m}^2/\text{kg}$, a value compatible with the Aster mission, where it is proposed a spacecraft with mass 150 kg [3] and an area near 1.5 m^2 . It is also considered the initial angular position of the Sun (v) with respect to the system being studied. Equation 1 presents the complete dynamic model:

$$F = F_G + F_{SRP} = G \left(-\frac{m_1}{r_1^2} \hat{r}_1 - \frac{m_2}{r_2^2} \hat{r}_2 \right) - C_R P_S \frac{A}{m} \frac{1}{r_S^2} \hat{r}_S \quad (1)$$

where m_1 and m_2 are the masses of M_1 and M_2 , r_1 is the distance between M_3 and M_1 , and r_2 is the distance between M_3 and M_2 . The criterion of the average distance minimization is to obtain the initial conditions of the spacecraft, for each orbit around M_2 , that gives the lowest average distance between these two bodies. The mean distance between M_3 and M_2 for a given time interval (T) is shown in Equation 2.

$$r_{avg} = \frac{1}{T} \int_0^T r_2(t) dt \quad (2)$$

Where r_2 is the distance between M_3 and M_2 . Then, it is possible to classify the orbits in order to obtain the initial conditions that minimize the distance between the two bodies and this is shown by the color maps presented in Figures 2 and 4. They show the distribution of the mean distances as a function the initial distance (D) from the vehicle with respect to the moon and the angle of reference (θ), for fixed components of the velocity v_x and v_y .

Next, the algorithm to find the orbits is described. Initially, it is assumed that Alpha (M_1) and Beta or Gamma (M_2) are aligned in an inertial frame and the spacecraft (M_3) is within a certain distance from the moon. Figure 1 shows the position of M_1 , M_2 , and M_3 with respect to an inertial frame and the initial conditions of M_3 with respect to M_2 , which identifies each orbit. The initial angular position of the Sun is given by v . This angle defines the direction of the solar radiation pressure: zero is when the Sun is aligned to the right and on the same line as M_1 and M_2 . Other values are obtained considering the counterclockwise direction for increasing v . Using Figure 1, it is also possible to determine the initial position and velocity of the spacecraft and then numerically integrate the equations of motion for the time interval of 30 days. The numerical method used to integrate these equations is the Runge-Kutta 78 and the integration step is variable. After each step, it is calculated if there was a collision between M_3 and M_2 , and then the average distance between them is calculated. The next step is to classify the orbits

according to the average distance to identify the initial conditions that keep the spacecraft in the smallest average distance from the moon.

3. Results and Discussion

The method presented here search for the initial conditions for orbits that survive for up to 30 days and that can be used by a spacecraft to travel around Beta and Gamma. It generates a large number of results. The results that will be presented here are those that are more interesting with respect to the lowest values of average distance between the spacecraft and Beta or Gamma and also those results related to the interference of the solar radiation pressure in the orbits. All the initial conditions and parameters were chosen by these criteria. Table 1 shows the numerical data of the bodies Alpha, Beta, and Gamma: mass, average diameter, semi-major axis, and orbital period used in the simulations. Initially, the orbits around Beta will be studied, and then this study is extended to Gamma.

Table 1. Physical and orbital components of the three bodies of the Asteroid 2001SN₂₆₃. [4]

Celestial body	Average diameter (km)	Mass (10^{10} kg)	Semi-major axis	Orbital period
Alpha	2.60	917.466	1.99 UA*	2.80 yr*
Beta	0.780	24.039	16.663 UA**	6.225 d**
Gamma	0.580	9.773	3.804 UA**	0.686 d**

*Around the Sun

**Around Alpha

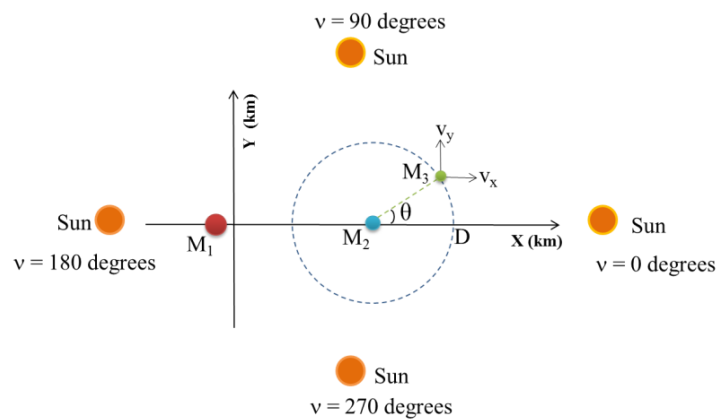


Figure 1. Geometry of the problem involving the three bodies and the initial conditions.

3.1. Study of orbits around Beta

Figures 2(a) to 2(c) show the color maps for orbits around Beta. They show the distribution of mean distances with the velocity components fixed $v_x = -0.1$ m/s and $v_y = -0.07$ m/s. The angle θ varies from 292 to 320 degrees and the initial distances vary from 0.874 to 1.20 km. In Figure 2(a) the solar radiation pressure is not considered, and in Figures 2(b) and 2(c) the solar radiation pressure is considered with $A/m = 0.01$ m²/kg for v equal to 180 and 270 degrees, respectively. As already explained, those values are used based in previous simulations, which showed that those values give orbits closer to the smaller asteroid considered. The initial conditions indicating collisions between the spacecraft and Beta in less than 30 days are indicated by the blank regions in the figures. Figures 2(a) to 2(c) shows that the largest

average distances occurs for the highest values of D , the initial distance, and the smallest mean distances occur for the lowest values of D . The solar radiation pressure substantially reduces the sets of initial conditions that generate orbits around Beta. This is proven by comparing the size of the white regions. In Figure 2(a), which does not consider the radiation pressure, this region corresponds to 1/4 of the total area of the graph, whereas in Figures 2(b) and 2(c), which consider the radiation pressure, this region covers about 2/3 of the total area. The figures have their minimum and maximum mean distance values: 0.874 and 1.476 km, 1.197 and 1.480 km, 0.968 and 1.430 km, respectively.

Next, a set of initial conditions of each color map is chosen. Figures 2(a) to 2(b) draw the trajectories around Beta. Figures 3(a), 3(b), and 3(c) show these trajectories, which have the velocities equal to $v_x = -0.1$ m/s and $v_y = -0.07$ m/s and the respective values of D and θ equal to: 0.97 km and 303 degrees, 1.12 km and 306 degrees, 1.01 km and 294 degrees. Beta is fixed at the origin of the reference system, and it is represented by the circular form. The circular shape for Beta was adopted for being the simplest and also consistent with the model.

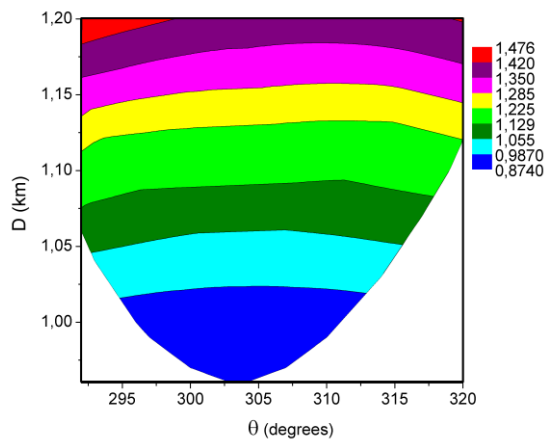
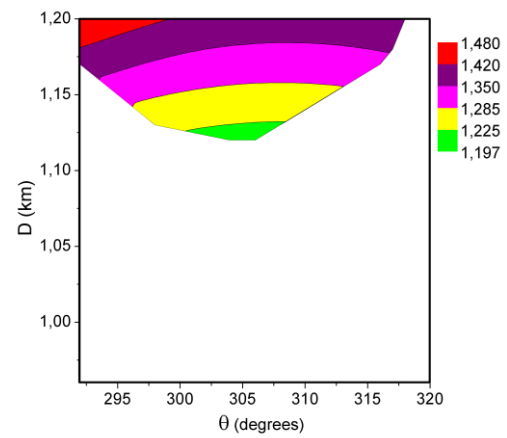
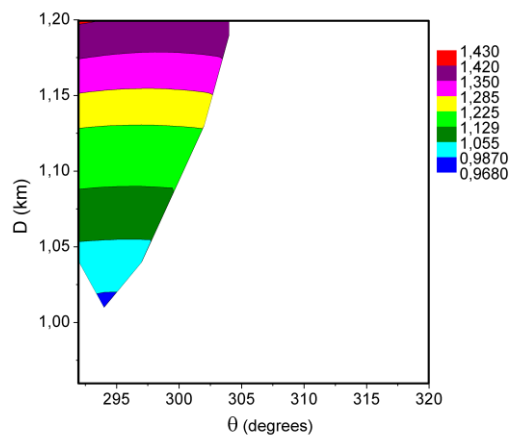
(a) $A/m = 0$ (without srp)(b) $A/m = 0.01 \text{ m}^2/\text{kg}$ and $v = 180$ degrees(c) $A/m = 0.01 \text{ m}^2/\text{kg}$ and $v = 270$ degrees

Figure 2. Distribution of the average distance values as a function of D (km) and θ (degrees) for fixed velocity values in $v_x = -0.1$ m/s and $v_y = -0.07$ m/s.

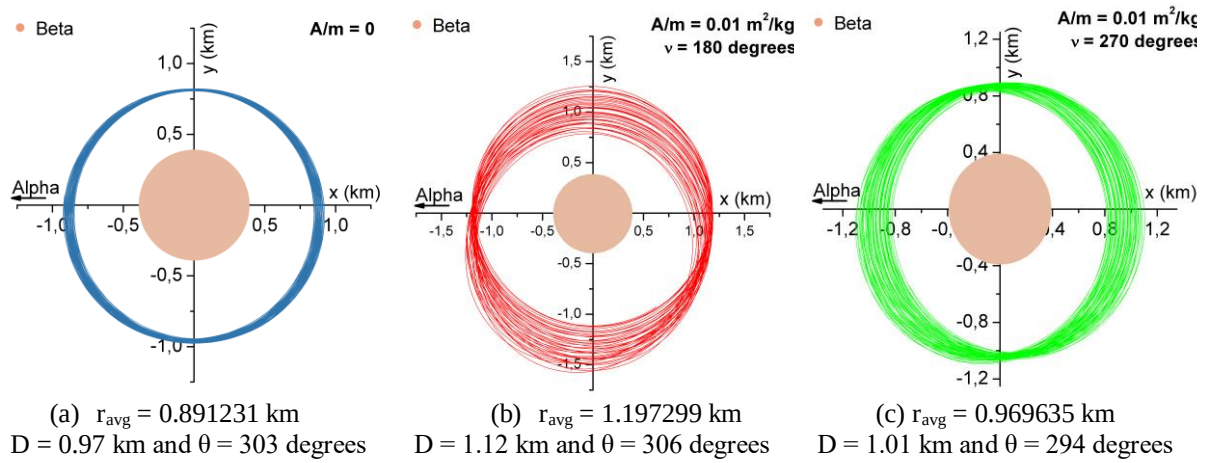


Figure 3. Trajectories around Beta for fixed velocity values in $v_x = -0.1$ m/s and $v_y = -0.07$ m/s.

Figures 3(a), 3(b) and 3(c) present three trajectories around Beta, which turn in the retrograde, clockwise direction. It turns out that they are quite stable and very close to Beta, at least 800 m away. In Figure 3(a), where only the gravitational interaction between Alpha and Beta is being considered, with $A/m = 0$, the average distance is 891.231 m. It is important to remember that Beta is about 38 times smaller than Alpha and it is located less than 17 km from it and, even then, these characteristics are sufficient for the existence of fairly cohesive orbits. For Figure 3(b), where the solar radiation pressure is considered with $A/m = 0.01$ m²/kg and $v = 180$ degrees, the average distance is 1.197299 km. In this case the orbit appears to deform transversely, in the direction of the y-axis, and no longer presents such a cohesive profile, especially in the upper and lower parts of the orbit. In Figure 3(c), where the solar radiation pressure is taken with $A/m = 0.01$ m²/kg and $v = 270$ degrees, the mean distance is 969.635 m, and the orbit deforms in the longitudinal direction of the x-axis. It also does not have such a cohesive profile, but now it occurs, particularly, on its sides. It behaves like the last two orbits had been "flattened" by the solar radiation pressure. It is possible to conclude, for these two orbits around Beta, that the solar radiation pressure deforms the shape of the trajectory in the transverse direction for $v = 180$ degrees, and in the longitudinal direction for $v = 270$ degrees, that is, the deformation of the orbit, for these two examples, is perpendicular to the initial angular position of the Sun (v) for a period of 30 days.

3.2. Study of orbits around Gamma

Figures 4(a) to 4(c) show the color maps for orbits around Gamma. They show the distribution of mean distances where the velocity components are fixed in $v_x = -0.04$ m/s and $v_y = -0.07$ m/s. The angle θ varies from 295 to 303 degrees and the initial distances vary from 781 to 910 m. In Figure 4(a) the solar radiation pressure is not considered, and in Figures 4(b) and 4(c) the solar radiation pressure is considered with $A/m = 0.01$ m²/kg for v equal to 90 and 180 degrees, respectively. The initial conditions indicating collisions between the spacecraft and Gamma in less than 30 days are indicated by the blank regions delimited by the edges of the figures.

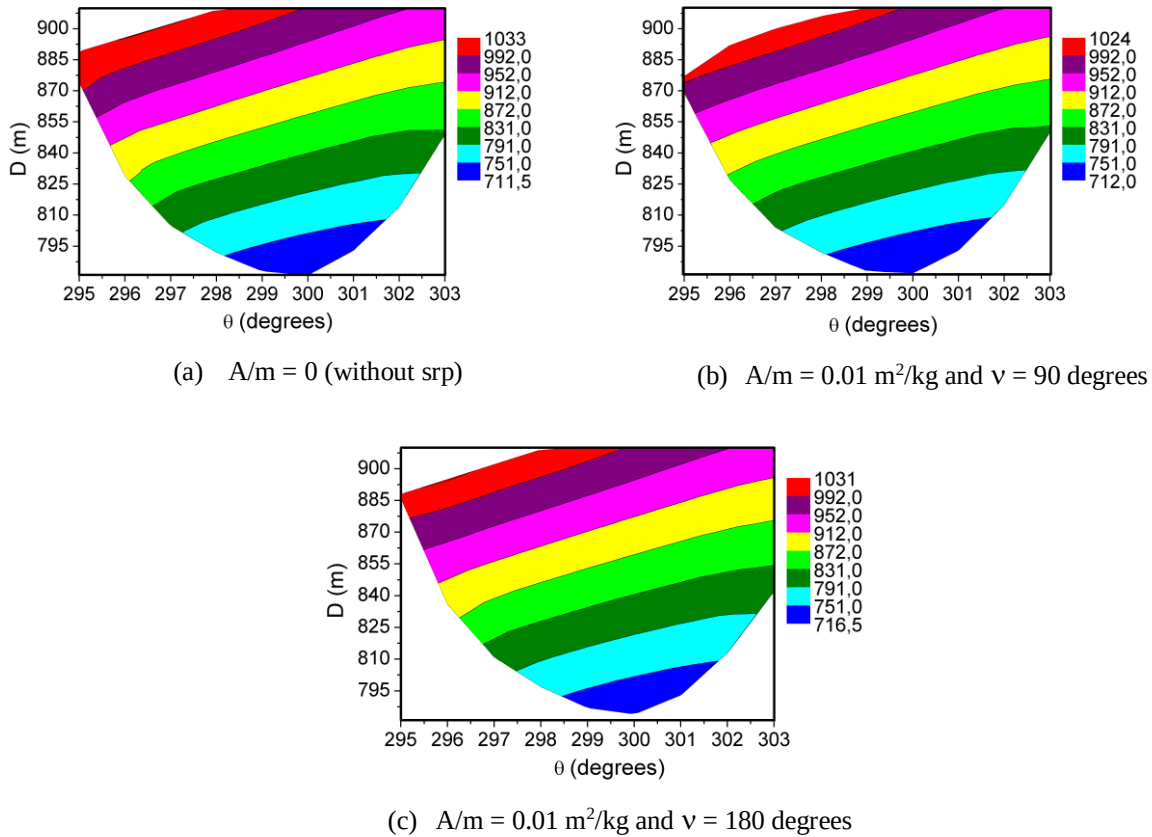


Figure 4. Distribution of the average distance values as a function of D (m) and θ (degrees) for fixed velocity values in $v_x = -0.04 \text{ m/s}$ and $v_y = -0.07 \text{ m/s}$.

Visually, Figures 4(a), 4(b), and 4(c) do not show large differences among them. The mean distance distribution occurs continuously varying from the smallest to the largest, as the initial distance D increases. The figures differ in their minimum and maximum mean distance values: 711 and 1033 m, 712 and 1024 m, 716 and 1031 m, respectively. The regions with the largest differences are located near the values of D between 860 and 890 m and θ between 295 and 296 degrees.

Figures 5(a), 5(b) and 5(c) show trajectories around Gamma considering the same set of initial conditions for the cases with and without radiation pressure, $A/m = 0$ and $A/m = 0.01 \text{ m}^2/\text{kg}$, for v equal to 90 e 180 degrees, respectively. The initial conditions are: $D = 787 \text{ m}$, $\theta = 299$ degrees, $v_x = -0.04 \text{ m/s}$ and $v_y = -0.07 \text{ m/s}$. Gamma is fixed at the origin of the reference systems, represented by a circular form in scale. The position of Alpha is indicated by an arrow to the left on the x-axis.

Figures 5(a), 5(b) and 5(c) present three trajectories rotating in a retrograde (clockwise) direction around Gamma. The profile of the trajectories is not as cohesive as those found around Beta, but they present average distances below 735 m. Figure 5(a) considers only the gravitational interaction between Alpha and Gamma and the average distance is 733.3133 m. Gamma is 100 times smaller than Alpha and it is located less than 4 km from it, so the trajectories suffer more influence from Alpha, compared to Beta. For the values of v of 90 and 180 degrees, respectively, the values of the mean distance were 731.8501 and 733.0861 m. The average distances did not show large differences, but it is noted that these values decreased and also a change of pattern of rotation in the format of the trajectories is observed compared to Figure 5(a), where the radiation pressure is not considered.

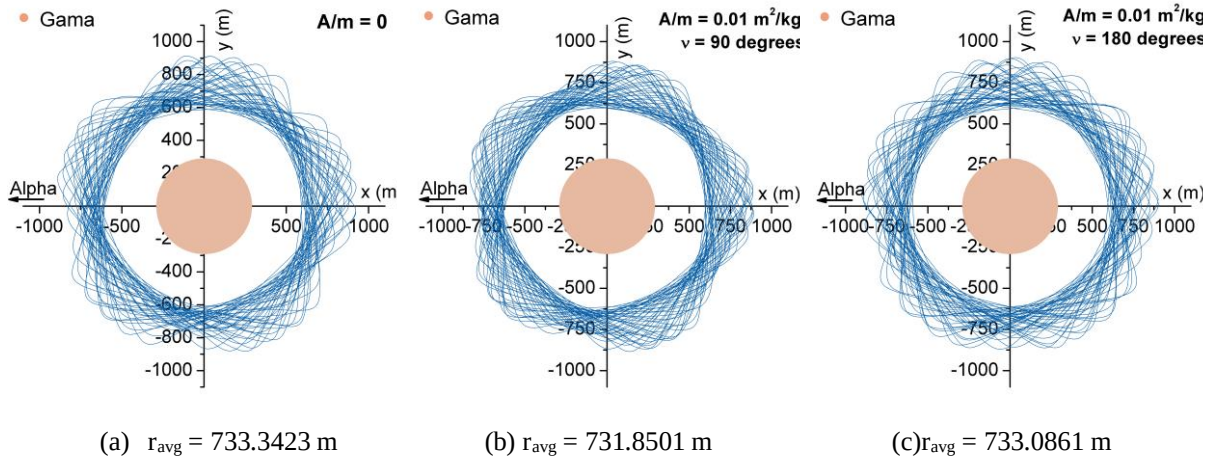


Figure 5. Trajectories around Gamma for initial conditions: $D = 787 \text{ m}$, $\theta = 299 \text{ degrees}$, $v_x = -0.04 \text{ m/s}$, and $v_y = -0.07 \text{ m/s}$.

4. Conclusion

Using the algorithm developed here, based in the circular planar model of the restricted three body problem and solar radiation pressure, it was found orbits around Beta and Gamma in the respective systems Alpha-Beta and Alpha-Gamma. It was also identified initial conditions that minimize the average distance between Beta and Gamma and the spacecraft using color maps. It was also possible to observe the influence of the solar radiation pressure on the shape of the trajectories and on the mean distance values for the Beta and Gamma. The results for the 30 day time period showed stable orbits around Beta and Gamma, with average distances below 1.5 km.

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References

- [1] ESA 2017 Missions to asteroids. <http://sci.esa.int/rosetta/54342-missions-to-asteroids/>. Accessed 15 March 2017
- [2] Sukhanov A, Velho H, Macau E, and Winter O C 2010 The Aster Project: Flight to a near-earth asteroid *Cosmic Res.*, pp 443–450
- [3] Macau E, Winter O C, Velho H, and Sukhanov A 2012 Missão Aster: Indo aonde ninguém jamais foi *Revista Ciência Hoje*, pp 20–25
- [4] Fang J, Margot J L, Brozovic M, Nolan M C, Benner L A M, and Taylor P A 2011 Orbits of near-earth asteroid triples 2001SN₂₆₃ and 1994 cc: Properties, origin, and evolution *Astron. J.*, **141** pp 154–168
- [5] Masago B Y P L, Prado A F B A, Chiaradia A P M, and Gomes V M 2016 Developing the precessing inclined bi-elliptical four-body problem with radiation pressure- to search for orbits in the triple asteroid 2001SN₂₆₃ *Adv. Space Res.*, **57** pp 962–982
- [6] Prado A F B A 2014 Mapping orbits around the asteroid 2001SN₂₆₃ *Adv. Space Res.*, **53** pp 877–889
- [7] Szebehely V 1967 *Theory of Orbits* Academic Press New York
- [8] Montenbruck O and Gill E 2001 *Satellite Orbits* Springer-Verlag Berlin Heidelberg, pp 77–79
- [9] Araújo R A N, Winter O C, Prado A F B A, and Sukhanov A 2012 Stability regions around the components of the triple system 2001SN₂₆₃ *Mont. Not. R. Astron. Soc.*, pp 3058–3073
- [10] Araujo R A N, Winter O C, and Prado A F B A 2015 Stable retrograde orbits around the triple system 2001SN₂₆₃ *Mont. Not. R. Astron. Soc.*, **449** pp 4404–4414
- [11] Prado A F B A 2015 A dynamical study of distant retrograde orbits in moons and double asteroids of the solar system *ISTS Kobe, Japan* **1** pp 1–6