Growth curve: an intelligent life history described by a mathematical model!

Manoel Garcia Neto
Ermias Kebreab

Introduction

A mystery! This is a classic answer to define what growth is (von Bertalanffy, 1952 p. 136).

The growth process is probably the most common occurrence and observed in several systems, biological or not (Burkhart & Tomé, 2012; Dadson et al., 2017; Parker, 2012).

And how does it occur? It is easily observable with the naked eye. So, it is feasible to certify that growth generally follows a sigmoid curve (S-shaped), this being a universal characteristic. Still, visually it is allowed to contemplate different phases in its path (Bertin, 2014; Pommerening & Grabarnik, 2019).

Thus, growth is characterized and clearly subdivided into different phases (e.g. lag, exponential, stationary, etc.) (Levert & Xia, 2001; Bukhman et al., 2015). In order to verify these different moments, measurements of size or proportions are recorded on several occasions along this route (von Bertalanffy, 1957; Paine, 2012).

In this way, through growth curves it is possible to describe changes in mass, size, volume, area, population and other units to represent the information of the wise behavior of growth over time with only a few parameters (Jørgensen et al., 2000; Pommerening & Grabarnik, 2019).

There are many candidate curves to represent the growth period, but they always adequately demonstrate the moments in which the transition between phases occurs (Bukhman et al., 2015; Paine, 2012; Zeide, 2004).

Therefore, in order to delve deeper into the essence of this mystery and contribute to answering other questions, it is essential to understand why several stages in the growth process are necessary, this question actually represents the essence of the mystery (Grimm et al., 2011). This mystery presents strategies and optimizations (Karkach, 2006).

In view of this, mathematical models are very appropriate to represent the route geometry in two dimensions (e.g. position × time) (Curran et al., 2010; Kebreab et al., 2010; Koya, & Goshu, 2013).

Mathematical Modeling

Mathematical modeling is considered a powerful tool to measure growth expansion, a fundamental and universally important process. Quantitative models allow simplifying complex realities, using only a few parameters (Pommerening & Muszta, 2015, 2016). This is the reason why it is widely applied in several areas that need to predict growth and its dynamics, as a function of time, requiring solid and well-distributed data in relation to the period studied (Zwietering & Besten, 2011)

The most common procedure for describing growth proposes to adjust the data in a mathematical model with few interpretable parameters, and the greater the complexity of the growth curve, the greater the number of parameters required. Thus, the ideal model should predict the experimental values with the minimum number of parameters and still offer a good prediction of growth behavior (Ricklefs, 1983).

Derivatives of position

In order to characterize different phases of a growth curve (life history), it is necessary to define the first, second and even third derivatives of the adopted model, to evaluate the growth process and its different phases and strategies (Bentea et al., 2017; Pommerening & Muszta, 2015, 2016).

The first derivative of the growth curve represents velocity and allows to characterize the inflection point, which divides the growth curve into a concave and then a convex moment (Figure 1). The second derivative, on the other hand, allows a good approximation of the traditional initial, growth and saturation phases (Easwaran, 2014; Garthright, 1991; Parker, 2012).

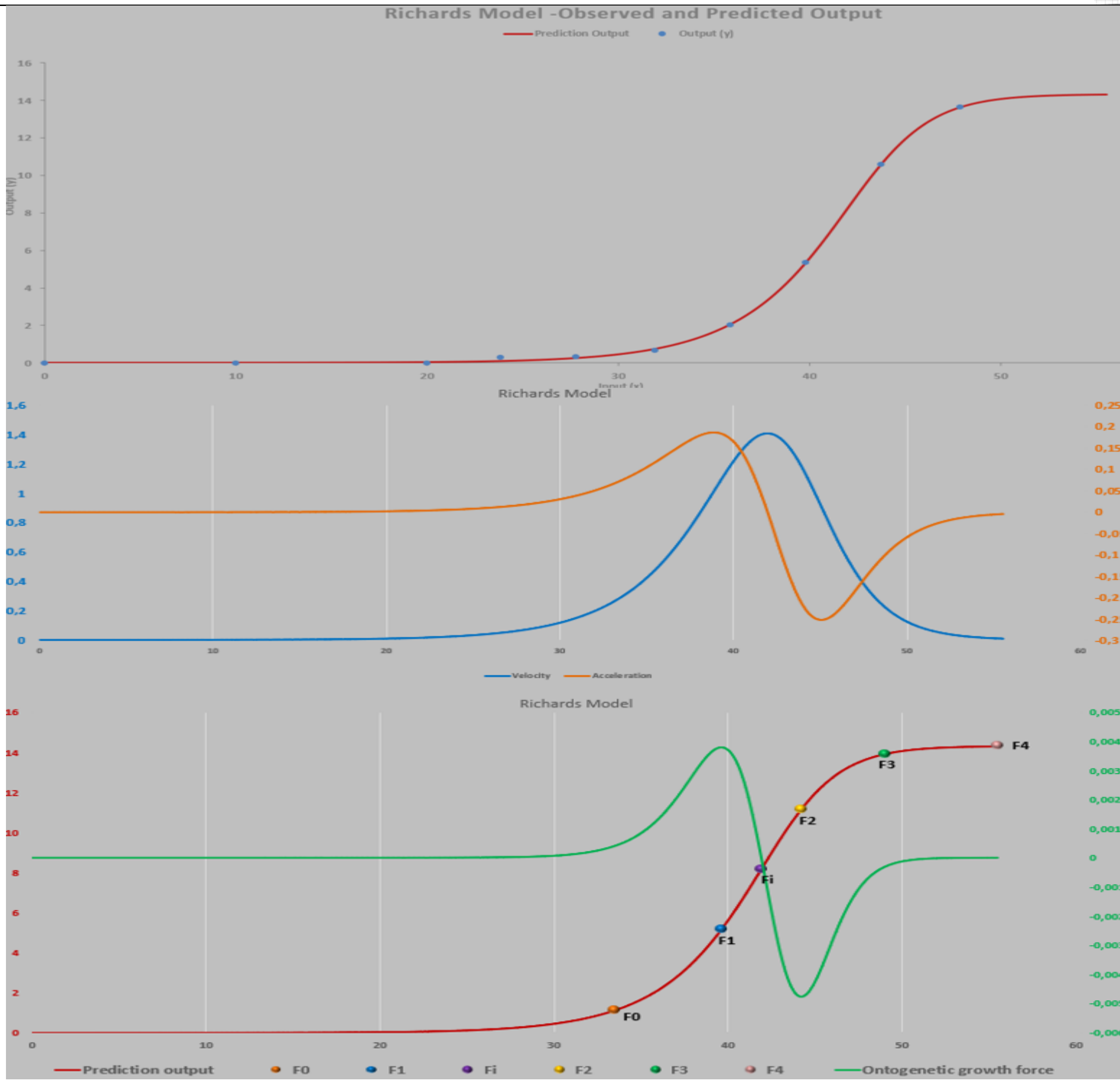

Figure 1. The first derivative of the growth curve represents velocity and allows to characterize the inflection point (Fi), which divides the growth curve into a concave and then a convex moment.

Jerk represents the derivative of acceleration, that is, three times the derivative of the position (Bentea et al., 2017; Kyriakopoulos & Saridis, 1988). This strange measure is useful in engineering, because when it manifests itself indicating discomfort in relation to passengers in a vehicle (Macfarlane & Croft, 2003; Eager et al., 2016). In amusement parks, Jerk is even appreciated, but on a daily basis it is a reason for liquids to be thrown out of its container, or damage to fragile structures such as egg transport (Jazar, 2011 p.53).

In other words, Jerk allows to measure the “vibration” of acceleration (Eager, 2018; Sandin, 1990; Shimojo, 2006), that is, a sudden change in acceleration (Altintas & Erkorkmaz, 2003; Biral et al., 2010; Dong et al., 2007; Schot, 1978). For this reason, the third derivative of the position shows its importance when measuring abrupt changes (Kyriakopoulos & Saridis, 1988).

Derivatives of force

Now, in order to measure abrupt changes in the ontogenetic growth force curve (Garcia-Neto et al., 2018), the PPFM spreadsheet proposes to use the resources of the third derivative of that ontogenetic growth force curve, the Snatch or Snap (Davidson & Ringwood, 2017; Mann et al., 2014; Palm et al., 2016; Potvin et al., 2001). Then, it is possible to define the sudden change that occurs in the ontogenetic growth force curve, precisely in the transition from the Lag phase to the exponential phase.

In this way, the Snatch is similar to Jerk in that it has the same ability to measure abrupt changes, being calculated by the force derivative (Easwaran, 2014).

Thus, the PPFM spreadsheet automatically calculates Snatch, using the same concept to calculate Tug (T) and Yank (Y) (Lin et al., 2019), from the ontogenetic growth force (Figure 2).

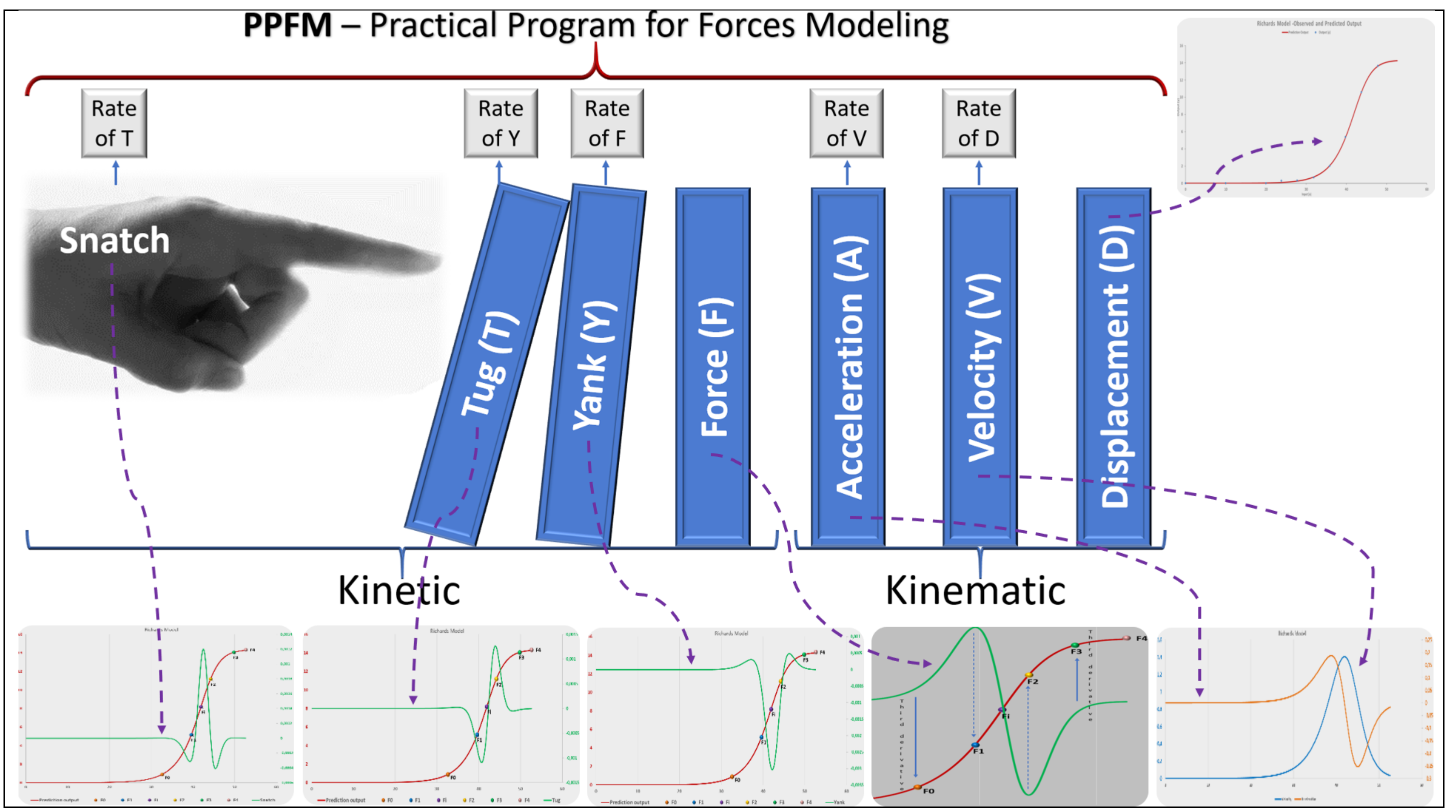

Figure 2. Derived from ontogenetic growth force (Snatch, Tug and Yank) and position (acceleration and velocity).

Growth stages

The typical sigmoid growth behavior is well known and studied, generally including three defined stages: lag phase, log or exponential phase and stationary phase. However, other phases are still possible: death phase, spurts phase, increasing exponential, quasi-linear phase, decreasing exponential and resilience phase (Dandurand & Shultz, 2010; Legan et al., 2002; Meredith et al., 2018). Thus, the most important thing is not only to characterize the geometric trend of growth, but to define the various phases and different strategies that describe its trajectory (Figure 3) (Levert & Xia, 2001).

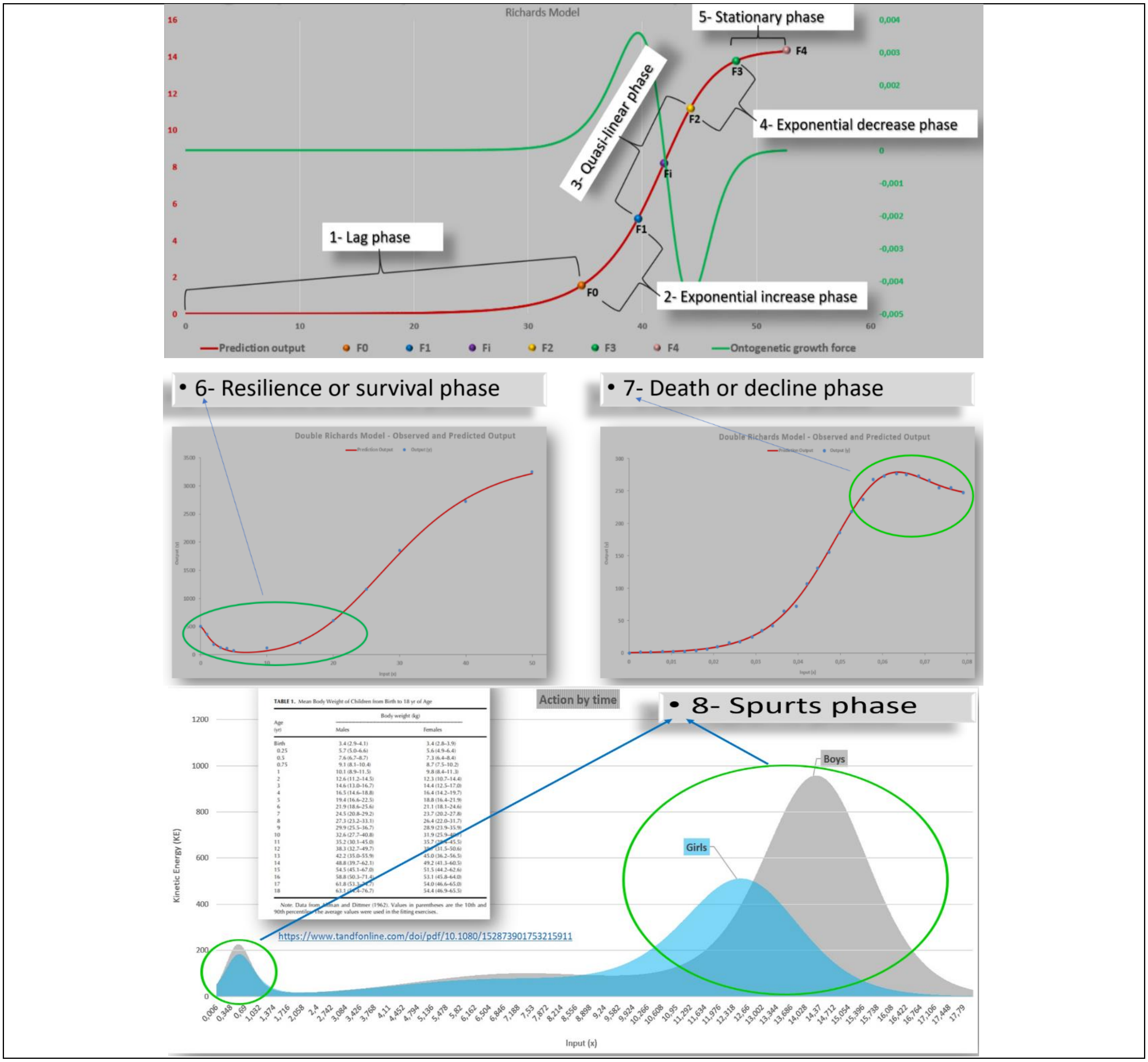

Figure 3. Eight possible growth phases identified by the PPFM spreadsheet.

The history of the growth process proves to be complex and adaptive, showing that tactics are necessary to overcome challenges involved in its path. This requires smart decisions, especially when resources are scarce (Trewavas, 2005). Thus, to maintain growth it is necessary to resist the various disturbances that may arise during the journey (Tonner et al., 2017). This is the reason why the growth curve is represented by an S format and not a J, and thus, it is characterized by several phases (Banavar et al., 2002; West et al., 2002).

In our three-dimensional world, only one dimension is not adequate enough to clarify the growth process, usually measured only by the accumulation of mass in relation to time. Thus, it is necessary to determine the amount of energy used in this expansion, for better growth clarity (Hong et al, 2018; Jørgensen et al., 2000).

The mass and time dimensions are well known in physics. However, a new interpretation of growth allows action (energy × time) to be reached (Figure 4), a link that has not yet been explored to better understand and explain the phases of growth and the wise path chosen for its expansion (Grandpierre, 2011).

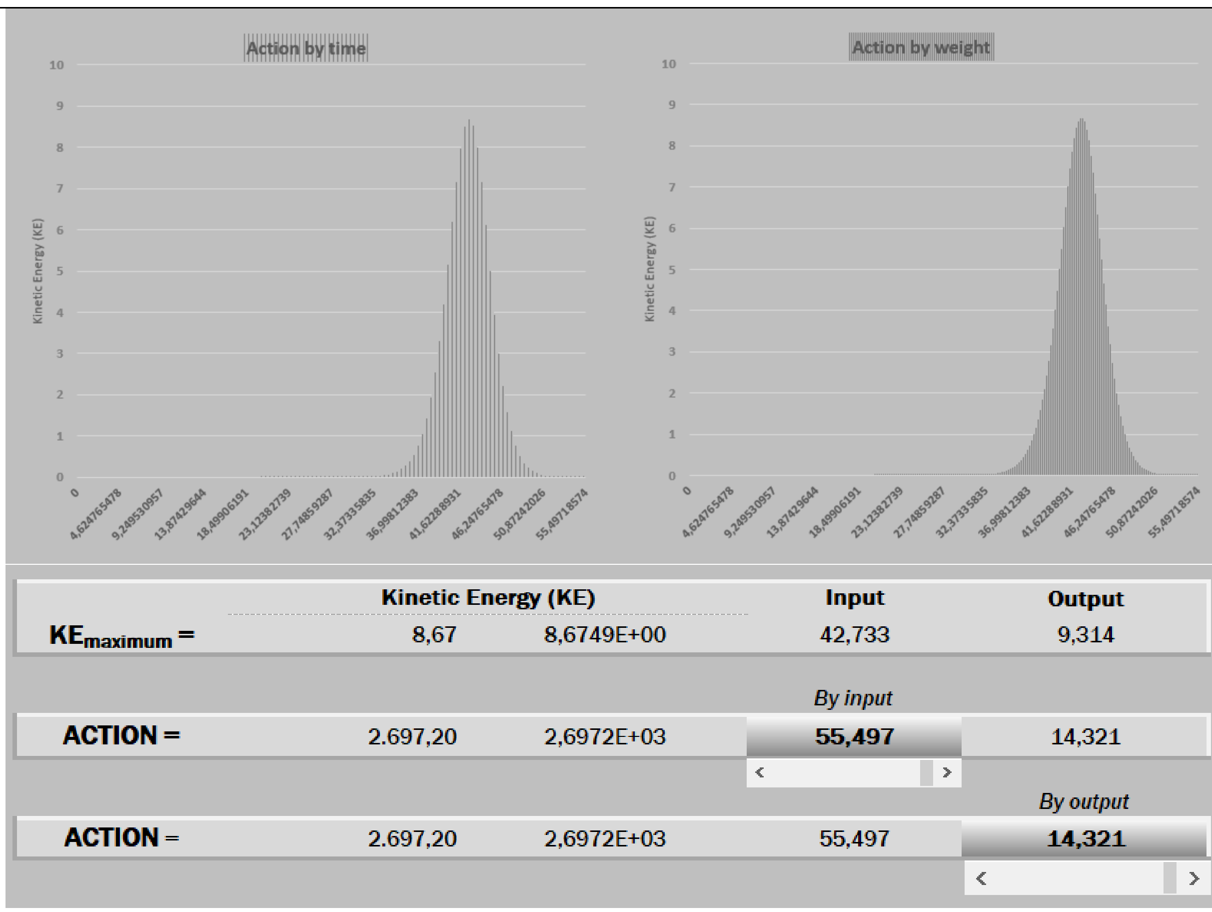

Figure 4. Principle of action (energy × time).

The action principle is an innovative form of numerical assessment of growth intelligence, which offers biology the opportunity to enter quantum mechanics, enabling a deeper understanding of nature. As a result, a new concept is proposed that emerges by uniting biology and physical theories (Mathew, 2014).

The action for being measured by the summation area over and under the kinetic energy curve (respectively – and +) allows to evaluate the challenging and mysterious phase of mortality, spurts and survival phases, characterizing the transition and the impact of these moments numerically (Figure 5) (Fekedulegn et al. 2007; Schentag et al., 1991).

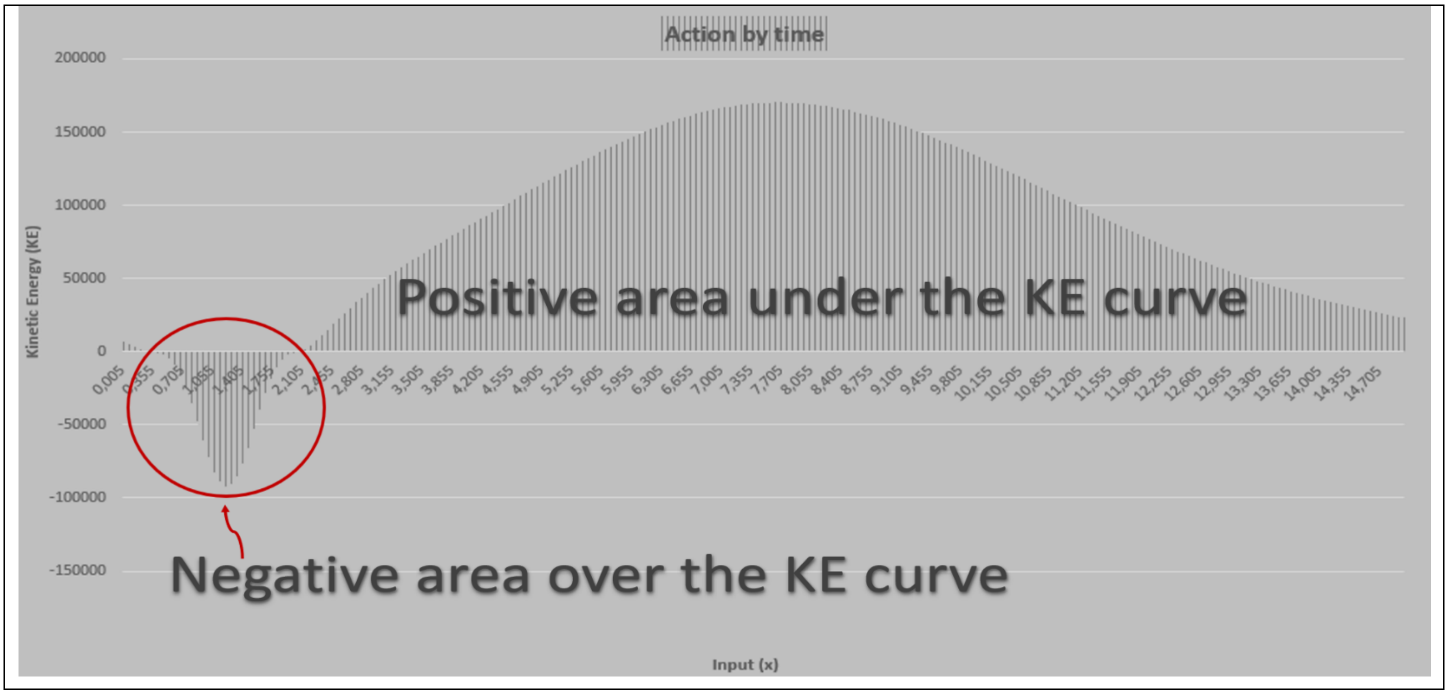

Figure 5. Summation area over (-) and under (+) the kinetic energy curve defines the action.

Furthermore, the action applied to biology enables a new reinterpretation of Newtonian mechanisms, integrating biology and physics, since many biological issues are not clarified only by traditional methods. In addition, the action principle allows for a better reflection on the biological processes involved in the complexity of growth (Simeonov et al., 2012).

Kinematics and Kinetics

From the application of the Newtonian point of view it is possible to define the critical points that define limits or the moments of transition between the different phases present in a growth curve (Simeonov et al., 2012). So, in addition to accurately estimating the parameters of a growth curve, it is also important to define the points that represent the transition between phases, as they are essential for a full understanding of the real characterization of growth (Levert & Xia, 2001).

Until now, only the geometry (effect) of growth development (kinematics) was studied. However, with applications of physics it is possible to detail the forces (kinetics / causes) that make this expansion feasible (Garfield, 2009).

Thus, to understand the reasons for growth, only the geometry of the growth curve is not sufficient (kinematic) (Fortney, 1983; Rice et al. 2017; Sutherland, 1997). Then, the reinforcement of physics (kinetic) comes in (Janzen & Mann, 2014; Lampe et al., 2004; Moir, 2009; Rodríguez-Rosell et al., 2018), which with great coherence allows to identify other moments of transition, such as the lag phase and the phase of death (Levert & Xia, 2001). A resilience phase is still possible (Carvalho et al., 2019; Firsov et al. 2001; Juan-García et al., 2017; Meredith et al., 2018; Rougé et al., 2013) and another called spurts (Ali et al., 2004; McFee et al., 2010)

Therefore, kinematics is strengthened by joining with the next link, which is kinetics. Such procedure allows to explore and celebrate more possibilities on the understanding of the expansion of growth, offering help to explain the intelligent path chosen for the expansion (Ben-Jacob et al., 2004; Calvo & Frantisek, 2015), for example, of the mass (m) in relation to time, allowing to define more precise moments of transition (lag phase for log phase and then to asymptotic phase) (Swain et al, 2016). For this reason, the evaluation of growth expansion, if addressed only by kinematics, would lose the essence of the optimization that occurs along the way.

In this way, the application of the principles of physics (kinematics and kinetics) allow new insights to evaluate growth models, by better characterizing the phase transitions and mainly resulting in the innovative calculation of the action, by summarizing in a single number, all the information of the system growth process (Mann et al., 2014; Sakthiselvan et al., 2019).

All of this, with the elegance of physics giving the best understanding of biology, by offering tools (biomathematics) that allow properties to emerge, hitherto hidden in the growth curve (Cormie et al., 2008; Haitao et al., 1997; Simeonov et al., 2012).

Thus, not only assess a geometric moment of growth (kinematics), but now the whole, through the global property defined by action, which is closely related to quantum mechanics (Fleming et al., 2011). In other words, the path chosen by an organism to grow reflects the most opportune and intelligent trajectory, which portrays the quantum mechanism (Garfield, 2009; Grandpierre, 2011).

Ontogenetic growth force curve

The ontogenetic growth force curve represents an innovative basis (Garcia-Neto et al., 2018), of great efficiency to evaluate a biological system or not, since it allows to measure very accurately the addition of new material (interest) continuously during the growth period, offering an economic measure finished in useful work (action) (Girtler et al., 2011; Gleiss et al., 2011). Thus, it makes it possible to define the real efficiency of the organism in relation to the production of the new biomass or material, that is, representing the efficiency index of the growth curve.

Consequently, the ontogenetic growth force curve is much more sensitive and appropriate to define the transition points of the phases of a growth curve, being automatically determined, making it possible to establish and quantify at least eight phases of the growth curve (Sibly et al., 2015).

Measuring the principle of action by summation area

Perhaps the most fascinating thing is to represent the entire growth process in just a single number, the action (energy × time). Action represents another mysterious topic, even for physics, but it applies perfectly to biology and the growth process (Grandpierre, 2009).

Recalling that the first derivative makes it possible to define the velocity (v) at each moment of time. So, again, with the help of the laws of physics, the formula $\mathrm{KE} = \frac{\mathbf{m} * \mathbf{v}^2}{\mathbf{2}}$ is applied (Kanski et al., 2015; Wong et al., 2018). Such a simple and majestic procedure now allows obtaining the kinetic energy (KE) required at each moment of the growth process. Hence the action (A = KE × time) that represents the summation area (A), that is, nothing more and nothing less than the useful work applied in the growth process (Girtler, 2011).

The area under or over the kinetic energy curve is obtained by adding the areas of the trapezoids, applying the Riemann sum principle (http://mathworld.wolfram.com/RiemannSum.html) (Shah et al., 2007). This procedure is performed automatically by Excel, which subdivides the period evaluated in 3000 equidistant parts, being calculated the individual area of each trapezoid (https://www.zweigmedia.com/RealWorld/Excel/tuts/RiemannSum.xls), resulting in the end in the added area of all of them (Jeger & Viljanen-Rollinson, 2001; Sprouffske & Wagner, 2016).

The area under or over the kinetic energy curve allows simplifying the analysis, making it possible for various information to be represented by a single value (Duan et al., 2012; Elahee & Poinapen, 2006; Pruessner et al., 2003; Schentag et al., 1991).

Area under or over the kinetic energy curve offers neutral results, in the sense that it does not necessarily depend on units, but only on dimensions (energy × time), and also because it allows both individual and group assessment. The most opportune is that the area is summarized in a single value with physical significance, by representing the action (energy × time) that makes it possible to estimate all the useful work applied to allow the growth process in relation to the period of time evaluated (Girtler, 2008; Rosen, 1986; Grandpierre, 2009).

Excel spreadsheet tools: graphs, macros, VBA and Solver

Excel is known and used worldwide, and is found in most personal computers. Among the several favorable attributes of Excel, we mention the use of macros and VBA (Visual Basic for Applications) that facilitate the automation of calculations, being friendly and with many graphical facilities, easy to manipulate data (John, 1998; Kazakis, 2019; Kemmer & Keller, 2010).

The referred software allows wide visualization of the inputs and outputs, in addition to accessibility of graphs to represent the results, allowing great practicality through its commands (autoexec) that allow the automation of the sequence of subroutines (Fylstra et al., 1998; Posavec et al, 2006).

The simplified reduction of data that represents the growth history in a few parameters and usually done through least-squares fitting (LSF) while minimizing the residual sound of squared (SSR) (Hossain et al., 2013; Kemer & Keller, 2010) through the Excel Solver supplement, which when using the generalized reduced gradient algorithm method, it allows a simple and intuitive procedure, but not always fast (Zhu & Chen, 2015).

However, the iterative principle is closely dependent on the initial values chosen to trigger the entire process (Afouxenidis et al., 2012). Thus, be very careful in evaluating whether the result offered by the Excel Solver supplement was not sensitive to find the really appropriate result, that is, if it only offered a mathematical solution. Thus, staying away from the biological reality or the studied process (garbage in - garbage out) (Hook et al 2011).

Thus, due to the robustness of Solver and its iterative processes (Fylstra et al., 1998; Walsh & Diamond, 1995), it is possible to obtain optimal modeling for the growth curves, when evaluating experimental data and their predictions, with wide applications (biological, chemical, economic, etc.). All this, without deep knowledge of mathematics or the need to manipulate formulas.

All models to be adjusted need to be assigned good parameters and coherent initial values, which will be the starting point for successive iterations, until the definition of optimal parameter values is feasible (Archontoulis & Miguez, 2015; Ward et al., 2001).

The sequence of longitudinal data collected to define the change in growth (mass or dimensions) should adequately represent the spatial interval evaluated, to characterize the asymptote very well (Bridson & Gould, 2000; Strathe et al., 2010; Zwietering et al., 1990). Thus, ensuring effective points that represent the studied spatial interval well is the basic and essential procedure to allow the growth curve parameters to be properly calculated (Pommerening & Grabarnik, 2019). Most of the time the lack of this accuracy is perceived visually.

PPFM - Practical Program for Forces Modeling

In order to contribute in a practical way to measure the different stages of growth, we now have a new tool called PPFM (Practical Program for Forces Modeling; https://sites.google.com/view/ppfm-spreadsheet/) (Figure 6).

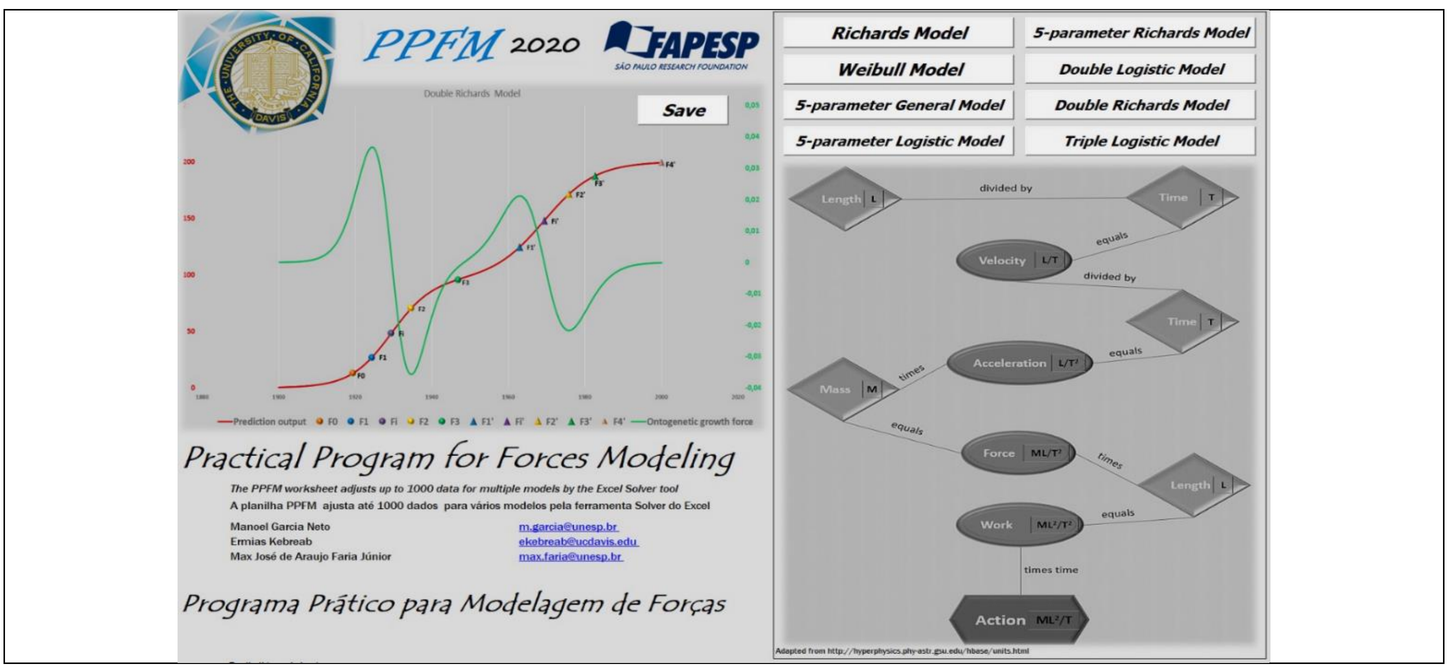

Figure 6. PPFM (Practical Program for Forces Modeling; https://sites.google.com/view/ppfm-spreadsheet/)

The PPFM spreadsheet allows the data to be friendly manipulated, with instant visualization, both of numerical results and graphs (Fujikawa, 2011; Kemmer & Keller, 2010).

Although the PPFM spreadsheet is robust, in order to obtain a reliable return, it is necessary to offer well-distributed data, which allow to describe and adequately represent all the phases contained in the growth behavior. Therefore, to avoid compromise and serious reservations about the quality of adjustments, all regions of the growth curve should have representative data (Barany & Roberts, 1995), characterizing the entire period evaluated very well. The ideal is to offer at least 10 to 12 points, and also appropriate initial values, to avoid mathematical solutions without biological coherence (Legan et al., 2002; McClure et al., 1994).

The adjustment of a model, in relation to the growth data, must be necessary to accurately describe the entire course of the event, being the responsibility of the modeler to determine which is the most appropriate function, among the candidates, to better represent the adjustment, depending on of the data offered (Brisbin et al., 1987; Kuhi et al,, 2003; López et al., 2004). Therefore, the PPFM spreadsheet provides models with 4, 5, 6, 8 or 9 parameters.

Therefore, to obtain a better model conversion condition and success in growth curve adjustments, three steps are recommended:

1- Based on a visual assessment, offer a consistent value for the asymptote and the inflection point;

2- Even after the spreadsheet has finalized its adjustment by Solver, it is advisable to check the residual graph to detect possible violations, due to an inappropriate accommodation of the growth curve parameter, which requires a new start, with new and more appropriate values;

3- In order to define the most appropriate model, the PPFM spreadsheet offers, in addition to the residual chart, the goodness-of-fit indicators residual (SD), the adjusted-R^2 (R^2_{adj}), corrected Akaike information criterion (AICc), Bayesian information criterion (BIC) and sum of squares of error (SSE), which makes it possible to assess how much of the data variation is not explained by the adjusted model (Figure 7).

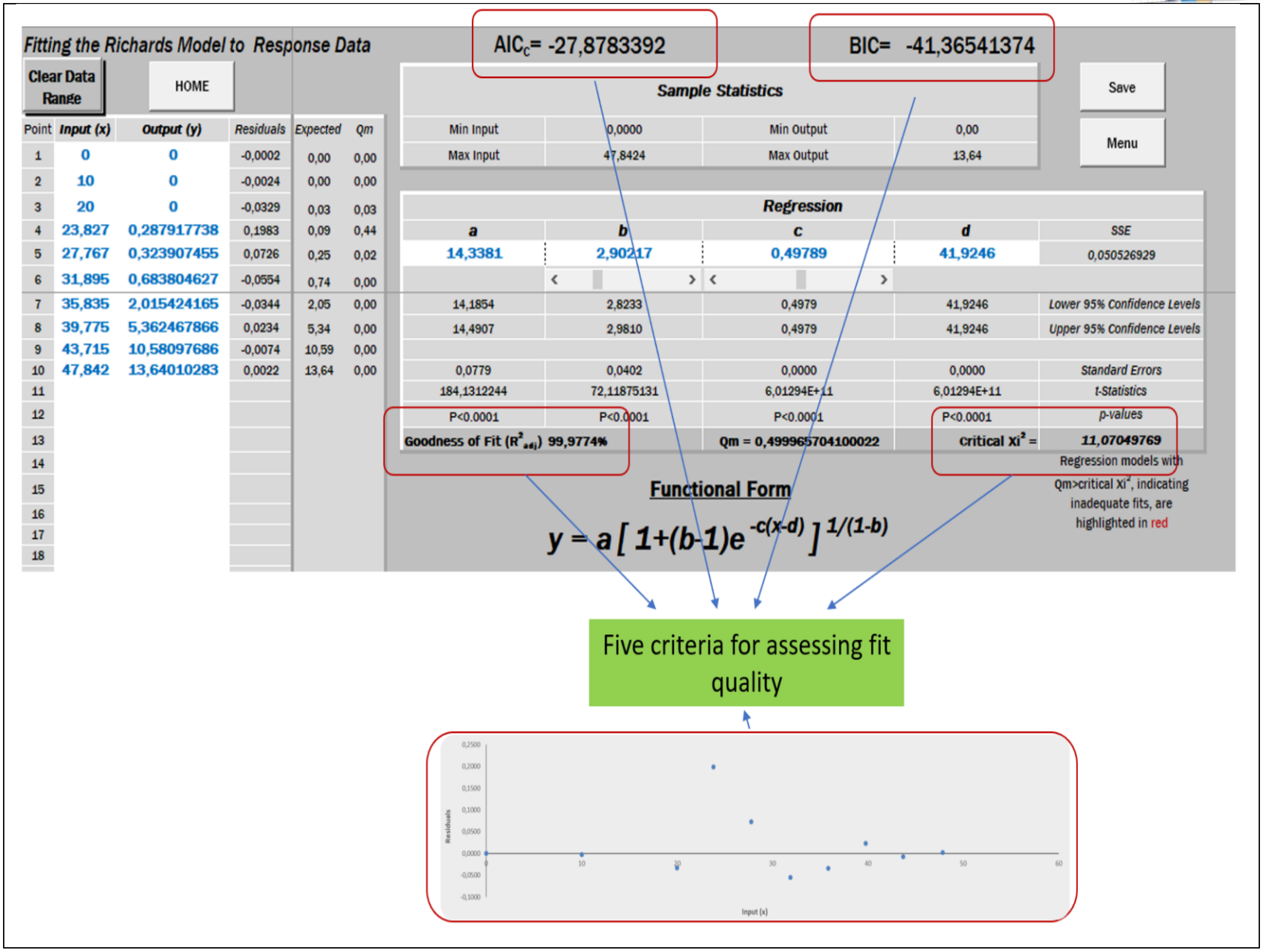

Figure 7. Five criteria for assessing fit quality.

The PPFM spreadsheet makes it possible to evaluate eight different models for modeling growth curves (Figure 8), using the Excel solver supplement, in a friendly and dynamic way, requiring the minimum of interventions, due to the fact that the functions are already inserted and the derivatives implemented, all simultaneously and automatically, with visual updated data and graph adjustments (Kemmer & Keller, 2010).

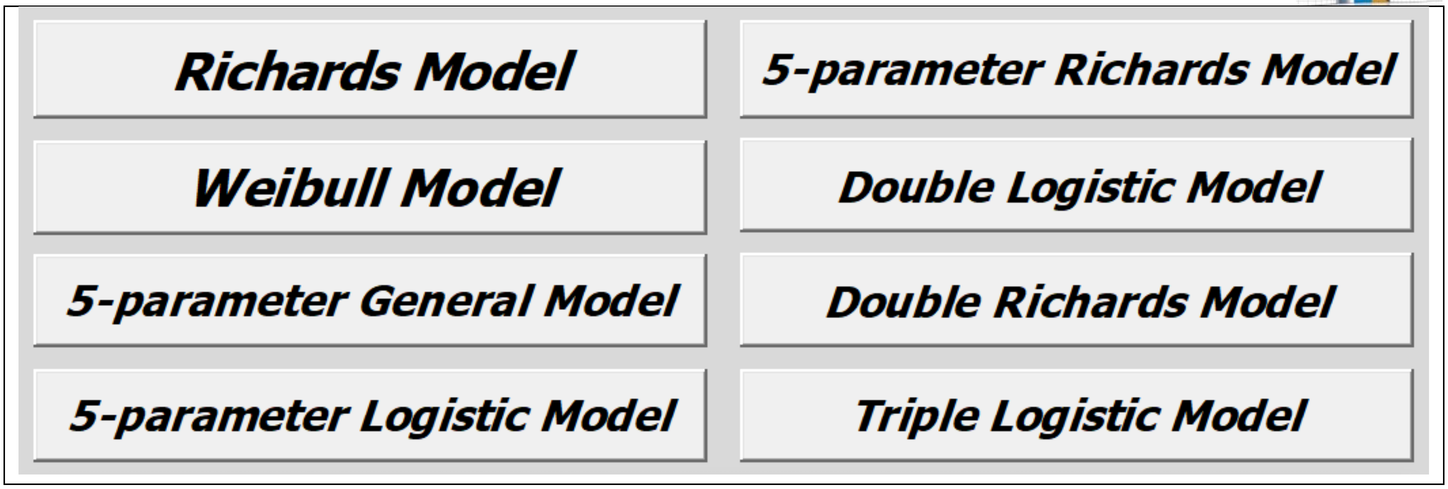

Figure 8. Mathematical models provided by the PPFM (Practical program for force modeling) spreadsheet. https://sites.google.com/view/ppfm-spreadsheet/

For some estimates, a longer time may be required for Solver to finish interactions. Despite this inconvenience, the PPFM spreadsheet solves adjustments for complex growth curves with unfolding phases of the growth process and finalizes the calculation by determining the area (energy × time) that represents the useful work (action) applied in the growth process, and all these steps with graphical visualization of these analyzes (Girtler, 2009ab, 2011; Rudnicki, 2009).

The PPFM spreadsheet was developed in the 2016 version of Excel. Therefore, it is also compatible for versions higher than 2016.

All models are adjusted by the method of least squares, through successive iterations that are initialized from initial values provided by the user, which must be consistent and realistic. Most of the time the simple graphic visualization allows to presume if the suggested initial data are appropriate or not (Kemmer & Keller, 2010).

Sometimes the model shows itself to be very sensitive to the suggested initial values, which impair or make it impossible to adjust by the solver. In this case, the appropriate and prudent option would be to change the initial values and proceed with a new optimization (Zach et al. 1984), aiming at good starting parameter values.

VBA macros and codes have been included in the PPFM spreadsheet to facilitate and streamline its use, making it possible to automatically invoke the add-in solver to minimize the sum of error squares, from iterations that adjust the initial parameters provided by the user, ending with the parameters estimated by the optimizer (Solver) in a simple, precise and somewhat fast way (Zhu & Chen, 2015).

However, a criterion is needed to choose the initial starting parameters, so as not to induce the spreadsheet to violate biology (Kebreab et al., 2010), even with an apparent mathematical success provided by the solver. Therefore, a coherent biological interpretation is required, which must be done with responsibility by the user.

Thus, using the facilitating attributes of the Excel spreadsheet, the period studied can be subdivided into 3000 equidistant parts. This procedure makes it possible to calculate the area over or under the kinetic energy curve as a function of the evaluated time (Girtler, 2011).

At that moment, it highlights another great virtue of this calculation, the possibility of measuring up to the unfavorable moment (area over the kinetic energy curve), in which the value is negative (Figure 9), that is, one can now also know and measure the moment of stress or death, which it allows a new approach to this history, by characterizing its different phases and strategies (Firsov et al, 2001).

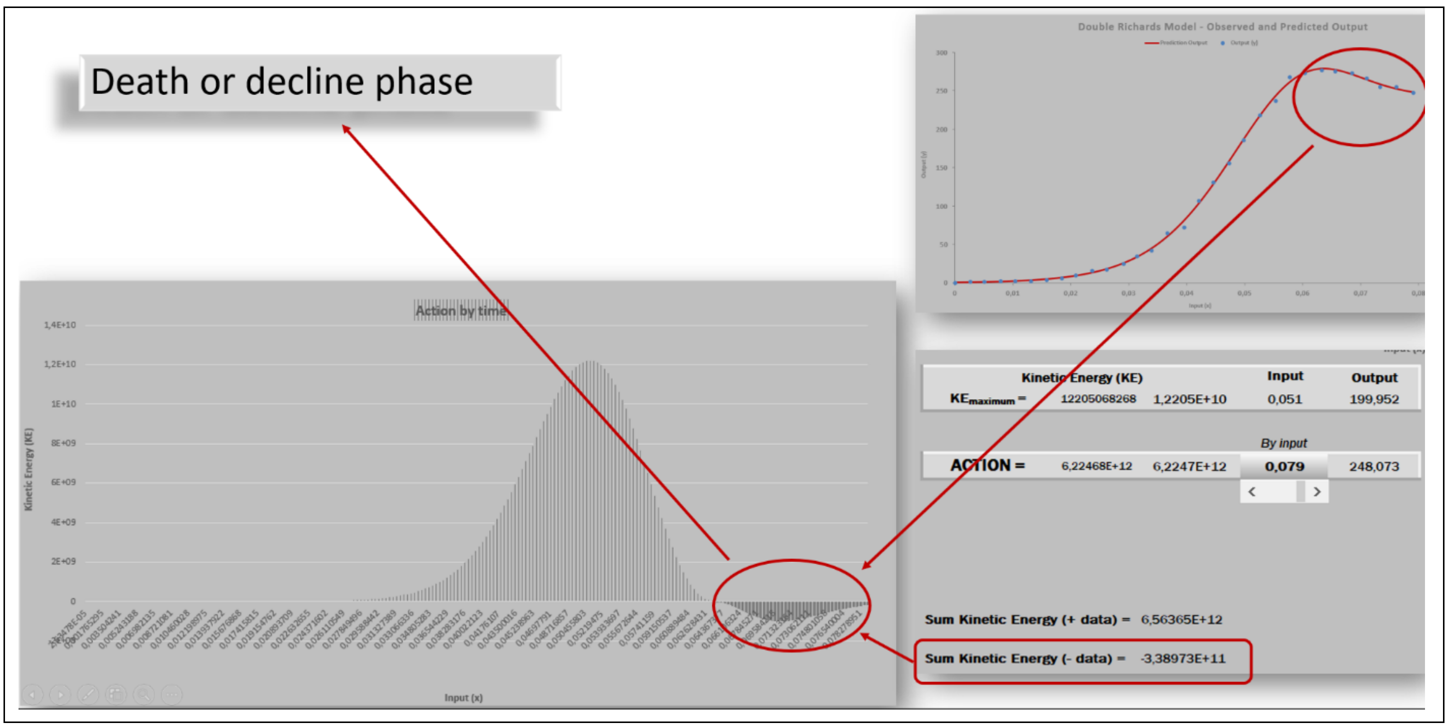

Figure 9. Death or decline phase.

In this way, this negative value, represented by the area over the kinetic energy curve, becomes a sensitive indicator for being sensitive to monitor the stress period very well (Fekedulegn et al., 2007; Vo et al., 2017). So, the action is an excellent indicator to monitor environmental stress, as it is flexible enough in characterizing and responding to this unfavorable effect at any time during the interval.

The kinetic energy is very sensitive to changes in the data of the growth curve and with clear biological meanings (Girtler, 2011, Grandpierre, 2009), which can be realistically interpreted as an effect of stress, showing very suitable for complex multiphasic growth behaviors and practical application, providing the understanding and measurement of the effect of environmental stress on growth.

Lag phase

This period is of great importance, especially for the food industry, which aims to extend this period indefinitely, avoiding or preventing possible contamination (Buchanal & Cygnarowicz, 1990; Gibson et al., 1987, 1988; Jiang et al., 2018). Such concern meets a growing appeal from the consumer that demands guarantees on the shelf life of the products consumed, for this it is extremely important to predict the end of the lag phase (Bath et al., 2002; Fukikawa & Kano, 2009).

The most classic definition of the end of the lag phase is the tangent that passes through the inflection point and cuts the lower asymptote, being attributed through the maximum second derivative of the growth curve, representing the maximum of the first derivative the inflection point (Baranyi & Pin, 1999; Buchanan & Cygnarowicz, 1990).

Defining the lag phase duration has been a great challenge (Baranyi et al, 1993, 1995; Fakruddin et al., 2011; Lambertini et al., 2010). Therefore, the second derivative of the growth function is suggested, the single point of maximum used to define the end of the lag phase and the single point of minimum for the completion of the growth phase. Other alternatives would be to apply the tangent to the inflection point, the introduction of new parameters in the growth curve itself (Fujikawa et al, 2004; Huang, 2013).

Even applying so many derivatives, the lag phase duration remains indeterminate, critical and mysterious (Bentea et al., 2017; Morris & Finke, 2009; Watzky & Finke, 1997)

However, from the ontogenetic growth force curve and its derivatives, it is possible to define more precisely the points F0, F1, F2 and F3 that they now characterize, with accuracy and applying robust principles of physics (Figure 10). F0 represents the moment that characterizes the exact transition between the lag phase to the exponential phase. Thus, the definition of this critical point by the PPFM spreadsheet offers a new indicator of the lag phase duration. However, it is essential that adequate data is ensured to characterize the referred phase (Baranyi et al 1993).

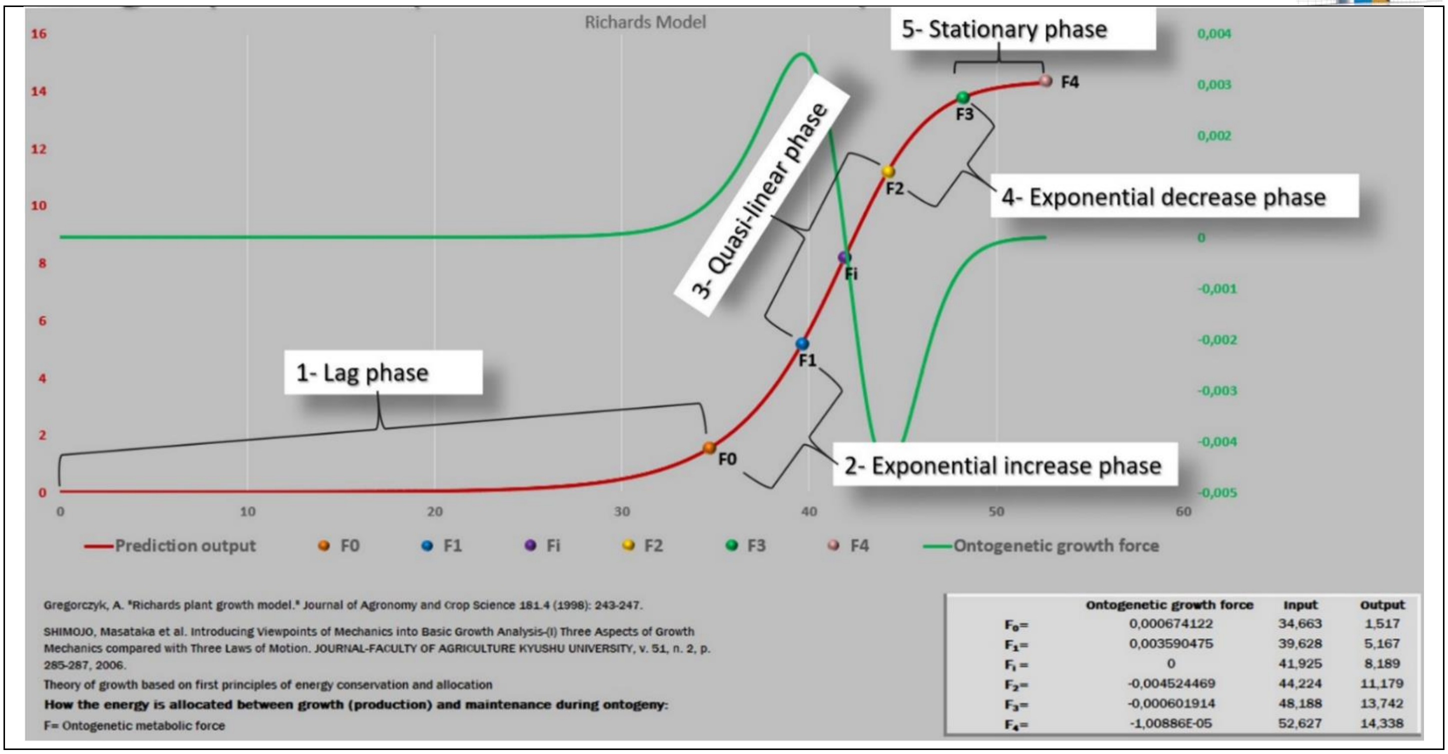

Figure 10. Phase and critical points defined by the PPFM (Practical Program for Forces Modeling; https://sites.google.com/view/ppfm-spreadsheet/) spreadsheet

Thus, the greatest innovation of the PPFM spreadsheet is in the procedure of defining the location of the points that end the lag phase, that is, the moment of transition between the end of the lag phase and the beginning of the growth phase (Dalgaard & Koutsoumanis, 2001).

This original concept offers new insights into the growth process and mainly by allowing to compare and discuss different and distinct behaviors between the growth curve (e.g. group subjected to favorable conditions and another to stressful conditions).

Resilience Phase

What characterizes the beginning of the resilience phase can be considered as the period of the growth curve in which survival is the priority and not the growth itself, requiring adaptations that prioritize survival, rather than growth (Mayne et al., 2015). And depending on the severity of the stress suffered, the growth becomes negative, reflecting in the area over the kinetic energy curve, capturing quantitatively the effect of the stressful condition (Figure 11).

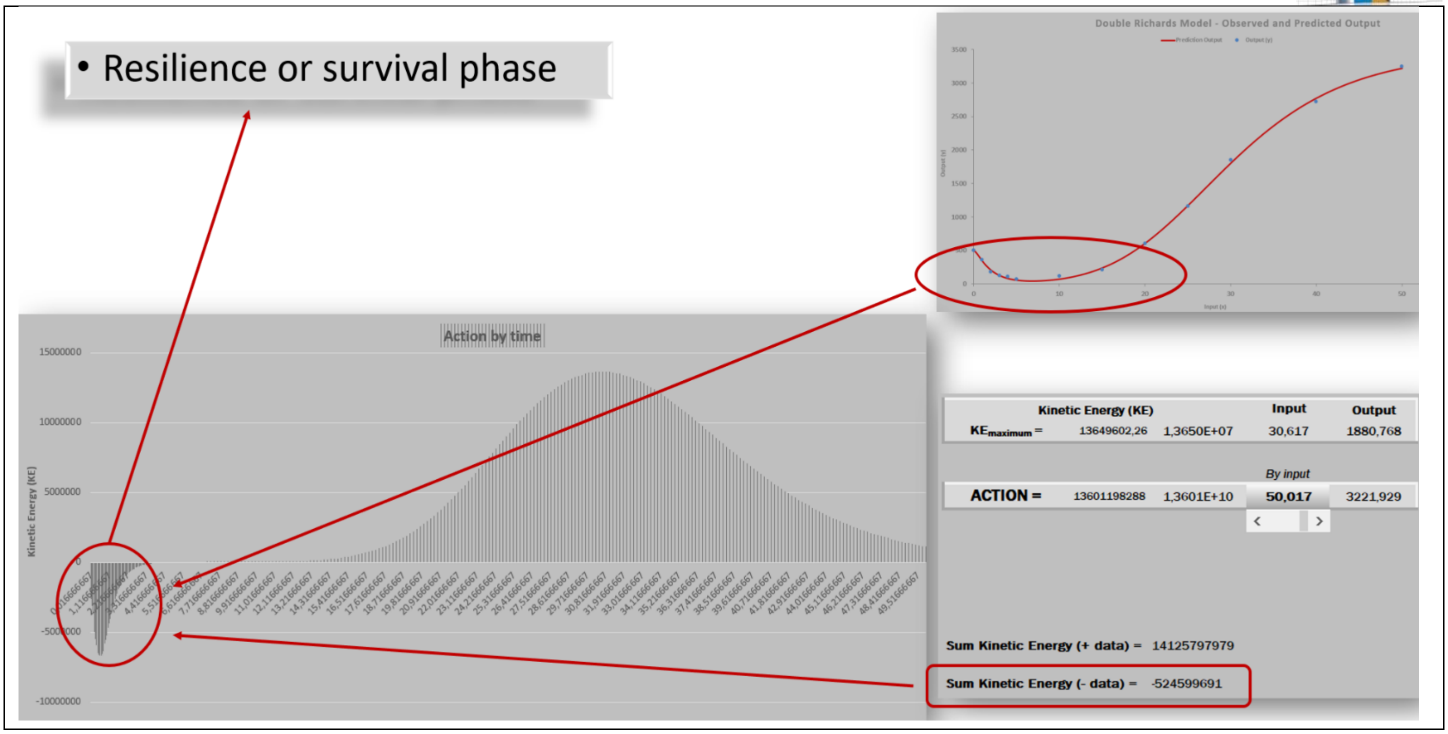

Figure 11. Resilience or survival phase.

Thus, retreat and recovery represent different aspects to assess the resilience phase of a growth process. Therefore, your information is complementary and not conflicting. That is, the first measures the impact and the time needed to get around the disorder, while the second, the time needed to recover (Pruitt & Kamau, 1993; Westerhoff et al., 2014).

Resistance is necessary, as well as smart strategies to promote growth, as there are always stressors opposing the journey (Ben-Jacob, 2009). But when the best strategy is to retreat, resilience manifests itself, characterizing the moment that the disturbance prevents the growth from continuing (Jabr, 2012; Mayne et al., 2015).

The great importance of a system being resilient is to allow and guarantee its sustainability (Juan-Garcia et al. 2017), to minimize the magnitude and duration of the disturbance. And so, maintain the continuity of growth by returning to the condition immediately prior to the stress suffered (stress conditions).

The robustness of the resilience (the regions where the area is over the kinetic energy curve) allows estimating the impact, numerically quantified, characterizing the moments of recoil and recovery induced by a stressful event of great magnitude, being a flexible index to represent the moment of resilience in a growth curve (Fekedulegn et al., 2007).

The PPFM spreadsheet allows to evaluate and quantify the capacity of an organism or system to absorb and recover, from an overtime impact of a stressor, to these adverse stressors, showing the ability to reduce the magnitude (area over the kinetic energy curve) to disturbance. The measurement of stress for being across the area, allows the comparison between different models, as it represents a neutral measure (Myerson et al., 2001; Pruessner et al., 2003).

Defining other growth curve transition points

Growth can be limited by a simple essential factor that also justifies the plateau phase. In relation to the subsequent decline observed after the saturation phase (plateau), it is generally justified by the accumulation of waste excretory products and by the decrease in the supply of nutrients, that is, environmental stressors (physical, chemical or competitive) (Burkhart & Tomé, 2012; Zeide, 1993).

Also, the same can be said about the moment of transition between decreasing exponential growth and the beginning of the asymptotic phase. If it were really defined, it would characterize the moment of the lag, growth and plateau phase, the geometric options offered (linear approximation), other mathematics (derivatives), resulting in good approximations, but without full success visually detected (Baty & Delignette-Muller, 2004).

To define the various mysterious and challenging transition points present in a growth curve (geometric description) the Newtonian point of view (description of history, its phases and strategies) is now applied (Simeonov et al., 2012). It is visually explicit that the point F1 represents exactly the beginning of the linear phase, and at the same time, the end of the exponential growth phase.

Point F2 marks the exact moment at the end of the linear period and the beginning of the decreasing exponential phase. The points F0, F1, F2 and F3 are endpoints that allow to mark the end of each phase of the growth curve, and thus, it allows to define its duration.

Then, also opportune for a better understanding of the growth process, are the beginning of the linear phase and its end (F1↔F2). Finally, point F3 corresponds to the moment when the decreasing exponential phase ends (F2↔F3) and starts the plateau phase (Bilge & Pekcan, 2013; Passos et al., 2012; Sedmák & Scheer, 2015).

However, it was necessary to use the derivatives of the ontogenetic growth force to find the points F0 and F3. Thus, the third derivative of the ontogenetic growth force curve (Snatch) was obtained.

The justification for using the third derivative is that the real moment when the lag phase ends and the phase of increasing exponential growth begins (point F0). Still, by Snatch it was possible to define the point F3, which marks the end of the phase of exponentially decreasing growth and the beginning of the plateau phase. Indeed, without the concepts of physics it would not be possible to find these points, which from now on are real and no longer mysterious, obscure or arbitrary (Bentea et al., 2017; Firsov et al 2001).

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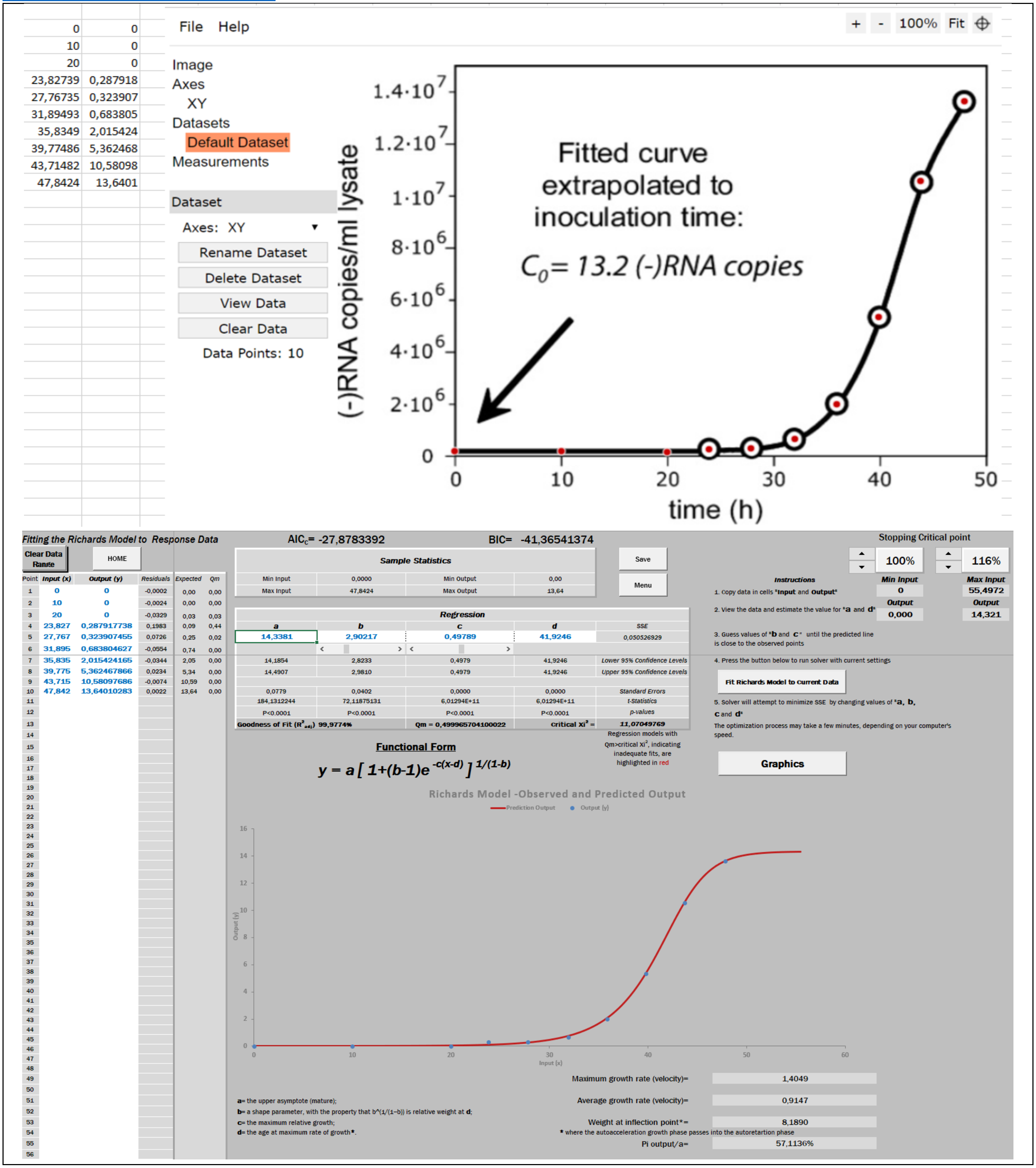
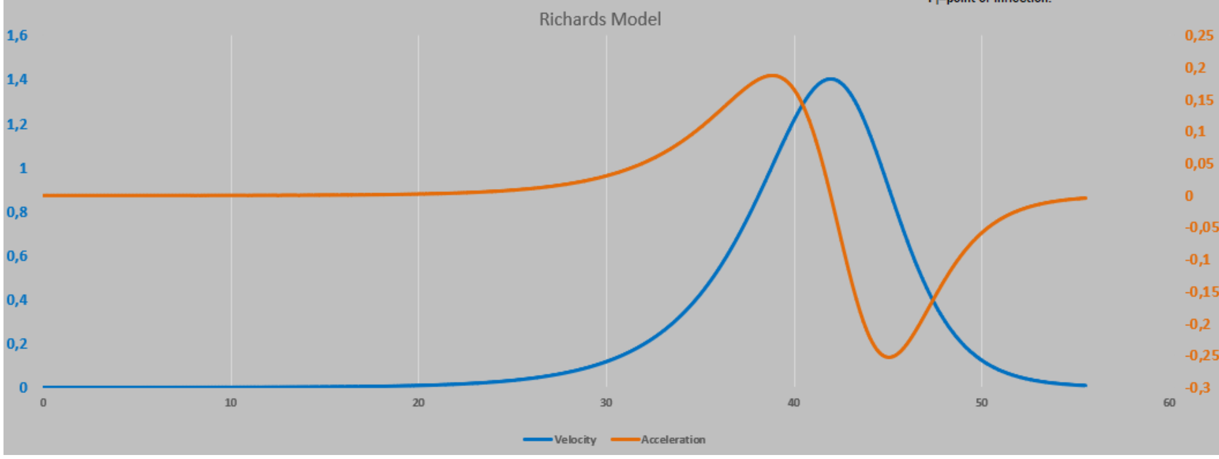
Appendix: examples of growth curves adjusted by PPFM spreadsheet

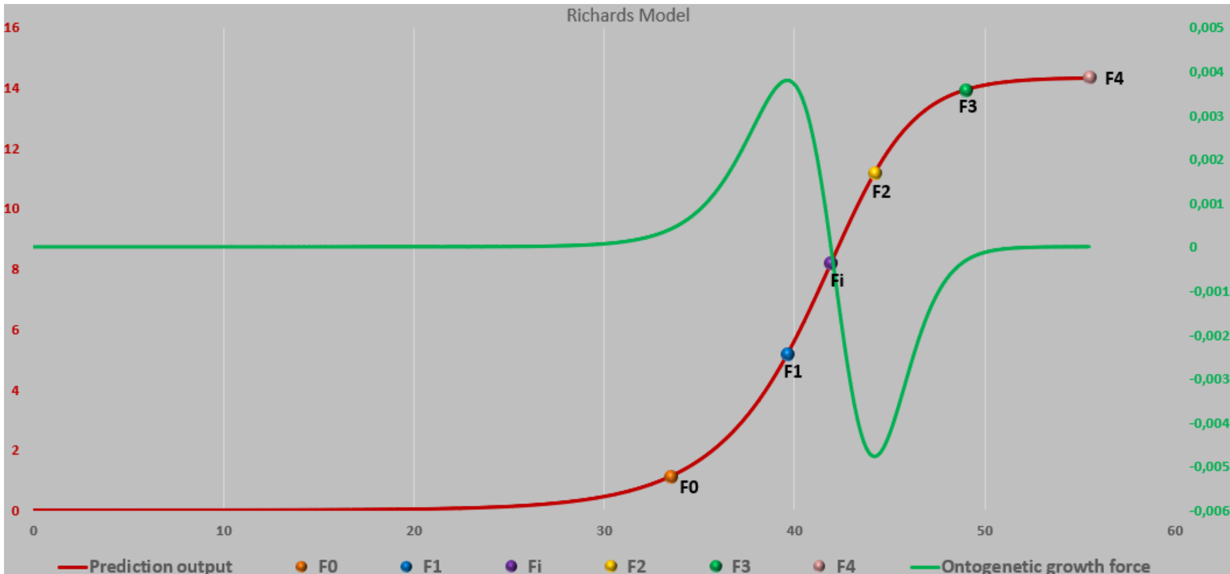
https://www.sciencedirect.com/science/article/pii/S0166093409004418
https://apps.automeris.io/wpd/



	Acceleration	Input	Output
P_1=	0,187034744	38,830	4,300
P_i=	0	41,925	8,189
P_2=	-0,252348191	45,027	11,987

P_1=maximum acceleration;
P_2=minimum acceleration;
P_i=point of inflection.





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Theory of growth based on first principles of energy conservation and allocation

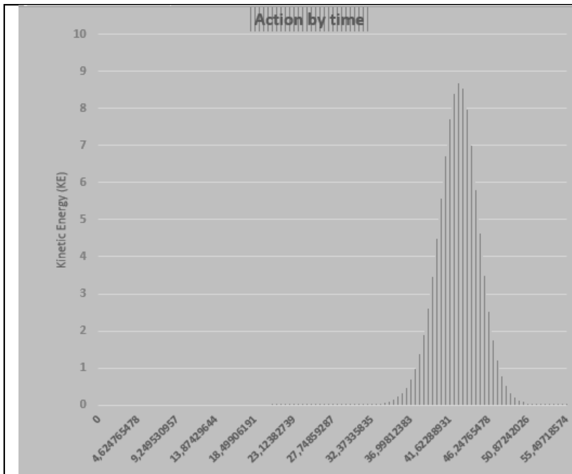
How the energy is allocated between growth (production) and maintenance during ontogeny:

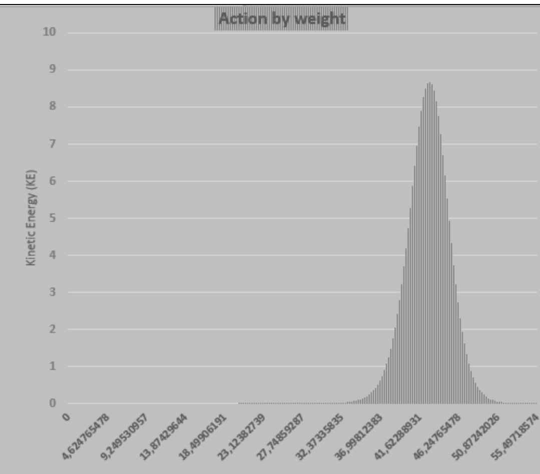
F= Ontogenetic metabolic force
m_1= mass of existing biomass
m_2=mass of new biomass (anabolism)
m=total biomass (m_1+m_2)
a_c=acceleration
$F=m*a_c$
$F=(m_1+m_2)*a_c$
$F=m_1*a_c + m_2*a_c$
m_1*a_c=Ontogenetic maintenance force
m_2*a_c=Ontogenetic growth force
§ mass (m_1 and m_2) can be used to determine the ontogenetic maintenance and growth forces

	Ontogenetic growth force	Input	Output
F_0=	0,000394144	33,483	1,118
F_1=	0,00378602	39,625	5,164
F_i =	0	41,925	8,189
F_2=	-0,004771714	44,213	11,167
F_3=	-0,000323511	48,989	13,930
F_4=	-6,30229E-07	55,497	14,338

F_0= lag phase duration;
F_1= maximum force;
Fi = "null force";
F_2 = minimum force;
F_3 = saturation point ;
F_4 = stability phase ≈ "a";
F = "Force that gets involved in the growth".

Exponential growth phase = F0 (1,12) to F1
Quasi-linear phase = F1 to F2
Exponential decay phase = F2 to F3
Saturation phase = F3 to F4
Stationary phase = > F4
F4 (Stability point ≈ "a") 14,3380621





	Kinetic Energy (KE)		Input	Output
$KE_{maximum}$ =	8,67	8,6749E+00	42,733	9,314
			By input	
ACTION =	2.697,20	2,6972E+03	**55,497**	14,321
				By output
ACTION =	2.697,20	2,6972E+03	55,497	**14,321**

Theory of growth based on first principles of energy conservation and allocation:

$$KE = m * v^2 / 2$$

KE= Kinetic Energy
m=total biomass
v=velocity

Power tells you how fast a job is done (J / s). The action, on the other hand, clarifies how great the performance of work is in a given period of time (J × s) (Girtler, 2011).
The action presents as quantitative base the dimension energy × time, integrating the evaluation of the entire interval of the event under consideration, from the beginning until its end endpoint (Grandpierre, 2011).
It should be noted that action must be understood as a property that characterizes the whole trajectory of growth, rather than as a single point along that path (Gribbin, 1998). From the above, it is essential to determine precisely the growth trajectory, and nothing more opportune than the flexibility of the sigmoid curves (Kebreab et al., 2010), when describing the three distinct phases: acceleration phase, linear phase and, finally, the saturation phase.
At last, the integral of the KE as a function of time yields the Action, which in a simple way would be the sum of all the useful work of the growth path.

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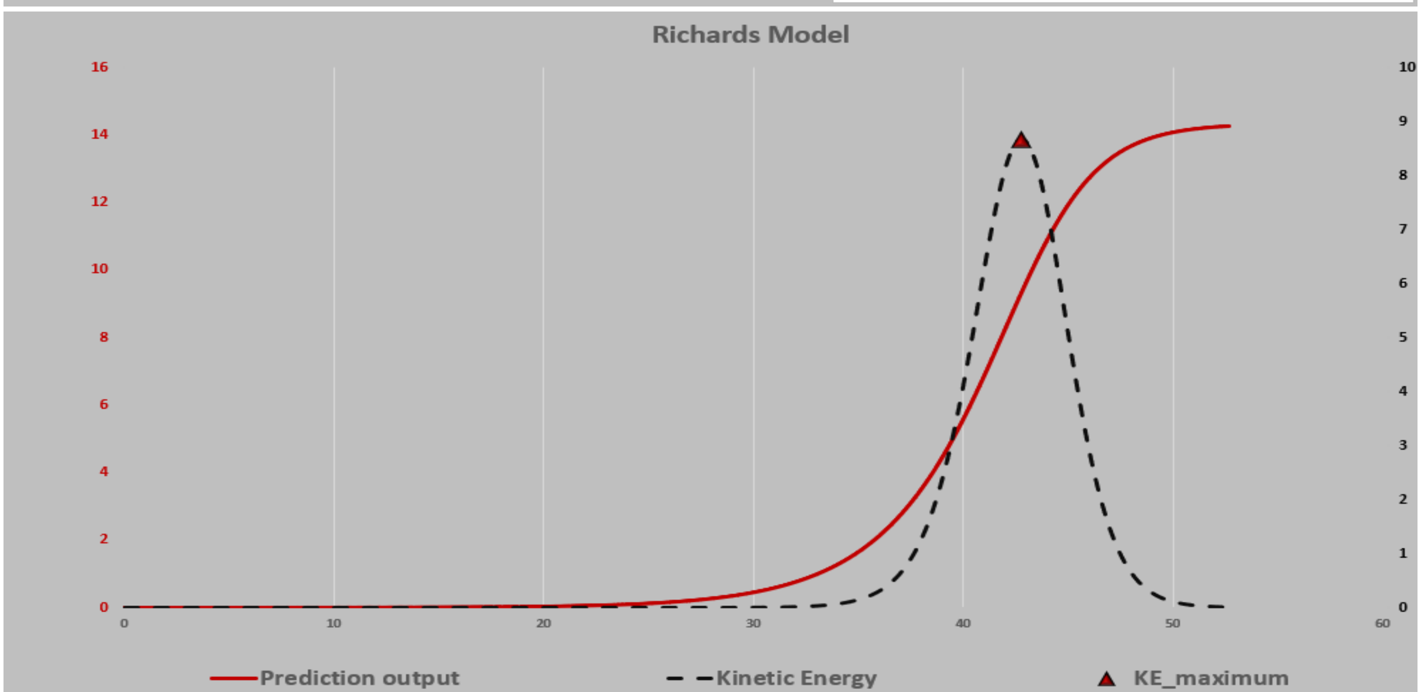
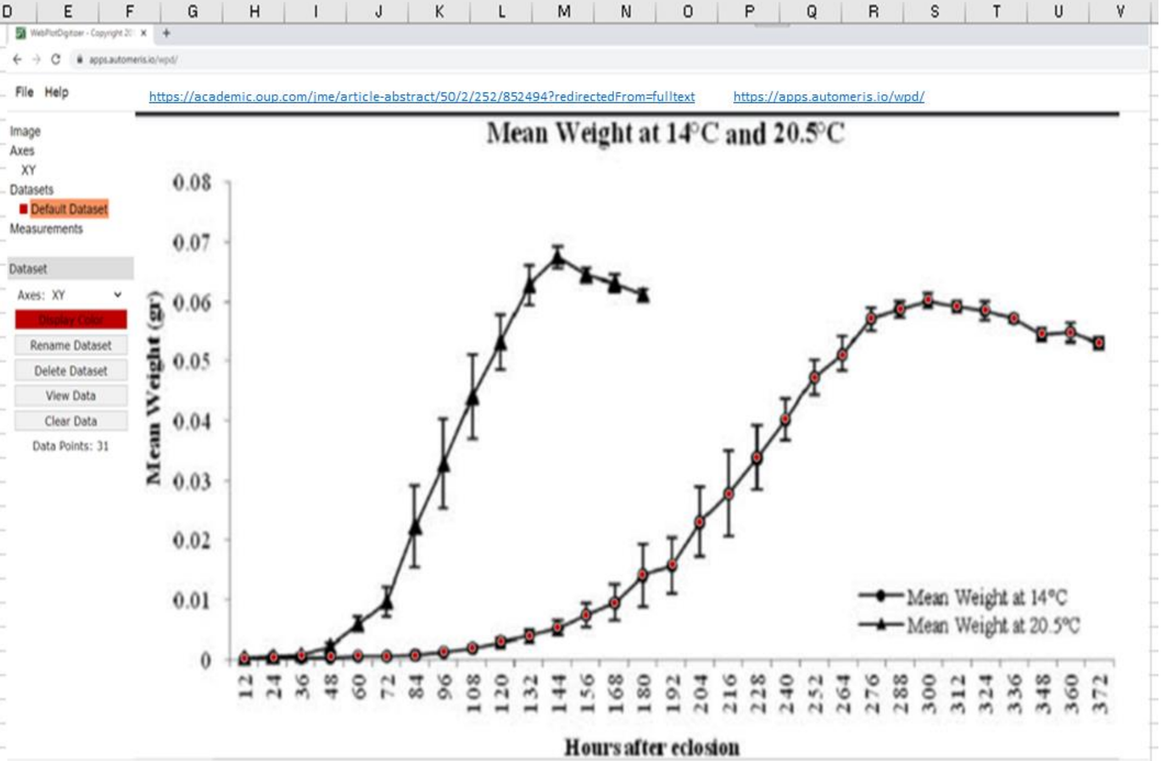


Sum Kinetic Energy (+ data) = 2843,868086

Sum Kinetic Energy (- data) = 0

https://academic.oup.com/jme/article/50/2/252/852494
https://apps.automeris.io/wpd/

	A	B	C
1	12	0,0003	0,3143
2	24	0,0005	0,4715
3	36	0,0006	0,6287
4	48	0,0006	0,6287
5	60	0,0008	0,7859
6	72	0,0008	0,7859
7	84	0,0008	0,7859
8	96	0,0014	1,4145
9	108	0,002	2,0432
10	120	0,0031	3,1434
11	132	0,0041	4,0864
12	144	0,0055	5,501
13	156	0,0075	7,5442
14	168	0,0096	9,5874
15	180	0,0143	14,303
16	192	0,016	16,031
17	204	0,0231	23,104
18	216	0,0278	27,819
19	228	0,0339	33,949
20	240	0,0404	40,393
21	252	0,0473	47,308
22	264	0,0511	51,081
23	276	0,0572	57,21
24	288	0,0588	58,782
25	300	0,0604	60,354
26	312	0,0593	59,253
27	324	0,0586	58,625
28	336	0,0572	57,21
29	348	0,0547	54,695
30	360	0,0549	54,853
31	372	0,0531	53,124



PPFM Action Phormia regina mg - Excel

Fitting the Double Richards Model to Response Data

AIC_c= -417,32 BIC= -412,98

Clear Data Range | HOME | Save | Menu

Point	*Input (x)*	*Output (y)*	*Residuals*	*Expected*	*Qm*
1	12	0,000314	0,0008	-5E-04	-0
2	24	0,000472	0,0009	-4E-04	-0
3	36	0,000629	0,0009	-2E-04	-0
4	48	0,000629	0,0007	-3E-05	-0
5	60	0,000786	0,0005	0,0003	0
6	72	0,000786	0,0001	0,0006	0
20	240	0,040393	0,0003	0,0401	0
21	252	0,047308	0,0002	0,0472	0
22	264	0,051081	-0,0021	0,0531	0
23	276	0,05721	0,0001	0,0571	0
24	288	0,058782	-0,0003	0,0591	0
25	300	0,060354	0,0009	0,0595	0
26	312	0,059253	0,0002	0,0591	0
27	324	0,058625	0,0005	0,0582	0
28	336	0,05721	0,0002	0,057	0
29	348	0,054695	-0,0010	0,0557	0
30	360	0,054853	0,0004	0,0544	0
31	372	0,053124	0,0000	0,0531	0
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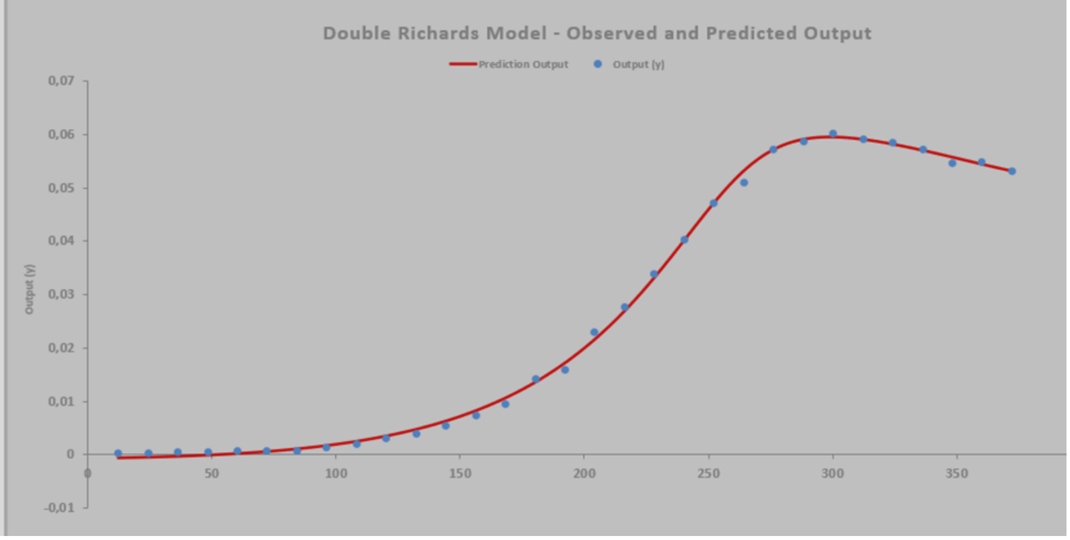
48					

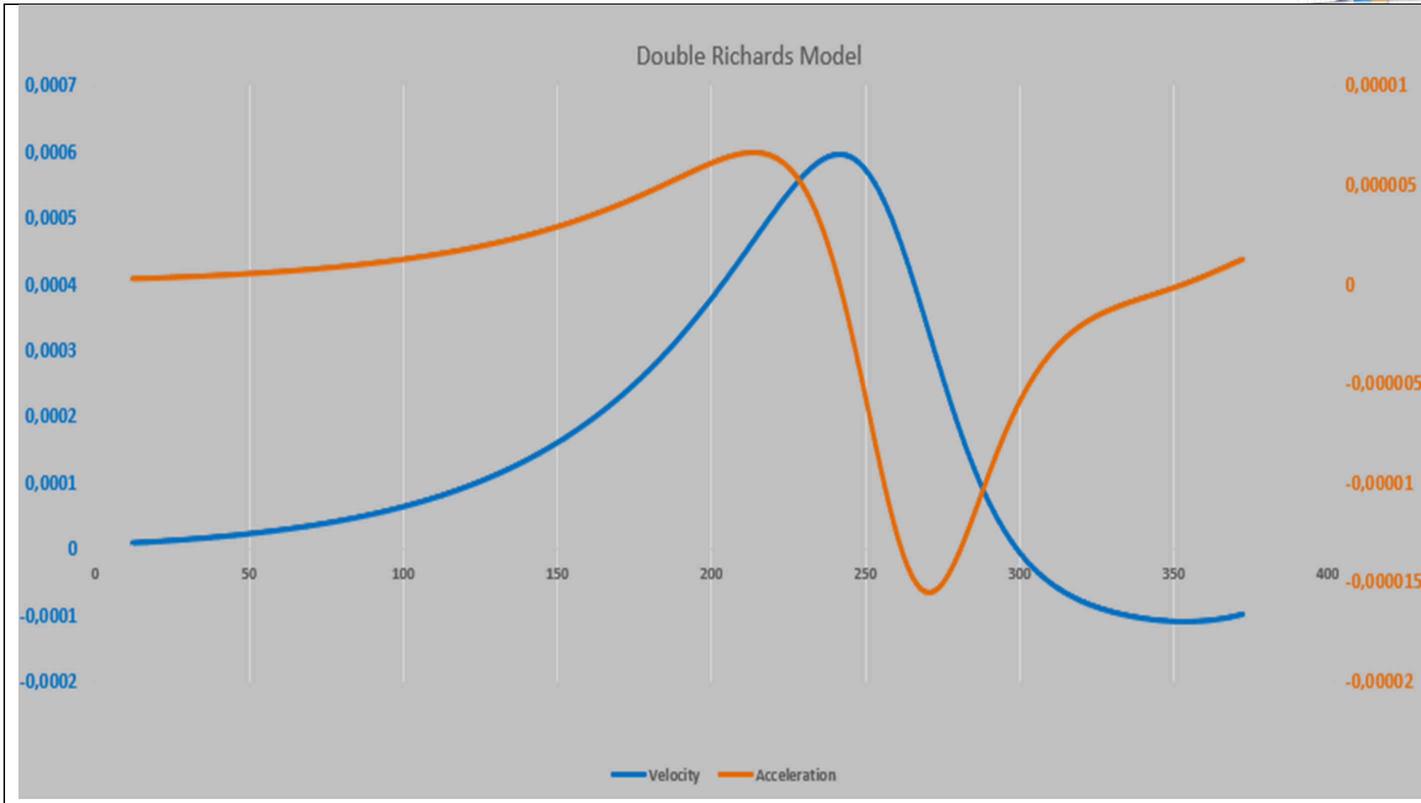
Sample Statistics

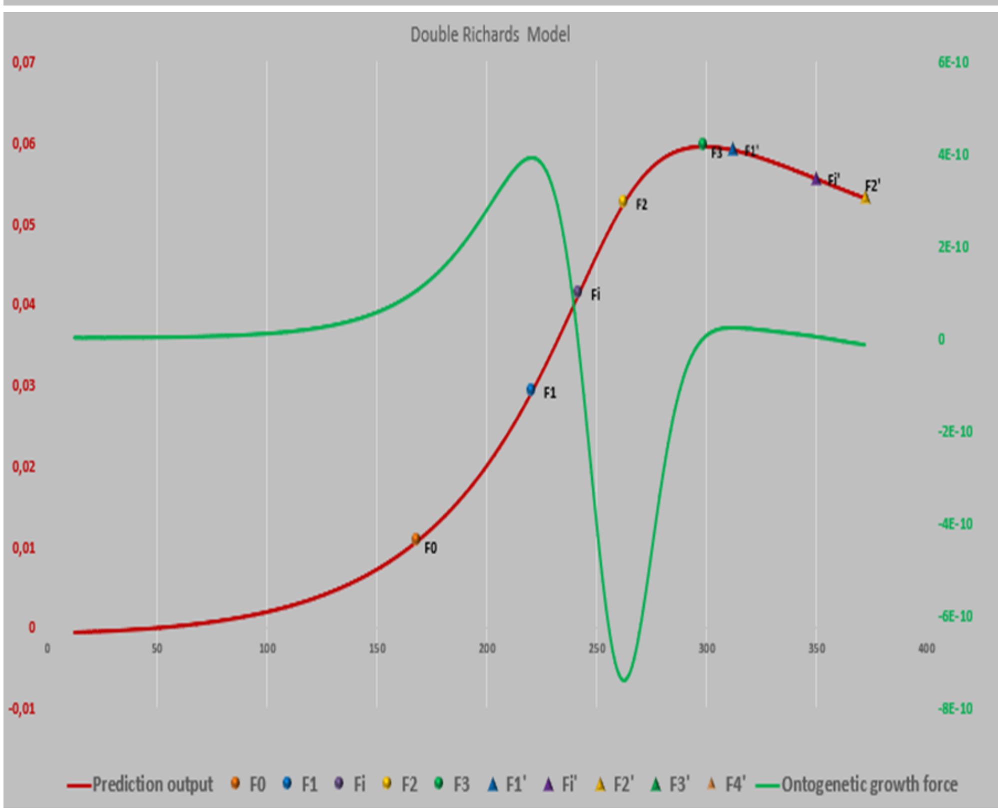
Min Input	12,0000	Min Output	0,00
Max Input	372,0000	Max Output	0,06

Regression

a	b	c	d	a'	b'	c'	d'	SSE
0,073986702	5,332814809	0,069723334	242,086098	0,049979932	9,494975417	0,058316745	349,878607	0,0000





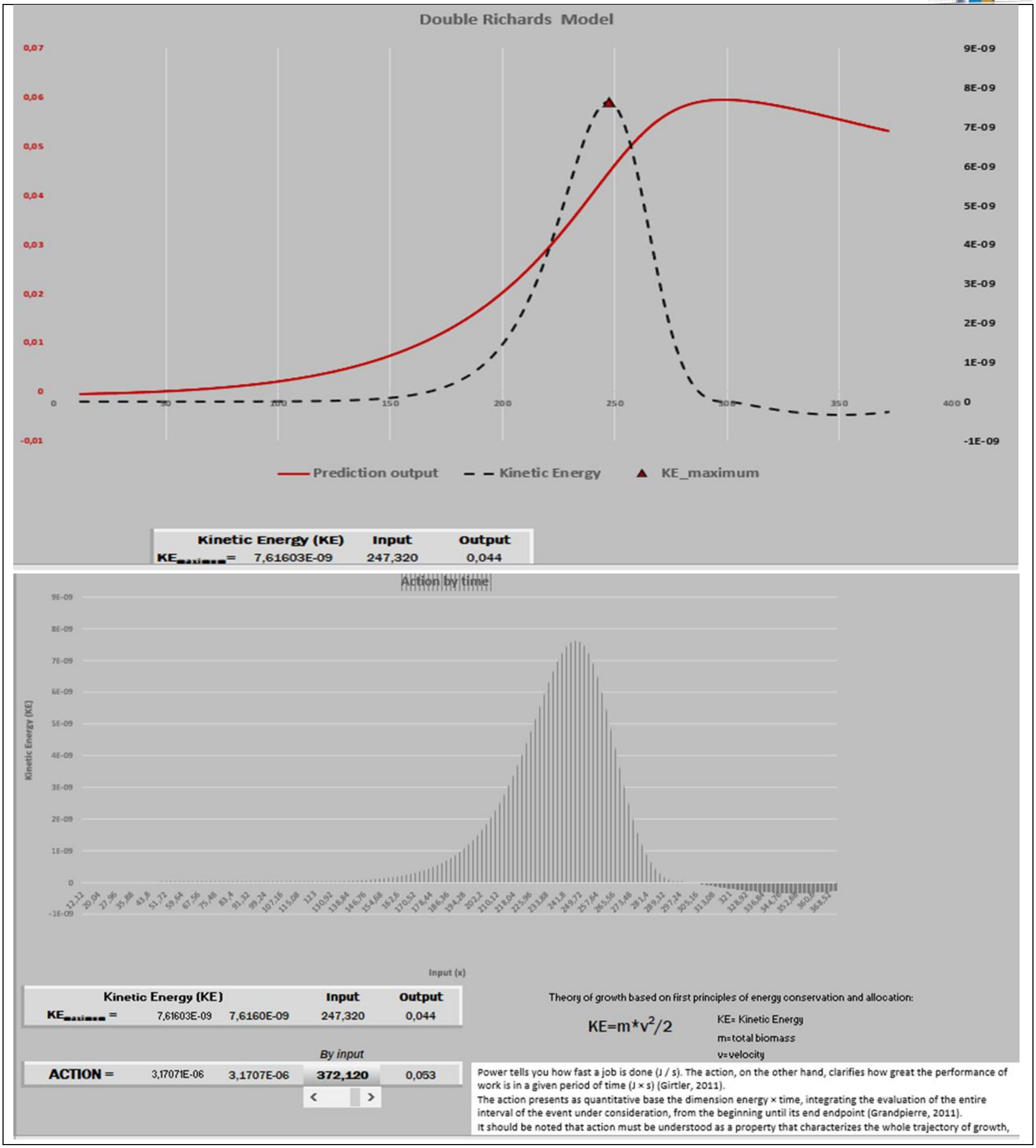


Ontogenetic growth force		Input	Output
F_0=	0	168,360	0,011
F_1=	3,8954E-10	220,560	0,029
F_i =	-2,797E-11	242,04	0,041
F_2 =	-7,404E-10	262,560	0,053
F_3 =	-3,039E-13	298,800	0,060
F_1'=	2,2231E-11	312,000	0,059
F_i'=	2,6333E-12	349,8	0,056
F_2'=	-1,474E-11	372,120	0,053
F_3'=			
F_4'=			-0,001

F_0= lag phase duration;
F_1 and F_1'= maximum force;
Fi and Fi'= "null force";
F_2 and F_2'= minimum force;
F_3 and F_3'= saturation point ;
F_4' = stability phase ≈ "a";
F = "Force that gets involved in the growth".

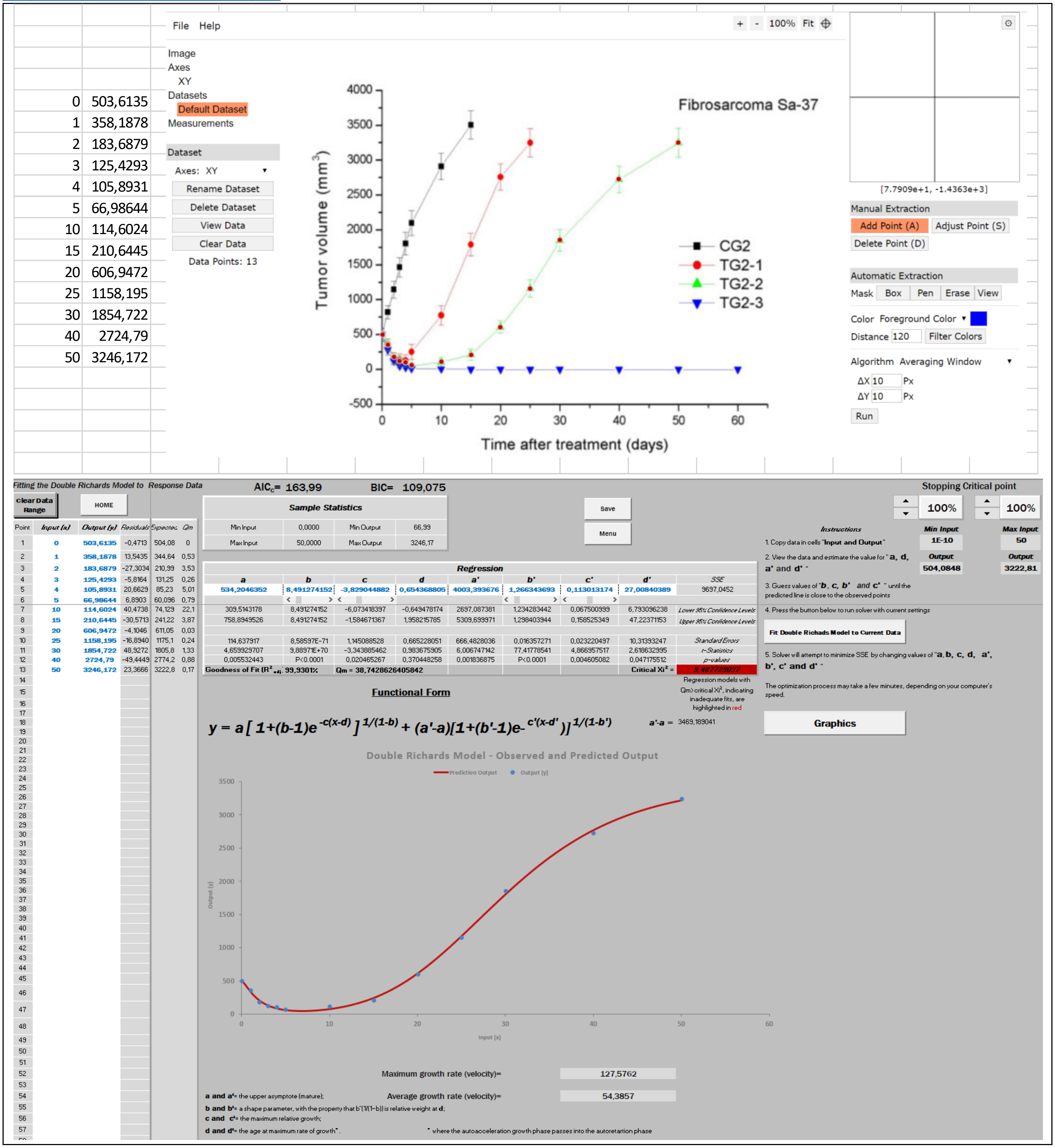
Exponential growth phase = F0 (0,01) to F1
Quasi-linear phase = F1 to F2 and F1' to F2
Exponential decay phase = F2 to F3 and F2' to F
Saturation point = F3 and F3'
F4' (Stability point ≈ " a' ") = -0,001

' Seconde equation

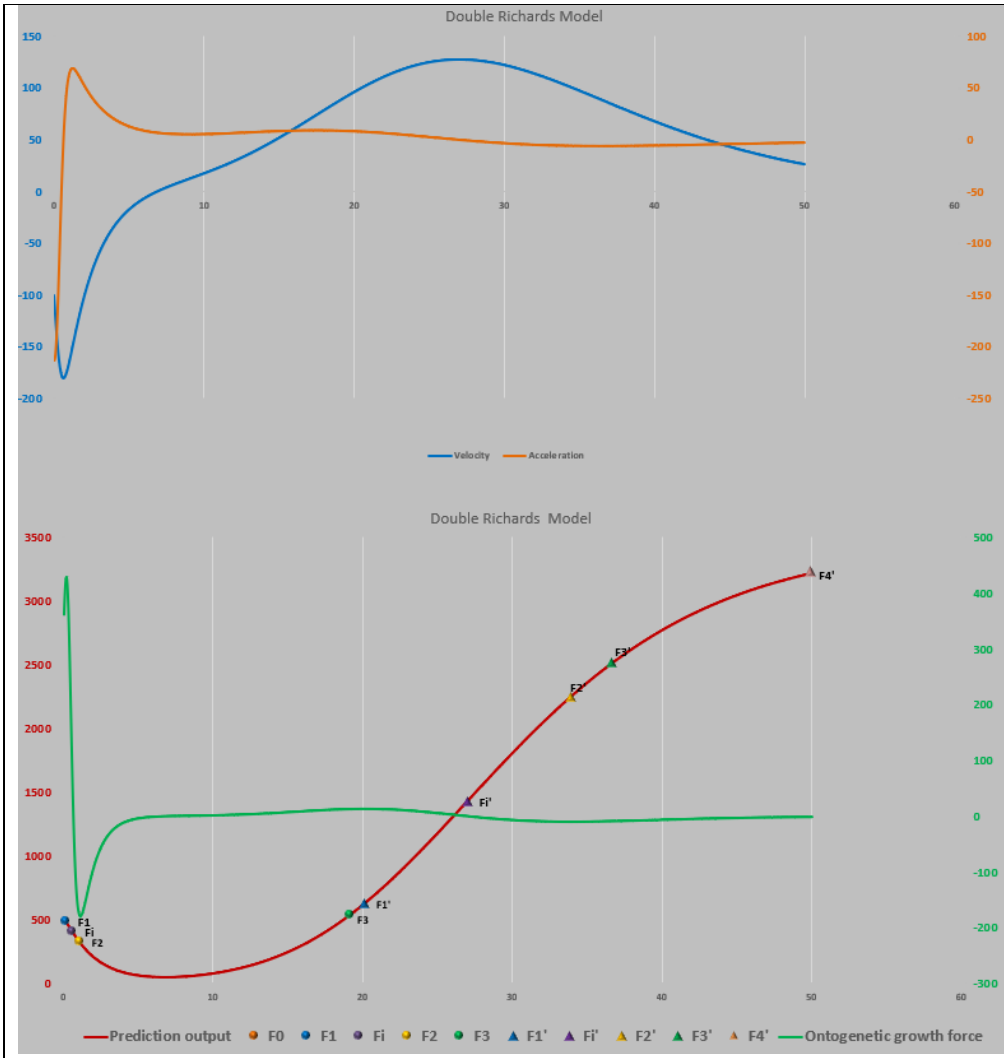


Double Richards Model
Prediction output
Kinetic Energy
KE_maximum
Kinetic Energy (KE)
Input
Output
KEmaximum = 7,61603E-09 247,320 0,044
Action by time
Kinetic Energy (KE)
Input (x)
Kinetic Energy (KE)
Input
Output
KEmaximum = 7,61603E-09 7,6160E-09 247,320 0,044
By input
ACTION = 3,17071E-06 3,1707E-06 372,120 0,053
Theory of growth based on first principles of energy conservation and allocation:
KE=m*v²/2
KE= Kinetic Energy
m=total biomass
v=velocity
Power tells you how fast a job is done (J / s). The action, on the other hand, clarifies how great the performance of work is in a given period of time (J × s) (Girtler, 2011).
The action presents as quantitative base the dimension energy × time, integrating the evaluation of the entire interval of the event under consideration, from the beginning until its end endpoint (Grandpierre, 2011).
It should be noted that action must be understood as a property that characterizes the whole trajectory of growth,

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC539271/
https://apps.automeris.io/wpd/



https://anemal.ucdavis.edu/
https://www.fmva.unesp.br/

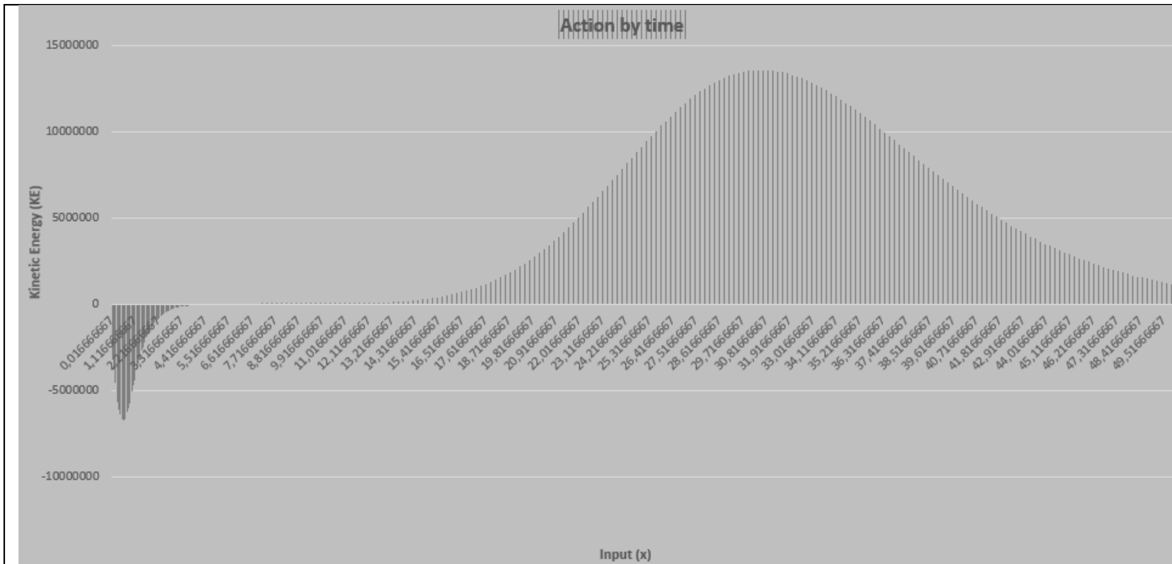


Ontogenetic growth force		Input	Output
F_0 =			
F_1 =	428,350799	0,183	482,898
F_i =	2,82155356	0,65	405,896
F_2 =	-178,59328	1,117	325,633
F_3 =	12,9026762	19,167	534,205
F_1'=	13,2096674	20,133	623,925
F_i'=	0,02331987	27	1428,458
F_2'=	-9,824505	33,883	2248,283
F_3'=	-8,8282948	36,666	2515,183
F_4'=	-1,2040161	50,017	3223,248

F_0= lag phase duration;
F_1 and F_1'= maximum force;
Fi and Fi'= "null force";
F_2 and F_2'= minimum force;
F_3 and F_3'= saturation point ;
F_4' = stability phase ≈ "a";
F = "Force that gets involved in the growth".

Exponential growth phase = F0 () to F1
Quasi-linear phase = F1 to F2 and F1' to F2
Exponential decay phase = F2 to F3 and F2' to F
Saturation point = F3 and F3'
F4' (Stability point ≈ " a' ") = ######

' Seconde equation



Kinetic Energy (KE)			Input	Output
$KE_{maximum}$ =	13558867,11	1,3559E+07	30,600	1878,545

Theory of growth based on first principles of energy conservation and allocation:

$$KE=m*v^2/2$$

KE= Kinetic Energy
m=total biomass
v=velocity

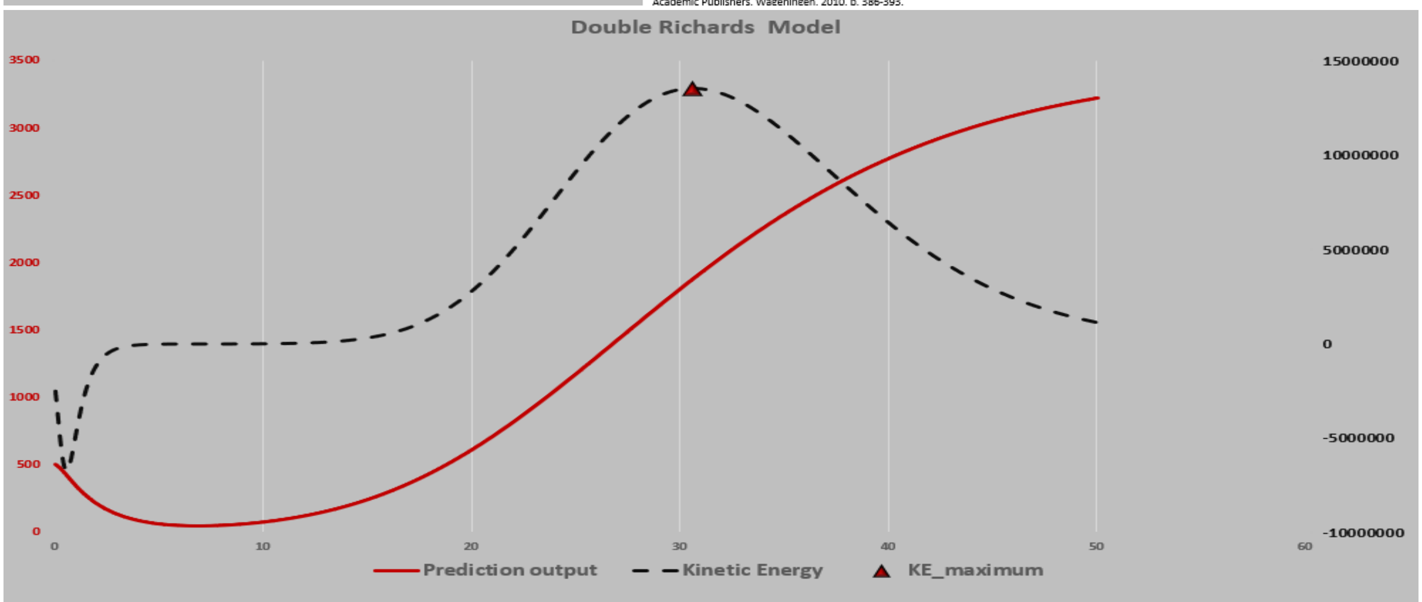
			By input	
ACTION =	13592907181	1,3593E+10	**50,017**	3223,248

Power tells you how fast a job is done (J / s). The action, on the other hand, clarifies how great the performance of work is in a given period of time (J × s) (Girtler, 2011).
The action presents as quantitative base the dimension energy × time, integrating the evaluation of the entire interval of the event under consideration, from the beginning until its end endpoint (Grandpierre, 2011).
It should be noted that action must be understood as a property that characterizes the whole trajectory of growth, rather than as a single point along that path (Gribbin, 1998). From the above, it is essential to determine precisely the growth trajectory, and nothing more opportune than the flexibility of the sigmoid curves (Kebreab et al., 2010), when describing the three distinct phases: acceleration phase, linear phase and, finally, the saturation phase.
At last, the integral of the KE as a function of time yields the Action, which in a simple way would be the sum of all the useful work of the growth path.

GIRTLER, Jerzy; KUSZMIDER, S.; PLEWIŃSKI, L. Valuation method for operation of crankshaftpiston assembly in combustion engines in energy approach. Journal of Polish CIMAC, v. 6, n. 1, p. 89-98, 2011.
GRANDPIERRE, Attila. The Biological Principle of Natural Sciences and the Logos of Life of Natural Philosophy: a Comparison and the Perspectives of Unifying the Science and Philosophy of Life. In: Phenomenology/Ontopoiesis Retrieving Geo-cosmic Horizons of Antiquity. Springer, Dordrecht, 2011. p. 711-727.
GRIBBIN, John. Q is for Quantum: Particle Physics from AZ. Universities Press, 1998.
KEBREAB, E. et al. Modelling the profile of growth in monogastric animals. In: Modelling nutrient digestion and utilisation in farm animals. Wageningen Academic Publishers. Wageningen. 2010. p. 386-393.

Sum Kinetic Energy (+ data) = 14116746927

Sum Kinetic Energy (- data) = -523839746



Kinetic Energy (KE)	Input	Output
$KE_{maximum}$= 13558867,11	30,600	1878,545