Spinodal decomposition in pure-gauge QCD

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Abstract. Spinodal decomposition in a model of pure-gauge $SU(2)$ theory that incorporates a deconfinement phase transition is investigated by means of real-time lattice simulations of the fully nonlinear Ginzburg-Landau equation. Results are compared with a Glauber dynamical evolution using Monte Carlo simulations of pure-gauge lattice QCD.

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Relativistic heavy-ion experiments produce highly excited hadronic matter. On the basis of quantum chromodynamics (QCD) one expects that at sufficiently high excitation energies this matter is composed by quarks and gluons that are not confined in the interior of hadrons. One important question in the study of the properties of the produced excited matter is the understanding of the dynamics of the quark-gluon deconfining process. One possible scenario [1] is that it proceeds via a process similar to spinodal decomposition of condensed matter physics [2]. Spinodal decomposition is an instability driven by infinitesimal amplitude, long wavelength fluctuations of an order parameter after a sudden temperature quench that brings the system into an unstable state. In pure gauge QCD, the equilibrium order parameter is the thermal average of the trace of the Polyakov loop in the fundamental representation, $\langle L_F \rangle$. For the $SU(2)$ gauge theory one can write [1]

$$L_F = \frac{1}{2}\mathrm{Tr}_F\, L\,, \qquad \text{where} \qquad L = \begin{pmatrix} e^{i\pi q} & 0 \\ 0 & e^{-i\pi q} \end{pmatrix}, \qquad 0 \le q \le 1\,. \tag{1}$$

For temperatures T below a critical value T_d, one has $\langle L_F \rangle = 0$, and for $T > T_d$, $\langle L_F \rangle \neq 0$. Like the magnetization of the Ising model, $\langle L_F \rangle$ is a nonconserved order parameter, in the sense that there is no associated conservation law.

In condensed matter problems the time evolution towards equilibrium of an nonconserved order parameter is characterized by a relaxation process and is commonly described by means of phenomenological macroscopic Ginzburg-Landau (GL) field equations with Langevin dynamics [2]

$$\frac{\partial \psi}{\partial t} = -\Gamma \frac{\delta S_{eff}[\psi]}{\delta \psi} + \varsigma\,, \tag{2}$$

where $S_{eff}[\psi]$ is the coarse grained effective action for the order parameter $\psi(\mathbf{x},t)$, Γ is a dissipation coefficient and ς is a noise term that mimics the thermal fluctuations of the system. The long-time equilibrium distribution $\rho[\psi_{eq}]$ at temperature T is given by

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$\rho[\psi] = \exp(-F[\psi]/T)$ where $F[\psi] = T\,S_{eff}[\psi]$ is the coarse-grained free-energy, or effective potential. The real time description of the Polyakov loop relaxation process directly from QCD is an impossible task at the moment. But one can use local Monte Carlo updating algorithms of lattice QCD simulations to imitate real time thermal fluctuations of the Polyakov loop, very much like the Glauber dynamical evolution [3] commonly used for spin systems [4]. This has been pursued some time ago by Miller and Ogilvie [1].

In this paper we present results of a real-time lattice simulation of the fully nonlinear GL equation of Eq. (2), and compare results with lattice Monte Carlo simulations of pure gauge $SU(2)$ theory. For the simulation of the GL equation we use the one-loop effective action $S_{eff}[\psi]$ [5], augmented with a phenomenological term to model the second order phase transition of $SU(2)$ pure gauge QCD [6]

$$S_{eff} = \int d^3x \left[\frac{\pi^2 T}{2g^2} (\nabla\psi)^2 + \frac{\pi^2 T^3}{12} \left(1-\psi^2\right)^2 - \frac{M^2 T}{4} \left(1-\psi^2\right) \right], \tag{3}$$

where $\psi(\mathbf{x},t)$ is related to q that parameterizes L in Eq. (1) by $q = (1-\psi)/2$. This action presents a second order deconfining phase transition at temperature $T_d = \sqrt{3/2}\,M/\pi$. At $T \ll T_d$, the minimum of the action density is at $\psi(\mathbf{x},t) = 0$. Now, if at $t = 0$ the temperature is rapidly increased to $T \gg T_d$, the system is brought to an unstable state and therefore will start "rolling down" to the two new minima of the effective action density. Therefore, at short times, when $\psi(\mathbf{x},t) \approx 0$, the ψ^4 term in Eq. (3) can be neglected and one can write the (noiseless) solution of Eq. (2) as

$$\psi(\mathbf{x},t \approx 0) = \int d^3k \; e^{i\mathbf{k}\cdot\mathbf{x} - \Gamma \frac{\pi^2 T}{g^2}\left[\mathbf{k}^2 - \frac{g^2 T^2}{3}\varepsilon(T)\right] t} \; \widetilde{\psi}(\mathbf{k},0), \tag{4}$$

where $\varepsilon(T) = 1 - T_d^2/T^2$, and $\widetilde{\psi}(\mathbf{k},0)$ is the Fourier transform of the $\psi(\mathbf{x},t=0)$. For $T \gg T_d$, one has $\varepsilon(T) \simeq 1$, and there will be an explosive exponential growth of long wavelengths modes, signalling the spinodal decomposition with a nonconserved order parameter. For a given temperature, there will exponential growth for modes with $k < k_c$, where $k_c^2 = \varepsilon(T)\,g^2 T^2/3$. Obviously there is a qualitative change with respect to the model Ref. [1] where$T_d = 0$, because spinodal decomposition will happen only for $T > T_d$. For $T \gg T_d$, the model of Ref. [1] and give the same short-time exponential growth of the order parameter.

As time increases, the order parameter increases and the linear approximation is no longer valid and the use of the complete effective action is necessary. From the point of view of the heavy-ions experiments, understanding the short-time behavior of $\langle L_F \rangle$ is important for knowing the time scales of the explosive growth of the order parameter compared to the expansion of the system. Since there is no hope that in a foreseeable future one will able to simulate an spectacular event like an heavy-ion simulation with lattice QCD, the use of phenomenological equations will be the only alternative available. The comparison between results of lattice QCD simulations and of real-time phenomenological equations like Eq. (2) is however essential for extracting parameters entering the phenomenological equations.

We solve the full nonlinear GL equation for $T = 2T_d$ and compare with a lattice simulation. Although we do not present results for dynamical critical exponents here, we are interested in the short-time growth of the order parameter to investigate the role of the nonlinearities. We use $T_d = 240$ MeV and $g = 3$, which gives $m_D(T) \simeq 210$ MeV for the Debye screening mass. We prepare several 64^3 lattices with random distributions around $\psi(\mathbf{x}, 0) \approx 0$ and then average the time-evolved solutions over all lattices. For the QCD simulation we thermalize several lattices with the heath-bath algorithm for $64^3 \times 4$ lattice sites with $\beta = 4/g^2 = 2.0$. From these thermalized lattices we evolve in "time" with $\beta = 3.0$ (which corresponds roughly to $T = 2T_d$) and then record the value of $\langle L_F \rangle$ for each heath-bath sweep, in the same lines of Ref. [1]. In Fig. 1 we present our results. On the left panel, are plotted the GL results on the right panel are the lattice QCD results.

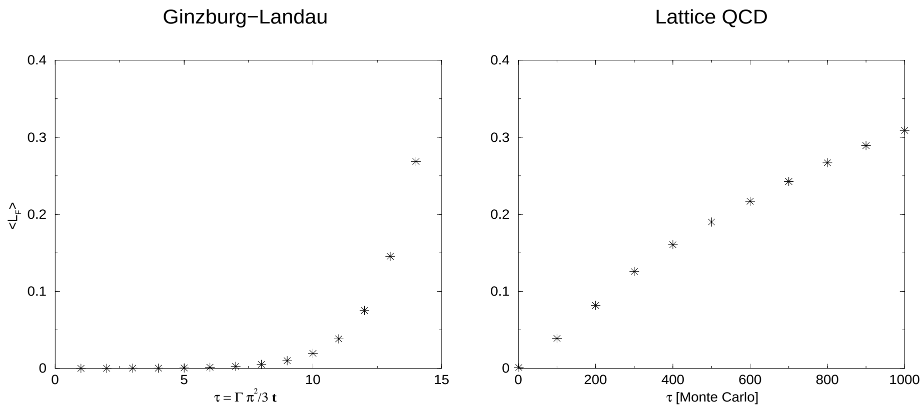

FIGURE 1. Left: GL equation. Right: lattice QCD.

Based on these results, one can make the point that in GL evolution, it seems that the nonlinearities start operating quite late, as compared to the MC time evolution. The MC early-time behavior seems nonlinear from the very beginning. While the early time evolution is exponential for the GL equation, in the MC evolution it seems to be power-law. The early-time nonlinearity might indicate that the dissipation constant Γ in a GL equation type of equation is not really a constant, but rather field dependent [7].

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