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Gravitational effects of Nix and Hydra in the external region of the Pluto–Charon system

P. M. Pires dos Santos,★ S. M. Giuliatti Winter and R. Sfair

UNESP-São Paulo State University, Guaratinguetá, CEP 12.516-410 SP, Brazil

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ABSTRACT

Two new companions to the Pluto–Charon binary system have been detected in 2005 by Weaver et al. These small satellites, named Nix and Hydra, are located beyond Charon's orbit. Although they are small when compared to Charon, their gravitational perturbations can decrease the stability of the external region (beyond Charon's orbit). The dynamical structure of this external region is analysed by numerically simulating a sample of particles under the gravitational effects of Pluto, Charon, Nix and Hydra. As expected the effects of Nix and Hydra decrease the external stable region. Agglomerates of particles can survive even after 10^5 orbital periods of the binary in some regions, such as coorbital to Nix and Hydra and between their orbits. We also analysed the effects of hypothetical satellites on the orbital evolution of Nix and Hydra in order to constrain an upper limit size. Some hypothetical satellites can be coorbital to Nix or Hydra without provoking any significant gravitational effects on them.

Key words: Kuiper belt: general – planetary systems.

1 INTRODUCTION

The first work to study the stability in the region of Pluto–Charon system was performed by Stern et al. (1994). They analysed the orbits of a sample of test particles under the gravitational effects of Pluto and Charon. Their results showed an unstable region interleaved between ∼1.8 A and 2.4A, where $A = 19\,600$ km, for test particles initially with inclination (I) and eccentricity (e) equal to zero. The integration time was 10^5 orbital periods of the binary ($T_{\mathrm{P-C}}$). An additional study was performed in order to verify how a massive hypothetical satellite can exist in the stable region of the system without causing any significant gravitational effects on Charon's eccentricity. Their results show that no satellites larger than $3 \times 10^{-4} m_{\mathrm{P-C}}$, where $m_{\mathrm{P-C}}$ is the mass of the binary, can exist in the internal (0.1–0.4A) and external (at 2A) regions without inducing an eccentricity on Charon's orbit larger than 10^{-3}.

Nagy, Sülli & Érdi (2006) analysed the dynamical structure of the phase space of the internal and external regions of the Pluto–Charon system through a set of numerical simulations of the spatial circular restricted three-body problem (Pluto–Charon particle). The gravitational effects of Nix and Hydra were not taken into account in the dynamical system. They numerically simulated a sample of particles initially located in the external region at semimajor axis a (relative to the barycentric reference frame) between 0.55B and 5B, where $B = 19\,571.4$ km. The eccentricities of the particles were taken from 0 to 0.3. The results (see their fig. 3), concerning particles for the planar case ($I = 0$), showed that for $a < 2.15B$ the system is unstable for all values of the eccentricity e for a time-span of 10^3 orbital periods of Charon. However, for $a \geq 2.15B$ there is a stable region depending on the value of e. Between Nix's and Hydra's orbits the eccentricity can reach values of up to 0.31.

An extension of this work has been presented by Sülli & Zsigmond (2009) through an analysis of the spatial elliptic restricted three-body problem for a time-span of 10^4 orbital periods of the binary. They generated stability maps for the $(a - e)$ and $(a - I)$ orbital element spaces near the satellites Nix and Hydra (taken as massless bodies) using three different complementary methods. As a result they obtained an unstable zone larger than the unstable zone derived from the circular restricted three-body problem. Both satellites, Nix and Hydra, are in stable regions. Nix might be in the 4 : 1 mean motion resonance if the values of the argument of pericentre or longitude of node falls in a certain range, while Hydra is not in the 6 : 1 mean motion resonance for arbitrary values of the argument of pericentre or longitude of nodes.

The encounter between the New Horizons mission and the Pluto system in 2015 will help to reconnaissance this peculiar system. The entire Hill sphere will be searched for additional rings and satellites during the approach phase (Young et al. 2008). Stern et al. (2006) proposed the possibility of rings in the Pluto system with a characteristic ring optical depth of $\tau = 5 \times 10^{-6}$ between Nix and Hydra. Steffl et al. (2006) analysed a set of *Hubble Space Telescope* data in order to search for additional satellites in the external and internal regions. By assuming spherical satellites and no limb darkening they ruled out (at the 90 per cent level of confidence) additional satellites with diameters larger than ∼50 km located

★E-mail: pos09032@feg.unesp.br

between the orbits of Nix and Hydra, and ∼36 km beyond Hydra's orbit. These values were calculated by assuming a very dark albedo of $\rho_v = 0.04$. If they assumed that these hypothetical satellites are as reflective as Charon, the values of the diameters decrease to 16 and 12 km, respectively.

The goal of this paper is to explore the external region taking into account the gravitational effects of Nix and Hydra on test particles. We also analysed the effects that a sample of small hypothetical satellites can induce on Nix's and Hydra's orbital eccentricities. These results can place an additional upper limit on the estimated size of these hypothetical satellites.

In the next section we present a sample of diagrams of semimajor axis versus eccentricity for particles in P-type orbits around the barycentre of Pluto–Charon system after approximately 10^5 orbital periods of the binary. In Section 3 we discuss the gravitational effects of a sample of small hypothetical satellites on Nix's and Hydra's orbital eccentricity. Our results are discussed in the last section.

2 DIAGRAMS (A-E)

Holman & Wiegert (1999) divided the orbits of a binary into three classes, following the designation given by Dvorak (1986): in the P-type orbits the particle is around the barycentre of the system, in the S-type orbits the particle is around one of the massive bodies and the third class is related to those particles around the Lagrangian triangular points, L_4 and L_5 (Murray & Dermott 1999). In this section we analyse the gravitational perturbations due to Nix and Hydra on particles in P-type orbits around the barycentre of the two massive bodies Pluto and Charon.

Table 1 gives the orbital parameters, diameters and masses of Charon, Nix and Hydra. The orbital parameters a, e, I, ω, Ω denote the semimajor axis, eccentricity, inclination, argument of pericentre and longitude of the ascending node, respectively. The orbital parameters of Charon are relative to Pluto and those of Nix and Hydra are relative to the barycentre of the system. The mass of the satellites was obtained assuming the density equals to 1.63 g cm^{-3}. Pluto's mass is taken to be equal to 1.304×10^{22} kg (Tholen et al. 2008). The orbital period of the binary is 6.3872 d (Tholen et al. 2008).

The orbital plane of Pluto–Charon was taken to be the reference plane. The numerical integrations have been carried out using the variable time-step Bulirsch–Stoer algorithm from the Mercury package (Chambers 1999). The perturbation of the Sun was neglected since we have simulated particles located up to 10^5 km.

Table 1. Satellite orbital elements and physical parameters used in the numerical simulations (derived from Tholen et al. 2008), at Epoch JD 245 2600.5. The orbital parameters a, e, I, ω, Ω denote the semimajor axis, eccentricity, inclination, argument of pericentre and longitude of the ascending node, respectively. The orbital parameters of Charon are relative to Pluto and the elements of Nix and Hydra are relative to the barycentre of the system. The masses of Charon, Nix and Hydra were obtained by assuming the density equals to 1.63 g cm^{-3}.

Parameters	Charon	Nix	Hydra
a (km)	19 570.3	49 240	65 210
e	0.0035	0.0119	0.0078
I (°)	96.168	96.190	96.362
ω (°)	157.9	244.3	45.4
Ω (°)	223.054	223.202	223.077
Diameter (km)	1212	88	72
Mass (kg)	1.520×10^{21}	5.8×10^{17}	3.2×10^{17}

2.1 Prograde orbits

To obtain the diagrams we use a set of 404 505 initial conditions varied in the following ways.

(i) the semimajor axis (relative to the barycentre of the system) was distributed from $1d$ to $5d$ ($d = 19\,570.$ km), with $\Delta a = 0.005d$.

(ii) The eccentricity assumed values from 0 to 0.2, with $\Delta e = 0.05$.

(iii) A set of 101 values of the argument of pericentre was randomly chosen between 0° and 360° for particles in prograde orbits. All particles started at the true anomaly equals to zero. The inclination was assumed to be 0 (prograde orbits) relative to the Pluto–Charon orbital plane.

The orbital evolution of the test particles are represented in Figs 1(a)–(c). We considered that the particle leaves the system when the distance between the particle and the barycentre is $10d$. When the distance between the particle and a massive body was less than the body's radius a collision was detected.

Fig. 1 shows the final values of the semimajor axis and eccentricity for a sample of particles, in prograde orbits, which survived after (a) 10 000 d (about $10^3 T_{\rm P-C}$), (b) 70 000 d (about $10^4 T_{\rm P-C}$) and (c) 650 000 d (about $10^5 T_{\rm P-C}$). The satellites Nix and Hydra are located at $2.516d$ and $3.332d$, respectively. Particles initially located at about $1d < a < 3d$ are represented in red, those particles initially at about $3d < a < 4d$ are in green and those at about $4d < a < 5d$ are represented in blue colour. Different colours can help to visualize the orbital evolution of the particles.

The instability region near Charon's orbit extends up to $2d$, as firstly verified by Stern et al. (1994). These particles leave the system in less than $10^3 T_{\rm P-C}$. Some clumps of particles start forming interior to Nix's orbit, coorbital to Nix and Hydra and in a small region between the orbits of these two small satellites. In Fig. 1(c) these clumps are clearly visible at about $1.8d$, $2d$ and $2.3d$ interior to Nix' s orbit. As discussed by Stern et al. (1994) these clumps might be associated to mean motion resonances with Charon. An agglomerate of particles can also be seen at $2.49d \le a \le 2.54d$ and $3.27d \le a \le 3.38d$, for values of $e \le 0.1$. Particles with a ranging between $2.78d$ and $3.01d$ (between the orbits of Nix and Hydra) also survived for this time-span of the integration. Particles with eccentricities larger than 0.1 were scattered due to close encounters with Nix and Hydra.

As can be seen in Fig. 1 the gravitational perturbations of both satellites Nix and Hydra on those test particles located beyond Hydra's orbit are small. Some structures appear to be associated with resonances between the particles and the satellites. Two bodies are in mean motion resonance when $n/n' = p/(p+q)$, where n and n' are their mean motions, p and q are small integers and q is of the order of the resonance. The mean motions are measured in the sidereal system (Sülli & Zsigmond 2009).

The locations of the mean motion resonance between the particle and the satellite Nix (top) and Hydra (bottom) are indicated in the figure. Particles near to a resonance had their eccentricities increased; this effect can be visualized, for example, for those particles located near the 3 : 2 mean motion resonance with Hydra.

Distinct regimes can be caused by the gravitational perturbations of an embedded satellite, in circular orbit around the primary, in a sample of massless particles. A chaotic zone, where the particles would be scattered by the gravitational perturbations of the satellite,

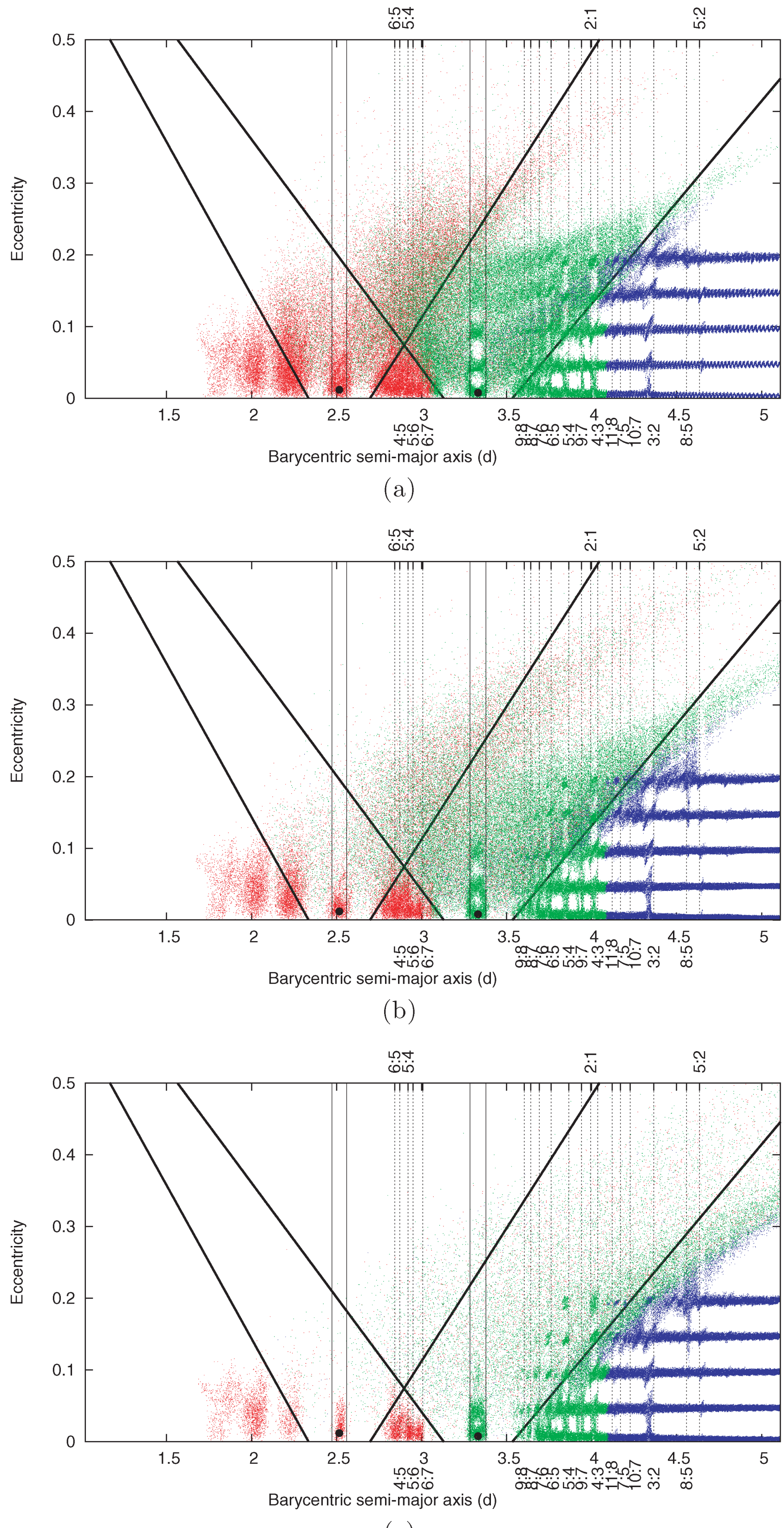
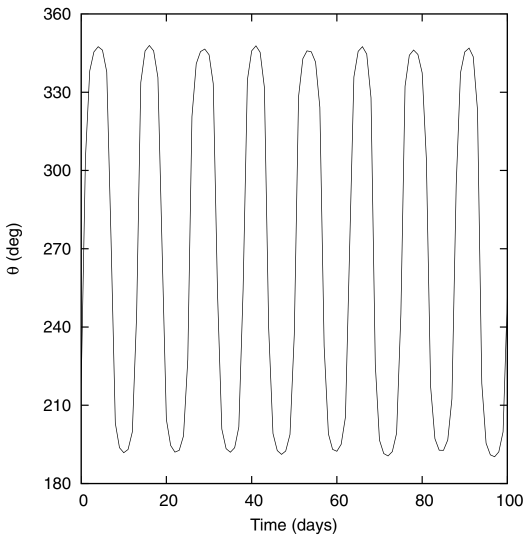


Figure 1. Diagram of the final semimajor axis and eccentricity for a set of test particles, in prograde orbits, after (a) 10 000 d, (b) 70 000 d and (c) 650 000 d. The satellites Nix and Hydra (represented by black circles) are located at 2.516d and 3.332d, respectively. The locations of the mean motion resonance between the particle and the satellite Nix (top) and Hydra (bottom) are indicated in this figure. Particles initially located at about $1d < a < 3d$ are represented in red, those particles initially at about $3d < a < 4d$ are in green and those at about $4d < a < 5d$ are represented in blue colour.



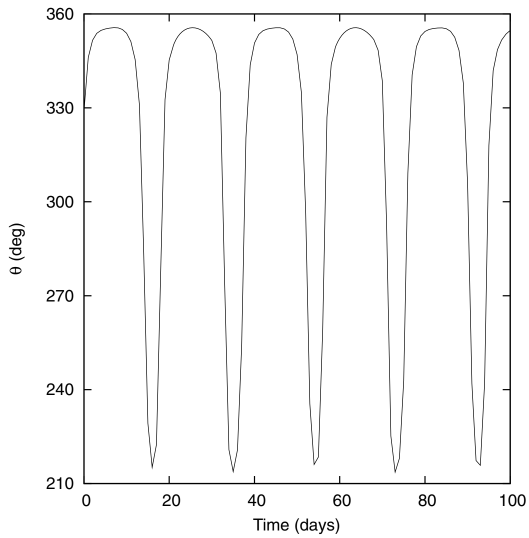


Figure 2. The angle θ versus time for one representative coorbital particle to (a) Nix located at 2.516*d* and (b) Hydra at 3.332*d*.

has a width (w_{ch}) given by (Wisdom 1980)

$$w_{ch} = 2 \times 1.3\mu^{2/7} a_{sat}, \quad (1)$$

where μ is the mass ratio between the mass of the satellite (m_{Nix} or m_{Hydra}) and the mass of the binary (m_{P-C}) and a_{sat} is the semimajor axis of the satellite. The width of the gap, derived from equation (1), generated by Nix is ~7000 km and by Hydra is ~7900 km. These values derived from the theory fit well the values shown in Fig. 1(c) for the eccentricity equals to 0.

Coorbital particles to Nix and Hydra have also survived after 10^5 T_{P-C}. From the circular restricted three-body problem the width of the coorbital region can be given by (Murray & Dermott 1999)

$$w \sim 0.5\mu^{1/3} a_{sat}. \quad (2)$$

From equation (2), the coorbital region generated by Nix is ~1700 km and by Hydra is ~1800 km. These values are represented by vertical lines, despite the fact that the width of the coorbital region, given by equation (2), is valid for the eccentricity of the particle equals to 0.

We have numerically simulated the agglomerates of particles located at 2.5*d* (near to Nix's orbit) and 3.3*d* (near to Hydra's orbit) in order to verify if the angle θ, the angular separation between the particle and the satellite, is smaller than 1° (Mourão 2001). Values of $\theta < 1°$ imply that the particle is no longer coorbital to the satellite. During the numerical integration, the majority of the particles remained coorbital to the satellites Nix and Hydra. As can be seen in Fig. 1, only a small amount of particles do not belong originally in the coorbital region. Fig. 2 shows the angle θ as a function of time of a representative coorbital particle to Nix (Fig. 2a) and to Hydra (Fig. 2b).

The lines shown in Fig. 1 represent the crossing lines. Those particles which enter the region limited by these lines are crossing the orbits of Nix or Hydra. Therefore, these particles will be ejected from the system or they will collide with the satellite.

Some particles cross the orbits of the satellites and can collide with them during the numerical simulation. Table 2 summarizes the numbers of collisions between the particles and the four massive bodies (Pluto, Charon, Nix and Hydra) for particles in prograde and retrograde (Section 2.2) orbits. Our results, concerning particles in prograde orbits, show that about 50 per cent of the initial set of the particles survived after 10^5 T_{P-C}, most of them were initially located beyond Hydra's orbit. About 32 per cent of the particles were ejected from the system.

Table 2. Result of the collisions between the test particle, in prograde and retrograde orbits, with Pluto, Charon, Nix and Hydra after 10^5 T_{P-C}, and the percentage of the particles ejected from the system.

		Prograde (per cent)	Retrograde (per cent)
Collisions	Pluto	8	0.6
	Charon	10	1.5
	Nix	5.5	11
	Hydra	5	11
	Total	29	24
Escape		32	5

2.2 Retrograde orbits

To obtain the diagram presented in Fig. 3 we use a set of 40 050 initial conditions varied in the following ways.

(i) The values of the semimajor axis and the eccentricity were the same as assumed for those particles in prograde orbits (Section 2.1).

(ii) The argument of pericentre have been randomly distributed into 10 different values. All particles started at the true anomaly equals to zero. The inclination was assumed to be 180° (retrograde orbits) relative to the Pluto–Charon orbital plane.

Fig. 3 shows the final values of the semimajor axis and eccentricity of a sample particles, in retrograde orbits, after 10^5 T_{P-C}. The lines across this figure represent the collision lines, as discussed in Section 2.1. Particles inside these lines may collide with one of the smaller satellites or may be ejected from the system, resulting in a less dense region.

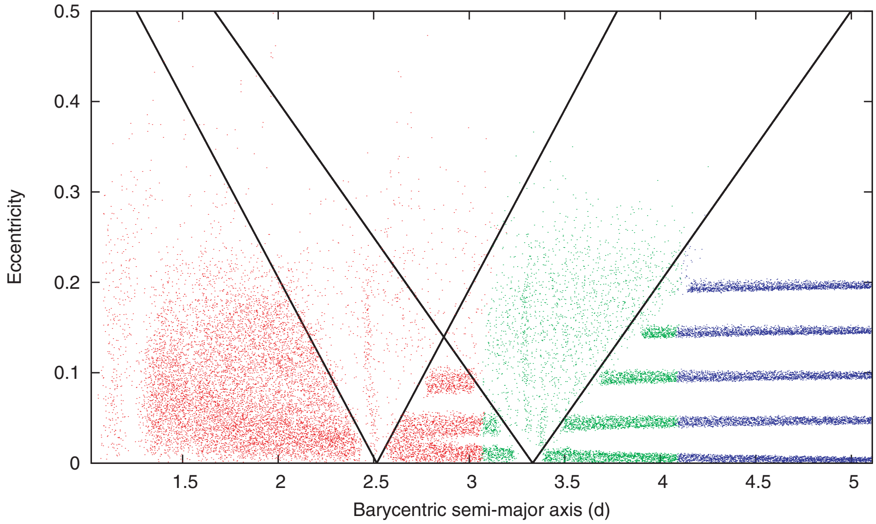


Figure 3. Diagram of the final semimajor axis and eccentricity for a set of test particles, in retrograde orbits, after $10^5 T_{\mathrm{P-C}}$. The satellites Nix and Hydra are located at 2.516d and 3.332d, respectively. Particles initially located at approximately $1d < a < 3d$ are represented in red, those particles initially at $3d < a < 4d$ are in green and those at $4d < a < 5d$ are represented in blue colour.

Comparison between Figs 1 and 3 shows that the stability region for those particles in retrograde orbits is larger than for particles in prograde orbits, as verified in Stern et al. (1994) for the region interior to Nix's orbit. Particles initially located interior to Nix, from $a \sim 1.31d$ to the inner boundary of Nix's collisional region, remained in this region. Their eccentricities reach values of up to 0.3.

An agglomerate of particles, initially between the orbits of Nix and Hydra, remained in this region. Beyond Hydra's orbit the stability region is larger than that obtained for particles in prograde orbits. Particles with $a \geq 3.4d$, this limit changes for different values of the eccentricity, suffer only a small gravitational perturbation of the satellites.

Table 2 presents the number of collisions between the particles and the massive bodies. Our results, concerning particles in retrograde orbits, show that about 24 per cent of the particles collided with one of the massive bodies and only 5 per cent are ejected from the system. Most of the particles survived after $10^5 T_{\mathrm{P-C}}$.

3 HYPOTHETICAL SATELLITES

In this section we analyse the gravitational effects of a sample of hypothetical satellites on the orbits of Nix and Hydra. These satellites were initially in circular, coplanar and prograde orbits around the barycentre of the system. A similar analysis was carried out by Stern et al. (1994) in order to constrain the mass of a hypothetical satellite near Pluto and Charon, without inducing any eccentricity in Charon's orbit larger than 10^{-3}.

The initial semimajor axis (relative to the barycentre of the system) of these hypothetical satellites (moonlets) were distributed from 2d to 5d, with $\Delta a = 0.005d$. All moonlets started at the true anomaly equals to zero. The orbital parameters of Pluto, Charon, Nix and Hydra are listed in Table 1. All bodies were treated as spherical.

The diameters of the hypothetical satellites were assumed to be between 2 km ($m = 6.8 \times 10^{12}$ kg) and 50 km ($m = 1.06 \times 10^{17}$ kg) for those satellites interior to Hydra's orbit, and between 2 and 36 km ($m = 3.9 \times 10^{16}$ kg) for those satellites exterior to Hydra's orbit. The masses of the hypothetical satellites were obtained by assuming the density equals to 1.63 g cm^{-3}. The stepsizes of the diameters of the hypothetical satellites are 2 km. This range of values was chosen based on the results presented by Steffl et al. (2006). They have ruled out additional satellites with diameters larger than 50 km located between the orbits of Nix and Hydra, and 36 km beyond Hydra's orbit. These values were obtained by assuming a very dark albedo of $\rho_v = 0.04$.

Fig. 4(a) depicts the results of the numerical simulations, where the values of the time (in days) are plotted as a function of the barycentric semimajor axis and radius (in km) of the hypothetical satellites. Inspection of this figure shows that interior to Hydra's orbit most of the satellites collided with Charon, Nix or Hydra, or were ejected from the system. 75 per cent of the satellites coorbital to Nix and Hydra, and 33 per cent of those satellites located between their orbits survived during the time of the integration (650 000 d). Beyond 3.6d all the hypothetical satellites remained in the system, since the gravitational effects of Charon, Nix and Hydra are less effective.

The available data are not sufficient to allow the precise definition of the orbital elements of Nix and Hydra. Precise orbital elements will be accomplished through the data sent by the New Horizons spacecraft during the period before and after its close approach with the Pluto system.

From Tholen et al. (2008), the osculating eccentricity variations as a function of time for Nix and Hydra are [0, 0.0272] and [0, 0.0179], respectively. Considering these results we constrained the size and position of the survived satellites (presented in Fig. 4a) to select only those satellites that did not induce any eccentricity larger than 10^{-3} in the orbits of Nix and Hydra.

During the numerical integrations the eccentricities of Nix and Hydra were monitored. Fig. 4(b) shows the location and size (in radius) of those hypothetical satellites that cause only a small increase (less than 10^{-3}) in the eccentricities of Nix and Hydra. Therefore hypothetical satellites can reside coorbital to Nix and Hydra, in a small region between their orbits and beyond Hydra's orbit. Their presence will affect the eccentricities of these two small satellites, but this effect cannot be detected by current technology.

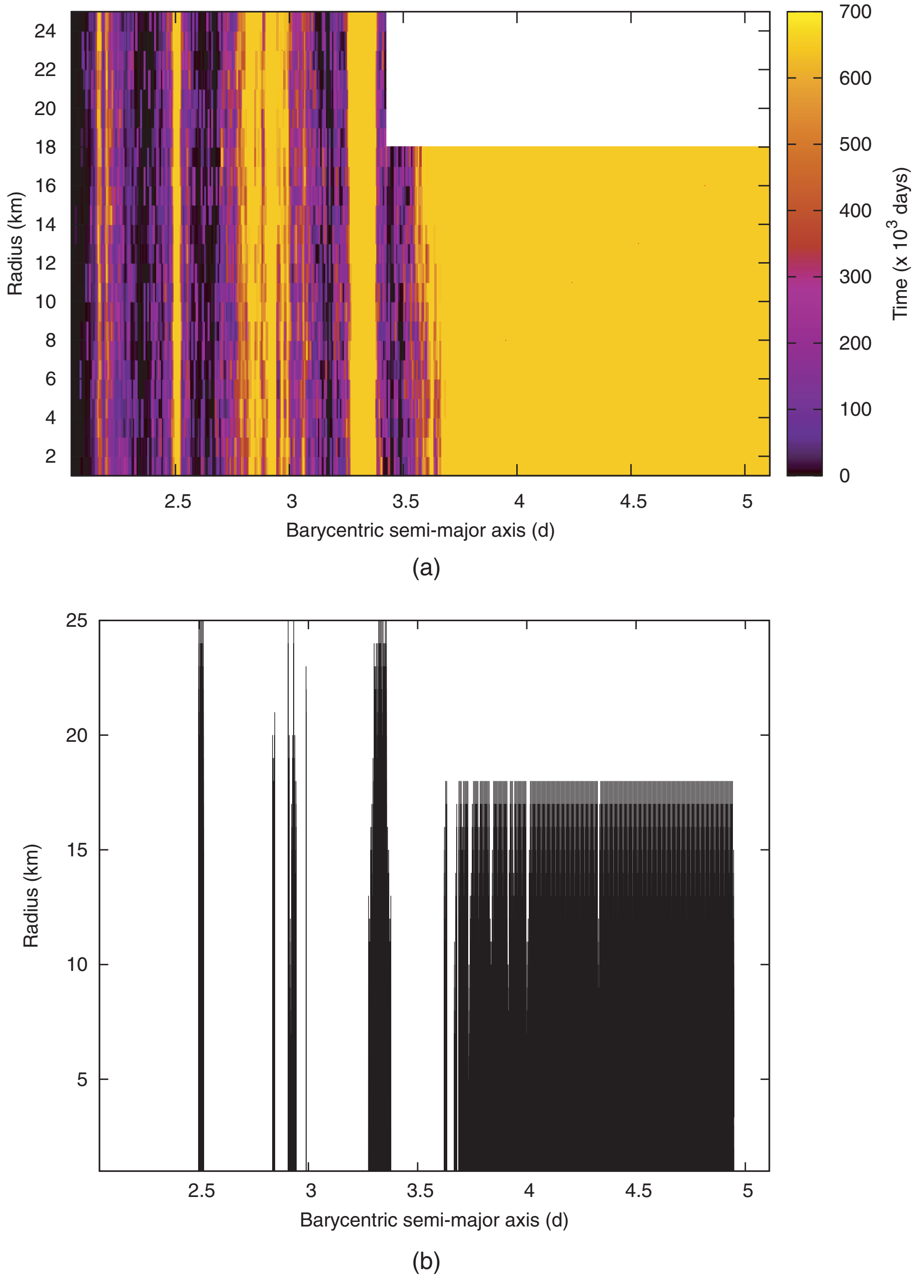


Figure 4. This figure presents the (a) values of the time (in days) as a function of the barycentric semimajor axis and radius (in km) of the hypothetical satellites and (b) those survived hypothetical satellites which cause only a negligible variation in the eccentricities of Nix and Hydra (less than 10^{-3}).

4 FINAL COMMENTS

Two new companions to the Pluto–Charon binary system have been detected in 2005 by Weaver et al. (2006). These small satellites, named Nix and Hydra, are located beyond Charon's orbit. Although they are small when compared to Charon, their gravitational perturbations can decrease the stability of the external region. The dynamical structure of this external region is analysed by numerically simulating a sample of particles in P-type orbits under the gravitational effects of Pluto, Charon, Nix and Hydra. The particles are separate into two groups: prograde and retrograde orbits. The presence of the small satellites Nix and Hydra, as expected, decreases the stable region beyond Charon's orbit. A simple analysis, taking into account the circular restricted three-body problem, can explain the formation of the agglomerates of particles, for particles in prograde orbits, near and coorbital to Nix and Hydra. Comparison between particles in prograde and retrograde orbits shows that the stability region for those particles in retrograde orbits is larger than for particles in prograde orbits.

Small unseen satellites can also exist in this system without inducing an eccentricity in the orbits of Nix and Hydra larger than 10^{-3}. These hypothetical satellites can be localized in the coorbital regions of Nix and Hydra, between their orbits and beyond Hydra's orbit.

The encounter between the New Horizons spacecraft and the Pluto system in 2015 will help to elucidate this peculiar quadruple system formed, up to now, by two massive bodies, Pluto and Charon, and two small satellites, Nix and Hydra.

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