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The Characteristics of Vibration Isolation System with Damping and Stiffness Geometrically Nonlinear

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Abstract. The paper concerns an investigation into the use of both stiffness and damping nonlinearity in the vibration isolator to improve its effectiveness. The nonlinear damping and nonlinear stiffness are both achieved by horizontal damping and stiffness as the way of the geometrical nonlinearity. The harmonic balance method is used to analyze the force transmissibility of such vibration isolation system. It is found that as the horizontal damping increasing, the height of the force transmissibility peak is decreased and the high-frequency force transmissibility is almost the same. The results are also validated by some numerical method. Then the RMS of transmissibility under Gaussian white noise is calculated numerically, the results demonstrate that the beneficial effects of the damping nonlinearity can be achieved under random excitation.

1. Introduction

Introducing an isolation mount between a source and a receiver is the most commonly adopted solution to reduce the level of transmitted vibrations, due to its economy and simple [1]. But the classic linear isolation system has two major defects: the one is the detrimental effect of damping on the high frequency transmissibility; the other one is that achieving low frequency vibration isolation using these types of isolators, a large static deflection is necessary [2, 3]. In this paper, an investigation is conducted into whether the improvements on such the two aspects can be made to an isolation system using the damping and stiffness nonlinearity [4].

Based on the recent research, the non-linear strategies can be roughly divided into two groups. The first method is introducing of nonlinear stiffness. Ibrahim has extensively reviewed such the nonlinear vibration isolation system [4]. Most studies focused on the reduction of the isolator's natural frequency using different types of nonlinear springs, for example, the cantilever spring [5] and the highly deformed continuous elastic beam spring [6]. Alabuzhev et al [7] proposed a simple and effective Quasi-Zero spring, which is achieved by positioning of auxiliary linear springs, it is found that using such Quasi-Zero spring the resonant frequency can be reduced without affecting the static stiffness. Brennan et al. [8] studied of the transmissibility for the isolation system with HSLDS and linear



damping. Further the jump-up and down frequencies of this type of nonlinear system with light damping are derived using the harmonic balance method [9]. Huang et al. [10] investigated the characteristics of a high-static-low-dynamic-stiffness (HSLDS) vibration isolation system, in which negative stiffness is realized by buckling Euler beams. Xu et al. [11] investigated the potential beneficial performance of a Multi-Direction Quasi-Zero-Stiffness vibration isolator with linear time-delayed active control. However, for the introduction of hardening or softening effects, the system must have enough damping to avoid instability.

The second type of nonlinear approach is the introduction of the damping nonlinearities. Ravindra and Mallik [12] firstly investigated the effects of Coulomb damping on a hardening Duffing system and then showed that nonlinear viscous damping can completely eliminate chaotic responses caused by the nonlinear spring [13]. Further Lang et al. [14] analytically studied the transmissibility of this nonlinear vibration isolation system, it is found that the trade-off between the resonant and high frequency transmissibility can be overcome by a cubic viscous damping. It is also demonstrated that nonlinear viscous damping is more effective in suppressing the resonance and jumps of a Duffing system with insignificant impact on the high frequency transmissibility [15]. Ho et al. analytically studied a theoretical single-degree-of-freedom nonlinear vibration isolation system only under harmonic excitation, which is concerned with the benefit of both spring and damping nonlinearity [16].

Despite considerable progress made in the study of the force transmissibility for the nonlinear vibration isolation system, there still remain challenges to be addressed. As mentioned before, most of the previously published works have been used the only stiffness or damping has nonlinearity. This paper proposed a novel nonlinear damping and stiffness vibration isolation system, in which the nonlinear damping and stiffness are both achieved by positioning of the two horizontal damping and stiffness. The analytical expression for the force transmissibilities of a particular nonlinear vibration isolation system is derived using Harmonic balance method. The model in current work is similar to that in [8, 17] and force transmissibility of that model has previously been considered in [8, 17]. However, in current work the horizontal damping are connected to the mass, rather than only horizontal stiffness to the mass as described in reference [8, 17]. The obtained expressions are employed to investigate the dynamics of the nonlinear isolation system which has both geometrical nonlinear stiffness and damping and to study whether it can outperform other existing systems. Also aims to analyze the force transmissibility of the nonlinear damping and stiffness isolation in more details. To this aim, we have studied the different dynamic behaviors of the nonlinear vibration isolation system subject to both harmonic and random excitation. This fresh insight to the isolation benefit of the nonlinear damping and stiffness mechanism subject to random excitation can be used for the practical vibration isolation designing.

The content of the article is organized as follows. In section 2 a mathematical model of the vibration isolation system based on both damping and stiffness nonlinearity is developed. The approximation solutions for the force transmissibility of the nonlinear vibration isolation system are derived, and how nonlinearity affected on the performance of the isolation is conducted. In section 3, the performance of the nonlinear damping and stiffness vibration isolation system is evaluated by the RMS value of the response using Gaussian white noise excitation. Finally the paper is closed in section 4 with some conclusions.

2. Force Transmissibility of Isolator with both Non-linear Damping and Non-linear Stiffness

Fig.1 shows a lumped parameter model of a nonlinear damping and stiffness vibration isolation system for a suspended mass m . The system is very similar to the classic nonlinear isolator described in reference [8, 17] in which there are two horizontal springs with stiffness k_h besides the vertical spring k_v , damper c_v , but here there are additional two horizontal dampers with damping c_h as positioning of the horizontal springs in reference [8, 17].

Of particular interest is the force transmissibility, which is the force transmitted to the rigid base due to a harmonic excitation force f_e applied to the mass. The horizontal springs introduce a geometric

stiffness nonlinearity, and the stiffness of these springs can be adjusted so that the linear natural frequency of the system is reduced, hence extending the frequency range of vibration isolation to lower frequencies.

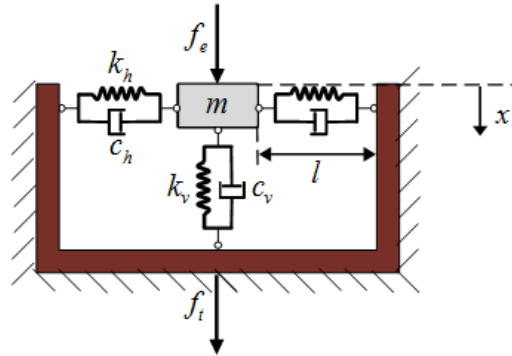


Figure 1. Schematic of a non-linear isolator that behaves as a Duffing oscillator.

When it is subject to harmonic forced excitation $f_e = F_e \cos(\omega t)$, the equation of motion for the system in Fig.1 is given by

$$m\ddot{x} + c_v\dot{x} + 2c_h \frac{x^2}{x^2 + l^2} \dot{x} + k_v x + 2k_h \left(1 - \frac{l_0}{(x^2 + l^2)^{1/2}} \right) x^3 = F_e \cos(\omega t) \quad (1)$$

where, l_0 is the initial length of the lateral springs and l is their length when they are in the horizontal position.

For $x < 0.2l$, Eq. (1) can be approximated and write as non-dimensional form

$$\hat{x}'' + 2\zeta_l \hat{x}' + 2\zeta_n \hat{x}^2 \hat{x}' + \alpha \hat{x} + \gamma \hat{x}^3 = \hat{F}_e \cos(\Omega \tau) \quad (2)$$

where $\hat{x} = (x/x_s)$, $x_s = (l_0^2 - l^2)^{1/2}$, $\zeta_l = \zeta_v$, $\zeta_n = \zeta_h (1 - \hat{l}^2)/\hat{l}^2$, $\zeta_v = \frac{c_v}{2m\omega_n}$, $\zeta_h = \frac{c_h}{2m\omega_n}$,

$\alpha = 1 - 2\hat{k}((1 - \hat{l})/\hat{l})$, $\gamma = \hat{k}((1 - \hat{l}^2)/\hat{l}^3)$, $\hat{l} = l/l_0$, $\hat{k} = k_h/k_v$, $\omega_n = \sqrt{k_v/m}$, $\hat{F}_e = F_e/k_v x_s$, $\Omega = \omega/\omega_n$,

$\tau = \omega_n t$, $(\cdot)' = d(\cdot)/d\tau$.

In this case $\hat{l}$ must be greater than or equal to a minimum value in order that the stiffness of the isolator does not become negative and have a snap-through characteristic. Applying the Harmonic Balance method and assuming a solution of the form $\hat{x} = \hat{X} \cos(\Omega \tau + \phi_F)$, with the term containing $\cos(3\Omega \tau)$ or $\sin(3\Omega \tau)$ neglected, leads to the frequency-amplitude relationship

$$\left((\alpha - \Omega^2) \hat{X} + \frac{3}{4} \gamma \hat{X}^3 \right)^2 + \left(2\zeta_l \Omega \hat{X} + \frac{1}{2} \zeta_n \Omega \hat{X}^3 \right)^2 = \hat{F}_e^2 \quad (3)$$

This is a quadratic equation in Ω^2 which can be solved to give

$$\Omega_{1,2} = \left(\alpha + \frac{3}{4}\gamma\hat{X}^2 - \frac{1}{2}\left(2\zeta_l + \frac{1}{2}\zeta_n\hat{X}^2\right)^2 \pm \frac{1}{\hat{X}}\left(\hat{F}_e^2 + \frac{1}{4}\left(\zeta_l + \frac{1}{4}\zeta_n\hat{X}^2\right)^4\hat{X}^2 - \left(\zeta_l + \frac{1}{4}\zeta_n\hat{X}^2\right)^2\left(\alpha\hat{X}^2 + \frac{3}{4}\gamma\hat{X}^4\right)\right)^{1/2} \right)^{1/2} \quad (4a,b)$$

which are the two stable branches (also referred to as resonant and non-resonant branches) of the frequency response curve.

The non-dimensional force transmitted through the nonlinear spring and the dashpot that comprises the isolator, is given by

$$\hat{f}_t = 2\zeta_l\hat{x}' + 2\zeta_n\hat{x}^2\hat{x}' + \alpha\hat{x} + \gamma\hat{x}^3 \quad (5)$$

Using the harmonic balance method the component of the transmitted force at the excitation frequency has the form $\hat{f}_t = \hat{F}_t \cos(\Omega\tau + \varphi_T)$, where the magnitude of the force is given by

$$\hat{F}_t = \hat{X} \left(\left(\alpha + \frac{3}{4}\gamma\hat{X}^2 \right)^2 + \Omega^2 \left(2\zeta_l + \frac{1}{2}\zeta_n\hat{X}^2 \right)^2 \right)^{1/2} \quad (6)$$

The force transmissibility is given by

$$|T_F| = \frac{\hat{F}_t}{\hat{F}_e} \quad (7)$$

Thus the magnitude of the transmissibility for the resonant and non-resonant branches can be determined by substituting for the two solutions for $\Omega_{1,2}$ in Eq. (4) into Eq. (6) and combining with Eq. (7) to give

$$|T_F|_1 = \frac{\hat{X} \left(\left(\alpha + \frac{3}{4}\gamma\hat{X}^2 \right)^2 + \Omega_1^2 \left(2\zeta_l + \frac{1}{2}\zeta_n\hat{X}^2 \right)^2 \right)^{1/2}}{\hat{F}_e} \quad (8a)$$

and

$$|T_F|_2 = \frac{\hat{X} \left(\left(\alpha + \frac{3}{4}\gamma\hat{X}^2 \right)^2 + \Omega_2^2 \left(2\zeta_l + \frac{1}{2}\zeta_n\hat{X}^2 \right)^2 \right)^{1/2}}{\hat{F}_e} \quad (8b)$$

The peak transmissibility corresponds to the peak displacement response, which can be determined by noting that Eq. (4a,b) are equal at this frequency, and hence

$\hat{F}_e^2 + \frac{1}{4}\left(\zeta_l + \frac{1}{4}\zeta_n\hat{X}^2\right)^4\hat{X}^2 - \left(\zeta_l + \frac{1}{4}\zeta_n\hat{X}^2\right)^2\left(\alpha\hat{X}^2 + \frac{3}{4}\gamma\hat{X}^4\right) = 0$. Note that the equation is hard to

solve and if the terms including ζ_n can be ignored, the value of the non-dimensional force $\hat{F}_e$ would increase. Then the equation becomes

$$\hat{F}_e'^2 + \frac{1}{4}\zeta_l^4 \hat{X}^2 - \zeta_l^2 \left(\alpha \hat{X}^2 + \frac{3}{4}\gamma \hat{X}^4 \right) = 0 \quad (9)$$

It's obvious that $\hat{F}_e'$ is larger than $\hat{F}_e$. Solving Eq. (9) gives

$$\hat{X}_{\max} = \left(\frac{2}{3\gamma} \left(-\alpha + \frac{1}{4}\zeta_l^2 + \frac{1}{\zeta_l} \left(\left(\alpha\zeta_l - \frac{1}{4}\zeta_l^3 \right)^2 + 3\gamma\hat{F}_e'^2 \right)^{1/2} \right) \right)^{1/2} \quad (10)$$

The frequency at which the transmissibility is a maximum can be calculated by substituting Eq. (10) into Eq. (8a) or (8b) to give

$$\Omega_{F\max} = \left(\frac{1}{2} \left(-\alpha + \frac{1}{4}\zeta_l^2 + \frac{1}{\zeta_l} \left(\left(\alpha\zeta_l - \frac{1}{4}\zeta_l^3 \right)^2 + 3\gamma\hat{F}_e'^2 \right)^{1/2} \right) + \alpha - 2\zeta_l^2 \right)^{1/2} \quad (11a)$$

If $\zeta_l \ll 1$ this simplifies to

$$\Omega_{F\max} = \frac{1}{\sqrt{2}} \left(\alpha + \left(\alpha^2 + \frac{3\gamma\hat{F}_e'^2}{\zeta_l^2} \right)^{1/2} \right)^{1/2} \quad (11b)$$

The maximum force that can be applied such that the peak in the response occurs at $\Omega=1$ can be determined by rearranging Eq. (11a) to give

$$\hat{F}_{e\max}' = \left(\frac{1}{3\gamma} \left(\left(2\zeta_l - \alpha\zeta_l + \frac{5}{4}\zeta_l^3 \right)^2 - \left(\alpha\zeta_l - \frac{1}{4}\zeta_l^3 \right)^2 \right) \right)^{1/2} \quad (12a)$$

If $\zeta_l \ll 1$ this simplifies to

$$\hat{F}_{e\max}' = 2\zeta_l \left(\frac{(1-\alpha)}{3\gamma} \right)^{1/2} \quad (12b)$$

which shows that the maximum force that can be applied such that the peak in the transmissibility does not occur at frequencies greater than $\Omega=1$. Note that, for practical purposes, the maximum value of the excitation force given in Eq. (11b) can be taken as the maximum force for which the theory given in this paper is applicable. If the force exceeds this value then the system may not respond in accordance with the assumption made in the derivation of the expressions given above.

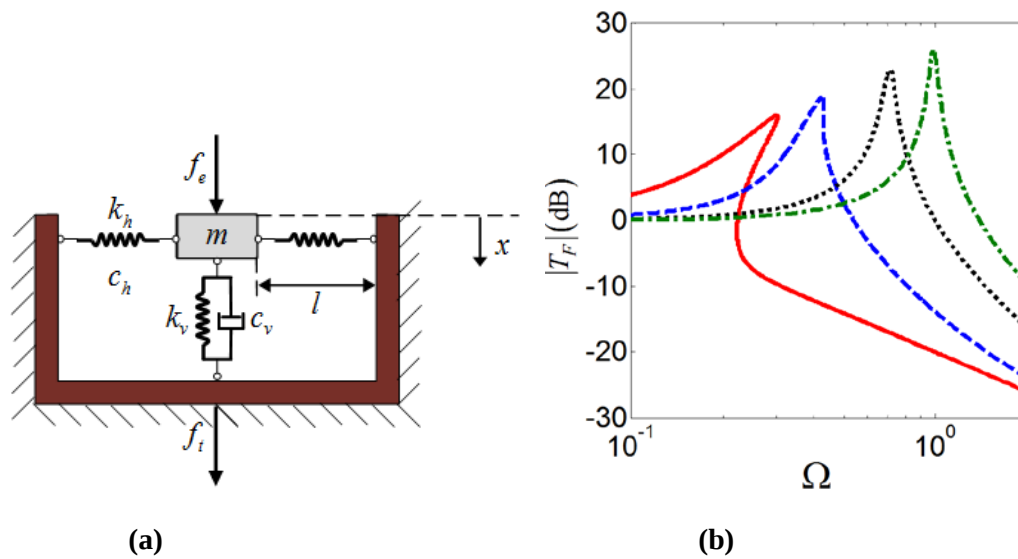


Figure 2. (a) Degradative model of a nonlinear isolator in which the horizontal springs k_h provide a softening effect to reduce the natural frequency of the system. (b) Force transmissibility changes with non-dimensional length of horizon springs $\hat{l}$. The parameters are $\hat{k} = 1$, $\zeta_v = 0.05$, $\zeta_h = 0.05$, $\hat{F}_e = 0.1\hat{F}_{e\max}$. Red solid line, $\hat{l} = 2/3$, blue dashed line, $\hat{l} = 0.7$, black dotted line, $\hat{l} = 0.8$, dark green dashed-dotted line, $\hat{l} = 0.9$.

Before the parametric study was carried out to illustrate how the horizontal damping affect the dynamic behavior, the benefits of horizontal stiffness on the vibration isolation performance is reviewed as shown in Ref. [8, 17]. Figure 2(a) shows the degraded model of such the nonlinear vibration isolation system. And the non-dimensional excitation force is set to $0.1\hat{F}_{e\max}$, the results of force transmissibility for different nonlinear stiffness by changing of $\hat{l}$ as shown in Figure 2(b). It can be seen that for each value of $\hat{l}$, the maximum occurs at a non-dimensional frequency, $\Omega < 1$ as expected. And the peak of the force transmissibility curve moves to a lower frequency so the isolation range is extended as $\hat{l}$ is decreased. Also the force transmissibility at high frequency is reduced. It is obvious that the smaller length ratio $\hat{l}$, the better the performance of the isolation system. But at the resonance frequency, the force transmissibility is still so large. So in this work two kinds of damper is added to the isolator to reduce the force transmissibility at resonance frequency, one is horizontal damper and the other one is vertical damper. Figure 3(a) and (b) shows the force transmissibility for different vertical damping ζ_v and horizontal damping ζ_h respectively, in which the non-dimensional excitation force is also set to $0.1\hat{F}_{e\max}$. It can be seen that as ζ_v increasing, the peak is reduced but in exchange, transmissibility at high frequency is increased, which is undesirable. While, as ζ_h is increased, the peak is also reduced, at the same time the behavior of isolator at high frequency is almost the same. It is obvious that transmissibility at high frequency is paid attention to so nonlinear damping is better than linear damping in this case. The reason for this is that displacement at resonance frequency is large so that nonlinear damping is large at resonance frequency but small at high frequency and linear damping do not change as displacement changing.

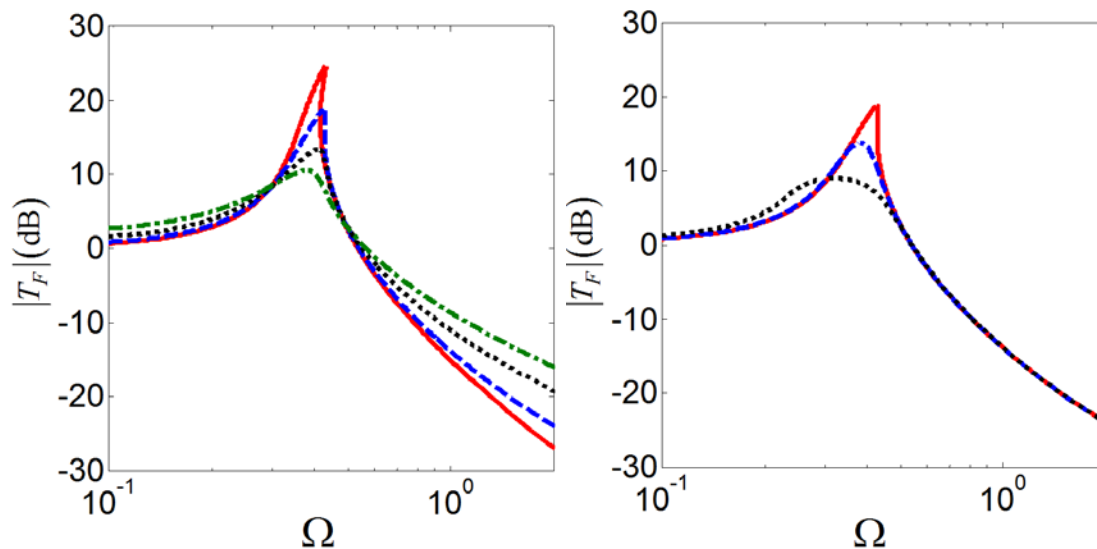


Figure 3 Force transmissibility changes with vertical damping ζ_v and horizon damping ζ_h . The parameters are $\hat{k}=1$, $\hat{l}=0.7$, $\hat{F}_e=0.1\hat{F}_{e\max}$. (a) changing of the vertical damping and the horizontal damping is fixed at $\zeta_h=0.05$: Red solid line, $\zeta_v=0.025$, blue dashed line, $\zeta_v=0.05$, black dotted line, $\zeta_v=0.1$, dark green dashed-dotted line, $\zeta_v=0.15$. (b) changing of the horizontal damping and the vertical damping is fixed at $\zeta_v=0.1$: Red solid line, $\zeta_h=0.05$, blue dashed line, $\zeta_h=0.5$, black dotted line, $\zeta_h=1$.

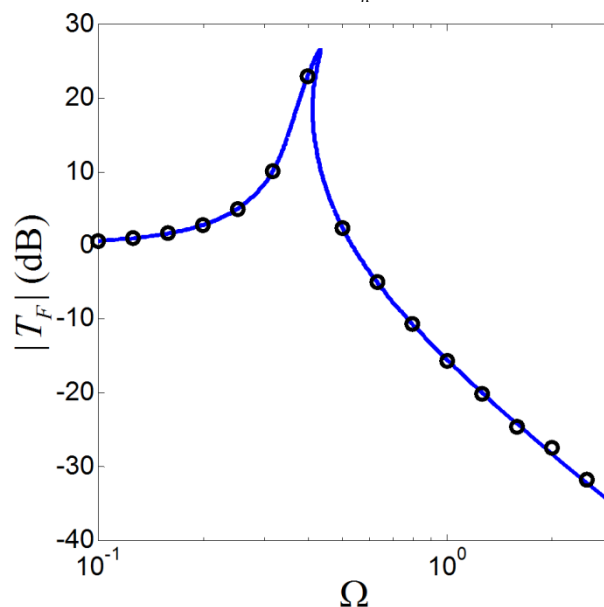


Figure 4 Illustration of the analytical and numerical solution for force transmissibility of the system when the parameters are $\hat{k}=1$, $\hat{l}=0.7$, $\zeta_h=0.05$, $\zeta_v=0.025$, $\hat{F}_e=0.1\hat{F}_{e\max}$. Harmonic balance method: blue solid line; Ruger-Kutta: black 'o'.

To check whether the employed harmonic balance method could correctly capture the dynamic behavior for the parameters chosen, the force transmissibility is plotted in figure 4 together with numerical results. It can be seen that there is reasonable agreement and so the observations made concerning figure 2 and figure 3 can be considered to be valid.

3. Isolation from random excitation

The above section shows that isolation system with both damping and stiffness nonlinearity behaves better than the one with only stiffness nonlinearity. Now the isolation system is introduced to reduce the vibration under random excitation which has Gaussian white-noise characteristic. When such the isolation system with both nonlinear stiffness and nonlinear damping is subject to Gaussian white-noise excitation, the governing equation can be given by

$$m\ddot{x} + c_v\dot{x} + 2c_h \frac{x^2}{x^2 + l^2} \dot{x} + k_v x + 2k_h \left(1 - \frac{l_0}{(x^2 + l^2)^{1/2}} \right) x^3 = \sigma W(t) \quad (13)$$

Where $W(t)$ is a unit white noise, namely, $E[W(t)W(t+\tau)] = \delta(\tau)$.

For $x < 0.3l$, Eq. (13) can be approximated and rewritten as non-dimensional form as

$$\hat{x}'' + 2\zeta_l \hat{x}' + 2\zeta_n \hat{x}^2 \hat{x}' + \alpha \hat{x} + \gamma \hat{x}^3 = \hat{\sigma} W(t) \quad (14)$$

where $\hat{\sigma} = \sigma/k_v x_s$. The equations for the first and second order moments on the basis of Gaussian closure are given by

$$\left\{ \begin{array}{l} \dot{m}_{10} = m_{01} \\ \dot{m}_{01} = -2\zeta_l m_{01} - 2\zeta_n m_{20} m_{01} - \alpha m_{10} - \gamma (3m_{10} m_{20} - 2m_{10}^3) \\ \dot{m}_{20} = 2m_{11} \\ \dot{m}_{11} = m_{02} - 2\zeta_l m_{11} - 2\zeta_n m_{20} m_{11} - \alpha m_{20} - \gamma (3m_{20}^2 - 2m_{10}^4) \\ \dot{m}_{02} = -4\zeta_l m_{02} - 4\zeta_n m_{20} m_{02} - 2\alpha m_{11} - 2\gamma (3m_{11} m_{20} - 2m_{10}^3 m_{01}) + 2\hat{\sigma}^2 \end{array} \right. \quad (15)$$

where $m_{ij} = E[\hat{x}^i \hat{x}'^j]$, $\hat{x}^i$ is the i th power of $\hat{x}$, $\hat{x}'^j$ is the j th power of $\hat{x}'$. Then these equations could be solved numerically for a short time step, and the obtained moments were used to achieve the two-dimensional Gaussian transition probability density. Probability density for displacement and velocity of the mass $p(\hat{x}, \hat{x}')$ can be calculated by path integration. And the non-dimensional force transmitted through the nonlinear spring and damper, is given by Eq. (5). Thus the approximate solution of probability density function of the non-dimensional transmitted force $p(\hat{f}_t)$ can be obtained. To check whether the path integration method correctly captures the stochastic dynamic behaviour for the parameters chosen, the probability density function for different ratios of $\hat{l}$, are plotted in Fig. 5 together with Monte-Carlo simulation results. It can be seen that there is reasonable agreement and so the observations made concerning figure 4 can be considered to be valid. It can be also found that the height of the peak for probability density function at $\hat{f}_t = 0$, is increased as the value of $\hat{l}$ decreasing, the reason for this is that the nonlinearity of the isolator is increased, then the probability for

transmitted force through the nonlinear spring and damper is decreased in probability as the value of $\hat{l}$ decreasing.

The approach for the force transmissibility is to calculate the Standard Deviation value of the transmitted force and form the ratio

$$|T_F| = \frac{S_d(\hat{f}_t)}{\hat{\sigma}} \quad (16)$$

Figure 6 shows the force transmissibility of the vibration isolation system by swiping intensity of Gaussian white-noise for different ratios of $\hat{k}$. It can be seen that as $\hat{\sigma}$ increasing the force transmissibility is increased. And the force transmissibility is decreased at low excitation but increased at high excitation, as $\hat{k}$ increasing. So the vibration isolation performance can be improved by nonlinear stiffness from harmonic excitation, but the performance of the vibration isolation from random excitation depends on the level of excitation noise. The force transmissibility of the vibration isolation system excited by Gaussian white-noise excitation for different ratios of ζ_h , are plotted in Fig.7. It can be seen that as $\hat{\sigma}$ increasing the force transmissibility is increased. And the force transmissibility is increased at any concerning intensity, as ζ_h increasing. So the performance of the vibration isolation from harmonic excitation can be improved but from random excitation is worsen as increasing nonlinear damping.

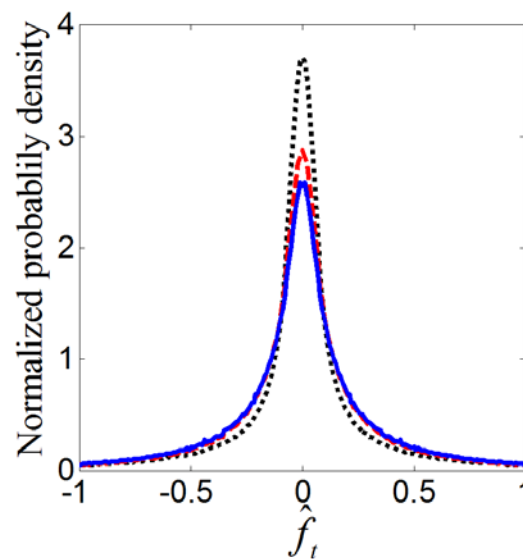


Figure 5 Probability density of non-dimensional transmitted force various with nonlinearity changing by $\hat{l}$. The parameters are the same with above figure, $\hat{\sigma} = 1$; Analytical results: black dotted line, $\hat{l} = 0.7$, red dashed line, $\hat{l} = 0.8$, blue solid line, $\hat{l} = 0.9$.

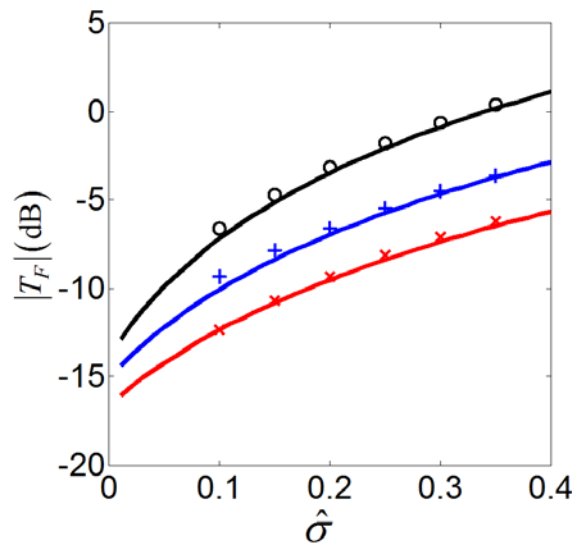


Figure 6 Force transmissibility changes with horizon damping ζ_h . The parameters are $\hat{l} = 0.7$, $\zeta_v = 0.05$, $\hat{k} = 1$. Red line, $\zeta_h = 0.5$, blue line, $\zeta_h = 0.25$, black line, $\zeta_h = 0.1$. black 'o', red 'x' and blue '+' are the results of Monte Carlo simulation.

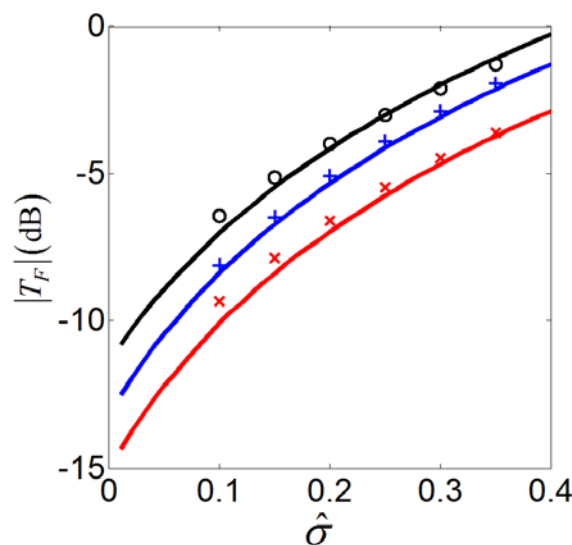


Figure 7 Force transmissibility changes with horizon damping ζ_v . The parameters are $\hat{l} = 0.7$, $\zeta_v = 0.05$, $\hat{k} = 1$. Red line, $\zeta_v = 0.05$, blue line, $\zeta_v = 0.025$, black line, $\zeta_v = 0.01$. black 'o', red 'x' and blue '+' are the results of Monte Carlo simulation.

4. Conclusion

In this work, a new vibration isolation system is proposed by exploiting the advantages of both nonlinear damping and stiffness. The harmonic balance method is a well-established technique for the analysis of such the vibration isolation systems under harmonic excitation. As nonlinear stiffness increasing, it was found that the isolation range is extended to lower frequencies. As nonlinear

damping increasing, it was found that the resonance response was reduced, at the same time high frequency transmissibility is almost the same. Further, such the isolator is introduced to reduce vibration under random excitation; the path integration method is used to analysis the force transmissibility. It was also found that as intensity of noise increasing the force transmissibility is increased, until to the maximum value, and then decreased. And the force transmissibility is increased at any concerning intensity, as nonlinear stiffness increasing. While the force transmissibility is increased for low levels of noise, and decreased for high levels of noise as nonlinear damping increasing.

Acknowledgements

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