

UNIVERSIDADE ESTADUAL PAULISTA - UNESP
Faculdade de Engenharia - Campus de Bauru

GABRIELLA FURLAN NEHEMY

**A PASSIVE SELF-TURNING VIBRATION NEUTRALISER FOR
DUAL FREQUENCY EXCITATION WITH ENHANCED
OPERATIONAL BANDWIDTH**

BAURU

2025

GABRIELLA FURLAN NEHEMY

**A PASSIVE SELF-TURNING VIBRATION NEUTRALISER FOR
DUAL FREQUENCY EXCITATION WITH ENHANCED
OPERATIONAL BANDWIDTH**

A thesis submitted to the São Paulo State University (UNESP), School of Engineering, Bauru, in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering.

Supervisor: Prof. Dr.

Paulo J. Paupitz Gonçalves

Co-supervisor: Prof. Dr.

Emiliano Rustighi

BAURU

2025

Nehemy, Gabriella Furlan.

A passive self-turning vibration neutraliser for dual frequency excitation with enhanced operational bandwidth / Gabriella Furlan Nehemy.

- Bauru, 2025

133 f. : il.

Tese (Doutorado)-Universidade Estadual Paulista (Unesp), Faculdade de Engenharia, Bauru

Orientador: Paulo José Paupitz Gonçalves

Coorientador: Emiliano Rustighi

1. vibration neutraliser. 2. non linear systems. 3. passive control. 4. auto-parametric excitation. I. Universidade Estadual Paulista. Faculdade de Engenharia. II. Título.

ATA DA DEFESA PÚBLICA DA TESE DE DOUTORADO DE GABRIELLA FURLAN NEHEMY, DISCENTE DO PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA MECÂNICA, DA FACULDADE DE ENGENHARIA - CÂMPUS DE BAURU.

Aos 28 de novembro de 2025, às 13h, por meio de Videoconferência, realizou-se a defesa de TESE DE DOUTORADO de GABRIELLA FURLAN NEHEMY, intitulada **A PASSIVE SELF-TUNING VIBRATION NEUTRALISER FOR DUAL FREQUENCY EXCITATION WITH ENHANCED OPERATIONAL BANDWIDTH**. A Comissão Examinadora foi constituída pelos seguintes membros: Prof. Dr. PAULO JOSÉ PAUPITZ GONÇALVES (Orientador(a) - Participação Virtual) do(a) Departamento de Engenharia Mecânica / UNESP / Câmpus de Bauru - FEB, Prof. Dr. FABRICIO CESAR LOBATO DE ALMEIDA (Participação Virtual) do(a) Departamento de Engenharia Mecânica / UNESP / Câmpus de Bauru - FEB, Prof. Dr. DIEGO FRANCISCO LEDEZMA RAMIREZ (Participação Virtual) do(a) Facultad de Ingeniería Mecánica y Eléctrica / Universidad Autonoma de Nuevo León, Professor Associado LEOPOLDO PISANELLI RODRIGUES DE OLIVEIRA (Participação Virtual) do(a) Departamento de Engenharia Mecânica / ESCOLA DE ENGENHARIA DE SÃO CARLOS-USP, Pós-doutorando VINÍCIUS GERMANOS CLEANTE (Participação Virtual) do(a) Departamento de Mecânica / UNESP - Faculdade de Engenharia de Ilha Solteira, Após a exposição pela doutoranda e arguição pelos membros da Comissão Examinadora que participaram do ato, de forma presencial e/ou virtual, a discente recebeu o conceito final: Aprovada . Nada mais havendo, foi lavrada a presente ata, que após lida e aprovada, foi assinada pelo(a) Presidente(a) da Comissão Examinadora.

Documento assinado digitalmente
 PAULO JOSE PAUPITZ GONCALVES
Data: 02/12/2025 12:00:45-0300
Verifique em <https://validar.iti.gov.br>

Prof. Dr. PAULO JOSÉ PAUPITZ GONÇALVES

To my sister

Acknowledgements

This dissertation was made possible by the contributions of several individuals and by the financial support provided by CAPES – Coordination for the Improvement of Higher Education Personnel, through the DS scholarship 88887.513387/2020-00 and the PDSE exchange programme 88881.690385/2022-01. I also acknowledge the financial support of CNPq – National Council for Scientific and Technological Development (Grant No 406594/2021-0) and FAPESP – São Paulo Research Foundation (Grant Nos 2018/15894-0 and 2020/00534-9).

My heartfelt gratitude goes to my supervisors, Paulo J. Paupitz Gonçalves and Emiliano Rustighi, for their constant support and encouragement. I am also deeply thankful to Michael J. Brennan for his guidance and valuable insights. I would like to particularly thank Emiliano Rustighi for welcoming me at the Università di Trento during my exchange period and for providing an excellent research environment. Thank you, Paulo, Emiliano and Mike for sharing your passion for research, challenging my thinking with important questions, and always being generous with your time. As Isaac Newton once said, "If I have seen further it is by standing on the shoulders of Giants."

Aos meus pais, Gisela e João, aos meus irmãos, Magali e Joaquim, e à minha avó, Nesmar, agradeço pelo apoio constante, pelo incentivo ao longo de toda a minha trajetória e pelos valores que me permitiram chegar até aqui. Obrigada por acreditarem no meu potencial, mesmo quando eu duvidava, e por comemorarem cada realização como se fosse a maior vitória do mundo. Esta conquista é resultado direto do exemplo de dedicação, coragem e determinação que sempre me ofereceram. Agradeço também à espiritualidade, cuja presença silenciosa me fortaleceu e iluminou em cada etapa deste caminho.

I am grateful to my dear friends and labmates Camila, Átila, Bruno, Milena, Pedro (*in memoriam*), for sharing my passion for research, productive discussions, collaboration, and companionship throughout this journey. Thank you for being so much more than colleagues. I couldn't have wished for a better team to share this journey.

Finally, I thank the members of the examining committee, Prof. Fabricio C. Lobato de Almeida, Prof. Diego F. Ledezma Ramirez, Prof. Leopoldo Pisanelli Rodrigues de Oliveira and Dr. Vinícius Germanos Cleante, for their careful evaluation and constructive feedback, which contributed significantly to the improvement of this work.

"The important thing is not to stop questioning. Curiosity has its own reason for existing."

Albert Einstein

Abstract

This thesis presents the development and analysis of a passive vibration neutraliser, designed as a beam-like structure mounted on a central roller bearing. The device addresses the narrow bandwidth limitation of conventional neutraliser by passively reorienting itself to one of two preferred angular positions depending only on the excitation frequency. The orthogonal stiffness distribution results in two distinct tuning frequencies, effectively broadening the operational range of vibration suppression. This research combined theoretical modelling, numerical simulations, and experimental validation. Initial analytical models with one and two degrees of freedom were used to establish the fundamental dynamics of vibration neutralisers. A frequency-domain model was then developed to predict the steady-state response for fixed orientations, with parameters identified through modal testing. To capture the full dynamic behaviour, including the self-turning effect, a five-degree-of-freedom time-domain model was proposed. This model incorporated rotational motion and harmonic base excitation, and was validated experimentally across three devices. The analysis showed that the self-rotation phenomenon can be interpreted as a form of auto-parametric excitation, driven by time-varying stiffness modulated at twice the excitation frequency. The results demonstrate that the device can passively adapt to changes in excitation frequency, autonomously adjusting its orientation to enhance vibration attenuation. This work introduces a new class of self-turning passive neutralisers, offering a significant advancement for applications requiring efficient vibration control over a broad frequency range.

Keywords: vibration neutraliser, non linear systems, passive control, auto-parametric excitation.

Resumo

Este trabalho apresenta o desenvolvimento e a análise de um neutralizador passivo de vibrações, baseado em uma viga retangular, montada sobre um rolamento central. O dispositivo foi concebido para superar a limitação de faixa estreita de atuação dos neutralizadores convencionais, por meio de uma rotação passiva para uma de duas posições preferenciais em resposta à frequência de excitação. A existência de duas direções ortogonais de rigidez permite que o sistema atue eficientemente em duas frequências de sintonização distintas, ampliando sua faixa efetiva de operação. A pesquisa utiliza modelagem teórica, simulações numéricas e validação experimental. Inicialmente, modelos analíticos de um e dois graus de liberdade foram empregados para descrever os princípios fundamentais do funcionamento de neutralizadores. Em seguida, foi desenvolvido um modelo no domínio da frequência para estimar a resposta em regime permanente com o dispositivo travado em diferentes orientações. Ensaios modais foram realizados para identificação dos parâmetros dinâmicos da estrutura. Para descrever o comportamento dinâmico completo, especialmente o fenômeno de auto-rotação, foi proposto um modelo no domínio do tempo com cinco graus de liberdade. Esse modelo incorporou os efeitos da rotação e da excitação harmônica da base, sendo validado experimentalmente para diversos protótipos. A análise revelou que o fenômeno de auto-rotação pode ser interpretado como uma excitação auto-paramétrica, sendo impulsionado por uma rigidez variável no tempo, modulada a duas vezes a frequência de excitação. Os resultados confirmam que o dispositivo é capaz de se adaptar passivamente à frequência de entrada, ajustando sua orientação de forma autônoma e ampliando sua faixa de atenuação vibratória. O trabalho contribui com uma nova classe de neutralizadores passivos, oferecendo um avanço conceitual e prático para aplicações sujeitas a excitações de frequência variável.

Palavras-chave: neutralizador de vibrações, sistemas não-lineares, controle passivo, excitação auto-paramétrica.

Contents

List of Tables	i
List of Figures	ii
Nomenclature	ix
1 Introduction	1
1.1 Literature review	5
1.1.1 Methods to increase bandwidth of vibration neutralisers	5
1.1.2 Research on neutralisers with variable stiffness elements	6
1.1.3 Utilization of smart materials	8
1.1.4 Active or semi-active control methods	11
1.1.5 Passive self-adaptation mechanisms	13
1.1.6 Centrifugal Pendulum Vibration Absorber	14
1.1.7 Remarks of the literature review	15
1.2 Objectives and contributions	15
1.3 Summary of the thesis	16
2 Fundamental principles and dynamic analysis of simplified vibration neutralisers	18
2.1 Single degree of freedom neutraliser	18
2.2 Two degrees of freedom neutraliser	21
2.3 Chapter summary	24
3 Frequency domain modelling and modal characterisation	25
3.1 Design and geometric features of the neutralisers	25
3.2 Development of the frequency domain model	29
3.3 Experimental identification of dynamic parameters	34
3.4 Chapter summary	39

4	Time domain model and experiments under harmonic excitation	41
4.1	Time domain model	41
4.2	Harmonic excitation with the beam free to rotate	45
4.3	Parametric study of neutraliser response under harmonic excitation	51
4.4	Chapter summary	59
5	Investigating self-rotation through auto parametric resonance	60
5.1	Introduction to auto parametric behaviour in mechanical systems	60
5.2	Physical mechanism of self-rotation	61
5.3	Modelling assumptions	62
5.4	Mathematical insights into self-rotation	62
5.5	Harmonic base excitation and coupling terms	63
5.5.1	Linearisation around key positions	64
5.6	Numerical interpretation of the coupling effects	66
5.7	Method of direct separation of motion	70
5.8	Comparison of self-rotation and classical auto parametric systems	71
5.9	Chapter summary	72
6	Antiresonance-based characterisation of the TC1 neutraliser	74
6.1	Modal characterisation of the TC1 neutraliser	74
6.2	Antiresonance variation with angular position	78
6.3	Influence of mass and stiffness ratios on anti resonance behaviour	81
6.4	Experimental investigation of self-turning in TC1	84
6.5	Colormap analysis of the TC1 self-turning behaviour	86
6.6	Chapter summary	87
7	Conclusions and suggestions for future work	88
7.1	Suggestions for future work	89
	References	90
	Appendix A Dynamic stiffness matrices	97
	Appendix B 3DOF time domain model	102

Appendix C Experimental investigation of rotational damping for improved time-
domain simulations 108

List of Tables

Table 3.1	Key characteristics of the neutralisers.	27
Table 3.2	Parameters of the TN2 neutraliser obtained through steady-state experiments	39
Table 6.1	Parameters of the TC1 neutraliser obtained through steady-state experiments	78

List of Figures

Figure 1.1	(a) Schematic representation of a dynamic vibration neutraliser (DVN), comprising a secondary mass m_N , damping element c_N , and stiffness k_N , forming a SDOF system. (b) Illustration of the DVN attached to a primary single-degree-of-freedom structure, characterized by mass m_S , damping c_S , and stiffness k_S . The point of attachment denotes the interface through which dynamic interaction occurs between the neutraliser and the main system. Adapted from Mead (1998).	3
Figure 1.2	Device for reducing vibrations and sounds. Adapted from Gustavsson (2011).	4
Figure 1.3	A picture of the neutraliser object of study.	5
Figure 1.4	A wideband neutraliser consisting of an array of N neutraliser. From Brennan (1997a).	6
Figure 1.5	Compound leaf spring. From Walsh and Lamancusa (1992).	6
Figure 1.6	Basic design of tuneable vibration absorber. From Keye et al. (2009).	7
Figure 1.7	(a) Elemental scheme of a series configuration ATVA and (b) test rig. From Rustighi (2010).	8
Figure 1.8	Schematic diagram of the ATVA developed. From Deng et al. (2006).	9
Figure 1.9	Photograph of the ATVA developed. From Deng et al. (2006).	9
Figure 1.10	Schematic design of the proposed MRE-based sandwich beam type ATVA. From Asadi Khanouki et al. (2020).	10
Figure 1.11	A beam-like TVA and a simplified 2DOF model. From Rustighi et al. (2005b).	11
Figure 1.12	Hybrid dynamic vibration absorber. From Wu et al. (2007).	12
Figure 1.13	Schematic of the vibration system with a nonlinear beam-slider structure as a neutraliser. From Yu et al. (2021).	14

Figure 2.1	Schematic representation of (a) a simplified host structure with a vibration neutraliser attached and (b) the dynamic model of the undamped vibration neutraliser.	19
Figure 2.2	Modulus of the accelerance excited by the spring of the undamped vibration neutraliser. (a) Original expression and its (b) dimensionless form. . . .	21
Figure 2.3	Neutraliser with 2DOF, system similar to the object of study, with a base and a tip mass.	21
Figure 2.4	Modulus of the accelerance at the base X_n of the undamped 2 DOF vibration neutraliser. (a) Original expression and its (b) dimensionless form. . .	24
Figure 3.1	Vibration neutraliser with varying stiffness, the object of study in this thesis.(a) Key components of the device, including the beam with rectangular cross section, tip masses, bearings, and base mass. (b) Reference frames: global coordinate system (Y_B , X_B , Y_N and X_N), local coordinate system (yy , xx and zz). The rotation angle θ defines the orientation of the beam relative to the base.	26
Figure 3.2	Photographs of the three vibration neutralisers. (a) TN1, with a traditional rectangular cross section. (b) TN2 and (c) TC1, both equipped with machined aluminium beams. The highlighted cross sections illustrate the geometrical differences, with TN2 and TC1 produced from circular bars and machined to achieve an approximately rectangular shape at the ends.	27
Figure 3.3	4-DOF neutraliser (a) attached to a host structure and (b) the dynamic model.	29
Figure 3.4	4DOF neutraliser (a) attached to a host structure and (b) the rotated dynamic model.	30
Figure 3.5	Forces decomposition from local to global coordinates with clock wise orientation.	31
Figure 3.6	Different angular positions (a)for $\theta = 0^\circ$, (b)for $\theta = 45^\circ$ and (c)for $\theta = 90^\circ$	33
Figure 3.7	Experimental rig used for the modal test. The TN2 neutraliser is mounted on a Modal Shop mini smartshaker, and the excitation force is measured using a PCB force sensor. Accelerometers were positioned at the base and tip masses (locations indicated) to capture the dynamic response, although they are not shown in the image.	35

Figure 3.8	Magnitudes of the accelerance responses at three angular positions ($\theta = 0^\circ, 45^\circ$, and 90°) in the global reference system ($X_B Y_B$). Subfigures (a) present the base accelerance, while subfigures (b) show the tip accelerance. Experimental results are represented by the thick grey lines, and model predictions by the thin blue lines. (dB ref. $1 \text{ N s}^2/\text{m}$).	37
Figure 3.9	Coherence functions obtained from the modal experiments at three angular positions $\theta = 0^\circ, 45^\circ$, and 90°). Subfigures (a) present the base coherence, while subfigures (b) show the tip coherence. Values close to 1 indicate high correlation between input and output signals, confirming measurement reliability across most of the frequency range.	38
Figure 4.1	Lumped-parameter model of the 5DOF system used in the time-domain analysis. The model includes the base mass m_B , the neutraliser mass m_N , and their respective displacements (x_B, y_B) and (x_N, y_N) . The neutraliser is supported by orthogonal linear springs and dampers (k_{xx}, c_{xx}) and (k_{yy}, c_{yy}) , aligned with the local coordinate system (xx, yy) , which rotates by an angle θ relative to the global frame. The rotational damping c_r opposes the relative rotation. A harmonic force $f(t)$ is applied at the base.	42
Figure 4.2	The rig used for the harmonic excitation. The neutraliser is attached to the shaker through a force sensor and has an accelerometer on the base. The high-speed camera and the light source are positioned to record the rotation of the neutraliser.	47
Figure 4.3	Snapshots of a recorded video showing a time-stories and highlighting the tracking grid used to capture the orientation of the beam.	48
Figure 4.4	Comparison between numerical and experimental results for harmonic excitation at 81 Hz.(a) Angular displacement of the neutraliser. The gray line represents the experimental response and the blue line corresponds to the numerical simulation. Sub figures (b-i) and (b-ii) show the experimental measurements of the tip displacement and base acceleration. And sub figures (c-i) and (c-ii) displays the corresponding numerical simulations of tip displacement and base acceleration.	49

Figure 4.5	Comparison between numerical and experimental results for harmonic excitation at 144 Hz.(a) Angular displacement of the neutraliser. The gray line represents the experimental response and the blue line corresponds to the numerical simulation. Sub figures (b-i) and (b-ii) show the experimental measurements of the tip displacement and base acceleration. And sub figures (c-i) and (c-ii) displays the corresponding numerical simulations of tip displacement and base acceleration.	50
Figure 4.6	Numerical colormap showing the final angular position of the neutraliser as a function of excitation frequency and force amplitude. The colour scale represents the angle θ . The dotted vertical lines indicate the first (f_{r1}) and second (f_{r2}) natural frequencies of the neutraliser base. The transition between colours exposes how the excitation conditions influence the preferred orientation of the device.	53
Figure 4.7	(a) Base accelerance curves for fixed angular positions of the neutraliser: 0° in blue and 90° in red. Solid lines indicate the configuration predicted by the numerical colormap as optimal for vibratio reduction in each frequency region. Sub figures (b) and (c) display the angular response of the neutraliser at selected excitation frequencies, with gray lines for experimental results and blue lines for numerical simulations. Specifically, (b-i) and (c-i) correspond to frequencies outside the antiresonance but still yielding a reduction in base acceleration (75 Hz and 135 Hz); (b-ii) and (c-ii) represent excitation at the antiresonances (81 Hz and 144 Hz); and (b-iii) and (c-iii) illustrate cases where excitation occurs outside the antiresonance region, resulting in increased base acceleration (90 Hz and 160 Hz).	56
Figure 4.8	Time domain results for excitation at (a) 75 Hz and (b) 90 Hz. Sub figures (a-i) and (b-i) display the angular displacement of the neutraliser, where the gray line represents the experimental measurement and the blue line the numerical simulation. Sub figures (a-ii) and (b-ii) present the experimentally measured base acceleration, while (a-iii) and (b-iii) show the corresponding numerical predictions.	57

Figure 4.9	Time domain results for excitation at (a) 135 Hz and (b) 160 Hz. Sub figures (a-i) and (b-i) display the angular displacement of the neutraliser, where the gray line represents the experimental measurement and the blue line the numerical simulation. Sub figures (a-ii) and (b-ii) present the experimentally measured base acceleration, while (a-iii) and (b-iii) show the corresponding numerical predictions.	58
Figure 5.1	Numerical simulation of the linearised terms governing the rotational dynamics of the neutraliser at two excitation frequencies: (a) 81 Hz and (b) 144 Hz. Sub plots (i) depicts the angular position $\theta(t)$, sub plots (ii) show the equivalent stiffness contribution $\tilde{k}A_1 [1 + \cos(2\omega t)]$, and sub plots (iii) present the equivalent moment $\tilde{k}A_2\theta [1 + \cos(2\omega t)]$. The results were obtained by evaluating the linearised expressions along trajectories generated from the original nonlinear model.	67
Figure 5.2	Spectrograms of the linearised excitation terms under harmonic base excitation at 81 Hz. (a-i) Spectrogram of the stiffness term. (a-ii) Spectrogram of the moment term.	68
Figure 5.3	Spectrograms of the linearised excitation terms under harmonic base excitation at 144 Hz. (a-i) Spectrogram of the stiffness term. (a-ii) Spectrogram of the moment term.	69
Figure 6.1	Experimental rig used for the modal test. The neutraliser is mounted on a Modal Shop mini smartshaker, and the excitation force is measured using a PCB force sensor. Accelerometers were positioned at the base and tip masses to capture the dynamic response.	75
Figure 6.2	Magnitudes of the accelerance responses of the TC1 at three angular positions ($\theta = 0^\circ, 45^\circ$, and 90°) in the global reference system ($X_B Y_B$). Subfigures (a) present the base accelerance, while subfigures (b) show the tip accelerance. Experimental results are represented by the thick grey lines, and model predictions by the thin blue lines. (dB ref. 1 Ns ² /m).	76

Figure 6.3	Coherence functions obtained from the TC1 modal experiments at three angular positions $\theta = 0^\circ$, 45° , and 90° . Subfigures (a) present the base coherence, while subfigures (b) show the tip coherence. Values close to 1 indicate high correlation between input and output signals, confirming measurement reliability across most of the frequency range.	77
Figure 6.4	(a) Evolution of the base antiresonance frequencies f_3 and f_4 as a function of the angular position θ , computed from the experimentally identified modal parameters of the TN2 neutraliser. The resonance frequencies f_{r1} and f_{r2} , which are independent of θ , are indicated as horizontal dashed lines for reference. Sub figures (b-i) to (b-iii) are the base accelerance for selected orientations: (b-i) for $\theta = 0^\circ$, (b-ii) for $\theta = 45^\circ$ and (b-iii) for $\theta = 90^\circ$. The corresponding resonance and antiresonance frequencies are highlighted with vertical dashed lines in each plot.	79
Figure 6.5	(a) Evolution of the base antiresonance frequencies f_3 and f_4 as a function of the angular position θ , computed from the experimentally identified modal parameters of the TC1 neutraliser. The resonance frequencies f_{r1} and f_{r2} , which are independent of θ , are indicated as horizontal dashed lines for reference. Sub figures (b-i) to (b-iii) are the base accelerance for selected orientations: (b-i) for $\theta = 0^\circ$, (b-ii) for $\theta = 45^\circ$ and (b-iii) for $\theta = 90^\circ$. The corresponding resonance and antiresonance frequencies are highlighted with vertical dashed lines in each plot.	80
Figure 6.6	(a) Influence of mass and stiffness ratio on the anti resonances. Anti resonances behaviour for a system with (a) $\kappa = 0.32$ and $\mu = 0.37$ and (b) $\kappa = 0.70$ and $\mu = 0.50$	83
Figure 6.7	Comparison between numerical and experimental results for harmonic excitation at 100 Hz.(a) Angular displacement of the neutraliser. The gray line represents the experimental response and the blue line corresponds to the numerical simulation. Sub figures (b-i) and (b-ii) show the experimental measurements of the tip displacement and base acceleration. And sub figures (c-i) and (c-ii) displays the corresponding numerical simulations of tip displacement and base acceleration.	85

Figure 6.8	Numerical colormap for the TC1 showing the final angular position of the neutraliser as a function of excitation frequency and force amplitude. The colour scale represents the angle θ . The dotted vertical lines indicate the first (f_{r1}) and second (f_{r2}) natural frequencies of the neutraliser base. The transition between colours exposes how the excitation conditions influence the preferred orientation of the device.	87
Figure B.1	The schematic of the lumped 3-DOF time-domain model without base mass and considering the outer ring of the bearing fixed.	103
Figure B.2	Time histories for fixed harmonic excitation. (a) excited at 295 Hz and (b) excited at 73 Hz. Sub figures (i) are the angular displacement, sub figures (ii) are the tip displacement and sub figures (iii) are the base displacements for the two experimental procedures.	105
Figure B.3	Elastic forces due to the mass displacement resulting in a torque.	106
Figure B.4	Numerical simulation for harmonic excitation at (a) the higher and (b) lower resonance frequency: (a) 71 Hz excitation with base displacement of 0.5 mm; (b) 300 Hz excitation with base displacement of 0.2 mm. The sub-figures are the (i) angular rotation and (ii) vertical mass displacement.	107
Figure C.1	Experimental rig used for the torsional damping experiment showing the neutraliser with the physical pendulum attached to the beam.	108
Figure C.2	Experimental results from the physical pendulum test. The blue line is the horizontal oscillatory motion extracted from the recorded movie. The red dots are the maximum amplitudes and the black dashed line the envelope of the movement.	109
Figure C.3	Mass-spring system with dry-friction. Adapted from Rao (2001).	110
Figure C.4	Comparison of experimental and numerical results for the system damped by dry friction. The gray thicker line represents the experimental data, while the blue thin line represents the numerical simulation.	110

Nomenclature

Designation	Explanation
Latin symbols	
a, A	scalar variables
$\mathbf{a}$	vectors
$\mathbf{A}$	matrices
$\mathbf{A}^T$	transpose of $\mathbf{A}$
$ \mathbf{A} $	modulus of $\mathbf{A}$
c	viscous damping
c_r	rotational damping
c_f	frictional coefficient
c_{xx}	horizontal orthogonal viscous damping
c_{yy}	vertical orthogonal viscous damping
$\mathbf{C}$	viscous damping matrix
$\mathbf{D}$	dynamic stiffness matrix
$\mathbf{D}_c$	damped dynamic stiffness matrix
E_k	kinetic energy
$f(t)$	harmonic force
F	amplitude force
$\mathbf{f}$	force vector
g	gravitational force
I	moment of inertia
j	imaginary number, $\sqrt{-1}$
k	stiffness
k_{xx}	horizontal orthogonal stiffness
k_{yy}	vertical orthogonal stiffness
$\mathbf{K}$	stiffness matrix
m	mass
$\mathbf{M}$	mass matrix
$\mathbf{q}$	displacements vector

Designation	Explanation
R	Rayleigh dissipation function
$\mathbf{T}$	coordinate transformation matrix
V	potential energy
$x(t)$	displacement response

Greek symbols

Γ_*	dimensionless accelerance
θ	angle of rotation
$\Theta(t)$	slow-varying component of motion
κ	stiffness ratio
μ	mass ratio
τ	fast timescale
$\Psi(t, \tau)$	fast-varying component of motion
ω	angular frequency
ω_n	n^{th} natural frequency
Ω_n	frequency ratio

Abbreviations

ATMD	Adaptive Tuned Mass Damper
ATVA	Active Tuned Vibration Absorber
CPVA	Centrifugal Pendulum Vibration Absorber
DOF	Degree Of Freedom
DVA	Dynamic Vibration Absorber
DVN	Dynamic Vibration Neutraliser
FRF	Frequency Response Function
MRE	Magnetorheological Elastomers
MDSM	Method of Direct Separation of Motions
PD controller	Proportional-Derivative controller
PLL	Phase-Locked Loop
SATVA	Semi-Active Tuned Vibration Absorber
SDOF	Single Degree Of Freedom

Designation	Explanation
SMA	Shape Memory Alloy
TMD	Tuned Mass Damper
TVA	Tuned Vibration Absorber

1 Introduction

Minimizing noise and vibrations is crucial in engineering and industrial domains for ensuring human comfort and safety. Extensive studies have been conducted to understand their impact on health in occupational environments (Hulshof and Veldhuijzen van Zanten, 1987). Noise pollution, also referred to as environmental noise, encompasses any unwanted or disruptive sound that detrimentally affects the health and well-being of both humans and other organisms. Noise is one of the most important occupational and environmental health hazards. One of its primary health implications is Noise Induced Hearing Loss (NIHL), the amount of hearing loss increases with noise intensity and duration of exposure Hong et al. (2013). The World Health Organization (WHO) (2018) has underscored environmental noise as a substantial health risk, contributing to the loss of approximately 1 million healthy years of life each year due to associated effects such as annoyance, sleep disturbances, and heart disease. The consequences of hearing loss are far-reaching and profound, presenting challenges in communication and causing delayed language development in children. This, in turn, leads to social isolation, loneliness, and frustration, significantly impacting older individuals struggling with hearing impairment. Effective vibration control becomes paramount in ensuring optimal conditions for individuals in such environments.

In the aviation industry, cabin noise presents a significant challenge, with main sources of aircraft interior noise originating from the power plant, propeller and engine. The attention to interior noise levels of propeller-driven airplanes declined in the late 1950s and early 1960s. However, interest returned in the 1970s due to the necessity to attenuate the high sound pressure levels produced during cruise speeds by the advanced turboprop powered aircraft (Wilby, 1996).

Despite the advantages of turboprop aircraft for commercial service in fuel efficiency, turboprops often generate more noise than jet planes, impacting passenger comfort and hindering their widespread acceptance. Regional jets remain preferred in the industry due to their superior flight performance, higher speed, and quieter, more comfortable cabins. However, it is noteworthy that while regional jets offer certain advantages, they also incur higher fuel costs compared to turboprops, with fuel expenses constituting 20% of their operational costs com-

pared to 13% for turboprops, as found by Babikian et al. (2002).

Research into reducing interior noise levels in aircraft has a long history. In their review of applications in interior noise control of propeller aircraft, Wright and Kidner (2004) conclude that while studies combining efforts to reduce interior noise levels in aircraft and advance vibration absorber technologies show promising results, the integration of adaptive vibration absorbers into standard noise control approaches for turboprop aircraft remains pending. In light of the importance of minimizing noise and vibrations in engineering and industrial contexts to ensure human well-being, the focus now turns to dynamic vibration absorbers and neutralisers. These devices are instrumental in countering the adverse effects of vibrations and reducing noise levels in various applications. The origin of vibration absorbers can be traced back to the innovative work of Frahm (1909), who secured a patent for the 'Device for Damping Vibrations of Bodies,' proposing an auxiliary body placed within or on the main body to manage disturbing vibrations which resonates in harmony with the main body vibrations.

Since the seminal work of Ormondroyd and Hartog (1928), the vibration absorber has been a base in engineering for nearly a century. It has emerged as a fundamental concept in vibration control and laid the foundation for modern vibration absorption and isolation technologies. Their contributions have been pivotal across diverse fields, from structural engineering to aerospace, significantly mitigating vibrations and their adverse effects in various applications.

A dynamic vibration neutraliser (DVN) is a comparatively small vibratory device designed to attach to a primary mechanical system or structure, aiming to reduce or control vibrations at the resonance (Den Hartog, 1985). The Figure 1.1 schematically illustrates the neutraliser configuration. In Figure 1.1(a), the neutraliser is depicted as consisting of a mass m_N , a spring k_N , and a viscous damper c_N . In Figure 1.1(b), the neutraliser is shown attached to a simple structure, represented by a mass m_s , a spring k_s , and a viscous damper c_s . The neutraliser system is excited by the motion of the main structure at the point of attachment and reacts back on it with a force that is proportional to its direct dynamic stiffness at that attachment point. Referring to the resonance frequency of the neutraliser as ω_N , at and near this frequency, the dynamic stiffness of the attachment point is extremely high, effectively resisting the motion imposed by the main structure. If the neutraliser has no damping, the dynamic stiffness at $\omega = \omega_N$ is infinite, completely preventing motion of the main structure at the attachment point (Mead, 1998).

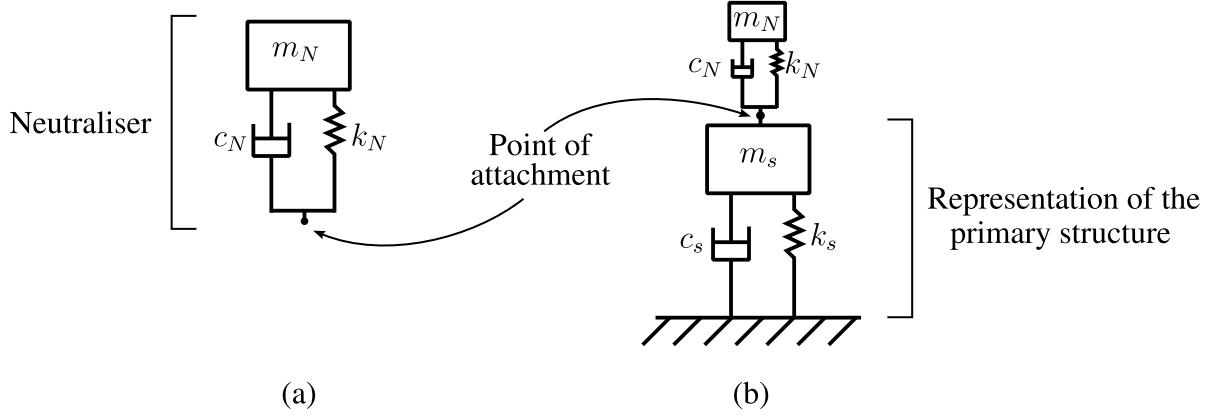


Figure 1.1: (a) Schematic representation of a dynamic vibration neutraliser (DVN), comprising a secondary mass m_N , damping element c_N , and stiffness k_N , forming a SDOF system. (b) Illustration of the DVN attached to a primary single-degree-of-freedom structure, characterized by mass m_s , damping c_s , and stiffness k_s . The point of attachment denotes the interface through which dynamic interaction occurs between the neutraliser and the main system. Adapted from Mead (1998).

The terms dynamic vibration absorber and dynamic vibration neutraliser are often mistaken. The terminology adopted in this work follows the definition by Brennan and Tang (2022), a device tuned to create a high impedance at a problematic forcing frequency is referred to as a DVN. Conversely, when tuned to a troublesome resonance frequency to introduce damping to the host structure, it is termed a 'dynamic vibration absorber' (DVA).

The vibration neutraliser is an important device used for vibration reduction in structures. However, its efficacy is constrained by its narrow operational bandwidth, limiting its ability to reduce vibrations across a wide frequency spectrum. While tuning a vibration neutraliser seems straightforward in theory, it often faces practical hurdles. For instance, in engineering scenarios involving rotating machinery or reciprocating machines, vibration source frequencies rarely remain constant. This inconsistency can compromise the neutraliser performance, or potentially convert the vibration neutraliser to a vibration amplifier as the forced vibration frequency shifts out of the anti-resonance band (Roberson, 1952). A viable solution involves implementing adaptation mechanisms to allow the neutraliser to adjust to varying excitation frequencies (Brennan, 1997b). This topic has been extensively studied, with the development of neutralisers with automatic tuning capabilities through adaptive systems. There are some review articles describing this work (Brennan, 2006; Ibrahim, 2008; Kela and Vähäoja, 2009). Furthermore, most of the currently available solutions to improve the tuning of vibration neutralisers and absorbers rely on active systems, which require significant additional costs for implementation and maintenance.

The use of vibration neutralisers as a noise control technique has been limited by the cost of commercial devices, the lengthy set-up time associated with tuning, the ability to vary the resonance frequency of the device in response to dynamic changes in the structure, and the inability to provide attenuation at multiple frequencies. In order to address this gap in the literature, the aim of this work is to develop and characterize a vibration neutraliser with broaden frequency range and two different tuning frequencies, with passive adjustment depending simply on the harmonic frequency of excitation. Applications of the device discussed in this work may include tuning a neutraliser to the blade passing frequency or its harmonics in propeller aircraft.

The neutraliser under discussion in this thesis draws inspiration from a design detailed in a patent by Gustavsson (2011), depicted in Figure 1.2. In the design, the stiffness element is a beam and has different stiffness with respect to the two main axes and connects to the rotary bearing. Additionally, due to the bearing, the stiffness element is capable of rotation around the axis of rotation, which is perpendicular to the two main axes. Mass elements are affixed to the ends of the rod. Moreover, further insights from numerical simulations and experimental findings were shared, with a focus on addressing scenarios typical of propeller aircraft, where noise frequencies vary with operational conditions (Gustavsson, 2016). However, the mechanisms that underpin the neutraliser proposed by Gustavsson (2011) are not fully understood.

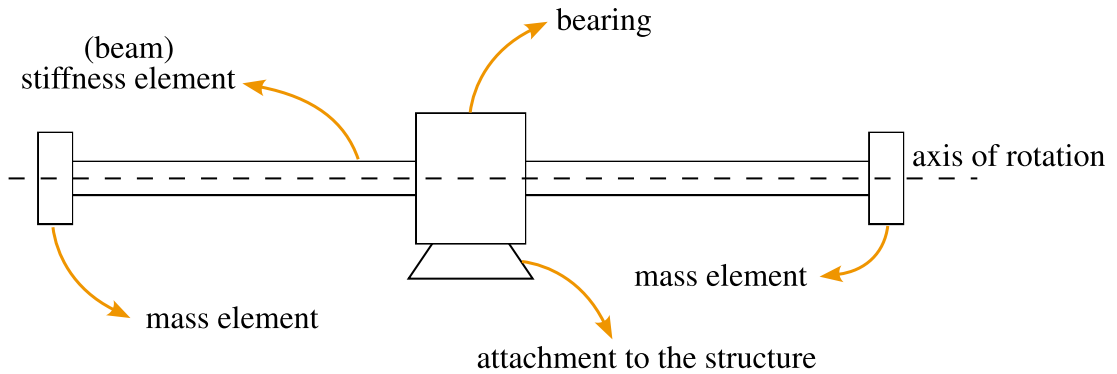


Figure 1.2: Device for reducing vibrations and sounds. Adapted from Gustavsson (2011).

The vibration neutraliser analysed in this thesis is depicted in Figure 1.3. The neutraliser is designed in a beam-like configuration, with the beam centrally attached to a roller bearing, which is fixed to a base connected to the host structure. Tip masses are attached at the ends of the beam. The beam serves as the stiffness component, and its rectangular cross-section contributes with two different orthogonal stiffness properties, which correspond to the tuning frequencies of the device. The ability of the beam to rotate around its central axis is crucial for effective vibration control, as it enables the neutraliser to resist vibrations differently based on

its orientation. This passive rotation occurs in response to the harmonic excitation frequency, with the neutraliser rotating to one of two orientations. When this rotation occurs, the vibration of the neutraliser increases while the vibration of the attachment point decreases, contributing to improved vibration control performance. To support the interpretation of the self-turning phenomenon, two complementary video recordings are provided in Nehemy (2024).

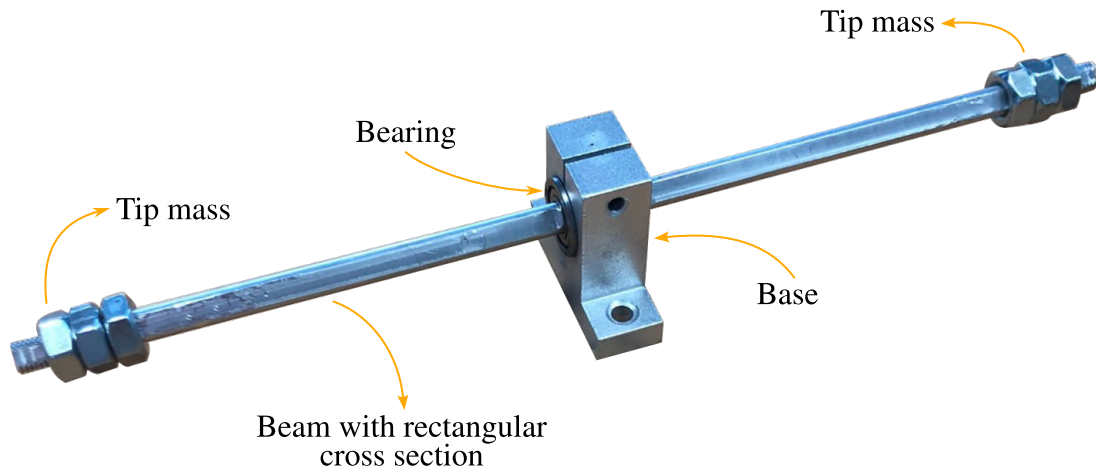


Figure 1.3: A picture of the neutraliser object of study.

1.1 Literature review

In the pursuit of overcoming the challenges outlined in the preceding section, a solution would require the neutraliser to adjust to the excitation frequency using an adaptation mechanism. Considerable research has been conducted in this topic. This literature review aims to provide a comprehensive overview of existing studies and advancements in adaptive dynamic vibration neutralisers and absorbers.

1.1.1 Methods to increase bandwidth of vibration neutralisers

Some studies in the literature propose methods to increase the bandwidth of vibration neutralisers. For instance, Brennan (1997a) proposes a wideband vibration neutraliser by arranging multiple neutralisers in parallel, as Figure 1.4, each with a slightly different natural frequency than its neighbouring partners. It is important to note that the total mass is kept constant, achieved by dividing the total mass by the number of neutralisers used to determine the mass of each individual neutraliser. As the number of neutralisers increases, so does the bandwidth of the device. This results in the impedance of a single neutraliser being spread across a wider

frequency range in the frequency domain.

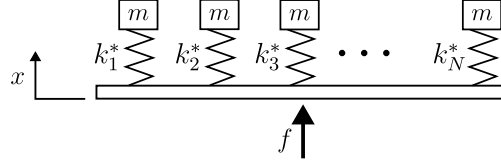


Figure 1.4: A wideband neutraliser consisting of an array of N neutraliser. From Brennan (1997a).

Additionally, the utilization of non-linearities serves to enhance effectiveness over a wider bandwidth. In their study on a non-linear vibration absorber with a hardening spring attached to a cantilever beam excited by a shaker, Hsu et al. (2013) found that the frequency response curve of the non-linear absorber effectively shifts the second resonant peak to a higher frequency, away from the tuned frequency. This observation suggests that the non-linear device exhibits robustness to mistuning when compared to a linear absorber with similar mass and damping. Specifically, it is the hardening characteristic of the device that bends the second resonant peak to a higher frequency. This non-linear effect mimics the impact of increasing the mass in a linear neutraliser, underscoring its efficacy in broadening the bandwidth (Brennan and Gatti, 2012).

1.1.2 Research on neutralisers with variable stiffness elements

The most common way of achieving a tunable neutraliser is to incorporate a variable stiffness element. Walsh and Lamancusa (1992) introduced a compound cantilever setup comprising two beams stacked on top of each other, which is based on the compound leaf spring shown in Figure 1.5. By adjusting the gap between these beams, they were able to alter the second moment of area of the cross-section, which in turn changed the flexural stiffness of the system.

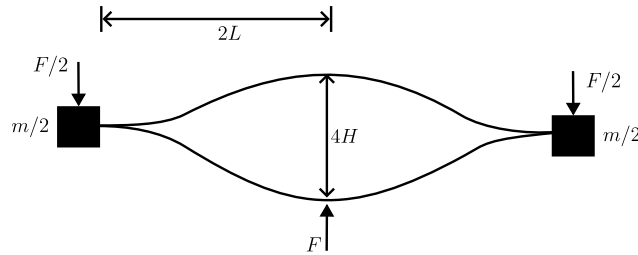


Figure 1.5: Compound leaf spring. From Walsh and Lamancusa (1992).

Longbottom et al. (1990) introduced a vibration absorber comprised by a mass supported by gas bellows, the stiffness is changed by altering the pressure in a set of rubber bellows. Xiang

and Shao (1992) came up with a different idea instead of using mechanical methods to provide the stiffness, they created a neutraliser that used an adjustable electromagnetic force to provide some of the stiffness needed. Franchek et al. (1996) developed a variable stiffness neutraliser by varying the effective number of coils in a helical spring. A device proposed by Varadarajan and Nagarajaiah (2004) consists of four spring elements arranged in a plane rhombus configuration which can change its stiffness continuously between maximum and minimum stiffness. In their work targeting turboprop aircraft, Keye et al. (2009) introduced a Tuned Vibration Absorber (TVA). They achieved a variable resonant frequency by applying pressure force to a pair of leaf springs, as Figure 1.6, effectively reducing their bending stiffness.

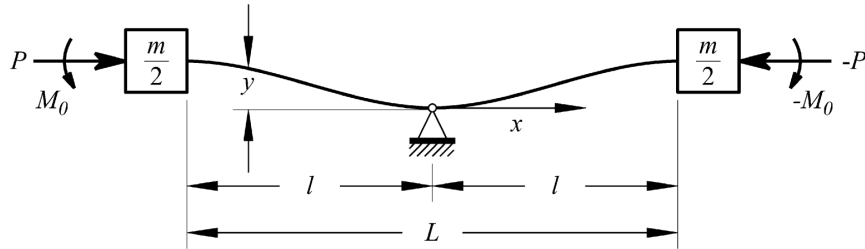


Figure 1.6: Basic design of tuneable vibration absorber. From Keye et al. (2009).

Now, focusing on the work of Bonello et al. (2005), proposed a Active Tuned Vibration Absorber (ATVA) where the stiffness element is formed from parallel curved beams. The ends of the beams are pivoted and are free to move in the vertical direction. Stiffness adjustment is achieved by changing the curvature of each beam. Experimental tests validated the effectiveness of this ATVA, especially its ability to promptly adapt to rapidly changing forcing frequencies.

In a related study, Rustighi (2010) proposed a variable stiffness tunable absorber based on a rotating spring element, that consists of two springs oriented perpendicular to each other and connected to a rotating mass, as Figure 1.7 (a). The stiffness of the absorber varies with the angle of rotation. Two experimental configurations were tested, Figure 1.7(b), and the widest tuning range was achieved by employing two motors to rotate the two spring elements attached to the same mass.

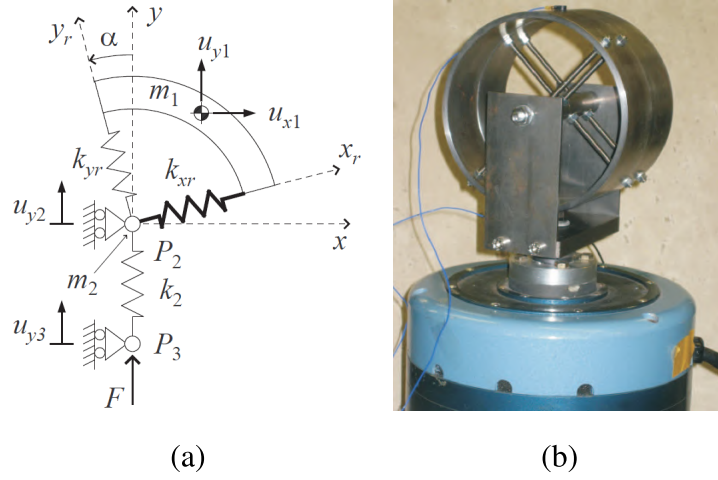


Figure 1.7: (a) Elemental scheme of a series configuration ATVA and (b) test rig. From Rustighi (2010).

Further, exploring the concept of a rotating stiffness element, Ghorbani-Tanha et al. (2011) presented a beam-like device with a rectangular beam as the stiffness element. The stiffness is changed in a smooth manner between minimum and maximum levels, representing the variations of moment of inertia of area as the beam is rotated. The rotation of the beam is achieved by an electric motor mounted at one end. When integrated into a Semi-Active Tuned Vibration Absorber (SATVA), it effectively reduced undesirable transient vibrations during the start-up of rotating machinery. Expanding on the concept of a rotational stiffness element, Minaei and Ghorbani-Tanha (2019) introduced a variable stiffness device employing ball bearings and springs. This device utilized an electric motor to adjust the orientation of the springs, thereby achieving the desired absorber stiffness.

1.1.3 Utilization of smart materials

Smart materials have also been used to realize ATVAs. Magnetorheological elastomers (MREs) are smart materials that exhibit reversible changes in their mechanical properties, such as stiffness and damping, in response to an applied magnetic field. Ginder et al. (2001), did pioneering work on the development of an adaptive tunable vibration absorber based on MRE. Deng et al. (2006) investigated, theoretically and experimentally, the application of MREs in a vibration absorber to understand their frequency-shifting capabilities and vibration absorption potential. The ATVA developed operates in shear mode and comprises several components: an oscillator, smart spring elements integrated with MREs, a magnet conductor, and two coils, as depicted in Figure 1.8. Attached to a supported beam, the ATVA's response to varying mag-

netic fields was studied to evaluate its frequency-shifting ability. White noise generated by a shaker excited the beam, while accelerometers monitored oscillations at both the oscillator and the base beam. A photograph of the developed ATVA is depicted in Figure 1.9. Both theoretical and experimental results indicate that the resonance frequency of the developed ATVA could be controlled by electrical currents and demonstrate that the ATVA developed has better performance than a conventional passive absorber in terms of its frequency-shift property and vibration absorption capacity.

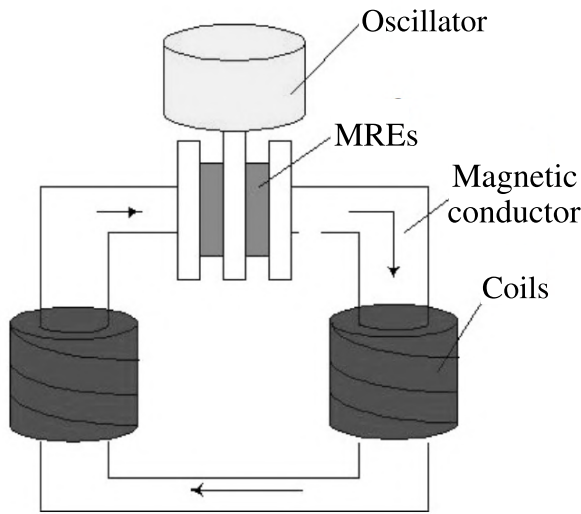


Figure 1.8: Schematic diagram of the ATVA developed. From Deng et al. (2006).



Figure 1.9: Photograph of the ATVA developed. From Deng et al. (2006).

Also working with MREs in shear mode, Hoang et al. (2009) introduced an ATVA for vehicle power train systems, demonstrating effective functionality within a frequency range of approximately 7 Hz to 70 Hz. Still on the topic of MREs, Sun et al. (2014) investigated a compact MRE absorber working in squeeze mode through experimental and theoretical research. The absorber comprises the oscillator, the smart spring element, the magnetic conductor and the coil. Four guide rods are used to keep the absorber stable. the squeeze MRE absorber was mounted onto a beam supported at each end to evaluate its vibration attenuation. The results showed that the squeeze working mode MRE absorber suppressed vibration from 37 Hz to 67 Hz.

Asadi Khanouki et al. (2020) investigated an ATVA with three layer sandwich beam with MRE in the core layer in three different configurations. Two electromagnets are placed on both ends of the beam to provide the magnetic field and act as the active mass of the absorber,

as Figure 1.10. The optimal configuration analysed, with U-shape eletromagnet, provided the highest frequency shift (42%) in the range of 5.73 to 8.14 Hz, while having a total mass of 596 g. To reinforce the structure while increasing the magnetic field density in semi-active MRE layers absorber, Rasooli et al. (2020) reinforced the sandwich beam with top, middle and bottom elastic steel plates. The results demonstrated a shift in natural frequency of about 9% under applied current of 6 Amp. However, despite the positive results in expanding the frequency range of absorbers with MREs, a limitation is that an external power source is required to control the magnetic field applied to the MREs.

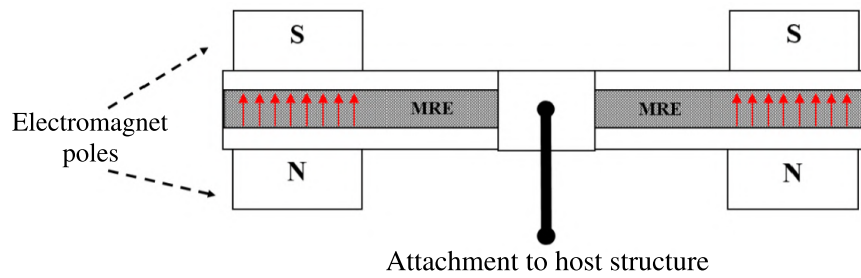


Figure 1.10: Schematic design of the proposed MRE-based sandwich beam type ATVA. From Asadi Khanouki et al. (2020).

While MREs have shown promising capabilities in enhancing the performance of vibration absorbers, researchers have also explored alternative smart materials such as Shape Memory Alloys (SMAs) for similar applications (Williams et al., 2002). SMAs offer distinct advantages due to their unique property of exhibiting changes in mechanical properties, particularly the Young's modulus, in response to temperature variations. Building on the use of SMAs in vibration absorbers, Rustighi et al. (2005b) designed a beam-like absorber incorporating an SMA wire sandwiched between two blocks of wood. By passing a current through the SMA, they achieved a change in stiffness, resulting in a tuning frequency variation of approximately 21.4%. However, the device exhibited a relatively long response time due to the thermal inertia of the SMA elements. Subsequently, Rustighi et al. (2005a) focused on the design and realization of an adaptive-passive controller for the SMA-based ATVA, building upon the same model. The PD controller (Proportional-Derivative controller) emerges as the most effective and reliable option, offering the added advantages of potentially lower cost and increased durability, particularly due to its feasibility for implementation as an analogue system.

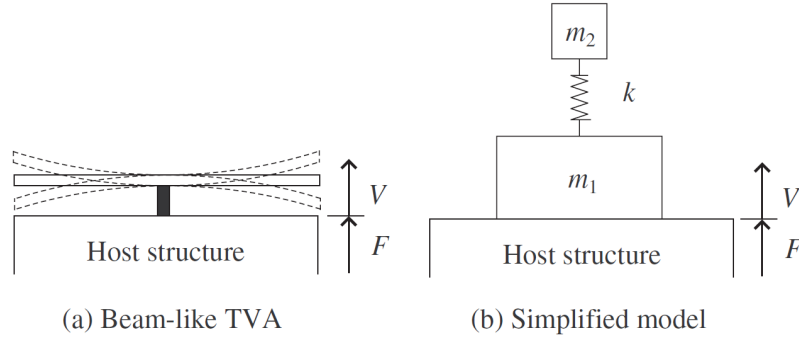


Figure 1.11: A beam-like TVA and a simplified 2DOF model. From Rustighi et al. (2005b).

To enhance the efficacy of a vibration absorber utilizing SMA at lower temperatures, Kumbhar et al. (2018) introduced MRE as an elastic spring in parallel with the SMA spring. This configuration aimed to facilitate a smoother and more precise adjustment in stiffness as required. The combination of two smart materials in the ATVA achieves smooth and continuous two-way tuning, widening the operating frequency range through frequency shifting. This results in a significant reduction in the vibration level of the main system. Despite the positive outcomes, it is important to note that SMA still requires an external power supply to heat the material and induce changes in the Young's modulus.

1.1.4 Active or semi-active control methods

Several studies have investigated the use of active control for vibration neutralisers and absorbers (Liu and Rouch, 1993; Nonami et al., 1994). Active control methods provide a more dynamic approach by actively adjusting parameters in real-time to counteract vibrations. The active control may involve reduce internal damping of the device (Kidner and Brennan, 1999), adaptive modification of internal forces (Long et al., 1998) classic control (Ezure et al., 1994) and fuzzy logic-based strategies (Hino et al., 1996; Kidner and Brennan, 2002). These active control systems often involve the integration of sensors, actuators, and control algorithms to monitor structural responses and apply corrective forces or displacements accordingly. By continuously adapting to changing conditions, these systems can effectively suppress vibrations across a broader frequency range and offer enhanced flexibility in vibration mitigation strategies. An example of such adaptability is virtual emulation, which is the process of simulating the behaviour, characteristics, or functionality of a physical system using computer software. A hybrid vibration absorber combining a passive resonator and a linear actuator is introduced by Wu et al. (2007). The actuator is controlled in such a way that it behaves like

an extra virtual mass-spring system with an adjustable frequency as depicted in Figure 1.12. By measuring how the primary structure moves and the acceleration of the actuator, the system can effectively cancel out unwanted vibrations caused by specific frequencies. Building upon this concept, Wu and Shao (2007) introduces an adaptive method where the virtual spring is tuned based on the phase difference between the acceleration of the primary body and the displacement of the virtual mass. This adaptation is facilitated by a phase detector with a low-pass filter, similar to a phase-locked loop (PLL). The dynamics of the virtual stiffness under this adaptation algorithm are analysed in detail based on a quasi-static assumption. It is noted that the disturbance frequency range must be known in order to design an effective controller.

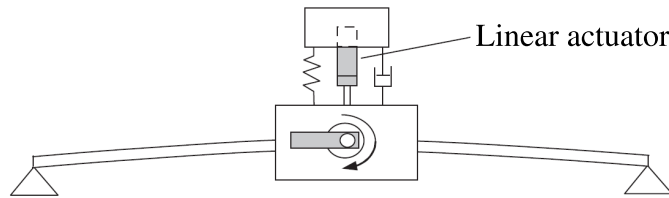


Figure 1.12: Hybrid dynamic vibration absorber. From Wu et al. (2007).

In their work, Hill and Snyder (2002) introduced a dual mass absorber for multiple frequencies with enlarged bandwidth. This innovative absorber configuration comprises two rods, one smooth and the other threaded, supporting equal masses on either side of a central section connected to the target structure. By employing a stepper motor, the threaded rods can be rotated to adjust the positions of the masses, thereby altering the absorber resonant frequencies. To achieve self-tuning capabilities, a low cost control system is utilized to dynamically adjust the mass positions, optimizing performance. The conclusions highlight the efficacy of this approach in attenuating vibrations across various frequencies, particularly on highly flexible structures. However, it also notes the limitation of this method on stiffer structures, where higher-order absorber modes are not sufficiently excited, thus affecting the effectiveness of vibration attenuation.

In their study, Carneal et al. (2004) explored the application of an ATVA to mitigate sound radiation from a simply supported plate. They investigated two different algorithms to assess effectiveness. The first algorithm involved 'tuning' the ATVA to match the excitation frequency of the plate, while the other utilized an acoustic cost function to optimize the resonant frequency of the ATVA. Results indicated that employing the acoustic cost function for resonant frequency optimization proved more effective in reducing the radiated sound field. Conversely, 'tuning' the ATVA to the excitation frequency only led to a reduction in vibration levels near the base of

the ATVA, and in some cases, it even increased sound levels.

In their work, Acar and Yilmaz (2013) introduced an adaptive absorber comprising a tension-adjustable string mechanism. By applying displacements to the end of the string, a broad spectrum of natural frequency variations can be obtained. This system incorporates a negative stiffness mechanism to facilitate string tension adjustment and a tuning controller algorithm to determine the string tension based on the main system amplitude and frequency. Through analytical, numerical, and experimental analyses, it was demonstrated that precise tuning of this system can be achieved with minimal displacement inputs. Moreover, the negative stiffness mechanism significantly reduces the force output requirement on the actuator, almost to zero. Overall, results indicated that the absorber effectively dampened vibrations across the main structure operational frequency range.

Bian and Gao (2017) employed a virtual semi-active vibration absorption technique to mitigate the vibrations of a nonlinear flexible arm. The absorber, realized through a servomotor, allows for adjustment of stiffness and damping coefficients using straightforward control algorithms, thereby emulating the behaviour of a physical vibration absorber. The spring tuning is achieved through variable position feedback gain, while damping is controlled via variable velocity feedback gain. Both numerical simulations and experimental analysis demonstrated the successful establishment of internal resonance within the flexible arm, leading to effective transfer and dissipation of vibrational energy by the virtual vibration absorber.

Shakib and Ghorbani-Tanha (2020) introduced a device featuring a spindle-shaped mass body, where the mass moment of inertia can be adjusted by moving a pair of rotating supports. This alteration of the natural frequency is achieved with low power consumption. The study demonstrates the device efficacy in mitigating undesirable vibrations during the start-up phase of rotating machinery, all while requiring significantly less physical mass compared to conventional absorbers. Moreover, the spindle adaptive tuned vibration absorber exhibits a rapid adaptation mechanism when compared to other adaptive-mass tuned vibration absorbers.

1.1.5 Passive self-adaptation mechanisms

In their investigation, Yu et al. (2021) explored the potential of utilizing the passive self-adaptation capability of a nonlinear beam-slider structure commonly employed in energy harvesting applications as a vibration neutraliser. The structure comprises a non-linear cantilever beam and a free movable slider shown in Figure 1.13. As the slider traverses along the beam,

it alters the mass distribution of the neutraliser, consequently transforming the frequency response function of the non-linear system. This alteration in mass distribution induces changes in the inertial force, which in turn influences the vibration response of the beam. The vibration of the beam and the movement of the slider form a closed-loop mechanism for passive self-adaptation. The study found that within the frequency range of 27 Hz to 31 Hz, the nonlinear neutraliser demonstrated effective vibration suppression with inherent self-adaptation capabilities. However, outside this frequency range, the vibration suppression was less significant, and the self-adaptation capacity of the system was not achieved.

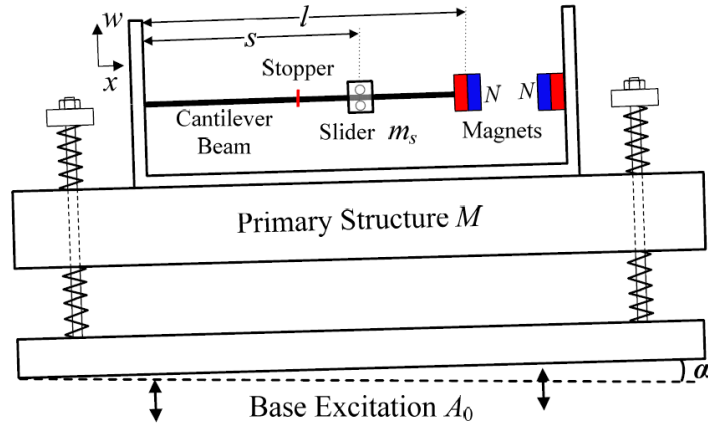


Figure 1.13: Schematic of the vibration system with a nonlinear beam-slider structure as a neutraliser. From Yu et al. (2021).

1.1.6 Centrifugal Pendulum Vibration Absorber

In 1935, Taylor (1936) introduced the Centrifugal Pendulum Vibration Absorber (CPVA), a passive self-tuning device aimed at mitigating torsional vibrations in geared radial aircraft-engine-propeller systems, typically resulting from torque fluctuations in rotating machinery. The CPVA's natural frequency varies directly with rotational speed, rendering it effective for reducing unwanted vibrations across a broad range of rotational speeds. Balachandran and Magrab (2018) derived the governing non-linear equations of the system. This absorber configuration consists of pendula suspended above the rotor, permitting oscillation in a plane perpendicular to the axis of rotation. By carefully selecting the suspension point and effective length of the pendula, the CPVA can be precisely tuned to dampen torsional oscillations at specific rotational speeds. Notably, CPVAs exhibit softening non-linear behaviour with increasing frequency, requiring a deliberate introduction of slight mistuning to ensure stable and reliable operation. Multiple CPVAs can be employed to mitigate or eliminate various torque harmonics

(Mitchiner and Leonard, 1991). Both CPVA and the object of study in this thesis share their passive nature. CPVAs are specifically designed for torsional vibration control. In contrast, this work focuses on a passive device adept at managing translational vibration.

1.1.7 Remarks of the literature review

In conclusion, the reviewed studies provide valuable insights into various approaches for enhancing the effectiveness of vibration neutralisers and absorbers. While methods such as parallel arrangement and utilization of non-linearities offer opportunities to broaden the bandwidth of devices, they often provide only a single frequency tuning capability over a wide range. Similarly, devices incorporating variable stiffness elements, including those utilizing smart materials such as SMAs or MREs, typically require an external power source to adapt the stiffness, limiting their applicability in certain scenarios. Moreover, active and semi-active control methods, while offering dynamic adjustment capabilities, necessitate the implementation of control algorithms, sensors, actuators, and external power sources, adding complexity and resource requirements to the system. To fill the existing gap in the literature the vibration neutraliser proposed here counts with an expanded frequency range and two different tuning frequencies, with passive adjustment depending simply on the harmonic forcing frequency.

1.2 Objectives and contributions

The main objective of this thesis is to develop and characterize a vibration neutraliser capable of passively adapting to a range of excitation frequencies, depending on the design of the neutraliser, thereby increasing its working bandwidth. This neutraliser operates autonomously without the need for control or an external power source, aiming to provide an effective solution for attenuating vibrations across a broader frequency range.

The specific objectives can be described as:

- 1** To identify and characterize the dynamic parameters of the vibration neutraliser, including mass, stiffness, damping, and natural frequencies.
- 2** To propose and develop a mathematical model capable of accurately describing the dynamic behaviour of the vibration neutraliser under harmonic excitation.

- 3 To explore and provide insights into the phenomenon of self-rotation exhibited by the vibration neutraliser, identifying the key mechanisms driving self-rotation.
- 4 To analyse and determine if specific geometric parameters of the vibration neutraliser, such as dimensions, shape, or configuration, significantly influence its functionality and performance across different excitation frequencies.

This work has contributed to the existing literature in each of these areas. Portions of this research have also been described in Nehemy et al. (2023, 2024).

1.3 Summary of the thesis

The thesis comprises seven chapters that progressively build upon each other to achieve the purpose of developing and characterising a vibration neutraliser with enhanced operational bandwidth.

Chapter 2 lays the theoretical basis by discussing the essential concepts in vibration attenuation and the primary models used to analyse passive neutralisers. These insights are important for understanding the mathematical and experimental models developed in the later stages of the work.

Chapter 3 develops a frequency-domain model describing the system's response when fixed at a given angular position. The model assumes a steady-state orientation and employs the neutraliser's geometric and mechanical parameters to derive analytical solutions. This formulation is supported by experimental modal testing, which provides estimates of stiffness, damping, and mass that validate the model's predictive capability.

These parameters are then incorporated into the time-domain model presented in Chapter 4. In this formulation, the rotation angle is free, enabling the simulation of the neutraliser's reorientation under harmonic excitation. The model captures both transient and steady-state behaviours and is validated through comparisons with experimental results across multiple prototypes.

Chapter 5 extends the analysis of the time-domain model to explore the self-rotation behaviour in depth. Using nonlinear analysis, linearisation procedures, and the Method of Direct Separation of Motions (MDSM), it is demonstrated that the neutraliser's reorientation arises from mechanisms consistent with auto-parametric resonance.

Chapter 6 expands the investigation to the influence of geometric design parameters by examining the effects of stiffness and mass ratios on the device's performance. Numerical and experimental studies are used to show how these ratios affect the system's ability to self-rotate. These results are important for deriving design guidelines and understanding the sensitivity of the system to parameter variations.

The work concludes with a summary of the findings and outlines directions for future research in Chapter 7.

2 Fundamental principles and dynamic analysis of simplified vibration neutralisers

This chapter offers a elementary exploration of vibration neutralisers, establishing a basis for the more advanced analyses presented later in this thesis. It begins by deriving the equations of motion for an undamped single-degree-of-freedom (SDOF) neutraliser. The discussion then progresses to introduce the mathematical framework for an undamped two-degree-of-freedom (2DOF) neutraliser. This structured progression supports a comprehensive investigation of vibration neutralisers, with the 2DOF configuration providing additional physical insight into the system's behaviour. Within the context of advanced engineering research, the chapter serves as a core platform for uncovering and scrutinizing the complexities of vibration neutralisers.

Central to this exploration is the Frequency Response Function (FRF), a widely employed metric in vibration engineering (Ewins, 2009). The FRF describes how a system responds in steady state at each excitation frequency. When examining the fundamental dynamics of a neutraliser, particularly under steady-state conditions, representing the equations of motion in the frequency domain proves advantageous. Three related functions, receptance (displacement-to-force), impedance (velocity-to-force), and accelerance (acceleration-to-force), are interconnected by frequency-domain integration and differentiation, offering a unified perspective on system behaviour.

In light of the above, this thesis emphasizes accelerance to maintain consistency with experimental practices, where data are primarily obtained in accelerance form. Thus, receptance expressions are converted into accelerance, ensuring that the analysis remains both practically relevant and aligned with standard measurement approaches.

2.1 Single degree of freedom neutraliser

A vibration neutraliser is a small device attached to a mechanical system or structure, Figure 2.1a. It is designed to reduce or control vibrations at a specific troublesome frequency. To

introduce the neutraliser's dynamics, consider the schematic in Figure 2.1b. This model consists of a mass element m_N (neutraliser mass - in kg) and a linear, massless spring with stiffness k_N (neutraliser stiffness - in N/m).

A harmonic force is applied at the end of the spring, represented as

$$f(t) = F e^{j\omega t},$$

where F is the amplitude of the excitation and ω is the angular frequency. The system is undamped, with an angular natural frequency

$$\omega_N = \sqrt{\frac{k_N}{m_N}},$$

expressed in rad/s. Under harmonic excitation, the steady-state response is also harmonic, which can be written as

$$x(t) = X e^{j\omega t}.$$

Newton's second law in the frequency domain leads to the following relations:

$$\begin{aligned} F &= k_N (X_n - X_2) \\ -\omega^2 m_N X_2 &= k_N (X_n - X_2) . \end{aligned} \quad (2.1)$$

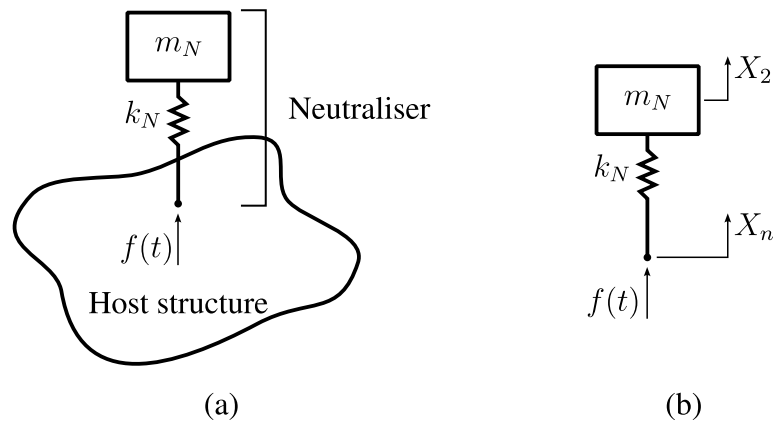


Figure 2.1: Schematic representation of (a) a simplified host structure with a vibration neutraliser attached and (b) the dynamic model of the undamped vibration neutraliser.

The receptance at the point of force application, $\frac{X_n}{F}$, is obtained by eliminating X_2 in

Equations (2.1):

$$\frac{X_n}{F} = \frac{-\omega^2 m_N + k_N}{-\omega^2 m_N k_N}. \quad (2.2)$$

Multiplying by $-\omega^2$ provides the accelerance (acceleration per unit force)

$$\frac{A_n}{F} = \frac{-\omega^2 m_N + k_N}{m_N k_N} \quad (2.3)$$

The natural frequency is the root of the numerator

$$\omega_N = \sqrt{\frac{k_N}{m_N}} \quad (2.4)$$

Figure 2.2a shows the modulus of the accelerance, as described by Equation (2.3). At the undamped neutraliser's natural frequency ω_N , the attachment point does not vibrate since the numerator of the accelerance expression becomes zero. A horizontal mass line appears before reaching ω_N , denoted by $|1/m_N|$. Beyond ω_N , the behavior shifts to a stiffness-controlled regime, indicated by $|\omega^2/k_N|$. Consequently, at low frequencies the mass term dominates, while at high frequencies the stiffness term becomes more dominant.

Introducing the frequency ratio $\Omega_n = \omega/\omega_N$ enables the definition of a dimensionless accelerance:

$$\Gamma^* = 1 - \Omega_n^2 \quad (2.5)$$

The natural frequency occurs at $\Omega_n = 1$. Figure 2.2b illustrates Γ^* . At low frequencies, the system is governed by the neutraliser's mass, so $\Gamma^* = 1$. Once the frequency surpasses ω_N , the expression $|\Omega_n^2|$ becomes dominant, indicating that stiffness begins to control the response. When the neutraliser is undamped and tuned precisely to the host structure's troublesome forcing frequency, a node forms at the attachment point, thereby cancelling vibrations (Reed, 2002). The neutraliser mass oscillates in opposition to the input force, generating a counter-acting force. As a result, the applied force on the host structure is effectively nullified at the attachment point, which remains motionless. In mathematical terms, if $\omega_N = \omega$, the numerator of Equation (2.2) vanishes, leading to $X_n = 0$.

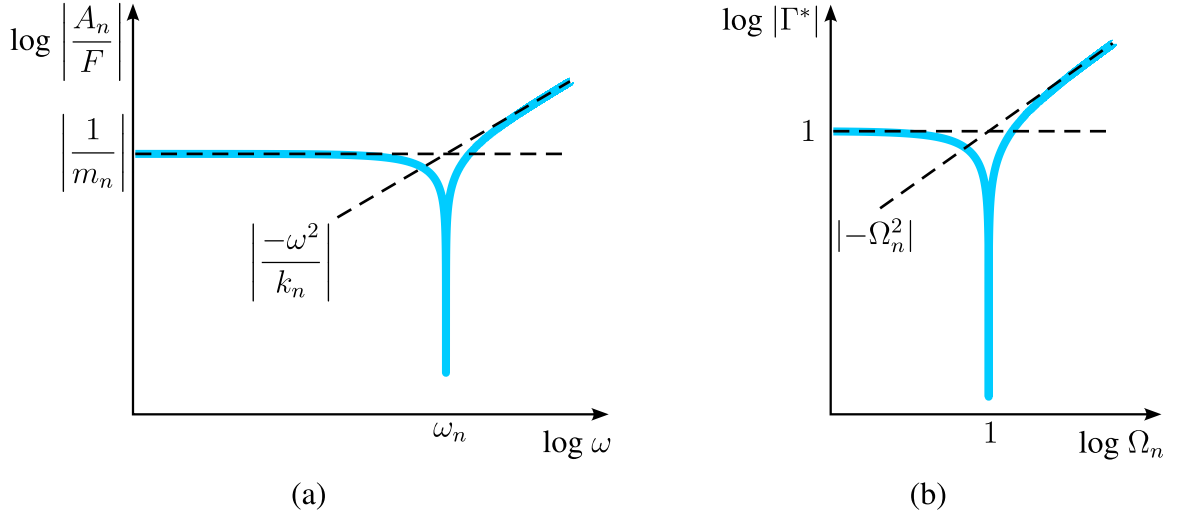


Figure 2.2: Modulus of the accelerance excited by the spring of the undamped vibration neutraliser. (a) Original expression and its (b) dimensionless form.

2.2 Two degrees of freedom neutraliser

Attaching a neutraliser to a host structure without introducing additional mass is inherently challenging. For this reason, the neutraliser proposed in this thesis incorporates both a base mass and a tip mass. The base facilitates attachment to the host structure, as shown in Figure 2.3a. A simplified model, depicted in Figure 2.3b, is introduced here to highlight key features. In this

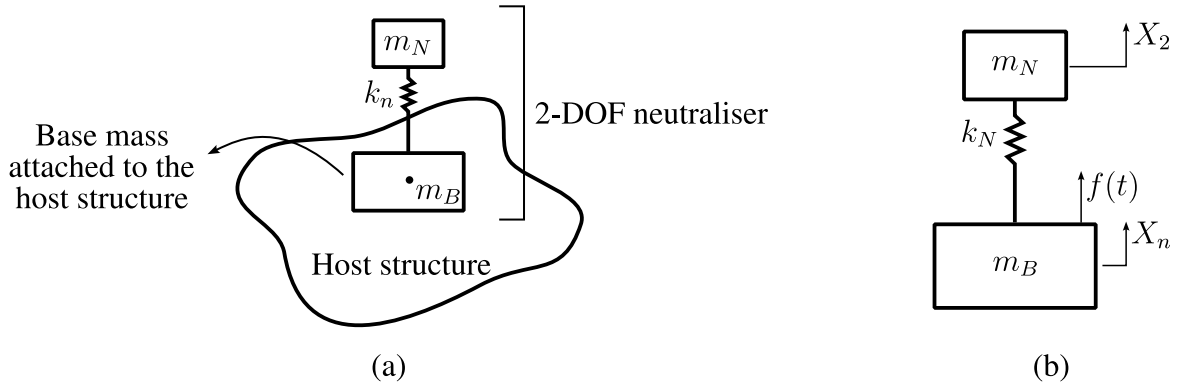


Figure 2.3: Neutraliser with 2DOF, system similar to the object of study, with a base and a tip mass.

system, a harmonic excitation is applied to the base mass m_B . As with the SDOF neutraliser, the steady-state response is also harmonic, with the displacement

$$x(t) = X e^{j\omega t}.$$

Newton's second law is used to derive the equations of motion in the frequency domain, accounting for interactions between the base mass and the neutraliser mass:

$$\begin{aligned} F &= k_N (X_2 - X_n) \\ -\omega^2 m_N X_2 &= k_N (X_n - X_2) \\ -\omega^2 m_B X_n &= k_N (X_2 - X_n) + F. \end{aligned} \quad (2.6)$$

Eliminating X_2 and substituting back into the equation for the base mass leads to

$$-\omega^2 m_B X_n = k_N \left(\frac{k_N X_n}{k_N - \omega^2 m_N} - X_n \right) + F. \quad (2.7)$$

which simplifies to the receptance

$$\frac{X_n}{F} = \frac{1}{\omega^2 m_B + \frac{k_N \omega^2 m_N}{k_N - \omega^2 m_N}} \quad (2.8)$$

The accelerance, defined as acceleration per unit force, follows from multiplying Equation (2.8) by $-\omega^2$:

$$\frac{A_n}{F} = \frac{k_N - \omega^2 m_N}{m_N k_N - \omega^2 m_B m_N + m_B k_N}. \quad (2.9)$$

The neutraliser's natural frequency is given by

$$\omega_N = \sqrt{\frac{k_N}{m_N}}. \quad (2.10)$$

A mass ratio, $\mu = \frac{m_B}{m_N}$, and a frequency ratio, $\Omega_n = \frac{\omega}{\omega_N}$, are introduced to obtain a dimensionless form of the accelerance in Equation (2.9). First, the stiffness k_N is replaced by $m_N \omega_N^2$, and m_N is factored out from both the numerator and denominator:

$$\frac{A_n}{F} = \frac{m_N (\omega_N^2 - \omega^2)}{m_N^2 \omega_N^2 + m_B m_N (\omega_N^2 - \omega^2)} \quad (2.11)$$

Substituting the frequency ratio Ω_n yields

$$\frac{A_n}{F} = \frac{m_N \omega_N^2 (1 - \Omega_n^2)}{m_N^2 \omega_N^2 + m_B m_N \omega_N^2 (1 - \Omega_n^2)} \quad (2.12)$$

Inserting the mass ratio $\mu = \frac{m_B}{m_N}$ into this expression:

$$\frac{A_n}{F} = \frac{1 - \Omega_n^2}{m_N \omega_N^2 (1 + \mu (1 - \Omega_n^2))} \quad (2.13)$$

Finally, dividing both numerator and denominator by $m_N \omega_N^2$ results in the dimensionless accelerance

$$\Gamma^* = \frac{1 - \Omega_n^2}{1 + \mu (1 - \Omega_n^2)} \quad (2.14)$$

Figure 2.4a presents the accelerance of the two-degree-of-freedom neutraliser with base and tip mass. The line preceding the anti-resonance is given by $|1/(m_N + m_B)|$. At low frequencies, the accelerance is controlled by both masses, m_B and m_N . Beyond the resonance, the asymptote becomes $|1/m_B|$, indicating that only the base mass governs the accelerance at high frequency. The anti-resonance ω_{AR} , is the root of the numerator in Equation (2.9), whereas the peak

$$\omega_{AR} = \sqrt{\frac{k_N(m_N + m_B)}{m_N m_B}} \quad (2.15)$$

corresponds to the root of its denominator.

In the dimensionless expression, illustrated in Figure 2.4b, the low and high frequency asymptotes are $|1/\mu|$ and $|1/(\mu + 1)|$, respectively. The valley and peak of the dimensionless accelerance occur at 1 and

$$\omega_{AR} = \sqrt{\frac{1 + \mu}{\mu}}, \quad (2.16)$$

obtained from the numerator and denominator of Equation (2.14).

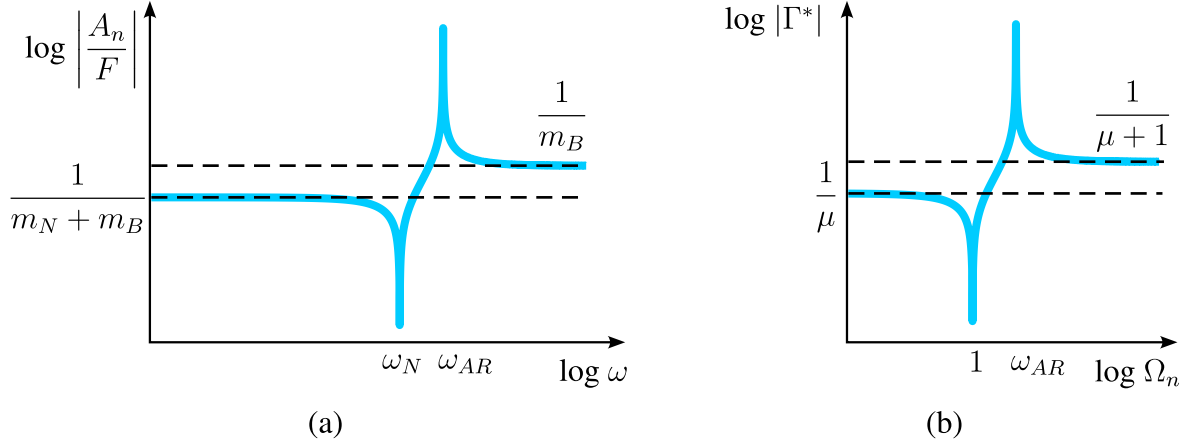


Figure 2.4: Modulus of the accelerance at the base X_n of the undamped 2 DOF vibration neutraliser. (a) Original expression and its (b) dimensionless form.

2.3 Chapter summary

This chapter establishes the foundation for the subsequent analysis of vibration neutralisers in this thesis. It begins with the derivation of the equations of motion, examining dynamic aspects of undamped SDOF and 2DOF neutralisers. The idea of a damping-free can create a node at its attachment point to counteract vibrations. A 2DOF neutraliser design with a base and tip mass is proposed to simplify attachment while minimizing additional mass on the host structure. These main concepts support the more advanced models and analyses presented in later chapters.

A brief mention was made regarding the tuning of neutralisers, though this topic will not be developed further in the thesis. Readers seeking a more detailed treatment of neutraliser tuning may consult works such as Den Hartog (1985), Mead (1998), and Jang et al. (2012).

3 Frequency domain modelling and modal characterisation

While Chapter 2 establishes the fundamental principles and dynamic behaviour of simplified vibration neutraliser models, particularly emphasizing a 2DOF system, this chapter provides a detailed presentation of the actual neutralisers developed and tested in the course of this work. Building upon the conceptual framework introduced earlier, a new model is proposed to represent the behaviour of these neutralisers.

The primary focus of this chapter is the analysis of the system's steady-state response under harmonic excitation, assuming the angular position of the beam remains fixed. This condition enables the formulation of the model in the frequency domain, expressed through FRFs.

3.1 Design and geometric features of the neutralisers

The vibration neutraliser, object of study of the thesis, is depicted in Figure 3.1a. This is a beam-type neutraliser, where a beam is the primary stiffness element. To achieve two different orthogonal stiffness, the geometry of the beam's cross section plays a critical role. A rectangular cross section naturally provides two different flexural stiffnesses. Two bearings are fixed to the base of the neutraliser through the outer ring and the beam is attached to the inner ring of the bearings, which allow the beam to rotate, alternating the direction of the cross section. The neutraliser mass, m_N , comprises the two tip masses, the inner part of the bearing and part of the beam mass. The base mass, m_B , includes the base, the outer part of bearing and part of the beam mass. The global (Y_B , X_B , Y_N and X_N) and local (yy , xx and zz) coordinates are indicated in Figure 3.1b and the neutraliser is designed to operate in the Y_B direction. The beam can rotate around the zz -axis, constituting the angle of rotation θ .

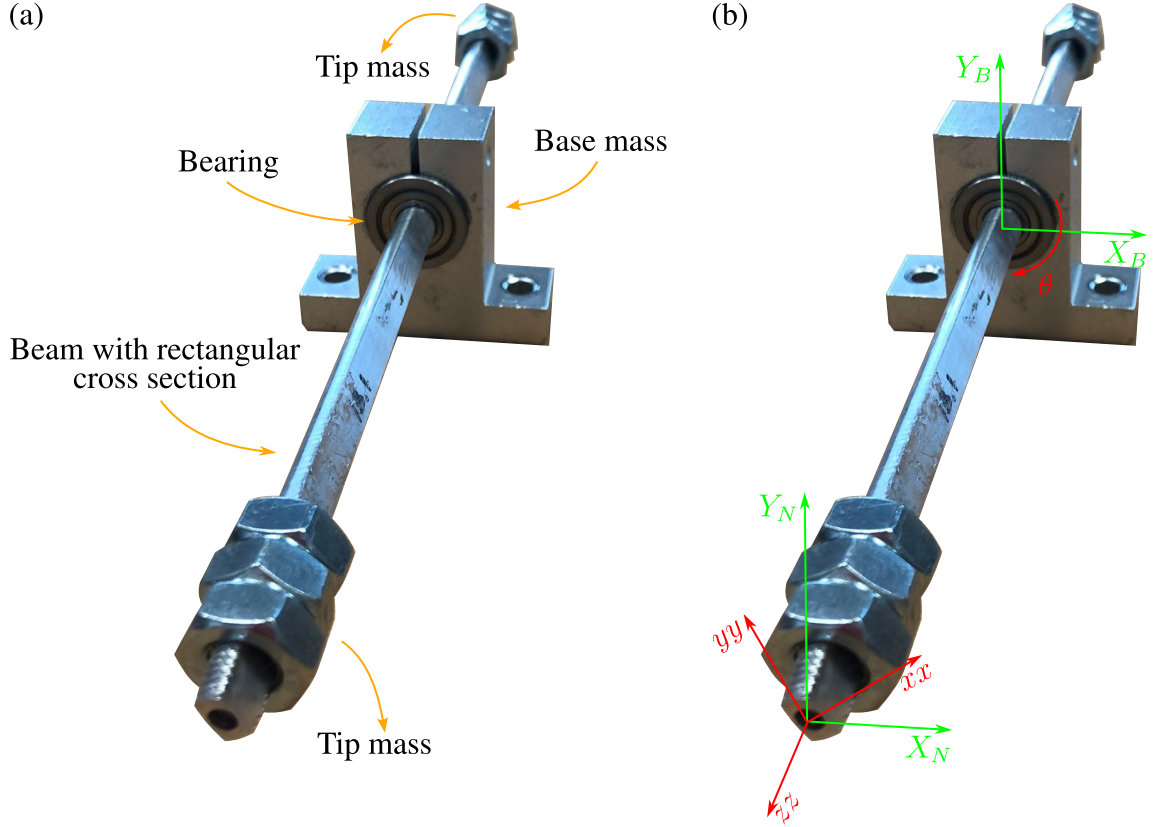


Figure 3.1: Vibration neutraliser with varying stiffness, the object of study in this thesis.(a) Key components of the device, including the beam with rectangular cross section, tip masses, bearings, and base mass. (b) Reference frames: global coordinate system (Y_B , X_B , Y_N and X_N), local coordinate system (yy , xx and zz). The rotation angle θ defines the orientation of the beam relative to the base.

Three different neutralisers were manufactured during the development of this work. The TN1, Figure 3.2a, the TN2, Figure 3.2b and the TC1, Figure 3.2c. They share some identical parts, as the base and the two roller bearings F699-ZZ (inner and outer diameters sizes of 9 and 20mm). The differences are the material and dimensions of the beam, as well as the tip masses weight. The cross section of each neutraliser is highlighted in the figure. From TN1 to TN2 and TC1 the fixing of the beam is improved. The cross sections of TN2 and TC1 are not truly rectangular. The beams were manufactured from a circular bar, machined at the ends to achieve an almost rectangular shape. In the middle, where the beam is attached to the bearing, the circular section was retained to enhance the fixation. This detail is showed in Figure 3.2b and 3.2c, and is described in the Table 3.1 as machined. The neutraliser TN1 comprises a steel beam with a total length of 193 mm and dimensions of 2.2 by 9 mm. The TN2 features an aluminium beam with a total length of 283 mm and dimensions of 4.5 by 9 mm. While TC1 consists of an aluminium beam with a total length of 323 mm and dimensions of 7 by 9 mm.

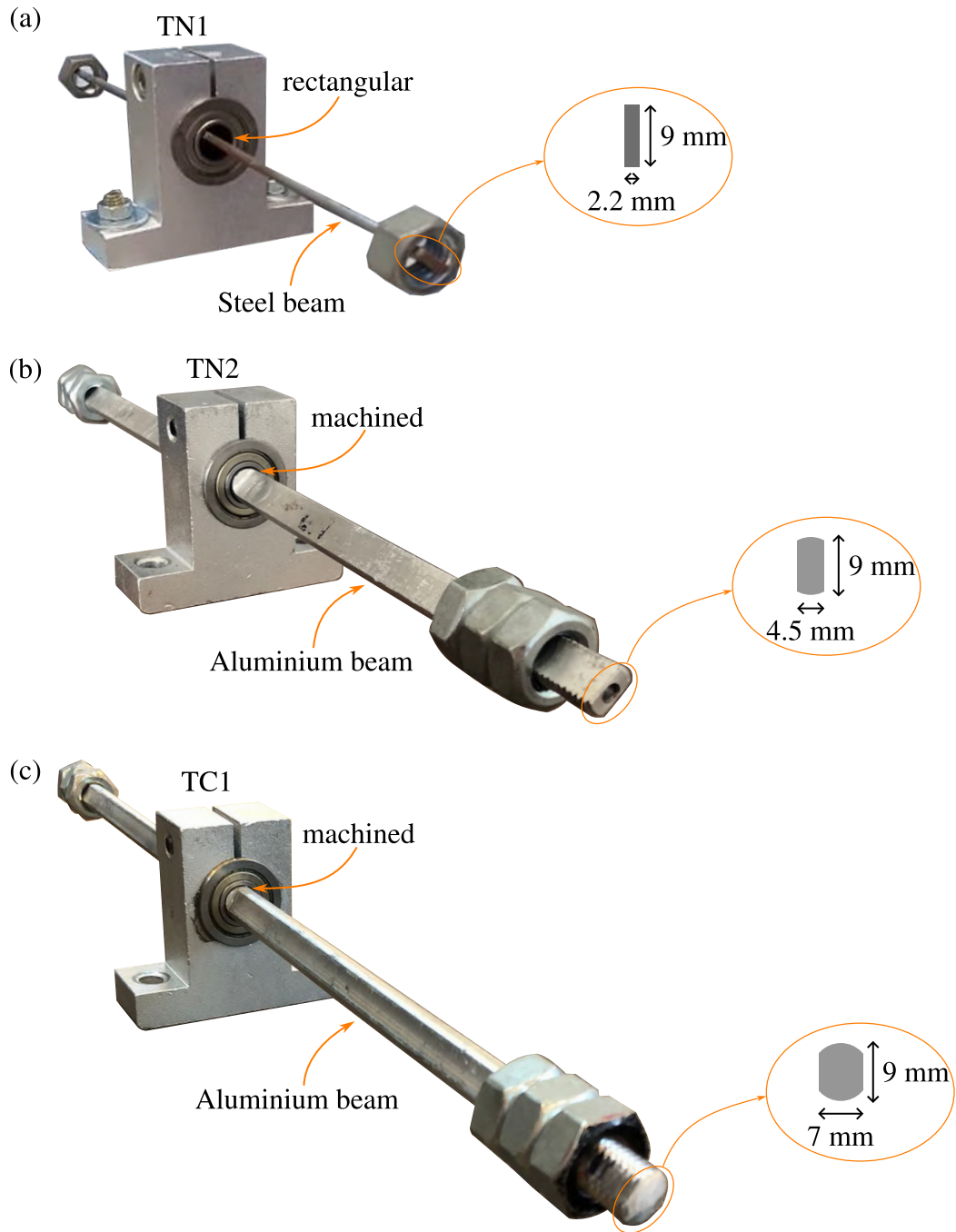


Figure 3.2: Photographs of the three vibration neutralisers. (a) TN1, with a traditional rectangular cross section. (b) TN2 and (c) TC1, both equipped with machined aluminium beams. The highlighted cross sections illustrate the geometrical differences, with TN2 and TC1 produced from circular bars and machined to achieve an approximately rectangular shape at the ends.

The key characteristics of the neutralisers are summarized in Table 3.1.

Table 3.1: Key characteristics of the neutralisers.

NEUTRALISER	Beam length	Height	Width	Material	Cross section
TN1	193 mm	9 mm	2.2 mm	steel	rectangular

TN2	283 mm	9 mm	4.5 mm	aluminium	machined
TC1	323 mm	9 mm	7 mm	aluminium	machined

The development of the vibration neutralisers and the corresponding theoretical models occurred in successive stages as previously explained in Section 3.1, with each iteration contributing to a deeper understanding of the system and improved experimental-model correlation.

The first neutraliser to be designed, manufactured, and tested was the TN1, Fig. 3.2(a). This device served as the basis for the initial modelling efforts. Two models were developed from TN1: a 4DOF frequency domain model and a 3DOF time domain model. Experimental tests, including modal and harmonic excitation, were conducted on TN1 to determine its dynamic parameters and validate the proposed models.

From the experimental analysis of TN1, two insights emerged. First, it was noted that the physical characteristics of the neutraliser could be improved. Second, a notable distinction on the method of excitation between the time domain model and the experiment suggested the need for model refinement.

Motivated by these observations, the two additional neutralisers — TN2 Fig. 3.2(b) and TC1 Fig. 3.2(c) — were designed and manufactured with modified geometric features aimed at improving performance. The frequency-domain model structure was retained, but the time-domain model was reformulated.

Both TN2 and TC1 underwent new experimental tests, including modal testing for parameter identification and harmonic excitation to evaluate the transient response. These experimental results informed the adjustment of the updated models, enabling more accurate comparisons between simulated and measured behaviour.

To maintain clarity and avoid confusion for the reader, all information related specifically to the first neutraliser (TN1) is presented separately in Appendix B.

TN2 is adopted as the reference neutraliser in the remainder of this thesis. It is used in all subsequent stages involving both frequency and time domain modelling, as well as in the comparison with experimental results.

Although TC1 shares similarities with TN2, its unique geometric characteristics merit separate discussion. Therefore, a dedicated discussion is later presented in Chapter 6 to address specific aspects of the TC1 model and its experimental validation.

3.2 Development of the frequency domain model

The proposed neutraliser model, presented in Figure 3.3a, is designed with two orthogonal stiffness, one acting in the vertical direction (k_{yy}) and the other in the horizontal direction (k_{xx}).

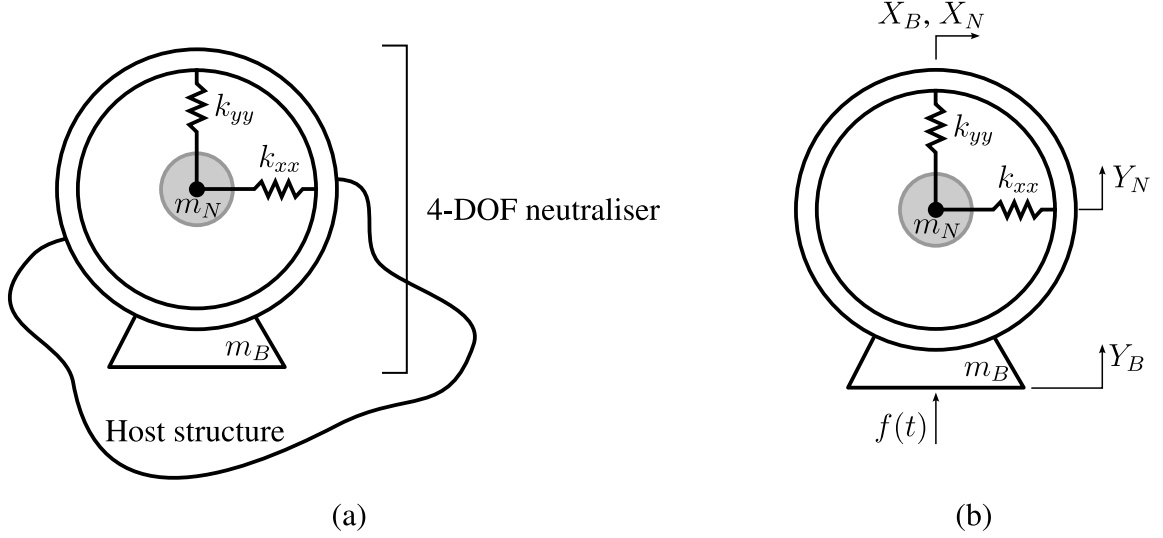


Figure 3.3: 4-DOF neutraliser (a) attached to a host structure and (b) the dynamic model.

This model shares similarities with the 2DOF neutraliser from Chapter 2, as the base mass that facilitates attachment to a host structure and the neutraliser mass m_N . A notable distinction lies in the incorporation of two additional degrees of freedom introduced by the horizontal stiffness k_{xx} . The displacements vector $\mathbf{q} = \{X_N, Y_N, X_B, Y_B\}^T$, where the subscript $()^T$ denotes the transpose of the vector, includes the new horizontal coordinates X_N and X_B . The mass matrix, $\mathbf{M}$, is defined by Equation (3.1), and the stiffness matrix, $\mathbf{K}$, by Equation 3.2.

$$\mathbf{M} = \begin{bmatrix} m_N & 0 & 0 & 0 \\ 0 & m_N & 0 & 0 \\ 0 & 0 & m_B & 0 \\ 0 & 0 & 0 & m_B \end{bmatrix}, \quad (3.1)$$

$$\mathbf{K} = \begin{bmatrix} k_{xx} & 0 & -k_{xx} & 0 \\ 0 & k_{yy} & 0 & -k_{yy} \\ -k_{xx} & 0 & k_{xx} & 0 \\ 0 & -k_{yy} & 0 & k_{yy} \end{bmatrix}. \quad (3.2)$$

The dynamic stiffness matrix is defined as

$$\mathbf{D}(\omega) = -\omega^2 \mathbf{M} + \mathbf{K}, \quad (3.3)$$

and a lumped parameter frequency domain model is developed, considering Figure 3.3b, as

$$\mathbf{D}\mathbf{q} = \mathbf{f} \quad (3.4)$$

the applied force $f(t) = F e^{j\omega t}$, where F represents the amplitude of the excitation force and ω denotes the angular frequency, is directed vertically at the base. The force vector is defined as $\mathbf{f} = \{0, 0, 0, F\}^T$.

Introducing the 4DOF varying stiffness neutraliser model, presented in Figure 3.4a, the unique feature of this design lies in the ability to rotate the stiffness components. Its angular position is given by the rotation angle θ showed in Figure 3.4b. The rotation is given by a local coordinate system, where the axes are defined by yy and xx , as depicted in the same figure. As the neutraliser mass and orthogonal stiffness rotate, the stiffness experienced at the base of the system in each direction undergoes changes. The resulting global stiffness depends on the specific orientation angle, θ , and the values assigned to each individual stiffness element, k_{xx} and k_{yy} .

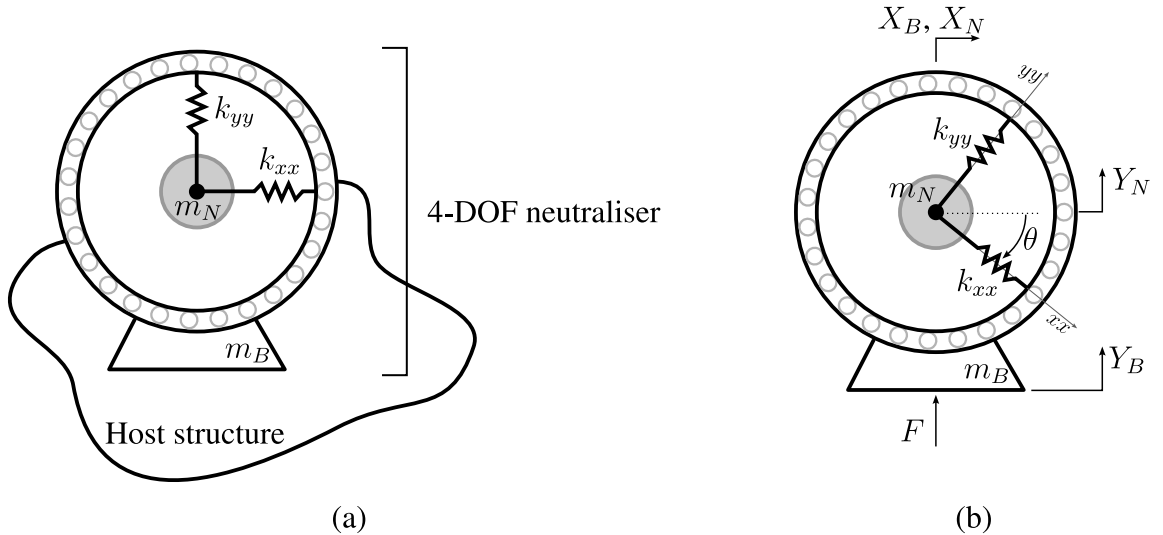


Figure 3.4: 4DOF neutraliser (a) attached to a host structure and (b) the rotated dynamic model.

To properly account for these directional variations, it is necessary to consider the relationship between local and global coordinate systems, a crucial aspect in the modelling of rotating mechanical systems. Initially, the stiffness matrix, Equation (3.2) is formulated assuming no ro-

tation, i.e., $\theta = 0^\circ$. However, as the neutraliser rotates,, the elastic forces must be re-expressed in the global reference frame of the system. This transformation is required because the orientation of the stiffness axes, governed by θ , alters the direction in which the restoring forces act. Therefore, the elastic forces originally described in the local coordinate system (yy and xx axes) must be transformed into the global coordinates of the structure, Y_B , X_B , Y_N and X_N , to accurately describe the system's response.

A force acting in the yy direction of the local coordinate system undergoes decomposition into two forces: one in the global Y direction and another in the global X direction. Similarly, a force in the xx direction of the local coordinate system is decomposed into two forces, with one contributing to the global X direction and the other to the global Y direction. This process is visually represented in Figure 3.5 and aligns with the matrix formulation in Equation (3.5).

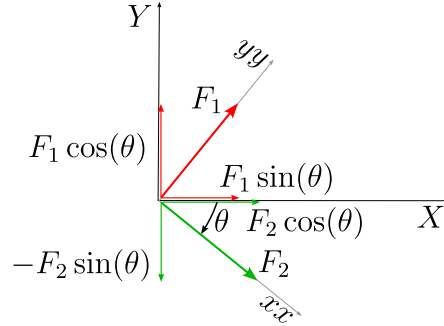


Figure 3.5: Forces decomposition from local to global coordinates with clock wise orientation.

Thus, the rotation of stiffness is achieved via a local to global coordinate transformation matrix $\mathbf{T}$, Equation (3.6), applied to the stiffness matrix $\mathbf{K}$, establishing the relationship between local and global coordinates. The dynamic stiffness matrix of the undamped system is given by Equation (3.7), and the detailed definition of its individual components may be found in Appendix A.

$$\begin{Bmatrix} xx \\ yy \end{Bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} \quad (3.5)$$

$$\mathbf{T} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & \cos(\theta) & -\sin(\theta) \\ 0 & 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \quad (3.6)$$

$$\mathbf{D}(\omega) = \mathbf{T}^T (-\omega^2 \mathbf{M} + \mathbf{K}) \mathbf{T} \quad (3.7)$$

Damping is incorporated into the model through the viscous damping matrix $\mathbf{C}$, as shown in Equation (3.8). This matrix includes the damping coefficients c_{xx} and c_{yy} which act in the horizontal and vertical directions, respectively, parallel to the corresponding stiffness components k_{xx} and k_{yy} .

$$\mathbf{C} = \begin{bmatrix} c_{xx} & 0 & -c_{xx} & 0 \\ 0 & c_{yy} & 0 & -c_{yy} \\ -c_{xx} & 0 & c_{xx} & 0 \\ 0 & -c_{yy} & 0 & c_{yy} \end{bmatrix} \quad (3.8)$$

With the inclusion of damping, the dynamic stiffness matrix of the system is modified accordingly, resulting in the formulation presented in Equation (3.9). The expressions for the individual terms of the damped dynamic stiffness matrix are detailed in Appendix A.

$$\mathbf{D}_C(\omega) = \mathbf{T}^T (-\omega^2 \mathbf{M} + j\omega \mathbf{C} + \mathbf{K}) \mathbf{T} \quad (3.9)$$

The complex amplitudes are calculated as $\mathbf{q} = (\mathbf{D}_C)^{-1} \mathbf{f}$, where $\mathbf{f}$ is the force vector defined as $\mathbf{f} = \{0, 0, 0, F\}^T$, and then transformed into accelerances with frequency domain differentiation, multiplying by $-\omega^2$.

In the steady state analysis, the angle θ is fixed. is considered fixed throughout the response. Three specific angular positions are of particular interest. When $\theta = 0^\circ$, Figure 3.6a, the global vertical stiffness is k_{yy} and the global horizontal stiffness is k_{xx} . At $\theta = 45^\circ$, Figure 3.6b, both global stiffness components are a combination of k_{xx} and k_{yy} due to the diagonal orientation of the beam. Finally, for $\theta = 90^\circ$, Figure 3.6c, the vertical stiffness becomes k_{xx} , and the horizontal stiffness becomes k_{yy} .

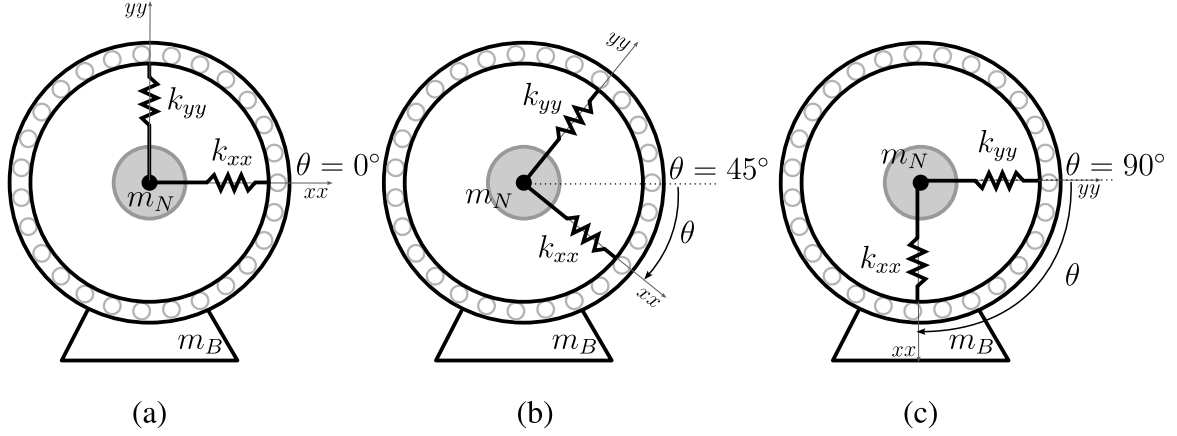


Figure 3.6: Different angular positions (a)for $\theta = 0^\circ$, (b)for $\theta = 45^\circ$ and (c)for $\theta = 90^\circ$.

When θ assumes any angle other than 0° and 90° , along with their multiples, the base acceleration displays two resonances and two anti resonances. In contrast, when θ equals 0° and 90° , and their multiples, the equations governing the vertical and horizontal directions become uncoupled. Consequently, the base resonance exhibits a singular resonance and a singular anti resonance. The dynamic stiffness matrices of the undamped and damped systems, Equations (3.7) and (3.9) respectively, can be simplified when θ equals 0° or 90° and these simplifications are presented in Appendix A.

The resonance frequencies of the base are analytically derived from the undamped model using Equation (3.7). By inverting the dynamic stiffness matrix and finding the roots of the denominator for each mode. The resonance frequencies, denoted as f_{r1} and f_{r2} , are defined by Equations (3.10) and (3.11), respectively, and remain independent of the angular position of the device.

$$f_{r1} = \frac{1}{2\pi} \left[\frac{k_{yy}(m_N + m_B)}{m_N m_B} \right]^{1/2} \quad (3.10)$$

$$f_{r2} = \frac{1}{2\pi} \left[\frac{k_{xx}(m_N + m_B)}{m_N m_B} \right]^{1/2} \quad (3.11)$$

Using the undamped model presented in Equation (3.7), the anti-resonance frequencies at the base can be determined by inverting the dynamic stiffness matrix and identifying the roots of the numerator associated with each mode. Unlike the resonance frequencies these expressions are more complex, as they explicitly depend on the angular position θ of the device.

In particular, for $\theta = 0^\circ$ and $\theta = 90^\circ$, the anti-resonances simplify to the expressions in

Equations (3.12) and (3.13), respectively. These cases represent the natural frequencies of the neutraliser's mass in each principal direction, where the stiffness components are decoupled. For other angular positions, the anti-resonance frequencies f_3 and f_4 vary with θ , reflecting the angle-dependent coupling of the orthogonal stiffnesses. A detailed investigation of the anti-resonance behaviour and its dependence on the neutraliser orientation is presented in Chapter 6.

$$f_3|_{\theta=0^\circ} = \frac{1}{2\pi} \sqrt{\frac{k_{yy}}{m_N}} \quad (3.12)$$

$$f_4|_{\theta=90^\circ} = \frac{1}{2\pi} \sqrt{\frac{k_{xx}}{m_N}} \quad (3.13)$$

3.3 Experimental identification of dynamic parameters

To identify the neutraliser parameters, a modal test was conducted using a Modal Shop mini smartshaker with integrated amplifier type K2007E01, as shown in Figure 3.7, together with a PCB-208C02 force sensor. White noise excitation was employed for its uniform spectral density, ensuring an even distribution of energy across the frequency range of interest and thereby enabling comprehensive excitation of all resonant modes. The base and tip mass accelerations were measured using PCB-352A24 accelerometers (not shown in the figure).

Data acquisition was performed using a Dewesoft SIRIUSi system at a dynamic sampling rate of 5 kHz, providing a measurement bandwidth of up to 1,953 Hz. A frequency resolution of 0.1 Hz was defined by specifying 25,000 spectral lines. Point and transfer accelerances (acceleration/force) were subsequently estimated with the H2 transfer function estimator in DewesoftX, specifically chosen to optimize resonance frequency determination by mitigating the influence of noise on the input measurement. A Hanning window with 50% overlap was applied for the time-to-frequency conversion, and 50 linear averages were taken to enhance the reliability of the measurements.

The PCB-352A24 accelerometers used in this study are general-purpose IEPE-type sensors. Although the technical specifications indicate a nominal operating frequency range starting at 0.5 Hz, their intermediate sensitivity of 10 mV/g, combined with the nature of IEPE electronics, limits the signal-to-noise ratio at low frequencies. This limitation arises because IEPE circuits

have a fixed electronic noise floor, and in low-sensitivity sensors, the electrical output generated under small accelerations often becomes comparable to the system noise, making reliable detection difficult. Additionally, the internal AC coupling acts as a high-pass filter, introducing amplitude and phase distortions in the lower frequency range. For these reasons, all subsequent results are presented from 20 Hz onwards, a frequency range in which the sensor response is stable and consistent with the structural modes of interest in this study.

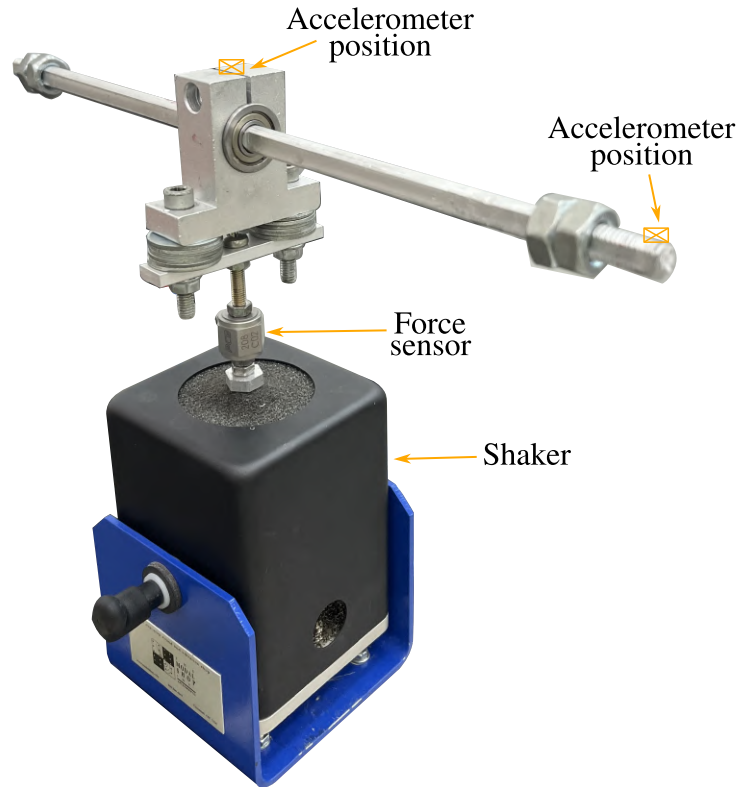


Figure 3.7: Experimental rig used for the modal test. The TN2 neutraliser is mounted on a Modal Shop mini smartshaker, and the excitation force is measured using a PCB force sensor. Accelerometers were positioned at the base and tip masses (locations indicated) to capture the dynamic response, although they are not shown in the image.

Figure 3.8 presents the comparison between the model and the experimental acceleration responses measured at the base and tip masses of the neutraliser. The thin blue lines represent the model predictions, while the thick grey lines correspond to the experimental data. Subfigures (i), (ii), and (iii) correspond to angular positions $\theta = 0^\circ$, $\theta = 45^\circ$ and $\theta = 90^\circ$, respectively. The corresponding coherence functions are shown in Figure 3.9, where values close to 1 indicate a strong correlation between input and output signals, thus confirming the reliability of the measurements across most of the frequency range.

For each angular configuration, the experimental results are compared with a steady-state model of the neutraliser, which assumes a fixed angular position consistent with the experi-

mental setup. Model parameters were estimated qualitatively by matching the low- and high-frequency asymptotes and aligning the resonance frequencies. Viscous damping was then manually adjusted to match the amplitude of the resonance peaks observed in the experimental data.

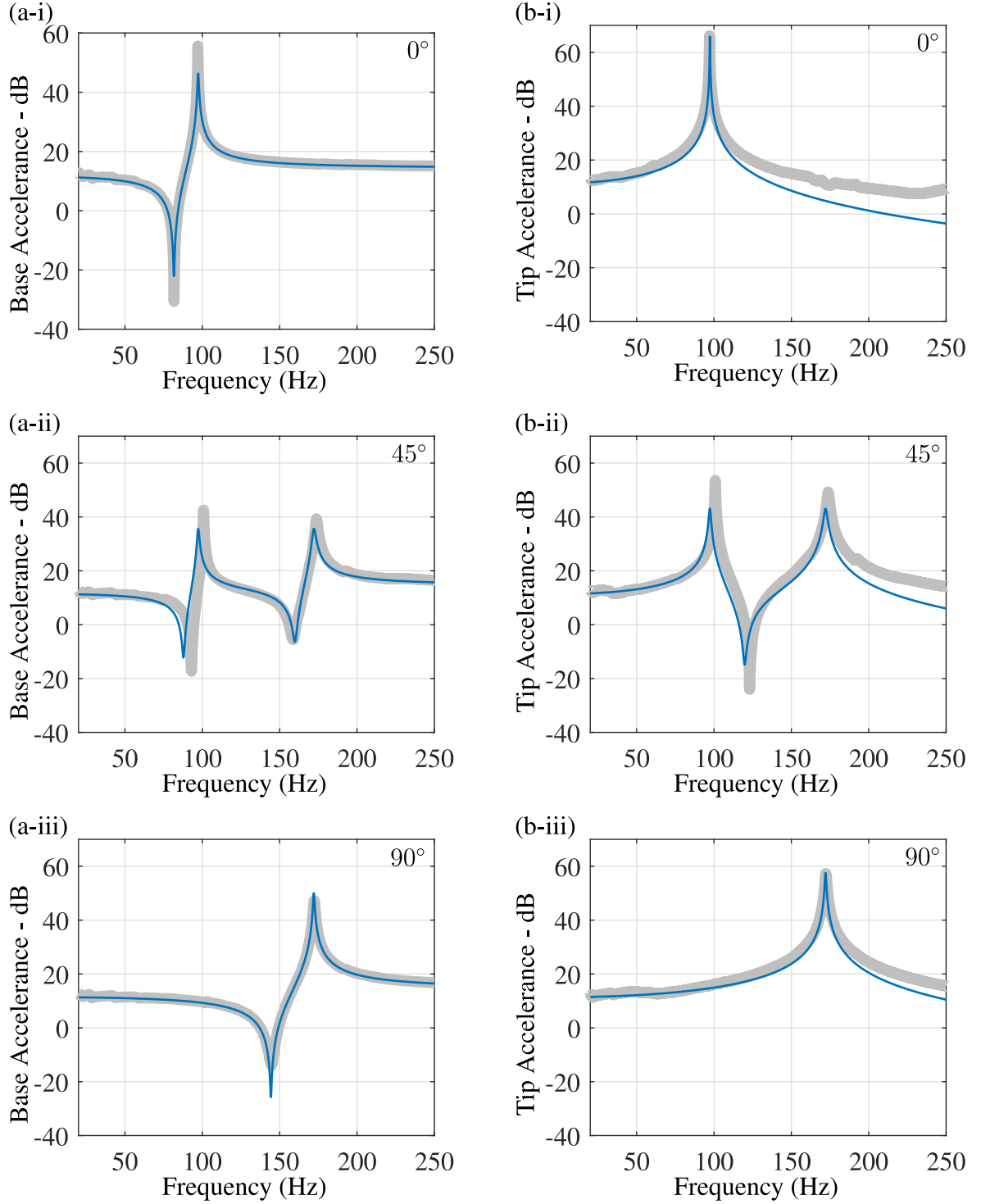


Figure 3.8: Magnitudes of the accelerance responses at three angular positions ($\theta = 0^\circ$, 45° , and 90°) in the global reference system ($X_B Y_B$). Subfigures (a) present the base accelerance, while subfigures (b) show the tip accelerance. Experimental results are represented by the thick grey lines, and model predictions by the thin blue lines. (dB ref. $1 \text{ N s}^2/\text{m}$).

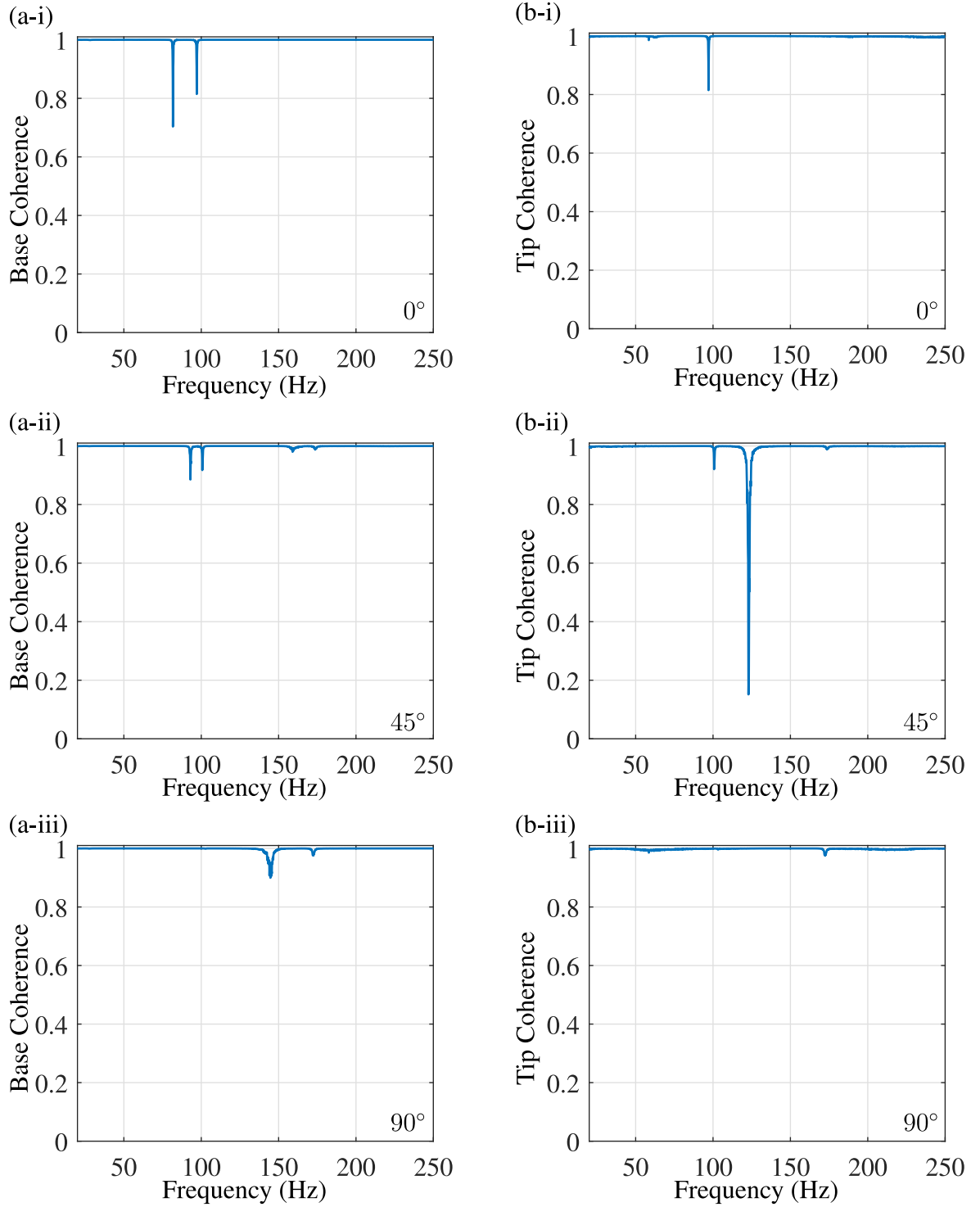


Figure 3.9: Coherence functions obtained from the modal experiments at three angular positions $\theta = 0^\circ$, 45° , and 90°). Subfigures (a) present the base coherence, while subfigures (b) show the tip coherence. Values close to 1 indicate high correlation between input and output signals, confirming measurement reliability across most of the frequency range.

The identified parameters of the model are shown in Table 3.2. Overall, the model shows good agreement with the experimental results, though some discrepancies are observed. One notable divergence occurs in the tip response at 45° , around 120 Hz, as shown in Figure 3.8(b-

ii) diverts from the experimental results. This divergence coincides with a significant drop in coherence, as seen in Figure 3.9(b-ii). The low coherence at this frequency is probably due to the poor mounting quality of the accelerometer, which was attached at the edge of the beam with minimal contact area. Such a weak attachment can lead to signal distortion or local decoupling, particularly at frequencies where the structural response is already low. Notably, the frequency in question corresponds to an anti-resonance, where the system's dynamic response is minimal. Under such conditions, the measured signal becomes highly susceptible to noise, reducing its correlation with the input and resulting in a marked drop in coherence.

Table 3.2: Parameters of the TN2 neutraliser obtained through steady-state experiments

Property	TN2
Stiffness in x direction k_{xx} [N/m]	57500
Stiffness in y direction k_{yy} [N/m]	18700
Base mass m_B [kg]	0.190
Neutraliser mass m_N [kg]	0.070

In the TN2 neutraliser, each antiresonance is deliberately tuned to counteract a specific resonance at a distinct angular position. When the device is oriented at $\theta = 0^\circ$, the antiresonance f_3 is positioned to suppress the second resonance frequency f_{r2} . Conversely, at $\theta = 90^\circ$, the antiresonance f_4 is tuned to counteract the first resonance frequency f_{r1} . This targeted tuning strategy ensures that the neutraliser effectively attenuates vibrations in two distinct frequency ranges, depending on its orientation. The neutraliser has the capability to autonomously rotate to either 0 or 90 degrees, depending on the excitation frequency. Consequently, in scenarios where the system experiences two distinct excitations, each operating at different frequencies, this neutraliser may self-turn to counteract both frequencies effectively.

3.4 Chapter summary

This chapter presented a detailed overview of the vibration neutralisers developed and investigated throughout this work. The structural configurations and geometric differences between the neutralisers were outlined, illustrating the design evolution aimed at enhancing dynamic performance. A frequency domain model was developed to describe the steady-state behaviour of the system under harmonic excitation. This model was then compared with experimental

results obtained from modal testing. Parameters were identified through frequency response measurements and adjusted to ensure consistency between the model and experiment. The strong agreement between numerical and experimental results validates the proposed model, thereby enabling the subsequent analyses. Notably, the neutraliser was shown to counteract distinct resonances at specific angular positions, allowing the device to autonomously adapt to different excitation frequencies. This capacity to attenuate vibrations at different frequencies through passive self-rotation represents a significant advancement in the operational bandwidth of vibration neutralisers. In the following chapter, the model is extended to the time domain to explore the dynamics associated with self-rotation and transient excitation.

4 Time domain model and experiments under harmonic excitation

In the preceding chapter, the vibration neutraliser with varying stiffness was introduced. The investigation began with the presentation of the geometric characteristics of the neutralisers, followed by the development of a frequency-domain model. Subsequently, the dynamic parameters were identified experimentally. A key feature of the model is that its dynamic behaviour depends on the angular orientation of the device, introducing time-dependence into the system response. In light of this, the present chapter shifts focus to a time-domain approach. A time-domain model is proposed to describe the performance of the neutraliser under harmonic excitation, allowing the beam to rotate freely. The chapter concludes with an experimental validation of the proposed model.

4.1 Time domain model

As mentioned in Section 3.1, the development of the time domain model began with a simplified 3DOF formulation, which is described in detail in Appendix B. Due to significant differences between the model and the experimental configuration, particularly in their methods of excitation, a more comprehensive 5DOF model was developed and is presented in this chapter.

The proposed lumped-parameter 5DOF model, illustrated in Figure 4.1, considers the base displacements (x_B, y_B) , the neutraliser displacements (x_N, y_N) , and the rotation of the local coordinate frame (xx, yy) , represented by the angular variable θ . The excitation force is harmonic, defined as $f(t) = Fe^{j\omega t}$. In this formulation, the elastic forces are functions of the device's angular orientation and thus vary with time.

The equations of motion are derived using Lagrange's method, as given in Equation (4.1), incorporating the Rayleigh dissipation function R , where n represents the number of degrees of freedom, and q_i stands for the independent generalized coordinates. E_k denotes the kin-

etic energy of the system, V represents the potential energy, with g being the gravitational force. The generalized force vector Q_{i-th} acts on the $i - th$ inertial element and is defined as $Q = [0; 0; 0; f(t)]$. The independent generalized coordinates describing the system's motion are given by $q = [\theta; x_N; y_N; x_B; y_B]$.

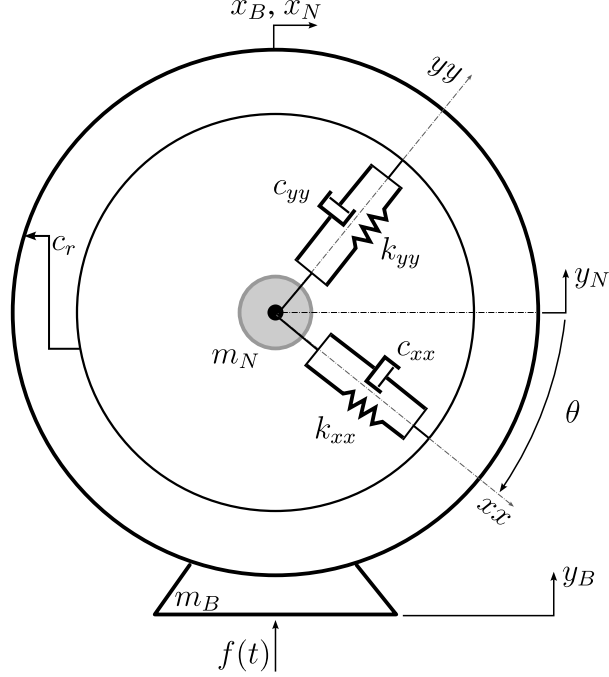


Figure 4.1: Lumped-parameter model of the 5DOF system used in the time-domain analysis. The model includes the base mass m_B , the neutraliser mass m_N , and their respective displacements (x_B, y_B) and (x_N, y_N) . The neutraliser is supported by orthogonal linear springs and dampers (k_{xx}, c_{xx}) and (k_{yy}, c_{yy}) , aligned with the local coordinate system (xx, yy) , which rotates by an angle θ relative to the global frame. The rotational damping c_r opposes the relative rotation. A harmonic force $f(t)$ is applied at the base.

$$\frac{d}{dt} \left(\frac{\partial E_k}{\partial \dot{q}_i} \right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} + \frac{\partial V}{\partial q_i} = Q_i, \quad i = 1, 2, \dots, n \quad (4.1)$$

Equation (4.2) defines the system kinetic energy involving the base and neutraliser masses and the inertia due to the neutraliser mass rotation. While Equation (4.3) represents the potential energy resulting from the strain energy stored in the springs and the gravitational field. To consider damping, Equation (4.4) incorporates the Rayleigh term using viscous damping.

The stiffness and viscous damping matrices remain consistent with the model presented in Chapter 3, as denoted by Equations (3.2) and (3.8), respectively. To transform the elastic and damping forces represented in the local coordinates $(yy$ and $xx)$, to the global coordinates of

the system (x_B, y_B, x_N, y_N) , the coordinate transformation matrix, given by (3.6), is applied. The moment of inertia I around the Z-axis comprises contributions from the beam, neutraliser mass moments, and the inner ring of the bearing. The superscript $()^T$ denotes the matrix or vector transpose.

$$E_k = \frac{m_N (\dot{x}_N^2 + \dot{y}_N^2)}{2} + \frac{m_B (\dot{x}_B^2 + \dot{y}_B^2)}{2} + \frac{I \dot{\theta}^2}{2} \quad (4.2)$$

$$V = mgy_N + m_B g y_B + \frac{1}{2} \mathbf{q}^T \mathbf{T}^T \mathbf{K} \mathbf{T} \mathbf{q} \quad (4.3)$$

$$R = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{T}^T \mathbf{C} \mathbf{T} \dot{\mathbf{q}} + \frac{1}{2} c_r \dot{\theta}^2 \quad (4.4)$$

$$\mathbf{K} = \begin{bmatrix} k_{xx} & 0 & -k_{xx} & 0 \\ 0 & k_{yy} & 0 & -k_{yy} \\ -k_{xx} & 0 & k_{xx} & 0 \\ 0 & -k_{yy} & 0 & k_{yy} \end{bmatrix}. \quad ((3.2))$$

$$\mathbf{C} = \begin{bmatrix} c_{xx} & 0 & -c_{xx} & 0 \\ 0 & c_{yy} & 0 & -c_{yy} \\ -c_{xx} & 0 & c_{xx} & 0 \\ 0 & -c_{yy} & 0 & c_{yy} \end{bmatrix} \quad ((3.8))$$

$$\mathbf{T} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & \cos(\theta) & -\sin(\theta) \\ 0 & 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \quad ((3.6))$$

By substituting Equations (4.2), (4.3), and (4.4) into Lagrange's Equation (4.1) with the follow-

ing variables are defined as:

$$\tilde{c} = c_{yy} - c_{xx}, \quad \tilde{k} = k_{yy} - k_{xx}, \quad \dot{z}_y = \dot{y}_B - \dot{y}_N, \quad z_y = y_B - y_N,$$

$$\dot{z}_x = \dot{x}_B - \dot{x}_N, \quad z_x = x_B - x_N, \quad f(t) = F \cos(\omega t)$$

five equations of motion can be derived as:

$$I\ddot{\theta} + c_r\dot{\theta} - \tilde{k} \left[z_x z_y \cos(2\theta) + \frac{1}{2}(z_y^2 - z_x^2) \sin(2\theta) \right] = 0 \quad (4.5)$$

$$m_N \ddot{x}_N + \frac{1}{2} \left[(\tilde{c}\dot{z}_y + \tilde{k}z_y) \sin(2\theta) - (\tilde{c}\dot{z}_x + \tilde{k}z_x) \cos(2\theta) + \tilde{c}\dot{z}_x + \tilde{k}z_x \right] - k_{yy}z_x = 0 \quad (4.6)$$

$$m_N \ddot{y}_N + \frac{1}{2} \left[(\tilde{c}\dot{z}_x + \tilde{k}z_x) \sin(2\theta) - (\tilde{c}\dot{z}_y + \tilde{k}z_y) \cos(2\theta) - \tilde{c}\dot{z}_y - \tilde{k}z_y \right] - c_{xx}\dot{z}_y - k_{xx}z_y + gm_N = 0 \quad (4.7)$$

$$m_B \ddot{x}_B - \frac{1}{2} \left[(\tilde{c}\dot{z}_y + \tilde{k}z_y) \sin(2\theta) + (\tilde{c}\dot{z}_x + \tilde{k}z_x) \cos(2\theta) + \tilde{c}\dot{z}_x + \tilde{k}z_x \right] + c_{yy}\dot{z}_x + k_{yy}z_x = 0 \quad (4.8)$$

$$m_B \ddot{y}_B - \frac{1}{2} \left[(\tilde{c}\dot{z}_x + \tilde{k}z_x) \sin(2\theta) - (\tilde{c}\dot{z}_y + \tilde{k}z_y) \cos(2\theta) - \tilde{c}\dot{z}_y - \tilde{k}z_y \right] + c_{xx}\dot{z}_y + k_{xx}z_y + gm_B = f(t) \quad (4.9)$$

During the development of the time-domain model, the rotational damping coefficient c_r emerged as a critical parameter, requiring manual adjustment to achieve agreement between numerical and experimental results. This parameter has a significant influence on the system's dynamic response, making its accurate representation essential. To this end, a dedicated experiment was designed to estimate a value of rotational damping that closely reflects the physical behaviour of the system, as detailed in Appendix C. The analysis discovered that the rotational damping is closely related to dry friction, which serves as the primary mechanism of energy dissipation under the observed conditions. Nonetheless, a viscous damping component was retained in the

model, as it had already demonstrated satisfactory results in earlier simulations and contributed to numerical stability.

To incorporate dry friction into the model, a friction coefficient c_f is introduced. The friction force is assumed to be proportional to the direction of motion and always acts in opposition to the angular velocity. Specifically, when the angular speed $\dot{\theta}$ is positive, the friction force acts in the negative direction with magnitude $+c_f$, resisting the rotation. Conversely, if $\dot{\theta}$ is negative, the force acts positively, still opposing the motion, and has magnitude $-c_f$. This behaviour is captured mathematically using the signal function, Equation (4.10), which ensures that the friction force always resists rotation regardless of its direction.

$$\text{sgn}(\dot{\theta}) = \begin{cases} 1 & \text{if } \dot{\theta} > 0, \\ 0 & \text{if } \dot{\theta} = 0, \\ -1 & \text{if } \dot{\theta} < 0. \end{cases} \quad (4.10)$$

The rotational equation of motion, originally presented in Equation (4.5), is thus modified to include the dry friction term, resulting in:

$$I\ddot{\theta} + c_r\dot{\theta} + c_f \text{sgn}(\dot{\theta}) - \tilde{k} \left[z_x z_y \cos(2\theta) + \frac{1}{2}(z_y^2 - z_x^2) \sin(2\theta) \right] = 0 \quad (4.11)$$

The inclusion of dry friction improves the physical realism of the model by helping to explain two important observed behaviours: (i) why the neutraliser eventually stops rotating once a steady-state is reached, and (ii) the existence of a threshold vibration amplitude required to initiate rotation. These effects, which are not fully captured by viscous damping alone, highlight the relevance of frictional forces in the dynamics of the device.

To confront the time domain model with the experimental results, the equations of motion, Equations (4.6) – (4.11), are numerically solved using a fourth-order Runge-Kutta method with fixed time-stepping. The results are presented together with the experimental data in Section 4.2.

4.2 Harmonic excitation with the beam free to rotate

The rig depicted in Figure 4.2 was used for the time domain experiments, where the beam was free to rotate and the neutraliser was subject to harmonic excitation. It depicts the neut-

raliser fixed to a Modal Shop Mini SmartShaker (model K2007E01) via a PCB-208C02 force sensor. Base acceleration $a_B(t)$ was measured using a PCB-352A24 accelerometer and recorded using a SIRIUSi data acquisition system (not depicted in the figure) at a sampling frequency of 5 kHz.

To prevent potential interference from the accelerometer with beam rotation, the angular displacement θ of the beam was captured using a camera Basler a2A1920-160umBAS with Basler Lens C23-1620-5M-P f16mm, operating at a rate of 800 frames per second. A tracking grid positioned at the tip of the beam (not shown in the figure) enabled the tracking of the angular position and displacement from the recorded movies using the software Tracker (Brown et al., 2020).

Lighting plays a fundamental role in object tracking, as it directly influences how features are perceived, extracted, and followed over time. The quality and consistency of illumination affect the contrast and visibility of the tracking grid, which are essential for accurate displacement extraction (Horn, 1986). To ensure uniform lighting conditions, a constant light source was employed using a LED panel, as shown in Figure 4.2. The panel was powered by a stable electrical source to prevent intensity fluctuations or flickering during high-speed video acquisition, thereby maintaining the visual consistency required for reliable motion tracking.



Figure 4.2: The rig used for the harmonic excitation. The neutraliser is attached to the shaker through a force sensor and has an accelerometer on the base. The high-speed camera and the light source are positioned to record the rotation of the neutraliser.

To correlate the experimental data acquired from the data acquisition system with the rotational measurements obtained from the high-speed camera, a manual synchronization procedure was adopted. Initially, the neutraliser was positioned at the desired initial angle, and the excitation frequency was set on the shaker. With the setup prepared, the data acquisition software was activated, followed immediately by the start of camera recording. The amplitude of the excitation force was then gradually increased from zero to a pre-defined value, previously established through preliminary tests to avoid sudden impulses in the shaker response.

Once the self-turning behaviour was completed, the video recording was stopped, followed by the termination of the data acquisition. During post-processing, the angular displacement of the beam was extracted from the recorded video using the software Tracker. Figure 4.3 depicts a time-stories extracted from a recorded video to highlight the tracking grid, two aligned markers positioned along the central axis of the beam, used to capture its orientation. From the video frames, the planar x and y displacements of each marker were extracted, and the rotation angle $\theta(t)$ was computed by evaluating the instantaneous orientation of the line connecting the two points, using the arctangent of their relative coordinates.

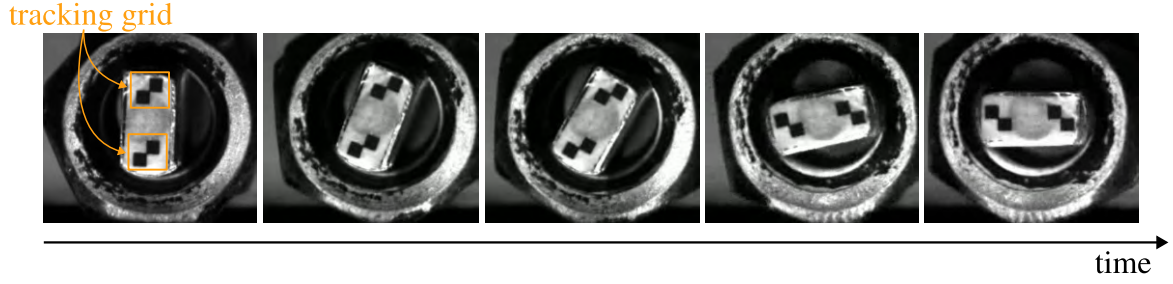


Figure 4.3: Snapshots of a recorded video showing a time-stories and highlighting the tracking grid used to capture the orientation of the beam.

Both the angular response and the accelerometer signals were imported into MATLAB and plotted over time. For synchronization, a visual alignment method was employed: by zooming in on the initial portion of the time signals, the moment when vibration began, identified by the initial increase in displacement amplitude from the video data and the corresponding rise in base acceleration from the sensor, was determined in both datasets. This reference point was used to define a common time origin, enabling temporal alignment between the two sources. It is worth noting that, since this synchronization relies on visual inspection, small timing inaccuracies may be present, introducing a degree of uncertainty in the correlation. However, this uncertainty is minor and does not significantly affect the analysis, and thus a more elaborate signal correlation method to determine the exact onset time in both datasets was assumed unnecessary.

Two experiments were carried out using the TN2 neutraliser to observe its self-turning behaviour under harmonic excitation. In the first experiment, the neutraliser was initially positioned at an angle close to 90° and excited at a constant frequency of 81 Hz. In the second, it started near 0° and was subjected to a harmonic force at a fixed frequency of 144 Hz.

The experimental results obtained in the two tests described above are now compared with the corresponding numerical simulations based on the time-domain model presented in the previous section. The goal is to assess the model's ability to capture the self-turning behaviour of the TN2 neutraliser under harmonic excitation.

The comparisons between the numerical simulations and the experimental results are shown in Figure 4.4 and Figure 4.5, corresponding to the excitation frequencies of 81 Hz and 144 Hz, respectively.

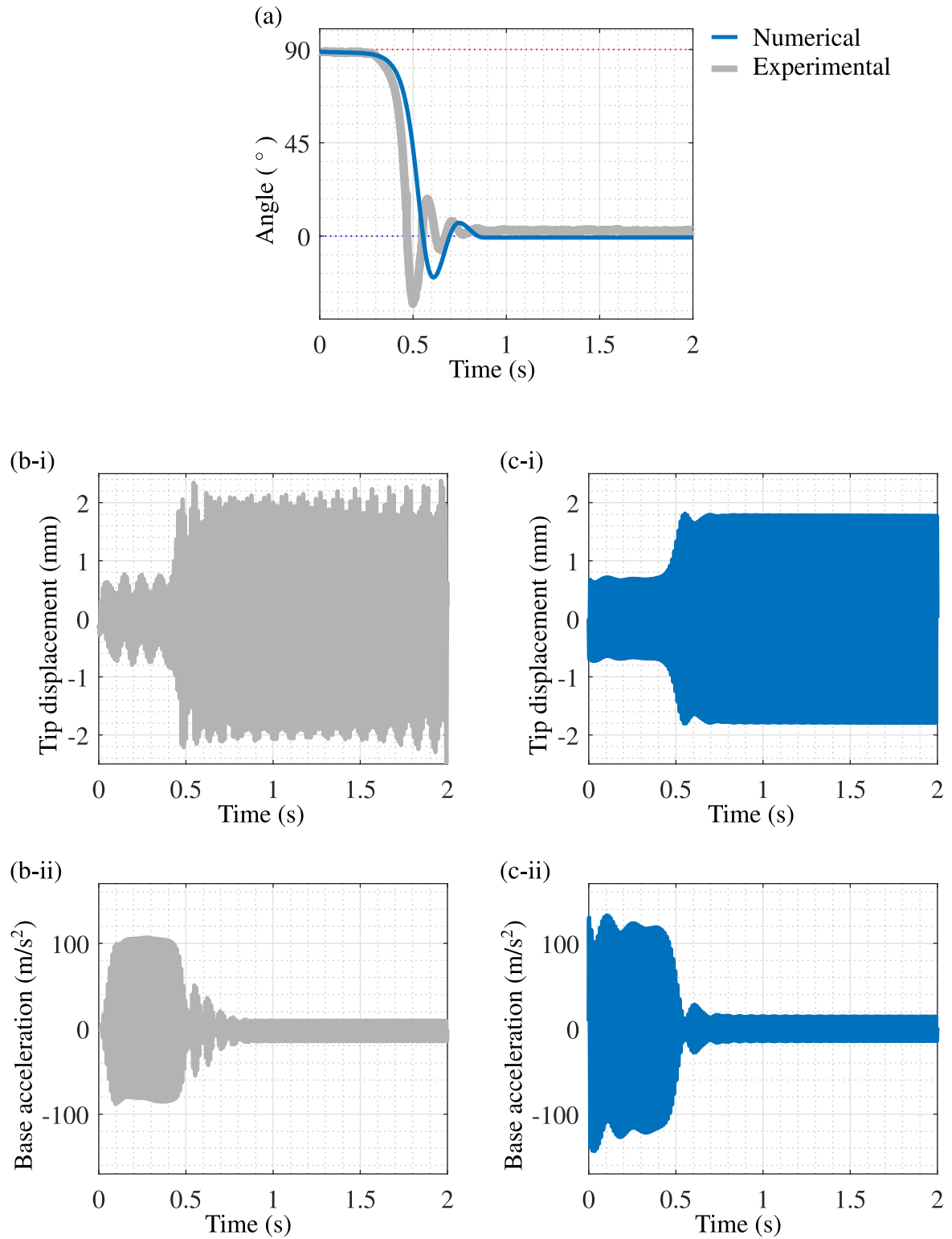


Figure 4.4: Comparison between numerical and experimental results for harmonic excitation at 81 Hz.(a) Angular displacement of the neutraliser. The gray line represents the experimental response and the blue line corresponds to the numerical simulation. Sub figures (b-i) and (b-ii) show the experimental measurements of the tip displacement and base acceleration. And sub figures (c-i) and (c-ii) displays the corresponding numerical simulations of tip displacement and base acceleration.

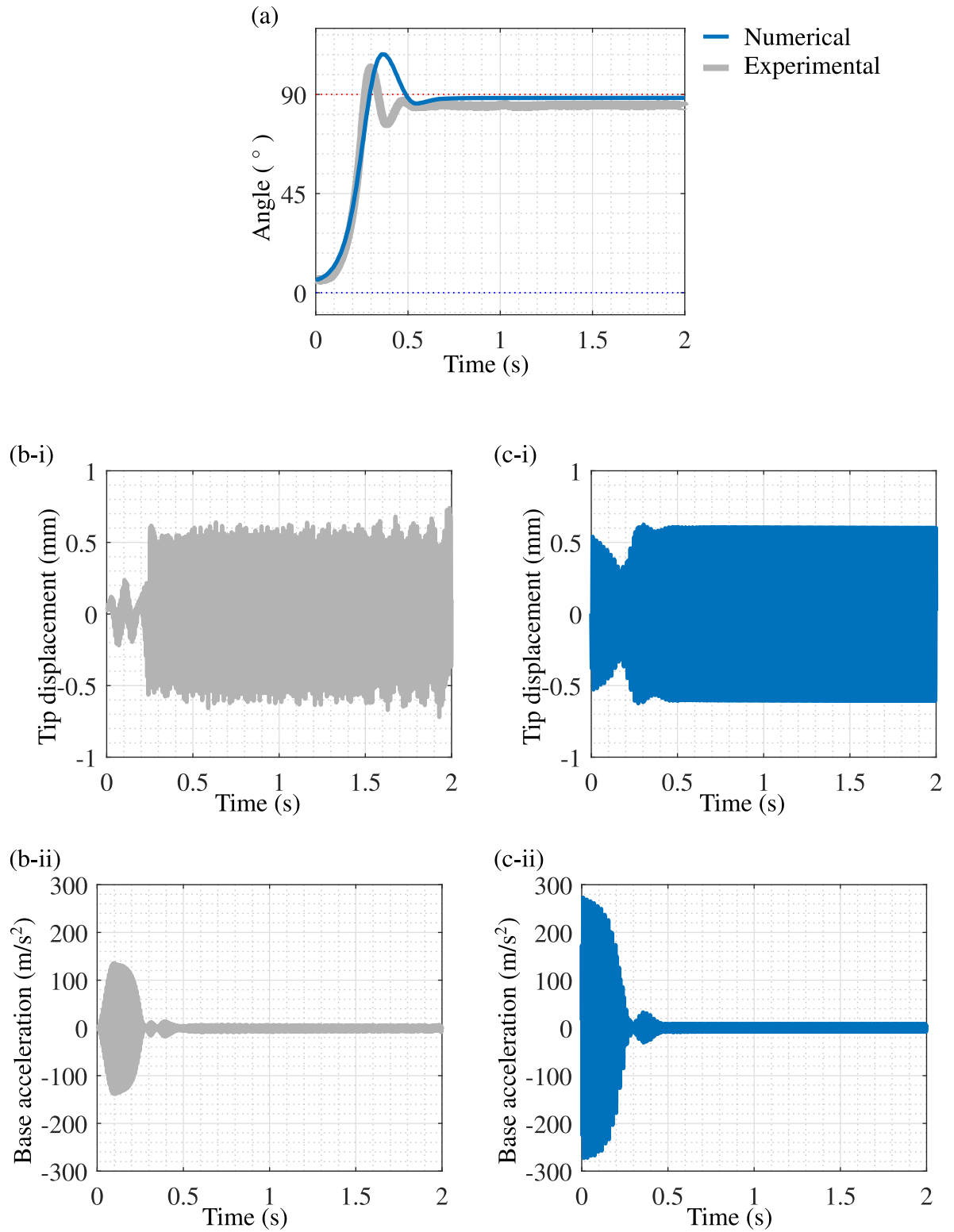


Figure 4.5: Comparison between numerical and experimental results for harmonic excitation at 144 Hz. (a) Angular displacement of the neutraliser. The gray line represents the experimental response and the blue line corresponds to the numerical simulation. Sub figures (b-i) and (b-ii) show the experimental measurements of the tip displacement and base acceleration. And sub figures (c-i) and (c-ii) displays the corresponding numerical simulations of tip displacement and base acceleration.

The numerical and experimental results show a good qualitative agreement in both test cases. In Figure 4.4, the neutraliser starts near 90° and rotates toward 0° , as expected. The angular position plot (a) shows that both responses follow a similar trajectory, with comparable rise time and final orientation. The experimental signal displays more pronounced oscillations during the transition, likely due to non-modelled physical imperfections or frictional variations not fully captured in the simplified model. However, the overall settling trend and final angle are well represented. Notably, the initial delay before the neutraliser begins to rotate can only be reproduced by including dry friction in the model, as it introduces a threshold force that must be overcome to initiate motion, an effect not achievable with viscous damping alone. Tip displacement and base acceleration also support the accuracy of the model. The simulated tip displacement (c-i) closely mirrors the experimental response (b-i), both in amplitude and frequency content. The base acceleration plots (b-ii) and (c-ii) exhibit similar transient and steady-state behaviour, indicating that the base response is correctly captured.

In Figure 4.5, the neutraliser initiates near 0° and rotates toward 90° , again exhibiting good agreement between experimental and numerical data. The angular response (a) aligns well with the experimental data, showing only slight discrepancies during the transient phase. Although some deviations are noted at the onset of motion, the final orientation and general trajectory are accurately captured. The inclusion of dry friction in the numerical model enables the simulation to reproduce a realistic stopping behaviour, where the final angle does not settle exactly at 90° , in a manner consistent with the experimental observations. This effect is also evident in the tip and base responses, where the numerical model captures the main trends of the measured data.

Altogether, the combination of viscous and dry friction in the model proves essential for reproducing the physical behaviour observed in the experiments, particularly the conditions required to trigger rotation and the mechanism by which it stops. While the analysis remains qualitative, the numerical model demonstrates a strong capacity to replicate the core features of the self-turning phenomenon.

4.3 Parametric study of neutraliser response under harmonic excitation

To evaluate the response of the neutraliser across a range of operating conditions, the system of equations (4.6)–(4.11) was solved numerically using a fourth-order Runge-Kutta with fixed

step. The implementation was carried out via the custom *ode4nr* function, developed in C++ and compiled as a MEX file for MATLAB to ensure efficient performance. The excitation was applied in the form of a harmonic force, $f(t) = F e^{j\omega t}$ with both frequency and amplitude varied systematically.

The excitation frequency ω ranged from 50 Hz to 250 Hz, increasing in steps of 2.5 Hz. Similarly, the force amplitude F was varied from 5 N to 50 N, also in 2.5 N increments. Each simulation was run for 100 seconds. The initial conditions were the same for all cases: an angular position $\theta_0 = 45^\circ$, and all displacements and velocities set to zero. After allowing sufficient time for the system to reach steady state, the final angular position θ was recorded for further analysis.

To represent the final angular position of the neutraliser as a function of excitation frequency and force amplitude, a colormap, Figure 4.6, was constructed based on the numerical integration of the time-domain model. For each combination of excitation parameters, the angular response $\theta(t)$ was recorded and analysed over the final 20 seconds of the simulation, corresponding to the steady-state regime.

Although the neutraliser rotates rapidly when excited with a sufficiently high force amplitude, a longer simulation time of 100 seconds was adopted to ensure that slower responses could also be captured. In particular, when the force amplitude is below the threshold required to promptly trigger self-rotation, the system may still reach the preferred angular position given sufficient time. To evaluate whether a steady-state orientation is eventually reached, the analysis is focused on the last 20 seconds of each simulation. This time window allows the system's final behaviour to be assessed independently of initial transients. The classification criteria applied to this interval help determine whether a preferred position emerges for a given combination of force and frequency. This approach proved essential because, in some specific cases, the simulations exhibited non-deterministic behaviour, such as continuous drift or persistent oscillations of the angle, which varied with the excitation parameters and duration of observation.

To construct the colormap the mean value of θ in the steady-state portion of the simulation was calculated and three classification criteria were established:

Reaches steady-state: If the final value of θ lies within a narrow tolerance of $\pm 2^\circ$ around the mean value, the system is considered to have reached a steady-state. The colormap is filled with the mean steady-state angle.

Oscillatory steady-state: If the final value lies within a wider range of $\pm 5^\circ$ around the mean,

it is assumed that the system oscillates around the mean value. In this case, the mean angle is also used in the colormap.

Non-converging or delayed response: If the final value falls outside this broader window, the system is considered to have not reached a well-defined steady-state within the simulated time. The last value of θ is used in the colormap.

Unclassified or non-deterministic behaviour: If none of the conditions are met the point is marked as undefined (NaN), and the corresponding region in the colormap appears grey.

Note that the first and second criteria are very similar in implementation and result in the same action within the colormap, as both assign the mean steady-state angle as the output. However, they reflect distinct behaviours of the neutraliser. After classification, a continuous colour scale was generated to map angular positions to colour values. The scale interpolates between blue (corresponding to 0°) and red (90°), with intermediate colours representing partial angular orientations. The resulting colormap thus provides a visual summary of the neutraliser's steady-state orientation under various excitation conditions, highlighting transitions between preferred angular positions and regions of unpredictable or non-deterministic behaviour.

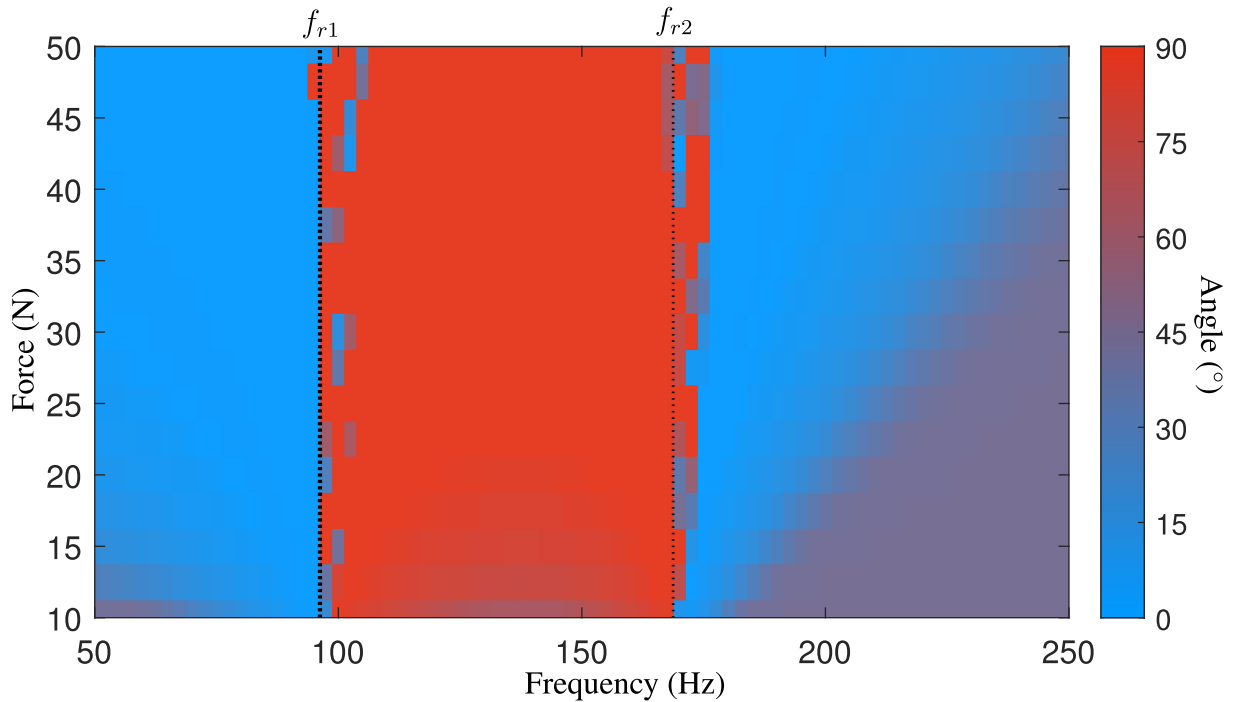


Figure 4.6: Numerical colormap showing the final angular position of the neutraliser as a function of excitation frequency and force amplitude. The colour scale represents the angle θ . The dotted vertical lines indicate the first (f_{r1}) and second (f_{r2}) natural frequencies of the neutraliser base. The transition between colours exposes how the excitation conditions influence the preferred orientation of the device.

The natural frequencies of the base structure influence the final orientation adopted by the neutraliser according to the excitation frequency and force amplitude. For excitation frequencies below the first natural frequency (f_{r1}) and above the second (f_{r2}), the neutraliser tends to align to 0° . In contrast, between f_{r1} and f_{r2} , it consistently aligns to 90° , indicating a clear frequency-dependent switching behaviour.

In addition, two regions of non-deterministic behaviour emerge near the transitions around f_{r1} and f_{r2} . These regions form triangular patterns in the colormap, originating at low force levels and expanding in width as the excitation force increases. This suggests that, under certain excitation conditions, the neutraliser does not settle into a unique steady-state orientation, potentially due to complex dynamic interactions.

The numerical colormaps predict the angular orientation to which the neutraliser will rotate for a given combination of excitation frequency and force amplitude. However, by analysing the frequency response of the base structure, it becomes possible to assess whether such a rotation effectively contributes to vibration attenuation. In other words, comparing the base accelerance for each fixed orientation (0° and 90°) allows one to determine if the angular position predicted by the colormap results in a reduction or amplification of the base vibration at that frequency. This analysis provides insight into the actual performance of the self-turning mechanism in terms of its effectiveness in mitigating structural vibrations.

In general terms, the numerical colormap shown in Figure 4.6 indicate that the neutralisers tend to rotate toward 0° before the first resonance f_{r1} and after the second resonance f_{r2} , and toward 90° in the frequency range between these two resonances. To assess whether these predicted orientations are effective in reducing vibration, Figure 4.7(a) presents the base accelerance for both fixed orientations. In regions where the colormap predicts a final angle of 90° , the base accelerance at this orientation is plotted as a solid red line. Conversely, when the predicted final angle is 0° , the corresponding accelerance is shown as a solid blue line. This approach enables a direct comparison between the numerically predicted orientation and the vibration attenuation performance of the system.

To experimentally evaluate the accuracy and effectiveness of these predictions, six test frequencies were selected to represent three distinct scenarios for each antiresonance. These scenarios include:

- 1 Excitation at the antiresonance, where the neutraliser rotates to the predicted angle and the base acceleration is reduced, Figures 4.7(b-ii) and (c-ii);

- 2 Excitation away from the antiresonance, still resulting in base vibration reduction, Figures 4.7(b-i) and (c-i);
- 3 Excitation away from the antiresonance, but leading to amplification of the base acceleration, Figures 4.7(b-iii) and (c-iii).

The excitation frequencies selected for the experimental evaluation comprised two frequencies corresponding to the antiresonances, at 81 Hz and 144 Hz, representing the first scenario. For the second scenario, excitation frequencies of 75 Hz and 135 Hz were chosen, which lie outside the antiresonance regions but still yield a reduction in base acceleration. Finally, excitation at 90 Hz and 160 Hz was employed to represent the third scenario, where operation away from the antiresonance results in amplification of the base acceleration.

Each case was tested for both tuned configurations of the neutraliser, corresponding to orientations of 0° and 90° , providing a broader understanding of how excitation frequency, final angular position, and vibration response are interrelated. The angular response analysis confirms that the neutraliser rotates to the numerically predicted angle, even when that orientation does not correspond to a minimum in base acceleration. This can be observed in the gray coloured frequency regions of Figure 4.7(a). To complement this analysis, Figure 4.8 presents the time domain base acceleration for 75 Hz and 90 Hz and Figure 4.9 presents the time domain base acceleration for 135 Hz and 160 Hz. The analysis for 81 Hz and 144 Hz were already examined in Figures 4.4 and 4.5.

The experimental base acceleration at 75 Hz and 135 Hz, shown in Figures 4.8(a-ii) and 4.9(b-ii), respectively, demonstrates a reduction in acceleration following the rotation of the neutraliser to the predicted angle. This behaviour is consistent with the corresponding numerical simulations in Figures 4.8(a-iii) and 4.9(b-iii). However, in the cases of 90 Hz and 160 Hz, frequencies that fall within the coloured region of Figure 4.7(a), the experimental base acceleration increases after the rotation. This amplification is not reflected in the numerical predictions, which continue to show a reduced or stable base response. The reasons why the neutraliser rotates to these angles despite increasing the base acceleration remain unclear; however, it is possible to anticipate and avoid such undesirable conditions by consulting the frequency-domain model, as shown in Figure 4.7(a).

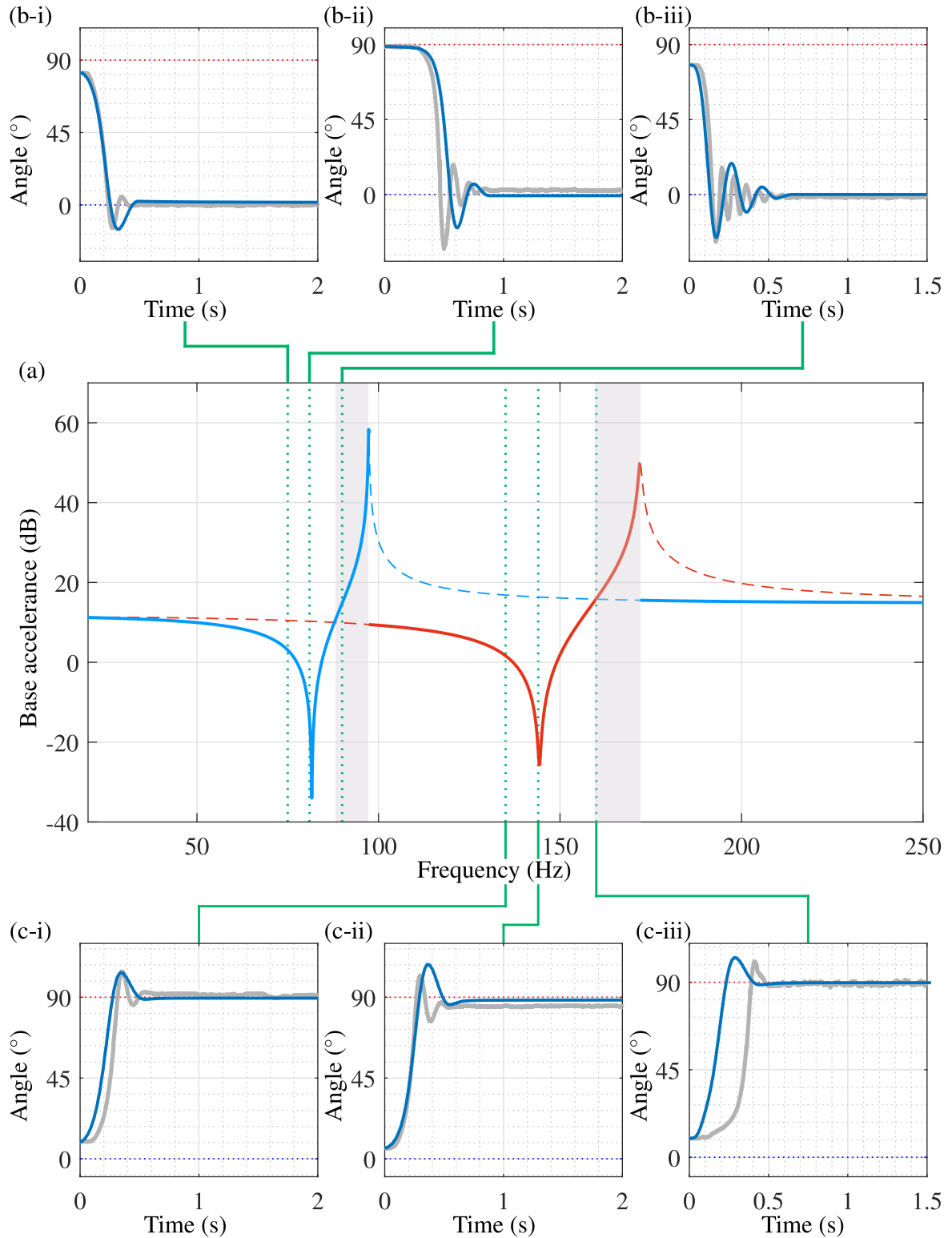


Figure 4.7: (a) Base acceleration curves for fixed angular positions of the neutraliser: 0° in blue and 90° in red. Solid lines indicate the configuration predicted by the numerical colormap as optimal for vibration reduction in each frequency region. Sub figures (b) and (c) display the angular response of the neutraliser at selected excitation frequencies, with gray lines for experimental results and blue lines for numerical simulations. Specifically, (b-i) and (c-i) correspond to frequencies outside the antiresonance but still yielding a reduction in base acceleration (75 Hz and 135 Hz); (b-ii) and (c-ii) represent excitation at the antiresonances (81 Hz and 144 Hz); and (b-iii) and (c-iii) illustrate cases where excitation occurs outside the antiresonance region, resulting in increased base acceleration (90 Hz and 160 Hz).

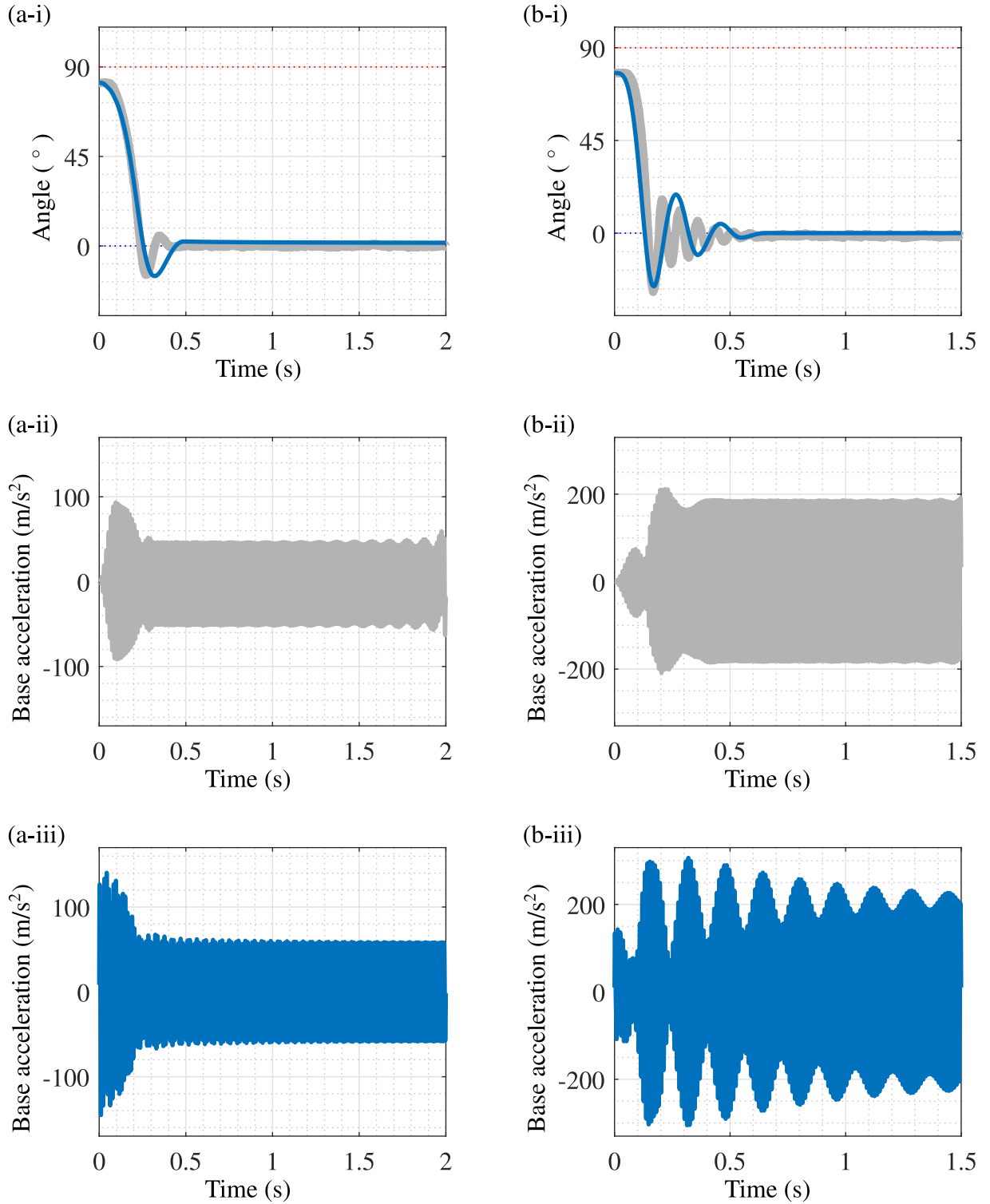


Figure 4.8: Time domain results for excitation at (a) 75 Hz and (b) 90 Hz. Sub figures (a-i) and (b-i) display the angular displacement of the neutraliser, where the gray line represents the experimental measurement and the blue line the numerical simulation. Sub figures (a-ii) and (b-ii) present the experimentally measured base acceleration, while (a-iii) and (b-iii) show the corresponding numerical predictions.

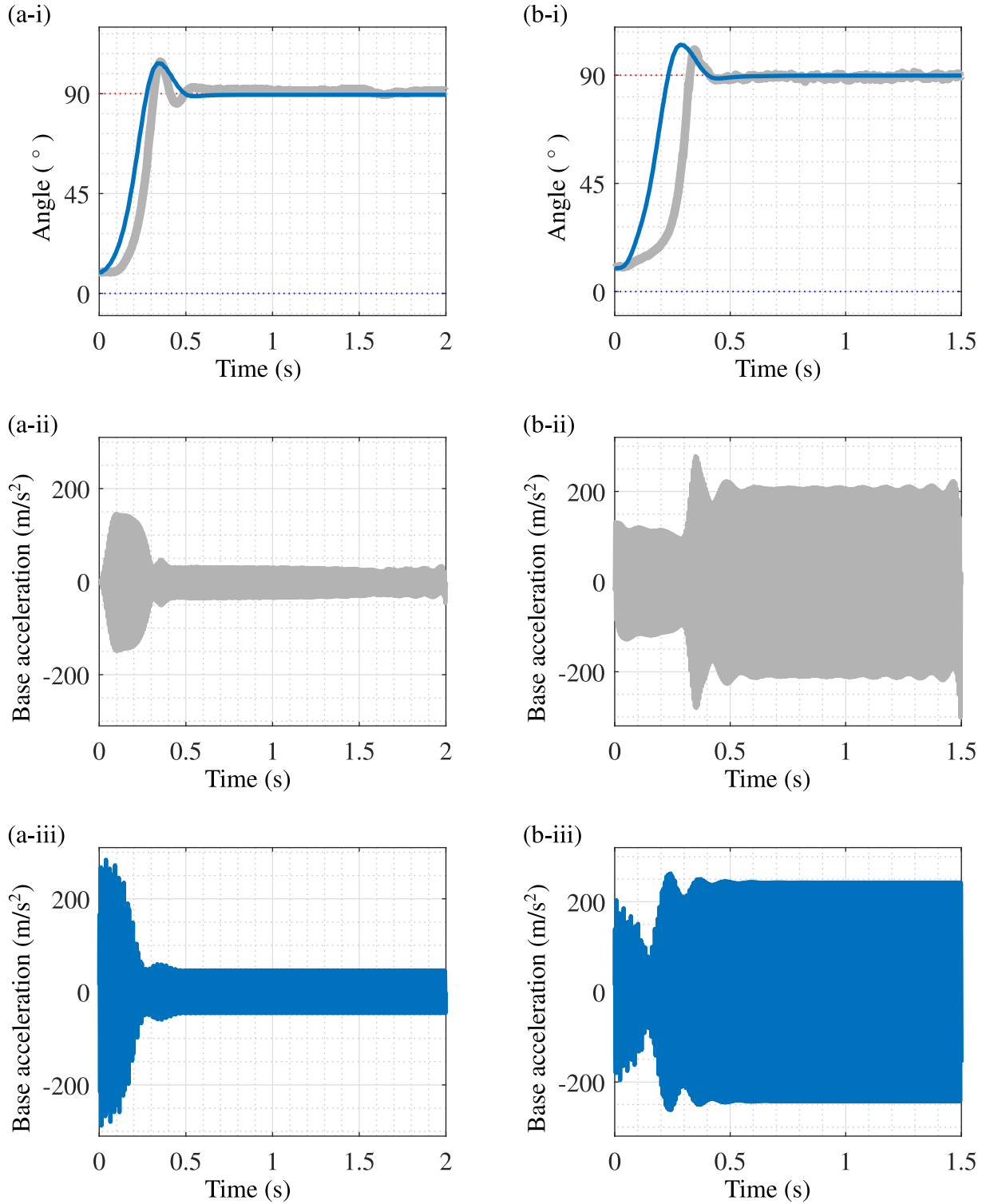


Figure 4.9: Time domain results for excitation at (a) 135 Hz and (b) 160 Hz. Sub figures (a-i) and (b-i) display the angular displacement of the neutraliser, where the gray line represents the experimental measurement and the blue line the numerical simulation. Sub figures (a-ii) and (b-ii) present the experimentally measured base acceleration, while (a-iii) and (b-iii) show the corresponding numerical predictions.

4.4 Chapter summary

This chapter presented the development and validation of a time domain model to describe the dynamic behaviour of the self-turning vibration neutraliser subject to harmonic excitation. The time domain approach was motivated by the need to capture the passive rotation of the neutraliser under harmonic excitation.

The model incorporates both viscous and dry friction damping, enabling it to replicate key experimental features such as delayed rotation and final angular position. Numerical simulations demonstrated strong qualitative agreement with experimental results, validating the model's self-turning mechanism.

A parametric study was also performed to map the steady-state orientation as a function of excitation frequency and force amplitude. The resulting colormap revealed frequency dependent switching behaviour and regions of non-deterministic response. When combined with frequency response analysis, the study showed that the predicted orientation does not always correspond to reduced base vibration, highlighting the importance of evaluating dynamic performance in conjunction with orientation behaviour.

5 Investigating self-rotation through auto parametric resonance

This chapter examines the physical and mathematical bases that give rise to self-rotation. Particular emphasis is placed on clarifying the dynamic coupling responsible for the energy exchange between modes and demonstrating that the observed behaviour is rooted in auto-parametric resonance mechanisms. Although the final orientation of the neutraliser is associated with a parametric stiffness modulation at twice the excitation frequency, the rotation itself is initiated by a transient time-varying moment generated by translational motion. The stiffness modulation determines where the system will settle, while the coupling-induced moment determines how the system gets there.

5.1 Introduction to auto parametric behaviour in mechanical systems

Auto-parametric resonance arises when the motion of one mode induces a time varying parameter, usually stiffness, in another mode, resulting in a energy exchange. The affected mode is not directly excited. However, its response arises due to the internal modulation imposed by the dominant mode. This behaviour has been historically illustrated through systems such as pendulums with oscillating supports, where vertical base motion alters the effective restoring force of the angular coordinate.

A classic example of a auto-parametric system is the pendulum with a moving support. This situation can occur due to external forces, such as when the pendulum's pivot is mounted on a vibrating platform. Periodic motion of the support can induce complex dynamics in the pendulum, potentially leading to parametric resonance. This problem was first introduced by Stephenson (1908) when he analysed the dynamic stability of an inverted pendulum with a vibrating suspension point. He showed that, under specific conditions, the normally unstable upright position could be made stable by applying vertical periodic forces at the suspension

point. This effect occurs when the vertical oscillations are sufficiently rapid and have a large enough amplitude. The concept is fundamental for understanding parametric excitation, where the stability of the system is influenced by time-varying parameters, such as the oscillating pivot, rather than by direct external forces (Stephenson, 1908).

The problem of the inverted pendulum was further explored in detail by P. L. Kapitza, who investigated how a rapidly oscillating support could stabilize the pendulum at its otherwise unstable equilibrium point. This problem, commonly referred to as Kapitza's pendulum, provides crucial insights into the stabilization of systems through parametric excitation (Kapitza et al., 1951).

Several researchers have further explored the dynamics of the pendulum with a moving support as a classical example to introduce the concepts of parametric resonance, including Blekhman (2000), Nayfeh and Mook (2008), and Thomsen et al. (2003).

The relevance of these classical researches lies not in their specific configurations, but in the mechanism they manifest, a fast oscillatory motion can reshape the effective potential experienced by a slower degree of freedom. In the neutraliser examined in this work, the roles are analogous. The translational motion induced by the base excitation plays the part of the primary oscillation, while the rotational coordinate behaves as the secondary, auto-parametrically influenced mode. The present system includes a coupling-induced moment that can inject angular momentum and thereby initiate rotation.

5.2 Physical mechanism of self-rotation

The self-rotation may be interpreted by separating two effects acting on the rotational coordinate:

1. A transient time-varying moment arising from stiffness asymmetry and translational motion. This moment exists only while the neutraliser is misaligned with its steady-state orientation. It is responsible for initiating rotation by transferring energy from translation to rotation.
2. A parametric stiffness modulation at twice the excitation frequency, which persists after the transient moment subsides. This modulation does not drive rotation but governs the stability of the possible equilibrium orientations, ensuring that one orientation becomes energetically favourable while the other is suppressed.

The moment causes rotation, while the parametric stiffness decides where the rotation stops. During the initial response, the translational motion generates a moment that acts on the angular coordinate. As the neutraliser rotates, the magnitude of this moment vanishes once alignment with a preferred stiffness axis is achieved. At this stage, the stiffness modulation continues to affect the angular dynamics, but only to maintain equilibrium rather than to induce motion.

5.3 Modelling assumptions

The mathematical developments in the next section rely on a set of assumptions intended to isolate the mechanisms of interest. The translational motion is assumed to be harmonic and dominated by the excitation frequency. In-phase responses are adopted for the two translational coordinates to facilitate analytical insight into the coupling terms. Although in practice small phase differences may exist, numerical simulations later in the chapter confirm that these deviations do not alter the qualitative nature of the self-rotation mechanism. The model therefore captures the dominant physics with sufficient fidelity, while remaining analytically tractable.

5.4 Mathematical insights into self-rotation

To investigate the mechanism that drives and sustains self-rotation, the rotational dynamics of the neutraliser are first expressed through its governing equation of motion, which includes inertial, damping, frictional, and stiffness-related terms.

$$I\ddot{\theta} + c_r\dot{\theta} + c_t \operatorname{sgn}(\dot{\theta}) - \tilde{k} \left[z_x z_y \cos(2\theta) + \frac{1}{2}(z_y^2 - z_x^2) \sin(2\theta) \right] = 0 \quad ((4.5))$$

This equation accounts for the rotational inertia $I\ddot{\theta}$, the viscous damping $c_r\dot{\theta}$, dry friction $c_t \operatorname{sgn}(\dot{\theta})$ and a nonlinear restoring torque dependent on both the angular displacement θ and the relative displacements z_x and z_y , modulated by the stiffness difference $\tilde{k} = k_{yy} - k_{xx}$.

It is evident from the equation that self-rotation is only possible if $\tilde{k} \neq 0$, i. e., if the system exhibits stiffness asymmetry.

5.5 Harmonic base excitation and coupling terms

The base excitation is assumed to be harmonic at frequency ω , which induces steady-state harmonic motion in the translational degrees of freedom. The relative displacements are expressed as

$$z_y(t) = Z_y \cos(\omega t), \quad z_x(t) = Z_x \cos(\omega t), \quad (5.1)$$

where Z_x and Z_y are the vibration amplitudes along the two axes. Although real responses may include small phase differences, adopting in-phase motion provides a transparent analytical description of the coupling mechanism and is sufficient to reproduce the relevant physics. These expressions lead to products of the form $z_x z_y$ and $z_y^2 - z_x^2$, which introduce terms proportional to $\cos(2\omega t)$. As a result, the nonlinear moment applied to the rotational coordinate contains a time-varying component at frequency 2ω , and a constant component associated with average stiffness asymmetry. The first component is responsible for parametric stiffness modulation, while the second contributes to a slowly varying or transient moment depending on the angular position.

Substituting these into the coupling terms of Equation (4.5):

$$z_x z_y = Z_x Z_y \cos^2(\omega t) = A_1 [1 + \cos(2\omega t)] \quad (5.2)$$

$$z_y^2 - z_x^2 = (Z_y^2 - Z_x^2) \cos^2(\omega t) = A_2 [1 + \cos(2\omega t)] \quad (5.3)$$

with the auxiliary constants defined as:

$$A_1 = \frac{Z_x Z_y}{2}, \quad A_2 = \frac{Z_y^2 - Z_x^2}{2}.$$

Replacing expressions (5.2) and (5.3) into (4.5) yields the following form:

$$\begin{aligned} I\ddot{\theta} + c_r \dot{\theta} + c_f \operatorname{sgn}(\dot{\theta}) - \tilde{k} \left[A_1 \cos(2\theta) + \frac{1}{2} A_2 \sin(2\theta) \right] \\ - \tilde{k} \left[A_1 \cos(2\omega t) \cos(2\theta) + \frac{1}{2} A_2 \cos(2\omega t) \sin(2\theta) \right] = 0 \end{aligned} \quad (5.4)$$

This structure matches the mathematical signature of auto-parametric systems, where a degree of freedom that is not directly excited experiences a parametric modulation induced by another coordinate. However, in contrast with classical cases, the present system also generates a time-dependent driving moment, which is the key factor responsible for initiating rotation.

5.5.1 Linearisation around key positions

The vibration neutraliser exhibits two preferred orientations, one aligned with the axis of higher translational stiffness and the other with the softer axis. These correspond approximately to $\theta = 0^\circ$ and $\theta = 90^\circ$. To gain insight into local stability and transient behaviour, the governing rotational equation, Eq. (5.4), is linearised about each position using first-order approximations for the trigonometric terms..

Case 1: Expansion around $\theta = 0$

For small angular deviations around $\theta = 0$, the trigonometric functions are approximated as:

$$\sin(2\theta) \approx 2\theta, \quad \cos(2\theta) \approx 1.$$

Substituting these into Equation (5.4) leads to the following expression:

$$I\ddot{\theta} + c_r\dot{\theta} + c_f \operatorname{sgn}(\dot{\theta}) - \tilde{k} [A_1 + A_2\theta] - \tilde{k} [A_1 \cos(2\omega t) + A_2 \cos(2\omega t)\theta] = 0 \quad (5.5)$$

rewriting the equation and grouping similar terms:

$$I\ddot{\theta} + c_r\dot{\theta} + c_f \operatorname{sgn}(\dot{\theta}) - \tilde{k}A_1 [1 + \cos(2\omega t)] - \tilde{k}A_2\theta [1 + \cos(2\omega t)] = 0 \quad (5.6)$$

In this expression, the term $\tilde{k}A_1 [1 + \cos(2\omega t)]$ represents a time-varying external moment acting on the angular coordinate, while the term $\tilde{k}A_2\theta [1 + \cos(2\omega t)]$ introduces a time-dependent stiffness.

Case 2: Expansion around $\theta = \pi/2$

Similarly, for small displacements around $\theta = \pi/2$, the trigonometric terms are approximated as:

$$\cos(2\theta) \approx -1, \quad \sin(2\theta) \approx -2(\theta - \pi/2).$$

These lead to the following approximations:

$$\begin{aligned}\cos(2\omega t) \cos(2\theta) &\approx \cos(2\omega t)(-1) = -\cos(2\omega t), \\ \cos(2\omega t) \sin(2\theta) &\approx \cos(2\omega t) \left[-2\left(\theta - \frac{\pi}{2}\right)\right] = -2\cos(2\omega t) \left(\theta - \frac{\pi}{2}\right).\end{aligned}$$

Substituting into Equation (5.4) yields:

$$\begin{aligned}I\ddot{\theta} + c_r\dot{\theta} + c_f \operatorname{sgn}(\dot{\theta}) \\ + \tilde{k} \left[A_1 + A_2 \left(\theta - \frac{\pi}{2} \right) + A_1 \cos(2\omega t) + A_2 \cos(2\omega t) \left(\theta - \frac{\pi}{2} \right) \right] = 0\end{aligned}\quad (5.7)$$

Grouping the similar terms:

$$I\ddot{\theta} + c_r\dot{\theta} + c_f \operatorname{sgn}(\dot{\theta}) + \tilde{k}A_2 [1 + \cos(2\omega t)] \left(\theta - \frac{\pi}{2} \right) + \tilde{k}A_1 [1 + \cos(2\omega t)] = 0 \quad (5.8)$$

To simplify the interpretation of this expression, introducing a new variable representing the angular deviation from equilibrium:

$$\alpha = \theta - \frac{\pi}{2}, \quad \dot{\alpha} = \dot{\theta}, \quad \ddot{\alpha} = \ddot{\theta}$$

Substituting this into Equation (5.8):

$$I\ddot{\alpha} + c_r\dot{\alpha} + c_f \operatorname{sgn}(\dot{\alpha}) + \tilde{k}A_2 [1 + \cos(2\omega t)] \alpha + \tilde{k}A_1 [1 + \cos(2\omega t)] = 0 \quad (5.9)$$

As in the previous case, the term $\tilde{k}A_1 [1 + \cos(2\omega t)]$ acts as a time-varying moment, while the term $\tilde{k}A_2 [1 + \cos(2\omega t)] \alpha$ introduces a parametric stiffness modulated by the angular deviation from the second equilibrium.

The linearised rotational equation takes the form

$$I\ddot{\theta} + c_r\dot{\theta} + c_t \operatorname{sgn}(\dot{\theta}) + k_{eq}(t)\theta = M_{eq}(t), \quad (5.10)$$

where $k_{eq}(t)$ is a time-varying stiffness and $M_{eq}(t)$ is an effective moment term. Both quantities are proportional to expressions of the form $[1 + \cos(2\omega t)]$ showing that the translational motion imposes a parametric excitation at twice the base frequency. The structure of the linearised equation demonstrates that the stiffness term persists regardless of angular motion, maintaining

a parametric signature even after the rotation stops, and the moment term is nonzero only while the orientation differs from the stable configuration. Thus, the linearisation clearly separates the rotation-driving and rotation-sustaining effects. As soon as the neutraliser aligns with the stable stiffness axis, the moment term tends to zero, leaving only the stiffness modulation to preserve the equilibrium.

5.6 Numerical interpretation of the coupling effects

The analytical observations are confirmed through numerical simulation of the full nonlinear model. Figure 5.1 presents a post-processed analysis of the numerical simulations performed in the previous section, in which the linearised torque and stiffness expressions were evaluated along the nonlinear trajectories of the system. The simulations correspond to two distinct excitation frequencies, 81 Hz (column a) and 144 Hz (column b), which lead to final orientations of $\theta = 0^\circ$ and $\theta = 90^\circ$, respectively.

In Figures 5.1(a-i) and (b-i), the angular position $\theta(t)$ demonstrates the neutraliser's self-orientation behaviour. At 81 Hz, the device transitions from 90° to 0° , while at 144 Hz the opposite occurs. Figures 5.1(a-ii) and (b-ii) display the equivalent stiffness term $\tilde{k}A_1 [1 + \cos(2\omega t)]$. Notably, this term becomes non zero immediately upon the onset of harmonic excitation and persists even after the angular position stabilises. This behaviour is consistent with the nature of stiffness modulation in auto-parametric systems, which continuously depends on both the angular displacement and the base excitation. In contrast, the moment term $\tilde{k}A_2\theta [1 + \cos(2\omega t)]$, shown in Figures 5.1(a-iii) and (b-iii), appears only during the transient rotation phase. It vanishes once the system reaches its steady-state orientation. This confirms that the time-varying moment acts as a driving mechanism for reorientation but ceases to contribute once the equilibrium configuration is attained.

Together, the results illustrate that self-rotation is initiated by the transient moment arising from the coupling of translational motions, whereas the stiffness term governs the system's restoring behaviour and persists as long as the harmonic base excitation is active. These findings reinforce the interpretation of the system as an auto-parametric oscillator with two stable orientations, each associated with a dominant translational energy direction.

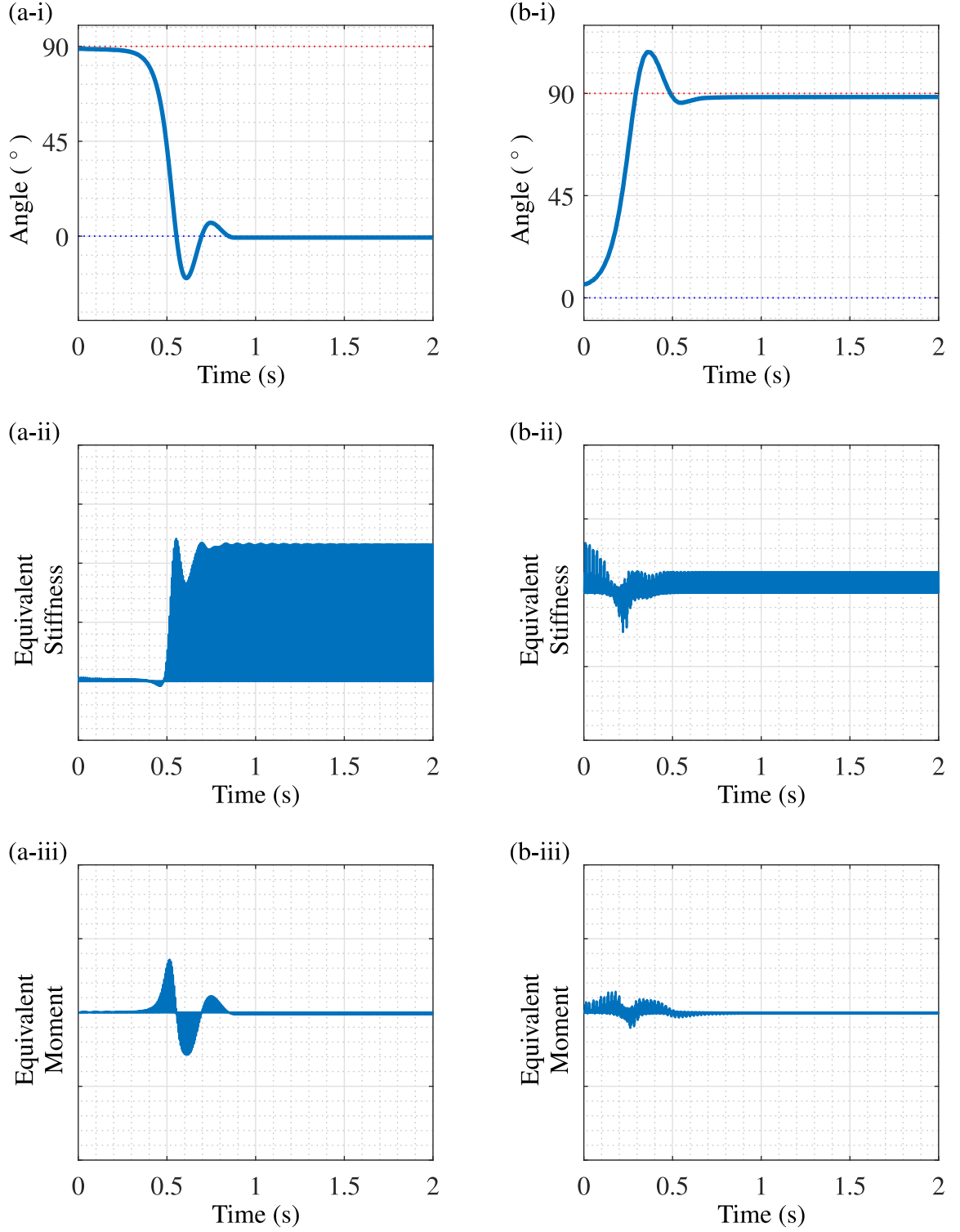


Figure 5.1: Numerical simulation of the linearised terms governing the rotational dynamics of the neutraliser at two excitation frequencies: (a) 81 Hz and (b) 144 Hz. Sub plots (i) depicts the angular position $\theta(t)$, sub plots (ii) show the equivalent stiffness contribution $\tilde{k}A_1 [1 + \cos(2\omega t)]$, and sub plots (iii) present the equivalent moment $\tilde{k}A_2\theta [1 + \cos(2\omega t)]$. The results were obtained by evaluating the linearised expressions along trajectories generated from the original nonlinear model.

To further investigate the nonlinear coupling effects that drive the self-rotation phenomenon,

the linearised terms representing the time-varying moment and stiffness were extracted from full nonlinear simulations. These terms, respectively associated with the expressions for the equivalent time-varying moment $\tilde{k}A_2\theta[1 + \cos(2\omega t)]$ and equivalent stiffness $\tilde{k}A_1[1 + \cos(2\omega t)]$, were analysed in the frequency domain using time-frequency spectrograms. The goal of this analysis is to confirm the presence of spectral components at twice the excitation frequency, a defining feature of parametric resonance. Simulations were performed for two excitation conditions, corresponding to base excitation frequencies of 81 Hz, Figure 5.2, and 144 Hz, Figure 5.3, which induce self-rotation towards the orthogonal equilibrium positions of the neutraliser.

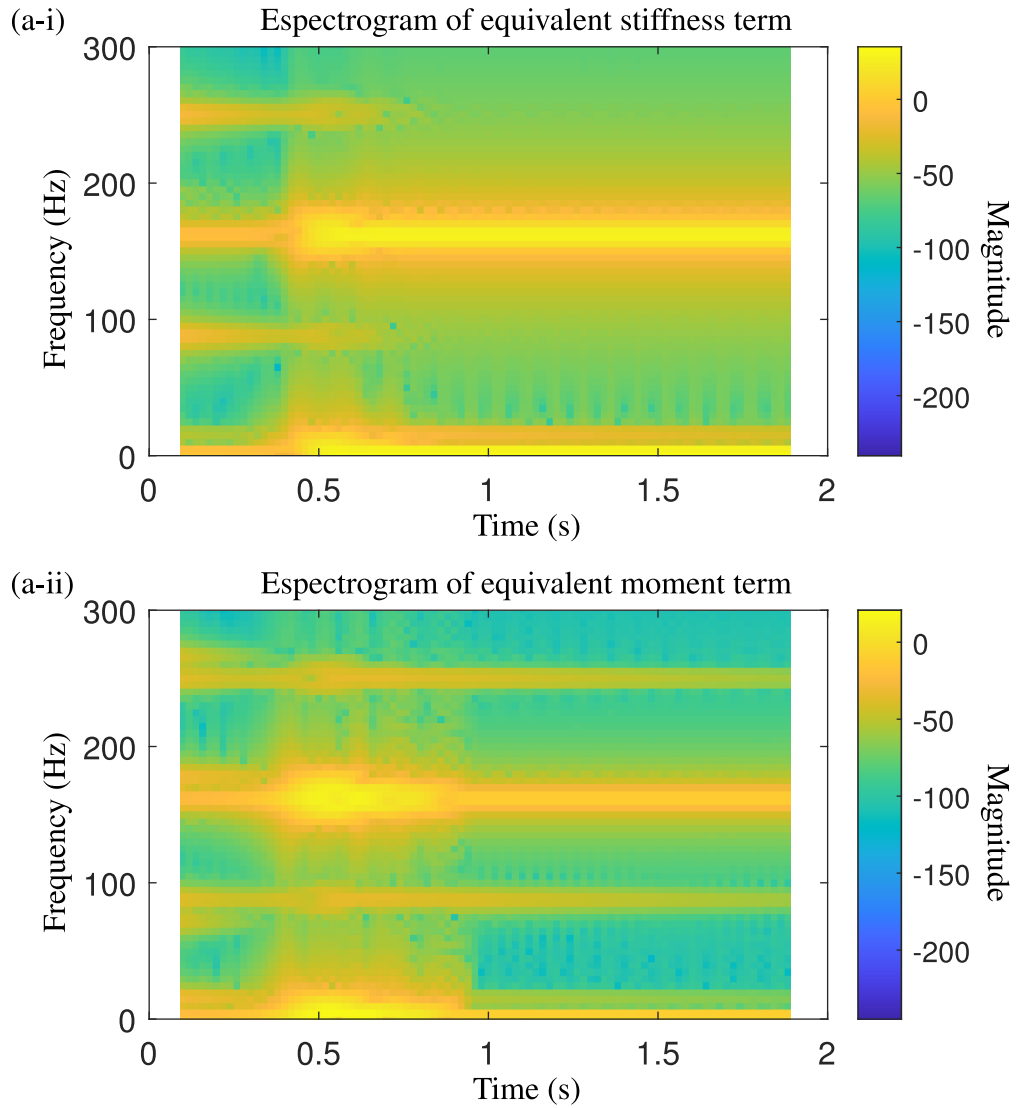


Figure 5.2: Spectrograms of the linearised excitation terms under harmonic base excitation at 81 Hz. (a-i) Spectrogram of the stiffness term. (a-ii) Spectrogram of the moment term.

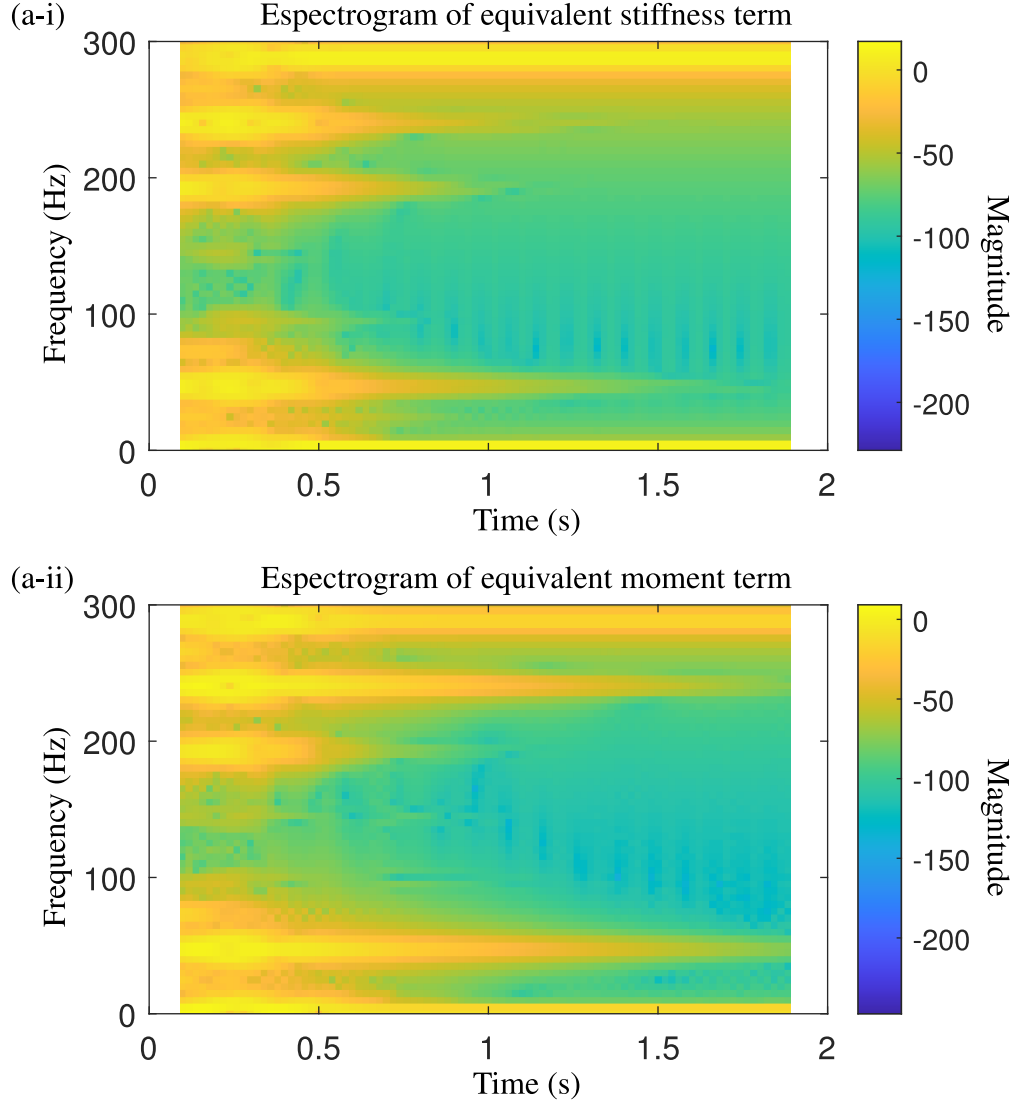


Figure 5.3: Spectrograms of the linearised excitation terms under harmonic base excitation at 144 Hz. (a-i) Spectrogram of the stiffness term. (a-ii) Spectrogram of the moment term.

A distinct spectral component at 162 Hz is observed in both plots of Figure 5.2, confirming the modulation of these terms at 2ω , characteristic of auto-parametric coupling. The equivalent moment term exhibits transient activation during the rotation interval, while the equivalent stiffness term remains continuously modulated after excitation onset. For the Figure 5.3, both terms exhibit strong spectral content at 288 Hz, supporting the presence of parametric excitation induced by the base motion. Similar to the previous case, the moment term is prominent only during the transient rotation, whereas the stiffness modulation persists throughout the time span.

The spectrograms provide numerically grounded evidence supporting the presence of auto-parametric excitation in the system. The energy transfer between translational and rotational modes is frequency-modulated, and its temporal distribution is closely linked to the timing of

the self-rotation and subsequent stabilization of the neutraliser. The transient activation of the moment term, observed only during the rotation interval, combined with the continuous presence of the stiffness modulation throughout the excitation period, suggests that the rotation is driven by an initial torque impulse and sustained by a persistent parametric stiffness modulation.

5.7 Method of direct separation of motion

The Method of Direct Separation of Motion (MDSM), introduced by Blekhman (2000), is a well-established analytical technique in nonlinear vibration theory, particularly suited for systems exhibiting distinct timescales in their dynamic response. Its fundamental principle is to decouple the fast and slow components of motion, enabling the derivation of averaged equations that govern the slow dynamics while incorporating the influence of the fast oscillations.

This methodology is especially effective in analysing systems subjected to high-frequency excitations or those with internal couplings that give rise to vibrational forces, effective forces that emerge from fast dynamics and act upon the slow motions. These vibrational forces are instrumental in explaining phenomena such as dynamic stabilisation and auto-parametric resonance, where high-frequency motions can sustain or modify the equilibrium of a slower subsystem, as exemplified by Kapitza's pendulum.

In the context of the present study, the MDSM provides a complementary viewpoint by isolating the slow rotational dynamics from the fast translational response induced by the base excitation. The angular displacement is represented as the sum of a slowly varying component and a small, fast fluctuation:

$$\theta(t) = \Theta(t) + \Psi(t, \tau). \quad (5.11)$$

where $\Theta(t)$ evolves over a slow time scale associated with the rotation, and $\Psi(t, \tau)$ contains fast oscillations at frequency ω or its harmonics. $\tau = \omega t$ denotes the fast timescale associated with the excitation frequency.

Substituting this decomposition into the rotational equation (5.4) and expanding the trigonometric terms using standard identities yields a full equation containing both slow and fast

components:

$$\begin{aligned}
I(\ddot{\Theta} + \ddot{\Psi}) + c_r(\dot{\Theta} + \dot{\Psi}) + \mu \operatorname{sgn}(\dot{\Theta} + \dot{\Psi}) - \tilde{k} \left[A_1 \cos(2\Theta) \cos(2\Psi) \right. \\
\left. - A_1 \sin(2\Theta) \sin(2\Psi) + \frac{1}{2} A_2 \sin(2\Theta) \cos(2\Psi) + \frac{1}{2} A_2 \cos(2\Theta) \sin(2\Psi) \right] \\
- \tilde{k} \left[A_1 \cos(2\omega t) \cos(2\Theta) \cos(2\Psi) - A_1 \cos(2\omega t) \sin(2\Theta) \sin(2\Psi) \right. \\
\left. + \frac{1}{2} A_2 \cos(2\omega t) \sin(2\Theta) \sin(2\Psi) + \frac{1}{2} A_2 \cos(2\omega t) \cos(2\Theta) \sin(2\Psi) \right] = 0. \quad (5.12)
\end{aligned}$$

In this expanded expression: Terms containing $\cos(2\omega t)$, $\cos(2\Psi)$ and $\sin(2\Psi)$ oscillate rapidly and represent the fast dynamics; Terms depending solely on $\Theta(t)$ define the slow response of the system.

To isolate the slow dynamics, the method proceeds by performing an averaging operation over a full period of the fast oscillation, defined as $T = \frac{2\pi}{\omega}$. For any fast-varying function $f(t)$, the average is given by:

$$\langle f \rangle = \frac{1}{T} \int_0^T f(t) d\tau. \quad (5.13)$$

Assuming that the slow dynamics evolve over a significantly longer time-scale than the excitation, all oscillatory terms involving the fast variables Ψ or $\cos(2\omega t)$ average out to zero. Thus, only the slow terms survive in the averaged equation. The resulting governing equation for the slow rotational dynamics becomes:

$$I\ddot{\Theta} + c_r\dot{\Theta} + \mu \operatorname{sgn}(\dot{\Theta}) - \tilde{k} \left[A_1 \cos(2\Theta) + \frac{1}{2} A_2 \sin(2\Theta) \right] = 0 \quad (5.14)$$

5.8 Comparison of self-rotation and classical auto parametric systems

The self-rotation behaviour exhibited by the vibration neutraliser shares key features with classical auto-parametric systems, such as parametric excitation at twice the driving frequency and the existence of multiple potential equilibria whose stability depends on the forcing conditions.

A central feature of auto-parametric systems is the transfer of energy between modes, which depends critically on the amplitude and frequency of excitation. In classical formulations, such

as those studied by Stephenson (1908) and later by Tondl (2000), this modal interaction can lead to instability thresholds and frequency multiplication effects. Analogously, in the neutraliser, the rotational motion does not arise from direct actuation, but rather through a secondary excitation mechanism induced by the oscillatory base motion. In the present system, this pathway alone would not generate sustained rotation, because the angular coordinate does not grow through parametric instability. Instead, rotation is initiated by a time-dependent moment that directly injects angular momentum into the rotational mode. The parametric stiffness, although essential, shows a supporting role as it shapes the equilibrium configuration and introduces frequency-doubling effects, but does not itself cause the rotation to begin.

One of the defining signatures of auto-parametric excitation is the emergence of dynamic responses at twice the primary excitation frequency. This effect is a consequence of the nonlinear coupling between the interacting modes. When a system is excited at a fundamental frequency ω , the coupling can give rise to internal responses not only at ω , but also at 2ω . In the case of the vibration neutraliser, this phenomenon is clearly observed in the term:

$$\tilde{k} A_2 \theta \cos(2\omega t + 2\phi_2)$$

which introduces a time-varying stiffness modulated at 2ω . This modulation is responsible for a parametric amplification effect that alters the stability of the rotational equilibrium points. Such behaviour mirrors what is classically understood in systems like the inverted pendulum with a vibrating base (Kapitza's pendulum), where rapid base oscillations create effective forces that reshape the potential landscape of the system.

In summary, the self-rotation mechanism of the neutraliser constitutes a practical instance of auto-parametric resonance. The nonlinear interaction between rotational and translational dynamics, the presence of excitation at 2ω , and the emergence of orientation-dependent stiffness all reflect fundamental traits of auto-parametric systems. This comparison not only validates the physical interpretation adopted in this work, but also situate the proposed neutraliser within a broader class of well-studied nonlinear dynamic systems.

5.9 Chapter summary

This chapter examined the self-rotation of a passive vibration neutraliser under harmonic base excitation and demonstrated that the phenomenon arises from nonlinear modal coupling

between translational and rotational degrees of freedom, made possible by stiffness asymmetry. The rotation is driven by a transient coupling-induced moment, which transfers energy from translation to rotation and initiates reorientation, while a persistent parametric stiffness modulation at twice the excitation frequency determines and stabilises the final orientation. Linearisation of the governing equation revealed that both time-varying moment and stiffness terms are present, although only the stiffness term remains once rotation ceases. Numerical simulations and spectrograms corroborated this behaviour by showing that the moment term is active only during the transient phase, whereas the stiffness modulation persists throughout excitation. The MDSM analysis further showed that the slow angular dynamics evolve within an effective potential containing two possible orientations, of which only one is stable for a given excitation.

6 Antiresonance-based characterisation of the TC1 neutraliser

In Chapter 3, while developing the frequency domain model, the resonance frequencies of the base were analytically derived from the undamped model using Equation (3.7), and defined as f_{r1} and f_{r2} by Equations (3.10) and (3.11), respectively. These resonance frequencies are independent of the angular position θ of the device. In contrast, the antiresonance frequencies exhibit a dependence on θ , such that only the cases for $\theta = 0^\circ$ and $\theta = 90^\circ$ were explicitly defined, as expressed in Equations (3.12) and (3.13). For arbitrary orientations, the base acceleration displays two resonance and two antiresonance frequencies. However, when θ assumes orthogonal orientations (0° , 90° , and their multiples), the equations governing motion in the vertical and horizontal directions become decoupled, resulting in a single identifiable resonance and a single antiresonance frequency.

As already said in Section 3.1, although TC1 shares several design principles with previously studied configurations it has unique dynamic behaviours. Given these distinctions, a dedicated investigation becomes necessary to isolate and analyse the effects introduced by the TC1 configuration. This chapter, therefore, focuses exclusively on the modelling and experimental validation of the TC1 neutraliser. Special attention is devoted to its frequency response behaviour under varying angular orientations.

6.1 Modal characterisation of the TC1 neutraliser

The experimental procedure adopted for the TC1 neutraliser followed the same methodology previously described for the TN2. The experimental rig is shown in Figure 6.1. A Modal Shop mini smartshaker (model K2007E01) with integrated amplifier and a PCB-208C02 force sensor were used to apply white noise excitation. Accelerations at the base and tip masses were measured using PCB-352A24 accelerometers, and data were acquired via a Dewesoft SIRIUSi system at 5 kHz. Frequency response functions were computed using the H2 estimator with a

Hanning window (50% overlap) and 50 linear averages, ensuring reliable identification of the modal parameters.

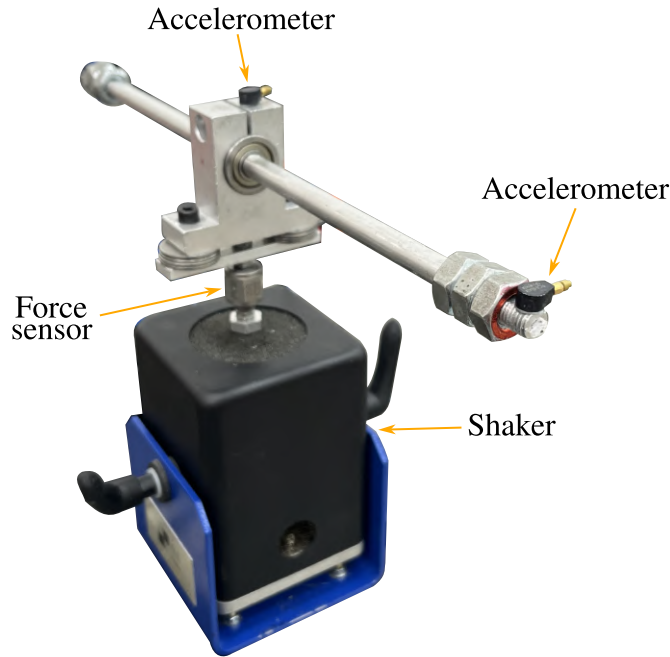


Figure 6.1: Experimental rig used for the modal test. The neutraliser is mounted on a Modal Shop mini smartshaker, and the excitation force is measured using a PCB force sensor. Accelerometers were positioned at the base and tip masses to capture the dynamic response.

Figure 6.2 compares the model predictions with the experimental accelerance responses measured at the base and tip masses of the TC1 neutraliser. In each sub plot, thin blue curves represent the numerical model, while thick grey curves correspond to the experimental measurements. Sub figures (i), (ii), and (iii) refer to angular positions $\theta = 0^\circ$, 45° and 90° , respectively. The associated coherence functions are shown in Figure 6.3, where values approaching unity confirm a high correlation between input and output signals, supporting the validity of the measurements over most of the frequency range.

The model employed for comparison corresponds to the steady-state response of the neutraliser at each fixed angular position, consistent with the experimental configuration. Parameter identification was performed qualitatively by fitting the low- and high-frequency asymptotic behaviour and aligning the resonance frequencies. Subsequently, viscous damping values were manually tuned to match the amplitude of the resonance peaks observed in the measured data.

The identified parameters of the model are shown in Table 6.1. Overall, the model shows good agreement with the experimental results, though some divergences are observed, mainly at the base and tip antiresonances, where is also observed a drop in the coherences of Figure

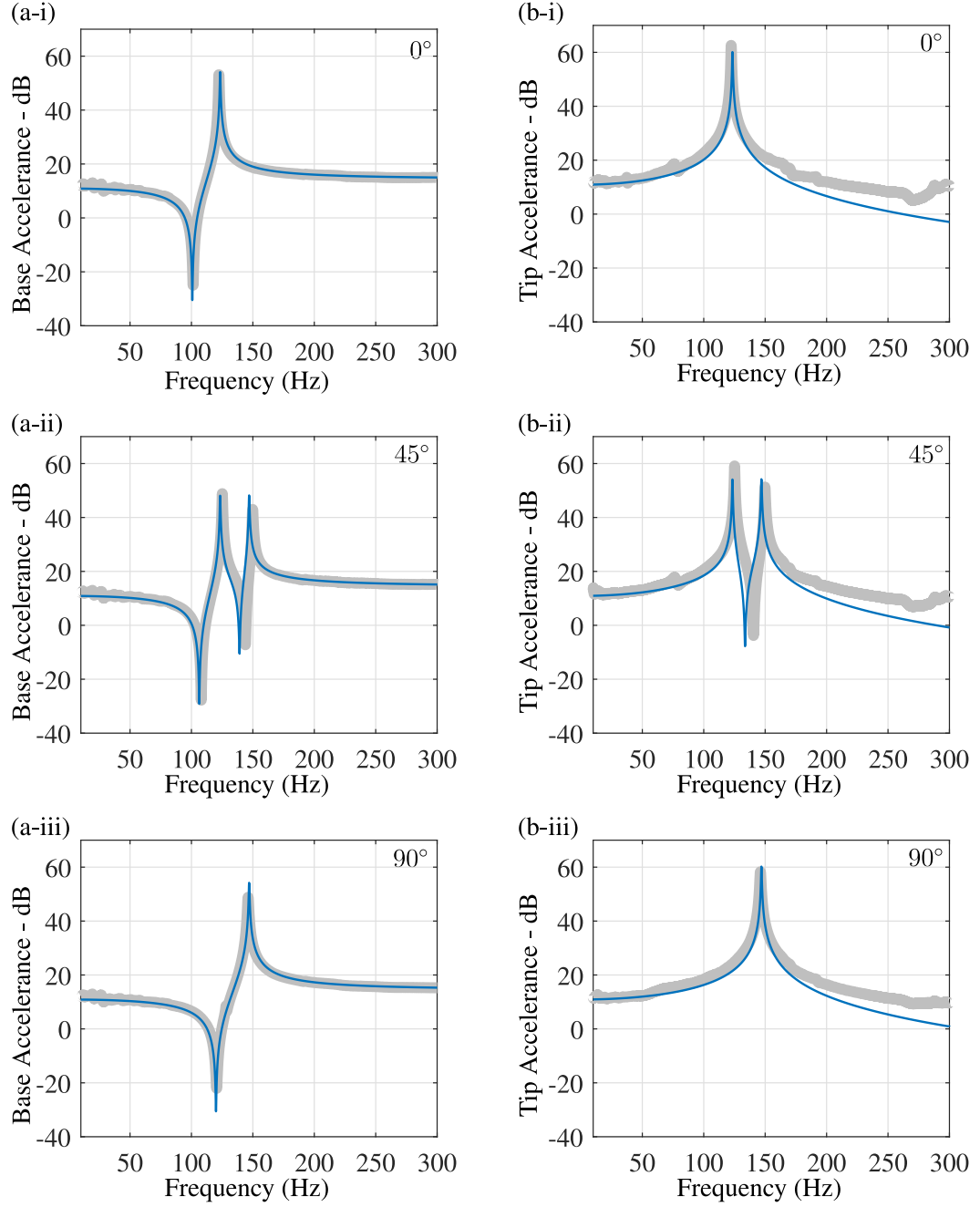


Figure 6.2: Magnitudes of the accelerance responses of the TC1 at three angular positions ($\theta = 0^\circ, 45^\circ$, and 90°) in the global reference system ($X_B Y_B$). Subfigures (a) present the base accelerance, while subfigures (b) show the tip accelerance. Experimental results are represented by the thick grey lines, and model predictions by the thin blue lines. (dB ref. $1 \text{ N s}^2/\text{m}$).

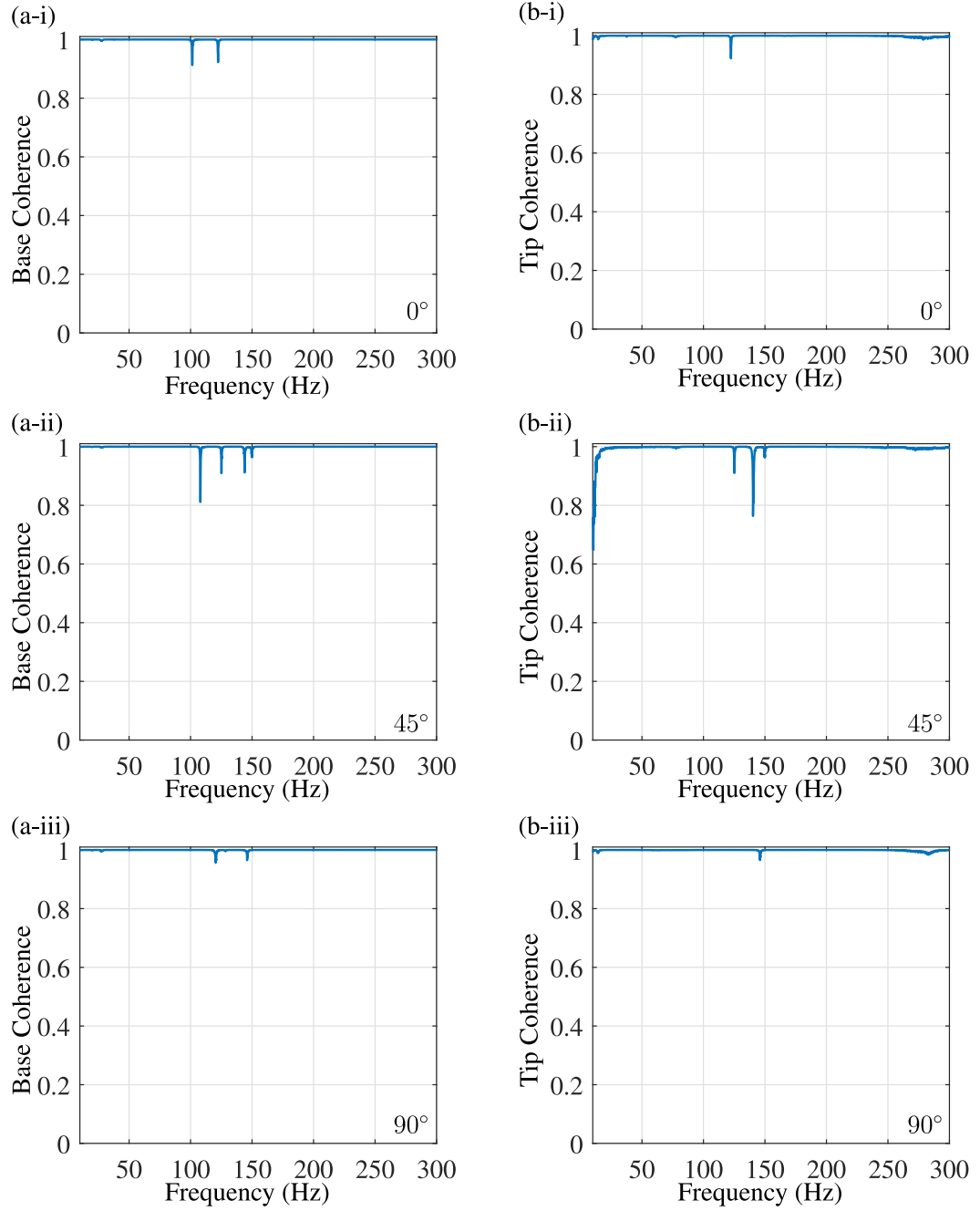


Figure 6.3: Coherence functions obtained from the TC1 modal experiments at three angular positions $\theta = 0^\circ$, 45° , and 90°). Subfigures (a) present the base coherence, while subfigures (b) show the tip coherence. Values close to 1 indicate high correlation between input and output signals, confirming measurement reliability across most of the frequency range.

6.3.

Table 6.1: Parameters of the TC1 neutraliser obtained through steady-state experiments

Property	TC1
Stiffness in x direction k_{xx} [N/m]	54000
Stiffness in y direction k_{yy} [N/m]	38050
Base mass m_B [kg]	0.190
Neutraliser mass m_N [kg]	0.095

6.2 Antiresonance variation with angular position

As previously discussed, the antiresonance frequencies of the base are intrinsically dependent on the angular orientation θ of the neutraliser. The variation of these antiresonance frequencies as a function of θ is investigated using the modal parameters for the TN2, experimentally identified in Chapter 3, in Figure 6.4. These parameters provide a reliable basis for characterizing the frequency response of the system and exploring how geometric aspect influences the dynamic decoupling between orthogonal directions.

Figure 6.4(a) depicts the variation of the antiresonances frequencies f_3 (orange line) and f_4 (green line) as a function of the angular position θ , confirming that the antiresonances are strongly dependent on the device orientation. And the two dashed black lines, correspond to the resonance frequencies f_{r1} and f_{r2} that are constant across the angular range, independent of the angular orientation. Each sub figure (b-i to b-iii) shows the magnitude of base acceleration in dB across frequency for a fixed angular orientation.

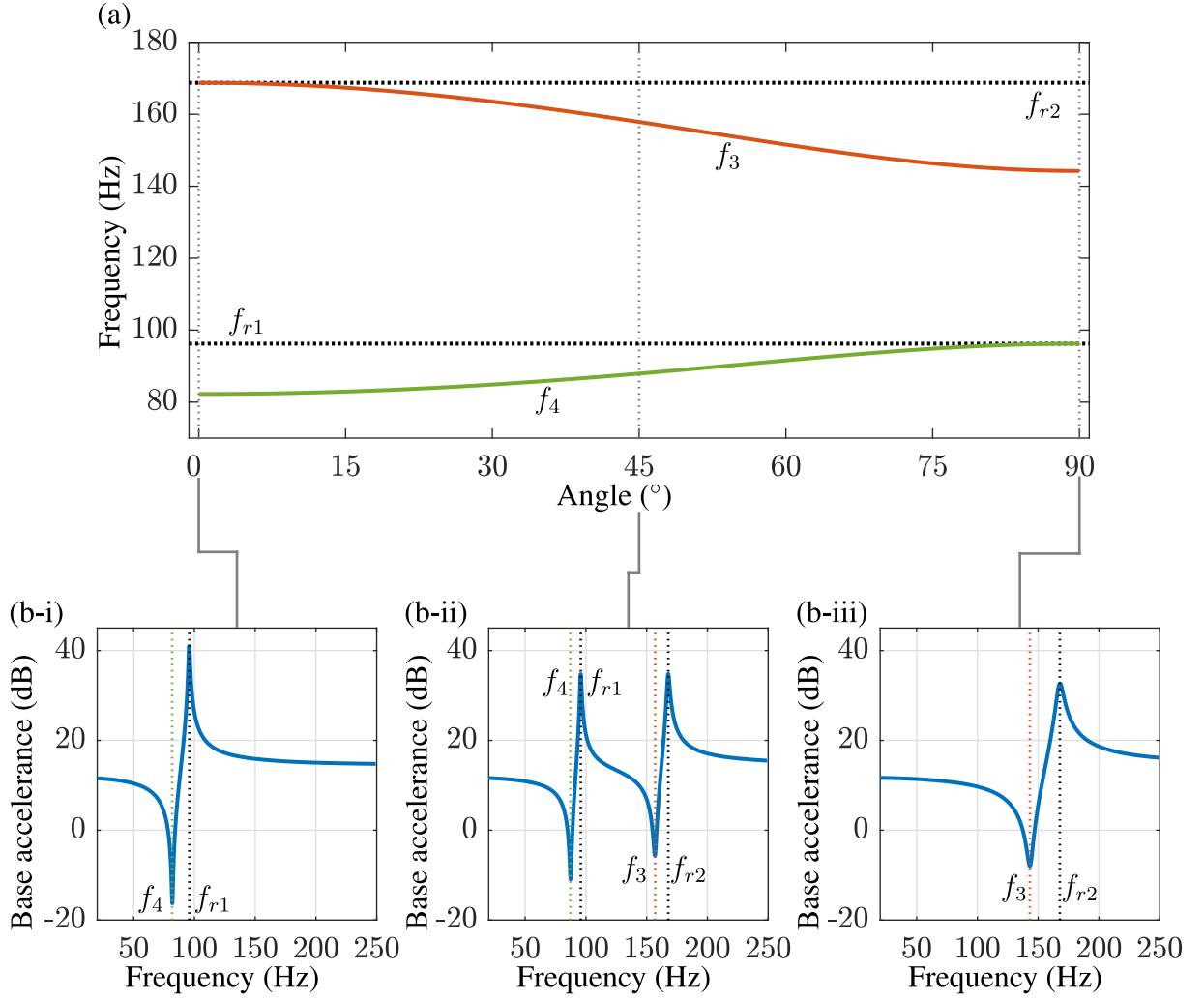


Figure 6.4: (a) Evolution of the base antiresonance frequencies f_3 and f_4 as a function of the angular position θ , computed from the experimentally identified modal parameters of the TN2 neutraliser. The resonance frequencies f_{r1} and f_{r2} , which are independent of θ , are indicated as horizontal dashed lines for reference. Sub figures (b-i) to (b-iii) are the base acceleration for selected orientations: (b-i) for $\theta = 0^\circ$, (b-ii) for $\theta = 45^\circ$ and (b-iii) for $\theta = 90^\circ$. The corresponding resonance and antiresonance frequencies are highlighted with vertical dashed lines in each plot.

The identified modal parameters for the TC1 neutraliser are used to investigate its antiresonance behaviour as a function of angular orientation, as illustrated in Figure 6.5. The evolution of the antiresonance frequencies f_3 (orange line) and f_4 (green line) with respect to the angular position θ is presented in Figure 6.5(a). In contrast, the resonance frequencies f_{r1} and f_{r2} , indicated by the two dashed black lines, remain constant over the full angular range, as they are not influenced by the neutraliser's orientation. Sub figures (b-i) to (b-iii) display the magnitude of the base acceleration as a function of frequency for fixed angular positions.

Unlike the TN2 neutraliser, where each antiresonance (f_3 and f_4) corresponds to cancelling a different resonance at specific angular positions (i.e., f_3 cancels f_{r2} at $\theta = 0^\circ$ and f_4 cancels

f_{r1} at $\theta = 90^\circ$), the TC1 neutraliser exhibits only one effective antiresonance. In this case, f_3 counteracts both resonances: it counteracts f_{r2} at $\theta = 0^\circ$ and f_{r1} at $\theta = 90^\circ$. Although the second antiresonance f_4 is present, it does not coincide with either resonance.

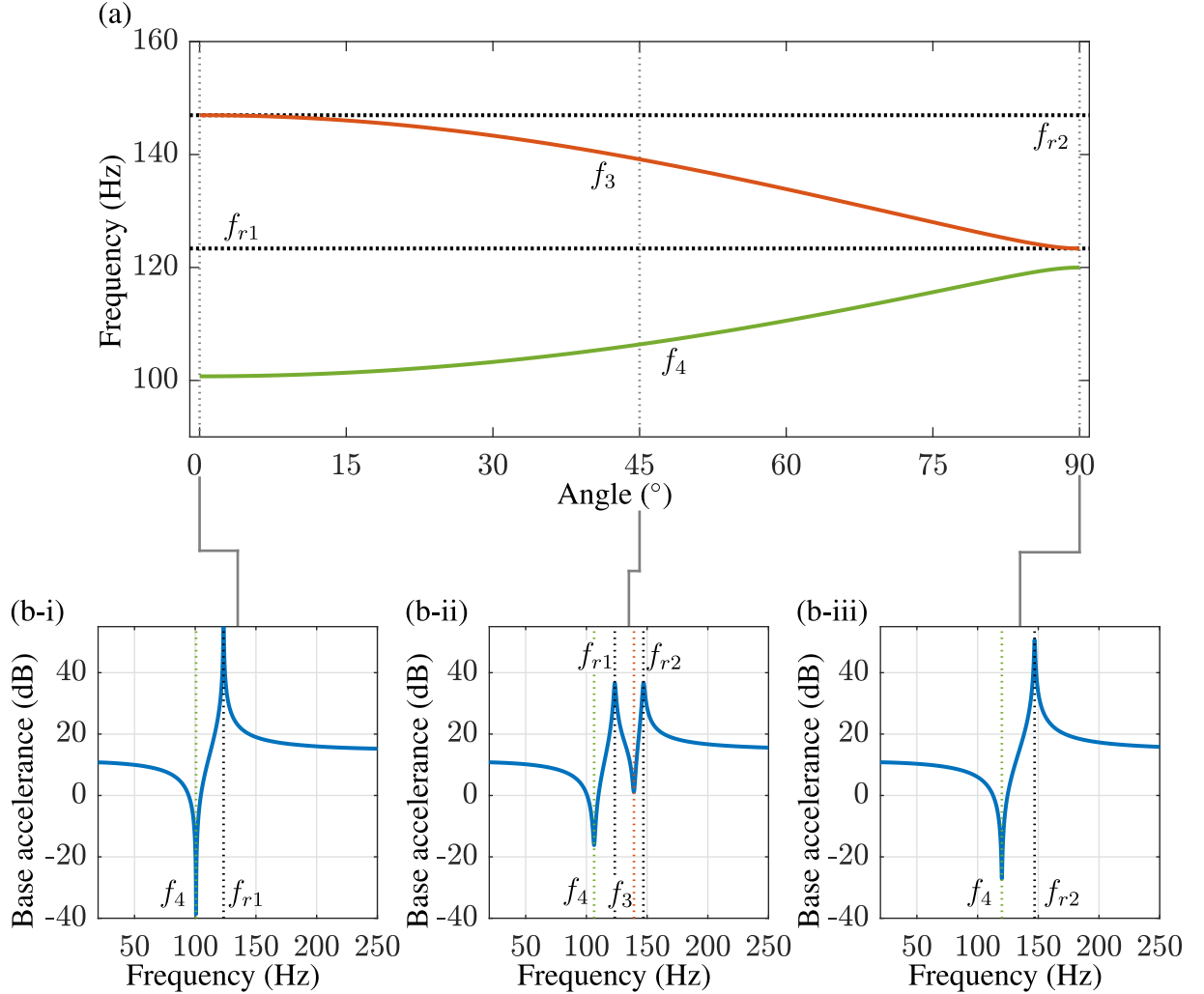


Figure 6.5: (a) Evolution of the base antiresonance frequencies f_3 and f_4 as a function of the angular position θ , computed from the experimentally identified modal parameters of the TC1 neutraliser. The resonance frequencies f_{r1} and f_{r2} , which are independent of θ , are indicated as horizontal dashed lines for reference. Sub figures (b-i) to (b-iii) are the base acceleration for selected orientations: (b-i) for $\theta = 0^\circ$, (b-ii) for $\theta = 45^\circ$ and (b-iii) for $\theta = 90^\circ$. The corresponding resonance and antiresonance frequencies are highlighted with vertical dashed lines in each plot.

6.3 Influence of mass and stiffness ratios on anti resonance behaviour

The influence of mass and stiffness ratios on the anti resonance behaviour shed light on the distinctive behaviour previously observed in the TC1 neutraliser and clarified why it did not perform in the same manner as the TN2. Motivated by this discrepancy, the investigation proceeded to examine whether the presence of only one effective antiresonance could be attributed to specific geometric configurations. This led to a more detailed parametric study aimed at understanding the influence of mass and stiffness ratios on the positioning of resonances and antiresonances. The focus in this section is to determine how such parameters govern the dynamic behaviour of the system and to identify conditions under which the existence of a single effective antiresonance emerges, offering deeper insights into the mechanisms behind the differing responses of each neutraliser design.

At 0° and 90° , the system demonstrates a singular resonance instead of two, contrasting with observations at other angles. The anti resonance at 90° , can vary, being either lower or higher than the resonance frequency f_{r1} depending on mass and stiffness ratios, μ and κ , respectively. There are two situations, in one of them, the antiresonance f_3 coincides the resonance f_{r1} when $\theta = 90^\circ$. In the other situation, is the anti resonance f_4 that coincides the resonance f_{r1} when $\theta = 90^\circ$. To examine this behaviour comprehensively, mass and stiffness ratios are defined by Equation 6.1, allowing them to vary within the range of zero to one.

$$\begin{aligned}\mu &= \frac{m_N}{m_B} \\ \kappa &= \frac{k_{yy}}{k_{xx}}.\end{aligned}\tag{6.1}$$

Given the potential for f_4 to align with f_{r1} , it is advisable to avoid this scenario, as it leads to a situation where the system exhibits only one tuning frequency, represented by the antiresonance f_3 . A simplified expression for the anti resonance at $\theta = 90^\circ$ is provided by Equation (3.13), emphasizing the necessity for it to surpass the resonance f_{r1} , as defined in Equation (3.10). This observation leads to the formulation of a simple criterion, expressed in Equation (6.6), which

ensures the desired relationship between anti resonance and resonance frequencies.

$$f_4|_{\theta=90^\circ} > f_{r1} \quad (6.2)$$

$$\left[\frac{k_{xx}}{m_N} \right]^{1/2} > \left[\frac{k_{yy}(m_N + m_B)}{m_N m_B} \right]^{1/2} \quad (6.3)$$

$$\frac{k_{xx}}{k_{yy}} > \frac{m_N(m_N + m_B)}{m_N m_B} \quad (6.4)$$

substituting μ and κ

$$\kappa > \frac{m_N^2 \mu}{m_N^2 (\mu + 1)} \quad (6.5)$$

$$\kappa > \frac{1}{\mu + 1} \quad (6.6)$$

In Figure 6.6, the influence of mass and stiffness ratios on the anti-resonances is explored, presenting a comparative analysis between TN2 and TC1, each characterized by distinct mass and stiffness ratios, providing insights into the influence of these parameters on system dynamics. Sub figures (b) and (c) provides an overhead view of the base accelerance for TN2 and TC1, respectively, illustrating variations in anti resonances across different angles. Together, these components contribute to a comprehensive understanding of the system's dynamic behaviour under varying conditions. The green area in Figure 6.6(a), ensures the desired relationship between antiresonance and resonance frequencies. In this representation, the variables μ and κ are confined within the defined green region, visually affirming the conditions for maintaining $f_4|_{\theta=90^\circ} > f_{r1}$.

The TC1 device operates differently from the other neutralisers and displays inconsistent behaviour. This disparity in performance may be attributed to differences in mass and stiffness ratios, as evidenced by its position within the gray region in Figure 6.6(a). To elaborate, multiple experimental tests were conducted on TC1, varying the excitation frequency, initial angle, and shaker amplitude, aiming to induce rotation. Although some rotation was observed, the device consistently settled at 0° regardless of the excitation frequency, indicating a lack of frequency-

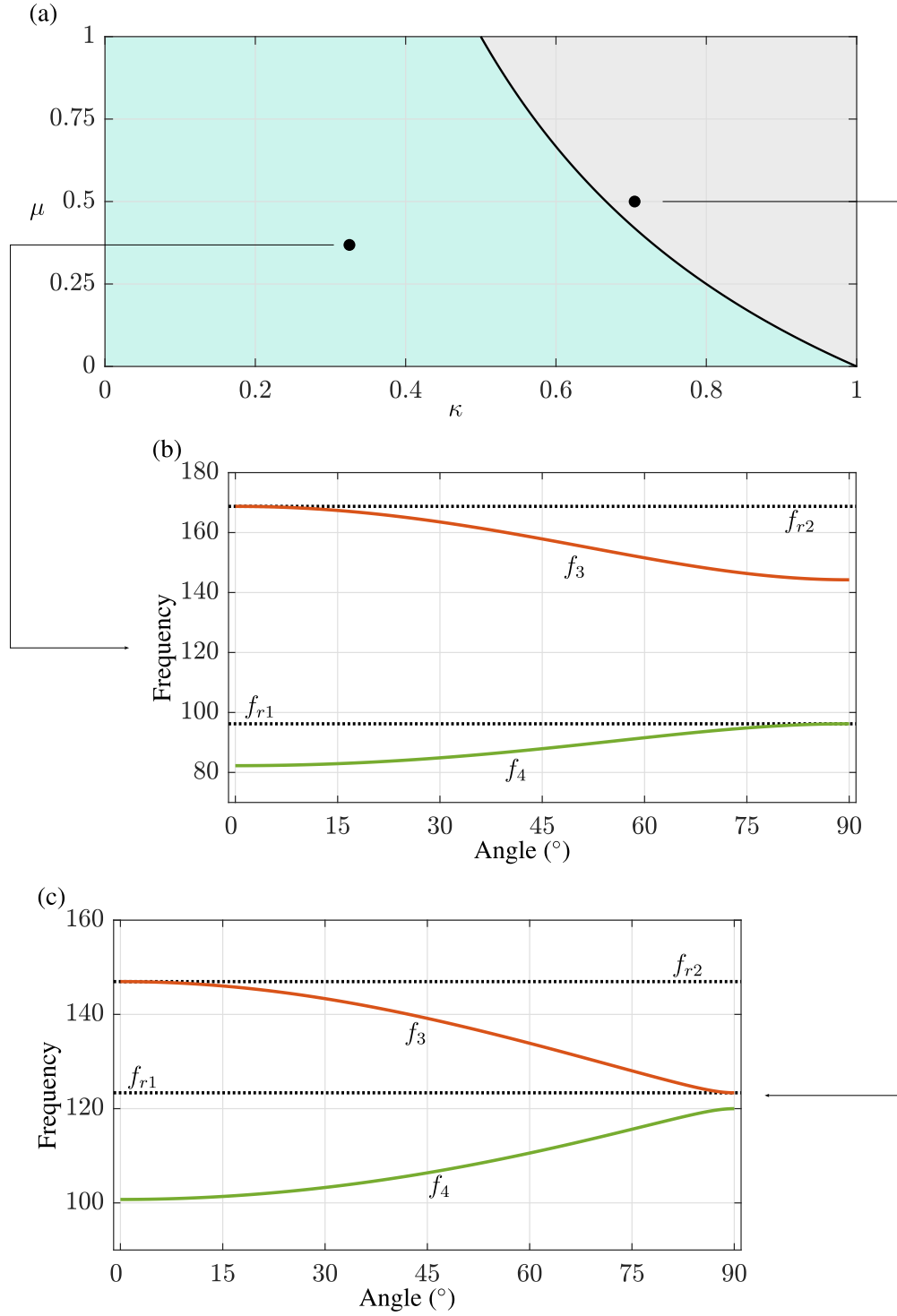


Figure 6.6: (a) Influence of mass and stiffness ratio on the anti resonances. Anti resonances behaviour for a system with (a) $\kappa = 0.32$ and $\mu = 0.37$ and (b) $\kappa = 0.70$ and $\mu = 0.50$

dependent tuning response. This behaviour may be explained by the fact that only one of its antiresonances (f_3) is responsible for counteracting both resonance frequencies, unlike the TN2 neutraliser, where each antiresonance is individually tuned to suppress a different resonance at a specific angular orientation.

6.4 Experimental investigation of self-turning in TC1

Following the validation of the time-domain model, the formulation presented in Section 4.1 was also applied to the TC1 neutraliser. Although this device displays a distinct behaviour, consistently rotating to 0° regardless of the excitation frequency, the time domain model accurately reproduces its self-turning response only when excitation occurs outside the region between the resonance frequencies f_{r1} and f_{r2} . To support this analysis, harmonic excitation experiments with the beam free to rotate were also conducted on the TC1, following the same procedure described for the TN2 neutraliser in Section 4.2. The results are exhibited in Figure 6.7.

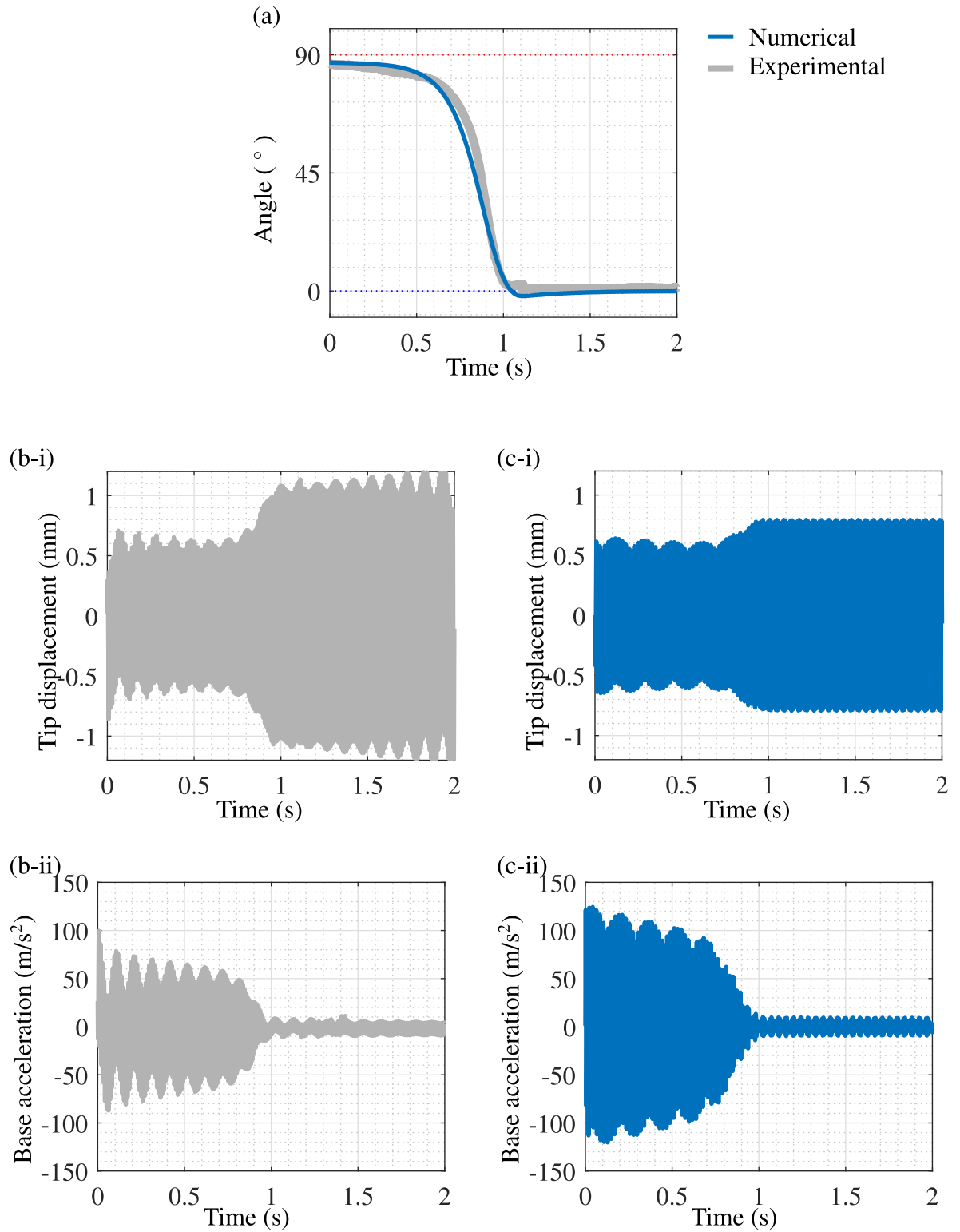


Figure 6.7: Comparison between numerical and experimental results for harmonic excitation at 100 Hz.(a) Angular displacement of the neutraliser. The gray line represents the experimental response and the blue line corresponds to the numerical simulation. Sub figures (b-i) and (b-ii) show the experimental measurements of the tip displacement and base acceleration. And sub figures (c-i) and (c-ii) displays the corresponding numerical simulations of tip displacement and base acceleration.

The experimental results and numerical simulations show strong qualitative agreement. Figure 6.7(a) presents the angular displacement, where both curves exhibit a comparable delay before the onset of rotation. Sub figures (b-i) and (c-i) display the tip displacement from the experiment and simulation, respectively, both presenting consistent dynamic behaviour throughout the excitation. Finally, sub figures (b-ii) and (c-ii) illustrate the base acceleration, which decreases following the conclusion of the self-turning motion, aligning well with the expected vibration attenuation effect.

6.5 Colormap analysis of the TC1 self-turning behaviour

Having confirmed that the time-domain model accurately predicts the self-turning behaviour of the TC1 neutraliser when excitation occurs outside the resonance band, the analysis was extended to a broader set of excitation conditions. In this context, the same parametric simulation procedure described in Section 4.3 was applied to the TC1 configuration, enabling a systematic evaluation of its dynamic response over varying force and frequency inputs. The resulting colormap is presented in Figure 6.8.

In addition to the fact that this neutraliser possesses only one effective antiresonance, the frequency range between its two resonance peaks is notably narrow. This limited interval is almost entirely dominated by non-deterministic behaviour, as observed in the simulations. These combined factors further explain why the TC1 device does not exhibit the same self-turning effectiveness as the other neutralisers investigated.

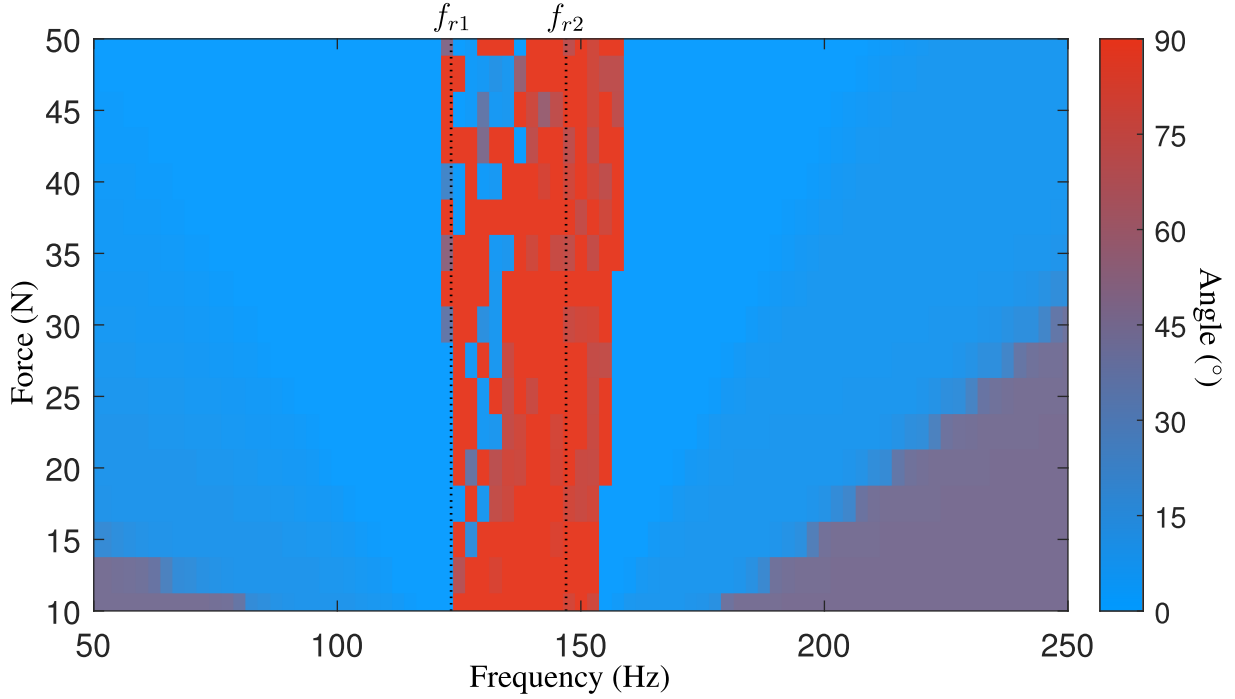


Figure 6.8: Numerical colormap for the TC1 showing the final angular position of the neutraliser as a function of excitation frequency and force amplitude. The colour scale represents the angle θ . The dotted vertical lines indicate the first (f_{r1}) and second (f_{r2}) natural frequencies of the neutraliser base. The transition between colours exposes how the excitation conditions influence the preferred orientation of the device.

6.6 Chapter summary

This chapter presented an extensive analysis of the TC1 neutraliser, highlighting its distinctive dynamic characteristics in comparison to the other devices studied. A frequency-domain model was validated against experimental modal data, revealing that, unlike TN2, the TC1 possesses only one effective antiresonance capable of cancelling both system resonances. This structural limitation, combined with a narrow frequency range between resonances, was found to be closely linked to the specific mass and stiffness ratios of the configuration. A parametric analysis further clarified the influence of these parameters on antiresonance behaviour, offering insights into the design constraints that led to the atypical response observed in TC1. Time-domain simulations and experiments demonstrated that, although the TC1 consistently self-rotates to 0° , the model accurately predicts this outcome when the excitation occurs outside the resonance region. Finally, a numerical colormap corroborated the limited effectiveness of the TC1 in achieving frequency-dependent orientation, reinforcing the understanding of how its geometric and dynamic properties restrict its vibration attenuation capabilities.

7 Conclusions and suggestions for future work

This thesis has introduced a passive vibration neutraliser composed of a beam-like structure with a central roller bearing, aimed at enhancing the robustness of neutralisers that are currently effective only within a narrow frequency range. Capable of autonomously turning itself to either of two angular positions when excited by a harmonic force, the neutraliser was experimentally characterized to elucidate its dynamic parameters. It exhibits two tuned frequencies corresponding to its alignment at 0 or 90°. The self-turning phenomenon was experimentally demonstrated, and a 5-DOF time-domain model was developed to describe the neutraliser behaviour under harmonic excitation in the time domain. Experimental results with TN1 and TN2 neutralisers corroborated the numerical model, highlighting the efficacy of the time-domain approach in studying self-turning behaviour. Additionally, linearisation of the angular position equation provides insights into the rotational moment, suggesting an auto-parametric system. This observation may provide a potential explanation for the self-turning behaviour exhibited by the neutraliser.

Analysis of the stiffness and mass ratios identified two types of neutralisers with distinct characteristics. The TN-type has two tuning frequencies, which are the anti-resonances f_3 or f_4 at angular positions of 0 or 90°, respectively. These frequencies depend on the stiffness characteristics (k_{yy} and k_{xx}) and the mass (m_N) of the neutraliser. When two distinct frequencies contribute to undesired dynamic responses, tuning the neutraliser to correspond with these frequencies offers an effective means of mitigating their influence. In this way, the neutraliser will self-turn to one of the orientations (0 or 90°), depending on which of the frequencies is active at the moment and exciting it. This allows the neutraliser to minimize unwanted vibrations at both troublesome frequencies, providing an effective solution for attenuating specific vibrations.

The numerical colormaps employ the 5-DOF numerical time domain model to obtain the angular response of each device. After obtaining the angular response, certain conditions are defined to obtain the colormap. Thus, it accounts for numerical errors besides the simplification of the conditions to define the colors printed on the map. Although simplified, the colormaps show a non-deterministic behaviour present in both neutralisers. This behaviour was confirmed

experimentally, but the reasons for it remain unclear. In TC-type neutralisers, the narrow range between f_{r1} and f_{r2} results in an unpredictable dynamic response within this narrow frequency band.

Despite the uncertainties presented thus far, the work discussed here opens the possibility of enhancing the robustness of neutralisers that are currently effective only within a narrow frequency range.

7.1 Suggestions for future work

Given the distinct behaviour and inconsistent performance of the TC1 device compared with the other neutralisers, future work should focus on conducting a more comprehensive experimental investigation of this configuration. Such studies could include steady-state analyses as well as tests under harmonic excitation, aiming to clarify the underlying mechanisms governing its dynamic response. These efforts would contribute to improving the accuracy and reliability of the model, thereby enabling a deeper understanding of the neutraliser's overall dynamic behaviour.

References

- Acar, M. and Yilmaz, C. (2013). Design of an adaptive–passive dynamic vibration absorber composed of a string–mass system equipped with negative stiffness tension adjusting mechanism. *Journal of Sound and Vibration*, 332(2):231–245.
- Asadi Khanouki, M., Sedaghati, R., and Hemmatian, M. (2020). Multidisciplinary design optimization of a novel sandwich beam-based adaptive tuned vibration absorber featuring magnetorheological elastomer. *Materials*, 13(10):2261.
- Babikian, R., Lukachko, S. P., and Waitz, I. A. (2002). The historical fuel efficiency characteristics of regional aircraft from technological, operational, and cost perspectives. *Journal of Air Transport Management*, 8(6):389–400.
- Balachandran, B. and Magrab, E. B. (2018). *Vibrations*. Cambridge University Press.
- Bian, Y. and Gao, Z. (2017). Nonlinear vibration absorption for a flexible arm via a virtual vibration absorber. *Journal of Sound and Vibration*, 399:197–215.
- Blekhman, I. I. (2000). *Vibrational mechanics: nonlinear dynamic effects, general approach, applications*. World Scientific.
- Bonello, P., Brennan, M. J., and Elliott, S. J. (2005). Vibration control using an adaptive tuned vibration absorber with a variable curvature stiffness element. *Smart Materials and Structures*, 14(5):1055.
- Brennan, M. (1997a). Characteristics of a wideband vibration neutralizer. *Noise Control Engineering Journal*, 45(5):201–207.
- Brennan, M. (1997b). Vibration control using a tunable vibration neutralizer. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 211(2):91–108.

- Brennan, M. J. (2006). Some recent developments in adaptive tuned vibration absorbers/neutralisers. *Shock and Vibration*, 13(4-5):531–543.
- Brennan, M. J. and Gatti, G. (2012). The characteristics of a nonlinear vibration neutralizer. *Journal of sound and vibration*, 331(13):3158–3171.
- Brennan, M. J. and Tang, B. (2022). *Virtual Experiments in Mechanical Vibrations: Structural Dynamics and Signal Processing*. John Wiley & Sons.
- Brown, D., Christian, W., and Belloni, M. (2020). Tracker Video Analysis and Modeling Tool. <https://physlets.org/tracker/>. Accessed: December 16, 2021.
- Carneal, J. P., Charette, F., and Fuller, C. R. (2004). Minimization of sound radiation from plates using adaptive tuned vibration absorbers. *Journal of Sound and Vibration*, 270(4-5):781–792.
- Craig Jr, R. R. and Kurdila, A. J. (2006). *Fundamentals of structural dynamics*. John Wiley & Sons.
- Den Hartog, J. P. (1985). *Mechanical vibrations*. Courier Corporation.
- Deng, H.-x., Gong, X.-l., and Wang, L.-h. (2006). Development of an adaptive tuned vibration absorber with magnetorheological elastomer. *Smart materials and structures*, 15(5):N111.
- Ewins, D. J. (2009). *Modal testing: theory, practice and application*. John Wiley & Sons.
- Ezure, K., Seto, K., et al. (1994). Vibration control of two-degree-of-freedom system using active dynamic absorber. *Transactions of the Japan Society of Mechanical Engineers*, 60:788–795.
- Frahm, H. (1909). Device for damping vibrations of bodies.
- Franchek, M., Ryan, M., and Bernhard, R. (1996). Adaptive passive vibration control. *Journal of Sound and Vibration*, 189(5):565–585.
- Ghorbani-Tanha, A., Rahimian, M., and Noorzad, A. (2011). A novel semiactive variable stiffness device and its application in a new semiactive tuned vibration absorber. *Journal of Engineering Mechanics*, 137(6):390–399.

- Ginder, J. M., Schlotter, W. F., and Nichols, M. E. (2001). Magnetorheological elastomers in tunable vibration absorbers. In *Smart structures and materials 2001: damping and isolation*, volume 4331, pages 103–110. SPIE.
- Gustavsson, M. (2011). A device for reducing vibrations and sounds.
- Gustavsson, M. (2016). A self-adaptive resonant device and its use for noise control in turbo-prop aircraft. In *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*, volume 253, pages 1353–1359. Institute of Noise Control Engineering.
- Hill, S. G. and Snyder, S. D. (2002). Design of an adaptive vibration absorber to reduce electrical transformer structural vibration. *J. Vib. Acoust.*, 124(4):606–611.
- Hino, J., Nanki, K., and Yoshimura, T. (1996). Semi-active dynamic absorber using fuzzy theory for a beam subjected to a moving load. *Control and Cybernetics*, 25:283–298.
- Hoang, N., Zhang, N., and Du, H. (2009). A dynamic absorber with a soft magnetorheological elastomer for powertrain vibration suppression. *Smart materials and structures*, 18(7):074009.
- Hong, O., Kerr, M. J., Poling, G. L., and Dhar, S. (2013). Understanding and preventing noise-induced hearing loss. *Disease-a-Month*, 59(4):110–118. Clinical Issues in Hearing Loss.
- Horn, B. (1986). *Robot vision*. MIT press.
- Hsu, Y.-S., Ferguson, N. S., and Brennan, M. J. (2013). The experimental performance of a nonlinear dynamic vibration absorber. In *Topics in Nonlinear Dynamics, Volume 1: Proceedings of the 31st IMAC, A Conference on Structural Dynamics, 2013*, pages 247–257. Springer.
- Hulshof, C. and Veldhuijzen van Zanten, B. (1987). Whole-body vibration and low-back pain: a review of epidemiologic studies. *International archives of occupational and environmental health*, 59:205–220.
- Ibrahim, R. (2008). Recent advances in nonlinear passive vibration isolators. *Journal of sound and vibration*, 314(3-5):371–452.
- Jang, S.-J., Brennan, M., Rustighi, E., and Jung, H.-J. (2012). A simple method for choosing the parameters of a two degree-of-freedom tuned vibration absorber. *Journal of Sound and Vibration*, 331(21):4658–4667.

- Kapitza, P. et al. (1951). Dynamic stability of a pendulum with an oscillating point of suspension. *Journal of experimental and theoretical physics*, 21(5):588–597.
- Kela, L. and Vähäoja, P. (2009). Recent studies of adaptive tuned vibration absorbers/neutralizers.
- Keye, S., Keimer, R., and Homann, S. (2009). A vibration absorber with variable eigenfrequency for turboprop aircraft. *Aerospace Science and Technology*, 13(4-5):165–171.
- Kidner, M. and Brennan, M. (1999). Improving the performance of a vibration neutraliser by actively removing damping. *Journal of Sound and Vibration*, 221(4):587–606.
- Kidner, M. and Brennan, M. (2002). Varying the stiffness of a beam-like neutralizer under fuzzy logic control. *J. Vib. Acoust.*, 124(1):90–99.
- Kumbhar, S. B., Chavan, S., and Gawade, S. (2018). Adaptive tuned vibration absorber based on magnetorheological elastomer-shape memory alloy composite. *Mechanical Systems and Signal Processing*, 100:208–223.
- Liu, K. and Rouch, K. E. (1993). An application of active dynamic absorber to the milling process and its experimental verification. In *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, volume 11771, pages 95–102. American Society of Mechanical Engineers.
- Long, T., Brennan, M., and Elliott, S. (1998). Design of smart machinery installations to reduce transmitted vibrations by adaptive modification of internal forces. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 212(3):215–228.
- Longbottom, C. J., Day, M. J., and Rider, E. (1990). A self tuning vibration absorber.
- Mead, D. J. (1998). *Passive vibration control*. Wiley.
- Minaei, A. and Ghorbani-Tanha, A. (2019). Optimal step-by-step tuning method for variable stiffness semiactive tuned mass dampers. *Journal of Engineering Mechanics*, 145(6):04019037.
- Mitchiner, R. and Leonard, R. (1991). Centrifugal pendulum vibration absorbers—theory and practice.

- Nayfeh, A. H. and Mook, D. T. (2008). *Nonlinear oscillations*. John Wiley & Sons.
- Nehemy, G., Rustighi, E., Gonçalves, P., and Brennan, M. (2024). Experimental investigation on a passive self-tuning vibration neutraliser for two distinct frequencies. In *Journal of Physics: Conference Series*, volume 2647, page 032006. IOP Publishing.
- Nehemy, G. F. (2024). Passive self-tuning vibration neutraliser [video]. <https://doi.org/10.5281/zenodo.14019282>).
- Nehemy, G. F., Rustighi, E., Gonçalves, P. J. P., and Brennan, M. J. (2023). A passive self-tuning vibration neutraliser using nonlinear coupling between the degrees of freedom. *Mechanical Systems and Signal Processing*, 185:109786.
- Nonami, K., Nishimura, H., and Cui, W. (1994). Disturbance cancellation control for vibration of multi-degree-of-freedom systems: Case of using active vibration absorber and active dynamic vibration absorber. *JSME international journal. Ser. C, Dynamics, control, robotics, design and manufacturing*, 37(1):86–93.
- Ormondroyd, J. and Hartog, D. (1928). Theory of the dynamic vibration absorber. *Transaction of the ASME*, 50:9–22.
- Rao, S. S. (2001). *Mechanical vibrations*.
- Rasooli, A., Sedaghati, R., and Hemmatian, M. (2020). A novel magnetorheological elastomer-based adaptive tuned vibration absorber: design, analysis and experimental characterization. *Smart Materials and Structures*, 29(11):115042.
- Reed, F. E. (2002). Dynamic vibration absorbers and auxiliary mass dampers. In Harris, C. and Piersol, A., editors, *Shock and vibration handbook*, volume 5th Edition, chapter 6. McGraw-Hill.
- Roberson, R. E. (1952). Synthesis of a nonlinear dynamic vibration absorber. *Journal of the Franklin Institute*, 254(3):205–220.
- Rustighi, E. (2010). A tuneable vibration absorber based on rotating stiffness elements.
- Rustighi, E., Brennan, M., and Mace, B. (2005a). Real-time control of a shape memory alloy adaptive tuned vibration absorber. *Smart Materials and Structures*, 14(6):1184.

- Rustighi, E., Brennan, M., and Mace, B. (2005b). A shape memory alloy adaptive tuned vibration absorber: design and implementation. *Smart materials and Structures*, 14(1):19.
- Shakib, A. and Ghorbani-Tanha, A. K. (2020). An innovative adaptive tuned vibration absorber with variable mass moment of inertia for mitigation of transient response of systems. *Structural Control and Health Monitoring*, 27(5):e2518.
- Stephenson, A. (1908). Memoirs and proceedings of the manchester literary and philosophical society 52, 1–10. *On a new type of dynamic stability*.
- Sun, S., Chen, Y., Yang, J., Tian, T., Deng, H., Li, W., Du, H., and Alici, G. (2014). The development of an adaptive tuned magnetorheological elastomer absorber working in squeeze mode. *Smart Materials and Structures*, 23(7):075009.
- Taylor, E. (1936). "eliminating" crankshaft torsional vibration in radial aircraft engines. *SAE Transactions*, pages 81–89.
- Thomsen, J. J., Thomsen, J. J., and Thomsen, J. (2003). *Vibrations and stability*, volume 2. Springer.
- Tondl, A. (2000). *Autoparametric resonance in mechanical systems*. Cambridge University Press.
- Varadarajan, N. and Nagarajaiah, S. (2004). Wind response control of building with variable stiffness tuned mass damper using empirical mode decomposition/hilbert transform. *Journal of engineering mechanics*, 130(4):451–458.
- Walsh, P. L. and Lamancusa, J. S. (1992). A variable stiffness vibration absorber for minimization of transient vibrations. *Journal of sound and vibration*, 158(2):195–211.
- Wilby, J. F. (1996). Aircraft interior noise. *Journal of Sound and Vibration*, 190(3):545–564.
- Williams, K., Chiu, G., and Bernhard, R. (2002). Adaptive-passive absorbers using shape-memory alloys. *Journal of Sound and Vibration*, 249(5):835–848.
- World Health Organization (2018). *Environmental noise guidelines for the European Region*. World Health Organization. Regional Office for Europe.

- Wright, R. and Kidner, M. (2004). Vibration absorbers: a review of applications in interior noise control of propeller aircraft. *Journal of Vibration and Control*, 10(8):1221–1237.
- Wu, S.-T., Chiu, Y.-Y., and Yeh, Y.-C. (2007). Hybrid vibration absorber with virtual passive devices. *Journal of sound and vibration*, 299(1-2):247–260.
- Wu, S.-T. and Shao, Y.-J. (2007). Adaptive vibration control using a virtual-vibration-absorber controller. *Journal of Sound and Vibration*, 305(4-5):891–903.
- Xiang, H. C. and Shao, M. (1992). Study on the frequency tracking damping vibration system with electro-magnetic natural frequency controllable dynamic vibration absorber. *Proceedings of First International Conference on Motion and Vibration Control*, page 681–688.
- Yu, L., Lan, C., Hu, G., Tang, L., and Yang, T. (2021). A nonlinear neutralizer with self-adaptation capability. In *Active and Passive Smart Structures and Integrated Systems XV*, volume 11588, pages 285–291. SPIE.

Appendix A

Dynamic stiffness matrices

In this appendix, the dynamic stiffness equations corresponding to the model introduced in Section 3.2 are presented for both undamped and damped systems. The provided equations offer a comprehensive representation of the dynamic stiffness of the system, facilitating a detailed understanding of its behaviour. The abbreviations $S_\theta = \sin \theta$ and $C_\theta = \cos \theta$ are used for brevity.

The undamped dynamic stiffness matrix, Equation (3.7), can be defined with its individual elements with D_{ij} referring to the element in the i -th row and j -th column of the matrix, as Equation (A.1)

$$\mathbf{D}(\omega) = \mathbf{T}^T (-\omega^2 \mathbf{M} + \mathbf{K}) \mathbf{T} \quad ((3.7))$$

$$\mathbf{D}(\omega) = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix} \quad (\text{A.1})$$

where the terms are given

$$\begin{aligned}
D_{11} &= k_{yy}S_\theta^2 + k_{xx}C_\theta^2 - m_N\omega^2 \\
D_{12} &= k_{xx}C_\theta S_\theta - k_{yy}C_\theta S_\theta \\
D_{13} &= -k_{yy}S_\theta^2 - k_{xx}C_\theta^2 \\
D_{14} &= k_{yy}C_\theta S_\theta - k_{xx}C_\theta S_\theta \\
D_{21} &= k_{xx}C_\theta S_\theta - k_{yy}C_\theta S_\theta \\
D_{22} &= k_{xx}S_\theta^2 + k_{yy}C_\theta^2 - m_N\omega^2 \\
D_{23} &= k_{yy}C_\theta S_\theta - k_{xx}C_\theta S_\theta \\
D_{24} &= -k_{xx}S_\theta^2 - k_{yy}C_\theta^2 \\
D_{31} &= -k_{yy}S_\theta^2 - k_{xx}C_\theta^2 \\
D_{32} &= k_{yy}C_\theta S_\theta - k_{xx}C_\theta S_\theta \\
D_{33} &= k_{yy}S_\theta^2 + k_{xx}C_\theta^2 - m_B\omega^2 \\
D_{34} &= k_{xx}C_\theta S_\theta - k_{yy}C_\theta S_\theta \\
D_{41} &= k_{yy}C_\theta S_\theta - k_{xx}C_\theta S_\theta \\
D_{42} &= -k_{xx}S_\theta^2 - k_{yy}C_\theta^2 \\
D_{43} &= k_{xx}C_\theta S_\theta - k_{yy}C_\theta S_\theta \\
D_{44} &= k_{xx}S_\theta^2 + k_{yy}C_\theta^2 - m_B\omega^2
\end{aligned}$$

This matrix $\mathbf{D}(\omega)$ may also be simplified for the specific positions $\theta = 0^\circ$, in Equation (A.2), and $\theta = 90^\circ$, in Equation (A.3). At this positions the directions X and Y are uncoupled.

$$\mathbf{D}(\omega) = \begin{bmatrix} k_{xx} - m_N\omega^2 & 0 & -k_{xx} & 0 \\ 0 & k_{yy} - m_N\omega^2 & 0 & -k_{yy} \\ -k_{xx} & 0 & k_{xx} - m_B\omega^2 & 0 \\ 0 & -k_{yy} & 0 & k_{yy} - m_B\omega^2 \end{bmatrix} \quad (\text{A.2})$$

$$\mathbf{D}(\omega) = \begin{bmatrix} k_{yy} - m_N \omega^2 & 0 & -k_{yy} & 0 \\ 0 & k_{xx} - m_N \omega^2 & 0 & -k_{xx} \\ -k_{yy} & 0 & k_{yy} - m_B \omega^2 & 0 \\ 0 & -k_{xx} & 0 & k_{xx} - m_B \omega^2 \end{bmatrix} \quad (\text{A.3})$$

The damped dynamic stiffness matrix given by (3.9), may be represented with its elements in the same way, as Equation (A.4).

$$\mathbf{D}_C(\omega) = \mathbf{T}^T (-\omega^2 \mathbf{M} + j\omega \mathbf{C} + \mathbf{K}) \mathbf{T} \quad ((3.9))$$

$$\mathbf{D}_C(\omega) = \begin{bmatrix} D_{C11} & D_{C12} & D_{C13} & D_{C14} \\ D_{C21} & D_{C22} & D_{C23} & D_{C24} \\ D_{C31} & D_{C32} & D_{C33} & D_{C34} \\ D_{C41} & D_{C42} & D_{C43} & D_{C44} \end{bmatrix} \quad (\text{A.4})$$

Which,

$$\begin{aligned}
D_{C11} &= j\omega (c_{yy}S_\theta^2 + c_{xx}C_\theta^2) + k_{yy}S_\theta^2 + k_{xx}C_\theta^2 - m_N\omega^2 \\
D_{C12} &= j\omega (c_{xx}C_\theta S_\theta - c_{yy}C_\theta S_\theta) + k_{xx}C_\theta S_\theta - k_{yy}C_\theta S_\theta \\
D_{C13} &= j\omega (-c_{yy}S_\theta^2 - c_{xx}C_\theta^2) - k_{yy}S_\theta^2 - k_{xx}C_\theta^2 \\
D_{C14} &= j\omega (c_{yy}C_\theta S_\theta - c_{xx}C_\theta S_\theta) + k_{yy}C_\theta S_\theta - k_{xx}C_\theta S_\theta \\
D_{C21} &= j\omega (c_{xx}C_\theta S_\theta - c_{yy}C_\theta S_\theta) + k_{xx}C_\theta S_\theta - k_{yy}C_\theta S_\theta \\
D_{C22} &= j\omega (c_{xx}S_\theta^2 + c_{yy}C_\theta^2) + k_{xx}S_\theta^2 + k_{yy}C_\theta^2 - m_N\omega^2 \\
D_{C23} &= j\omega (c_{yy}C_\theta S_\theta - c_{xx}C_\theta S_\theta) + k_{yy}C_\theta S_\theta - k_{xx}C_\theta S_\theta \\
D_{C24} &= j\omega (-c_{xx}S_\theta^2 - c_{yy}C_\theta^2) - k_{xx}S_\theta^2 - k_{yy}C_\theta^2 \\
D_{C31} &= j\omega (-c_{yy}S_\theta^2 - c_{xx}C_\theta^2) - k_{yy}S_\theta^2 - k_{xx}C_\theta^2 \\
D_{C32} &= j\omega (c_{yy}C_\theta S_\theta - c_{xx}C_\theta S_\theta) + k_{yy}C_\theta S_\theta - k_{xx}C_\theta S_\theta \\
D_{C33} &= j\omega (c_{yy}S_\theta^2 + c_{xx}C_\theta^2) + k_{yy}S_\theta^2 + k_{xx}C_\theta^2 - m_B\omega^2 \\
D_{C34} &= j\omega (c_{xx}C_\theta S_\theta - c_{yy}C_\theta S_\theta) + k_{xx}C_\theta S_\theta - k_{yy}C_\theta S_\theta \\
D_{C41} &= j\omega (c_{yy}C_\theta S_\theta - c_{xx}C_\theta S_\theta) + k_{yy}C_\theta S_\theta - k_{xx}C_\theta S_\theta \\
D_{C42} &= j\omega (-c_{xx}S_\theta^2 - c_{yy}C_\theta^2) - k_{xx}S_\theta^2 - k_{yy}C_\theta^2 \\
D_{C43} &= j\omega (c_{xx}C_\theta S_\theta - c_{yy}C_\theta S_\theta) + k_{xx}C_\theta S_\theta - k_{yy}C_\theta S_\theta \\
D_{C44} &= j\omega (c_{xx}S_\theta^2 + c_{yy}C_\theta^2) + k_{xx}S_\theta^2 + k_{yy}C_\theta^2 - m_B\omega^2
\end{aligned}$$

When $\theta = 0^\circ$ or $\theta = 90^\circ$, the matrix (A.1) simplifies to (A.5), and the terms are given by equations (A.6) for $\theta = 0^\circ$ and (A.7) for $\theta = 90^\circ$.

$$\mathbf{D}_C(\omega) = \begin{bmatrix} D_{C11} & 0 & D_{C13} & 0 \\ 0 & D_{C22} & 0 & D_{C24} \\ D_{C31} & 0 & D_{C33} & 0 \\ 0 & D_{C42} & 0 & D_{C44} \end{bmatrix} \quad (\text{A.5})$$

$$D_{C11} = -m_N\omega^2 + j\omega c_{xx} + k_{xx} \quad (\text{A.6a})$$

$$D_{C13} = -j\omega c_{xx} - k_{xx} \quad (\text{A.6b})$$

$$D_{C22} = -m_N\omega^2 + j\omega c_{yy} + k_{yy} \quad (\text{A.6c})$$

$$D_{C24} = -j\omega c_{yy} - k_{yy} \quad (\text{A.6d})$$

$$D_{C31} = -j\omega c_{xx} - k_{xx} \quad (\text{A.6e})$$

$$D_{C33} = -m_B\omega^2 + j\omega c_{xx} + k_{xx} \quad (\text{A.6f})$$

$$D_{C42} = -j\omega c_{yy} - k_{yy} \quad (\text{A.6g})$$

$$D_{C44} = -m_B\omega^2 + j\omega c_{yy} + k_{yy} \quad (\text{A.6h})$$

$$D_{C11} = -m_N\omega^2 + j\omega c_{yy} + k_{yy} \quad (\text{A.7a})$$

$$D_{C13} = -j\omega c_{yy} - k_{yy} \quad (\text{A.7b})$$

$$D_{C22} = -m_N\omega^2 + j\omega c_{xx} + k_{xx} \quad (\text{A.7c})$$

$$D_{C24} = -j\omega c_{xx} - k_{xx} \quad (\text{A.7d})$$

$$D_{C31} = -j\omega c_{yy} - k_{yy} \quad (\text{A.7e})$$

$$D_{C33} = -m_B\omega^2 + j\omega c_{yy} + k_{yy} \quad (\text{A.7f})$$

$$D_{C42} = -j\omega c_{xx} - k_{xx} \quad (\text{A.7g})$$

$$D_{C44} = -m_B\omega^2 + j\omega c_{xx} + k_{xx} \quad (\text{A.7h})$$

Appendix B

3DOF time domain model

This appendix documents the initial development of a simplified 3DOF time-domain model employed during the early stages of the study. Although later refined into the more representative 5DOF formulation presented in Chapter 4, the 3DOF model offers valuable insights into the fundamental rotational dynamics and their coupling with translational motion. It is included here for reference purposes and for readers interested in the initial modelling approach.

A notable distinction between the model and the experimental setup lies in the method of excitation. While the experimental configuration employs a force input applied by a shaker table, the 3DOF model uses prescribed base motion as excitation. This choice was made to reduce model complexity while preserving the essential features of the system's dynamic behaviour.

The dynamic response observed in experimental tests with the TN1 neutraliser under harmonic excitation can be qualitatively reproduced using this lumped-parameter model. The model considers translational motions along the X and Y axes and a rotational degree of freedom around the Z -axis, denoted by θ . Unlike the earlier linear model used to derive accelerance transfer functions at fixed orientations in Chapter 3, this formulation is inherently nonlinear. The nonlinearity arises from including θ as a dynamic variable, making the coordinate transformation matrix time-dependent. The presence of trigonometric terms in this transformation introduces strong geometric nonlinearities into the equations of motion.

The schematic representation of the model is shown in Figure B.1. The model assumes the base is fixed and does not include its mass. The mass m_N representing the neutraliser is subjected to vertical harmonic displacement in the y -direction, described by $q_N^{(Y)} = A \cos(\omega t)$, where A is the amplitude and ω the excitation frequency.

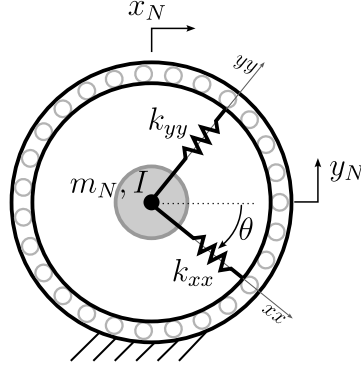


Figure B.1: The schematic of the lumped 3-DOF time-domain model without base mass and considering the outer ring of the bearing fixed.

The equations of motion are derived using Lagrange's method:

$$\frac{d}{dt} \left(\frac{\partial E_k}{\partial \dot{q}_i} \right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial V}{\partial q_i} = 0, \quad i = 1, 2, \dots, n \quad (\text{B.1})$$

Where n represents the number of degrees of freedom, and q_i stands for the independent generalized coordinates used to describe the motion, defined as $q = [\theta; x_N; y_N]$. In this context, E_k denotes the kinetic energy of the system, V represents the potential energy, with g being the gravitational force.

The system's kinetic energy includes contributions from translational and rotational motion:

$$E_k = \frac{m_N (\dot{x}_N^2 + \dot{y}_N^2)}{2} + \frac{I \dot{\theta}^2}{2} \quad (\text{B.2})$$

The potential energy includes gravitational and elastic energy terms:

$$V = m_N g y_N + \frac{1}{2} \begin{Bmatrix} x_N \\ y_N - A \cos(\omega t) \end{Bmatrix}^T \begin{bmatrix} k_{11}(\theta) & k_{12}(\theta) \\ k_{21}(\theta) & k_{22}(\theta) \end{bmatrix} \begin{Bmatrix} x_N \\ y_N - A \cos(\omega t) \end{Bmatrix} \quad (\text{B.3})$$

where $\begin{bmatrix} k_{11}(\theta) & k_{12}(\theta) \\ k_{21}(\theta) & k_{22}(\theta) \end{bmatrix} = \begin{bmatrix} k_{xx} \cos^2(\theta) + k_{yy} \sin^2(\theta) & (k_{xx} - k_{yy}) \sin(\theta) \cos(\theta) \\ (k_{xx} - k_{yy}) \sin(\theta) \cos(\theta) & k_{yy} \cos^2(\theta) + k_{xx} \sin^2(\theta) \end{bmatrix}$. Adding

viscous damping, the resulting equations of motion are:

$$\begin{aligned}
I\ddot{\theta} + c_t\dot{\theta} + \frac{1}{2}(k_{xx} - k_{yy})[-\sin(2\theta)x_N^2 + 2\cos(2\theta)x_Ny_N + \sin(2\theta)y_N^2] \\
= \frac{1}{4}(k_{xx} - k_{yy})\{-\sin(2\theta)A^2 + 4A[\cos(2\theta)x_N + \sin(2\theta)y_N]\cos(\omega t) \\
-A^2\sin(2\theta)\cos(2\omega t)\}
\end{aligned} \tag{B.4}$$

$$\begin{aligned}
m_N\ddot{x}_N + c_x\dot{x}_N + [k_{xx}\cos^2(\theta) + k_{yy}\sin^2(\theta)]x_N + (k_{xx} - k_{yy})\cos(\theta)\sin(\theta)y_N \\
= (k_{xx} - k_{yy})\cos(\theta)\sin(\theta)A\cos(\omega t)
\end{aligned} \tag{B.5}$$

$$\begin{aligned}
m_N\ddot{y}_N + c_y\dot{y}_N + [k_{xx}\sin^2(\theta) + k_{yy}\cos^2(\theta)]y_N + (k_{xx} - k_{yy})\cos(\theta)\sin(\theta)x_N \\
= [k_{xx}\sin^2(\theta) + k_{yy}\cos^2(\theta)]A\cos(\omega t)
\end{aligned} \tag{B.6}$$

Equation (B.4) is nonlinear due to trigonometric coupling, while Equations (B.5) and (B.6) are linear in form but coupled through the stiffness terms. The angular position θ modulates the stiffness matrix, introducing auto-parametric behaviour, as described in (Nayfeh and Mook, 2008; Thomsen et al., 2003).

To visually demonstrate the neutraliser functionality and self-turning response to an excitation force, two experiments were carried out using the TN1 neutraliser. In the first experiment, the neutraliser was set at an angle of $\theta = 90^\circ$, and the system was then subjected to a fixed-frequency excitation at 73 Hz, corresponding to the antiresonance f_4 when $\theta = 0^\circ$. For the second experiment, the neutraliser was initially positioned with an angle of $\theta = 0^\circ$ and exposed to a harmonic force at a fixed frequency of 295 Hz, corresponding to the antiresonance f_3 when $\theta = 90^\circ$.

The results of these experiments are displayed in Figure B.2, where sub-figures (a) represent the first test and (b) the second. In Figure B.2(a-i), the angular position of the beam varies from 0 to approximately 100° . Simultaneously, there is an increase in the tip mass displacement, as shown in Figure B.2(a-ii), while the base displacement experiences a notable reduction, as indicated in Figure B.2(a-iii). For the second test, the angular position of the beam is depicted in Figure B.2(b-i), changing from 90 to approximately 7° . Both the tip (Figure B.2(b-ii)) and the base (Figure B.2(b-iii)) displacements exhibit a similar behaviour to the previous test. Thus, the neutraliser effectively mitigates the vibration of a host structure, as evidenced by the reduction in base displacement.

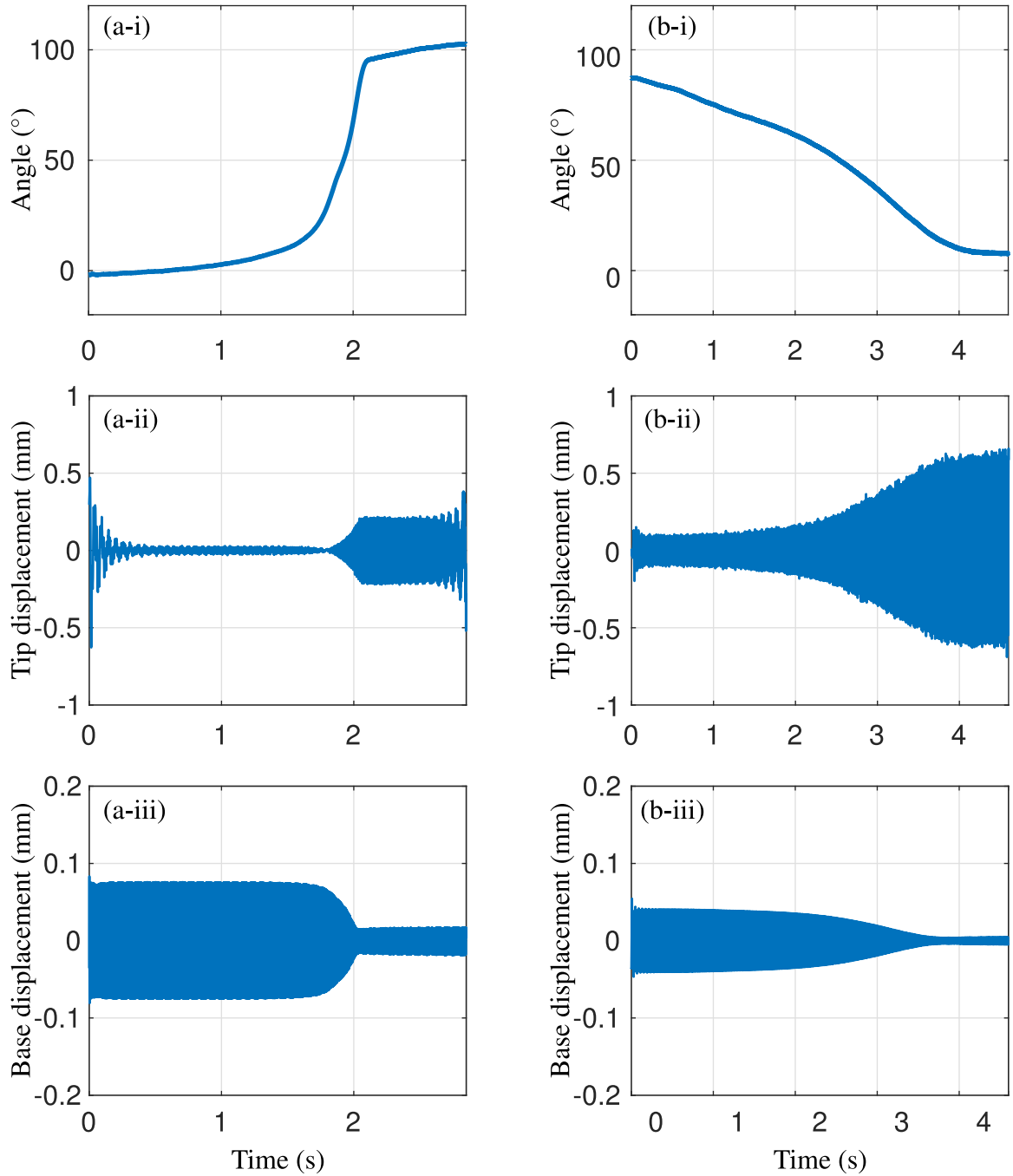


Figure B.2: Time histories for fixed harmonic excitation. (a) excited at 295 Hz and (b) excited at 73 Hz. Sub figures (i) are the angular displacement, sub figures (ii) are the tip displacement and sub figures (iii) are the base displacements for the two experimental procedures.

When the mass is displaced, the resulting elastic forces induce a torque on the bearing's inner ring. As shown in Figure B.3, the torque arises from the moment arm between the force application point and the centre of rotation, with magnitude depending on the square of the displacement:

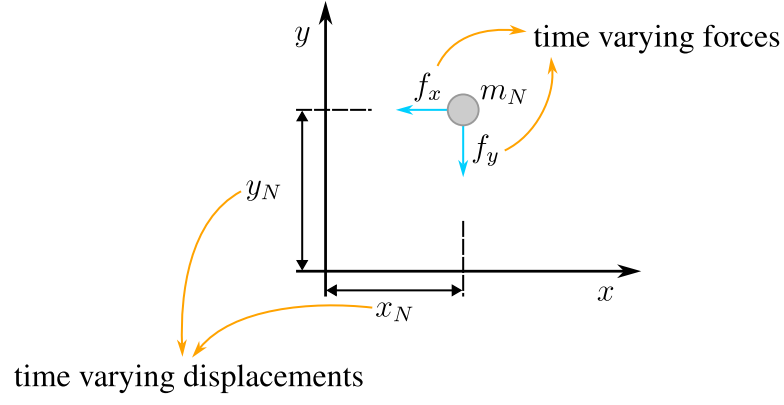


Figure B.3: Elastic forces due to the mass displacement resulting in a torque.

$$\begin{aligned} f_x &= [k_{xx} \cos^2(\theta) + k_{yy} \sin^2(\theta)] x_n + (k_{xx} - k_{yy}) \cos(\theta) \sin(\theta) y_N \\ f_y &= [k_{xx} \sin^2(\theta) + k_{yy} \cos^2(\theta)] y_n + (k_{xx} - k_{yy}) \cos(\theta) \sin(\theta) x_N \end{aligned} \quad (\text{B.7})$$

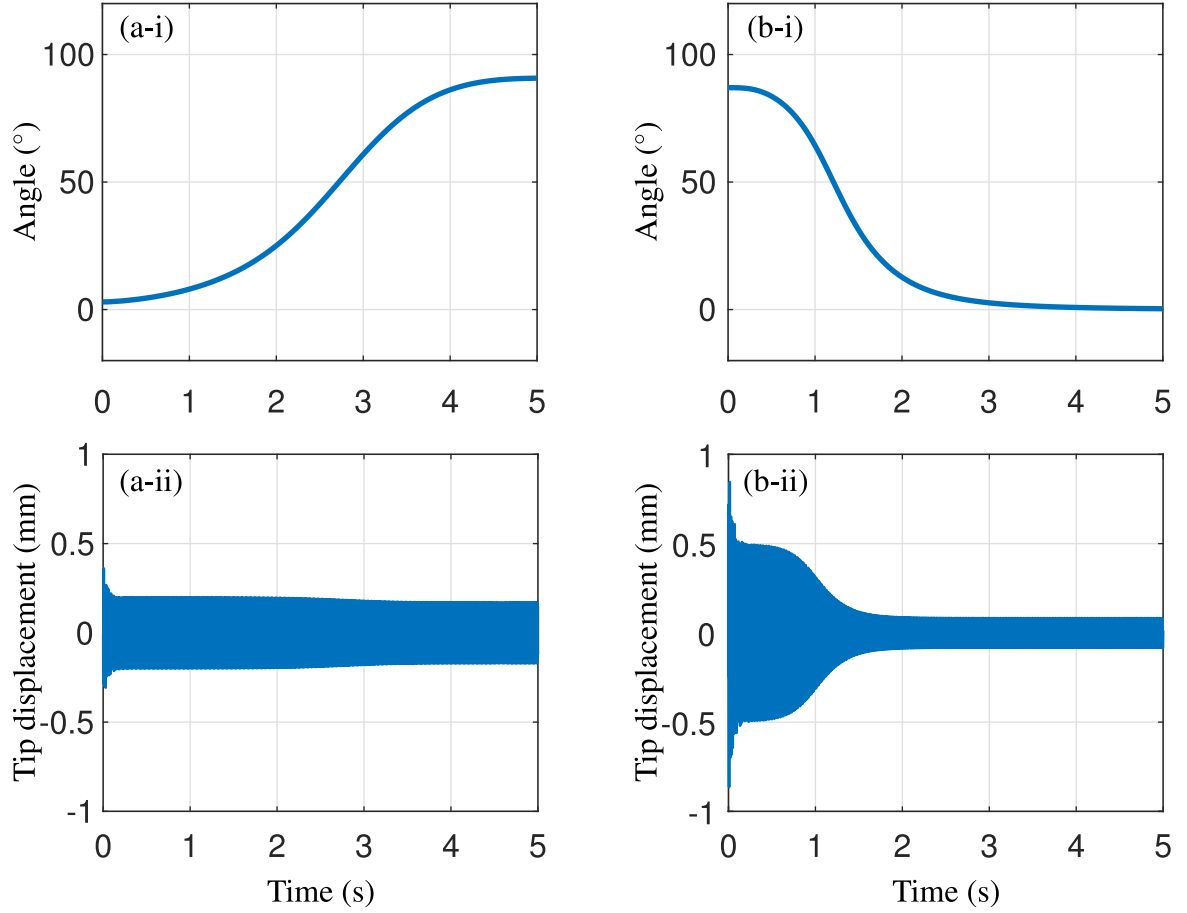


Figure B.4: Numerical simulation for harmonic excitation at (a) the higher and (b) lower resonance frequency: (a) 71 Hz excitation with base displacement of 0.5 mm; (b) 300 Hz excitation with base displacement of 0.2 mm. The sub-figures are the (i) angular rotation and (ii) vertical mass displacement.

The TN1 device, which has the highest stiffness asymmetry among the prototypes, generates a larger initial torque. This explains why TN1 exhibits self-rotation from initial positions near $\theta = 0^\circ$ or 90° , as seen in Figures B.2(a-i) and B.2(b-i). This model, despite its simplicity, offers insight into the rotational dynamics and illustrates the role of stiffness asymmetry in generating torque. Its qualitative predictions serve as a bridge for the more comprehensive 5DOF time-domain model.

Appendix C

Experimental investigation of rotational damping for improved time-domain simulations

During the development of the time-domain model, the rotational damping coefficient c_r proved to be a critical parameter, exerting a significant influence on the system's dynamic response. In preliminary simulations, this parameter had to be manually tuned to reconcile numerical predictions with experimental observations, highlighting its sensitivity and importance in accurately capturing the behaviour of the neutraliser. Given its impact, a reliable experimental determination of c_r became essential to ensure a physically grounded and predictive model.

To this end, a dedicated experimental procedure was developed to investigate and quantify the rotational damping under conditions representative of the actual operating environment. This appendix presents the methodology and results of that investigation. The ultimate objective is to refine the damping representation in the numerical model, reducing reliance on empirical fitting and improving the overall fidelity of the simulation with respect to real system dynamics.

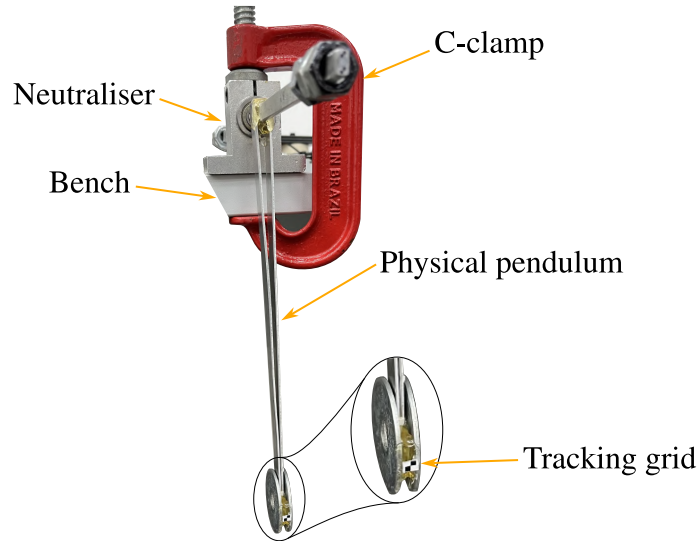


Figure C.1: Experimental rig used for the torsional damping experiment showing the neutraliser with the physical pendulum attached to the beam.

Figure C.1 shows a picture of the experimental set-up, where the base of the neutraliser was fixed on a table using a clamp, and a physical pendulum was attached to the beam, which rotated freely around the bearing. A tracking grid was placed at the mass of the physical pendulum to

enable tracking in the recorded video. With the pivot at the bearing, it becomes possible to deduce damping characteristics through a free vibration model.

The base of the neutraliser was fixed, and the physical pendulum was attached to the beam of the neutraliser. The pendulum was released from a small angle θ_0 from rest. It oscillated until it reached equilibrium. The motion was recorded by a Basler a2A1920 160umBAS camera at a frame rate of 720 frames per second. Subsequently, the horizontal component of the oscillatory movement was extracted from the recorded video using Tracker software (Brown et al., 2020). The results of the free vibration are depicted in Figure C.2 with the blue line.

Furthermore, an envelope analysis was performed on the free vibration response of the physical pendulum. This analysis involved pinpointing the maximum amplitudes of the oscillatory motion over time, depicted as red dots in Figure C.2. By extracting these extreme values, the envelope curve was delineated, representing the boundary of the oscillations, as indicated by the black dashed line in Figure C.2.

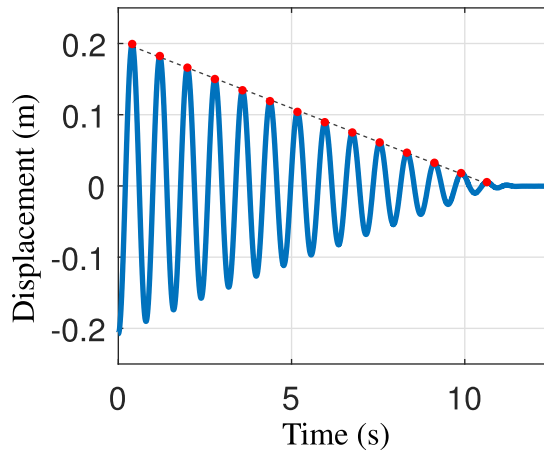


Figure C.2: Experimental results from the physical pendulum test. The blue line is the horizontal oscillatory motion extracted from the recorded movie. The red dots are the maximum amplitudes and the black dashed line the envelope of the movement.

The system damped by dry friction (or Coulomb friction) behaves similarly to an undamped SDOF system with a shift in its equilibrium position at the end of each half-cycle. A distinguishing feature of the response is that the amplitude decays linearly with time, unlike the exponential decay seen in systems with viscous damping (Craig Jr and Kurdila, 2006). The equation of movement for a simple mass-spring system under dry-friction, Figure C.3, is given by Equation (C.1) (Rao, 2001).

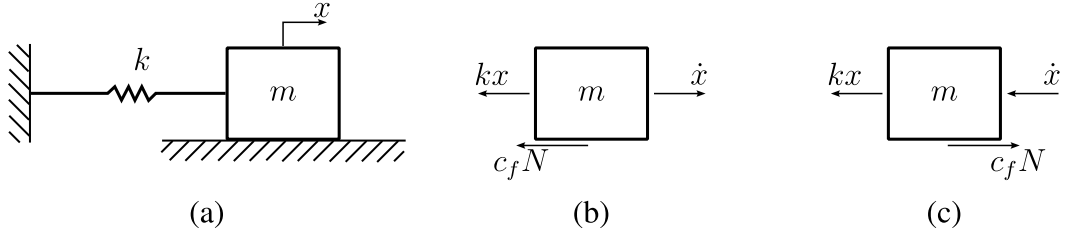


Figure C.3: Mass-spring system with dry-friction. Adapted from Rao (2001).

$$m\ddot{x} + c_f \text{sgn}(\dot{x}) + kx = 0, \quad (\text{C.1})$$

where $\text{sgn}(y)$ is the signal function, which is defined as Equation (C), and evaluates the direction of the mass velocity. When the mass changes direction, so does the velocity, and the friction force opposes this movement. c_f is the friction coefficient.

$$\text{sgn}(y) = \begin{cases} 1 & \text{if } y > 0, \\ 0 & \text{if } y = 0, \\ -1 & \text{if } y < 0. \end{cases}$$

To model a system subjected to dry friction, numerical simulations were conducted, and the results are compared with experimental data. Remarkably, the comparison between the experimental results and the model predictions yielded a perfect match, indicating a high level of agreement between the two. This comparison is depicted in Figure C.4.

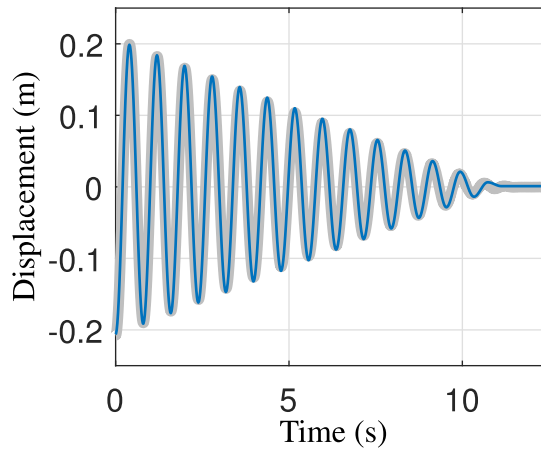


Figure C.4: Comparison of experimental and numerical results for the system damped by dry friction. The gray thicker line represents the experimental data, while the blue thin line represents the numerical simulation.

Whenever components slide relative to each other, such as in the case of a bearing, dry-friction damping arises internally. In this scenario, dry friction acts as the primary mechanism responsible for dissipating energy from the system.