5th International Symposium on Environmental Geotechnology and Global Sustainable Development, Belo Horizonte, Minas Gerais, Brazil, from August 17–23, 2000.

COD 175
ID 101

Variations in drainage basins analyses through the methods Box-Counting and SPA-BOX

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ABSTRACT

Drainage basins, like other plane images, can be analyzed by the fractal dimension; satellites images or images produced at soil level in topographical surveys have been analyzed through some methods that determine the fractal dimension, but, that technology is still not available for most technicians in the area. The objective of this work was to compare two methods, Box Counting and SPA-BOX, which determine the fractal dimension of images of drainage basins. The obtained results of fractal dimension (D) through the Box-Counting method wer 1.2912 ± 0.0528,1.1515 ± 0.0357, 1.2033 ± 0.0609,1.1853 ± 0.0552,1.1579 ± 0.0162,1.1616 ± 0.0125,1.1872 ± 0.0583,1.1650 ±0.0190. For SPA-BOX method the following values of D were obtained: 1.2192 ± 0.0049,1.2280 ± 0.0040,1.2521 ± 0,0040,1.2229 ± 0.0021,1.2330 ± 0.0025,1.1868 ± 0.0035,1.2468 ± 0.0044, 1.2281 ± 0.0016, for the drainage basins from 1 to 8 respectively. From these results, it was possible to infer that Box-Counting

can lead to experimental mistakes, mainly, in relation to the insertion of the illustrations to be analyzed, while SPA-BOX is more accurate in that kind of analysis, but it has the inconvenience of being more difficult in its execution.

KEYWORDS: drainage basins, fractals, fractal dimension.

INTRODUCTION

There are many areas in which the chaos and fractal concepts have been applied. However, practical methods to be applied in different areas of knowledge need to be developed. Spadotto's formula, the MC-D and the SPA-BOX were developed to facilitate the understanding of chaos and fractals' concepts and to provide applications of this new science in different research areas and technology. The planialtimetric maps and general plane images of drainage basins can be analyzed through the fractal analysis, as in the case of the Box - Counting method. However, some care should be taken in those types of work.

One of the greatest discoveries of the present times was the fractal dimension, in opposition to the whole or Euclidean dimension. In 1975, some works began to be published in this area. It was the case of PAUMGARTNER et. al. (1981) who compared surface and volume of cellular membranes with fractal dimension, and the following results were found: external mitochondrial membrane D=1.09 in the cross-section and D=2.09 in the space, internal mitochondrial membrane D=1.54 in the cross - section and D=2.54 in the space, and endoplasmatic reticulum D=1.72 in the cross - section and D=2.72 in the space.

Besides those observations, the authors related the fractal dimension obtained in cross-section or area (D2) with the fractal dimension obtained in space or volume (D3) in the following equation: D2=D3–1.

The fractal concepts reached the applicability when MANDELBROT (1982) published The Fractal Geometry of Nature. According to the author and corroborated by many others, nature is fractal and the Euclidean geometry cannot characterize it correctly. In practice, this implies a lot of changes such as: the possibility of characterizing objects and natural phenomena. Many research results obtained in the past need to be reviewed under the Chaos and Fractal Science. Agreeing with the first change is easier, but the second change caused resistance to the popularization of this knowledge, as told by Mandelbrot. In MANDELBROT

(1982) a fractal is defined as a group to which the *Hausdorff-Besicovitch* dimension exceeds the topologic dimension. The definition got a more practical aspect when it was determined that a fractal is a form made up of similar parts in the whole and in some ways. It is understood that a fractal is defined in limits (although its iteration tends to the infinite) and that a group can assume the form of a multifractal system . However, these are not the only fractal fundamental characteristics; an object or behavior is fractal when it presents the following characteristics: self-similarity, scale independence, infinite complexity, generated by simple feedback process and it is linked with non - linear processes or gap.

The more the techniques are developed for the work in fractal geometry, the more developed the applications are, even in new areas of human knowledge.

LOSA & NONNENMACHER (1996) worked with a graphic method to determine the fractal dimension (yardsticks method) starting from the equation $L(\varepsilon)=N(\varepsilon).(\varepsilon)$, where $L(\varepsilon)$ is the cell borderline perimeter, (ε) is the unit of measurement (yardsticks) to measure the perimeter and $N(\varepsilon)$ the yardsticks number. The equation applied for the graph is $\log[L(\varepsilon)/l_o]=(1-D)\log[\varepsilon/l_o]$, being l_o the reference scale and does not influence D. In the y-axis, $\log[L(\varepsilon)/l_o]$ in the x-axis, $\log[\varepsilon/l_o]$; the inclination of the straight line is 1-D; fractal dimension is obtained from it. The same authors state that besides the yardsticks method, the Box-Counting can be used.

The objective of this work was to compare two determinating methods of fractal dimension, Box-Counting and SPA-BOX, in analysis of drainage basins, offering bases for future works on Sustainable Development and Geotechnology.

ANALYSIS

The fractal dimension was determined by SPA-BOX. This method fixes a 2 mm checkered paper on which the illustration (graph or image) to be analyzed is placed. The base for SPA-BOX as well as for SPA-INDEX is Spadotto's formula to determine the fractal dimension (D), that is $D=\log V_2/\log^b\sqrt{M}$. In SPA-BOX the number of boxes is counted in the largest diameter (l), and l_2 is the same as "M" or apparent mass; V_2 or apparent volume is obtained counting the boxes of the illustrations borders or following the irregularities of a graph, if it is the case. Four repetitions were done for each image, and the results were analyzed in the computer program Origin 4.1.

According to SPADOTTO et. al. 1997, the results of fractal analysis with reference in the second dimension should vary between 2 and 1. For areas, as in the case of PAUMGARTNER et. al. (1981), when they worked with surfaces of cellular membranes (plane images as the images of the drainage basins), and by the definition of MANDELBROT (1982), the values of fractal dimension for areas should be between 1 and 2.

SPA-BOX, like Box-Counting, allows to determine the fractal dimension of images on surfaces. The practical characteristic and the possibility of simple explanations put SPA-BOX as an interesting method to obtain the fractal dimension. The use of a fixed 2 mm checkered paper solves the problem of the size of the illustration to be inserted. However, deeper knowledge of the phenomenon to be analyzed is needed. In any cases, according to most of the authors and LOSA & NONNENMACHER (1996), irregularities through an unit of measure are analyzed. The method Box-Counting allows the images to be analyzed faster. However, the results presented in the table 1 show that this method is less accurate than SPA-BOX.

RESULTS

The results of this work are presented in the Table 1. In this table, the medium values of fractal dimension are presented (D) for the drainage basins, which are considered the drainage nets from planialtimetric maps.

Table 1. It shows the comparative result among the analyses of fractal dimension (D) done through Box-Counting and SPA-BOX, for the same drainage basins.

Basins	1	2	3	4	5	6	7	8
SPA-BOX								
D (mean)	1.2192	1.2280	1.2512	1.2229	1.2330	1.1868	1.2468	1.2281
Se	0.0049	0.0040	0.0040	0.0021	0.0025	0.0035	0.0044	0.0016
Box-Counting								

D (mean)	1.2912	1.1516	1.2034	1.1853	1.1579	1.1616	1.1873	1.1650
Se	0.0528	0.0357	0.0609	0.0552	0.0162	0.0125	0.0583	0.0190

CONCLUSIONS

The Box-Counting method and the SPA-BOX method allowed to find values of fractal dimension for drainage basins starting from representative planialtimetric maps of drainage nets. The method SPA-BOX showed to be more accurate, better differentiating the drainage basins, but, its execution is slow, although it is simple. The Box-Counting method is simple and fast in its application, but, it presents serious difficulties in the illustrations insertion and provokes larger operational mistakes.

ACKNOWLEDGEMENTS

Support: FREA, SEBRAE/ANPEI, IBD, ZAZ-Botucatu, Univ. de Bonn.

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