J Control Autom Electr Syst (2016) 27:317–327
DOI 10.1007/s40313-016-0234-6

A Node-Depth Encoding-Based Tabu Search Algorithm for Power Distribution System Restoration

Waldemar Pereira Mathias-Neto[1] · **José Roberto Sanches Mantovani**[1]

Received: 17 June 2015 / Revised: 3 December 2015 / Accepted: 7 February 2016 / Published online: 26 February 2016

Abstract This paper presents a new algorithm to solve the distribution power system restoration problem based on a joint application of tabu search (TS) algorithm and the node-depth encoding (NDE). The integration of NDE, its operators, and the TS algorithm results in a methodology that combines the best of each technique. The main purpose of the proposed meta-heuristic approach is to minimize the costs involved in the restoration process while electrical and operational constraints are met. Simulation results for three scenarios of a modified IEEE 37-node test case are presented. The results show the computational performance and the robustness of the proposed algorithm.

Keywords Tabu search algorithm · Node-depth encoding · Power distribution system · Restoration

List of Symbols

Sets

B	Set of busses of the system;
S	Set of circuits;
T	Set of substation power transformers.

Parameters

$I_{\mathrm{km}}^{\mathrm{MAX}}$	Maximum current limit for branch km;
$S_{k,t}^{\mathrm{MAX}}$	Maximum apparent power limit for transformer t, located at node k;
V^{MIN}	Lower voltage magnitude limit;
V^{MAX}	Upper voltage magnitude limit;
$\mathrm{OC}_{\mathrm{km}}$	Operating cost of switching device located in branch km;
NDDC_k	Non-distributed demand cost for node k;
SC_k	Social cost by load shedding at node k;
g_{km}	km-branch conductance;
b_{km}	km-branch susceptance;
P_k	Active power of the load at node k;
Q_k	Reactive power of the load at node k;
ω_{km}^{i}	Initial status of branch km, assumes 1 if active and zero otherwise.

Variables

P_{km}	Active power flows through the branch km;
Q_{km}	Reactive power flows through the branch km;
$P_{k,t}$	Active power flows through the transformer t, located at node k;
$Q_{k,t}$	Reactive power flows through the transformer t, located at node k;
V_k	Voltage at node k;
ω_k	Binary variable for status of node k, assumes 1 if energized and zero otherwise;
ω_{km}	Binary variable for status of branch km, assumes 1 if active and zero otherwise.

✉ Waldemar Pereira Mathias-Neto
waldemarmathias@gmail.com

José Roberto Sanches Mantovani
mant@dee.feis.unesp.br

[1] LaPSEE - Power System Planning Lab, Department of Electrical Engineering, UNESP - Univ Estadual Paulista, Campus of Ilha Solteira, Ilha Solteira, São Paulo, Brazil

1 Introduction

The modern overhead power distribution systems are designed to operate with high reliability indices. However, power outages may occur at any time and caused by several factors, such as windstorms, trees or vehicle collision with the network. Therefore, to ensure minimal impact of these interruptions in network reliability indices, out-of-service areas, which are not affected by the permanent fault, should be re-energized as fast as possible. The process of system re-energizing is called restoration.

The main purpose of restoration process is to restore the largest amount of load, as fast as possible, through a sequence of switching actions. The existence of several switchgears, strategically allocated in the network, allows the meshed distribution systems work with radial topologies. Therefore, performing a sequence of maneuvers, automatically or manually, may change the radial grid topology and restore the loads previously without a power supply.

The distribution system restoration problem is reported in the literature as combinatorial, nonlinear with real and integer and discrete variables, highly restricted, multimodal and large-scale (Li et al. 2014). The combinatorial explosion and the nonlinearity of the problem make it difficult to obtain a solution for real systems using classical optimization techniques, at reasonable times, for the real-time control. Thus, non-classical methods such as heuristics, meta-heuristics and expert algorithms have been proposed in the literature for solving the distribution restoration problem.

The heuristics and experts algorithms were the first computational methods developed to solve the restoration problem (Liu et al. 1988; Aoki et al. 1989). Genetic algorithms (GA) and evolutionary techniques were the first meta-heuristics dedicated to solving the problem and the most intensely researched until now (Nara 1992; Fukuyama and Chiang 1995; Mathias-Neto et al. 2010). Others modern techniques were also used for the restoration problem, such as ant colony algorithm (ACO) (Mohanty et al. 2003; Chin and Su 2005), simulated annealing (SA), particle swarm optimization (PSO), tabu search (TS) (Toune et al. 1998; Mori and Ogita 2002; Mori and Muroi 2011), among others (Nagata and Sasaki 2002; Li et al. 2009). However, some studies suggest that the TS has better compatibility with the problem and consequently presents better results than others consolidated techniques such as GA and SA (Mori and Ogita 2002; Toune et al. 2002; Mori and Muroi 2011).

The challenge in using meta-heuristics as an optimization technique for distribution restoration problem is to find a proper encoding. This encoding must be able to represent the distribution system in a computationally efficient manner and, simultaneously, to be flexible and compatible with optimization technique. Therefore, a encoding designed this way results in an algorithm that requires less processing time and is suitable for real-time application.

A computationally efficient encoding for the representation of graph-oriented problems is the node-depth encoding (NDE) (Delbem et al. 2004). The NDE was used together with evolutionary algorithm for solving the network reconfiguration problems, and the outcome was satisfactory (Santos et al. 2010).

In this paper, a new optimization technique to solve the distribution systems restoration problem based on a joint use of the node-depth encoding and TS meta-heuristic is presented. To increase the efficacy of algorithm, a new operator to the NDE and a new control technique for accelerating the convergence of the algorithm TS are developed. The TS algorithm with this feature is a contribution of this work. The approach is general and can be applied to any mathematical model of the restoration problem without major adjustments.

This paper is structured as follows: Section 2 introduces the mathematical model, objective function, and constraints. Section 3 explains the basic concepts of NDE, its operators, and the new concepts developed in this work. Section 4 presents the integration of the TS algorithm and the NDE to meet the objectives described in Sect. 2, how to find an initial solution to algorithm, and how to build the neighborhood structure. Section 5 contains the simulation results for three scenarios of a modified IEEE 37-bus test system, and Sect. 6 presents the concluding remarks.

2 Problem Formulation

In this study, the problem is formulated by an objective function and eight constraints. The objective function consists of the summation of costs involved in restoring process (RC). The RC includes the financial losses associated with unmet demand (NDDC), restoration operational costs (OC), and interruption social cost (SC). Equations (2), (3), and (4) represent the components of the objective function.

The main constraints of the problem are: maximum current flow in conductors and maneuvering equipment, maximum power flow in the power transformers, the minimum voltage magnitude in all nodes of primary distribution network, power flow equations, load balancing and operational radiality. The set of Eqs. (5)–(12) represents these constraints.

$$\min \mathrm{RC} = \mathrm{OC} + \mathrm{NDDC} + \mathrm{SC} \tag{1}$$

2.1 Objectives

A1) Restoration operating cost

$$\mathrm{OC} = \sum_{\mathrm{km}} \mathrm{OC}_{\mathrm{km}} \cdot \left| \omega_{\mathrm{km}} - \omega_{\mathrm{km}}^{i} \right| \tag{2}$$

A2) Non-distributed demand cost

$$\mathrm{NDDC} = \sum_{k} \mathrm{NDDC}_{k} \cdot P_{k} \cdot (1 - \omega_{k}) \tag{3}$$

A3) Interruption social cost

$$\mathrm{SC} = \sum_k \mathrm{SC}_k \cdot (1 - \omega_k) \tag{4}$$

2.2 Constraints

B1) The maximum current flow in the equipment and conductors must be kept below its operating limit:

$$0 \le \frac{P_{\mathrm{km}}{}^2 + Q_{\mathrm{km}}{}^2}{|V_k|^2} \le I_{\mathrm{km}}^{\mathrm{MAX}^2} \cdot \omega_{\mathrm{km}} \tag{5}$$

B2) The maximum power flow in the substations transformers must be kept below its operating limit

$$P_{k,t}{}^2 + Q_{k,t}{}^2 \le S_{k,t}^{\mathrm{MAX}^2} \tag{6}$$

B3) The system voltage levels must be kept within acceptable ranges, predefined by utility regulators:

$$V^{\mathrm{MIN}} \le V_k \le V^{\mathrm{MAX}} \tag{7}$$

B4) Active power flow equation

$$P_{\mathrm{km}} = V_k^2 \cdot g_{\mathrm{km}} - V_k \cdot V_m \cdot g_{\mathrm{km}} \tag{8}$$

B5) Reactive power flow equation

$$Q_{\mathrm{km}} = -V_k^2 \cdot b_{\mathrm{km}} + V_k \cdot V_m \cdot b_{\mathrm{km}} \tag{9}$$

B6) Active power balance equations

$$\sum P_{\mathrm{mk}} - \sum P_{\mathrm{km}} + \sum_T P_{k,t} = P_k \tag{10}$$

B7) Reactive power balance equations

$$\sum Q_{\mathrm{mk}} - \sum Q_{\mathrm{km}} + \sum_T Q_{k,t} = Q_k \tag{11}$$

B8) Radiality complementary constraint

$$\sum w_{\mathrm{km}} \le \dim(B) - \sum w_{\mathrm{km}}^i \tag{12}$$

$$k, m \in B$$
$$\mathrm{km}, \mathrm{mk} \in S$$
$$t \in T$$

The mathematical model ensures the connectivity of the subgraph solution through Eqs. (10) and (11), and number of branches less than nodes by Eq. (12). However, the proposed meta-heuristic, as shown in Sect. 5, uses NDE operators to build the neighborhood of TS algorithm. These operators, when applied to the optimization approach, ensure the compliance to Eq. (12), and thereafter, only radial solutions are built.

3 Node-Depth Representation

The node-depth encoding is a mathematical manner for representing graphs through a set properly organized in tables. In NDE, any tree of a graph can be represented by its nodes and depths concerning to the root node in a single table. Consequently, one or more tables grouped represent a forest F.

The concepts associated with NDE, developed by Delbem et al. (2004), were applied to solve the reconfiguration and restoration problems of distribution power systems (Santos et al. 2010; Kaewmanee and Sirisumrannukul 2011; Sanches et al. 2014). In both studies, the problem was solved using evolutionary algorithms and combining two NDE operators. These operators were previously called preserve ancestral operator (PAO) and change ancestral operator (CAO) by their features.

Another operator is developed in this paper and added to the list of NDE operators. This new operator, called CUT, is introduced to adjust the optimization technique to the restoration problem. The PAO, CAO, and CUT operators are described in details in Sects. 3.1, 3.2, and 3.3, respectively. An improvement in the PAO and CAO operators, to suit them to the proposed methodology, is presented in Sect. 3.4.

3.1 PAO Operator

The PAO operator was developed to introduce small changes in forest F composed of, at least, two trees. These changes are performed by a simple cut in a tree and its subsequent transfer to another of forest F. Figure 1 shows an example of the application.

In this example, the $T1$ and $T2$ trees belong to forest F. The subtree highlighted in $T1$ is pruned in "p" and then moved from $T1$ to $T2$. However, to execute this move, the node "a" in $T2$ must be adjacent to the "p," i.e., it is necessary that there exists an edge that connects both nodes in the graph of F.

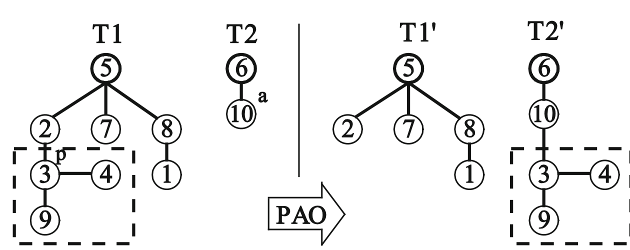

Fig. 1 Example of PAO operator application

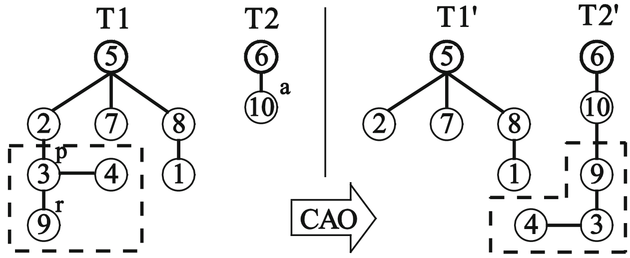

Fig. 2 Example of CAO operator application

3.2 CAO Operator

The CAO operator was developed to perform larger and more significant changes in forest *F*. In addition to cut a tree of *F* and the consequent transfer to another tree of the forest, the subtree pruned is reordered according to a new root "*r*." This reordering significantly modifies the *F* topology. In Fig. 2, an application example is illustrated.

In this example, the highlighted subtree is pruned in "*p*" and transferred from *T*1 to *T*2. However, it is reordered on new root "*r*." The node "*a*" in *T*2 must be adjacent to the "*r*" of the *T*1, i.e., it is necessary that there exists an edge that connects both nodes in the graph of *F*.

3.3 CUT Operator

The CUT operator is proposed in this work, among others, to adjust the optimization method for restoration problem. In real distribution systems operation, after faults, constraint violations of the problem associated with network overloads may occur. To avoid network overloads, selective load shedding is a common practice at operation centers of power utilities. These load shedding are achieved by optimization technique applying the CUT operator in a tree of forest *F*. The CUT operator, unlike the others operators, must be applied in a single tree of *F*. This operator has the characteristic of increasing the number of trees of the forest *F*. Figure 3 shows an application example.

In this example, the highlighted subtree is pruned in "*p*" and a new tree *T*2' has been added to the forest. The number of trees present in *F* is increased by one.

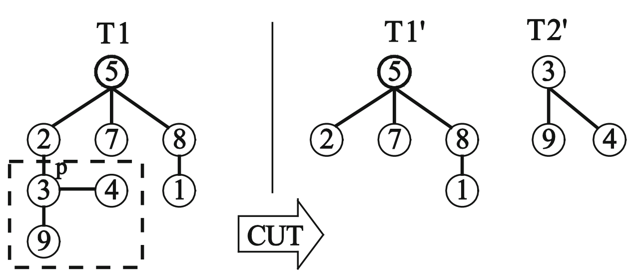

Fig. 3 Example of CUT operator application

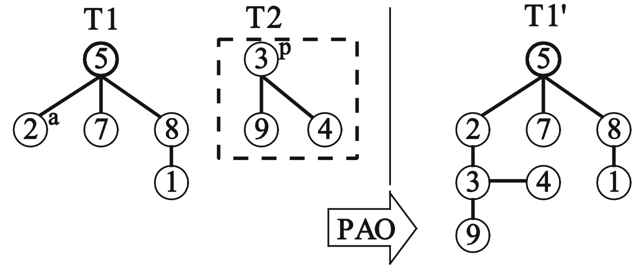

Fig. 4 Example of pruning on root node by PAO operator

3.4 PAO and CAO Improvement

The PAO and CAO operators were developed for the planning of networks and then were applied to the power distribution reconfiguration problems (Delbem et al. 2004; Santos et al. 2010). Although the reconfiguration problem can be similar in some ways with the restoration problems, they have completely different goals. For reconfiguration problem, the load shedding does not make sense; therefore, the operators CAO and PAO maintain constant the number of trees of forest *F* during the optimization process. The introduction of the CUT operator changes this condition, and it is necessary to improve the PAO and CAO operators.

The improvement in both operators is based on the release of pruning constraint on the root node of a tree previously cut. Thus, a tree that previously pruned by the CUT operator can return to its original configuration through the PAO and CAO operators. In Fig. 4, an example of the return to the original configuration through PAO operator has been shown.

In this example, the forest modified by the CUT operator in the previous example (Fig. 3) returns to its original configuration by applying the PAO operator. The pruning was done in the root node of the *T*2, and this subtree moved to *T*1. Therefore, the number of trees of *F* is reduced by one.

4 Tabu Search Algorithm

The tabu search algorithm was introduced by Fred Glover for solving combinatorial optimization problems (minimizing or maximizing) and was applied in several knowledge areas with satisfactory results (Glover 1989, 1990; Glover et al. 2007). The effectiveness of TS is related to its adaptive memory structure called "tabu list" (TL), a dynamically managed list during the optimization process that allows the meta-heuristic escape from local minima and widely explore the search space. The low dependence between the initial solution and incumbent solution and the small number of parameters that need to be tuned are the main advantages of TS compared to other meta-heuristics.

On the other hand, the TS tends to require high computational time for solving constrained problems in real applications. The high processing times are principally attributed to the generation and evaluation of successive neighborhoods.

Therefore, the definition of good neighborhood structure with diversification characteristics (ability to explore a lot of the search space and escape from local minima) and intensification (ability to perform search in the most attractive areas) is essential to design algorithms with good performance.

In this work, the combination of the three NDE operators is used to build the neighborhood of the TS algorithm. The NDE and its operators ensure some advantages to the algorithm with respect to other heuristic techniques and metaheuristics:

1) Maintaining the radial topology throughout the search process. The application of PAO, CAO, or CUT operator in a radial topology always produces radial topologies;
2) The operators of NDE introduce different levels of network changes. The PAO operator performs minor changes when applied to a solution; the CAO operator produces intermediate changes; the CUT introduces major and significant changes in the network topology. The joint use of the three operators to establish the structure of neighborhood ensures the diversification and intensification characteristics;
3) Tables are computationally efficient to handle and to store large amount of data;
4) Backward/forward sweep power flow routines (Shirmohammadi 1988; Baran and Wu 1989) can be implemented using NDE (Sanches 2013). These routines, when using node-depth encoding, do not require successive reordering to evaluate each new network topology.

The radial topology of overhead distribution systems is a premise assumed for the development of this work. This constraint is represented by Eqs. (10)–(12), and it is met through node-depth encoding and its operators. Other assumptions, such as the knowledge of grid fault location and the distribution system state, are also considered by the proposed methodology. The Shirmohammadi's (1988) load flow subroutine is used to define the network status.

In the following items, the proposed algorithm, the way to choose an initial solution, and the generation of the neighborhood through the PAO , CAO and CUT operators are considered in detail.

4.1 Proposed TS Algorithm

The proposed TS algorithm can be implemented as follows.

i. Define the initial conditions, set the iteration counter k = 1, the initial solution x^0 and the incumbent $x^* = x^0$.
ii. Determine the neighborhood S $(x^k) = \{x_i^k : i = 1, \ldots, \text{n}\}$ by applying the PAO, CAO, and CUT operators in x^k. These operators must be applied with the weights α, β and γ, respectively, where $\alpha + \beta + \gamma = 1$ and $\alpha > \beta > \gamma$. This step is explained in detail in Sect. 4.3.
iii. Calculate $c(x_i^k)$ for all generated solutions. The notation $c(x_i^k)$ is the objective function of the problem.
iv. Order $S(x^k) : c(x_i^k) \le c(x_{i+1}^k)$.
v. Select the neighbor which is not in tabu list x_i^k(or that meets the criteria of aspiration) with lower cost $c(x_i^k)$. Do $x_0^{k+1} = x_i^k$.
vi. If $c(x_i^k) < c(x*)$, do $x^* = x_i^k$.
vii. If a stopping criterion is satisfied, go to step *ix*, otherwise, go to step *viii*. The overall number of iterations (K) is used.
viii. Update the tabu list and return to step *ii*.
ix. Print the best solution found.

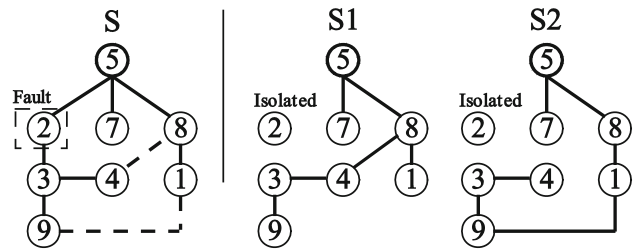

Fig. 5 The system S and its two initial solutions

4.2 Initial Solution

The initial solution to the TS algorithm is a feasible (or infeasible) configuration of the distribution system after isolating the faulted section. The feasibility of constraints B1 to B7 is not required, and the algorithm will be responsible for re-establish it (if exists); however, the feasibility of radiality constraint B8 is mandatory.

The node-depth encoding requires the compliance of radiality constraint, and its operators also require to generate the neighborhoods $S(x^k)$ during the search process. Furthermore, electrical constraints violations can be managed by the TS algorithm under the search process. Therefore, its feasibility is not required.

In Fig. 5, a hypothetical distribution system S and its two initial solutions are illustrated. The sections 3, 4, and 5 are out of service after a permanent fault on section 2.

The supply to the sections 3, 4, and 5 can be restored through the *tie switches* located between the sections 4 and 8 or sections 9 and 1. Both the initial solutions are suitable; however, techniques for generating good quality of initial solutions can be used to accelerate the convergence of the TS algorithm (Toune et al. 1998).

4.3 TS Neighborhood and Improvements

The neighborhood of the TS algorithm is built by applying n times the node-depth encoding operators to current solution

```
for i=1 to n
    op = rand( ) :  {op ∈ ℝ: 0 ≤ op ≤ 1  }
    if (op <= α)
        x^k_i = PAO(x^k_0)
    else if  (α < op <= α+β)
        x^k_i = CAO(x^k_0)
    else if  (α+β < op <= 1)
        x^k_i = CUT(x^k_0)
    end if
end for
```

Fig. 6 Neighborhood generation pseudocode

```
for i=1 to n
    op = rand( ) :  {op ∈ ℝ: 0 ≤ op ≤ 1  }
    ng = rand( ) :  {ng ∈ ℤ: 0 ≤ ng ≤ i-1}
    if (op <= α)
        x^k_i = PAO(x^k_ng);
    else if  (α < op <= α+β)
        x^k_i = CAO(x^k_ng)
    else if  (α+β < op <= 1)
        x^k_i = CUT(x^k_ng)
    end if
end for
```

Fig. 7 Evolutionary neighborhood strategy pseudocode

x^k. A new solution x_i^k is generated in each new application. Therefore, mathematically, it can be represented as (13):

$$S(x^k) = \{\ldots, x_{i-1}^k = \text{PAO}(x^k), x_i^k = \text{CAO}(x^k), \ldots, \\ x_{i+1}^k = \text{CUT}(x^k), \ldots, x_n^k = \text{PAO}(x^k)\} \qquad (13)$$

The operator to be applied to the i-th generation at k-th iteration is randomly selected. However, the weights between applications of PAO, CAO, and CUT operators should remain constant and equal to α, β, and γ, respectively.

The use of appropriate values for α, β, and γ parameters, such that $\alpha + \beta + \gamma = 1$ and $\alpha > \beta > \gamma$, is essential for the proper performance of the methodology and ensures the properties of diversification and intensification of TS algorithm. The PAO operator, by their design, holds only minor changes to the current solution and should be applied on a larger scale to x^k. This operator represents the search intensification process in nearby regions. On the other hand, the CUT operator introduces significant changes to the topology of the current solution and therefore should be applied less frequently. This operator is used to escape from local optimal. The CAO operator introduces intermediate changes in system topology.

In Fig. 6, the pseudocode used to generate the neighborhood of the tabu search algorithm is shown.

To accelerate the convergence of the TS algorithm, an enhancement, called *evolutionary neighborhood,* to the code presented in Fig. 6 is developed. The evolutionary neighborhood extends the explored search space on a single tabu iteration and accelerates the convergence of TS. The strategy is to apply the NDE operators on the $(i - 1)$ last solutions generated for all k-iterations. Thus, x_i^k solution will be obtained by application of the operators $x_0^k, x_1^k, \ldots, x_{i-2}^k$ or x_{i-1}^k.

Evolutionary neighborhood is an approach that can be used in the initial solutions, whenever there is stagnation of the search process or during all iterations of the algorithm. In Fig. 7, pseudocode for the generation of neighborhood based on evolutionary neighborhood strategy is illustrated.

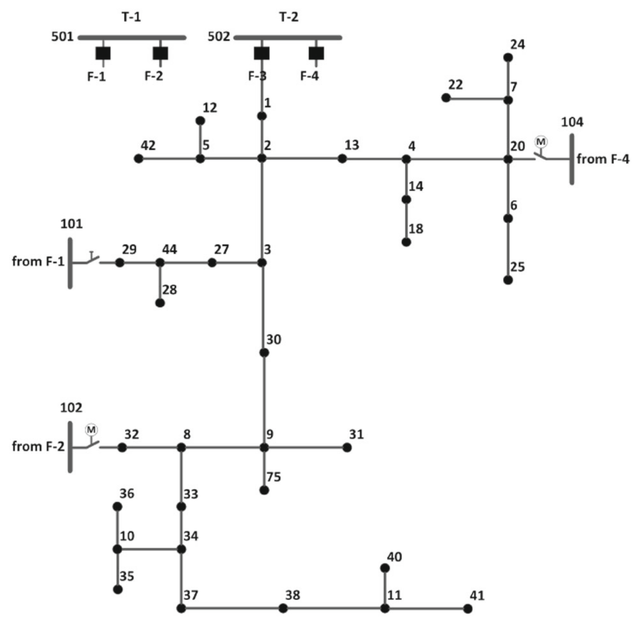

Fig. 8 Topology of modified IEEE 37 test system

5 Test and Results

The proposed algorithm was implemented in the programming language C++ and executed on a personal computer with Intel® Core™ i7 processor and 4 GB RAM.

5.1 Test System

The IEEE 37-bus radial distribution system is used to evaluate the proposed algorithm with some changes (IEEE 2015). At original system, a new transformer, three feeders, and three normally open tie switches were added and the existing voltage regulator was removed. It is assumed that there is one sectionalizing switch at every branch of the system. The system topology is illustrated in Fig. 8.

The rated power of transformers T-1 and T-2 were kept at 2.5 MVA; the nominal system voltage was kept at 4.8 kV; loads, impedances and ampacity of conductors as shown in the “Appendix” (Table 5). All system switches are remote controlled, but the switch 101-29 is manually operated.

Table 1 TS algorithm parameters

Parameter description	Symbol	Value
Neighborhood size	n	50
Max. number of iterations	K	1000
Tabu list size	–	10
Proportionality among the operators	α	0.45
	β	0.30
	γ	0.25

Table 2 Best solutions found (Case 1)

#	RC	Sequence of switching operations
1	30	30-9 102-32 104-20
2	30	3-30 102-32 104-20
3	30	2-3 102-32 104-20
4	30	2-13 102-32 104-20
5	30	13-4 102-32 104-20

5.2 Simulation Parameters

The proposed TS algorithm has two well-defined parameters sets. The first group is related to the meta-heuristic, such as neighborhood size, number of iterations, and tabu list size. These parameters were set based on experiments performed during the calibration algorithm step and kept constant in all scenarios. The second group is result of TS and NDE integration and composed by α, β, and γ parameters. These parameters were calibrated to maintain a balance between intensifying search in nearby regions and the diversification of solutions to escape from local optima. In Sect. 5.4, additional considerations to setup parameters α, β, and γ are provided. In Table 1, the parameters values used to run the proposed algorithm are listed.

The operational cost (OC_{km}) for manual switching was established in 100 currency units (c.u.), and the remote controlled switches in 10 c.u. The social cost (SC_k) was fixed in 1,000,000 c.u. The minimum and maximum voltages (V^{MIN}, V^{MAX}) were defined in 0.92 and 1.05 p.u., respectively. The non-distributed demand cost ($NDDC_k$) was set at 500 c.u. per unsupplied kW.

5.3 Simulations Scenarios

To evaluate the proposed algorithm, three different scenarios involving the modified IEEE 37-bus test system were considered. However, in all scenarios, a permanent fault on the node 1 was considered. This fault causes the power outage into entire feeder, and unfaulted sections need to be restored.

To restore the unsupplied consumers, the support feeders F-1, F-2, and F-4 are available, which are interconnected by nodes 101, 102, and 104, respectively. The spare capacity of support feeders is constrained by its ampacity and respective power transformers limits (as "Appendix").

The initial solution to the TS algorithm was defined by the switch closure located among nodes 20-104 and opening the switch among 1-2 and feeder circuit breaker. The choice of the F-4 support feeder was random among others available feeders (F-1 and F-2). It is worth mentioning that this initial solution has current constraint violations (belong the branches 502-104-20-4-13) and voltage constraint violations on several nodes (downstream node 13). In addition, only for simplifications purpose, the opening maneuvers to isolate the faulted section were not considered in the calculations of the objective function (RC).

5.3.1 *Case 1: Substation Transformers with Rated Power*

In this case, the test system was considered as presented in "Appendix." The main objective is evaluating the behavior of the optimization technique versus switching costs.

For this scenario, the algorithm was able to restore all out-of-service loads through three switchings. Table 2 shows the best solutions found by the algorithm for this case with their respective restoration costs. These solutions use the F-2 and F-4 support feeders. The F-1 feeder was not used due to the higher cost of switching between nodes 101-29 (manual switch).

The simulation time for this scenario remained between 219 and 477 ms, including 196 ms needed to perform preprocessing steps (reading data network, coding data as NDE and evaluating the initial solution). In Fig. 9, the chart time-to-target is shown for this scenario with and without evolutionary neighborhood strategy. The chosen target is a solution with restoration costs equal or less than 30 c.u. (Aiex et al. 2007).

It can be observed that in 80 % of the simulations, an optimal solution was found in <330 ms. The number of iterations required for finding a solution for this scenario was also low.

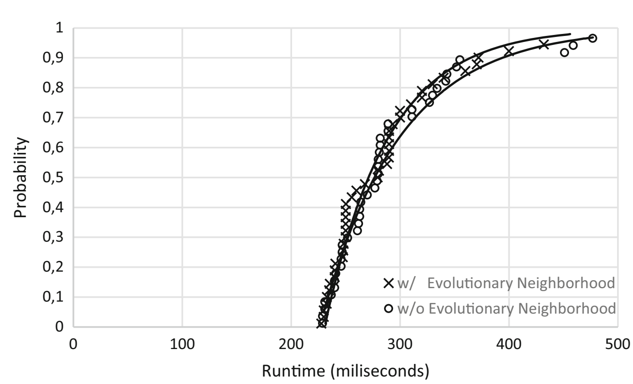

Fig. 9 Time-to-target graph for Case 1

In 80 % of the simulations, a good quality solution was found until the second iteration and not exceeded 6 iterations in all simulations performed.

The concept of evolutionary neighborhood did not contribute significantly to reduce the time processing to this scenario. Both techniques found the target solutions in similar time probability. It can be explained by low difficulty level of this case (without power constraints). The following test case will demonstrate the expressive convergence improvement introduced by the proposed strategy.

5.3.2 Case 2: Substation Transformers with Reduced Rated Power

In this scenario, the system is considered as previous; however, the rated power in the *T*-1 and *T*-2 transformers is reduced to 600 and 900 kVA, respectively. Therefore, the constraint represented by Eq. (6) is switched on and it will cause unavoidably load shedding at the distribution system.

For this scenario, the algorithm found solutions with objective function values among 255,180 and 292,650. These simulations got factible solutions with restoration rate among 70.67 and 74.43 % of amount the total load shedding. The best solution found for Case 2 is shown in Fig. 10.

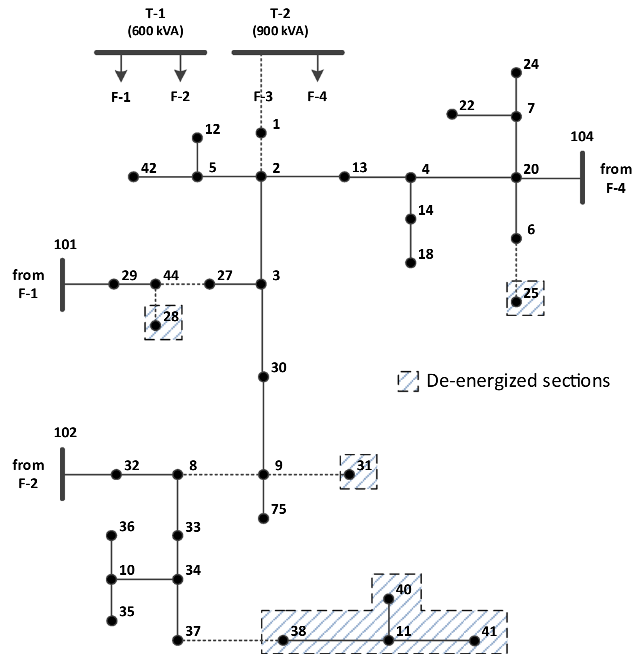

Fig. 10 Best solution found for Case 2

This solution involves the load shedding of seven sections (510 kW), eight switching operations of remote-controlled switches, and a manual switch. In other words, the loads are shed by opening four sectionalizing switches and the feeders *F*-1, *F*-2, and *F*-4 are isolated each other by opening another two switches. Additionally, the system is restored by closing 3 tie switches. At this post-restoration topology, the system minimal voltage was 0.95 p.u. and both transformers were operating at almost rated power. Numerically, *T*-1 and *T*-2 transformers were operating with 599.04 and 890.40 kVA, respectively. The current constraint has not been activated.

Other solutions were also found for this scenario. In Table 3, the best and near-best solutions found to this case are showed. These solutions are ordered by their restoration costs (RC).

The number of iterations required for the algorithm to reach a target solution was higher than the previous case. The algorithm took, on average, 40 iterations more when using the proposed strategy for neighborhood generating, and 273 iterations more, when not. Consequently, the runtime was also higher. A target solution was achieved, at 80 % of the simulations, in 1.91 s when using the evolutive neighborhood, and in 13.4 s when the strategy was not used. In Fig. 11, the time-to-target graph for the scenario 2 is illustrated. The selected target is a solution with restoration costs equal or less than 300,000 currency units.

It can be seen that evolutive neighborhood has considerable influence on the convergence speed of algorithm. It can be explained as consequence of increasing variability of solutions in each algorithm iteration and, as result, fast exploitation of search space. Numerically, the proposed strategy for neighborhood generation is up to 700 % faster than conventional generation on 80 % of simulations.

5.3.3 Case 3: Substation Transformers with Reduced Rated Power and Priority Customer

In this test case, the system was considered as previous case; however, in order to reduce the power in *T*-1 and *T*-2, a priority customer was added in node 38. The inclusion of the

Table 3 Best and near-best solutions found (Case 2)

#	RC	Sequence of switching operations
1	255,180	44-27 8-9 6-25 9-31 104-20 37-38 102-32 44-28 101-29
2	255,180	3-27 8-9 5-42 104-20 37-38 102-32 44-27 44-28 101-29
3	255,180	3-27 8-9 9-31 104-20 37-38 102-32 44-27 44-28 101-29
4	262,670	44-27 9-30 14-18 104-20 101-29 9-31 34-37 102-32
5	262,670	44-29 9-30 2-5 104-20 101-29 9-31 37-38 102-32

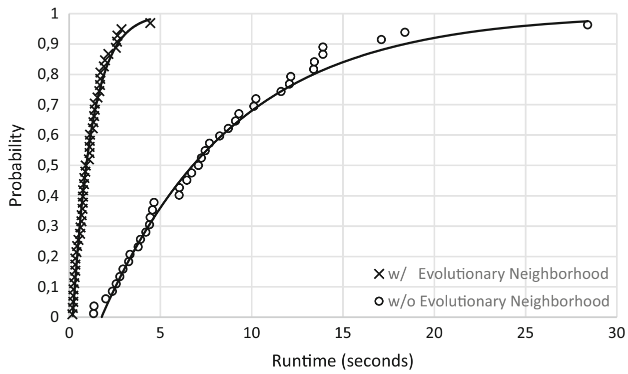

Fig. 11 Time-to-target graph for Case 2

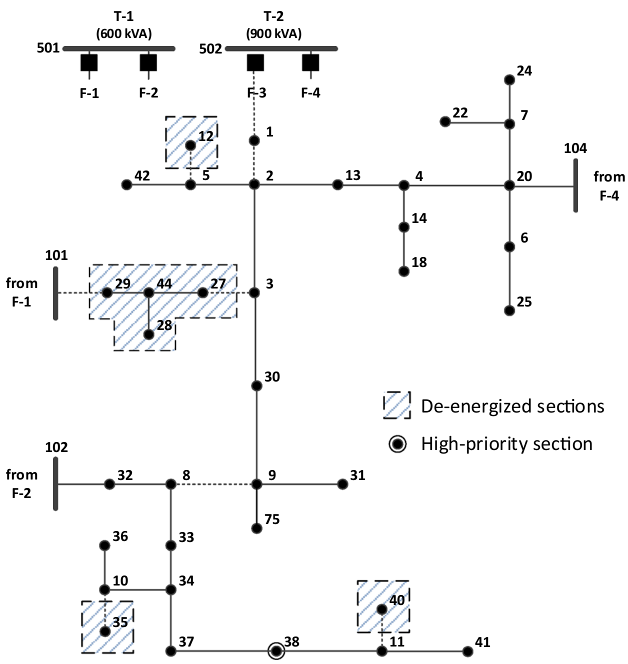

Fig. 12 Best solution found for Case 3

priority consumer makes the problem more complex and difficult to solve. The best solution found for this scenario is shown in Fig. 12. The solution involves the load shedding of seven sections (525 kW) and the switching of seven remote-controlled tie switches (four switchings for load shedding, one for segregate the feeders, and two for restore the system).

In Table 4, other solutions found by the algorithm for this scenario are shown, and, as expected, none of solutions include the node 38 in the restoration plan. However, the included priority consumer has degraded the objective function compared to the previous case. The restoring costs were 3 % higher than previous case, restoring 15 kW less.

The number of iterations required for the algorithm to reach a target solution and, consequently, the simulation time of this scenario were higher than the previous one. The target solution was found, up to 80 % of the simulations, with

Table 4 Best and near-best solutions found (Case 3)

#	RC	Sequence of switching operations
1	262,570	3-27 5-12 8-9 104-20 10-35 11-40 102-32
2	262,570	3-27 9-31 8-9 104-20 10-36 11-38 102-32
3	262,570	3-27 9-31 8-9 104-20 10-35 11-40 102-32
4	262,580	44-28 44-29 5-2 8-9 104-20 11-40 10-35 102-32
5	262,580	44-29 4-42 3-30 104-20 11-40 10-35 8-9 102-32

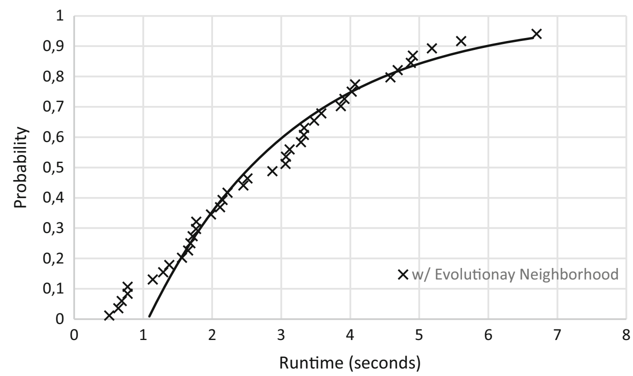

Fig. 13 Time-to-target graph for Case 3

approximately 4.52 seconds, and it is 76 % higher than runtime found in Case 2. In Fig. 13, the chart time-to-target for scenario 3 is shown. The chosen target is a solution with restoration costs equal or less than 450,000 currency units.

The target value was increased up to 450,000 currency units in this case to attempt to reach the target without evolutionary neighborhood strategy. However, the algorithm without the strategy did not reach the target. Therefore, it confirms that proposed strategy for neighborhood generating is useful to speed up the convergence of tabu search algorithm.

5.4 TS Performance as Function of α, β, and γ Parameters

It was observed that the parameters α, β, and γ have direct influence on the quality of solutions found by the proposed TS algorithm. However, this influence is also conditioned on system topology and the set of active constraints in the search process of the proposed algorithm. For example, in Case 1, only current and voltage constraints (Eqs. 5 and 7) were active and, concomitantly, reconfigurations in the topology of the distribution system were able to circumvent these constraints, not requiring load shedding. Thus, successive runs of PAO and CAO operators became necessary to find a good solution, and therefore, assigning large values to parameters α and β is more interesting for fast convergence of the algorithm.

On the other hand, highly constrained systems or whose changes in the topology do not eliminate the restrictions of the problem, such as illustrated in scenarios 2 and 3, and the use of CUT operator becomes necessary. Thus an increase in the value of parameter γ is essential. However, the use of excessively high values of γ can result in successive cuts in the graph of the system and degradation of the current solution. In general, values among 15–25 % are considered as a good starting point for setting γ.

The parameters α and β showed no major inference in the algorithm convergence. As starting point for the setup, these parameters suggest values between 30 and 50 %, with β slightly greater than α.

6 Conclusions

This paper presented a new optimization technique to the distribution restoration problem based on the node-depth encoding and TS algorithm. Additionally, it was proposed a new operator for the NDE and a technique to accelerate the convergence of tabu search as a meta-heuristic algorithm.

The combined use of the three operators of NDE for the generation of neighborhood structure of TS algorithm is the main contribution of this work. The joint use of the NDE and TS has the advantage of maintaining the radial network topology throughout the optimization process, ensures the intensification and diversification characteristics, and allows storing and processing data more efficiently. The development of the CUT operator has made it possible to solve highly constrained systems.

The numerical results illustrate the viability of the proposed optimization technique, and they show technically feasible solutions and compatible processing time for real-time applications. The results for highly restricted systems, as Cases 2 and 3, demonstrate the robustness of the algorithm. Flexibility is another benefit of the proposed algorithm, i.e., any mathematical model for distribution restoration problem can be used, as well as higher level control techniques can be used for specific purposes.

Acknowledgments The authors gratefully acknowledge the Ilha Solteira Education and Research Foundation—FEPISA (Grant 011/2011), São Paulo Research Foundation—FAPESP (Grant 2013/23590-8) and National Counsel of Technological and Scientific Development—CNPq (Grant 305371/2012-6) for their economic support for this project.

7 Appendix

The modified IEEE-37-bus test system used by this paper to run all the simulations is shown at Fig. 5.

Table 5 The modified IEEE-37-bus test system

FB	TB	*R*	*X*	*A*	OC	*P*	*Q*
502	1	92.7	66.6	286	10	120	45
1	2	81.6	48.7	286	10		
2	3	526.7	185	286	10		
3	30	148	71.9	245	10	90	30
30	9	49.3	24	245	10		
9	31	148	71.9	138	10	90	30
9	75	49.3	24	138	10	30	15
9	8	78.9	38.3	245	10		
8	33	78.9	38.3	185	10	90	30
33	34	138.1	67.1	185	10	45	15
34	37	157.8	76.7	138	10	150	60
37	38	98.7	47.9	138	10	120	60
38	11	98.7	47.9	138	10		
11	40	79.8	28	138	10	90	45
11	41	98.7	47.9	138	10	45	15
2	13	88.8	43.1	245	10	90	30
13	4	128.2	62.3	185	10		
4	20	197.3	95.8	185	10	90	30
20	6	148	71.9	138	10		
6	25	111.7	39.2	138	10	45	30
20	7	367.1	128.9	138	10		
7	22	47.9	16.8	138	10	165	30
7	24	303.3	106.5	138	10	45	30
4	14	31.9	11.2	138	10	45	15
14	18	207.5	72.9	138	10	30	15
2	5	159.6	56	138	10		
5	12	95.8	33.6	138	10	90	30
5	42	127.7	44.8	138	10	90	45
3	27	95.8	33.6	245	10	45	30
27	44	69.1	33.5	245	10	45	15
44	28	79.8	28	138	10	120	60
44	29	111.7	39.2	245	10	45	15
8	32	127.7	44.8	245	10	45	15
34	10	207.5	72.9	138	10		
10	35	79.8	28	138	10	90	30
10	36	510.7	179.3	138	10	45	15
502	104	185.4	133.2	286	10		
104	20	0.1	0.1	185	10		
501	101	92.7	66.6	286	10		
101	29	0.1	0.1	245	100		
501	102	92.7	66.6	286	10		
102	32	0.1	0.1	286	10		

Legend: FB: from bus; TB: to bus; R: branch resistence (mΩ); X: branch reactance (mΩ); A: maximum branch ampacity (A); OC: cost of branch switching (c.u.); P: three-phase active power (kW) at TB;Q: three-phase reactive power (kvar) at TB

References

Aiex, R. M., Resende, M. G. C., & Ribeiro, C. C. (2007). TTT plots: A perl program to create time-to-target plots. *Optimization Letters*, *1*, 355–366.

Aoki, K., Nara, K., Itoh, M., & Satoh, T. (1989). A new algorithm for service restoration in distribution systems. *IEEE Transactions on Power Delivery*, *4*(3), 1832–1839.

Baran, M., & Wu, F. (1989). Network reconfiguration in distribution systems for loss reduction and load balancing. *IEEE Transactions on Power Delivery*, *4*(2), 1401–1407.

Chin, H. & Su Y.-S. (2005). Application of the ant-based network for power system restoration. In *Transmission and distribution conference and exhibition: Asia and Pacific* (pp. 1–5).

Delbem, A. C. B., Carvalho, A. DE, Policastro, C. A., Pinto, A. K., Honda, K. & Garcia, A. C. (2004). Node-depth encoding for evolutionary algorithms applied to network design. In *Genetic and evolutionary computation conference* (GECCO-2004) (pp. 678–687). Berlin.

Fukuyama, Y., & Chiang, H. (1995). A parallel genetic algorithm for service restoration. In *International joint conference of the fourth IEEE international conference on fuzzy systems and the second international fuzzy engineering symposium* (Vol. 1, pp. 275–282).

Glover, F. (1989). Tabu search—Part I. *ORSA Journal on Computing*, *1*(3), 190–206.

Glover, F. (1990). Tabu search—Part II. *ORSA Journal on Computing*, *2*(1), 4–32.

Glover, F., Laguna, M., & Marti, R. (2007). Principles of tabu search. In T. F. Gonzalez (Ed.), *Handbook of approximation algorithms and metaheuristics*. Boca Raton: Chapman & Hall/CRC.

IEEE. Distribution test feeders. http://ewh.ieee.org/soc/pes/dsacom/testfeeders/. Accessed 01 June 2015.

Kaewmanee, J., & Sirisumrannukul, S. (2011). Service restoration in distribution system using fuzzy decision algorithm and node-depth encoding (pp. 893–896).

Li, J., Ma, X.-Y., Liu, C.-C., & Schneider, K. P. (2014). Distribution system restoration with microgrids using spanning tree search. *IEEE Transactions on Power Systems*, *29*(6), 3021–3029.

Li, X. D., Xu, Y. Q., & Zhang, L. (2009). Distribution service restoration with DGs based on multi-agent immune algorithm. In *2nd international conference on power electronics and intelligent transportation system (PEITS)* (Vol. 2, pp. 1–4).

Liu, C., Lee, S. J., & Venkata, S. S. (1988). An expert system operational aid for restoration and loss reduction of distribution systems. *IEEE Transactions on Power Systems*, *3*(2), 619–626.

Mathias-Neto, W. P., Leao, F. B., & Mantovani, J. R. S. (2010). Distribution system restoration in a DG environment using a heuristic constructive multi-start algorithm. In *Transmission and distribution conference and exposition: Latin America (T D-LA)* (pp. 86–91).

Mohanty, I., Kalita, J., & Das, S. (2003) Ant algorithms for the optimal restoration of distribution feeders during cold load pickup. In *Swarm Intelligence Symposium* (pp. 132–137).

Mori, H., & Muroi, T. (2011). Application of probabilistic Tabu Search to distribution system service restoration. In *IEEE international symposium of circuits and systems (ISCAS)* (pp. 1037–1040).

Mori, H., & Ogita, Y. (2002). A parallel tabu search based approach to optimal network reconfigurations for service restoration in distribution systems. In *Proceedings of the international conference on control applications* (Vol. 2, pp. 814–819).

Nagata, T., & Sasaki, H. (2002). A multi-agent approach to power system restoration. *IEEE Transactions on Power Systems*, *17*(2), 457–462.

Nara, K. (1992). Implementation of genetic algorithm for distribution systems loss minimum re-configuration. *Transactions on Power Systems*, *7*(3), 1044–1051.

Sanches, D. (2013). Algoritmos Evolutivos Multi-Objetivo para Reconfiguraçao de Redes em Sistemas de Distribuiçao de Energia Elétrica. http://www.teses.usp.br/teses/disponiveis/18/18154/tde-26032013-080436/en.php. Accessed 01 Nov 2014.

Sanches, D. S., London Junior, J. B. A., & Delbem, A. C. B. (2014). Multi-objective evolutionary algorithm for single and multiple fault service restoration in large-scale distribution systems. *Electric Power Systems Research*, *110*, 144–153.

Santos, A. C., Delbem, A. C. B., London, J. B. A., & Bretas, N. G. (2010). Node-depth encoding and multiobjective evolutionary algorithm applied to large-scale distribution system reconfiguration. *IEEE Transactions on Power Systems*, *25*(3), 1254–1265.

Shirmohammadi, D. (1988). A compensation-based power flow method for weakly meshed distribution and transmission networks. *IEEE Transactions on Power Systems*, *3*(2), 753–762.

Toune, S., Fudo, H., & Genji, T. (1998) A reactive tabu search for service restoration in electric power distribution systems. In *Proceedings of IEEE international conference on evolutionary computation* (pp. 763–768).

Toune, S., Fudo, H., Genji, T., Fukuyama, Y., & Nakanishi, Y. (2002). Comparative study of modern heuristic algorithms to service restoration in distribution systems. *IEEE Transactions on Power Delivery*, *17*(1), 173–181.