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The minimal 3-3-1 model with a spectator sextet

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Abstract

We consider the minimal 3-3-1 model with a heavy scalar sextet and realize, at the tree level, an effective dimension five interaction that contributes to the mass of the charged leptons. We give the scalar mass spectra of the model and analyze under which conditions the fields in the scalar sextet are heavy. With the effective dimension five operator and a triplet we obtain the correct charged lepton masses. In this case the neutrino masses arise by a type I seesaw mechanism and we also show how the Pontecorvo-Maki-Nakawaga-Sakata matrix arises. The conditions under which it is possible to have a stable vacuum at tree level are also given.

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I. INTRODUCTION

Almost all the extensions of the standard model (SM) include extra scalar multiplets: complex [1] or real singlets [2], two doublets [3], Hermitian [4] and/or non-Hermitian triplets [5, 6]. Moreover, usually extra scalar multiplets are introduced in a given model just in order to give masses to the neutrino or/and charged leptons. Hence, they may have small vacuum expectation values (VEV). However, this usually implies that they may be light neutral scalars which can be easily ruled out by phenomenology. In two Higgs doublets this is not the case when there is a positive quadratic term $\mu^2 > 0$, which behaves like a positive mass square term in the scalar potential. In this case this parameter may dominate the contributions to the masses of the multiplets' members which are almost mass degenerated. For instance, in the context of models with inert doublets, see [7] and [8].

The 3-3-1 models are intrinsically multi-Higgs models. For instance, in the minimal 3-3-1 model (here denoted by m331 for short) the charged leptons gain mass from a triplet and a sextet and the neutrino gain Majorana masses only through the sextet $(\mathbf{6}, 0)$ [9–11]. On the other hand, in the model with heavy charged [12] or neutral leptons [13], only triplets are needed if right-handed neutrinos are introduced and the type-I seesaw mechanism is implemented.

Because of the scalar sextet the scalar potential in the m331 becomes more complicated and for this reason in Ref. [14] was pointed out that the sextet can be omitted if a dimension five effective operator involving only triplets is introduced in order to give mass to the charged leptons and neutrinos. The m331 model without the sextet was called reduced m331 model [15]. However, as in the case of the SM, the question now is, how does this effective operator arise at tree and/or the loop level [16–18]. The mechanism for generating at tree level effective dimension five operators for the case of the neutrino masses, in the context of the m331 model, was given in Ref. [19].

In this paper we show that the sextet is just a way to generate, at the tree level, the effective five dimensional operator proposed in Ref. [14] in order to give to the charged leptons the correct mass. This happens if all fields in this multiplet are heavy and its neutral components gain a small (s_2^0) or a zero (s_1^0) VEV. In m331 we study, for the first time, the conditions upon the dimensionless coupling constants that imply a scalar potential bounded from below.

The outline of this paper is the following. In Sec. II we give the scalar representation content of the m331 model and the scalar potential of the model. In Sec. III we obtain the mass spectra of the model under the conditions of a $Z_7 \otimes Z_3$ discrete symmetries. In Sec. IV we obtain the charged lepton and neutrino masses and the corresponding Pontecorvo-Maki-Nakawaga-Sakata (PMNS) matrix. Our conclusions appear in Sec. V. The conditions for a stable minimum, at tree level, of the scalar potential are obtained in the Appendix A.

II. THE SCALAR SECTOR IN THE M331 MODEL

The scalar potential in several 3-3-1 models was considered in Ref. [20]. Here however, we will study only the m331 model in the situation in which the sextet gain a small VEV and there is no explicit total lepton number violation in the scalar potential, which is avoided by an appropriate discrete symmetry.

In the m331 model the scalar sector is composed of a sextet $S \sim (\mathbf{6}, 0)$ and three triplets: $\eta = (\eta^0, -\eta_1^-, \eta_2^+)^T \sim (\mathbf{3}, 0)$, $\rho = (\rho^+, \rho^0, \rho^{++})^T \sim (\mathbf{3}, 1)$ and $\chi = (\chi^-, \chi^{--}, \chi^0)^T \sim (\mathbf{3}, -1)$, where (x, y) refer to the $(SU(3)_L, U(1)_X)$ transformations. The triplet η and the sextet S

$$S = \begin{pmatrix} s_1^0 & \frac{s_1^-}{\sqrt{2}} & \frac{s_2^+}{\sqrt{2}} \\ \frac{s_1^-}{\sqrt{2}} & S_1^{--} & \frac{s_2^0}{\sqrt{2}} \\ \frac{s_2^+}{\sqrt{2}} & \frac{s_2^0}{\sqrt{2}} & S_2^{++} \end{pmatrix}. \quad (1)$$

We can write the $SU(3)$ multiplets above in terms of the $SU(2)$ ones:

$$\eta = \begin{pmatrix} \Phi_\eta \\ \eta_2^+ \end{pmatrix} \sim (\mathbf{3}, 0), \quad \rho = \begin{pmatrix} \Phi_\rho \\ \rho^{++} \end{pmatrix} \sim (\mathbf{3}, 1), \quad \chi = \begin{pmatrix} \Phi_\chi \\ \chi^0 \end{pmatrix} \sim (\mathbf{3}, -1). \quad (2)$$

and

$$S = \begin{pmatrix} T & \frac{\Phi_s}{\sqrt{2}} \\ \frac{\Phi_s^T}{\sqrt{2}} & S_2^{--} \end{pmatrix}, \quad S^* = \begin{pmatrix} T^* & \frac{\Phi_s^*}{\sqrt{2}} \\ \frac{\Phi_s^\dagger}{\sqrt{2}} & S_2^{++} \end{pmatrix}, \quad (3)$$

where, under the $SU(2) \otimes U(1)_Y$ group

$$\Phi_\eta = \begin{pmatrix} \eta^0 \\ -\eta_1^- \end{pmatrix}, \quad \Phi_\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \end{pmatrix}, \quad \Phi_\chi = \begin{pmatrix} \chi^- \\ \chi^{--} \end{pmatrix}, \quad \Phi_s = \begin{pmatrix} s_2^+ \\ s_2^0 \end{pmatrix}, \quad (4)$$

are doublets with the weak hypercharge $Y = -1, +1, -3, +1$, and

$$T = \begin{pmatrix} s_1^0 & \frac{s_1^+}{\sqrt{2}} \\ \frac{s_1^+}{\sqrt{2}} & S_1^{--} \end{pmatrix}, \quad (5)$$

	$Q_{(1,2)L}$	Q_{3L}	U_{aR}	D_{aR}	Ψ_{aL}	ν_{aR}	η	χ	ρ	S	j_{mR}	J_R
Z_7	1	ω^6	ω^5	ω^3	ω^2	ω^6	ω^3	ω^6	ω^5	ω^4	ω^6	ω^2
Z_3	1	w^2	w	w	w	w	w	1	w	w	1	w

TABLE I: Transformation properties of the fermion and scalar fields under $Z_7 \otimes Z_3$.

is a triplet with $Y = 2$; the $SU(2)$ singlets $\eta_2^+, \rho^{++}, \chi^0, S_2^{--}$ have $Y = +2, +4, 0, +4$, respectively.

The total lepton number assignment in the scalar sector is [21]

$$L(T^*, \eta_2^-, \Phi_\chi, \rho^{--}, S_2^{--}) = +2, \quad L(\Phi_{\eta,\rho,s}, \chi^0) = 0. \quad (6)$$

Notice that the only scalar doublet carrying lepton number is Φ_χ and both members of the doublet have electric charge, for this reason, always $\langle \Phi_\chi \rangle = 0$. The existence of scalars carrying lepton number imply the possibility of explicit breaking of this quantum number in the scalar potential. It is possible to avoid these terms by imposing an appropriate discrete symmetry. We show one possibility in the Table I.

Since the complex triplet T and the singlet S_2^{++} (under $SU(2)$) carry lepton number, if there are no lepton number violating terms in the scalar potential, these scalars do not mix with $\Phi_{\eta,\rho,\chi}$. As we will show below, there is some range of the parameters space that allow $\langle s_1^0 \rangle = 0$ and $\langle s_2^0 \rangle \approx 0$. In this situation the neutral scalar s_1^0 does not participate in the spontaneous symmetry breaking and s_2^0 has a small effect on the vector and charged lepton masses. At this stage, active neutrinos are massless and the charged leptons gain a rather small mass. However, these scalar fields are heavy and the charged leptons gain the appropriate mass through the interaction with the triplet η and an effective interactions involving the triplets ρ and χ , in a similar way as in the standard model a non-Hermitian scalar triplet generates at tree level the neutrino masses through the interaction $(1/\Lambda)\phi^0\phi^0\nu\nu$ [16], see Fig. 1.

The most general scalar potential involving the three triplets and the sextet is [21]

$$V(\eta, \rho, \chi, S) = V^{(2)} + V^{(3)} + V^{(4a)} + \dots + V^{(4e)}, \quad (7)$$

where

$$\begin{aligned}
V^{(2)} &= \sum_{x=\eta,\rho,\chi,S} \mu_x^2 \text{Tr}(X^\dagger X), \\
V^{(3)} &= \frac{1}{3!} f_1 \epsilon_{ijk} \eta_i \rho_j \chi_k + f_2 (\chi^T S^* \rho + \rho^T S^* \chi) + f_3 \eta^T S^* \eta + \frac{f_4}{3!} \epsilon_{ijk} \epsilon_{mnl} S_{im}^* S_{jn}^* S_{kl}^*, \\
V^{(4a)} &= a_1 (\eta^\dagger \eta)^2 + a_2 (\rho^\dagger \rho)^2 + a_3 (\chi^\dagger \chi)^2 + \chi^\dagger \chi (a_4 \eta^\dagger \eta + a_5 \rho^\dagger \rho) + a_6 (\eta^\dagger \eta) (\rho^\dagger \rho) \\
&\quad a_7 (\chi^\dagger \eta) (\eta^\dagger \chi) + a_8 (\chi^\dagger \rho) (\rho^\dagger \chi) + a_9 (\eta^\dagger \rho) (\rho^\dagger \eta) + [a_{10} (\chi^\dagger \eta) (\rho^\dagger \eta) + H.c.], \\
V^{(4b)} &= b_1 \chi^\dagger S \hat{\chi} \eta + b_2 \rho^\dagger S \hat{\rho} \eta + b_3 \eta^\dagger S [\hat{\chi} \rho - \hat{\rho} \chi] + H.c., \\
V^{(4c)} &= c_1 \text{Tr}[\hat{\eta} S \hat{\eta} S] + c_2 \text{Tr}[\hat{\rho} S \hat{\rho} S] + H.c., \\
V^{(4d)} &= d_1 (\chi^\dagger \chi) \text{Tr} S S^* + d_2 [(\chi^\dagger S)(S^* \chi)] + d_3 (\eta^\dagger \eta) \text{Tr}(S S^*) + d_4 \text{Tr}[(S^* \eta)(\eta^\dagger S)] \\
&\quad + d_5 (\rho^\dagger \rho) \text{Tr} S S^* + d_6 \text{Tr}[(S^* \rho)(\rho^\dagger S)], \\
V^{(4e)} &= e_1 (\text{Tr} S S^*)^2 + e_2 \text{Tr}(S S^* S S^*), \tag{8}
\end{aligned}$$

and we have defined in the $V^{(b)}$ and $V^{(c)}$ terms $\hat{x}_{ij} = \epsilon_{ijk} x_k$ being $x = \eta, \rho, \chi$. Notice also that $S^\dagger = S^*$ since S is a symmetric matrix. The conditions for having a bounded from below potential in Eq. (8), under the conditions in Table I, are given in the Appendix A.

Concerning the vacuum alignment and the conservation of the lepton number L , five possibilities can be considered (see also Ref. [21])

1. Explicit L violation and $\langle s_{1,2}^0 \rangle \neq 0$ and arbitrary. This is the most general case.
2. Explicit lepton number violation in the scalar potential and $\langle s_1^0 \rangle = 0$ and $\langle s_2^0 \rangle \neq 0$ at tree level, but $\langle s_1^0 \rangle \neq 0$ by loop corrections [12, 22].
3. No explicit L violation, $\langle s_1^0 \rangle = 0$ and $\langle s_2^0 \rangle = 0$. Notwithstanding, the latter condition is not stable under radiative corrections unless a fine tuning is imposed.
4. No explicit lepton number violation but $\langle s_1^0 \rangle \neq 0$ and $\langle s_2^0 \rangle \neq 0$. In this case there is a triplet Majoron that has been ruled out by the Z invisible width [23].
5. No explicit L violation and $\langle s_1^0 \rangle = 0$ but $\langle s_2^0 \rangle \neq 0$. Although L is conserved, there is violation of the numbers $L_{e,\mu,\tau}$. In this case $\langle s_1^0 \rangle = 0$ is stable at tree and higher order level [11, 12].

Here we will consider the case 5), with $\langle s_1^0 \rangle = 0$ and $\langle s_2^0 \rangle / v_W \ll 1$, where v_W is the electroweak scale. This case occurs if the constraint $v_W^2 = \sum v_i^2 = 246(\text{GeV})^2$ (note that

$i = \rho^0, \eta^0, s_1^0, s_2^0$) is saturated with the v_η^2 and v_ρ^2 as in Ref. [24]. Moreover, as we said before, in order to simplify the scalar potential we impose a Z_7 discrete symmetry which forbid the L violating terms, $f_3, f_4, a_{10}, b_3, c_2 = 0$, but also c_1 and $b_{1,2}$ are forbidden if we impose the an additional discrete Z_3 symmetry. See the Table I.

III. SCALAR MASS SPECTRA IN THE MODEL

Let us consider the scalar potential in Eq. (8) with the $Z_7 \otimes Z_3$ symmetries given in Table I. We make as usual $y^0 = (1/\sqrt{2})(v_y + X_y^0 + iI_y^0)$, where $y = \eta^0, \rho^0, \chi^0$ and s_2^0 . The constraint equations obtained by imposing that $\partial V/\partial x^0 = 0$, being V is the potential in (8) and x^0 the neutral fields, are given by

$$\begin{aligned}
v_{s_2} \left[4\mu_S^2 + 2d_3v_\eta^2 + (2d_5 + d_6)v_\rho^2 + 2(2e_1 + e_2)v_{s_2}^2 + (2d_1 + d_2)v_\chi^2 + \frac{f_2v_\rho v_\chi}{v_{s_2}} \right] &= 0, \\
v_\chi \left[4\mu_\chi^2 + 2a_4v_\eta^2 + 2a_5v_\rho^2 + (2d_1 + d_2)v_{s_2}^2 + 4a_3v_\chi^2 + 2v_\rho \left(\frac{\sqrt{2}f_1v_\eta + f_2v_{s_2}}{v_\chi} \right) \right] &= 0, \\
v_\eta \left[2\mu_\eta^2 + 2a_1v_\eta^2 + a_6v_\rho^2 + d_3v_{s_2}^2 + a_4v_\chi^2 + \frac{\sqrt{2}f_1v_\rho v_\chi}{v_\eta} \right] &= 0, \\
v_\rho \left[\mu_{2\rho} + \frac{a_5}{2}v_\chi^2 + \frac{a_6}{2}v_\eta^2 + \frac{1}{2} \left(d_5 + \frac{1}{2} \right) v_{s_2}^2 + \frac{f_1v_\eta v_\chi}{\sqrt{2}v_\rho} + \frac{f_2v_{s_2}v_\chi}{2v_\rho} \right] &= 0. \tag{9}
\end{aligned}$$

Notice that no VEV can be zero unless $f_1 = f_2 = 0$. However, if this were the case, the scalar potential has a non Abelian symmetry larger than the rest of the Lagrangian. Hence, $f_1, f_2 \neq 0$ in order that the gauge symmetry of the scalar potential is the same as the other terms of the Lagrangian.

The mass square matrix of the CP even real scalar in the basis $(X_\eta^0, X_\rho^0, X_\chi^0, X_{s_2}^0, X_{s_1}^0)M^2(X_\eta^0, X_\rho^0, X_\chi^0, X_{s_2}^0, X_{s_1}^0)^T$ decomposes as $5 \times 5 = 4 \times 4 + 1 \times 1$, where the matrix elements, after using the constraint equations in (9), are given by (here

we write for simplicity $R = M^2$)

$$\begin{aligned}
R_{11} &= \frac{1}{4v_\eta} \left[8a_1v_\eta^3 - 2\sqrt{2}f_1v_\chi v_\rho \right] \\
R_{12} &= a_6v_\eta v_\rho + \frac{f_1v_\chi}{\sqrt{2}} \approx \frac{1}{\sqrt{2}}f_1v_\chi \\
R_{13} &= \frac{f_1v_\rho}{\sqrt{2}} + a_4v_\eta v_\chi \approx a_4v_\eta v_\chi \\
R_{14} &= d_3v_\eta v_{s_2} \approx 0 \\
R_{22} &= -\frac{1}{2v_\rho} \left[-4a_2v_\rho^3 + \sqrt{2}f_1v_\eta v_\chi + f_2v_{s_2}v_\chi \right] \approx -\frac{f_1v_\eta v_\chi}{\sqrt{2}v_\rho} \\
R_{23} &= \frac{f_1v_\eta}{\sqrt{2}} + \frac{f_2v_{s_2}}{2} + a_5v_\rho v_\chi \approx a_5v_\rho v_\chi \\
R_{24} &= \frac{1}{2} \left((2d_5 + d_6)v_{s_2}v_\rho + f_2v_\chi \right) \approx \frac{1}{2}f_2v_\chi \\
R_{33} &= \frac{1}{2v_\chi} \left[4a_3v_\chi^3 - \sqrt{2}f_1v_\eta v_\rho - f_2v_\rho v_{s_2} \right] \approx 2a_3v_\chi^2 \\
R_{34} &= \frac{1}{2} \left(f_2v_\rho + ((2d_1 + d_2)v_{s_2})v_\chi \right) \approx 0 \\
R_{44} &= \frac{1}{4v_{s_2}} \left[4(2e_1 + e_2)v_{s_2}^3 - 2f_2v_\rho v_\chi \right] \approx -\frac{v_\rho}{2v_{s_2}}f_2v_\chi
\end{aligned} \tag{10}$$

and a 1×1 matrix:

$$\begin{aligned}
R_{55} &= -\frac{1}{4v_{s_2}} \left[2e_2v_{s_2}^3 - 2d_4v_\eta^2v_{s_2} + d_6v_\rho^2v_{s_2} + d_2v_\chi^2v_{s_2} + 2f_2v_\rho v_\chi \right] \\
&\approx -\frac{1}{4}d_2v_\chi^2 - \frac{f_2v_\rho v_\chi}{2v_{s_2}}
\end{aligned} \tag{11}$$

On the right hand side, after the \approx sign, we have kept only contributions proportional v_χ and $1/v_{s_2}$. It is possible to see from (10) that the CP even neutral scalar related to the sextet component s_2^0 , dominated by the R_{44} entry, is heavy with a mass dominated by the term $-\frac{v_\rho}{2v_{s_2}}f_2v_\chi$, with $f_2 < 0$. The CP even scalar related to s_1^0 is also heavy with a mass given by the R_{55} , dominated by the term $d_2v_\chi^2$, even if this field does not get a VEV.

The CP odd neutral scalar have a mass matrix as follows (in the basis $(I_\eta^0, I_\rho^0, I_\chi^0, I_{s_2}^0)$) also decomposes as $5 \times 5 = 4 \times 4 + 1 \times 1$, where

$$\left(\begin{array}{cccc}
-\frac{v_\chi f_1 v_\rho}{\sqrt{2} v_\eta} & -\frac{f_1 v_\chi}{\sqrt{2}} & -\frac{f_1 v_\rho}{\sqrt{2}} & 0 \\
& -\frac{(\sqrt{2} f_1 v_\eta + f_2 v_{s_2}) v_\chi}{2 v_\rho} & -\frac{f_1 v_\eta}{\sqrt{2}} - \frac{f_2 v_{s_2}}{2} & -\frac{1}{2} (f_2 v_\chi) \\
& & -\frac{v_\rho (\sqrt{2} f_1 v_\eta + f_2 v_{s_2})}{2 v_\chi} & -\frac{1}{2} (f_2 v_\rho) \\
& & & -\frac{f_2 v_\chi v_\rho}{2 v_{s_2}}
\end{array} \right) \tag{12}$$

and

$$\begin{aligned}
m_{I_{s_1^0}}^2 &= \frac{1}{4v_{s_2}}[-2e_2v_{s_2}^3 + 2d_4v_\eta^2v_{s_2} - d_6v_\rho^2v_{s_2} - d_2v_\chi^2v_{s_2} - 2f_2v_\rho v_\chi] \\
&\approx -\frac{1}{4}d_2v_\chi^2 - \frac{v_\rho}{2v_{s_2}}f_2v_\chi, \quad \frac{v_\rho}{2v_{s_2}} \gg d_2, \quad f_2 < 0.
\end{aligned} \tag{13}$$

Hence, also the CP odd part of s_1^0 is heavy, $m_{s_2}^2 \approx -(v_\rho/2v_{s_2})f_2v_\chi$, although this field does not get a VEV.

Moreover, we have also verified that the matrix in (13) and also those of the singly and doubly charged in the basis $(\rho^+, \eta_1^+, h_2^+)$, $(\chi^+, \eta_2^+, h_1^+)$ and $(\chi^{--}, \rho^{--}, H_1^{--}, H_2^{--})$ have the correct number of Goldstone bosons.

We can use numerical values for the parameters in the mass matrices. For instance, choosing $a_1 = 0.5, a_2 = 0.999, a_3 = 0.3, a_7 = a_8 = 0.5, a_9 = 0.9, d_2 = d_4 = d_6 = 0.1, e_1 = e_2 = 0.3$, and using the minimal conditions of Case 2 in the Appendix we can choose: $a_4 = a_6 = a_5 = d_1 = d_5 = 0$. Moreover the constants with dimension of mass (in GeV) $f_1 = -2.1, f_2 = -7.0$, and $v_\chi = 1500$. The $f_{1,2}$ constants are naturally smaller than the $SU(3)$ scale since if they were zero the symmetry of the potential would be larger than $SU(3) \otimes U(1)$.

With the values of the constants above, we obtain the following masses (all masses below are in GeV): CP even neutral scalar, 124.9, 241.3, 1162,5314,5324. Notice that, the lightest scalar have a mass compatible with the LHC resonance and that $\text{Re } s_2^0$ has the main projection in a neutral scalar with mass of the order of 5 TeV. The same happens with $\text{Re } s_1^0$. Then, it is possible to have heavy fields in the sextet even with small v_{s_2} and $v_{s_1} = 0$. This justified our scheme which consider that the fields in the sextet generate the effective Lagrangian in Eq. (14).

In the CP odd sector we obtain the masses 0, 0, 102,5309,5324. We see again that $\text{Im } s_{1,2}^0$ have the main projection in the heavy pseudo-scalar.

In the singly charged scalars, for the fields that do not carry L charge, the mass matrices are written in the basis $(\rho^+, \eta_1^+, s_2^+)$ and the eigenvalues are 0, 194, 5325. Notice that s_2^+ is the charged scalar in the $SU(2)$ doublet, Φ_s in Eq. (4), of the sextet and it is heavy as well. In the sector of charged scalar carrying L charges with mass matrix written in the basis $(\chi^+, \eta_2^+, s_1^+)$, where s_1^+ is the charged scalar in the triplet T in Eq. (5), we obtain the following eigenvalues 0, 760,5309. Finally in the doubly charged scalar sector with mass matrices in the basis $(\chi^{--}, \rho^{--}, S_1^{--}, S_2^{--})$ the eigenvalues are 0, 757,5319, 5329.

IV. THE LEPTON MASSES AND THE PMNS MATRIX

The possibility that the lepton masses are generated by an effective dimensional operator was presented in Ref. [14]. Nevertheless, we will consider the case when sextet is introduced and generates an effective non-renormalizable interaction. The latter situation arises when the member of the sextet are very heavy and one the neutral ones gain a zero VEV and the other a small VEV. The Yukawa interactions are given by

$$\begin{aligned}
-\mathcal{L}_1^{lep} = & -\frac{1}{2}\epsilon_{ijk} \overline{(\Psi_{ia})^c} G_{ab}^\eta \Psi_{jb} \eta_k + \frac{1}{2\Lambda_l} \overline{(\Psi_a)^c} \tilde{G}_{ab}^s (\chi^* \rho^\dagger + \rho^* \chi^\dagger) \Psi_b \\
& + \overline{(\Psi_{aL})} (G^\nu)_{ab} \nu_{aR} \eta + \overline{(\nu_{aR})^c} (M_R)_{ab} \nu_{bR} + H.c.,
\end{aligned} \tag{14}$$

Being the second term of the first line of Eq. (14) the dimension five operator generated by the loop in Figure 1. We will assume, for the sake of simplicity, that M_R is diagonal and that $M_3 = M > m_{R1}, m_{R2}$, and $M_R^{-1} = (1/M)\bar{M}_R$ where $\bar{M} = \text{diag}(r_1, r_2, 1)$ and $r_1 \equiv M/m_{R1}, r_2 \equiv M/m_{R2}$. In this case we have

$$M_{ab}^\nu \approx -\frac{v_\eta^2}{2} G_{ab}^\nu \frac{\bar{M}_R}{M} G^{\nu T}, \quad M_{ab}^l = G_{ab}^\eta \frac{v_\eta}{\sqrt{2}} + \frac{1}{\Lambda_l} \tilde{G}_{ab}^s v_\rho v_\chi. \tag{15}$$

where $\Lambda_\eta = M$ and Λ_l is another mass scale related to the origin of the dimension five interaction. If it is the sextet through the interaction $\overline{(\Psi_{ai})^c} G_{ab}^s \Psi_{jb} S_{ij}$ and the f_2 trilinear term in the scalar potential involving ρ, χ and S , as it was shown in here, then we have $1/\Lambda_l = f_2/m_{\sigma_2}^2$ and $\tilde{G}^s = G^s$.

We will assume, for the sake of simplicity, that M_R is diagonal and that $M_3 = M > m_{R1}, m_{R2}$, and $M_R^{-1} = (1/M)\bar{M}_R$ where $\bar{M} = \text{diag}(r_1, r_2, 1)$ and $r_1 \equiv M/m_{R1}, r_2 \equiv M/m_{R2}$. In this case we have

$$M_{ab}^\nu \approx -\frac{v_\eta^2}{2} G_{ab}^\nu \frac{\bar{M}_R}{M} G^{\nu T}, \quad M_{ab}^l = G_{ab}^\eta \frac{v_\eta}{\sqrt{2}} + \frac{1}{\Lambda_S} G_{ab}^s v_\rho v_\chi. \tag{16}$$

where $\Lambda_\eta = M$ and Λ_S is another mass scale.

These mass matrices are diagonalized as follows:

$$\hat{M}^\nu = V_L^{\nu T} M^\nu V_L^\nu, \quad \hat{M}^l = V_L^{l\dagger} M^l V_L^l, \tag{17}$$

where $\hat{M}^\nu = \text{diag}(m_1, m_2, m_3)$ $\hat{M}^l = \text{diag}(m_e, m_\mu, m_\tau)$. The relation between symmetry eigenstates (primed) and mass (unprimed) fields are $l'_{L,R} = V_{L,R}^l l_{L,R}$ and $\nu'_L = V_L^\nu \nu_L$, where $l'_{L,R} = (e', \mu', \tau')_{L,R}^T$, $l_{L,R} = (e, \mu, \tau)_{L,R}^T$ and $\nu'_L = (\nu_e \nu_\mu \nu_\tau)_L^T$ and $\nu_L = (\nu_1 \nu_2 \nu_3)_L$. For the charged leptons we have $V_L^{l\dagger} M^l M^{l\dagger} V_L^l = (\hat{M}^l)^2$ and $V_R^{l\dagger} M^l M^l V_R^l = (\hat{M}^l)^2$.

In the following we assume $v_\chi \approx \Lambda_S$ and, as in Ref. [24], $v_\rho \sim 54, v_\eta \sim 240$ GeV. The neutrino mass matrix is as in Eq. (16). We will solve simultaneously the following equations:

$$\hat{M}^\nu = V_L^{\nu T} G_{ab}^\nu V_L^\nu \frac{v_\eta^2}{\Lambda_\eta}, \quad V_L^{l\dagger} M^l M^{l\dagger} V_L^l = V_R^{l\dagger} M^{l\dagger} M^l V_R^l = (\hat{M}^l)^2, \quad (18)$$

and the values for the charged lepton masses obtained are (in MeV) $(m_e, m_\mu, m_\tau) = (0.509648, 105.541, 1775.87)$ and for the neutrinos masses (in eV) $(m_1, m_2, m_3) = (0.051, -0.0194, 0.0174)$ which are consistent with $\Delta m_{23}^2 = 2.219 \times 10^{-3} (\text{eV})^2$ and $\Delta m_{21}^2 = 7.5 \times 10^{-5} (\text{eV})^2$. These values for the masses arise from the following values for the Yukawa matrices: $v_\eta^2/M = 0.33$ eV, $G_{11}^\nu = 0.109$, $G_{12}^\nu = 0.097$, $G_{13}^\nu = 0.101$, $G_{22}^\nu = 0.09$, $G_{23}^\nu = -0.02$, $G_{33}^\nu = 0.0106$ in the neutrino sector and If $G_{11}^s = -0.0453$, $G_{12}^s = -0.0076$, $G_{13}^s = -0.0008$, $G_{22}^s = 0.0015$, $G_{23}^s = 0.0001$, $G_{33}^s = 1.84 \times 10^{-5}$, $G_{12}^\eta =$, $G_{13}^\eta = G_{13}^\eta = -0.00001$ in the charged lepton sector. The only way to avoid the latter fine tuning is to consider v_η smaller but in the context of the Ref. [24] this VEV is already fixed.

We obtain

$$V_L^\nu \approx \begin{pmatrix} -0.24825 & -0.57732 & 0.77786 \\ 0.73980 & -0.40539 & 0.53698 \\ -0.62535 & -0.70877 & 0.32647 \end{pmatrix} \quad (19)$$

and

$$V_L^l \approx \begin{pmatrix} -0.00985 & 0.01457 & -0.99984 \\ -0.31848 & -0.94787 & -0.01067 \\ 0.94788 & -0.31833 & -0.01398 \end{pmatrix}, \quad V_R^l \approx \begin{pmatrix} 0.00501 & 0.00716 & 0.99996 \\ 0.00261 & 0.9910 & -0.00717 \\ 0.99998 & -0.00265 & -0.00499 \end{pmatrix} \quad (20)$$

Notice that we have defined the lepton mixing matrix as $V_{PMNS} = V_L^\nu V_L^{l\dagger}$, it means that this matrix appears in the charged currents coupled to W^- . We obtain from Eqs. (19) and (20)

$$|V_{PMNS}| \approx \begin{pmatrix} 0.826 & 0.548 & 0.130 \\ 0.506 & 0.618 & 0.602 \\ 0.249 & 0.563 & 0.788 \end{pmatrix}. \quad (21)$$

which is in agreement, within 3σ , with the experimental data given in Ref. [25],

$$|V_{PMNS}| \approx \begin{pmatrix} 0.795 - 0.846 & 0.5713 - 0.585 & 0.126 - 0.178 \\ 0.4205 - 0.543 & 0.416 - 0.730 & 0.579 - 0.808 \\ 0.215 - 0.548 & 0.409 - 0.725 & 0.567 - 0.800 \end{pmatrix}. \quad (22)$$

and we see that it is possible to accommodate all lepton masses and the PMNS matrix. We do not consider CP violation here.

V. CONCLUSIONS

The possibility that the sextet can be “avoided” in the m331 model was considered in Ref. [14]. Here we have considered the same, but now we have shown how the sextet can induce the effective dimension five operator that is responsible for generating the charged lepton masses. This is similar to the case of the generation of the dimension five interactions $(\bar{L}H^\dagger)(HL)$ in the extension of the standard model with a complex scalar triplet. The existence of several mechanisms to generate this interaction in the context of the standard model have been shown in literature [16–18]. Notwithstanding, the effective operator in (14) can be originated by the effects of higher dimension operators.

In this paper we have shown that there is a range of the parameters in which the neutral fields in the sextet are heavy and the effective operator in Eq. (14) is generated by diagrams like that in Fig. 1. The heavy sextet case justified the approximation in the so called reduced minimal 3-3-1 [14, 15]. In fact, all extra scalars in the model are heavy except for two neutral scalars that correspond to the fields in a two Higgs doublet extension of the SM. We also show the conditions under which we have a weak copositivity of the scalar potential and generate the vacuum stability at tree level. It is interesting to study the same problem at the one-loop level. For instance, in a model with two doublets (one of them inert) taking into account a neutral scalar with a mass of 125 GeV, the stability of the vacuum was shown in Ref. [26].

In order to obtain the correct mass for the charged leptons, besides the interaction in (14), it is necessary to consider the interactions with the triplet η . For neutrinos, as we are considering that $v_{s_1} = 0$, we can introduce right-handed neutrinos to generate the type-I seesaw mechanism. With the unitary (orthogonal if we neglect phases) matrices that diagonalize the mass matrices in the lepton sector it is possible to accommodate a realistic Pontecorvo-Maki-Nakawaga-Sakata matrix. See Ref. [27] for details where were also considered the constraints on the masses of the extra particles in m331 model coming from lepton violation processes.

Finally, in the present context we would like to say that the neutral scalar ρ^0 it is which

has the larger projection on the neutral escalar with a mass of near 125 GeV [24].

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Appendix A: Stability of the scalar potential

The scalar potential has to be bounded from below to ensure its stability. In the SM to ensure the stability of this potential is easy, we just have to ensure that $\lambda > 0$. In theories that increases the number of scalars it is more difficult to ensure that the potential is bounded from below, in all directions. The following cases are based on the copositivity matrix presented in Eq. (A2) below. Each case correspond to different sign possibilities of the matrix elements. Also, for every case, all diagonal elements have to be positive. Here we follow Refs. [28, 29]. A scalar potential has a quadratic form in the quadratic couplings in the form $A_{ab}\phi_a^2\phi_b^2$ if the matrix A_{ab} is copositive is possible to ensure that the potential has a global minimum, to do this analysis, we can ignore any terms with dimensionful couplings, mass or soft terms, since in the limit of large field values, terms with dimension smaller than four are negligible in comparison with the quartic part of the scalar potential V^4 . As well, we will assume the conditions from the Table (I), so that $f_3, f_4, a_{10}, b_1, b_2, b_3, c_1, c_2 = 0$. Taking the potential in Eq. (8) we redefine the triplets and the sextet as:

$$|H_i|^2 = h_i^2, \quad H_i^\dagger H_j = h_i h_j r_{ij} e^{i\theta_{ij}} \quad (\text{A1})$$

where $H_{1,2,3,4} = \eta, \rho, \chi, S$. The parameters r_{ij} and θ_{ij} are not physical parameters, ranging from 0 to 1 and 0 to 2π respective. They are used to analyze the 4-field direction by demanding the maximization of the parameter space. Then, they should be set to values which allows most parameter space.

Rewriting the quartic terms of the potential in Eq. (8), and using Eqs. (A1) and (I), we find that the matrix A in the basis $(h_1^2, h_2^2, h_3^2, h_4^2)$ is written as:

$$A = \begin{pmatrix} a_1 & a_6 + a_9 r_{12}^2 & a_4 + a_7 r_{13}^2 & d_3 r_{14}^2 \\ a_6 + a_9 r_{12}^2 & a_2 & a_5 + a_8 r_{23}^2 & d_5 + d_6 r_{24}^2 \\ a_4 + a_7 r_{13}^2 & a_5 + a_8 r_{23}^2 & a_3 & d_1 + d_2 r_{34}^2 \\ d_3 r_{14}^2 & d_5 + d_6 r_{24}^2 & d_1 + d_2 r_{34}^2 & e_1 + e_2 \end{pmatrix}. \quad (\text{A2})$$

In order to analyze the copositivity of the matrix A above we have to choose values for the r_{ij} that minimize the entries of the matrix. For the off-diagonal elements which involve sums, two cases are relevant: if both coupling constants are positive/negative, the minimum comes from choosing $r_{ij} = 0$; if the constants have opposite signs, the minimum comes from $r_{ij} = 1$. For the elements $A_{14} = A_{41}$, the minimum is obtained with $r_{14} = 0$. For the sake of simplicity, we will assume all coupling constants on each entry to have the same sign, therefore we make all $r_{ij} = 0$ and consider six cases for the copositivity.

Below are shown the different conditions under which the potential is bounded from below. All indices i, j, k, l, \dots are fixed and different from each other. Also, we want to remind the reader that all diagonal elements from matrix A given in Eq. (A2) should be positive to have copositivity.

Case 1. All A_{ij} positive.

All couplings positive, and $E = e_1 + e_2 > 0$.

Case 2. $A_{ij} \leq 0$ with i, j and the other entries positive.

1. If $a_6 \leq 0$ we have $a_1 a_2 - a_6^2 > 0$
2. If $a_4 \leq 0$, then $a_1 a_3 - a_4^2 > 0$
3. If $a_5 \leq 0$, then $a_2 a_3 - a_5^2 > 0$
4. If $d_5 \leq 0$, then $-d_5^2 + a_2 E > 0$
5. If $d_1 \leq 0$, then $-d_1^2 + a_3 E > 0$

For each of the cases above, $a_1 E > 0$ has also to be true.

Case 3. $A_{ij} \leq 0$, $A_{kl} \leq 0$, and the other entries positive.

1. If $a_5 \leq 0$ and $a_6 \leq 0$ we have $a_1 a_2 - a_6^2 > 0$ and $a_2 a_3 - a_5^2 > 0$
2. If $a_6 \leq 0$ and $d_5 \leq 0$, then $a_1 a_2 - a_6^2 > 0$ and $-d_5^2 + a_2 E > 0$

3. If $a_6 \leq 0$ and $d_1 \leq 0$, then $a_1a_2 - a_6^2 > 0$ and $-d_1^2 + a_3E > 0$

4. If $a_4 \leq 0$ and $d_5 \leq 0$, then $a_1a_3 - a_4^2 > 0$ and $-d_5^2 + a_2E > 0$

5. If $a_4 \leq 0$ and $d_1 \leq 0$, then $a_1a_3 - a_4^2 > 0$ and $-d_1^2 + a_3E > 0$

6. If only $a_5 \leq 0$, then $a_2a_3 - a_5^2 > 0$ and $a_1E > 0$

7. If $a_5 \leq 0$ and $d_1 \leq 0$, then $a_2a_3 - a_5^2 > 0$ and $-d_1^2 + a_3E > 0$

Case 4 As in case 3: $A_{ij} \leq 0$, $A_{ik} \leq 0$ and the other entries positive

1. $a_4 \leq 0$ and $a_6 \leq 0$, then $a_1a_5 + \sqrt{(a_1a_3 - a_4^2)(a_1a_2 - a_6^2)} > 0$

2. If only $a_6 \leq 0$, and $a_1d_5 + \sqrt{(a_1a_2 - a_6^2)a_1E} > 0$

3. If only $a_4 \leq 0$, and $a_1d_1 + \sqrt{(a_1a_3 - a_4^2)a_1E} > 0$

4. $a_5 \leq 0$ and $d_5 \leq 0$, and $a_2d_1 + \sqrt{(a_2a_3 - a_5^2)[-d_5^2 + a_2E]} > 0$

Case 5. $A_{ij} \leq 0$, $A_{jk} \leq 0$, $A_{ik} \leq 0$ and the other entries positive

1. If $a_4, a_5, a_6 \leq 0$, then

$$\begin{aligned} \sqrt{a_1a_2} + a_6 > 0, \quad \sqrt{a_1a_3} + a_4 > 0, \quad \sqrt{a_2a_3} + a_5 > 0, \\ a_1a_2a_3 - a_2a_4^2 - a_1a_5^2 + 2a_4a_5a_6 - a_3a_6^2 > 0 \end{aligned} \quad (\text{A3})$$

2. If only $a_6, d_5 \leq 0$, then

$$\begin{aligned} \sqrt{a_1a_2} + a_6 > 0, \quad \sqrt{a_1E} > 0, \quad d_5 + \sqrt{a_2E} > 0, \\ (a_1a_2 - a_6^2)E - a_1d_5^2 > 0 \end{aligned} \quad (\text{A4})$$

3. If only $a_4, d_1 \leq 0$, then

$$\begin{aligned} \sqrt{a_1a_3} + a_4 > 0, \quad \sqrt{a_1E} > 0, \quad d_1 + \sqrt{a_3E} > 0, \\ (a_1a_3 - a_4^2)E - a_1d_1^2 > 0 \end{aligned} \quad (\text{A5})$$

4. If $a_5, d_1, d_5 \leq 0$, then

$$\begin{aligned} \sqrt{a_2a_3} + a_5 > 0, \quad d_5 + \sqrt{a_2E} > 0, \quad d_1 + \sqrt{a_3E} > 0, \\ -a_2d_1^2 + 2a_5d_1d_5 - a_3d_5^2 + a_2a_3e_1 - a_5^2e_1 + a_2a_3e_2 - a_5^2e_2 > 0, \end{aligned} \quad (\text{A6})$$

Case 6. $A_{ij} \leq 0$, $A_{ik} \leq 0$, $A_{il} \leq 0$, and the other entries positive

1. If only $a_4, a_6 \leq 0$, then

$$\begin{aligned}
& a_1 a_2 - a_6^2 > 0, \quad a_1 a_3 - a_4^2 > 0, \quad -a_1 E > 0, \\
& a_1 a_5 - a_4 a_6 + \sqrt{(a_1 a_3 - a_4^2)(a_1 a_2 - a_6^2)} > 0, \\
& a_1 d_5 + \sqrt{(a_1 a_2 - a_6^2)[a_1 E]} > 0, \\
& a_1 d_1 + \sqrt{(a_1 a_3 - a_4^2)[a_1 E]} > 0, \\
& -a_1^2 \{a_1 [a_2 (d_1^2 - a_3 E) + a_3 d_5^2 + a_5^2 E - 2a_5 d_1 d_5] \\
& + E [a_2 a_4^2 + a_6 (a_3 a_6 - 2a_4 a_5)] - (a_6 d_1 - a_4 d_5)^2\} > 0
\end{aligned} \tag{A7}$$

2. If $a_5, a_6, d_5 \leq 0$, then

$$\begin{aligned}
& a_1 a_2 - a_6^2 > 0, \quad a_2 a_3 - a_5^2 > 0, \quad -d_5 + a_2 E > 0, \\
& a_2 a_4 - a_5 a_6 + \sqrt{(a_2 a_3 - a_5^2)(a_1 a_2 - a_6^2)} > 0, \\
& -a_2^2 \{-a_6^2 d_1^2 + 2a_4 a_6 d_1 d_5 + a_1 a_3 d_5^2 - a_4^2 d_5^2 + a_3 a_6^2 E \\
& + a_5^2 a_1 E + 2a_5 (-a_1 d_1 d_5 - a_4 a_6 E) \\
& + a_2 [+a_1 d_1^2 + a_4^2 E - a_3 a_1 E]\} > 0
\end{aligned} \tag{A8}$$

3. If $a_4, a_5, d_1 \leq 0$, then

$$\begin{aligned}
& a_1 a_3 - a_4^2 > 0, \quad a_2 a_3 - a_5^2 > 0, \quad -d_1 + a_3 E > 0, \\
& -a_4 a_5 + a_3 a_6 + \sqrt{(a_1 a_3 - a_4^2)(a_2 a_3 - a_5^2)} > 0, \\
& -a_4 d_1 + \sqrt{(a_1 a_3 - a_4^2)[-d_1^2 + a_3 E]} > 0, \\
& -a_3^2 \{-a_6^2 d_1^2 + 2a_4 a_6 d_1 d_5 + a_1 a_3 d_5^2 - a_4^2 d_5^2 + a_3 a_6^2 E \\
& + a_5^2 a_1 E + 2a_5 (-a_1 d_1 d_5 - a_4 a_6 E) \\
& + a_2 + a_1 d_1^2 + a_4^2 E - a_3 a_1 E)\} > 0
\end{aligned} \tag{A9}$$

4. If only $d_1, d_5 \leq 0$, then

$$\begin{aligned}
& a_1 E > 0, \quad -d_5^2 + a_2 E > 0, \quad -d_1^2 + a_3 E > 0, \\
& a_6 E + \sqrt{[a_1 E] [-d_5^2 + a_2 E]} > 0, \\
& a_4 E + \sqrt{[a_1 E] [-d_1^2 + a_3 E]} > 0, \\
& -d_1 d_5 + a_5 E + \sqrt{[-d_5^2 + a_2 E] [-d_1^2 + a_3 E]} > 0, \\
& E^2 \{ -a_1 [a_2 (d_1^2 - a_3 E) + a_3 d_5^2 + a_5^2 E - 2a_5 d_1 d_5] - E [a_2 a_4^2 + a_6 (a_3 a_6 - 2a_4 a_5)] \\
& + (a_6 d_1 - a_4 d_5)^2 \} > 0
\end{aligned}
\tag{A10}$$

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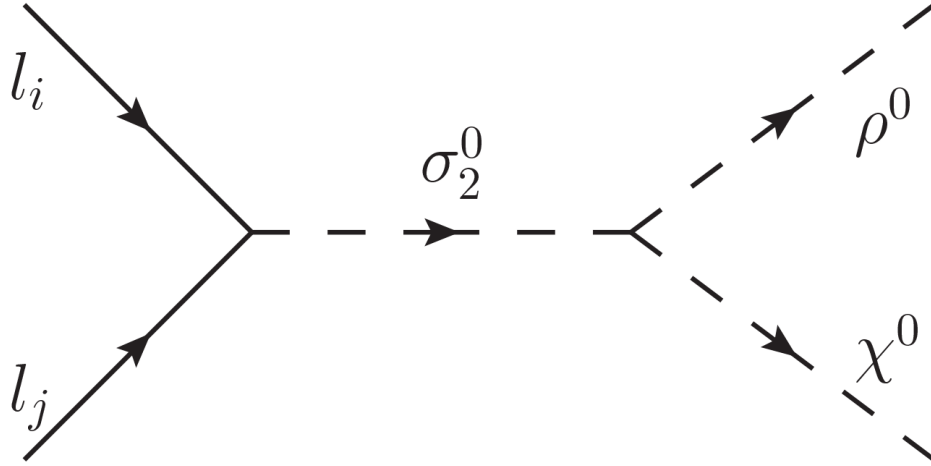


FIG. 1: Tree level realization of the effective dimension five operator in Eq. (14) with heavy scalar sextet.