5th International Symposium on Environmental Geotechnology and Global Sustainable Development, Belo Horizonte, Minas Gerais, Brazil, from August 17–23, 2000.

COD 176
ID 102

Fractal Dimension Characterization for Soils Through Practical Mc-D Method

Anselmo Jose Spadotto
R. Com. Pedro Stefanini, 290 – CEP: 18.609-590 – Botucatu, SP – Brasil
UNESP/FREA/IBD spadotto@surfnet.com.br
Marcos L. P. Tse
R.. Vicente da Rocha Torres, 1050 CEP: 18603-580 – Botucatu, SP - Brasil
UNESP tuama20@hotmail.com
Wilson Nishimura
R. Dr. Cardoso de Almeida, 2015 CEP: 18.602-130 – Botucatu, SP - Brasil
SEBRAE-SP wilsonn@sebraesp.com.br
Simone Guimarães Cornelsen
R. Dr. Cardoso de Almeida, 2015 CEP: 18.602-130 – Botucatu, SP - Brasil
ANPEI wilsonn@sebraesp.com.br
Joaquim Odilon Pereira
R. Andre de Barros, 548 – CEP: 85.815-150 – Cascavel, PR - Brasil
UNIOESTE jodilon@zaz.com.br

ABSTRACT

Fractal dimension has been an important instrument to characterize porous materials like soils, allowing to expand knowledge of soil properties, such as porosity and water infiltration. Alterations in the well-known values of fractal dimension of soils may indicate, for example, contamination or erosion susceptibility. Prior published works characterizing soils by using fractal dimension do not incorporate the practical character of the MC-D method, presented in this paper. The main purpose of the present paper is to characterize soils by using the fractal dimension through the practical MC-D method. Three types of soils were chosen, Alfisol, Oxisol and Inceptisol, and the results were, respectively: 2.72±0.02, 2.48±0.01 and 2.39±0.01. According to the literature, the values for fractal dimension can be associated to basic soil characteristics, making it possible a better understanding of the soil mechanical behaviour. One great advantage of this method is the fact, that the incorporation of the MC-D method into the fractal dimension analysis does not take much time and technicians, even in field conditions, could perform it.

KEYWORDS: soil, fractal, porous materials.

INTRODUCTION

By applying the concepts and the Spadotto formula to establish the fractal dimension, as shown in the Equation 1.

$$(D = \log V_2 / \log {}^{b}\sqrt{} M \text{ or } D = \log V_2 / \log {}^{b}\sqrt{} (d_2 * V_2) \qquad (1)$$

where,

D = Fractal Dimension

V_2= Apparent volume in the Fractal Dimension

V_1 = Apparent volume in the whole Dimension

M = Apparent mass

d_2 = Apparent fractal density

${}^{b}\sqrt{}$ = Dimensional Reducer

log = Logarithm Function that works as an adjuster, transforming a non- linear system into linear.

It was obtained the exact values of fractal dimensions for the classic models of fractals. Yet, the formula flexibility

$$(D = \log V_1 / \log {}^{b}\sqrt{} (d_2 * V_2) \qquad (2)$$

allows the accurate determination of the whole dimensions, if the models above are transformed into Euclidean or integers. A vast field of researches and technological development are opening up with the perspectives evidenced in this work.

The Theory of Fractals has been used in countless research and industrial areas, making possible to obtain new parameters that are useful today for the improvement of the techniques that exists nowadays.

Mandelbrot (1982), in his book that is the base for the understanding of the fractals, present them with the following fundamental characteristics: self-similarity, same dimension in any scale and gap.

The work proposed by Spadotto & Guerrini (1996) facilitates the didactics involved in this area. In this case, it is assumed the existence of dimensional greatness named “apparent”, to differentiate from those already established by the traditional Physics. Thus, the application of that new terminology would lead to the concept of an apparent mass on an apparent volume originating an apparent density. Spadotto & Guerrini (1996) contributed to the widespread of fractals applications, when they defined fractal unit (FU) and fractal matrix (FM) and a generic formula to determine the fractal dimension . In this work, the authors emphasized that the fractal dimension can be determined by the formula:

$$D = \text{whole/portion or } D = FM/FU \tag{3}$$

where,

FM = Fractal matrix

FU = Fractal unit

D = Fractal Dimension

that is to say, a relation of the whole with the part of a fractal system. Later on, Spadotto et al., (1996) defined that the practical formula for the determination of the fractal dimension for porous materials is $D=\log V_2/\log\sqrt[3]{M}$, where V_2 is the compacted volume, and M is the mass corresponding to this volume. The results obtained for fractal dimension in this work, in Spadotto et al., (1997 a) and Spadotto et al., (1997 b) confirmed the applicability of this formula.

For the adjustments of the formulas and concepts, researchers developed some fractals that became classic and references for new studies starting from Mandelbrot. These exact fractals were named after their creators: Cantor Dust, Kock Curve, Sierpinsky Triangle, Davi Star , Duerer Fractal , Sierpinsky Carpet, Menger Sponge . These fractals present the following fractal dimensions, respectively: 0.63,1.26,1.58,1.77,1.84,1.89,2.73.

Up to 1989, the researches found difficulties in the characterization of a quantification, which the fractal dimension could make it possible. For three different soil types, Ahl & Niermeyer (1989) obtained values of D ranging from 2.1 to 2.8 and suggest that their measures quantified the space of own filling of the soil structure. However, the comparison made by Toledo et al. (1990), using the electronic microscope, obtained dimensions for the sandstone porous wall ranging from 2.57 to 2.87, and suggests that the values of D are still underestimated.

Rieu & Sposito (1991 a), determining the resultant of the relation between the density and the mass of soil aggregates, found values of D = 2.88 for sand + clay, D=2.91 for mud of sand and D=2.95 for clay. The authors suggest that soil aggregates can be fractal objects and that the spaces of the pores in plastic soils can exhibit characteristics of structure of a medium fragmented fractal, however, never surpassing the value 3.

Young & Crawford (1991) examined the theory of the fractals in relation to the structure of the soil with applications in agricultural practices. In soils handled with different systems of cultures, they showed that D varied from 2.75 to 2.93, and they also recommend the use of the fractal dimension to quantify their heterogeneity, agreeing with the original definition of Mandelbrot. The same authors made it clear that the comparison of values of D determines, above all, the validity of the method for analysis of the pores in the rugous wall.

In the same time, Scott & Stephen (1992) analyzed the hypothetical mass distribution for the determination of the fractal dimension relating the mass percentage and the ray of the particles. The authors obtained values of D between 2.40 and 2.96, and they increased as the ray decreased and the mass increased. Thus, they established a relation between the clay percentages with the sand obtaining the sequence:

(sand + clay): 80+10, D=2.96; 40+40, D=2.88; 20+50,D=2.76; 10+70, D=2.66;0+90, D = 2.41.

This work has as objectives: a) Presenting a basic formula to determine the fractal dimension; b) Presenting a practical method to determine the fractal dimension in porous materials such as soils, as a subsidy to study and develop technology for the maintainable development.

ANALYSIS

It was taken 3 types of soils from Botucatu region : Alfisol, Oxisol and Inceptisol. Most of the concepts described below are new or not well known; therefore, for the application in this work they should be incorporated into this methodology. The sequence that begins is self-explanatory, being its details reserved for discussion. The term “apparent” is used to differ from the definitions restricted to the Physics. The units of measures for this work correspond to the amounts in generic terms. The conventions to be utilized are the following:

a) “$^b\sqrt{}$ ” = Dimensional Reducer, that reduces any dimension to the dimension “1 “. For lengths, b = 1; for areas b=2 and for volumes, b=3.

b) log = Logarithm Function that works as an adjuster, transforming a non- linear system into linear.

c) $D= \log FM/\log {}^b\sqrt{FM}$

d) $FM =V_1$ = apparent volume in the whole dimension, being used to designate this dimension.

e) $FM =V_2$ = apparent volume in the fractal dimension, being used to determine the fractal dimension (any amount in the fractal object).

f) M = apparent mass (any amount in the generating object)

G) $d_2 =M/ V_2$ = apparent fractal density

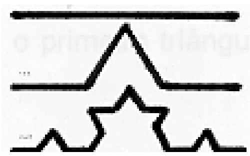
h) $FU = {}^b\sqrt{V_1} = {}^b\sqrt{M} = {}^b\sqrt{(d_2 xV_2)}$

i) $Di = \log V_1/\log {}^b\sqrt{V_1}=\log V_1/\log {}^b\sqrt{M}= \log V_1 /\log {}^b\sqrt{(d_2 xV_2)}$

j)$D= \log V_2/ \log {}^b\sqrt{V_1}=\log V_2/ \log {}^b\sqrt{M}= \log V_2 /\log {}^b\sqrt{(d_2 xV_2)}$

k)$D= \log V_1$or $V_2 /\log {}^b\sqrt{(d_2 xV_2)}$ $D=\log V_1$ ou $V_2/ \log {}^b\sqrt{M}$ (Spadotto formula)

The fractals mentioned below are classic models and are widespread in this area. With the purpose of adjusting the methodology, the Spadotto formula can be applied to determine the fractal dimension of these models, clarifying that this same formula can be used to determine the Euclidean dimension, which are demonstrated as follows. To facilitate the nomenclature, Di was adopted to name the whole dimension and D for the fractal.



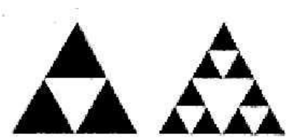
Before bending the line:

$Di = \log 3/\log^{1}\sqrt{0.75} * 4 \therefore Di = 1.00$

After bending the line:

$D = \log 4 / \log^{1}\sqrt{0.75} * 4 \therefore Di = 1.26$

Figure 1: Kock Curve



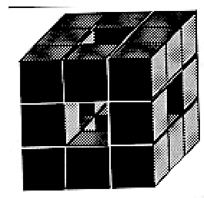
In the generating equilateral triangle:

$Di = \log 4/\log \sqrt{1.333} * 3 \therefore Di = 2.00$

After the central triangle removal:

$D = \log 3/\log \sqrt{1.333} * 3 \therefore D = 1.58$

Figure 2: Sierpinsky Triangle



For the generator:

$Di = \log 27 / \log \sqrt[3]{1.35 \times 20} \therefore Di = 3.00$

After the 7 small cubes removal:

$D = \log 20 / \log \sqrt[3]{1.35 * 20} \therefore D = 2.73$

Figure 3: Menger Sponge

METHOD FOR POROUS MATERIALS

With the purpose of determining the fractal dimension in porous materials, for example the soil , the practical method was developed through cubes, designated as MC-D (Spadotto et al., 1996 b)

ANALYSIS OF THE DATA

The deviations of the repetitions were analyzed in the computer program called Microcal Origin 4.1. For great differences, it was made the practical option for interpreting the efficiency results in percentage. For the exact results (of classic models), there was no need of analysis besides the direct comparison.

One of the keys for the understanding and practical work with fractal dimension is in the work proposed by Spadotto & Guerrini (1996). In that work the following concepts were established: fractal unit (FU) is any amount reduced to the dimension “1” and, fractal matrix (FM) corresponds to the whole of the fractal system . It should be observed that, in these cases, it was not preset any unit of measure, but, it was inferred that the fractal unit has a relation with the fractal matrix. That relation, corresponds to the following formulas:

D = whole / portion (4)

Or,

$D = \log FM/\log FU$ (5)

However, for better visualization of the fractal phenomenon , it should be understood that the fractal unit can be formed from other units, which were denominated structural fractal units (SFU). Thus, a fractal matrix and a fractal unit form a fractal, and the latter can be formed by small structures or small parts-each one of these small parts is the SFU. Aiming at better visualizing the topics of this discussion, some of the models of classic fractals are presented with the topics discussed in this section. It is understood, then, the case of porous materials, such as soil or solid ingredients of rations, where countless grains or particles compose the fractal unit, being each one of them a SFU.

When a new formula is propose and it presents as results the exact values of those models of classic fractals, this formula can be used for this purpose. This is the case of the present work, with the formula here proposed, $D = \log V_2/\log \sqrt[b]{d_2 \, x V_2}$ or $D = \log V_2/\log \sqrt[b]{M}$. Yet, to show the validity and flexibility of this formula, in the same chart the values of the Euclidean and whole dimensions are presented (Di). So , the fractal models were transformed in to entire and on the same ones it was applied the formula proposed here. Once again, the obtained results were exactly the same of the literature and of the Physics, therefore, the concepts and the formula proposed can encompass the traditional Euclidean dimension.

The method to determine the fractal dimension through cubes (MC-D), has as foundations the formula already previously discussed and the possibility to mold porous material, such as soils, into cubes. The efficiency of the new concepts and have the proposed formula were already proved, making necessary, then to discuss MC-D.

The results for the adjustments of MC-D that are shown in chart 2, which agree with the last four works previously mentioned and with Ahl & Niermeyer (1989), Toledo et al. (1990), Rieu & Sposito (1991), Young & Crawford(1991), Tyler & Wheatcraft (1992), Scott & Stephen (1992). What is clear with the results of this work and with all the works mentioned in this section, if the particles are considered to have spherical shapes, the fractal dimension varies inversely with their diameter – larger diameter is equivalent to the smallest fractal dimension. The chart 2 represents well this situation, because the soil has particles similar to spheres, although, not exactly. Once the Inceptisol has larger particles than Oxisol and, this one, has larger particles than Alfisol. It is important to make it clear, once again, that these sizes of particles cannot be inferred by the apparent density (d_2) for the reasons discussed above. Similarly to the results obtained in this work, Rieu & Sposito(1991a) found values of D = 2.88 for sand + clay, D=2.91 for a mixture with more clay and D=2.95 for clay. Scott developed similar work & Stephen (1992), finding that the fractal dimension had relation inverse to the radius of the soil particles.

The concepts, the formula and all methodology applied in MC-D can be expanded. In this work, also, it is suggested that this technology can be applied for non-deformed samples of porous materials. For this, it is necessary to calculate the volume of a non-deformed sample, that can be, for example, a cylinder, and to apply the same Spadotto formula and concepts here proposed. This way, it can be evaluated the real situation of the soil or other porous material, even so, it should be considered that the system formed in this way can be multi-fractal, better, formed by more than one fractal system or fractal. In this case, the author of this work recommends the use of the term apparent fractal dimension , as already proposed in Spadotto & Guerrini (1996), since the values could be the combination of several fractal dimensions and surpassing the value 3.

Fractal dimension is already used in several areas of knowledge and will expand more. In the beginning, is was used to classify the forms and phenomenon, separately. However, this didactic need of isolating the fractal systems should be replaced by methods of analysis that facilitate to work with multi-fractals systems, as it is here proposed with the Spadotto formula and with MC-D and their expansions.

RESULTS

The chart 1 shows the whole dimension results(Di), of fractal dimension (D) and of apparent density (d_2) of the fractals models already consecrated in the literature, obtained by the Spadotto formula, which was applied to the classic models of fractals.

Chart 1: Values of Di, D and d_2 for the classics models (at the level 1 of iteration)

Models of Fractals	$D=\log V_1/\log^{b}\sqrt{d_2} * V_2$	Result (Di or D)	d_2
	$D= \log V_2/ \log^{b}\sqrt{d_2} * V_2$		
Cantor Dust	D=log3/log3	1.00	1.00
Cantor Dust	D=log2/log3	0.63	1.50
Kock Curve	Di=log3/log3	1.00	1.00
Kock Curve	D=log4/log3	1.26	0.75
Sierpinsky Triangle	$Di=\log4/\log\sqrt{4}$	2.00	1.00
Sierpinsky Triangle	$D=\log3/\log\sqrt{4}$	1.58	1.33
Davi Star	$Di=\log9/\log\sqrt{9}$	2.00	1.00
Davi Star	$D=\log7/\log\sqrt{9}$	1.77	1.29
Duerer Fractal	$Di=\log7/\log\sqrt{7}$	2.00	1.00
Duerer Fractal	$D=\log6/\log\sqrt{7}$	1.84	1.17
Sierpinsky Carpet	$Di=\log9/\log\sqrt{9}$	2.00	1.00
Sierpinsky Carpet	$D=\log8/\log\sqrt{9}$	1.89	1.12
Menger Sponge	$D=\log27/\log^{3}\sqrt{27}$	3.00	1.00
Menger Sponge	$D=\log20/\log^{3}\sqrt{27}$	2.73	1.35

The values found here are exacts, when compared with the original results of these models.

In this case, d_2 have not unit for being of apparent density

MC-D applied for soils

Chart 2: Fractal dimensions(D) and apparent densities (d_2) of the soils

Types of soils	$D= \log V_2/ \log^{b}\sqrt{d_2} * V_2$	d_2 (g/cm^3)
Inceptisol	2.39±0.01	1.65±0.01
Oxisol	2.48±0.01	1.55±0.01
Alfisol	2.72±0.02	1.24±0.01

The deviations are given by the standard mistake of the average, in 10 repetitions

CONCLUSIONS

In function of the results obtained and under the conditions of the present work, it may be concluded that:

- The Spadotto formula $D= \log V_2 / \log \sqrt[b]{M}$ with its derivations $D= \log V_2 / \log \sqrt[b]{d_2 * V_2}$ and $D = \log V_1 / \log \sqrt[b]{d_2 * V_2}$ is used to determine the fractal dimension and the whole or Euclidean dimension.
- O MC-D allows to determine the fractal dimension in porous materials such as the soils, for example , the relationship between fractal dimension and soil fragmentation.
- A large field of researches and development of new technologies can be open with the methods here presented, for examples, soil water movement, variations of the humidity of the soil and variations of volume of the soil.

ACKNOWLEDGEMENTS

Support: FREA, SEBRAE/ANPEI, IBD, ZAZ-Botucatu, Univ. de Bonn.

REFERENCES

AHL, C. & NIEMEYER, J. The fractal dimension of the pore-volume inside soils. *Zeitchrift für Pflanzenernährun und Bodenkunde.* v.152, p. 457-458, 1989.

FEDER, J. *Fractals.* Plenum Press. New York. 1988. 283p.

MANDELBROT, B. B. *The Fractal Geometry of Nature*. W. H. Freeman, San Francisco. 1982. 468p.

PERFECT, E; RASIAH, V.; KAY, B. D. Fractal dimensions of soil aggregate-size distributions calculated by number and mass. *Soil Sci. Am. J.* v.56, p.1407 - 1409, 1992.

RIEU, M. & SPOSITO, G. Fractal fragmentation, soil porosity, and soil water properties: I. Theory. *Soil Sci. Soc. Am. J.* v.55, p. 1231 - 1238, 1991a.

SCOTT, W. T. & STEPHEN, W. W. Fractal scaling of soil particle-size distributions: analysis and limitations. *Soil Sci. Soc. Am. J.,* v.56, p. 362 -369, 1992.

SPADOTTO, A. J.; GUERRINI, IVAN A. Fractais: Conceitos e paradigma através de estudos em materiais estruturados. In: REUNIÃO ANUAL DA SBPC, 48, 1996. São Paulo. Anais... São Paulo: Sociedade Brasileira para o Progresso da Ciência, 1996. p.561.

SPADOTTO, A. J.; GUERRINI, IVAN A.; REZENDE, M. A. Método prático para a determinação da dimensão fractal aplicado nas áreas agronômica e zootécnica. In: I SIMPÓSIO NACIONAL DE INSTRUMENTAÇÃO AGROPECUÁRIA, 1, 1996. São Carlos. Anais... São Carlos, 1996b.

SPADOTTO, A. J.; GUERRINI, IVAN A.; REZENDE, M. A. Desenvolvimento de um método didático para a determinação da dimensão fractal. In: REUNIÃO ANUAL DA SBPC, 49, 1997. Belo Horizonte. Anais... Belo Horizonte, 1997a. p.49.

SPADOTTO, A. J.; GUERRINI, IVAN A.; REZENDE, M. A. Análise da relação entre a dimensão fractal e densidade em materiais porosos de interesse agronômico e zootécnico. . In: REUNIÃO ANUAL DA SBPC, 49, 1997. Belo Horizonte. Anais... Belo Horizonte, 1997b. p.119.

SPADOTTO, A. J.; GUERRINI, IVAN A.; SERAPHIM, O. J. Uso racional de energia no processamento de minerais para alimentação animal através do uso da dimensão fractal. In: CONGRESSO BRASILEIRO DE ENGENHARIA AGRÍCOLA, 26, 1997. Campina Grande, Anais... Campina Grande, 1997c.

SPADOTTO, A. J.; GUERRINI, IVAN A.;REZENDE,M. A;MIRANDA, J.R. Practical methods for determination of fractal dimension in porous material. In: WORD CONGRESS ON MEDICAL PHYSICS AN BIOMEDICAL ENGINEERING, 1997. Nice-French Riviera, Anais... Nice-French Riviera, 1997d.

TOLEDO, P. G.; NOVY, R.A.; DAVIS, H. T.; SCRIVEN, L. E. Hydraulic conductivity of porous media at low water content. *Soil Sci. Soc. Am.J.,* v.54, p. 673 - 679, 1990.

TYLER, S.W. & WHEATCRAFT, S. W. Fractal scaling of soil particle-size distributions: analysis and limitations. *Soil Sci. Soc. Am. J.,* v.56, p. 362-369, 1992.

YOUNG, I. M. & CRAWFORD, J. W. The fractal structure of soil aggregates: its measurement and interpretation. *Journal Soil Sci.,* v.42, p. 187 - 192, 1991.