

Brane cosmic string compactification in Brans-Dicke theoryM. C. B. Abdalla,^{1,*} M. E. X. Guimarães,^{2,†} and J. M. Hoff da Silva^{1,‡}¹*Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona 145 01405-900 São Paulo, SP, Brazil*²*Departamento de Matemática, Universidade de Brasília, Asa Norte 70910-900, Brasília-DF, Brazil*

(Received 15 March 2007; published 16 April 2007)

We investigate an alternative compactification of extra dimensions using local cosmic string in the Brans-Dicke gravity framework. In the context of dynamical systems it is possible to show that there exist a stable field configuration for the Einstein-Brans-Dicke equations. We explore the analogies between this particular model and the Randall-Sundrum scenario.

DOI: [10.1103/PhysRevD.75.084028](https://doi.org/10.1103/PhysRevD.75.084028)

PACS numbers: 11.25.-w, 04.50.+h, 12.60.Fr

The search of an alternative compactification of extra dimensions is a very interesting tool to analyze the hierarchy problem in string theory. Among other physical implications, they provide a beautiful scenario of our universe, inserting it on a brane [1,2]. In the Randall-Sundrum (RS) model there are two 3-branes in an orbifold with $\mathbb{Z}_2$ symmetry in five dimensions [3]. A warp factor in the metric shows how this type of model can help us with the hierarchy problem, and beyond it, the model is a 5- D realization of Horava-Witten solution [4]. Many physical effects related to the extra dimensions can be calculated using the standard brane-world gravity [5]. However, from the point of view of topological defects, these models use global domain walls to generate the branes [6]. But cosmologists call our attention to cosmological problems related to global defect like domain walls [7]. In this vein an interesting alternative (global strings) was given in [8] that provides in six dimensions all the necessary structure; three-brane plus transverse space.

On the other hand, scalar-tensorial theories, in particular, the Brans-Dicke [9], stand for many reasons as a good laboratory to develop these kinds of physical models. First, they show a robust relation with gravity coming from fundamental string theory at low energy. Second, they can be tested by experimental observations. Besides, theoretically we hope that the additional scalar field plays an important role in the distribution of masses of the fundamental particles.

A particular and important characteristic of the models like RS and the one found in [8,10] is that the topological defect can compactify the spacetime around it by itself, leading to a singularity. Topological domain walls and global strings effectively do it, but if one introduces a time dependent factor on the metric in the case of a global string the solution presents a cosmological event horizon free of singularity [11]. However this event horizon is not stable [12]. In Einstein's theory only global defects have this property. However, when we look at the equations of a

local cosmic string in the Brans-Dicke theory they seem to have that property, except for the fact that there is no singularity at GUT scale [13]. In other words, local cosmic string in the Brans-Dicke theory is similar, in this context, to global string in Einstein's gravity.

Our effort here is to present a model of alternative compactification using a local string in the Brans-Dicke theory. In the course of development we shall use several approximations and by using a dynamical system argument we arrive at a model similar to the one in [8,10]. Nevertheless, we would like to stress that the main difference between the previous works and ours is that we use local cosmic string in the framework of a low energy string theory while they use a global string in Einstein's theory. Besides, the space is topologically different since we end up with a de Sitter behavior, and the additional scalar field plays an important role in the warp factor.

We start with the equation

$$R^\mu_\nu = 2\partial^\mu\sigma\partial_\nu\sigma + \varepsilon\left(T^\mu_\nu - \frac{\delta^\mu_\nu}{p+1}T\right) - \frac{2\delta^\mu_\nu\Lambda}{p+1}, \quad (1)$$

which comes from the Brans-Dicke action in $p+3$ dimensions in the Einstein frame. In (1), Λ is the cosmological constant, σ is the scalar field, T^μ_ν is the stress tensor of a cosmic string, T is the trace of the stress tensor and $\varepsilon =$

$-\frac{1}{M_{p+3}^{p+3}}$, where M_{p+3} is the analogue of the Planck mass in $p+3$ dimensions. The equations that make the relationship between the two frames (the Jordan-Brans-Dicke and the Einstein frames) are $w = \frac{1}{4\beta^2} - \frac{3}{2}$ (the Brans-Dicke parameter), $\tilde{\phi} = \frac{1}{\varepsilon}e^{-2\beta\sigma}$ and $\tilde{g}_{\mu\nu} = e^{2\beta\sigma}g_{\mu\nu}$. The physical quantities (i.e., the quantities defined in the JBD frame) are the tilde ones. Note that in the Einstein's frame the stress tensor is not conserved.

In order to calculate the metric we expect that the spacetime has the same symmetry of the source, and since our source is a cosmic string, we write down the ansatz

$$ds^2 = e^A(-dt^2 + dz_i^2) + dr^2 + e^C d\theta^2, \quad (2)$$

where $i = 1 \dots p$ and A and C are functions of r only. So,

*Electronic address: mabdalla@ift.unesp.br

†Electronic address: marg@unb.br

‡Electronic address: hoff@ift.unesp.br

all the bulk structure has a cylindrical symmetry. Now we turn to tell a little bit more about the source.

The standard model for a gauge cosmic string is the $U(1)$ invariant Lagrangean

$$L = \frac{1}{2}(D_\mu \Phi)(D^\mu \Phi)^* + \lambda(\Phi^* \Phi - \eta^2) + \frac{F_{\mu\nu} F^{\mu\nu}}{16\pi}, \quad (3)$$

where $D_\mu = \partial_\mu + ieA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The ansatz to generate a stringlike solution is $\Phi = \eta X e^{i\theta}$ and $A_\mu = \frac{1}{e}(P-1)\partial_\mu \theta$. Here X and P are also functions of r only. Now, two important remarks: first, when the masses of gauge and scalar fields are equal, the fields have a behavior, far from the string in Einstein theory, given by [14]

$$\begin{aligned} e^C &\rightarrow r^2(1 - 4G\eta^2)^2, \\ P &\rightarrow 2\sqrt{2}(1 - 4G\eta^2)\gamma r^{1/2}e^{-2\sqrt{2}r}, \\ X &\rightarrow \gamma r^{-1/2}e^{-2\sqrt{2}r}, \end{aligned} \quad (4)$$

where γ is a constant that can be determined by some boundary condition. Besides, it is possible to show that in the Brans-Dicke theory the stress tensor associated to (3) in Einstein frame is [13]

$$\begin{aligned} T_t^t &= T_z^z = -\frac{\eta^2}{2}(X'^2 e^{2\beta\sigma} + e^{-C} X^2 P^2 e^{2\beta\sigma} \\ &\quad + 2(X^2 - 1)^2 e^{4\beta\sigma} + \frac{1}{8}e^{-C} P'^2), \\ T_r^r &= \frac{\eta^2}{2}(X'^2 e^{2\beta\sigma} - e^{-C} X^2 P^2 e^{2\beta\sigma} - 2(X^2 - 1)^2 e^{4\beta\sigma} \\ &\quad + \frac{1}{8}e^{-C} P'^2), \\ T_\theta^\theta &= \frac{\eta^2}{2}(-X'^2 e^{2\beta\sigma} + e^{-C} X^2 P^2 e^{2\beta\sigma} - 2(X^2 - 1)^2 e^{4\beta\sigma} \\ &\quad + \frac{1}{8}e^{-C} P'^2), \end{aligned} \quad (5)$$

where prime means derivative with respect to r .

A remarkable characteristic of the solution found by Gundlach and Ortiz [13] is that the string solution in the Brans-Dicke theory can be obtained using an amazing approximation far enough from the core, so that (4) is valid, in such way that $e^{\beta\sigma} \simeq 1$ is still a good approximation. We shall use it here. Note that in the components of stress tensor it means $\sigma \sim cte$, i.e. the contribution of a scalar Brans-Dicke field, as a source, is too small compared with that one coming from the cosmic string, even in the $r \gg$ region. Since we are interested in an exotic compactification model, our system will be analyzed in the $r \gg$ region. Taking everything into account, i.e., substituting the expressions (4) into (5) and using $e^{\beta\sigma} \simeq 1$ we have, at $1/r$ order,

$$\begin{aligned} T_t^t - \frac{T}{p+1} &\simeq 16\left(\frac{1-p}{1+p}\right)\eta^2\gamma^2\frac{e^{-4\sqrt{2}r}}{r}, \\ T_r^r - \frac{T}{p+1} &= T_\theta^\theta - \frac{T}{p+1} \simeq \frac{32\eta^2\gamma^2}{p+1}\frac{e^{-4\sqrt{2}r}}{r}. \end{aligned} \quad (6)$$

So, the Einstein-Brans-Dicke equation. (1), after some

calculation and calling $f(r) = \frac{32\epsilon\eta^2\gamma^2}{p+1}\frac{e^{-4\sqrt{2}r}}{r}$, gives

$$\begin{aligned} (p+1)\left(A'' + \frac{A'^2}{2}\right) + C'' + \frac{C'^2}{2} \\ \simeq -4\sigma'^2 - 2f(r) + \frac{4\Lambda}{p+1}, \end{aligned} \quad (7)$$

$$C'' + \frac{C'^2}{2} + \frac{(p+1)}{2}A'C' \simeq -2f(r) + \frac{4\Lambda}{p+1}, \quad (8)$$

$$A'' + \frac{(p+1)}{2}A'^2 + \frac{A'C'}{2} \simeq f(r)(p-1) + \frac{4\Lambda}{p+1}. \quad (9)$$

It is easy to note that even in such approximation the equations above are not simple. We shall perform, as in [8], a dynamical system formulation and then an analysis in the phase plane. We start with the variables

$$x = pA' + C', \quad y = C'. \quad (10)$$

Equations (7) and (8), together, give ($f(r) \equiv f$)

$$-4\sigma'^2 = (p+1)\left(A'' + \frac{A'^2}{2} - \frac{A'C'}{2}\right), \quad (11)$$

and substituting A'' from (9) we have

$$-4\sigma'^2 = (p+1)\left((p-1)f + \frac{4\Lambda}{p+1} - \frac{pA'^2}{2} - A'C'\right). \quad (12)$$

Then, taking into account that $\frac{x^2 - y^2}{2p} = \frac{pA'^2}{2} + A'C'$, we arrive at

$$4\sigma'^2 = -(p^2 - 1)f - 4\Lambda + \frac{(p+1)}{2p}(x^2 - y^2). \quad (13)$$

The dynamical system (10) can be studied in the (x, y) plane and the relation (13) is a consistency equation that plays an important role later on. From (8) it is easy to see that

$$y' = -2f + \frac{4\Lambda}{p+1} - \frac{1}{2p}[xy(p+1) - y^2], \quad (14)$$

and (7), (9), and (13) led to

$$x' = (p+1)(p-2)f + 4\Lambda - \frac{1}{2p}[(p+1)x^2 - xy]. \quad (15)$$

Equations (14) and (15) form a nonautonomous dynamical system that is characterized by two critical points

$$(\bar{x}, \bar{y})_\pm = \pm\Delta\left(4\Lambda + f(p+1)(p-2), \frac{4\Lambda}{p+1} - 2f\right), \quad (16)$$

where $\Delta = \left(\frac{2(p+1)}{4\Lambda(p+2) + f(p^2-3)(p+1)}\right)^{1/2}$, and the $(\bar{x}, \bar{y})_+$ and

$(\bar{x}, \bar{y})_-$ are the attractor and repeller points, respectively. Basically it means that there is at least one stable configuration for the fields (attractor). Looking at (2) one notes that if $p = 3$ we have a brane-world picture, with a transverse space (r, θ) . Beyond this, the attractor point in the phase plane gives the integral equations to the fields in $r \gg \lambda$, i. e.

$$\sigma'^2 = \frac{-4f(\Lambda + 2f)}{5\Lambda + 6f}, \quad (17)$$

$$C' = \left(\frac{2}{5\Lambda + 6f}\right)^{1/2} (\Lambda - 2f), \quad (18)$$

$$A' = \left(\frac{2}{5\Lambda + 6f}\right)^{1/2} (\Lambda + 2f). \quad (19)$$

From Eq. (17) we note that, since $f < 0$ (because $\varepsilon < 0$), the cosmological constant is positive and must obey the following inequality:

$$\Lambda > 2|f(r)|. \quad (20)$$

It is a quite remarkable relation. First, it tells that we are, in fact, dealing with a de Sitter space. Second, this is a new characteristic of models of this kind. Like in Ref. [8], there are two branches and the system never goes to the repeller. In a more explicit way, Eq. (20) gives

$$\Lambda > 16|\varepsilon|\eta^2\gamma^2 \frac{e^{-4\sqrt{2}r}}{r}, \quad (21)$$

which leads to a very small cosmological constant.

The next step is to analyze how this model can help in solving the hierarchy problem. Then, after coming back to the physical frame we have

$$ds^2 = W(r)(\eta_{\mu\nu}dx^\mu dx^\nu) + e^{2\beta\sigma}dr^2 + H(r)d\theta^2,$$

where $W(r) = e^{2\beta\sigma+A}$, $H(r) = e^{2\beta\sigma+C}$ and A , C and σ are the fields which asymptotic behavior is found in (17)–(19). Explicitly, the function $W(r)$ intervenes directly in the masses generated by the Higgs mechanism, just like in [3]. Nevertheless, we should emphasize here that there is a

new possible adjustment provided by the Brans-Dicke scalar field. However, the way it appears can not tell, in a rigorous way, its influence in the Higgs mechanism once the equations are only valid far from the brane. By all means, it is clear that inclusion of the Brans-Dicke field opens a new possibility.

It is easy to note from Eqs. (17)–(19) that if one implements the limit $f(r) \rightarrow 0$, the scalar field becomes constant, and the fields A and C are equal, apart of a constant. When this occurs the functions $W(r)$ and $H(r)$ become equal and we arrive into a new brane-world picture, given by

$$ds^2 = e^{k\Lambda^{1/2}r}(\eta_{\mu\nu}dx^\mu dx^\nu + d\theta^2) + dr^2, \quad (22)$$

where k , in general, depending on the dimension and the constants, had been absorbed in new variables (x^μ, θ, r) . Again, Eq. (22) is valid only in $r \gg \lambda$ region (far from the string) and we stress the minuteness of the cosmological constant. Of course, the shape found in (22) can also be obtained solving the Einstein's equations with cylindrical symmetry without any source and free of approximations, however there is no physical reason to do it.

We conclude this work stressing that we believe that this is a promising field of research, i.e. to realize brane-gravity on this model, analyzing how the extra-dimensions influence in the calculation of measurable physical amounts [15]. Special techniques used in other models cannot be used here, like among others, Israel junction conditions [16] at the brane that makes explicitly use of the $\mathbb{Z}_2$ symmetry.

The combination of a local cosmic string and the low energy string theory such as Brans-Dicke seems to be a good alternative model for compactifying extra dimensions from both physical and topological points of view.

The authors would like to thank Professors A. A. Bytsenko and J. A. Helayel-Neto for useful comments on this manuscript. J. M. Hoff da Silva would like to acknowledge CAPES-Brazil for financial support. M. E. X. Guimarães and M. C. B. Abdalla acknowledge CNPq for support.

-
- [1] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125**, 136 (1983); **125**, 139 (1983).
 - [2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D **59**, 086004 (1999).
 - [3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).
 - [4] P. Horava and E. Witten, Nucl. Phys. B **460**, 506 (1996).
 - [5] R. Maartens, Living Rev. Relativity **7**, 7 (2004).
 - [6] A. Lukas, B. A. Ovrut, K. S. Stelle, and D. Waldram, Phys. Rev. D **59**, 086001 (1999).
 - [7] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, 1994).
 - [8] P. Bowcock, C. Charmousis, and R. Gregory, Class. Quant. Grav. **17**, 4745 (2000); R. Gregory, Phys. Rev. Lett. **84**, 2564 (2000).
 - [9] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

- [10] A.G. Cohen and D.B. Kaplan, Phys. Lett. B **470**, 52 (1999).
- [11] R. Gregory, Phys. Rev. D **54**, 4955 (1996).
- [12] A. Wang and J.A.C. Nogales, Phys. Rev. D **56**, 6217 (1997).
- [13] C. Gundlach and M. Ortiz, Phys. Rev. D **42**, 2521 (1990); M.E.X. Guimarães, Class. Quant. Grav. **14**, 435 (1997).
- [14] B. Linet, Phys. Lett. A **124**, 240 (1987).
- [15] M. Cavaglia, Int. J. Mod. Phys. A **18**, 1843 (2003); P. Kanti, Int. J. Mod. Phys. A **19**, 4899 (2004); S. Creek, R. Gregory, P. Kanti, and B. Mistra, Class. Quant. Grav. **23**, 6633 (2006).
- [16] W. Israel, Nuovo Cimento B **44S10**, 1 (1966).