Shear viscosity of a pion gas resulting from $\rho\pi\pi$ and $\sigma\pi\pi$ interactions

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We have evaluated the shear viscosity of pion gas with vanishing chemical potential taking into account its scattering with the low mass resonances σ and ρ during propagation in the medium. The thermal width (or collisional rate) of pions is calculated from $\pi\sigma$ and $\pi\rho$ loop diagrams using effective interactions in the real-time formulation of finite temperature field theory. A very small value of shear viscosity by entropy density ratio (η/s), close to the quantum bound, is obtained which approximately matches the range of values of η/s used by Niemi *et al.* [H. Niemi, G. S. Denicol, P. Huovinen, E. Molnár, and D. H. Rischke, Phys. Rev. Lett. **106**, 212302 (2011); Phys. Rev. C **86**, 014909 (2012)] to fit the elliptic flow data from the Relativistic Heavy Ion Collider.

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I. INTRODUCTION

To explain the elliptic flow parameter v_2 extracted from data collected at the Relativistic Heavy Ion Collider (RHIC) [1–7], hydrodynamical calculations [8–12] as well as some transport calculations [13–16] suggest that the matter produced in the collisions is likely to have a very small ratio of shear viscosity to entropy density η/s. Recent studies [17–22] have shown that η/s may reach a minimum in the vicinity of a phase transition— for earlier studies, see, e.g., Ref. [23]. In this context, the smallness of this minimum value with respect to its lower bound, $\eta/s = 1/4\pi$, commonly known as the Kovtun-Son-Starinets (KSS) or quantum bound [24], assumes particular significance. Again from the recent work of Niemi *et al.* [25], the transverse momentum p_T dependence on the elliptic flow parameter extracted from the RHIC data is highly sensitive to the temperature dependence of η/s in hadronic matter, and is almost independent of the viscosity in the quark-gluon plasma (QGP) phase. This result attributes extra importance to the microscopic calculations of the viscosity of hadronic matter in recent years [26–43], though these investigations began some time ago [44,45].

The calculations based on kinetic theory (KT) approaches in Refs. [36,40,44,45] predicted a shear viscosity η of pionic matter that increases with T, whereas using a Kubo approach, Lang *et al.* [39] predicted η to decrease with T. For the interaction of pions in the medium, Lang *et al.* [39] used lowest order chiral perturbation theory (χPT), which describes well the experimental data on π-π cross sections up to center-of-mass energies of $\sqrt{s} = 0.500$ GeV. For higher energies, resonances, particularly σ and ρ, become important and the iteration of the amplitude (unitarization) is necessary to describe the data. In the χPT approach, σ and ρ resonances in π-π scattering can be generated dynamically under unitarization. Fernandez-Fraile *et al.* [31] showed that under unitarization, χPT predicts η increasing with T in the Kubo approach — without unitarization, η decreases with T. Again in Ref. [32], it was shown that a KT approach leads to an η of the pionic medium that increases with T when a phenomenological interaction is used, while a decreasing function of T is obtained when using χPT in that same approach. An increasing trend of η with T was also observed by Mitra *et al.* [40,41], who incorporated a medium dependent π-π cross section in the transport equation for a pion gas. They also found a significant effect of a temperature-dependent pionic chemical potential [41]. Again, the question of the magnitude of η is also an unsettled issue. For example, near the critical temperature $T_c \simeq 0.175$ GeV, the authors of Refs. [32,39] predicted an $\eta \approx 0.001$ GeV^3; in Refs. [31,40,45], $\eta = 0.002 - 0.003$ GeV^3; and in Refs. [26,27], $\eta = 0.4$ GeV^3.

From these considerations, it is evident that the issue of the temperature dependence of hadronic shear viscosity is still a matter of debate and warrants further investigation. Motivated by this, we have calculated η of a pion gas using an effective Lagrangian for $\pi\pi\sigma$ and $\pi\pi\rho$ interactions, which may be treated as an alternative way to describe π-π cross sections up to the $\sqrt{s} = 1$ GeV [40,41] beside the unitarization technique [31]. Using real-time thermal field theory we calculated the in-medium pion correlator to obtain the thermal width, a necessary ingredient to calculate η. We also estimated the temperature dependence of the shear viscosity to entropy density ratio η/s of the pionic gas and compared our results to others of the recent literature. Although the hadronic matter that is formed in heavy-ion collisions at RHIC is comprised of more hadrons than pions only, our study nevertheless is of relevance to the real situation as, at least in the central rapidity region, pions are the dominant component of the hadronic fluid.

In the next section, we present the formalism used to evaluate the shear viscosity of a pion gas. Our numerical results are presented in Sec. III and in Sec. IV we present the summary and conclusions.

II. FORMALISM

Let us start with the standard expression of the shear viscosity for pion gas:

$$\eta = \frac{\beta}{10\pi^2} \int \frac{d^3k\, \boldsymbol{k}^6}{\Gamma_\pi(\boldsymbol{k},T)\,\omega_k^2}\, n(\omega_k)[1 + n(\omega_k)], \tag{1}$$

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where

$$n(\omega_k) = \frac{1}{e^{\beta\omega_k} - 1}, \tag{2}$$

is the Bose-Einstein distribution function for a temperature $T = 1/\beta$, with $\omega_k = (\boldsymbol{k}^2 + m_\pi^2)^{1/2}$, and $\Gamma_\pi(\boldsymbol{k},T)$ is the thermal width of π mesons in hadronic matter at temperature T. We note that this expression can be derived either with the Kubo formalism [46] using a retarded correlator of the energy-momentum tensor [47], or with a kinetic approach using the Boltzmann equation in the relaxation-time approximation [48]. In both approaches, to evaluate $\Gamma_\pi(\boldsymbol{k},T)$ one needs the interactions of the pions *in medium*. Here, we pursue the use of retarded correlators — for an explicit derivation see Ref. [47].

As mentioned previously, from lowest order χPT, the estimated π-π cross section in free space is well in agreement with the experimental data up to the center-of-mass energy $\sqrt{s} = 0.5$ GeV. Beyond this value of $\sqrt{s}$, the σ and ρ resonances play an essential role in explaining the data. On unitarization, the σ and ρ resonances are generated dynamically [31] in the amplitude. An alternative way, which we follow in the present paper, is to incorporate these resonances by using the effective interaction for $\pi\pi\sigma$ and $\pi\pi\rho$ interactions:

$$\mathcal{L} = g_\rho\, \boldsymbol{\rho}_\mu \cdot \boldsymbol{\pi} \times \partial^\mu \boldsymbol{\pi} + \frac{g_\sigma}{2} m_\sigma \boldsymbol{\pi} \cdot \boldsymbol{\pi}\, \sigma, \tag{3}$$

where the coupling constants g_ρ and g_σ are fixed from their experimental decay widths. We use this effective Lagrangian to calculate the contributions of the $\pi\rho$ and $\pi\sigma$ loops to the self-energy of the π meson at finite temperature. The contributions coming from the interactions of the pions in medium, which are the relevant ones for $\Gamma_\pi(\boldsymbol{k},T)$ in Eq. (1), can be obtained from the imaginary part of the retarded pion correlator $\Pi_\pi^R(k)$ evaluated at the π-meson pole, $k = (k_0 = \omega_k, \boldsymbol{k})$. In real-time thermal field theory, this relationship can be expressed as [49,50]

$$\begin{aligned}\Gamma_\pi(\boldsymbol{k},T) &= -\frac{1}{m_\pi} \mathrm{Im}\, \Pi_\pi^R(k)|_{k_0=\omega_k} \\ &= -\tanh\left(\frac{\beta k_0}{2}\right) \frac{1}{m_\pi} \mathrm{Im}\, \Pi_\pi^{11}(k)|_{k_0=\omega_k}.\end{aligned} \tag{4}$$

For clarity of presentation, we start considering the correlator in the narrow-width approximation, in which the widths of the σ and ρ resonances are neglected. At one-loop order — see Fig. 1 — one can write

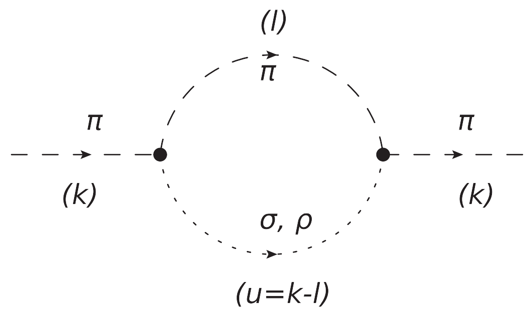

FIG. 1. One-loop self-energy diagram of a pion.

$$\Pi_\pi^{11}(k) = \Pi_\pi^{11}(k,\sigma) + \Pi_\pi^{11}(k,\rho), \tag{5}$$

with

$$\Pi_\pi^{11}(k,u) = -i \int \frac{d^4 l}{(2\pi)^4} L(k,l)\, D^{11}(l,m_l)\, D^{11}(u,m_u), \tag{6}$$

for each loop ($\pi\sigma$ or $\pi\rho$), where $m_l = m_\pi$, $u = k - l$, and $m_u = m_\sigma$ for the $\pi\sigma$ loop and $m_u = m_\rho$ for the $\pi\rho$ loop. The propagators $D^{11}(l)$ are given by

$$D^{11}(l) = \frac{-1}{l^2 - m_l^2 + i\eta} + 2\pi i\, n(\omega_l)\, \delta\big(l^2 - m_l^2\big), \tag{7}$$

where $n(\omega_l)$ is the Bose-Einstein distribution given in Eq. (2) and

$$L(k,l) = -\frac{g_\sigma^2 m_\sigma^2}{4}, \tag{8}$$

for the $\pi\sigma$ loop and

$$\begin{aligned}L(k,l) = -\frac{g_\rho^2}{m_\rho^2} \big\{&k^2\big(k^2 - m_\rho^2\big) + l^2\big(l^2 - m_\rho^2\big) \\ &- 2\big[(k\cdot l)\, m_\rho^2 + k^2\, l^2\big]\big\},\end{aligned} \tag{9}$$

for the $\pi\rho$ loop.

Using Eq. (7) in Eq. (6), one can perform the l_0 integration and, from the relation between $\mathrm{Im}\,\Pi^R$ and $\mathrm{Im}\,\Pi^{11}$ in Eq. (4), one obtains

$$\begin{aligned}\mathrm{Im}\, \Pi_\pi^R(k,u) = \int \frac{d^3 l}{32\pi^2 \omega_l \omega_u} \{&L(k,l)|_{l_0=\omega_l} [(1 + n(\omega_l) + n(\omega_u))\delta(k_0 - \omega_l - \omega_u) - (n(\omega_l) - n(\omega_u))\delta(k_0 - \omega_l + \omega_u)] \\ &+ L(k,l)|_{l_0=-\omega_l} [(n(\omega_l) - n(\omega_u))\, \delta(k_0 + \omega_l - \omega_u) - (1 + n(\omega_l) + n(\omega_u))\, \delta(k_0 + \omega_l + \omega_u)]\}.\end{aligned} \tag{10}$$

The Dirac delta functions provide branch cuts in the k_0 axis, identifying the different kinematic regions where the imaginary part of the pion self-energy acquires nonzero values. The relevant term for the in-medium decay width is the one proportional to $n(\omega_l) - n(\omega_u)$, which is due to the interactions of in-medium pions only and vanishes in a vacuum. The relevant branch cut, the Landau cut, is the region $-[\boldsymbol{k}^2 + (m_u - m_\pi)^2]^{1/2} \leqslant k_0 \leqslant [\boldsymbol{k}^2 + (m_u - m_\pi)^2]^{1/2}$; it gives

$$\begin{aligned}\Gamma_\pi^{\mathrm{nw}}(\boldsymbol{k},T,u) = &\frac{1}{16\pi|\boldsymbol{k}|m_\pi} \int_{\omega_+}^{\omega_-} d\omega\, L(\omega) \\ &\times [n(\omega) - n(\omega_k + \omega)],\end{aligned} \tag{11}$$

where the superscript nw indicates that this expression is obtained in the narrow-width approximation, and

$$\omega_\pm = \frac{R^2}{2m_\pi^2}(-\omega_k \pm |\boldsymbol{k}|\, W), \tag{12}$$

with $R^2 = 2m_\pi^2 - m_u^2$ and $W = \left(1 - 4m_\pi^4/R^4\right)^{1/2}$, and

$$L(\omega) = L\left(k_0 = \omega_k, \boldsymbol{k}, l_0 = -\omega, |\boldsymbol{l}| = \sqrt{\omega^2 - m_\pi^2}\right). \tag{13}$$

The physical interpretation of the Landau cut contributions is straightforward [51]. During the propagation of π^+, it may disappear by absorbing a thermalized π^- from the medium to create a thermalized ρ^0 or σ. Again the π^+ may appear by absorbing a thermalized ρ^0 or σ from the medium as well as by emitting a thermalized π^-. $n_l(1+n_u)$ and $n_u(1+n_l)$ are the corresponding statistical probabilities of the forward and inverse scatterings, respectively. By subtracting them, one gets the factor $(n_l - n_u)$ in Eq. (11).

Next, to take into account the widths of the resonances, we use the spectral representations of the σ and ρ propagators in Eq. (6) — see, e.g., Refs. [52,53]. This results in a folding of the narrow-width expression for $\Gamma_\pi(\boldsymbol{k},T,m_u)$:

$$\Gamma_\pi(\boldsymbol{k},T,m_u) = \frac{1}{N_u}\int_{(m_u^-)^2}^{(m_u^+)^2} dM^2\, \rho_u(M)\, \Gamma_\pi^{\rm nw}(\boldsymbol{k},T;M), \tag{14}$$

where $\Gamma^{\rm nw}(\boldsymbol{k},T;M)$ is the narrow-width expression given in Eq. (11), with m_u replaced by M; $\rho_u(M)$ is the spectral density

$$\rho_u(M) = \frac{1}{\pi}{\rm Im}\left[\frac{-1}{M^2 - m_u^2 + iM\Gamma_u(M)}\right], \tag{15}$$

and N_u is the normalization

$$N_u = \int_{(m_u^-)^2}^{(m_u^+)^2} dM^2\, \rho_u(M). \tag{16}$$

$\Gamma_u(M)$, $u = \sigma,\rho$, are the spectral widths of the mesons:

$$\Gamma_\sigma(M) = \frac{3g_\sigma^2 m_\sigma^2}{32\pi M}\left(1 - \frac{4m_\pi^2}{M^2}\right)^{1/2}, \tag{17}$$

$$\Gamma_\rho(M) = \frac{g_\rho^2 M}{48\pi}\left(1 - \frac{4m_\pi^2}{M^2}\right)^{3/2}. \tag{18}$$

In the integration limits, $m_u^\pm = m_u \pm 2\,\Gamma_u^0$, with $\Gamma_\sigma^0 = \Gamma_\sigma(M = m_\sigma)$ and $\Gamma_\rho^0 = \Gamma_\rho(M = m_\rho)$. In view of Eq. (5), the total pionic width is the sum

$$\Gamma_\pi(\boldsymbol{k},T) = \Gamma_\pi(\boldsymbol{k},T,\rho) + \Gamma_\pi(\boldsymbol{k},T,\sigma). \tag{19}$$

A quantity closely related to the thermal width is the mean free path

$$\lambda_\pi(\boldsymbol{k},T) = \frac{|\boldsymbol{k}|}{\omega_k\, \Gamma_\pi(\boldsymbol{k},T)}. \tag{20}$$

Phenomenologically, the analysis of this quantity is interesting for getting further insight in the propagation of pions in medium; in particular, it allows to know the values of typical pion momenta that are responsible for dissipation in medium, as we shall discuss in the next section. On the theoretical side, this quantity is interesting as [54] $\lambda_\pi \equiv 1/\Gamma_\pi$ in the chiral limit $m_\pi = 0$; as such, the m_π dependence of λ_π provides insight on the effects due to explicit chiral symmetry breaking [39].

TABLE I. The mass m_σ (in GeV) and vacuum width Γ_σ^0 (in GeV) of the σ resonance taken from Refs. [57–59], from which the corresponding coupling constants g_σ are extracted.

	m_σ	Γ_σ^0	g_σ
Set 1 (BES) [57]	0.390	0.282	5.82
Set 2 (E791) [58]	0.489	0.338	5.73
Set 3 (PDG min) [59]	0.400	0.400	6.85
Set 4 (PDG max) [59]	0.550	0.700	7.03

III. RESULTS AND DISCUSSION

Let us first consider the separate contributions of the $\pi\rho$ and $\pi\sigma$ loops to the imaginary part of the pion self-energy as a function of the invariant mass $m^2 = k_0^2 - |\boldsymbol{k}|^2$ for fixed values of temperature $T = 0.150$ GeV and three-momentum $|\boldsymbol{k}| = 0.300$ GeV — the results are shown in Fig. 2. We have used here the following set of parameters: $m_\pi = 0.140$ GeV, $m_\rho = 0.770$ GeV, $\Gamma_\rho^0 = 0.150$ GeV, and $g_\rho = 6$. The parameters for the σ resonance are those of Set 1 in Table I.

In Fig. 2, the dashed lines clearly indicate the sharp ends of the Landau cuts at $m = m_\rho - m_\pi = 0.630$ GeV for the $\pi\rho$ loop [Fig. 2(a)] and at $m = m_\sigma - m_\pi = 0.250$ GeV for the $\pi\sigma$ loop [Fig. 2(b)]. These sharp ends turn into smooth falloffs at large values of m due to the folding with the spectral functions of the σ and ρ resonances. This large-m effect does not affect $\Gamma_\pi(\boldsymbol{k},T)$ as this quantity is calculated at $m = m_\pi$. However, folding does affect $\Gamma_\pi(\boldsymbol{k},T)$ via a large effect induced by the

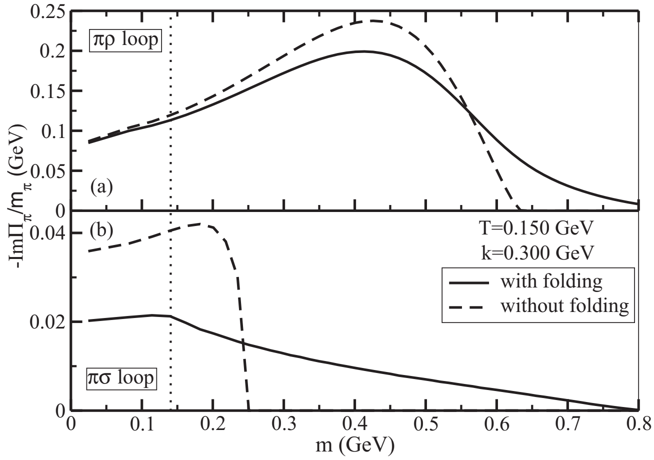

FIG. 2. The imaginary part of pion self-energy from (a) $\pi\rho$ and (b) $\pi\sigma$ loops as a function of the invariant mass $m = \sqrt{k_0^2 - |\boldsymbol{k}|^2}$ for fixed values of temperature $T = 0.150$ GeV and three-momentum $|\boldsymbol{k}| = 0.300$ GeV. The vertical dotted line indicates the on-shell value $m = m_\pi$. Parameters are as follows: $m_\pi = 0.140$ GeV, $m_\rho = 0.770$ GeV, $\Gamma_\rho^0 = 0.150$ GeV, $g_\rho = 6$, and Set 1 in Table I for parameters of σ resonance.

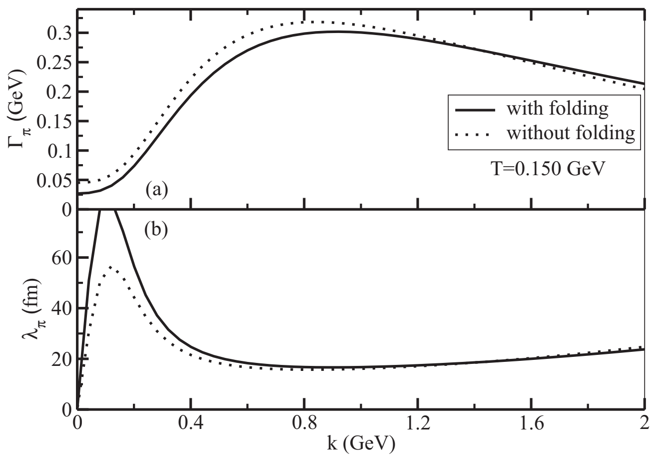

FIG. 3. Momentum dependence of (a) the thermal width and (b) of the mean free path for a fixed value of temperature $T = 0.150$ GeV. Parameters are the same as in Fig. 2.

$\pi\sigma$ channel; at $m = m_\pi$, folding decreases the contribution of the $\pi\sigma$ loop by 50% as compared to the corresponding contribution in the narrow-width approximation. This does not come as a surprise, as the σ resonance has a large width, while the width of the ρ is not as large. One should also notice that numerically, the contribution of the ρ resonance to Γ_π is one order of magnitude larger than the one from the σ loop at $m = m_\pi$. However, as we shall see shortly, this does not mean that one can neglect the σ resonance altogether.

Next, we consider the momentum dependence of thermal width and of the mean free path for a fixed temperature. The results are shown in Fig. 3. First of all, one sees that the effects of folding are not big when considering the joint contributions of the $\pi\rho$ and $\pi\sigma$ loops — this is due to the combined facts that the width of ρ has only a mild effect and the dominance of the $\pi\rho$ loop over the $\pi\sigma$ loop. One also sees that the value of λ_π is very big for momenta $0.100 \text{ GeV} \leqslant |\boldsymbol{k}| \leqslant 0.300$ GeV, but for $|\boldsymbol{k}| \geqslant 0.400$ GeV the value of the mean free path varies very little, reaching an average value of $\lambda_\pi \simeq 25$ fm. In a typical relativistic heavy ion collision at RHIC, the size of the hadronic systems produced after freeze-out varies between 20 and 40 fm. Therefore, scattering processes with center-of-mass momenta larger than $|\boldsymbol{k}| = 0.400$ GeV are those responsible for dissipation in the medium, at least for the chosen temperature $T = 0.15$ GeV.

In Fig. 4 we present results for the temperature dependence of the thermal width [Fig. 4(a)] and of the mean free path [Fig. 4(b)] for a fixed value of momentum $|\boldsymbol{k}| = 0.300$ GeV. Clearly, folding does not much affect the temperature dependence of these quantities. The reason for this is the same as for their momentum dependence: the dominance of the contribution of the $\pi\rho$ loop over that from $\pi\sigma$ loop. The figure also shows that only temperatures larger than $T = 0.120$ GeV give a mean free path smaller than the typical size of the hadronic system produced in a typical heavy ion collision at RHIC.

Of course, the viscosity of the pion gas is determined not only by the value of Γ_π (or λ_π), which is given basically by the π-π interaction; it depends also on the momentum distribution of the in-medium pions, which is determined by the temperature in the Bose-Einstein distribution. In Fig. 5 we present the results for the temperature dependence of η. Interestingly, we see that the $\pi\rho$ and $\pi\sigma$ contributions play a complementary role in η to be nondivergent in the higher ($T > 0.100$ GeV) and lower ($T < 0.100$ GeV) temperature regions, respectively. The lesson here is that the consideration of both resonances in π-π scattering is strictly necessary to obtain a smooth, nondivergent η for temperatures below the critical temperature $T_c \simeq 0.175$ GeV. Moreover, though η at very low temperatures ($T < 0.020$ GeV) tends to become very large in the narrow-width approximation [Fig. 5(a)], this trend disappears after taking into account the the widths of the resonances [Fig. 5(b)].

In Fig. 6, we have compared our results to the earlier results in the Kubo approach by Fernandez-Fraile *et al.* [31] and Lang *et al.* [39], along with previous results obtained by some of us [40] in a KT approach. In the KT approaches

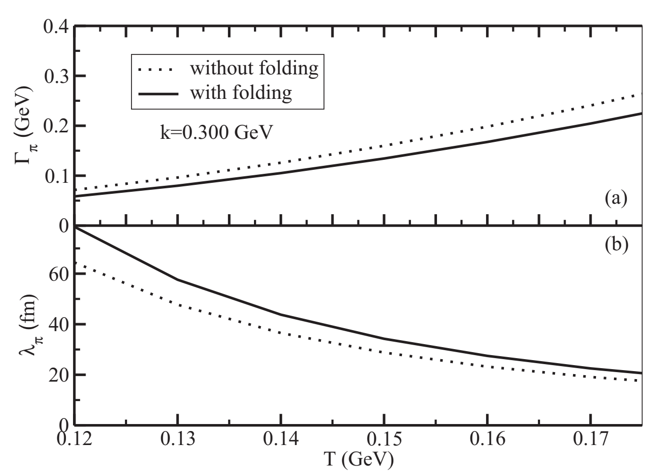

FIG. 4. Temperature dependence of (a) the thermal width and (b) of the mean free path for a fixed value of momentum $|\boldsymbol{k}| = 0.300$ GeV. Parameters are the same as in Fig. 2.

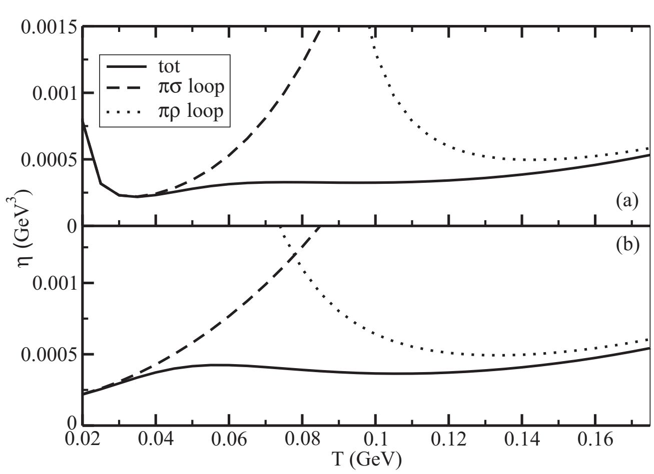

FIG. 5. Temperature dependence of η from the $\pi\sigma$ (dashed lines) and $\pi\rho$ (dotted lines) loops. The results (a) without and (b) with folding.

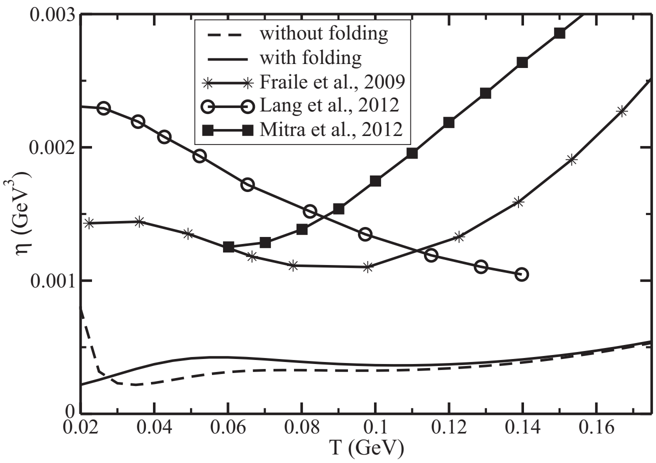

FIG. 6. Results of η versus T obtained in this work compared to some other results by Lang *et al.* [39], Fraile *et al.* [31], and Mitra *et al.* [40].

of Refs. [36,40,44,45], the predicted η is a monotonically increasing function of temperature in the temperature range $0.100\ \text{GeV} < T < 0.175\ \text{GeV}$ and vanishing baryon chemical potential ($\mu = 0$). The results of Lang *et al.* [39] obtained with the Kubo approach indicate an η decreasing in that same temperature range. Similar trends were obtained by Fernadez-Fraile *et al.* [31] with the Kubo-approach without unitarization of Γ, but the trend is reversed when dynamically generated (through unitarization) ρ and σ resonances come into play. Our calculations, based on an effective Lagrangian taking into account the low-mass σ and ρ resonances, found a similar trend of an increasing η with T for $T > 0.100$ GeV, although smaller in magnitude and slope, lending support to other calculations that take into account those resonances.

Now we concentrate on the sensitivity of our predictions associated with the phenomenological uncertainty of the parameters of the σ resonance. The results presented above have been obtained by choosing (arbitrarily) the parameters of Set 1 shown in Table I. Although longstanding controversies about the properties of this resonance seem to be settling to a consensus [55], recent literature [56] still shows conflicting values for those properties, as one can see in Table I. We have explored the impact of the different values for the σ parameters; the results are shown in Fig. 7. As can be seen, all sets predict η to be small, although parameter sets with smaller widths predict smaller η's at low temperatures; for $T > 0.1$ GeV, all sets predict essentially the same result.

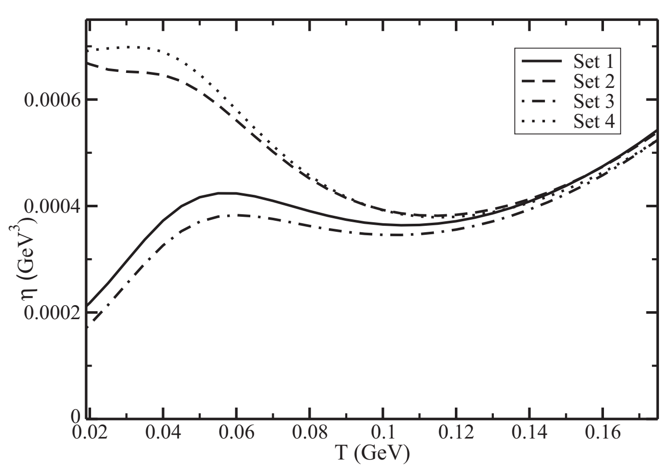

FIG. 7. The band of uncertainty of η in the low temperature domain for different sets of m_σ, $\Gamma(m_\sigma)$ and g_σ from Table I.

Finally, we estimate the temperature dependence of shear viscosity to entropy density ratio η/s in our model. In the calculation of the entropy density

$$s = 3\beta \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\left(\omega_k + \frac{\boldsymbol{k}^2}{3\omega_k}\right) n(\omega_k), \tag{21}$$

where $\omega_k = \sqrt{\boldsymbol{k}^2 + m_\pi^{*2}(\boldsymbol{k},T)}$, we have explored the effect of the loops in the real part of Π_π^R on the effective pion mass $m_\pi^* = \sqrt{m_\pi^2 + \text{Re}\Pi_\pi^R(k_0 = \omega_k, \boldsymbol{k}, T)}$. The variation of m_π^* with T for two different values of $\boldsymbol{k}$ is shown in Fig. 8(b). As can be seen, the effect is not big, at most 15% for the highest values of momentum and temperature. The effect of this change in the pion mass on the entropy density is marginal and can be safely neglected.

The dependence of the ratio η/s on T is shown in Fig. 8(a), where the results of some other groups are also included for comparison. Compared to the results of Lang *et al.* [39] and Fernadez-Fraile *et al.* [31], our η/s values are closer to the results obtained by Chakraborty *et al.* [21] in the framework of the relaxation time approximation, where $m_\sigma = 0.900$ GeV is considered as input in the Linear sigma model. However, those values become five times larger when they take $m_\sigma = 0.600$ GeV [21]. During this comparison, we should keep in mind that the different groups have normalized their $\eta(T)$ by different $s(T)$. Lang *et al.* [39] have taken the entropy density of the interacting pion gas at a two-loop order whereas Fernadez-Fraile *et al.* [31] adopted $s(T)$ of ideal pion gas like us. In Chakraborty *et al.* [21] the entropy density has been obtained from the pressure evaluated in a mean field approach

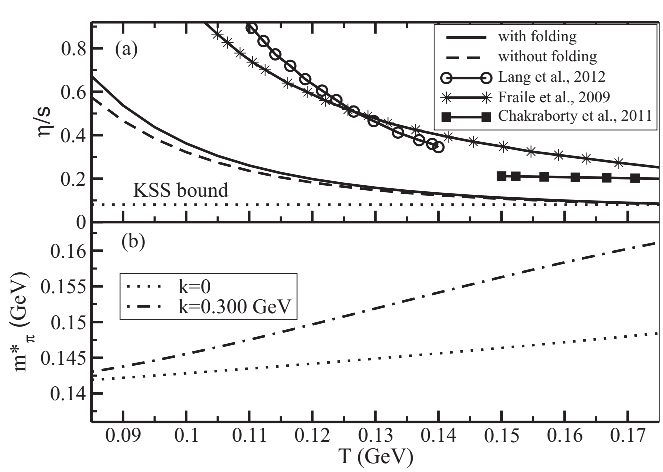

FIG. 8. (a) η/s versus T compared to other results by Lang *et al.* [39], Fraile *et al.* [31], and Chakraborty *et al.* [21] and also the KSS bound (dotted line). (b) Dependence of the effective pion mass m_π^* with temperature T at two different values of three-momentum $|\boldsymbol{k}|$.

in which the equation of motion and effective mass equations were solved self-consistently. The form of the entropy density remains unchanged and corresponds to that of a free gas with an effective mass. Our results respect the KSS bound $\eta/s \leqslant 1/4\pi$, as indicated by the dotted line. We recall that Niemi *et al*. [25] in their investigation of $v_2(p_T)$ of RHIC data, used an $\eta/s(T)$ from Ref. [34] that is in the same range of our results shown in the figure. This also lends support to the validity of our phenomenological analysis in which σ and ρ resonances play a decisive role in the dissipation properties of the pion gas.

IV. SUMMARY

We have calculated the shear viscosity of a pion gas at finite temperature and vanishing chemical potential taking into account the low-mass resonances σ and ρ on pion propagation in medium. The thermal width Γ_π is calculated from one-loop pion self-energy at finite temperature in the framework of real-time thermal field theory. We evaluated the contributions of $\pi\sigma$ and $\pi\rho$ loops to the pion self-energy with the help of an effective Lagrangian for the $\sigma\pi\pi$ and $\rho\pi\pi$ interactions. To take into account the widths of σ and ρ resonances, we folded the zero-widths self-energies with their spectral functions. We have seen a complementary role played by the $\pi\sigma$ and $\pi\rho$ loops in producing a smooth temperature dependence for η.

We also explored the impact of uncertainties in the parameters of the σ resonance on our results. Using the range of σ mass ($m_\sigma = 0.400$–0.550 GeV) and width ($\Gamma_\sigma = 0.400$–0.700 GeV) from the latest Particle Data Group (PDG) compilation [59], we obtained smaller values for η at low temperatures than those when using the earlier PDG values [60] ($m_\sigma = 0.400$–1.200 GeV and $\Gamma_\sigma = 0.600$–1.00 GeV). For temperatures larger than 0.1 GeV, all parameter sets give essentially the same value for η.

Our estimated temperature dependence for the ratio η/s respects the KSS bound $\eta/s \leqslant 1/4\pi$, and comes very close to the bound for temperatures near the critical temperature $T_c = 175$ MeV. It agrees with the results of the authors of Refs. [33,34]. From the recent work by Niemi *et al*. [25], the elliptic flow parameter $v_2(P_T)$ of RHIC data prefers such small values of $\eta/s(T)$ for hadronic matter. The results seem to provide experimental justification to the microscopic calculations of shear viscosity that include $\sigma\pi\pi$ and $\rho\pi\pi$ interactions, as the one performed in the present work.

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