# Stabilizing the invisible axion in 3-3-1 models

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By introducing local $Z_N$ symmetries with $N=11,13$ in two 3-3-1 models, it is possible to implement an automatic Peccei-Quinn symmetry, keeping the axion protected against gravitational effects at the same time. Both models have a $Z_2$ domain wall problem and the neutrinos are strictly Dirac particles.

## I. INTRODUCTION

Recently, observations of the core mass distribution in the cluster of Galaxies Abell 2029 using the NASA's Chandra X-ray Observatory suggest the existence of cold dark matter (CDM) [1]. On the other hand, the Wilkinson Microwave Anisotropy Probe measurements of the cosmic microwave background temperature anisotropy and polarization are also consistent with CDM and a positive cosmological constant [2]. Although the exact nature of the CDM is not known yet, candidates for this sort of matter are elementary particles such as neutralinos or the invisible axion [3]. However, early invisible axion models [4,5] are unstable against quantum gravitational effects [6], which may generate a large axion mass and also spoil the value of the $\bar{\theta}_{\text{eff}}$ parameter. One way to stabilize the axion is by considering large discrete gauge symmetries in the sense of Ref. [7] as was done in the multi-Higgs-boson extension of the standard model [8], the 3-3-1 model [9], or the supersymmetric model [10]. The search for dark matter is of course related to the search for new physics beyond the standard model which in turn is related to the existence of new fundamental energy scales. In the literature, the most easily recognized fundamental energy scales are those related to supersymmetry, the neutrino masses, grand unification, and superstring theory.

In this vein, it is worth recalling once more that it has been known for a long time that the measured value of the electroweak mixing angle $\sin^2\theta_W(M_Z)=0.23113 \lesssim 1/4$ appears to obey, at an energy scale $\mu$, an $SU(3)$ symmetry in such a way that $\sin^2\theta_W(\mu)=1/4$ [11]. Hence, if the value of $\sin^2\theta_W$ is not an accident, it may be considered as an indication of a new fundamental energy scale of the order of a few TeVs. Notwithstanding, in models with $SU(3)$ electroweak symmetry there is trouble when we try to incorporate quarks. A solution to this issue is to introduce an extra $U(1)$ factor such as in 3-3-1 models [12,13], to embed the model in a Pati-Salam-like model [14], or even to embed it in theories of TeV gravity [15].

Independently of the axion or dark matter issues, 3-3-1 models are interesting possibilities, on their own, for physics at the TeV scale. At low energies they coincide with the standard model and some of them give at least a partial explanation of some fundamental questions that are accommodated but not explained by the standard model. For instance, (i) in order to cancel the triangle anomalies the number of

generations must be three or a multiple of three; (ii) the model of Ref. [12] predicts that $(g'/g)^2=\sin^2\theta_W/(1-4\sin^2\theta_W)$; thus there is a Landau pole at the energy scale $\mu$ at which $\sin^2\theta_W(\mu)=1/4$, and according to recent calculations $\mu\sim 4$ TeV [16]; (iii) the quantization of the electric charge [17] and the vectorial character of the electromagnetic interactions [18] do not depend on the nature of the neutrinos, i.e., whether they are Dirac or Majorana particles; and (iv) the model possesses $\mathcal{N}=1$ supersymmetry naturally at the $\mu$ scale [19]. If right-handed neutrinos are considered to transform nontrivially, 3-3-1 models [12,13] can be embedded in a model with 3-4-1 gauge symmetry in which leptons transform as $(\nu_l,l,\nu_l^c,l^c)_L\sim(\mathbf{1},\mathbf{4},0)$ under each gauge factor [20].

Models with $SU(3)$ [or $SU(4)$] symmetry may have doubly charged vector bosons. These types of bileptons may be detected by measuring the left-right asymmetries in Møller scattering [21], for instance, at the E158 SLAC experiments (which use 48 GeV polarized electrons scattering off unpolarized electrons in a liquid hydrogen target [22]); or in future lepton-lepton accelerators. It is interesting that the weak interaction's parity nonconservation has never been observed in lepton-lepton scattering. Those asymmetries may also be used for seeking a heavy neutral $Z'^0$ vector boson, which is also a prediction of these models, in $e\mu$ collisions [23]. Singly and doubly charged vector bileptons may also be produced in $e^-\gamma$ [24] or $\gamma\gamma$ [25] or hadron [26] colliders. New heavy quarks are also part of the electroweak quark multiplets in the minimal model representation. They are singlets under the standard model $SU(2)_L\otimes U(1)_Y$ group symmetry. In some versions their electric charge is different from the usual one, so that it can be used to distinguish such a model from their viable competitors. In fact, the $p\bar{p}$ production and decay of these exotic quarks at the energies of the Tevatron have been studied in Ref. [27] where a lower bound of 250 GeV on their masses was found. This sort of model is also predictive with respect to neutrino masses [28]; the models can implement the large mixing angle Mikheyev-Smirnov-Wolfenstein solution to the solar neutrino issue [29], and also the almost bimaximal mixing matrix in the lepton sector [30].

Summarizing, from the present experimental data, say those from the CERN $e^+e^-$ collider LEP, 3-3-1 models are safe if the symmetry breaking from 3-3-1 to 3-2-1 occurs at the level of TeVs; however, they have rich phenomenological

consequences as we mentioned above. It will be interesting to search for some of the new particles that are present in these models, as extra Higgs scalars, exotic quarks, and vector bileptons, at the energies of the upgraded DESY *ep* collider HERA and Tevatron [26,31]. The scalar sectors are equivalent to multi-Higgs-boson extensions of the standard model; for instance, under $SU(2)_L \otimes U(1)_Y$ the model with three triplets has two doublets and several non-Hermitian singlets, while the model with a sextet has three doublets, a complex triplet, and several complex singlets. In particular, the neutral singlet ($\chi^0$) is Z-phobic [its coupling with $Z^0$ vanishes when the scale of the $SU(3)_L$ symmetry goes to infinity] and for this reason it evades the LEP constraints. For a finite $SU(3)_L$ energy scale there are corrections that can be calculated by using the oblique *S*, *T*, and *U* radiative parameters which constrain the allowed masses for the leptoquarks and bileptons [32]. These masses are of the same order of magnitude, a few TeV, as those allowed by the running of the coupling constants. Through the condition $\sin^2\theta_W(\mu)=0.25$, the running is sensitive to a new degree of freedom. Hence, the masses of exotic scalars and bileptons run from hundreds of GeV to a few TeV [33]. We will return to this point later.

Turning back to the axion, the interesting point is that a Peccei-Quinn (PQ) symmetry [34] is almost automatic in the classical Lagrangian of 3-3-1 models. It is only necessary to avoid a trilinear term in the scalar potential by introducing a $Z_2$ symmetry [35]. Unfortunately, even in this case the PQ symmetry is explicitly broken by gravity effects. In order to stabilize the axion, and at the same time automatically implement the PQ symmetry, we must introduce local discrete symmetries $Z_N$. In fact, recently it was shown that in a version of the Tonasse and Pleitez 3-3-1 model [13] it is possible to implement both symmetries $Z_{13}$ and PQ automatically; thus the axion is naturally light and there is no domain wall problem [9].

We will consider in this work two 3-3-1 models in which only the known leptons transform nontrivially under the gauge symmetry, as in Ref. [12], but we add also right-handed neutrinos and exotic charged leptons transforming as singlets. In one model (model A) we consider a scalar sextet but it is possible to use only three scalar triplets (model B). Both models admit a large enough discrete $Z_N$ symmetry, implying a natural light invisible axion.

## II. THE AXION IN TWO 3-3-1 MODELS

We will consider two versions of the 3-3-1 model of Ref. [12]. In model A we use three scalar triplets and a sextet, while in model B we avoid the scalar sextet. In both models we introduce also a scalar singlet, $\phi \sim (\mathbf{1},\mathbf{1},0)$, and lepton singlets.

The quark and lepton sectors have the same representation content in both models. We have quarks transforming, under $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$, as follows: $Q_{mL} = (d_m, u_m, j_m)_L^T \sim (\mathbf{3}, \mathbf{3}^*, -1/3)$, $m=1,2$; $Q_{3L} = (u_3, d_3, J)_L^T \sim (\mathbf{3}, \mathbf{3}, 2/3)$, and the corresponding right-handed components in singlets, $u_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, 2/3)$, $d_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, -1/3)$, $\alpha = 1,2,3$; $J_R \sim (\mathbf{3}, \mathbf{1}, 5/3)$; $j_{mR} \sim (\mathbf{3}, \mathbf{1}, -4/3)$; the leptons are the known ones and transform as triplets $(\mathbf{3}_a, 0)$, $\Psi_{aL} = (\nu_a, l_a, l_a^c)_L^T$, $a$

$= e, \mu, \tau$, and we also add right-handed neutrinos and a charged lepton in the singlets $\nu_{aR} \sim (\mathbf{1},\mathbf{1},0)$, $E_{L,R} \sim (\mathbf{1},\mathbf{1}, -1)$ [36,37]. The scalar sector, in the minimal version, has only three triplets $\chi = (\chi^-, \chi^{--}, \chi^0)^T$, $\rho = (\rho^+, \rho^0, \rho^{++})^T$, $\eta = (\eta^0, \eta_1^-, \eta_2^+)^T$, transforming as $(\mathbf{1},\mathbf{3},-1),(\mathbf{1},\mathbf{3},1)$, and $(\mathbf{1},\mathbf{3},0)$, respectively, and a scalar singlet $\phi \sim (\mathbf{1},\mathbf{1},0)$.

With the quark and scalar multiplets above we have the Yukawa interactions

$$-\mathcal{L}_Y^q = \bar{Q}_{iL}(F_{i\alpha}u_{\alpha R}\rho^* + \tilde{F}_{i\alpha}d_{\alpha R}\eta^*) + \lambda_{im}\bar{Q}_{iL}j_{mR}\chi^*$$
$$+ \bar{Q}_{3L}(G_{1\alpha}u_{\alpha R}\eta + \tilde{G}_{1\alpha}d_{\alpha R}\rho) + \lambda_1\bar{Q}_{3L}J'_{1R}\chi + \text{H.c.,} \tag{1}$$

where repeated indices mean summation.

### A. Model with a scalar sextet (model A)

In this model we add a scalar sextet $S \sim (\mathbf{1},\mathbf{6},0)$ with the following electric charge assignment:

$$S = \begin{pmatrix} \sigma_1^0 & h_1^- & h_2^+ \\ h_1^- & H_1^{--} & \sigma_2^0 \\ h_2^+ & \sigma_2^0 & H_2^{++} \end{pmatrix}, \tag{2}$$

and we will assume that only $\sigma_2^0$ gets a nonzero vacuum expectation value in order to give the correct mass to the known charged leptons plus a mixing with the heavy leptons ($K_a$ and $K_a'$ terms below). The Yukawa interactions in the lepton sector are given by

$$-\mathcal{L}_Y^l = H_{ab}^\nu \bar{\Psi}_{aL}\nu_{bR}\eta + H_{ab}^l \bar{\Psi}_{aL}S(\Psi_{bL})^c + K_a\bar{\Psi}_{aL}E_R\rho$$
$$+ K_a'\chi^T\bar{E}_L(\Psi_{aL})^c + G_E\bar{E}_L E_R\phi + \text{H.c.,} \tag{3}$$

where $H_{ab}^l$ is a symmetric matrix in the generation space; we have omitted $SU(3)$ indices. Neutrinos are strictly Dirac particles since the total lepton number will also be automatically conserved.

Next we impose a $Z_{13}$ discrete symmetry under which the fields transform as $Q_{iL} \to \omega_2^{-1}Q_{iL}$, $Q_{3L} \to \omega_0 Q_{3L}$, $u_{\alpha R} \to \omega_3 u_{\alpha R}$, $d_{\alpha R} \to \omega_5^{-1}d_{\alpha R}$, $J_R \to \omega_4 J_R$, $j_{mR} \to \omega_6^{-1}j_{mR}$, $\Psi_L \to \omega_6\Psi_L$, $E_L \to \omega_3 E_L$, $\nu_R \to \omega_4^{-1}\nu_R$, $E_R \to \omega_1 E_R$, $\eta \to \omega_3^{-1}\eta$, $\rho \to \omega_5\rho$, $\chi \to \omega_4^{-1}\chi$, $S \to \omega_1^{-1}S$, $\phi \to \omega_2\phi$, where $\omega_k = e^{2\pi i k/13}, k=0,\ldots,6$. Notice that if $N$ is a prime number the singlet $\phi$ can transform under this symmetry with any assignment (but the trivial one), otherwise we have to be careful with the way we choose the singlet $\phi$ to transform under the $Z_N$ symmetry. This symmetry implies that the lowest order effective operator that contributes to the axion mass is $\phi^{13}/M_{\text{Pl}}^9$ which gives a mass of the order $(v_\phi)^{11}/M_{\text{Pl}}^9$ and also keeps the $\bar{\theta}$ parameter small as discussed in Ref. [9].

The most general scalar potential invariant under the gauge and $Z_{13}$ discrete symmetries is

$$V_{331}^{(A)} = V_{\text{H}} + (\lambda_{\phi 1}\phi\epsilon^{ijk}\eta_i\rho_j\chi_k + \lambda_{\phi 2}\chi^T S^\dagger\rho\phi^* + \text{H.c.}), \tag{4}$$

where $V_H$ denotes the Hermitian terms of the potential. This scalar potential has the correct number of Goldstone bosons and an axion field.

After imposing the $Z_{13}$ symmetry defined above we have that both the total lepton number $L$ and the PQ symmetry are automatic. The PQ charge assignment is as follows:

$$u'_L = e^{-i\alpha X_u} u_L, \quad d'_L = e^{-i\alpha X_d} d_L, \quad l'_L = e^{-i\alpha X_l} l_L,$$

$$\nu'_L = e^{-i\alpha X_\nu} \nu_L, \quad j'_L = e^{-i\alpha X_j} j_L, \quad J'_L = e^{-i\alpha X_J} J_L,$$

$$E'_L = e^{-i\alpha X_{E_L}} E_L, \quad E'_R = e^{-i\alpha X_{E_R}} E_R, \tag{5}$$

and in the scalar sector we have the following PQ charges:

$$\eta^0: \quad -2X_u = 2X_d = X_\nu - X_{\nu_R},$$

$$\eta^-_1: \quad -(X_u + X_d) = X_u + X_d = X_l - X_{\nu_R},$$

$$\eta^+_2: \quad -(X_J + X_u) = X_j + X_d = -(X_l + X_{\nu_R}),$$

$$\rho^0: \quad 2X_u = -2X_d = X_l - X_{E_R},$$

$$\rho^+: \quad -(X_u + X_d) = X_u + X_d = X_\nu - X_{E_R},$$

$$\rho^{++}: \quad -(X_J + X_d) = X_j + X_u = -(X_l + X_{E_R}),$$

$$\chi'^-: \quad -(X_u + X_J) = X_d + X_j = X_\nu + X_{E_L},$$

$$\chi^{--}: \quad -(X_d + X_J) = X_u + X_j = X_{E_L} + X_l,$$

$$\chi^0: \quad -2X_J = 2X_j = X_{E_L} - X_l, \quad \phi: \quad -2X_j,$$

$$\sigma^0_1: \quad X_d + 3X_j = 2X_\nu, \quad h^-_1: \quad 3X_j - X_d = X_l + X_\nu,$$

$$h^+_2: \quad 4X_j = -X_l + X_\nu, \quad \sigma^0_2: \quad 4X_j - 2X_d = 0,$$

$$H^{--}_1: \quad 3(X_j - X_d) = 2X_l, \quad H^{++}_2: \quad 5X_j - X_d = -2X_l. \tag{6}$$

From Eqs. (6) we obtain the relations $X_j = -X_J = \frac{1}{2}X_d = -\frac{1}{2}X_u = -\frac{2}{3}X_l = -\frac{2}{3}X_{\nu_R} = \frac{2}{5}X_\nu = \frac{2}{5}X_{E_R} = 2X_{E_L} = -\frac{1}{2}X_\phi$.

Notice that the mass scale related to the exotic charged lepton $E$, up to an arbitrary dimensionless constant $G_E \sim O(1)$, is related to the PQ energy scale since the requirement of the symmetries of the model imposes the Yukawa couplings in Eq. (3).

Notice that, at the energy scale below the breakdown of the $SU(3)_L$ symmetry, this model has scalar multiplets transforming under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ as follows: two doublets $(\rho^+, \rho^0) \sim (\mathbf{1,2}, +1)$, $(\eta^0, \eta^-) \sim (\mathbf{1,2}, -1)$ and a non-Hermitian triplet $(H^{--}, h^-_1, \sigma^0_1) \sim (\mathbf{1,3}, -2)$. With the lighter scalar multiplets, and the usual degrees of freedom, the energy scale at which $\sin^2\theta_W = 0.25$ is 5.2 TeV. The doublets $(\chi^-, \chi^{--}) \sim (\mathbf{1,2}, -2)$, $(h^+_2, h^0) \sim (\mathbf{1,2}, +1)$ and the extra vector bosons have masses proportional to $v_\chi$; the lepton singlet $E$ has a mass of the order of $v_\phi$. More details will be given elsewhere.

## B. Model with three scalar triplets (model B)

In this model we do not introduce the scalar sextet and the Yukawa interactions are

$$-\mathcal{L}^l_Y = H^\nu_{ab}\overline{(\Psi)}_{aL}\nu_{bR}\eta + H^l_{ab}\epsilon_{ijk}\overline{(\Psi)}^c_{iaL}\Psi_{jbL}\eta_k$$
$$+ K_a\overline{\Psi}_{aL}E_R\rho + K'_a\chi^T\overline{E_L}(\Psi_{aL})^c + G_E\overline{E}_L E_R\phi^*$$
$$+ \text{H.c.}, \tag{7}$$

where $H^l_{ab}$ is now an antisymmetric matrix. In both Yukawa interactions above, a general mixing is allowed in each charge sector. As in the previous model, neutrinos are strictly Dirac particles. The charged leptons gain mass as in Ref. [37].

If we want to implement a given texture for the quark and lepton mass matrices we have to introduce more scalar triplets, and a larger $Z_N$ symmetry will be possible in the model.

Let us introduce a $Z_4$ symmetry with parameters denoted by $\tilde{\omega}_0$, $\tilde{\omega}_1$, $\tilde{\omega}_1^{-1}$, and $\tilde{\omega}_2 \equiv \tilde{\omega}_2^{-1}$. $u_{\alpha R}$, $Q_{iL}$, and $\nu_{aR}$ transform with $\tilde{\omega}_1$, $d_{\alpha R}$, $Q_{3L}$, $\Psi_{aL}$, $E_R$, $\chi$, and $\phi$ transform with $\tilde{\omega}_1^{-1}$, $\eta$ transforms with $\tilde{\omega}_2$, and all the other fields remain invariant, i.e., transform with $\tilde{\omega}_0$. After $Z_4$ is imposed, the total lepton number $L$ and the PQ and $Z_{11}$ symmetries are all automatically implemented in the Yukawa sector and in the scalar potential. The most general scalar potential is then

$$V^{(B)}_{331} = V_H + (\lambda\phi\epsilon^{ijk}\eta_i\rho_j\chi_k + \text{H.c.}). \tag{8}$$

The following $Z_{11}$ symmetry is automatically implemented in both the Yukawa interactions and in the scalar potential: $Q_{iL} \to \omega_3 Q_{iL}$, $Q_{3L} \to \omega_0 Q_{3L}$, $u_{\alpha R} \to \omega_4 u_{\alpha R}$, $d_{\alpha R} \to \omega_1^{-1} d_{\alpha R}$, $J_R \to \omega_5^{-1} J_R$, $j_{mR} \to \omega_3^{-1} j_{mR}$, $\Psi_L \to \omega_2\Psi_L$, $E_L \to \omega_3 E_L$, $\nu_R \to \omega_5^{-1}\nu_R$, $E_R \to \omega_1 E_R$, $\eta \to \omega_4^{-1}\eta$, $\rho \to \omega_1\rho$, $\chi \to \omega_5\chi$, $\phi \to \omega_2^{-1}\phi$. It happens that, in addition to the $Z_{11}$ symmetry, the $U(1)_{PQ}$ and the conservation of the total lepton number are also automatic, i.e., a consequence of the gauge symmetry and renormalizability of the model, in the interactions in Eqs. (1), (7), and (8). The PQ charge assignments for the fermions in the model are as in Eq. (5); and in the scalar sector we have constraints equations as in the previous subsection. In this case, proceeding as in the model A, we obtain the relations $X_d = X_u = 0$ and $X_l = X_{\nu_R} = X_\nu = X_{E_R} = -\frac{1}{2}X_j = \frac{1}{2}X_J = -\frac{1}{3}X_{E_L} = \frac{1}{4}X_\phi$. Notice that for leptons the PQ transformations are not a chiral symmetry. As in model A, we see from Eq. (7) that the mass scale related to the singlet charged lepton $E$ is related to $v_\phi$. Moreover, in this model we have that $\sin^2\theta_W = 0.25$ at 4 TeV.

## III. CONCLUSIONS

We have built two invisible axion models in which the axion is naturally light and protected against quantum gravity effects. In model A, the $Z_{13}$ symmetry has to be imposed but in model B the $Z_{11}$ symmetry is automatically implemented in the classical Lagrangian after imposing a $Z_4$ symmetry. With a $Z_{13}$ symmetry the axion is protected from

gravitational effects even if $v_\phi \approx 10^{12}$ GeV but, with a $Z_{11}$ symmetry, $v_\phi \lesssim 10^{10}$ GeV. In both models the PQ symmetry is automatically implemented in the classical Lagrangian in the sense that it is not imposed on the Lagrangian but is just a consequence of the particle content of the model, its gauge invariance, renormalizability, and Lorentz invariance.

We would like to stress the strong constraint put on model building by the approach proposed in Refs. [8,9]. Once the symmetry $Z_N$ is used, automatic or imposed, there is no choice for new interactions. In this vein, in both models neutrinos are strictly Dirac particles, and for this reason both models will be ruled out if the neutrinos turn to be Majorana particles, say, by observation of the neutrinoless double beta decay.

In the PQ solution to the strong $CP$ problem the quark contributions to $\bar\theta$ are such that $\bar\theta \rightarrow \bar\theta - 2\alpha \Sigma_f X_f$, where $f$ denotes any quark. In both models we have $\bar\theta \rightarrow \bar\theta - 2\alpha X_j$

and we have the domain wall problem related to $Z_2 \subset U(1)_{PQ}$ for $X_j = 2$.

Concerning the CDM, we would like to call attention to a new possible candidate: a light and stable scalar in nonsupersymmetric models [38,39]. Although the mass of this scalar is in the range 32 GeV $\lesssim m < M_Z$, the model is still compatible with the LEP data since the lightest scalar is almost a singlet under $SU(2)_L \otimes U(1)_Y$, and it may be mistaken for a light, $m_\chi \lesssim 50$ GeV [40], or for the usual, $m_\chi \gtrsim 50$ GeV, neutralino. We recall that the latter bound comes from LEP2 searches for the corresponding chargino $m_{\chi^\pm} \gtrsim \sim 100$ GeV.

## ACKNOWLEDGMENTS

[1] A.D. Lewis, D.A. Buote, and J.T. Stocke, Astrophys. J. **586**, 135 (2003).

[2] C.L. Bennett *et al.*, Astrophys. J., Suppl. Ser. **148**, 1 (2003).

[3] M. Srednicki and N.J.C. Spooner, in Particle Data Group, K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).

[4] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981).

[5] J.E. Kim, Phys. Rev. Lett. **43**, 103 (1979).

[6] R. Holman, T.W. Kephart, and S.-J. Rey, Phys. Rev. Lett. **71**, 320 (1993).

[7] L.M. Krauss and F. Wilczek, Phys. Rev. Lett. **62**, 1221 (1989).

[8] Alex G. Dias, V. Pleitez, and M.D. Tonasse, Phys. Rev. D **69**, 015007 (2004).

[9] Alex G. Dias, V. Pleitez, and M.D. Tonasse, Phys. Rev. D **67**, 095008 (2003).

[10] K.S. Babu, I. Gogoladze, and K. Wang, Phys. Lett. B **560**, 214 (2003).

[11] S. Weinberg, Phys. Rev. D **5**, 1962 (1972).

[12] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992); P. Frampton, Phys. Rev. Lett. **69**, 2889 (1992); R. Foot, O.F. Hernandez, F. Pisano, and V. Pleitez, Phys. Rev. D **47**, 4158 (1993).

[13] V. Pleitez and M.D. Tonasse, Phys. Rev. D **48**, 2353 (1993); J.C. Montero, F. Pisano, and V. Pleitez, *ibid.* **47**, 2918 (1993); R. Foot, H.N. Long, and T.A. Tran, *ibid.* **50**, 34 (1994); W.A. Ponce, Y. Giraldo, and L.A. Sanchez, *ibid.* **67**, 075001 (2003).

[14] S. Dimopoulos and D.E. Kaplan, Phys. Lett. B **531**, 127 (2002).

[15] S. Dimopoulos, D.E. Kaplan, and N. Weiner, Phys. Lett. B **534**, 124 (2002); I. Gogoladze, Y. Mimura, and S. Nandi, *ibid.* **554**, 81 (2003).

[16] P.H. Frampton and T.W. Kephart, hep-ph/0306053; P.H. Frampton, hep-th/0306029.

[17] C.A. de S. Pires and O. Ravinez, Phys. Rev. D **58**, 035008 (1998).

[18] C.A. de S. Pires, Phys. Rev. D **60**, 075013 (1999).

[19] J.C. Montero, V. Pleitez, and M.C. Rodriguez, Phys. Rev. D **65**, 035006 (2002).

[20] F. Pisano and V. Pleitez, Phys. Rev. D **51**, 3865 (1995).

[21] J.C. Montero, V. Pleitez, and M.C. Rodriguez, hep-ph/0204130, and references therein.

[22] See http://www.slac.stanford.edu/exp/e158

[23] J.C. Montero, V. Pleitez, and M.C. Rodriguez, Phys. Rev. D **58**, 097505 (1998).

[24] E.M. Gregores, A. Gusso, and S.F. Novaes, Phys. Rev. D **64**, 015004 (2001); D.V. Soa, T. Inami, and H.N. Long, hep-ph/0304300.

[25] G. Tavares-Velasco and J.J. Toscano, Europhys. Lett. **53**, 465 (2001); Phys. Lett. B **472**, 105 (2000).

[26] B. Dion, T. Grégoire, D. London, L. Marleau, and H. Nadeu, Phys. Rev. D **59**, 075006 (1999).

[27] P. Das, P. Jain, and D.W. McKay, Phys. Rev. D **59**, 055011 (1999).

[28] T. Kitabayashi and M. Yasue, Phys. Rev. D **63**, 095006 (2001); T. Kitabayashi, *ibid.* **64**, 057301 (2001).

[29] T. Kitabayashi and M. Yasue, Phys. Lett. B **508**, 85 (2001).

[30] J.C. Montero, C.A. de S. Pires, and V. Pleitez, Phys. Rev. D **65**, 095001 (2002).

[31] M. Kuze and Y. Sirois, Prog. Part. Nucl. Phys. **50**, 1 (2003).

[32] P.H. Frampton and M. Harada, Phys. Rev. D **58**, 095013 (1998); H.N. Long and T. Inami, *ibid.* **61**, 075002 (2000).

[33] D. Ng, Phys. Rev. D **49**, 4805 (1994); P. Jain and S.D. Joglekar, Phys. Lett. B **407**, 151 (1997).

[34] R.D. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. D **16**, 1791 (1977).

[35] P.B. Pal, Phys. Rev. D **52**, 1659 (1995).

[36] T.V. Duong and E. Ma, Phys. Lett. B **306**, 307 (1993).

[37] J.C. Montero, C.A. de S. Pires, and V. Pleitez, Phys. Rev. D **65**, 093017 (2002).

[38] Alex G. Dias, V. Pleitez, and M.D. Tonasse, hep-ph/0204121.

[39] D. Fregolente and M.D. Tonasse, Phys. Lett. B **555**, 7 (2003); H.N. Long and N.Q. Lan, Europhys. Lett. **64**, 571 (2003).

[40] A. Bottino, F. Donato, N. Fornengo, and S. Scopel, Phys. Rev. D **69**, 037302 (2004).