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Vortex-Antivortex Pair Production in Perturbed Dipolar Bose-Einstein Condensate

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Abstract. The dynamics of vortex-antivortex pair production with the associated critical velocities are investigated in perturbed dipolar Bose-Einstein condensates (BECs), by using a quasi-two-dimensional mean-field Gross-Pitaevskii (GP) model. By exploring the range of dipolar interaction strengths, it is also verified the regime in which turbulent behaviors can be observed. In the present contribution, we consider the emission of vortex and antivortex pairs in dipolar BECs produced by circularly moving blue detuned laser, simulated by a two-dimensional Gaussian obstacle. The critical velocities of the moving obstacle for vortex-antivortex nucleation, which emerge as regular pairs or cluster, are determined by numerical simulations, considering a BEC of dysprosium atoms with repulsive contact and dipolar interactions.

1. Introduction

The emergence of Bose-Einstein condensates (BECs) in chromium (^{52}Cr) [1, 2], dysprosium (^{164}Dy) [3] and erbium (^{168}Er) [4] accompanied by long-range dipole-dipole (DD) interactions superimposed on the short-range contact inter-atomic collisions have impacted the investigations of ultracold quantum gases [5–8]. The anisotropic character and long-range nature of DD interactions endows the dipolar BECs (DBECs) with several distinct features such as the subordination of stability on the trap geometry [2], roton-maxon character of the excitation spectrum, vortices and vortex stripes [9], new dispersion relations for elementary excitations [10], novel quantum phases, explicit and hidden vortices [11], specific vortex-antivortex pairs [12–14], etc. Among the several studies, the dynamical stabilization was explored by time modulation of scattering length and using the interplay between nonlinear interactions [15–17]. Tuning contact interactions by means of the Feshbach resonance is an important tool in analyzing the properties of DBECs [5,6]. In particular, DBECs with pure long-range interactions can be made by tuning the contact interactions to zero. Noticeable is the crescent interest in binary dipolar mixtures, exploring miscible and immiscible regimes [18], with an experimental realization (using erbium and dysprosium) reported in [19]. Along with these studies, the intensive investigations on rotational properties and vortex dynamics with binary dipolar atoms [20, 21] aim to provide a platform to access plenty of other many-body quantum phenomena, such as the possible creation of long-lived quantum-droplet states and expected connections with superfluidity in BEC. On this regard, opening new avenues to explore other possible exotic regimes of dipolar quantum matter, it is worthwhile to point out the recent reported realization of BEC with dipolar sodium-cesium (NaCs) molecules [22], following investigations to stabilize ultracold gases of these strongly dipolar systems [23].

Within our interest to explore the production of quantum vortices in dipolar gases, using pancake-like quasi-two-dimensional (quasi-2D) formalism, we can briefly review some well-known general properties of quantum vortices in BEC [7], which can be created only at a minimum critical angular velocity, when energetically favorable, different from their occurrence in normal fluids. The critical rotation for vortex nucleation in realistic experiments can be larger due to dynamical instabilities at the boundaries. In superfluid and superconductor phase transitions, quantum vortices play a prominent role, as known since discoveries related to Helium superfluidity [24] and at high-temperature superconductors [25]. Beyond that, they are commonly observed and investigated in a wide range of contexts going beyond cold-atom physics, such as on exciton-polariton condensates [26], on polariton superfluids [27], and in optics [28]. Quantized vortices have been nucleated in BEC through several techniques, such as rotating the trapping potential or thermal cloud (in experiments consistent with following up numerical analysis [11–14,29]), stirring the condensate with a blue- or red-detuned laser beam [30], moving a condensate past a defect [31], rapid quench phase transitions of a cooling condensate [32], or decay instability of a soliton [33]. More recently, vortex nucleations have been applied to lattice configurations [34], by predicting vortex positions along low-density paths separating the sites. Within recent reported experiment, by applying a moving Gaussian obstacle in a BEC [35], the authors have shown that a tangle of quantized vortices can be produced. As known from classical fluid dynamics, vortex tangles are understood as a signature of turbulence, with the structure of turbulence being already associated with the flow of incompressible viscous fluids by Kolmogorov [36].

Next section, we set the basic quasi-2D model formalism for a dipolar BEC, followed by the main results provided in section 3, with our conclusions in section 4.

2. The 2D dipolar time-dependent formalism

At ultra-low temperatures, the Gross-Pitaevskii (GP) formalism for a dipolar BEC confined in a pancake-like trap potential, after reduction from three to 2D, and considering a time-dependent stirring external potential in addition to the usual 2D trap potential, can be written as (for details, see Refs. [37–39]):

$$\mathrm{i}\frac{\partial\psi}{\partial t} = \left(-\frac{1}{2}\nabla_\rho^2 + \frac{\rho^2}{2} + V_G + g|\psi|^2 + g_{dd}\int \frac{d^2k_\rho}{4\pi^2} e^{i\mathbf{k}_\rho\cdot\tilde{\rho}}\tilde{n}(\mathbf{k}_\rho)\widetilde{V}^{(d)}(\mathbf{k}_\rho)\right)\psi, \quad (1)$$

where $\psi \equiv \psi(\boldsymbol{\rho}, t)$ is the dimensionless 2D wave function, normalized to one, $g \equiv \sqrt{8\pi\lambda}a_sN/\ell_\rho$ is the 2D contact interaction parameter, with a_s the two-body scattering length, N the number of atoms, with $\ell_\rho \equiv \sqrt{\hbar/(m_A\omega_\rho)}$ the length unit given in terms of the transversal trap frequency ω_ρ ($m_A = Am_n$ is the mass of an atom with A nucleons of mass m_n). The dipolar interaction term, in the 2D dimensions, is given by a momentum-space 2D integral on the Fourier transformed density $\tilde{n}(\mathbf{k}_\rho)$ and dipolar interaction $\widetilde{V}^{(d)}(\mathbf{k}_\rho)$ ($\mathbf{k}_\rho$ being the transversal 2D momentum) with g_{dd} being the dipolar parameter, that can be written as $g_{dd} = 3N(a_{dd}/\ell_\rho)$, where defined as dipole length, $a_{dd} \equiv \mu_0\mu^2/(12\pi\hbar\omega_\rho\ell_\rho^2)$ (μ is the atom magnetic moment, with μ_0 being the permeability of free space). As detailed in [39] the dipolar interaction in momentum space is given by a momentum function $h_{2d}^{\perp}(\mathbf{k}_\rho) = 2 - 3\sqrt{\frac{\pi}{2\lambda}}k_\rho \exp\left(\frac{k_\rho^2}{2\lambda}\right)\mathrm{erfc}\left(\frac{k_\rho}{\sqrt{2\lambda}}\right)$, which is multiplied by a factor $(3\cos^2\alpha - 1)/2$, where α is defined by the magnetic field applied to the system and the direction of the magnetic moment μ_A of the atoms. Therefore, effectively, the dipolar factor can be tuned, from maximum positive ($\alpha = 0$) to negative values ($\alpha = 90°$), by adjusting this

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angle. In (1), V_G represents the time-dependent moving Gaussian obstacle,

$$V_G(x,y,t) \quad \equiv \quad A_0 \exp\left(-\frac{[x-x_0(t)]^2+[y-y_0(t)]^2}{2\sigma^2}\right), \tag{2}$$

which is modeling a possible experimental realization with a stirring mechanism that uses a laser-detuned 2D obstacle moving circularly within a fixed radius r_0 and given frequency ν. In the above Gaussian distributions, $x_0(t) \equiv r_0\cos(\nu\, t)$ and $y_0(t) \equiv r_0 \sin(\nu\, t)$ are giving the instant position of the obstacle in the 2D plane, with σ being the corresponding standard radial deviation (close to half-width of the distribution). The amplitude (strength) of the perturbation A_0 we can assume as a percentage of the stationary chemical potential.

3. Vortex nucleation by stirring Gaussian potential

This section is concerned with the main results on the nucleation of vortex-antivortex pairs and the dynamics in the dipolar BEC by the stirring potential. To obtain a stable ground state density profile, in our simulations the Eq. (1) is first solved by using imaginary time ($t \to -it$), without the Gaussian obstacle ($A_0 = 0$). Next, this solution is evolved in real-time with the obstacle ($A_0 \neq 0$). Once considered fixed the shape of the Gaussian model for the obstacle (amplitude and width), with its radial position inside the condensate, the principal model parameters are the strength of the DDI and the angular frequency ν of the obstacle, implying in a corresponding linear velocity $v = \nu r_0$.

For the computational analysis, to investigate the vortex nucleation and dynamics of the vortex pair productions in dipolar BECs, we need to combine the usual split-step Crank-Nicholson method with the Fast Fourier Transform approach [40]. In the numerical approach, we have used a 256×256 grid size with $\Delta x = \Delta y = 0.1$ for both x and y (units ℓ_ρ), with time step $\Delta t = 0.0001\omega_\rho^{-1}$. Along with our numerical simulations, motivated by possible experimental realization considering the dipolar ^{164}Dy, we also kept fixed the following parameters related to the atom-atom interactions and stirring Gaussian model: $a_{dd} = 131\, a_0$, $N = 1.5 \times 10^4$, $r_0 = 4\ell_\rho$, and $\sigma = 1.5\ell_\rho$, where $a_0 = 5.29177 \times 10^{-11} m$ is the Bohr radius and $\ell_\rho = 1 \times 10^{-6} m$. With two-body scattering length enough repulsive such that the effect of the dipolar interaction can better be evaluated by changing the tilting DDI angle α. The radial position of the obstacle (r_0) inside the condensate was arbitrarily chosen at an approximate average distance between the center and the border limits of the condensate, considering that the critical velocities for the production of vortex-pairs are given by $v_c = r_0\nu_c$. By varying the r_0 position inside the main part of the condensed fluid, one can verify that no vortex pairs can be produced when $r_0 \to 0$.

In Fig. 1, by assuming the ^{131}Dy, with $a_{dd} = 131a_0$ and the dipolar angle $\alpha = 0$, and considering $a_s = 50a_0$ for the the contact interaction, the dynamical production of vortex pairs, generated by a circularly moving obstacle is represented by a series of time snap-shot panels, considering given values of the amplitude A_0 (in terms of the chemical potential μ) and rotational frequency ν. The time is represented by the columns, considering up two loops of the obstacle, from $\nu t = 0.5\pi$ [a$_1$ (first column)] till $\nu t = 4\pi$ [a$_6$ (sixth column)], as indicated. Results for three values of constant A_0 are shown, as indicated: $A_0 = 0.5\mu$ (upper row), $A_0 = \mu$ (middle row), and $A_0 = 1.5\mu$ (lower row). Therefore, as in all the cases we have the obstacle at the same radial position r_0, the time evolution of the three rows are indicating the vortex-pair production dependence on the amplitude.

In Fig. 2, for a direct comparison, in the same way we are representing snap-shot results in close similarity as being represented in Fig. 1, with all parameters being the same except the nonlinear ones. The contact interaction in this case is fixed to zero, $a_s = 0$ (relying in the possibilities to apply Feshbach techniques to change a_s), with the dipolar interactions given by $a_{dd} = 131a_0$, with $\alpha = 21.8°$. The main idea in showing these panels was to verify the vortex-pair production in case of a pure dipolar system. By increasing the value of the tilting angle α

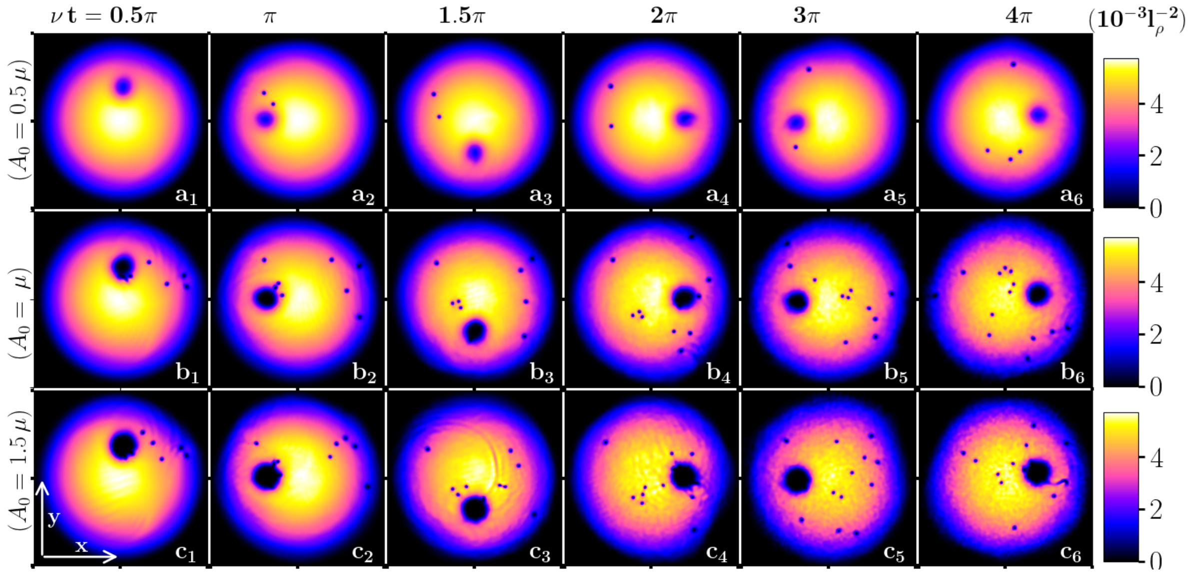

Figure 1. (color online) Contour plots of the density showing the time evolution for the vortex nucleation by the moving Gaussian obstacle in the dipolar BEC with $a_s = 50\,a_0, a_{dd} = 131\,a_0$ (assuming ^{164}Dy) and dipolar tilting angle $\alpha = 0$. Panels in $1^{st}(a_i)$, $2^{nd}(b_i)$, and $3^{rd}(c_i)$ rows are for Gaussian amplitudes, $A_0 = 0.5\mu, \mu$, and 1.5μ, respectively (μ being the chemical potential). See also Table 1. The x and y 2D coordinates runs from -10ℓ_ρ to 10ℓ_ρ for each panel.

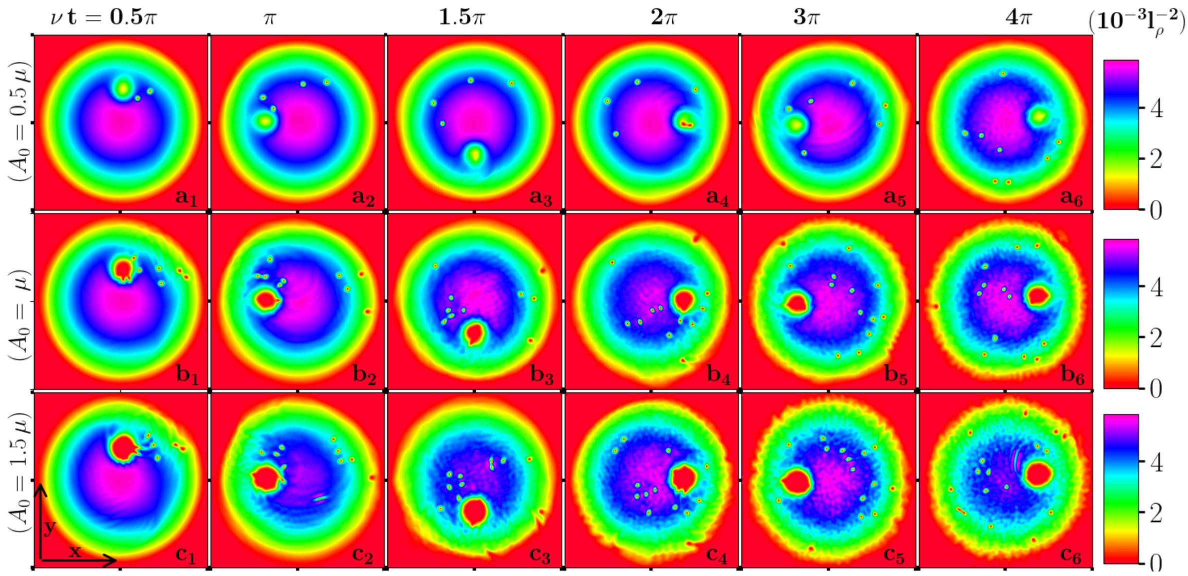

Figure 2. (color online) Similar as in Fig.1 for the case of pure-dipolar system, with $a_s = 0$, $a_{dd} = 131\,a_0$ and $\alpha = 21.18$. See also the parameters in Table 1.

from zero (as in Fig. 1) to 21.8°, the purpose was to keep about the same valeu the chemical potential in both cases represented in Figs. 1 and 2. By comparing the vortex production and dynamics of both figures, we have a clear indication that dipolar systems may have a reduced critical limit for vortex production, as well as more fluctuations in the fluid along the dynamics. For higher values of A_0, more visible is the effect of long-range interactions in the dynamics.

In the following Table 1, we are resuming the physical quantities and interaction parameters being considered in both the cases, for easy comparison.

Table 1. Physical quantities and interaction parameters considered in Figs. 1 and 2. The constants are $N = 1.5\times10^4$, $r_0 = 4\ell_\rho$, and $\sigma = 1.5\ell_\rho$, $a_0 = 5.29177\times10^{-11}m$ and $\ell_\rho = 1\times10^{-6}m$.

Description	a_s	$(a_{dd},\ \alpha)$	$\mu(\hbar\omega_\rho)$	$E(\hbar\omega_\rho)$	$\sqrt{\langle\rho^2\rangle}(\ell_\rho)$	$\lvert\psi\rvert^2_{max}(\ell_\rho^{-2})$
Figure 1	$50a_0$	$(131a_0,\ 0°)$	56.98	37.85	6.11	0.006
Figure 2	0	$(131a_0,\ 21.18°)$	56.98	37.85	6.11	0.006

4. Conclusions

We have studied the vortex-antivortex pair nucleation and dynamics in dipolar Bose-Einstein condensates (BECs), by using a quasi-two-dimensional mean-field Gross-Pitaevskii equation, in which the 2D trap potential is affected by a circularly moving blue detuned laser, simulated by a two-dimensional Gaussian obstacle inside the fluid. The presented results are extending to dipolar BEC of ^{131}Dy the previous results published in [39] and also discussed in [41], in which it was considered the erbium as the sample dipolar atom. As verified by our results, in which we compare results obtained for pure dipolar systems with the case that we have both contact and dipolar nonlinear interactions, both with about the same static physical quantities (chemical potential, energy and root-mean-square radius), the effect due the long-range dipolar interactions can be clearly verified by the minimum critical rotational velocity for the emission of vortex pairs (smaller than in case we have non-zero contact interactions), as well as in the subsequent dynamics of vortex pairs inside the quantum fluid.

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