

Ultra peripheral heavy ion collisions and the energy dependence of the nuclear radius

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Abstract

To estimate realistic cross sections in ultra peripheral heavy ion collisions we must remove effects of strong absorption. One method to eliminate these effects make use of a Glauber model calculation, where the nucleon–nucleon energy dependent cross sections at small impact parameter are suppressed. In another method we impose a geometrical cut on the minimal impact parameter of the nuclear collision ($b_{\min} > R_1 + R_2$, where R_i is the radius of ion ‘ i ’). In this last case the effect of a possible nuclear radius dependence with the energy has not been considered in detail up to now. Here we introduce this effect showing that for final states with small invariant mass the effect is negligible. However when the final state has a relatively large invariant mass, e.g., an intermediate mass Higgs boson, the cross section can decrease up to 50%.

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Collisions at relativistic heavy ion colliders like the relativistic heavy ion collider RHIC/Brookhaven and the large hadron collider LHC/CERN (operating in its heavy ion mode) are mainly devoted to the search of a quark–gluon plasma in central nuclear reactions. In addition to this important feature of heavy-ion colliders, ultra peripheral collisions may give rise to a huge luminosity of photons opening the possibilities of studying two-photon and other interactions as reviewed in Refs. [1–3]. In the early papers on peripheral heavy ion collisions the effect of strong absorption was not taken into account. The separation of the strong interactions effects was solved by using impact parameter space methods in Refs.

[4–6]. In order to obtain a truly peripheral photon–photon interaction one has to remove completely the central collisions, i.e., we must enforce that in the cross section calculation the minimum impact parameter, $b_{\min}$, should be larger than $R_1 + R_2$, where R_i is the nuclear radius of the ion ‘ i ’ [4]. The photon distributions can be described using the equivalent-photon approximation (EPA) with the requirement of minimum impact parameter (or geometric cut) discussed above [3,6].

The above method is not the only manner to avoid events where hadronic particle production overshadows the γ – γ interaction, i.e., events where the nuclei physically collide. An alternative is to use the Glauber model for heavy ion collisions [7]. It is a semiclassical model picturing the nuclei moving in a straight path along the collision direction, and gives the nucleus–nucleus interaction in terms of the interaction between

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the constituent nucleons and nuclear density distributions. If we write the cross section for the collision of two nucleus A and B as a function of the impact parameter (b), the elastic (el) peripheral cross section will be given by

$$\sigma_{\text{el}} = \int d^2b [1 - \exp(-AB\sigma_0 T_{AB}(b)/2)]^2, \quad (1)$$

where A and B are the nucleon numbers, σ_0 is the total nucleon–nucleon cross section and

$$T_{AB}(b) = \int \frac{dQ^2}{(2\pi^2)} F_A(Q^2) F_B(Q^2) e^{iQb}, \quad (2)$$

where $F_{A(B)}$ are nuclear form factors. Eq. (1) and the form (2) for $T_{AB}(b)$ are valid only if one can neglect the finite range of the nuclear interaction. If at higher energies the total cross section increases both due to strength and due to the range the equation for $T_{AB}(b)$ should take this into account. The exponential factor in Eq. (1) is the one responsible for the suppression of the inelastic collisions. The σ_0 total nucleon–nucleon cross section that appears in Eq. (1) is known to be dependent on the energy. Actually the increase of hadron–hadron total cross sections have been theoretically predicted many years ago [8] and these predictions have been accurately verified by experiment [9]. For instance, the proton–proton total cross section roughly double as we go from a few GeV up to the Tevatron energies.

In ultra peripheral heavy ion collisions it is clear how this energy dependence of the cross section enters in the Glauber approximation. However the same is not true when we compute the cross sections with the EPA and the requirement of a minimum impact parameter. It seems that cross sections in very peripheral heavy ion collisions calculated within the Glauber method turn out to be slightly different from the ones computed with the geometric cut [10].

The nuclear radius certainly expands with the increase of the energy in the same way as the proton expands, and this expansion should be implemented in the geometrical cut calculation of peripheral heavy ion collisions. As far as we know this effect has not been discussed in detail in the literature, and it is the purpose of this Letter to introduce the energy dependence of the nuclear radius in the calculations of peripheral heavy ion collisions when the geometric cut method is used.

In order to introduce the energy dependence of the nuclear radius in the calculations of peripheral heavy ion collisions we start discussing a standard computation of the photon distribution in the ion with the geometric cut method. The photon distribution in the nucleus can be described using the Weizsäcker–Williams approximation (or EPA) in the impact parameter space. Denoting by $F(x)dx$ the number of photons carrying a fraction between x and $x + dx$ of the total momentum of a nucleus of charge Ze , we can define the two-photon luminosity through

$$\frac{dL}{d\tau} = \int_{\tau}^1 \frac{dx}{x} F(x) F(\tau/x), \quad (3)$$

where $\tau = \hat{s}/s$, $\hat{s}$ is the square of the center of mass (c.m.s.) system energy of the two photons and s of the ion–ion system. The total cross section of the process $ZZ \rightarrow ZZX$ is

$$\sigma(s) = \int d\tau \frac{dL}{d\tau} \hat{\sigma}(\hat{s}), \quad (4)$$

where $\hat{\sigma}(\hat{s})$ is the cross section of the subprocess $\gamma\gamma \rightarrow X$. There remains only to determine $F(x)$. In the literature there are several approaches for doing so, and we choose the conservative and more realistic photon distribution of Ref. [6]. Cahn and Jackson [6], using a prescription proposed by Baur [4], obtained a photon distribution which is not factorizable. However, they were able to give a fit for the differential luminosity which is quite useful in practical calculations:

$$\frac{dL}{d\tau} = \left(\frac{Z^2 \alpha}{\pi} \right)^2 \frac{16}{3\tau} \xi(z), \quad (5)$$

where $z = 2MR\sqrt{\tau}$, M is the nucleus mass, R its radius and $\xi(z)$ is given by

$$\xi(z) = \sum_{i=1}^3 A_i e^{-b_i z}, \quad (6)$$

which is a fit resulting from the numerical integration of the photon distribution, accurate to 2% or better for $0.05 < z < 5.0$, and where $A_1 = 1.909$, $A_2 = 12.35$, $A_3 = 46.28$, $b_1 = 2.566$, $b_2 = 4.948$, and $b_3 = 15.21$. For $z < 0.05$ we use the expression (see Ref. [6])

$$\frac{dL}{d\tau} = \left(\frac{Z^2 \alpha}{\pi} \right)^2 \frac{16}{3\tau} \left(\ln \left(\frac{1.234}{z} \right) \right)^3. \quad (7)$$

Eq. (3) is written in a factorised form, which of course is valid only if one neglects the exclusion of central collisions into account. Therefore Eq. (3) is not the most general form [3,6], and the same is true for Eq. (5). The calculation assumes that the same radius R is used for both ions $b_{\min} = 2R$ but also to have a cutoff for the individual impact parameter b_1 and b_2 (which either is necessary to eliminate final state interaction, or which takes into account the form factor effects, that is, the decrease of the charge inside the nucleus). Especially when looking, for instance, an intermediate mass Higgs boson production or other non-strongly interacting particles there is no reason to assume that the size of the individual cutoff radii for b_1 and b_2 scales in the same way as $b_{\min}$. Therefore the calculation overestimates the dependence on R a bit.

The condition for realistic peripheral collisions ($b_{\min} > R_1 + R_2$) is present in the photon distributions showed above. To obtain the above equations an elastic Gaussian form factor and an energy independent nuclear radius giving by $R_{\text{ion}} = 1.2 A^{1/3}$ fm have been used. A more accurate Woods–Saxon distribution for symmetric nuclei would produce some small deviations, but for our purposes the expressions for the luminosity described above are enough. However the expression for the nuclear radius is exactly the one we believe that should be changed by its energy dependent expression, and the problem is to have a phenomenologically sensible expression for the nuclear radius increase with the energy.

In the heavy ions colliders nucleus like Au and Pb will collide with a great amount of energy, going from 200 GeV/nucleon (Au at RHIC) up to 5.5 TeV/nucleon (Pb at LHC), and the ultra peripheral collisions can be computed with the help of the photon distribution described above. If the ion radius increase with the energy, the value corresponding to $b_{\min}$ will also become greater, and consequently the cross section must decrease. This is easily seen in the many examples calculated in the literature where the cross section for a given process is concentrated at some moderate impact parameter and decreases when b increases. Of course the Lorentz factor is also important to determine this behavior. Therefore, if we introduce the energy dependence in the nuclear radius we expect lower rates for a given process than those obtained in the usual calculations, and this effect, even

if it is small, could be important if we have a high precision measurement.

The authors of Ref. [11] modelled the particle production process in ultrarelativistic heavy ion collisions in terms of an effective scalar field produced by the colliding objects, in their work they showed that the nuclear cross sections increase with the energy due to a logarithmic increase of the nuclear radius with the energy. We shall use this reference to obtain a relation between the nuclear radius and the incident energy that is the following:

$$R_H^2(s) = 1 + 2 \frac{d}{R'} \gamma_E + (\delta + 1) \frac{d}{R'} \ln \left(\frac{A \sqrt{s}}{\varepsilon_0} \right). \quad (8)$$

$R' = R_P + R_T \simeq 2.4 A^{1/3}$ fm ($R_P(R_T)$ means projectile (target) and we assume $R_P = R_T = R_{\text{ion}}$), $\sqrt{s}$ is the energy of the projectile nucleus in the laboratory rest frame. The nuclear density for a nucleus A at distance x from its center is modelled by a Woods–Saxon distribution for symmetric nuclei,

$$\rho_{\text{WS}}(x) = \frac{\rho_0}{(1 + \exp[\frac{(x-R_{\text{ion}})}{d}])}, \quad (9)$$

where $d = 0.549$ fm, and ρ_0 can be found when the Wood–Saxon density is normalized by the condition $\int d^3x \rho(x) = A$. And ε_0 is equal to

$$\varepsilon_0 = M_Z d \left[\frac{16}{\pi^2 g^2 \rho_0^4} \frac{R' (R_T R_P)^{(\delta-7)/4}}{d^3 \Gamma^2(\frac{\delta+1}{4})} \right]^{2/(\delta+1)}. \quad (10)$$

M_Z is the nuclear mass. The coupling constant g and the parameter for the mass spectrum δ were estimated in Ref. [11] and they are equal to $\delta = -0.56$ and $g = 3.62 \text{ fm}^{(7+\delta)/2}$.

The radius $R_H^2(s)$ appearing in Eq. (8) at small energies gives a nuclear radius larger than R_{ion} , for this reason we have assumed the following normalization

$$R^2(s) = \frac{R_H^2(s) R_{\text{ion}}^2}{R_H^2(s = M_Z^2)}, \quad (11)$$

where $R_{\text{ion}} = 1.2 A^{1/3}$ fm. With this normalization factor we assure that when the ion energy is equal to its mass, the nuclear radius will be equal to R_{ion} . It is the radius given by Eq. (11) that should be used in Eqs. (5)–(7). Typical values for R_{ion} and $R(s)$ are showed in Table 1.

To show the effects of the nuclear radius dependence on the energy we computed production of leptons pairs (muons and taus) and resonances formed

Table 1

Values for R_{ion} and $R(s)$, Eq. (11), in fm. The energies ($\sqrt{s}$) are in TeV/nucleon

Ion	$\sqrt{s}$	R_{ion}	$R(s)$
Au	0.2	6.98	7.29
Ca	7.2	4.10	4.75
Pb	5.5	7.11	7.61

by the photon–photon fusion. In the resonance case we considered the η_c meson and an intermediate mass Higgs boson with a mass equal to 115 GeV. We computed the cross sections for two cases: in one the nuclear radius is energy independent (and equal to R_{ion}). In the second case the radius obeys Eq. (11).

For an invariant mass of the photon pair above the threshold $\sqrt{\hat{s}} > 2m_l$, a lepton pair can be produced in two-photon collisions ($\gamma\gamma \rightarrow l^+l^-$) and the lowest order QED cross section for this subprocess is given by [3]

$$\sigma(\gamma\gamma \rightarrow l^+l^-) = \frac{4\pi\alpha^2}{\hat{s}}\beta_l \left[\frac{(3-\beta_l^4)}{2\beta_l} \ln\left(\frac{1+\beta_l}{1-\beta_l}\right) - 2 + \beta_l^2 \right], \quad (12)$$

where $\beta_l = \sqrt{1-4m_l^2/\hat{s}}$ is the velocity of the pair in the $\gamma\gamma$ rest frame, m_l is the lepton mass, $\sqrt{\hat{s}}$ is the c.m. system energy of the two photons and α is the fine-structure constant. Using this elementary cross section in Eq. (4) we obtained the rates shown in Table 2. The calculation was performed for three different ions with different beam energies, the one of RHIC (Au) and the ones expected at LHC (Ca and Pb). The cross sections were integrated in a bin of energy equal to $1 < \sqrt{s} < 10$ GeV. The third column of Table 2 shows the cross section computed with a constant nuclear radius and the fourth column the one with the energy dependent radius. For the three different ions the cross sections decrease when we consider the energy dependent radius described by the Eq. (11). In all the cases the decrease is smaller than 10% and is negligible considering the theoretical and experimental uncertainties involved in the problem.

In Table 3 it is possible to see the results when the subprocess analyzed is $\gamma\gamma \rightarrow \tau^+\tau^-$ with $2m_\tau < \sqrt{\hat{s}} < 10$ GeV. The general behavior of the τ pair production cross sections is very similar to the one observed previously in Table 2. Of course, the cross

Table 2

Cross sections of the process $ZZ\gamma\gamma \rightarrow ZZ\mu^+\mu^-$. The cross sections σ_{R_i} ($\sigma_{R(s)}$) given in the third (fourth) column are the ones computed with the energy independent (dependent) radius. The last column shows the ratio between the third and fourth columns. The cross sections are in mbarn and the energies ($\sqrt{s}$) are in TeV/nucleon

Ion	$\sqrt{s}$	$\sigma_{R_{\text{ion}}}$	$\sigma_{R(s)}$	Ratio
Au	0.2	2.127	1.947	1.09
Ca	7.2	0.643	0.588	1.09
Pb	5.5	106.4	101.3	1.05

Table 3

The same as in Table 2, but for the subprocess $\gamma\gamma \rightarrow \tau^+\tau^-$

Ion	$\sqrt{s}$	$\sigma_{R_{\text{ion}}}$	$\sigma_{R(s)}$	Ratio
Au	0.2	6.972×10^{-4}	5.727×10^{-4}	1.22
Ca	7.2	5.176×10^{-3}	4.604×10^{-3}	1.12
Pb	5.5	0.759	0.718	1.05

sections for producing tau pairs are smaller. However the collision of Au–Au and Ca–Ca are now more sensitive to the energy dependence of the nuclear radius, producing an effect larger than 10%. The rates for tau pairs production in Pb collision with a c.m. energy equal to 5.5 TeV/nucleon, with and without energy dependence in the ion radius are not so different. As we shall discuss later the larger cut that we perform in the impact parameter when we consider the energy dependent radius removes photons of larger energy. Therefore for final states with larger invariant masses we may expect a larger effect.

Let us now consider the case of heavy resonances. To estimate the production of one resonance R formed by a photon–photon fusion in peripheral heavy ion collisions we use the following elementary cross section in Eq. (4),

$$\sigma(\gamma\gamma \rightarrow R) = \frac{8\pi^2}{M_R s} \Gamma(R \rightarrow \gamma\gamma) \delta\left(\tau - \frac{M_R^2}{s}\right), \quad (13)$$

where M_R is the resonance mass and $\Gamma(R \rightarrow \gamma\gamma)$ its decay width into two photons. In Table 4 we show the results obtained for two-photon production of η_c in peripheral heavy ion collisions with $M_{\eta_c} = 2.979$ GeV and $\Gamma(\eta_c \rightarrow \gamma\gamma) = 6.6$ keV. The ratio of the cross sections considering the two scenarios are 1.16 and 1.11 for Au and Ca ions, respectively, and 1.06 for the Pb ion. Finally, in Table 5 it can be observed the values corresponding to the subprocess $\gamma\gamma \rightarrow H$

Table 4

The same as in Table 2, but for the subprocess $\gamma\gamma \rightarrow \eta_c$

Ion	$\sqrt{s}$	$\sigma_{R_{\text{ion}}}$	$\sigma_{R(s)}$	Ratio
Au	0.2	2.147×10^{-3}	1.846×10^{-3}	1.16
Ca	7.2	2.897×10^{-3}	2.614×10^{-3}	1.11
Pb	5.5	0.437	0.413	1.06

Table 5

The same as in Table 2, but for the subprocess $\gamma\gamma \rightarrow H$

Ion	$\sqrt{s}$	$\sigma_{R_{\text{ion}}}$	$\sigma_{R(s)}$	Ratio
Ca	7.2	9.970×10^{-10}	6.789×10^{-10}	1.47
Pb	5.5	1.854×10^{-8}	1.387×10^{-8}	1.34

with $M_H = 115$ GeV, where we used the Higgs boson two-photon decay width found in Ref. [12]. We do not show the result for RHIC energies because it is too small. The values of Table 5 indicate that the production cross sections for both ions are strongly affected by the inclusion of a radius described by Eq. (11). In the case of Ca collision with a c.m. energy of 7.2 TeV/nucleon the cross sections decrease nearly to half of the value obtained in the case of a energy independent radius. The situation is less drastic for the Pb ion with $\sqrt{s} = 5.5$ TeV/nucleon, but the ratio is still large ($= 1.34$). This is the only situation that we investigated where the Pb collision is clearly sensitive to the use of Eq. (11) (or to the energy dependence of the nuclear radius).

The fact that a sharp cutoff in impact parameter space at b_{min} should be replaced by a smooth one was already discussed in Ref. [2]. Comparing the Glauber model calculation with the one with a sharp cutoff we could expect significant deviations present at the upper end of the invariant mass distribution. Looking at the photon luminosity we see that only the smallest impact parameter contribute significantly to the events with large invariant masses. Imposing the cut on b_{min} but now with the radius described by Eq. (11) we obtain a more realistic calculation of the very peripheral heavy ion collisions.

In conclusion, we discussed the two different ways to compute cross sections for ultra peripheral heavy ion collisions. In the Glauber method it is quite clear how the increase with the energy of the nucleon–nucleon cross section enters in the calculation. In the calculation with the geometrical cut imposed on

the impact parameter, the nucleon, as well as the nuclear, radius expansion with the energy was not introduced up to now. It was noticed in Ref. [10] that there was a difference between the two methods. The difference was small and had some dependence on the invariant mass of the final states. The work of Ref. [11] prescribe a very precise way to introduce the nuclear radius dependence with the energy.

We believe that the estimative of the cross sections in ultra peripheral collisions with the geometrical cut method just changing the radius independent of the energy by the one dependent of the energy will give realistic predictions for any invariant mass of the final state. The effect is of order of 50% for an intermediate mass Higgs boson. Turning the problem the other way around we may also say that if the ultra peripheral collisions are measured with high precision, we may have a new way to study the increase of the nuclear radius with the energy. To do so we just have to measure the cross sections for very known final states with small and large invariant masses with high precision, there should be a decrease of the cross sections as a function of the invariant mass as we go to larger and larger energies.

Note added

Some comments on the effects discussed in this Letter were also made by Klein and Nystrand in [13], where the Fig. 3 gives the reduction in gamma–gamma luminosity (for gold at RHIC) for a Glauber calculation of hadronic interactions compared to the one with geometrical cut.

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