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VÍCTOR HUGO RIBEIRO

MODELLING OF THE FATIGUE CRACK INITIATION AND PROPAGATION BEHAVIOUR IN METALS AND ALLOYS

Ilha Solteira
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PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA MECÂNICA

VÍCTOR HUGO RIBEIRO

MODELLING OF THE FATIGUE CRACK INITIATION AND PROPAGATION BEHAVIOUR IN METALS AND ALLOYS

A thesis submitted to the Faculdade de Engenharia de Ilha Solteira - UNESP in partial fulfillment of the requirements for obtaining the Doctorate in Mechanical Engineering. Knowledge area: Solid Mechanics

Advisors: Aparecido Carlos Gonçalves
José António Fonseca de Oliveira Correia

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This thesis is especially dedicated

to my family.

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“*The ear that heareth the reproof of life abideth among the wise.*
He that refuseth instruction despiseth his own soul, but he that heareth reproof getteth understanding.”

Proverbs 15:31-32

RESUMO

A fadiga é uma das principais causas de falhas e também de fraturas mecânicas em detalhes estruturais sob carregamento cíclico. Existem vários modelos para descrever o comportamento de fadiga de baixo ciclo com base em critérios de dano por deformação, como Coffin-Manson-Morrow, utilizado na fase iniciação de trinca. A lei de Paris só é aplicável na região estável de crescimento de trinca (região II), e como consequência, surgiram diversos modelos dela derivados para contornar essa limitação, aumentando o número de variáveis e parâmetros necessários e, consequentemente, tornando a solução onerosa. Outros modelos como UniGrow, Huffman, Peeker, entre outros, descrevem as fases de iniciação e propagação de trincas de fadiga, adotando o conceito de reinicializações sucessivas de trincas (incrementos) com base em abordagens locais.

Nesta pesquisa, a modelagem de fadiga de baixo ciclo baseada na abordagem de Huffman usando a densidade de energia de deformação e considerando a densidade de discordâncias é investigada e discutida. Para isso, várias metodologias para avaliar a resistência à fadiga de baixo ciclo com base na abordagem de Huffman e explorar diferentes parâmetros de densidade de deslocamento são sugeridas: (i) densidade de discordâncias crítica associada a maior amplitude de deformação; (ii) valor médio da densidade de discordâncias dos dados experimentais de fadiga disponíveis e, (iii) predição estocástica de Monte Carlo(MC) considerando a variabilidade da densidade de discordâncias e o coeficiente de endurecimento por deformação cíclica. Dados experimentais de fadiga das ligas de Al1050, Al6061-T651 e AlMgSi0.8 da literatura são usados neste estudo. E ainda, é feita uma comparação entre os dados experimentais de fadiga e as curvas de tensão-vida com base nas várias metodologias sugeridas.

Adicionalmente, é apresentada uma comparação do efeito de vários modelos de fechamento/abertura de trinca no comportamento de crescimento de trinca por fadiga da liga de Al6061-T651. Os modelos de fechamento de trinca em consideração são: Elber; Katcher-Kaplan; Clerivet-Bathias; Schijve; Zhang; Newman; Savaidis; Codrington-Kotousov; Correia. Uma comparação entre esses modelos é feita.

Além disso, foi sugerida modelagem do comportamento de crescimento de trinca por fadiga para a liga 6061-T651 através do modelo Huffman com base na densidade de energia de deformação das discordâncias considerando os efeitos das tensões residuais à frente da ponta da trinca(x), onde essas tensões estão relacionadas ao dano por fadiga de um incremento de fissura Δa, como parâmetro do calibrador. Duas abordagens sugeridas por Noroozi e Huffman são estudadas e discutidas. Na modelagem do comportamento de crescimento de trinca por fadiga, a influência da densidade de energia de deformação calculada para os valores de densidade de discordâncias crítica associada a maior amplitude de deformação e o valor médio da densidade de discordâncias para os resultados de fadiga experimentais disponíveis também foram considerados nesta investigação. Os valores de densidade de discordâncias estudados não influenciam significativamente o comportamento de propagação de trincas por fadiga. Conclui-se também que o procedimento de consideração dos efeitos das tensões residuais influencia o calibrador, Δa, não sendo possível concluir qual é o melhor método para descrever os efeitos das tensões residuais.

Palavras-chave: ligas de alumínio; fadiga de baixo ciclo; taxa de crescimento de trincas; efeitos do fechamento de trincas; energia de deformação; densidade de discordâncias; efeitos de tensões residuais.

ABSTRACT

Fatigue is the main causes of mechanical fractures in structural details under cyclic loading. There are several models to describe low-cycle fatigue behavior based on strain damage criteria, such as Coffin-Manson-Morrow, which is used in the crack initiation phase. The Paris law is only applicable in the stable region of crack growth (region II), and, as a consequence of this, several models have emerged derived from it emerged to overcome this limitation, increasing the number of variables and parameters required and, consequently, making the solution costly. Other models such as UniGrow, Huffman, Peeker, among others, describe the initiation and propagation phases of fatigue cracks, adopting the concept of successive crack restarts (increments) based on local approaches.

In this research, low-cycle fatigue modeling based on the Huffman approach using strain energy density and considering dislocation density is investigated and discussed. To this end, several methodologies to evaluate low-cycle fatigue resistance based on the Huffman approach and exploring different dislocation density parameters are suggested: (i) critical dislocation density associated with greater strain amplitude; (ii) average value of dislocation density from available experimental fatigue data and, (iii) stochastic Monte Carlo (MC) prediction considering the variability of dislocation density and the cyclic strain hardening coefficient. Experimental fatigue data of Al1050, Al6061-T651 and AlMgSi0.8 alloys from the literature are used in this study. Furthermore, a comparison is made between the experimental fatigue data and the stress-life curves based on the various suggested methodologies.

Additionally, a comparison of the effect of various crack closure/opening models on the fatigue crack growth behavior of Al6061-T651 alloy is presented. The crack closure models under consideration are: Elber; Katcher-Kaplan; Clerivet-Bathias; Schijve; Zhang; Newman; Savaidis; Codrington-Kotosov; and Correia. A comparison between these models is done.

Furthermore, modeling of the fatigue crack growth behavior for the Al6061-T651 alloy was suggested using the Huffman model based on the strain energy density of the dislocations considering the effects of residual stresses ahead of the crack tip(x), where these stresses are related to the fatigue damage of a crack increment Δa, as a calibrator parameter. Two approaches suggested by Noroozi and Huffman are studied and discussed. In modeling fatigue crack growth behavior, the influence of the calculated strain energy density for the critical dislocation density values associated with the largest strain amplitude and the average dislocation density value for available experimental fatigue results were also considered in this investigation. The dislocation density values studied do not significantly influenced the fatigue crack propagation behavior. It is also concluded that the procedure for considering the effects of residual stresses influences the calibrator, Δa, and it is not possible to conclude which is the best method to describe the effects of residual stresses.

Keywords: aluminium alloys; low-cycle fatigue; crack growth rates; crack closure effects; strain energy; dislocation density; residual stress effects.

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NOMENCLATURE

LATIN

a	Crack length
a^*	Normalized crack length
a_1, a_i	Initial crack size, crack size increment
a_n	Distance of load-line to notch tip
a_0	Notch size
Δa	Calibration parameter
Δa_i	Crack range increment
A	Elongation at fracture, Codrington-Kotousov model parameter
A_0, A_1, A_2, A_3	Newman's or Savaidis model parameters
b	Fatigue strength exponent
$\vec{b}$	Burger's vector
B	Thickness, Codrington-Kotousov model parameter
c	Fatigue ductility exponent
$C, C_1, C_2, C_C, C_{el}, C_{pl}, C_F, C_{HS}, C_{MOD}, C_P, C_W$	Material constant; parameter related to endurance limit in probabilistic fatigue model, Codrington-Kotousov model parameter
COV	Coefficient of variation
d	Distance of load-line from edge of the plate
D	Huffman fatigue damage parameter
$\frac{da}{dN}, \frac{da^*}{dN^*}$	Fatigue crack growth rate
e	Eccentricity of load-line, error
$\bar{e}$	Mean error
E	Modulus of elasticity or Young´s modulus
E_{steel}	Elastic modulus of steel
$f, f()$	Function of the stress intensity factor range; frequency; yield function
$F(), F^{-1}()$	Cumulative distribution function
f_y	Higher yield strength
f_u	Ultimate tensile strength
F_T	Geometric factor for tension
F_B	Geometric factor for bending
F_{max}	Maximum force
F_{min}	Minimum force
ΔF	Force range
H	Distance between the points of load application
K	Stress intensity factor; Applied stress intensity factor; Strain hardening coefficient; Material constant

$\boldsymbol{K_0}$	Stress intensity factor for $R = 0$.
$\boldsymbol{K_L}$	Limiting $\boldsymbol{K_{max}}$ value for detectable closure
$\boldsymbol{K'}$	Cyclic strength coefficient
$\boldsymbol{K_C}, \boldsymbol{K_{IC}}$	Fracture toughness
$\boldsymbol{K_{mat}}$	Material toughness measured by stress intensity factor
$\boldsymbol{K_{max}}$	Maximum stress intensity factor
$\boldsymbol{K_{max,applied}}$	Applied maximum stress intensity factor
$\boldsymbol{K_{max,tot}}$	Total maximum stress intensity factor
$\boldsymbol{K_{min}}$	Minimum stress intensity factor
$\boldsymbol{K_{op}}$	Opening Stress intensity factor
$\boldsymbol{K_r}, \boldsymbol{K_{residual}}$	Residual stress concentration factor
$\boldsymbol{\Delta K}$	Stress intensity factor range
$\overline{\boldsymbol{\Delta K}}$	Walker equivalent stress intensity range
$\boldsymbol{\Delta K_I}$	Stress intensity factor range in mode I
$\boldsymbol{\Delta K_{I,Tension}}$	Tension component of stress intensity factor range in mode I
$\boldsymbol{\Delta K_{I,Bending}}$	Bending component of stress intensity factor range in mode I
$\boldsymbol{\Delta K_{app}}, \boldsymbol{\Delta K_{applied}}$	Applied stress intensity factor range
$\boldsymbol{\Delta K_{cl}}$	Stress intensity factor range at closuring
$\boldsymbol{\Delta K_{eff}}$	Effective stress intensity factor range
$\boldsymbol{\Delta K_{eff,0}}$	Effective stress intensity factor range for $R = 0$
$\boldsymbol{\Delta K_{eff,max}}$	Maximum Effective stress intensity factor range
$\boldsymbol{\Delta K_{eff,min}}$	Minimum effective stress intensity factor range
$\Delta \boldsymbol{K_{op}}$	Stress intensity factor range at opening
$\boldsymbol{\Delta K_{tot}}$	Total stress concentration factor range
$\boldsymbol{\Delta K_{th,eff}}$	Effective threshold ΔK_{eff}
$\boldsymbol{\Delta K_{up,eff}}$	Effective limiting ΔK_{eff}
$\Delta \boldsymbol{K^*_{eff}}$	Normalized ΔK_{eff}
$\Delta \boldsymbol{K^*_{th,eff}}$	Normalized $\Delta K_{th,eff}$
$\Delta \boldsymbol{K^*_{up,eff}}$	Normalized $\Delta K_{up,eff}$
$\boldsymbol{\Delta K_{th}}$	Threshold of stress intensity factor range
$\boldsymbol{\Delta K_{th,0}}$	Threshold of stress intensity factor range for $R = 0$
$\boldsymbol{\Delta K_{th,steel}}$	Threshold of stress intensity factor range of steel
$\boldsymbol{\Delta K_{up}}$	Ultimate stress intensity factor range
$\Delta \boldsymbol{K^*}$	Normalized ΔK
$\Delta \boldsymbol{K^{*+}}$	Normalizing variable of ΔK
$\Delta \boldsymbol{K^*_{th}}$	Normalized ΔK_{th}
$\Delta \boldsymbol{K^*_{up}}$	Normalized ΔK_{up}
$\boldsymbol{J_{mat}}$	Material toughness measured by J-methods
$\boldsymbol{\Delta J}$	Strain energy release density
$\boldsymbol{\Delta J_{up}}$	Ultimate value of J-integral range
$\boldsymbol{\Delta J_{th}}$	Threshold value of the cyclic J-integral

$\Delta \mathbf{J}^{*+}$	Normalizing variable of ΔJ
$\Delta \mathbf{J}_{th}^{*}$	Normalized ΔJ_{th}
$\Delta \mathbf{J}_{up}^{*}$	Normalized ΔJ_{up}
L, L_1, L_2	Characteristic length
L_{0p}	Length at fracture
$m, m_1, m_2, m_C,$ $m_{el}, m_{pl}, m_F, m_{HS},$ m_{MOD}, m_P, m_W	Material constant
n	Strain hardening exponent, number of specimens
n'	Cyclic strain-hardening exponent
N	Number of cycles (fatigue life)
N^*	Normalized number of cycles
N_0	Reference number of cycles; Threshold value of lifetime
N_E	Experimental number of cycles
N_f	Number of cycles to failure
N_i	Number of cycles in crack initiation; Number of cycles of the specimen i
N_P	Predicted number of cycles
$2N_f$	Reversals to failure
$2N_{f,i}$	Reversals to failure of the specimen i
p	Material constant; Probabilistic to failure
$P1, P2 \ldots P5$	Transversal direction specimen identification
P_{max}	Maximum force
P_{min}	Minimum force
ΔP	Eccentric applied force range
r	Crack tip polar coordinate, radius
R, R_σ	Stress ratio, radius
R_{eff}	Effective stress R-ratio
$R_{eff,1}$	Deterministic effective stress R-ratio for first cycle measurement
$R_{eff,s}$	Deterministic effective stress R-ratio for stabilized measurement
R_ε	Strain ratio
$R_{p,0.2}$	Monotonic 0.2% proof stress
R_m	Ultimate tensile stress
R^2	Determination coefficient
S_{max}	Maximum stress
$S_{op,phen}$	Opening stress
T	Thickness
SWT	Smith, Watson and Topper fatigue damage parameter
U, U_{SIF}	Quantitative closure parameter; Effective stress intensity ratio
U_1	Quantitative closure parameter for the first cycle

U_d	Dislocation strain energy
$U_{d,1}$	Deterministic quantitative closure parameter for the first cycle measurement
$U_{d,s}$	Deterministic quantitative closure parameter for the stabilised measurement
U_e	Tensile elastic strain energy density
U_i	Quantitative closure parameter related to crack increment i
U_s	Quantitative closure parameter for stabilised measurement
U_{th}	Quantitative closure parameter for the first crack length
U_{up}	Quantitative closure parameter for the last crack length
U_p^*	Complimentary plastic strain energy density
W	Characteristic length, width
$W1, W2 \dots W5$	Longitudinal direction specimen identification
x	Distance from the crack tip, calibration parameter
$\overline{x}$	Mean value
X	Cartesian component
Y	Cartesian component, function depending on geometric and loading conditions
Z	Reduction in cross-section at failure

GREEK

α	Material constant; scale parameter of 2-parameter Weibull and Logistic distribution; relation of crack length and specimen width
β	Shape parameter of the 2-parameter Weibull and Logistic distribution
$\gamma, \gamma_{th}, \gamma_w$	Curve fitting parameter; Fatigue crack growth equation exponent, parameter of the statistical Gumbel distribution
δ	Scale parameter of Weibull model; Elementary material blocks of length
δ^*	Elementary material blocks of length in the process zone
ε	Strain
ε_a	Amplitude Strain; Principal in-plane strain (axial direction)
ε_f'	Fatigue ductility coefficient
ε_w	Walker strain
$\Delta\varepsilon$	Strain range
$\Delta\varepsilon^E$	Elastic strain range
$\Delta\varepsilon^P$	Plastic strain range
η	Material property, Codrington-Kotousov non-dimensional parameter
υ	Poisson ratio
ρ	Dislocation density
ρ^*	Notch tip radius; Elementary material block size

$\boldsymbol{\rho_c}$	Critical blunt tip radius, critical dislocation density
$\boldsymbol{\rho_i}$	Dislocation density for each tested specimen
$\boldsymbol{\rho_m}$	Mean dislocation density of available experimental fatigue results
$\boldsymbol{\sigma}$	Stress, standard deviation of Normal distribution
$\boldsymbol{\sigma_0}$	Cyclic yield stress; Stress fatigue limit
$\boldsymbol{\sigma_a}$	Stress amplitude
$\boldsymbol{\sigma_{a,i}}$	Stress amplitude obtained during low-cycle fatigue tests for the specimen i
$\boldsymbol{\sigma_f, \sigma_o}$	Flow stress
$\boldsymbol{\sigma'_f}$	Fatigue strength coefficient
$\boldsymbol{\sigma_M, \sigma_{max}}$	Maximum stress
$\boldsymbol{\sigma_{max,i}}$	Maximum stress amplitude obtained during low-cycle fatigue tests for the specimen i
$\boldsymbol{\sigma^*_M}$	Normalized maximum stress
$\boldsymbol{\sigma_m, \sigma_{min}}$	Minimum stress, mean stress
$\boldsymbol{\sigma^*_m}$	Normalized minimum stress
$\boldsymbol{\sigma_{med}}$	Mean stress
$\boldsymbol{\sigma_{op}}$	Opening stress
$\boldsymbol{\sigma_x}$	Elastic stress along the x direction
$\boldsymbol{\sigma_{xx}, \sigma_{yy}}$	Analytical elastoplastic stress distributions
$\boldsymbol{\sigma_y}$	Yield stress; Elastic stress along the y direction
$\boldsymbol{\sigma_r, \sigma_{residual}}$	Residual stress
$\boldsymbol{\Delta\sigma}$	Stress range
$\boldsymbol{\Delta\sigma^*}$	Normalized stress range
$\boldsymbol{\Delta\sigma_0}$	Endurance fatigue limit
$\boldsymbol{\Delta\sigma_T}$	Tensile stress range
$\boldsymbol{\Delta\sigma_B}$	Bending stress range
$\boldsymbol{\sigma_{\varepsilon_p=0.05}}$	Stress at 0.05 plastic strain
$\boldsymbol{\sigma_{\varepsilon_p=0.001}}$	Stress at 0.001 plastic strain
$\boldsymbol{\Delta\kappa}$	Fatigue crack growth driving force
$\boldsymbol{\mu}$	Mean value of Normal distribution

ACRONYMS

AA	Aluminum association
ASME	American Society of Mechanical Engineers
ASTM	American Society for Testing and Materials
BS	British standards
CAPES	Coordenação de Aperfeiçoamento de Pessoal de Nível Superior

CCS	Castillo-Canteli-Siegele model
COV	Coefficient of variation
CT	Compact tension specimen
CTOD	Crack tip opening displacement
DIC	Digital image correlation
DNVGL	Det Norske Veritas (Norway) and Germanischer Lloyd (Germany)
EC3	Eurocode 3: Design of steel structures
EDM	Electrical discharge machining
ESE(T)	Single-edge cracked geometry
FCG	Fatigue crack growth
FEA	Finite element analysis
FEUP	Faculdade de Engenharia da Universidade do Porto
IFSP	Instituto Federal de São Paulo
iLAPS	Improved LAPS
LAPS	Long and physically short crack model
LCF	Low cycle fatigue
LEFM	Linear elastic fracture mechanics
MC, M.C.	Monte Carlo
NASA/FLAGRO	NASGRO computer program for fracture control analysis
SENB	Single-edge notched bending specimen
STP	Special technical publication (ASTM)
SWT	Smith-Watson-Topper (damage parameter)
TEM	Transmission electron microscopy
UNESP	Universidade Estadual Paulista “Júlio de Mesquita Filho”

CONTENTS

CHAPTER I

1 INTRODUCTION

1.1 MOTIVATION

Fatigue is the main cause of mechanical failures and fractures in mechanical components when subjected to load fluctuations[1-3]. Its studies had begun in the early nineteenth century and today it is still much addressed by the scientific and technical community. Despite the many advances, the infinity of proposed models, and its huge application in design codes[4-6], fatigue modeling is always linked to experimental results due to the high complexity of the phenomenon. In view of the recent evolution of experimental techniques[7-9], what is seen most in fatigue is the development of models that depend less and less on experimental data, either because of the scarcity of data, high occupancy of available equipment, or its high testing cost.

In this context, this research started with the development of a low-cost workbench (methodology) combining an experimental-analytical analysis to characterize the fatigue crack growth behaviour for the ESE(T) specimens of small thickness. A comparison between the crack growth constants (e.g. slope *m*) obtained in the low-cost workbench combining an experimental-analytical analysis and application of the ASTM E647 standard is done. A low-cycle fatigue modelling supported by the Huffman approach based on strain energy density considering the variability of the density of dislocations was suggested. In addition, the crack closure effects on fatigue crack propagation behaviour is also studied and discussed. Finally, the modelling of the fatigue crack growth behaviour based on the strain energy density approach suggested by Huffman[10], taking into account residual stress effects, is presented. In this investigation, experimental data on low cycle fatigue and fatigue crack propagation of 1050/1100[11], 6061-T651[12,13], and AlMgSi0.8[11] aluminium alloys were used.

1.2 OBJECTIVES

The general objective of this research project thesis is to investigate and discuss the modelling of the fatigue crack initiation and propagation behaviour in metals and alloys. To achieve this goal the main objectives are the following:

- To create a simple low-cost useful workbench, to establish a measurement setup for studying the fatigue crack propagation; To use the conventional machines to describe the fatigue crack propagation, the tensile test, hardness and microstructure of the selected material; and finally, to discuss and compare the results obtained in the workbench and conventional system.
- To investigate and discuss the low-cycle fatigue modelling based on the Huffman approach using the strain energy by suggesting different approaches for the dislocation density parameter. To apply Monte Carlo stochastic simulations for obtaining the strain-life parameters, fatigue strength and ductility coefficients, and to generate probabilistic fields for the low-cycle fatigue behaviour of metals. To analyse the various suggested methodologies using available experimental data from the literature, and compare them with each other.
- To compare and discuss the effects of various crack closure/opening models on the fatigue crack growth behaviour of 6061-T651 aluminium alloy and additionally to compare the models to experimental results of crack closure available in the literature.
- To model the fatigue crack growth behaviour for the 6061-T651 aluminium alloy through the Huffman fatigue crack growth approach based on the strain energy density from dislocations considering the residual stress effects, where a comparison with the fatigue crack growth data is done.

1.3 ORGANIZATION OF THESIS

This thesis is organized by: an introduction (Chapter I); a review on fatigue crack growth modelling (Chapter II); an experimental program to characterize the fatigue crack growth of 1100 aluminium alloy (Chapter III); a low-cycle fatigue modelling supported by strain energy density-based Huffman model considering the variability of dislocation density to (Chapter IV); an application and discussion of various crack closure models to predict fatigue crack growth in 6061-T651 aluminium alloy (Chapter V); a fatigue crack growth modelling supported by strain energy density-based Huffman model considering the residual stress effects (Chapter VI); and, finally, the main conclusions and future works (Chapter VII).

The thesis begins with an Introduction (Chapter I), presenting the motivation, the objectives, thesis organization and research contribution. The scope of the introductory part of this thesis is to describe and integrate each piece of this project and give the reader a broad view of the organization of this research.

Then, in Chapter II, a review on fatigue crack growth modelling is presented, where the following topics were addressed – fatigue crack growth laws, fatigue crack growth laws including crack closure effects and fatigue crack propagation based on local damage criteria.

Next, Chapter III is presented, whose title is **an experimental program to characterize the fatigue crack growth of 1100 aluminium alloy**, containing a useful low-cost workbench to describe fatigue crack propagation of very thin aluminium alloy specimens using digital image correlation. The chemical composition, hardness measurements, tensile strength, fatigue crack propagation, and microscopy analysis for Al1100 and Al1050 are also assessed. Furthermore, a comparison between the crack growth constants obtained in the low-cost workbench combining an experimental-analytical analysis and application of the ASTM E647 standard, considering the thickness effects, was also done.

In Chapter IV, entitled **Low-cycle fatigue modelling supported by strain energy density-based Huffman model considering the variability of dislocation density**, it is suggested various methodologies to evaluating low-cycle fatigue strength based on Huffman approach and exploring different dislocation density parameters: the critical dislocation density, the mean value of dislocation density of available experimental fatigue data, and a Monte Carlo stochastic prediction considering the variability of dislocation density and the cyclic strain hardening coefficient. Monte Carlo stochastic simulations are also used to obtain the strain-life parameters, fatigue strength and ductility coefficient, and to generate probabilistic fields for the low-cycle fatigue behaviour. Experimental fatigue data of aluminium alloys were used to apply and compare the suggested methodologies.

Chapter V, proposes **an application and discussion of various crack closure models to predict fatigue crack growth in 6061-T651 aluminium alloy**. At first, a review of fatigue crack growth laws considering the regions of applicability, and laws including crack closure effects is presented. Then, a comparison of the effect of various crack closure/opening models on the fatigue crack growth behaviour is done. Experimental data obtained from literature is compared with these models using deterministic quadratic relation. It was possible to observe that few models presented good fit to experimental crack closure results for the first or stabilised cycles, and many models are inappropriate to do the same.

In Chapter VI, the **modelling of the fatigue crack growth behaviour for the 6061-T651 aluminium alloy through the Huffman fatigue crack growth approach based on the strain energy density** from dislocations considering the residual stress effects was suggested. The Huffman fatigue crack growth modelling is based on the cyclic stress-strain behaviour of the material as well as the local elastoplastic stresses and strains obtained for a distance ahead of the crack tip (x), where those stresses are related to the fatigue damage of a crack increment Δa, as calibrator parameter. The calculations of the elastoplastic stresses and strains are done using Neuber's or Glinka's approach. Two approaches supported by the Noroozi and Huffman's suggestions to consider the residual stress effects were studied and discussed. Besides, in the modelling of the fatigue crack growth behaviour, the influence of the strain energy density calculated for values of critical dislocation density driven by the highest strain amplitude specimen and the mean value of the dislocation density for the available experimental fatigue results was also considered in this investigation.

And finally, in Chapter VII, the main conclusions and future works are presented. It starts with a summary of the main results obtained in each section, and then, it lists the contributions that were made and future steps for the continuity of this research.

1.4 RESEARCH CONTRIBUTIONS

The research contributions presented in this research project thesis are the following:

- A low-cost useful workbench to characterize the fatigue crack propagation in thin specimens;
- Fatigue crack propagation behaviour, mechanical properties, microstructure of commercially pure aluminium were evaluated;
- A comparison of the effect of various crack closure/opening models on the fatigue crack growth behaviour of 6061-T651 aluminium alloy was presented.
- Various fatigue crack closure/opening models were explored.

- Deterministic quadratic relations between U vs. R and R_{eff} vs. R, for the 6061-T651 aluminium alloy, were suggested.
- Low-cycle fatigue behaviour based on the Huffman approach, 1050, 6061-T651 and AlMgSi0.8 aluminium alloys, was studied and discussed.
- Variability of dislocation density on the low-cycle fatigue behaviour was introduced.
- A Monte Carlo stochastic procedure for probabilistic low-cycle fatigue modelling was suggested.
- A comparison between the experimental fatigue data and strain-life curves, 1050, 6061-T651 and AlMgSi0.8 aluminium alloys, is done.
- Fatigue crack growth modelling based on the Huffman FCG approach based on strain energy density and residual stress effects, for the fatigue data of the 6061-T651 aluminium alloy, was done.
- Strain energy density for values of critical dislocation density driven by the highest strain amplitude specimen and the mean value of the dislocation density for the available experimental fatigue results was discussed.
- Residual stress effects based on Noroozi and Huffman's suggestions were studied.

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CHAPTER VII

7 CONCLUSIONS

7.1 CONCLUDING REMARKS

This thesis was dedicated to contributing to: - development of a low-cost workbench (methodology) combining an experimental-analytical analysis to characterize the fatigue crack growth behaviour for the ESE(T) specimens of small thickness (0.3mm) made of 1100 aluminium alloy and constant amplitude tests up to 100N. A comparison between the crack growth constants obtained in the low-cost workbench combining an experimental-analytical analysis and application of the ASTM E647 standard was done.; low-cycle fatigue modelling supported by strain energy density-based Huffman model considering the variability of dislocation density applied to the experimental fatigue data of 1050, 6061-T651, and AlMgSi0.8 aluminium alloys; - application and discussion of various crack closure models to predict fatigue crack growth in 6061-T651 aluminium alloy; and, - fatigue crack growth modelling supported by strain energy density-based Huffman model considering the residual stress effects for the 6061-T651 aluminium alloy.

An experimental programme to characterize the fatigue crack growth of 1100 aluminium alloy

Understanding the current political and scientific scenario in which future researchers are inserted in and knowing the difficulty in obtaining financial support, studies involving less-time-consuming experiments and low-cost workbenches are always welcome. Therefore, this investigation also aims to assess FCG data from the proposed workbench combining an experimental-analytical analysis for the 1100 aluminium alloy specimens, where results seem to be promising for fatigue crack propagation from the stable crack growth (Paris region II). However, in the actual configuration, it is limited to very thin specimens and for non-structural metals. Additionally, the chemical composition, hardness measurements, tensile strength, fatigue crack propagation, and microscopy analysis for Al1100 and Al1050, were also assessed. The results obtained are in line with those recommended for the materials under study. A comparison between the crack growth constants obtained in the low-cost workbench combining an experimental-analytical analysis and application of the

ASTM E647 standard, considering the thickness effects, was done. The results obtained for both approaches, although for different thicknesses (0.3mm and 3mm, respectively, low-cost workbench and ASTM E647 standard), showed consistent and aligned values for the Paris law constants.

Low-cycle fatigue modelling supported by strain energy density-based Huffman model considering the variability of dislocation density

The main conclusions on the low-cycle fatigue modelling supported by strain energy density-based Huffman model considering various addressed methodologies, applied to the experimental fatigue data of 1050, 6061-T651, and AlMgSi0.8 aluminium alloys[1] are given by

- The various methodologies studied were able to model the low-cycle fatigue behaviour based on the strain energy density approach and a good agreement between the experimental fatigue data and the modelled curve is found.
- The strain-life parameters, σ_f' and ε_f', were estimated based on the Huffman model through the critical dislocation density (ρ_c), the mean value of the dislocation density (ρ_m), and the dislocation density (ρ) stochastically considered.
- Among the various methodologies used, the strain-life parameters, σ_f' and ε_f', estimated by strain energy density-based Huffman approach through the mean value of dislocation density (ρ_m) and the dislocation density (ρ) stochastically considered lead to better results when the comparison is made with experimental fatigue data since according to the literature, the dislocation structure changes with the total strain amplitude.
- The output discrete distributions generated by Monte Carlo (MC) stochastic simulation for the strain-life parameters, σ_f' and ε_f', shows that there is not a single probability distribution function suitable to describe each of these variables, for the metal materials under consideration.
- The Weibull and Logistic distribution functions showed the best fitting when compared with the output distributions obtained by stochastic prediction for the fatigue strength coefficient, σ_f', and the fatigue ductility coefficient, ε_f', respectively, for the 1050 aluminium. While, for the 6061-T651 aluminium alloy, the Normal and Weibull distributions exhibited the best fitting with the MC

simulation-based output distributions for the fatigue strength coefficient, σ_f', and the fatigue ductility coefficient, ε_f', respectively.

- The probabilistic strain-life curves generated through the Huffman approach using the Monte Carlo stochastic simulation to estimating the output distributions of the strain-life parameters σ_f' and ε_f' are presented. It can be concluded that the Monte Carlo stochastic prediction allowed the generation of a probabilistic strain-life field through the ρ random variable (ρ is the dislocation density), where the experimental fatigue data are between the 5% and 95% failure probability curves.

Application and discussion of various crack closure models to predict fatigue crack growth in 6061-T651 aluminium alloy

The main conclusions on a comparative study of the fatigue crack propagation rates (da/dN) versus effective stress factor range (ΔK_{eff}) data, in 6061-T651 aluminium alloy, calculated through the various crack closure models under consideration can be summarized as follows[2]:

- The deterministic values of the quantitative parameter, U, and the effective stress ratio, R_{eff}, based on the first cycle and stabilised measurements were obtained. Also, the values of U and R_{eff} were calculated for each crack closure model under consideration.
- Correia's model[3] presents good results for U and R_{eff} when compared to the deterministic data based on stabilised measurements, where the mean values of the error index module are of 2.46% and 6.73%, respectively. Newman's model also shows good results in terms of U and R_{eff}, presenting error index modules of 6.64% and 21.93%, respectively, when the comparison is made with deterministic data based on stabilised measurements. When the same comparison is made with the deterministic values based on the 1st cycle measurements, the Newman's[4], Codrington-Kotousov's[5] and Katcher and Kaplan's[6] models present a good agreement, where the mean values of the error index module of 7.34%, 5.41% and 6.30%, and 12.19%, 8.27% and 9.97%, respectively. The other models also studied based on the 1st cycle and stabilised measurements have mean values of the error index module, $\bar{e}$, greater than 20%, and are not considered adequate to describe the crack closure effects.

- The assumptions suggested in the Hudak and Davidson's[7] investigations and later also recommended by Ellyin[8] were confirmed by the use of the Correia's model[3], where the crack closure quantitative parameter is a function of applied stress R-ratio, maximum stress intensity factor (or ΔK), and driven by threshold stress intensity factor range, ΔK_{th}. Also, Correia's crack closure model based on experimental data suggest the limit stress intensity factor (K_L) in estimating the quantitative parameter (U) to describe the crack closure effects. In this study, the quantitative parameter (U) was estimated by using the maximum stress intensity factor (K_{max}) for stabilised cycles.

Fatigue crack growth modelling supported by strain energy density-based Huffman model considering the residual stress effects

The main conclusions on the fatigue crack growth modelling for the 6061-T651 aluminium alloy supported by the strain energy density-based Huffman approach considering the residual stress effects ahead of the crack tip are the following[9]:

- An elastoplastic stress/strain analytical analysis at the crack tip region based on the multiaxial Neuber[10] and Glinka's[11] rules was assumed, where the both approaches revelled similar elastoplastic stress distributions.
- A comparison on the elastoplastic stress distributions between the numerical results from the finite element modelling of the CT specimen and the analytical results based on Neuber and Glinka's rules was done. A good agreement for the maximum elastoplastic stresses was found.
- The comparison on the residual stress distributions between numerical and analytical solutions showed deviations that increase for a stress ratio equal 0.5, revelling that the analytical solutions overestimate the residual stresses.
- For values of critical dislocation density of 5.90E+15 and 6.088E+15 m^{-2}, both modelling analyses of the fatigue crack propagation behaviour based on the strain energy density approach, considering the residual stress effects estimated by Noroozi et al.[23-25] or Huffman suggestions[10,12], exhibited very similar results.
- The critical dislocation density, driven by the highest strain amplitude specimen or by mean value of the dislocation density for the available experimental fatigue results, does not influence significantly the fatigue crack propagation behaviour modelled by the strain energy density approach.

- For both procedures to consider the residual stress effects, Noroozi and Huffman suggestions, the fatigue crack growth constants obtained from the modelling based on the strain energy density approach are very similar with the exception of the Δa calibration parameter. It is concluded that the procedure for considering the residual stress effects influences the Δa calibrator. Therefore, it is not possible to conclude which is the most recommended method to describe the residual stress effects, since there is no measured data to support such analysis.
- The used approach allows to model the fatigue crack propagation behaviour for the near-threshold and Paris regions.

7.2 FUTURE WORKS

This study tries to cover the most relevant aspects of fatigue crack initiation and propagation modelling based on the strain energy density of aluminium alloys, as well as to discuss the residual stress effects ahead of the crack tip, i.e.; the concept of virtual crack closure effects, where various issues and challenges still need to be investigated in next steps. Future works can be considered as the following:

An experimental programme to characterize the fatigue crack growth of 1100 aluminium alloy

The low-cost workbench approach combining an experimental-analytical analysis to characterize the fatigue crack growth behaviour needs to be able to perform tests in accordance with ASTM E647 standard, test thicker specimens, measure the crack closure, and implement an automatic measurement system of fatigue crack propagation rates, applied stress intensity factor ranges, and virtual crack closure effects.

Low-cycle fatigue modelling supported by strain energy density-based Huffman model considering the variability of dislocation density

A comparison between the dislocation densities obtained based on the strain energy density approach and experimental techniques, such as special metallographic procedures, X-ray diffraction, and transmission electron microscopy (TEM), must be performed. In addition, a study on the evolution of the dislocation density as a function of the level of strain amplitude and number of cycles to failure must be carried out. Adequate relationships between the dislocation density (ρ) and the cyclic stress-strain parameters, considering that this parameter may vary throughout the low-cycle fatigue test for each strain amplitude, supported eventually by stochastic analysis, must be found. Further studies should be done on the use of approaches based on the strain energy density to model the fatigue crack initiation and propagation phases.

Application and discussion of various crack closure models to predict fatigue crack growth in 6061-T651 aluminium alloy

It can be stated that the simultaneous monitoring of CTOD-based experimental measurements of the crack closure effects and getting the crack tip stress–strain field based on digital image correlation (DIC) measurements, supported by analytical/numerical solutions, seems to be a good way to describe the fatigue crack growth correctly, as suggested by Ferreira-Castro-Meggiolaro[13,14]. This analysis may be relevant to evaluate the residual stress effects and find a relationship with the concept of virtual crack closure, since the phenomena may be related.

Fatigue crack growth modelling supported by strain energy density-based Huffman model considering the residual stress effects

The residual stress effects need to be determined experimentally by digital image correlation, where the stresses and strains ahead of the crack tip for some crack sizes and different stress ratios can be estimated. This analysis will be used to understand what residual stress approaches (Noroozi[15] or Huffman[12] suggestions) are more suitable to model the fatigue crack propagation behaviour. Therefore, the fatigue crack propagation calibration parameters (Δa, x) to be estimated by applying the approach based on the Huffman strain energy density may be seen to be representative of the metals under study. Another aspect to be explored is perhaps the probabilistic fatigue crack propagation modelling using the strain energy density approach through a stochastic analysis, where the parameters of the cyclic stress-strain curve, mainly the cyclic strain hardening coefficient (K'), are described by probability density functions.

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