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QCD sum rule study for a possible charmed pentaquark $\Theta_c(3250)$

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We use QCD sum rules to study the possible existence of a $\Theta_c(3250)$ charmed pentaquark. We consider the contributions of condensates up to dimension 12 and work at leading order in α_s. We obtain $m_{\Theta_c} = (3.29 \pm 0.13)$ GeV, compatible with the mass of the structure seen by *BABAR* Collaboration in the decay channel $B^- \to \bar{p}\Sigma_c^{++}\pi^-\pi^-$. The proposed state is compatible with a previous proposed pentaquark state in the anticharmed sector.

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I. INTRODUCTION

Recently, the *BABAR* Collaboration has reported [1] the observation of unexplained structures in the $B^- \to \bar{p}\Sigma_c^{++}\pi^-\pi^-$ decay channel. In particular, they observed three enhancements in the $\Sigma_c^{++}\pi^-\pi^-$ invariant mass distribution at 3.25, 3.80 and 4.20 GeV [1]. We shall refer to these signals $\Theta_c(3250)$, $\Theta_c(3800)$ and $\Theta_c(4200)$, respectively. There are already theoretical calculations interpreting the $\Theta_c(3250)$ enhancement as a possible $D_0^*(2400)N$ molecular state [2,3]. In this note we follow a different approach, and we use the QCD sum rules (QCDSR) [4–6] to try to interpret $\Theta_c(3250)$ enhancement as a charmed pentaquark.

There are already some calculations for charmed pentaquarks. Based on simple theoretical considerations, Diakonov has predicted the masses of the exotic antidecapentaplet of charmed pentaquarks [7]. In his model, the lightest members of this multiplet are explicitly exotic doublets, $cuud\bar{s}$ and $cudd\bar{s}$, with mass about 2.42 GeV. The cryptoexotic $cudd\bar{u}$ pentaquark should have a mass around 140 MeV heavier. Since the accuracy of this prediction is ~150 MeV, Diakonov's prediction for the mass of the $cudd\bar{u}$ pentaquark is ~50 MeV smaller than the observed enhancement. Using the Skyrme soliton model Wu and Ma have studied the exotic pentaquark states with charm and anticharm [8]. In their approach, they obtained a mass around 2.70 GeV for both $cudd\bar{u}$ and $uudd\bar{c}$ states.

In 2004, the H1 experiment at DESY announced [9] the observation of a possible anticharmed pentaquark $uudd\bar{c}$ with a mass of 3099 MeV. Using the chiral doublers scenario, the authors in Ref. [10] succeeded in describing this state as an anticharmed pentaquark. The first QCDSR calculation for this state was done in Ref. [11]. The authors have found a mass around 3.10 GeV, supposing that the anticharmed pentaquark can be described by a current with two light diquarks and one anticharm quark. Since for a charmed $cudd\bar{u}$ pentaquark one needs a light diquark, a heavy-light diquark and a light antiquark to describe it, and since light diquarks are supposed to be very bound states [12] and heavy-light diquarks less bound [13], we expect the mass of the charmed pentaquark to be bigger than the mass of the anticharmed pentaquark and, therefore, compatible with the observed $\Theta_c(3250)$ enhancement.

II. TWO-POINT CORRELATION FUNCTION

A possible current describing a charmed neutral pentaquark with quark content $[cudd\bar{u}]$, which we call Θ_{1c}, is given by

$$\eta_{1c} = \varepsilon^{abc}(\varepsilon^{aef}\mathbf{u}_e^T C\gamma_5 \mathbf{d}_f)(\varepsilon^{bgh}\mathbf{c}_g^T C\gamma_5 \mathbf{d}_h)C\gamma_5 \bar{\mathbf{u}}_c^T, \quad (1)$$

where $a, b, \ldots$ are color indices, C is the charge conjugation matrix and in bold letters are the respective quark fields. We have considered two scalar diquarks since they are supposed to be more bound than the pseudoscalars [12]. However, since the study presented in [12] is related with the light diquarks, one could also have a current describing another pentaquark, Θ_{2c}, with a scalar light diquark and a pseudoscalar heavy-light diquark as follows:

$$\eta_{2c} = \varepsilon^{abc}(\varepsilon^{aef}\mathbf{u}_e^T C\gamma_5 \mathbf{d}_f)(\varepsilon^{bgh}\mathbf{c}_g^T C \mathbf{d}_h)C\bar{\mathbf{u}}_c^T, \quad (2)$$

like the current used in [11] for $\Theta_{\bar{c}}$. It is known that there is no one-to-one correspondence between these currents to the respective pentaquark states. By using Fierz transformation [14], both currents in Eqs. (1) and (2) can be rewritten as a linear combination of baryon-meson

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molecular-type currents. However, in the Fierz transformation of a pentaquark current, each molecular component contributes with suppression factors that originate from picking up the correct Dirac and color indices [14]. This means that if the physical state is a molecular state, it would be best to choose a molecular type of current so that it has a large overlap with the physical state. Similarly for a pentaquark state it would be best to choose a tetraquark current.

The sum rule for both currents (1) and (2) is constructed from the two-point correlation

$$\Pi(q) = i\int d^4x e^{iq\cdot x}\langle 0|T[\eta_c(x)\bar{\eta}_c(0)]|0\rangle = \Pi_1(q^2) + \not{q}\Pi_2(q^2), \tag{3}$$

where Π_1 and Π_2 are two invariant independent functions. In the phenomenological side, we parametrize the spectral function using the standard duality ansatz: "one resonance" + "QCD continuum". The QCD continuum starts from a threshold s_0 and comes from the discontinuity of the QCD diagrams. Transferring its contribution to the QCD side of the sum rule, one obtains the Borel/Laplace sum rules:

$$\begin{aligned} |\lambda_{\Theta_c}|^2 m_{\Theta_c} e^{-m_{\Theta_c}^2/M_B^2} &= \int_{m_c^2}^{s_0} ds e^{-s/M_B^2}\rho_1(s), \\ |\lambda_{\Theta_c}|^2 e^{-m_{\Theta_c}^2/M_B^2} &= \int_{m_c^2}^{s_0} ds e^{-s/M_B^2}\rho_2(s), \end{aligned} \tag{4}$$

where $\rho_i = \frac{1}{\pi}\,\mathrm{Im}\Pi_i(s)$ are the spectral densities whose expressions are given in the Appendix. In Eq. (4), λ_{Θ_c} and m_{Θ_c} are the pentaquark residue and mass, respectively; M_B^2 is the sum rule variable. One can estimate the pentaquark mass from the following ratios ($i = 1, 2$):

$$\begin{aligned} \mathcal{R}_i &= \frac{\int_{m_c^2}^{s_0} ds s e^{-s/M_B^2}\rho_i(s)}{\int_{m_c^2}^{s_0} ds e^{-s/M_B^2}\rho_i(s)}, \\ \mathcal{R}_{12} &= \frac{\int_{m_c^2}^{s_0} ds e^{-s/M_B^2}\rho_1(s)}{\int_{m_c^2}^{s_0} ds e^{-s/M_B^2}\rho_2(s)}, \end{aligned} \tag{5}$$

where at the M_B^2-stability point we have $m_{\Theta_c} \simeq \sqrt{\mathcal{R}_i} \simeq \mathcal{R}_{12}$.

III. NUMERICAL RESULTS

For a consistent comparison with other results obtained using the QCDSR approach, we have considered the same QCD parameters used in Refs. [6,15,16], which are listed in Table I. We shall take the heavy-quark mass in the range covered by the running mass, $\bar{m}_c(m_c)$, and on-shell mass because of its ambiguous definition when working to lowest order of perturbative QCD.

It is worth mentioning that, for both currents η_{1c} and η_{2c}, we have found a substantial M_B^2 instability in the $\mathcal{R}_{12}$ sum rule evaluation. Although $\mathcal{R}_{12}$ is, in general, the sum rule used for light baryons [17,18], this is not the case for heavy baryons where, in general, the sum rule $\mathcal{R}_i$ works better [19–21]. Therefore, in this work, we only considered the results from $\mathcal{R}_i$. As pointed out in Ref. [18], for multiparticle currents there could be strong cancellation of neighboring dimension contributions. Such cancellations can take place until dimension 10–12. To take into account such possible strong cancellations, we consider condensates up to dimension 12 in the Wilson's Operator Product Expansion (OPE). However one should notice that the most relevant dimension-12 operator, $\langle\bar{q}q\rangle^4$, only contributes to $\mathcal{R}_1$. Therefore, in the sum rule $\mathcal{R}_2$ we work up to dimension 10.

TABLE I. QCD input parameters.

Parameters	Values
m_c	(1.23–1.47) GeV
$\langle\bar{q}q\rangle$	$-(0.23 \pm 0.03)^3$ GeV^3
$\langle g_s^2G^2\rangle$	(0.88 ± 0.25) GeV^4
$\langle g_s^3G^3\rangle$	(0.58 ± 0.18) GeV^6
$m_0^2 \equiv \langle\bar{q}Gq\rangle/\langle\bar{q}q\rangle$	(0.8 ± 0.1) GeV^2

A. Θ_{2c} pentaquark state

We start our analysis with the current η_{2c}. As mentioned above, we calculate the mass related to this current using only the results from the $\mathcal{R}_1$ and $\mathcal{R}_2$ sum rules. Considering the $\mathcal{R}_2$ sum rule, we show in Fig. 1(a) the relative contributions of the terms in the OPE, for $\sqrt{s_0} = 4.70$ GeV. From this figure, we see that the contribution of the dimension-10 condensate is smaller than 20% of the total contribution for values of $M_B^2 \geq 2.7$ GeV^2, which indicates the starting point for a good OPE convergence. In Fig. 1(b), we also see that the pole contribution is bigger than the continuum contribution only for values $M_B^2 \leq 3.1$ GeV^2. Therefore, we can fix the Borel window as $(2.7 \leq M_B^2 \leq 3.1)$ GeV^2. Then we can evaluate the ground state mass, which is shown as a function of M_B^2 in Fig. 1(c). We conclude that there is a very good M_B^2 stability in the determined Borel window, which is indicated through the parentheses. Varying the value of the continuum threshold in the range $\sqrt{s_0} = (4.70 \pm 0.10)$ GeV, and other parameters as indicated in Table I, we get

$$m_{\Theta_{2c}} = (4.15 \pm 0.09)\ \text{GeV}. \tag{6}$$

We now consider the $\mathcal{R}_1$ sum rule for the current η_{2c}. The comparison between the two sum rules is shown in Fig. 2(a), considering the Borel range $(2.0 \leq M_B^2 \leq 6.0)$ GeV^2 and $\sqrt{s_0} = 4.70$ GeV. As one can see from this figure, both sum rules present an excellent M_B^2 stability and provide results which are in complete agreement with each other. It is important to mention that the M_B^2 stability for the $\mathcal{R}_1$ sum rule only could be achieved with the inclusion of the dimension-11 and -12 operators. We

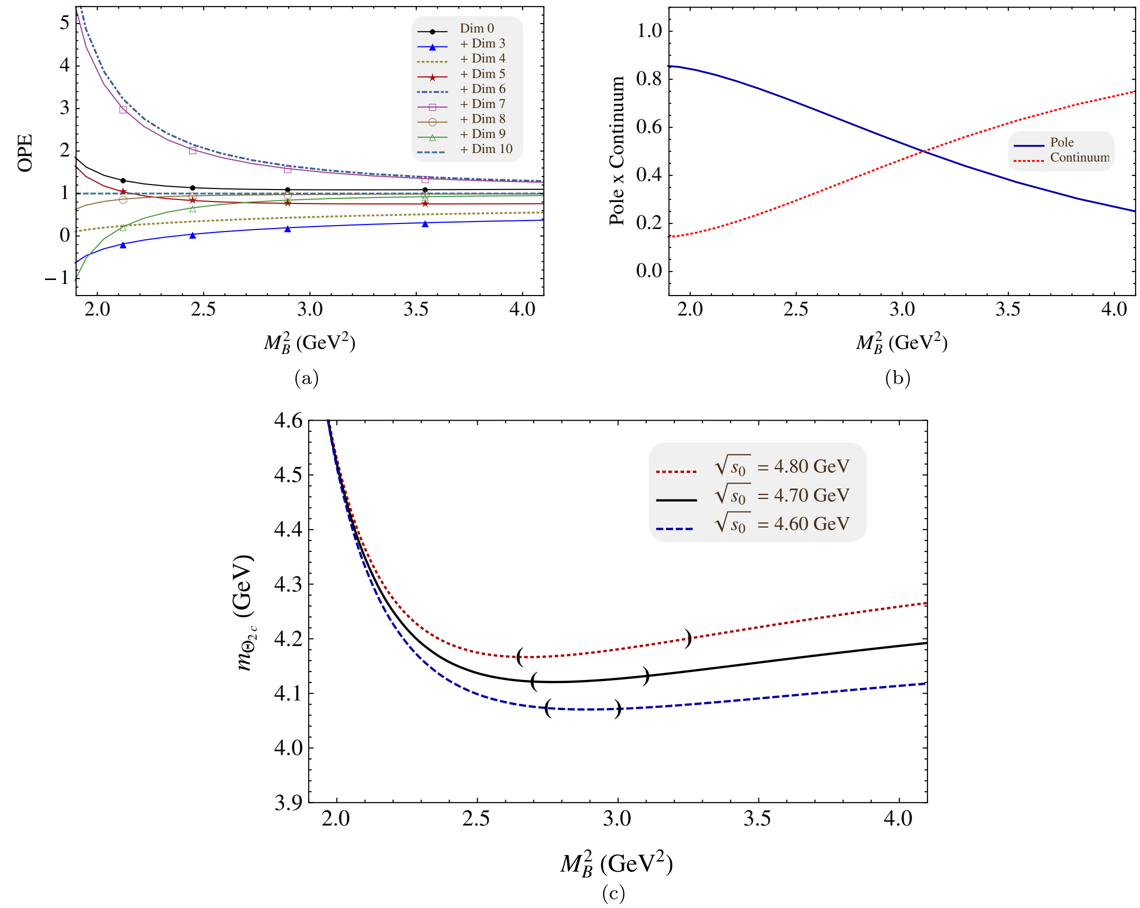

FIG. 1 (color online). $\mathcal{R}_2$ sum rule analysis using the pentaquark current η_{2c}. We have considered contributions up to dimension 10 in the OPE, using $m_c = 1.23$ GeV. (a) OPE convergence in the region $(2.0 \leq M_B^2 \leq 4.0)$ GeV^{-2} for $\sqrt{s_0} = 4.70$ GeV. We plot the relative contributions starting with the perturbative contribution and each other line represents the relative contribution after adding of one dimension in the OPE expansion. (b) The relative pole and continuum contributions for $\sqrt{s_0} = 4.70$ GeV. (c) The mass as a function of the sum rule parameter M_B^2, for different values of $\sqrt{s_0}$. For each line, the region bounded by parentheses indicates a valid Borel window.

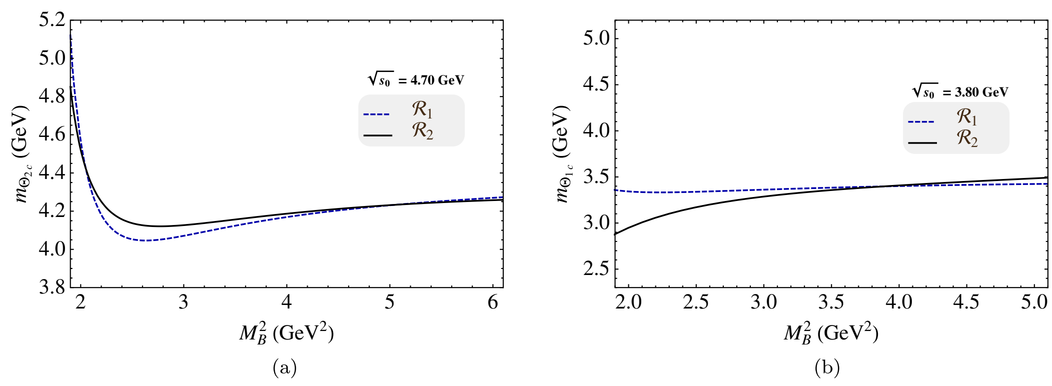

FIG. 2 (color online). The comparison between the mass results evaluated with the $\mathcal{R}_1$ (dashed line) and $\mathcal{R}_2$ (solid line) sum rules. (a) Θ_{2c} mass, in the region $(2.0 \leq M_B^2 \leq 6.0)$ GeV^2 for $\sqrt{s_0} = 4.70$ GeV. (b) Θ_{1c} mass, in the region $(1.5 \leq M_B^2 \leq 5.5)$ GeV^2 for $\sqrt{s_0} = 3.80$ GeV.

conclude that using the results of both sum rules, $\mathcal{R}_1$ and $\mathcal{R}_2$, we obtain a robust sum rule prediction for the Θ_{2c} pentaquark mass (in GeV):

$$m_{\Theta_{2c}} = 4.14(4)_{s_0}(6)_{m_c}(9)_{\langle\bar{q}q\rangle}(1)_{m_0^2}(4)_{SR} \\ = 4.14 \pm 0.12, \tag{7}$$

where, in the first line of Eq. (7), we have indicated the uncertainties due to the variation of each parameter. The last uncertainty (indicated by $_{SR}$) is related to the differences between the results from both sum rules. The obtained mass is surprisingly compatible with one of the unexplained structures observed by *BABAR* Collaboration [1] at 4.2 GeV. Therefore, from a sum rule point of view, such a Θ_{2c} pentaquark state with an internal structure composed by a scalar light diquark and a pseudoscalar heavy-light diquark could be a good candidate to explain the $\Theta_c(4200)$ enhancement.

B. Θ_{1c} pentaquark state

In the case of the current η_{1c}, we also obtain consistent results from both $\mathcal{R}_1$ and $\mathcal{R}_2$ sum rules. For simplicity, we only present the analysis related to the $\mathcal{R}_1$ since it contains a better M_B^2 stability than $\mathcal{R}_2$, but the uncertainty due to the choice of the sum rule is accounted for. The comparison between both sum rules are shown in Fig. 2(b). The pole dominance and OPE convergence are shown in Figs. 3(a) and 3(b), respectively. From these figures, we can fix the Borel window as $(2.0 \leq M_B^2 \leq 2.5)$ GeV^2, for $\sqrt{s_0} = 3.80$ GeV, where the OPE convergence starts when the contribution of the dimension-12 condensate is smaller than 10% of the total contribution. As one can see from Fig. 3(c), we also obtain a good M_B^2 stability inside the respective Borel window, providing reliable results from this sum rule. Varying the continuum threshold in the range $\sqrt{s_0} = 3.85 \pm 0.15$ GeV, and the other parameters as indicated in Table I, we get (in GeV)

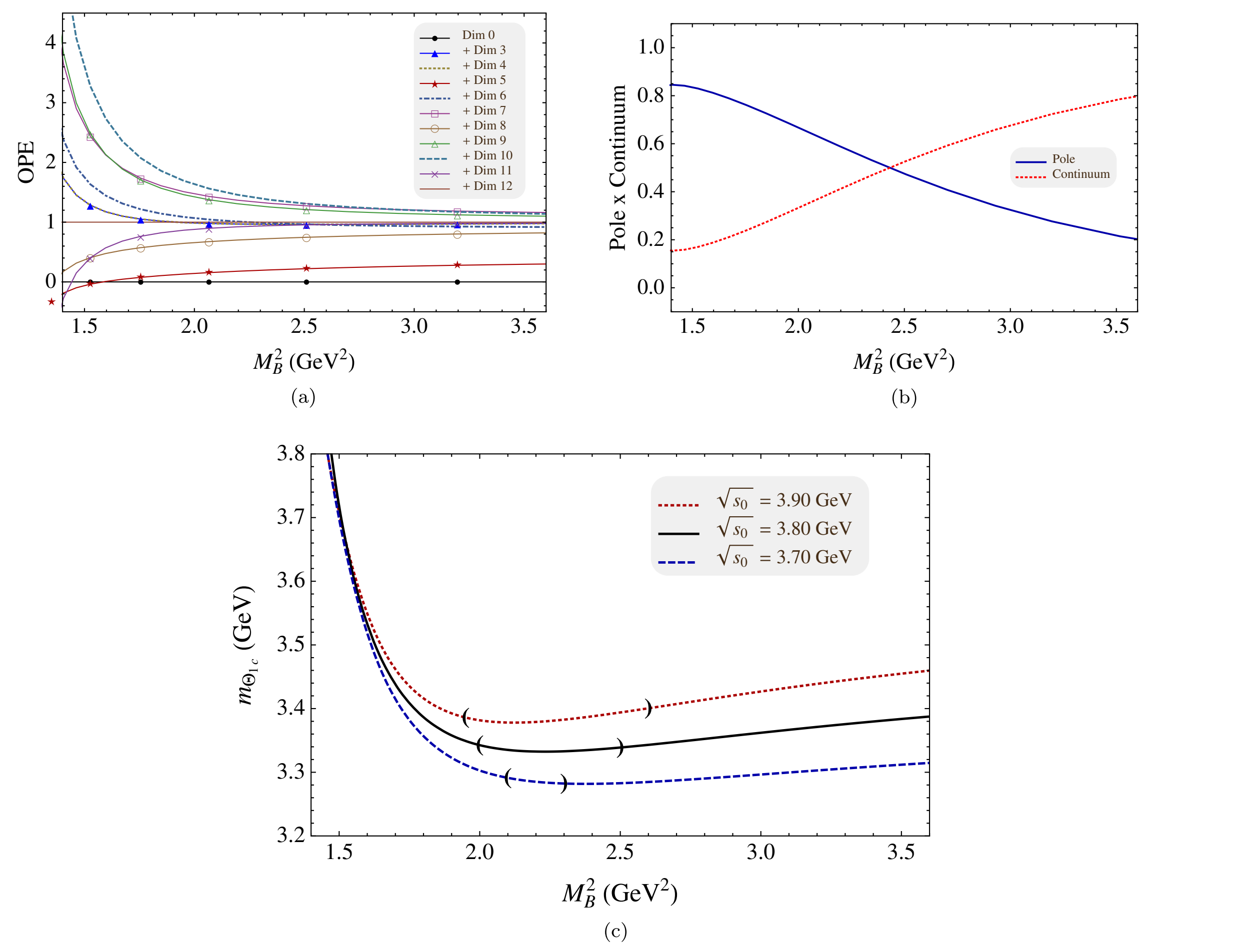

FIG. 3 (color online). $\mathcal{R}_1$ sum rule analysis using the pentaquark current $\eta_{\Theta_{1c}}$. We have considered contributions up to dimension 12 in the OPE, using $m_c = 1.23$ GeV. (a) OPE convergence in the region $(1.5 \leq M_B^2 \leq 3.5)$ GeV^{-2} for $\sqrt{s_0} = 3.80$ GeV. We plot the relative contributions starting with the perturbative contribution and each other line represents the relative contribution after adding of one dimension in the OPE expansion. (b) The relative pole and continuum contributions for $\sqrt{s_0} = 3.80$ GeV. (c) The mass as a function of the sum rule parameter M_B^2, for different values of $\sqrt{s_0}$. For each line, the region bounded by parentheses indicates a valid Borel window.

$$m_{\Theta_{1c}} = 3.29(5)_{s_0}(5)_{m_c}(7)_{\langle\bar{q}q\rangle}(3)_{m_0^2}(8)_{SR} = 3.29 \pm 0.13. \quad (8)$$

This value for the mass is compatible with the first signal observed in Ref. [1] at 3.25 GeV. Therefore, we conclude that the $\Theta_c(3250)$ state can also be described by a pentaquark containing two scalar diquarks in its internal structure. It is very interesting to notice that we get a smaller mass with the current with two scalar diquarks, when compared with the current with one scalar and one pseudoscalar diquarks. Although we have one light and one heavy-light diquark, our results follow the phenomenology obtained by Shuryak [12] for the light diquarks.

As argued before, pentaquark currents can be rewritten in terms of a sum over baryon-meson molecule-type currents, by using Fierz transformation. Thus, it is interesting to compare our results with those found in Ref. [3], where the author evaluates the sum rule for the $D_0^*(2400)N$ molecule. Indeed, the result of Ref. [3] is in agreement with our result in Eq. (8), which was obtained with the current in Eq. (1). However, there are some points in the analysis done in Ref. [3] that deserve consideration. In particular, to obtain a mass compatible with the 3.25 GeV enhancement observed by *BABAR*, the author of Ref. [3] had to release the criteria of pole dominance and the usual good OPE convergence. In doing so, the analysis inevitably led to a misleading definition of the Borel window, fixed as $(2.0 \leq M_B^2 \leq 3.0)\ \mathrm{GeV}^2$ for the $D_0^*(2400)N$ molecule. Besides, one can see that there is also no M_B^2 stability in such a Borel window. Therefore, we believe that if the author of Ref. [3] had imposed pole dominance, good OPE convergence and Borel stability in his analysis, he would have obtained a bigger value for the mass of the $D_0^*(2400)N$ current.

IV. CONCLUSIONS

In conclusion, we have presented a QCDSR calculation for the two-point function of two possible pentaquark states, whose internal structure is composed of two scalar diquarks, for Θ_{1c}, and a scalar light diquark plus a pseudoscalar heavy-light diquark, for Θ_{2c}. As expected from phenomenology [12], we get a smaller mass with the current η_{1c} containing two scalar diquarks, in comparison with the current η_{2c} containing one scalar and one pseudoscalar diquark. Also, we get a bigger mass for the Θ_{2c} pentaquark state when comparing with the one studied in Ref. [11], where the authors considered for the $\Theta_{\bar{c}}$ state a current with two light diquarks and one anticharm quark. Indeed, this result is in agreement with the expectation that heavy-light diquarks are less bound than light diquarks [13]. Our findings strongly suggest that at least two enhancements observed by *BABAR* Collaboration, with a peak at 3.25 and 4.20 GeV, decaying into $\Sigma_c^{++}\pi^-\pi^-$, could be understood as being such pentaquarks.

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APPENDIX: SPECTRAL DENSITIES

The spectral densities expressions for the charmed neutral pentaquarks, Θ_{1c} and Θ_{2c}, described by the currents in Eqs. (1) and (2), respectively, have been calculated up to dimension-12 condensates, at leading order in α_s. At these dimensions in the OPE one can safely neglect the contributions proportional to $\langle g_s^2 G^2\rangle$ and/or $\langle g_s^3 G^3\rangle$ condensates.

To keep the heavy-quark mass finite, we use the momentum-space expression for the heavy-quark propagator. We calculate the light-quark part of the correlation function in the coordinate space, and we use the Schwinger parameters to evaluate the heavy-quark part of the correlator. To evaluate the d^4x integration in Eq. (3), we use again the Schwinger parameters, after a Wick rotation. Finally we get integrals in the Schwinger parameters. The results of these integrals are given in terms of logarithmic functions, from where we extract the spectral densities and the limits of the integration. The same technique can be used to evaluate the condensate contributions.

For the $\not{q}$ structure of the correlation function (3), we get

$$\rho_2^{\mathrm{pert}}(s) = -\frac{1}{5^2\cdot 3\cdot 2^{15}\pi^8}\int_0^\Lambda d\alpha\,\frac{\alpha^5\mathcal{H}_\alpha^5}{(1-\alpha)^4},$$

$$\rho_2^{\langle\bar{q}q\rangle}(s) = (-1)^{j+1}\frac{m_c\langle\bar{q}q\rangle}{3^2\cdot 2^{11}\pi^6}\int_0^\Lambda d\alpha\,\frac{\alpha^4\mathcal{H}_\alpha^3}{(1-\alpha)^3},$$

$$\rho_2^{\langle G^2\rangle}(s) = -\frac{\langle g_s^2G^2\rangle}{5\cdot 3^2\cdot 2^{21}\pi^8}\int_0^\Lambda d\alpha\,\frac{\alpha^3\mathcal{H}_\alpha^2}{(1-\alpha)^4}[32m_c^2\alpha^2 + 5\mathcal{H}_\alpha(1-\alpha)(52-33\alpha)],$$

$$\rho_2^{\langle\bar{q}Gq\rangle}(s) = (-1)^{j+1}\frac{m_c\langle\bar{q}Gq\rangle}{3\cdot 2^{15}\pi^6}\int_0^\Lambda d\alpha\,\frac{\alpha^3\mathcal{H}_\alpha^2}{(1-\alpha)^3}(19-23\alpha),$$

$$\rho_2^{\langle\bar qq\rangle^2}(s)=\frac{\langle\bar qq\rangle^2}{3\cdot 2^7\pi^4}\int_0^\Lambda d\alpha\,\frac{\alpha^2\mathcal{H}_\alpha^2}{1-\alpha},$$

$$\rho_2^{\langle G^3\rangle}(s)=-\frac{\langle g_s^3G^3\rangle}{5\cdot 3^2\cdot 2^{20}\pi^8}\int_0^\Lambda d\alpha\,\frac{\alpha^4\mathcal{H}_\alpha}{(1-\alpha)^4}[4m_c^2(95-91\alpha)+\mathcal{H}_\alpha(285-281\alpha)],$$

$$\rho_2^{\langle\bar qq\rangle\langle G^2\rangle}(s)=(-1)^{j+1}\frac{m_c\langle\bar qq\rangle\langle g_s^2G^2\rangle}{3^3\cdot 2^{15}\pi^6}\int_0^\Lambda d\alpha\,\frac{\alpha^2}{(1-\alpha)^3}[4m_c^2\alpha^2+3\mathcal{H}_\alpha(49-\alpha(119-74\alpha))],$$

$$\rho_2^{\langle\bar qq\rangle\langle\bar qGq\rangle}(s)=\frac{\langle\bar qq\rangle\langle\bar qGq\rangle}{3\cdot 2^{12}\pi^4}\int_0^\Lambda d\alpha\,\frac{\alpha\mathcal{H}_\alpha}{(1-\alpha)}(70-73\alpha),$$

$$\rho_2^{\langle\bar qq\rangle^3}(s)=(-1)^j\frac{m_c\langle\bar qq\rangle^3}{3^2\cdot 2^3\pi^2}\int_0^\Lambda d\alpha\,\alpha,$$

$$\rho_2^{\langle G^2\rangle\langle\bar qGq\rangle}(s)=(-1)^{j+1}\frac{m_c\langle g_s^2G^2\rangle\langle\bar qGq\rangle}{3^3\cdot 2^{17}\pi^6}\left\{\int_0^\Lambda d\alpha\,\frac{3\alpha}{(1-\alpha)^2}(39-\alpha(89-66\alpha))-\int_0^1 d\alpha\,\frac{16m_c^2\alpha^3}{(1-\alpha)^3}\delta\left(s-\frac{m_c^2}{1-\alpha}\right)\right\},$$

$$\rho_2^{\langle\bar qq\rangle\langle G^3\rangle}(s)=(-1)^{j+1}\frac{m_c\langle\bar qq\rangle\langle g_s^3G^3\rangle}{3^3\cdot 2^{14}\pi^6}\left\{\int_0^\Lambda d\alpha\,\frac{3\alpha^3}{(1-\alpha)^3}(9-8\alpha)-\int_0^1 d\alpha\,\frac{m_c^2\alpha^3}{(1-\alpha)^4}(6-5\alpha)\delta\left(s-\frac{m_c^2}{1-\alpha}\right)\right\},$$

$$\rho_2^{\langle\bar qGq\rangle^2}(s)=\frac{\langle\bar qGq\rangle^2}{3^2\cdot 2^{13}\pi^4}\int_0^\Lambda d\alpha(57-70\alpha),$$

$$\rho_2^{\langle G^2\rangle\langle\bar qq\rangle^2}(s)=\frac{\langle g_s^2G^2\rangle\langle\bar qq\rangle^2}{3^3\cdot 2^{13}\pi^4}\left\{\int_0^\Lambda d\alpha(91-61\alpha)-\int_0^1 d\alpha\,\frac{16m_c^2\alpha^2}{(1-\alpha)^2}\delta\left(s-\frac{m_c^2}{1-\alpha}\right)\right\},$$

$$\rho_2^{\langle\bar qq\rangle^2\langle\bar qGq\rangle}(s)=(-1)^{j+1}\frac{m_c\langle\bar qq\rangle^2\langle\bar qGq\rangle}{3^2\cdot 2^9\pi^2}\int_0^1 d\alpha\,\frac{(48-67\alpha)}{(1-\alpha)}\delta\left(s-\frac{m_c^2}{1-\alpha}\right),$$

$$\rho_2^{\langle\bar qq\rangle^4}(s)=0,$$

where the integration limit is given by $\Lambda=1-m_c^2/s$. We also have used the definition $\mathcal{H}_\alpha=m_c^2-(1-\alpha)s$, and $j=1$, 2 for the currents η_{1c} and η_{2c}, respectively.

For the 1-structure, we get

$$\rho_1^{\rm pert}(s)=0,$$

$$\rho_1^{\langle\bar qq\rangle}(s)=(-1)^{j+1}\frac{\langle\bar qq\rangle}{3^2\cdot 2^{11}\pi^6}\int_0^\Lambda d\alpha\,\frac{\alpha^3\mathcal{H}_\alpha^4}{(1-\alpha)^3},$$

$$\rho_1^{\langle G^2\rangle}(s)=0,$$

$$\rho_1^{\langle\bar qGq\rangle}(s)=(-1)^{j+1}\frac{\langle\bar qGq\rangle}{3\cdot 2^{11}\pi^6}\int_0^\Lambda d\alpha\,\frac{\alpha^2\mathcal{H}_\alpha^3}{(1-\alpha)^2},$$

$$\rho_1^{\langle\bar qq\rangle^2}(s)=-\frac{m_c\langle\bar qq\rangle^2}{3\cdot 2^7\pi^4}\int_0^\Lambda d\alpha\,\frac{\alpha^2\mathcal{H}_\alpha^2}{(1-\alpha)^2},$$

$$\rho_1^{\langle G^3\rangle}(s)=0,$$

$$\rho_1^{\langle\bar qq\rangle\langle G^2\rangle}(s)=(-1)^{j+1}\frac{\langle\bar qq\rangle\langle g_s^2G^2\rangle}{3^3\cdot 2^{17}\pi^6}\int_0^\Lambda d\alpha\,\frac{\alpha\mathcal{H}_\alpha}{(1-\alpha)^3}[64m_c^2\alpha^2+3\mathcal{H}_\alpha(1-\alpha)(142-85\alpha)],$$

$$\rho_1^{\langle\bar qq\rangle\langle\bar qGq\rangle}(s)=-\frac{m_c\langle\bar qq\rangle\langle\bar qGq\rangle}{3\cdot 2^{11}\pi^4}\int_0^\Lambda d\alpha\,\frac{\alpha\mathcal{H}_\alpha}{(1-\alpha)^2}(35-43\alpha),$$

$$\rho_1^{\langle\bar qq\rangle^3}(s)=(-1)^j\frac{\langle\bar qq\rangle^3}{3^2\cdot 2^3\pi^2}\int_0^\Lambda d\alpha\,\mathcal{H}_\alpha,$$

$$\rho_1^{\langle G^2\rangle\langle\bar qGq\rangle}(s)=(-1)^{j+1}\frac{\langle g_s^2G^2\rangle\langle\bar qGq\rangle}{3^2\cdot 2^{17}\pi^6}\int_0^\Lambda d\alpha\,\frac{1}{(1-\alpha)^2}[16m_c^2\alpha^2+3\mathcal{H}_\alpha(1-\alpha)(13+6\alpha)],$$

$$\rho_1^{\langle\bar{q}q\rangle\langle G^3\rangle}(s) = (-1)^{j+1}\frac{\langle\bar{q}q\rangle\langle g_s^3 G^3\rangle}{3^3\cdot 2^{15}\pi^6}\int_0^\Lambda d\alpha\frac{\alpha^2}{(1-\alpha)^3}[2m_c^2(57-53\alpha)+\mathcal{H}_\alpha(171-167\alpha)],$$

$$\rho_1^{\langle\bar{q}Gq\rangle^2}(s) = -\frac{m_c\langle\bar{q}Gq\rangle^2}{3\cdot 2^{13}\pi^4}\int_0^\Lambda d\alpha\left(\frac{19-35\alpha}{1-\alpha}\right),$$

$$\rho_1^{\langle G^2\rangle\langle\bar{q}q\rangle^2}(s) = -\frac{m_c\langle g_s^2G^2\rangle\langle\bar{q}q\rangle^2}{3^3\cdot 2^{12}\pi^4}\left\{\int_0^\Lambda d\alpha\left(\frac{65-\alpha(193-152\alpha)}{(1-\alpha)^2}\right)-\int_0^1 d\alpha\frac{8m_c^2\alpha^2}{(1-\alpha)^3}\delta\left(s-\frac{m_c^2}{1-\alpha}\right)\right\},$$

$$\rho_1^{\langle\bar{q}q\rangle^2\langle\bar{q}Gq\rangle}(s) = (-1)^j\frac{\langle\bar{q}q\rangle^2\langle\bar{q}Gq\rangle}{3\cdot 2^9\pi^2}(16+m_c^2/s),$$

$$\rho_1^{\langle\bar{q}q\rangle^4}(s) = -\frac{m_c\langle\bar{q}q\rangle^4}{54}\int_0^1 d\alpha\frac{m_c^2\tau}{(1-\alpha)^2}\delta\left(s-\frac{m_c^2}{1-\alpha}\right).$$

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