Can we measure the $R\phi^2$ coupling parameter at tree-level scattering?

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The lowest order invariant amplitudes for the $2 \to 2$ processes concerning the action $S[g,\phi] = \int d^4x\sqrt{-g}[2R/\kappa^2 + \frac{1}{2}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \lambda R\phi^2)]$ are computed. It is found that these results do not depend on the value of λ; therefore, it is impossible to measure the $R\phi^2$ coupling parameter at tree-level scattering in this case. It is also shown that the theory described by the above action is equivalent at the classical level to Einstein's theory with a massless minimally coupled scalar field provided that $1 + (\lambda\kappa^2\phi^2/4) > 0$. Some consequences for cosmology and black holes are discussed.

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It has been suggested by various authors that the action for gravity should contain, in addition to the Einstein action, certain nonminimal functionals of the scalar field. There are many reasons for these suggestions: the necessity to soften the divergences of the stress tensor [1], the possibility of a theoretical explanation of Mach's principle [2], the incorporation of the spontaneous symmetry-breaking mechanism into gravity [3], the construction of nonsingular models for the Universe [4], the need to maintain renormalizability in quantum field theory in curved space [5], and so on [6].

The candidates for such nonminimally coupled actions contain, in general, a term of the form $\lambda R\phi^2$, which incidentally is the only possible local term involving a dimensionless coupling between the scalar field ϕ and the curvature scalar R [5]. Accordingly, here we shall concentrate our attention on theories described by the action functional

$$S[g,\phi] = \int d^4x\sqrt{-g}\left[\frac{2R}{\kappa^2} + \frac{1}{2}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \lambda R\phi^2)\right] , \tag{1}$$

with $\kappa^2 = 32\pi G$ in natural units. The Ricci tensor is defined by $R_{\mu\nu} = -\partial_\alpha\Gamma^\alpha_{\mu\nu} + \cdots$, and the metric convention is $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$.

Generally the coupling parameter λ in Eq. (1) is chosen to be zero (minimal coupling) or $-1/6$ (conformal coupling) for the sake of simplicity. Nevertheless, there is no *a priori* reason why it could not have any other real value. So, which value of λ should we use in our ordinary calculations? To answer this question a scheme must be devised which, at least in principle, allows us to gain a feel for this parameter λ. Since the $2 \to 2$ processes concerning action (1) for which λ will very likely enter the invariant amplitudes are only the scalar-scalar and graviton-scalar scattering, we shall calculate in the following the lowest order invariant amplitudes for these processes. Of course, there is no need to consider the annihilation of scalars into gravitons since, as is well known, the invariant amplitude for this process can be promptly inferred from the one concerning the graviton-scalar scattering.

The usual procedure in quantum gravity for obtaining the vertex functions in the absence of fermions is to start with the action functional describing the matter fields and to write the metric tensor $g_{\mu\nu}(x)$ as

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x) , \tag{2}$$

where $\eta_{\mu\nu}$ is the flat space metric and $h_{\mu\nu}(x)$ is the graviton field. Consequently, the Feynman rules for scalar-graviton interactions are obtained from the action for a gravitational nonminimally coupled scalar field:

$$S_S = \int d^4x\sqrt{-g}\frac{1}{2}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2 + \lambda R\phi^2] ,$$

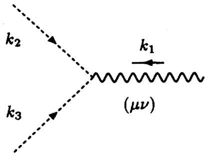
expanding around flat space using Eq. (2). This leads to

$$\begin{aligned}S_S = \int d^4x\Big\{&\frac{\kappa}{4}[h(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2) - 2h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2\lambda(\Box h - \partial_\alpha\partial_\mu h^{\alpha\mu})\phi^2]\\
&+\frac{\kappa^2}{16}[(h^2 - 2h^\alpha_\beta h^\beta_\alpha)(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2) + 4(2h^\mu_\alpha h^{\alpha\nu} - hh^{\mu\nu})\partial_\mu\phi\partial_\nu\phi\\
&+\lambda\phi^2(16h^{\alpha\beta}\eta^{\mu\nu}\partial_\nu\partial_\beta h_{\alpha\mu} + 8\partial_\alpha h^{\alpha\beta}\partial_\mu h^\mu_\beta - 8h^{\alpha\beta}\partial_\alpha\partial_\beta h - 8h^{\alpha\beta}\Box h_{\alpha\beta} - 8\partial_\alpha h\eta^{\mu\nu}\partial_\nu h^\alpha_\mu\\
&-6\eta^{\mu\nu}\partial_\nu h^{\alpha\beta}\partial_\mu h_{\alpha\beta} + 4\eta^{\mu\nu}\partial_\nu h^{\alpha\beta}\partial_\beta h_{\alpha\mu} + 4h\Box h - 4h\partial_\alpha\partial_\mu h^{\alpha\mu} + 2\eta^{\alpha\beta}\partial_\alpha h\partial_\beta h)]\Big\} ,\end{aligned}$$

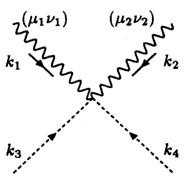
where the action for the free scalar field has been omitted.

From the previous expression the Feynman rules for the elementary vertices may readily be deduced. These are shown in Fig. 1.

Let us now analyze the gravitational scattering of identical massive scalars with arbitrary $R\phi^2$ coupling. The Feynman diagrams for the process $\phi(p_1)\phi(p_2) \to \phi(p_3)\phi(p_4)$ in lowest order are displayed in Fig. 2, and



$$\frac{ik}{2}[-\eta_{\mu\nu}(k_2\cdot k_3+m^2+2\lambda k_1{}^2)+k_{2\mu}k_{3\nu}+k_{3\mu}k_{2\nu}+2\lambda k_{1\mu}k_{1\nu}]$$



$$\begin{aligned}\frac{ik^2}{4}\{&(k_3\cdot k_4+m^2)(\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}+\eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2})\\&-\eta_{\nu_1\mu_2}[k_{3\mu_1}k_{4\nu_2}+k_{4\mu_1}k_{3\nu_2}+\tfrac{\lambda}{2}(k_{1\nu_2}k_{2\mu_1}+k_{1\mu_1}k_{2\nu_2})]\\&-\eta_{\nu_1\nu_2}[k_{3\mu_1}k_{4\mu_2}+k_{4\mu_1}k_{3\mu_2}+\tfrac{\lambda}{2}(k_{1\mu_2}k_{2\mu_1}+k_{1\mu_1}k_{2\mu_2})]\\&-\eta_{\mu_1\mu_2}[k_{3\nu_1}k_{4\nu_2}+k_{4\nu_1}k_{3\nu_2}+\tfrac{\lambda}{2}(k_{1\nu_2}k_{2\nu_1}+k_{1\nu_1}k_{2\nu_2})]\\&-\eta_{\mu_1\nu_2}[k_{3\nu_1}k_{4\mu_2}+k_{4\nu_1}k_{3\mu_2}+\tfrac{\lambda}{2}(k_{1\mu_2}k_{2\nu_1}+k_{1\nu_1}k_{2\mu_2})]\\&+\lambda[2(k_1^2+k_2^2)+3k_1\cdot k_2](\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}+\eta_{\mu_1\nu_2}\eta_{\nu_1\mu_2})\}\end{aligned}$$

FIG. 1. Feynman rules for scalar-graviton interactions with arbitrary $R\phi^2$ coupling. The wavy lines represent gravitons, and the dashed lines stand for scalars. In the calculation concerning the quadrilinear vertex we have made use of the fact that $\partial_\nu h^{\mu\nu}=0$ and $h=0$, since we are only interested in physical gravitons.

the corresponding invariant amplitude is given by

$$\begin{aligned}|\mathcal{M}_{\phi\phi\to\phi\phi}|^2=\frac{\kappa^4}{64}\Bigg[&\frac{t^2+u^2}{s}+\frac{s^2+u^2}{t}+\frac{s^2+t^2}{u}\\&-12m^4\left(\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right)+8m^2\\&-8\lambda m^2(5+6\lambda)\Bigg]^2,\end{aligned}\tag{3}$$

where s, t, and u are the usual Mandelstam variables. It is clear that in the massless limit, which is the case for the action (1), the invariant amplitude becomes totally independent of the parameter λ. Equation (3) with $\lambda=0$ agrees with the result of Peet [7].

The evaluation of the invariant amplitude for scalar-graviton scattering is rather involved. The corresponding Feynman diagrams are shown in Fig. 3. We made use of the formalism of Refs. [8,9] to calculate the helicity amplitudes in this case, obtaining

$$\begin{aligned}|\mathcal{M}_{\phi g\to\phi g}(++)|^2&=|\mathcal{M}_{\phi g\to\phi g}(--)|^2\\&=\frac{\kappa^4}{16}\frac{(su-m^4)^4}{t^2(s-m^2)^2(u-m^2)^2},\\|\mathcal{M}_{\phi g\to\phi g}(+-)|^2&=|\mathcal{M}_{\phi g\to\phi g}(-+)|^2\\&=\frac{\kappa^4m^8}{16}\frac{t^2}{(s-m^2)^2(u-m^2)^2},\end{aligned}$$

where the $\pm$ signs refer to the initial and final graviton helicities. We can see that even for massive scalars the invariant amplitude does not depend on λ. Our result agrees with the one given by Berends and Gastmans [10], who have computed the elastic scattering of gravitons on massive scalars for gravitational minimally coupled scalars ($\lambda=0$).

Thus, we come to the unexpected conclusion that the tree-level invariant amplitudes for the $2\to 2$ processes concerning action (1) ($\phi-\phi$ and $\phi-g$ scattering) are completely independent of the $R\phi^2$ coupling parameter, which directly implies that this parameter cannot be measured at tree-level scattering in this case.

The preceding semiclassical result strongly suggests that there ought to be transformations which eliminate the nonminimal coupling from action (1) at no expense. Accordingly, we introduce the following reparametrization in the aforementioned action:

$$\bar{g}_{\mu\nu}(x)=\left(1+\frac{\lambda\kappa^2}{4}\phi^2\right)g_{\mu\nu}(x),$$

where $1+\frac{\lambda\kappa^2}{4}\phi^2$ is supposed to be positive. Then we

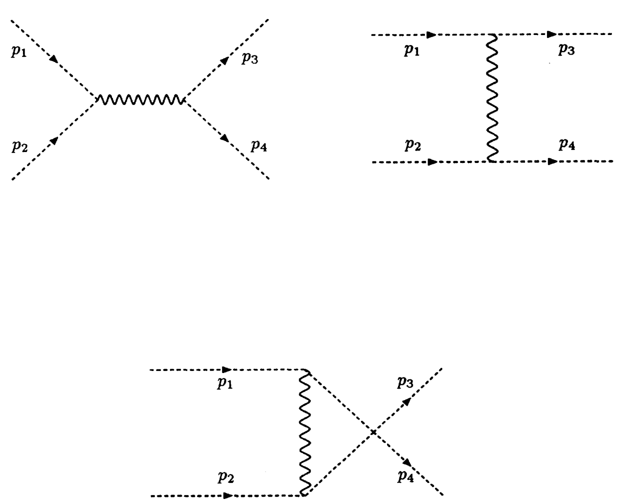


FIG. 2. Feynman diagrams for scalar-scalar scattering.

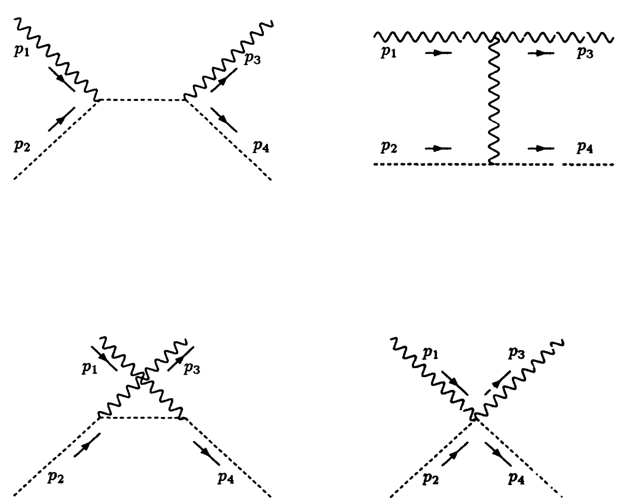


FIG. 3. Feynman diagrams for the process $g\phi\to g\phi$.

have

$$S[g,\phi] = \int d^4x\sqrt{-\bar{g}}\left[\frac{2\bar{R}}{\kappa^2} + \frac{1}{2}F(\phi)\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] , \quad (4)$$

where

$$F(\phi) \equiv \frac{1 + \frac{\lambda\kappa^2}{4}(1+6\lambda)\phi^2}{(1+\frac{\lambda\kappa^2}{4}\phi^2)^2} .$$

The nonlinearity with respect to the scalar field ϕ may, by means of a further field redefinition, be removed from (4). Indeed, from the differential relation

$$\bar{g}^{\mu\nu}(\sqrt{F}\partial_\mu\phi)\sqrt{F}\partial_\nu\phi = \bar{g}^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\phi} , \quad (5)$$

where $\bar{\phi}(\phi)$ is a new scalar field, we easily obtain the equation

$$\frac{d\bar{\phi}(\phi)}{d\phi} = \sqrt{F} ,$$

which can be trivially integrated with the help of the boundary condition $\bar{\phi}(\phi)|_{\phi=0} = 0$. Hence, Einstein's theory with a nonminimally coupled scalar field ϕ and Einstein's theory with a minimally coupled scalar field $\bar{\phi}$,

$$S[g,\phi] = \int d^4x\sqrt{-\bar{g}}\left[\frac{2\bar{R}}{\kappa^2} + \bar{g}^{\mu\nu}\partial_\mu\bar{\phi}\partial_\nu\bar{\phi}\right] ,$$

which are usually considered as totally different, are actually one and the same theory provided that $1+\lambda\kappa^2\phi^2/4 > 0$. What about the physical significance of this constraint? To answer this question we write the field equations obtained by variation of the action (1) in the suggestive form

$$G^{\mu\nu} = \mathcal{H}[\lambda(g^{\mu\nu}\Box - \nabla^\mu\nabla^\nu)\phi^2 + \tfrac{1}{2}g^{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi - \partial^\mu\phi\partial^\nu\phi] , \quad (6a)$$

$$\Box\phi - \lambda R\phi = 0 , \quad (6b)$$

where

$$\mathcal{H} \equiv \frac{\kappa^2}{1+\frac{\lambda\kappa^2\phi^2}{4}}$$

is an effective Einstein constant which will be positive if and only if $1 + \lambda\kappa^2\phi^2/4 > 0$. Consequently, the adoption of the above-mentioned constraint leads to Einstein's standard formula $G^{\mu\nu} = \mathcal{H}T^{\mu\nu}$, where $T^{\mu\nu}$ is given by the right-hand side of Eq. (6a) and $\mathcal{H} > 0$. It is worth mentioning, in passing, that the problem concerning the classical equivalence of $\lambda R\phi^2$ theories has been discussed by some authors in the last few years, although in a different context from the one in hand [11].

To determine the cosmological effect of the $\lambda R\phi^2$ term for $\mathcal{H} > 0$, we may study the behavior of (6), with $\lambda = 0$, on a Friedmann-Robertson-Walker (FRW) metric. Using dynamical system techniques, for instance, it is easy to show that the resulting solutions reach a singularity in a finite proper time, as is expected from the singularity theorems of Hawking and Penrose, since the ordinary scalar field obeys the basic assumption of these theorems. Bekenstein's bouncing universe [4], which was examined by Deng and Mannheim [4], is no exception to this rule: in fact, the value of the effective Einstein constant for this Einstein-conformal scalar solution ($\lambda = -\frac{1}{6}$) is negative ($\mathcal{H} < 0$). Thus, all FRW solutions concerning Eq. (6) are singular for $\mathcal{H} > 0$.

Since there are no static black holes in the framework of Einstein-ordinary scalar field theory [12], we can promptly infer that $\lambda R\phi^2$ black holes have no hair provided that $\mathcal{H} > 0$. It is straightforward to show that $\mathcal{H} < 0$ for Bekenstein's black hole with a scalar charge [4,13]. Incidentally, this black hole was proved to be unstable under monopole perturbations [14].

We believe that the semiclassical result presented in this Brief Report could be generalized in order to take into account quantum corrections.

Finally, we would like to mention the fact that a cosmological constant, a scalar self-interaction, or even other matter would reveal the $R\phi^2$ coupling.

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[1] C. G. Callan, S. Coleman, and R. Jackiw, Ann. Phys. (N.Y.) **59**, 42 (1970).
[2] E. A. Lord, *Tensors, Relativity and Cosmology* (McGraw-Hill, New Delhi, India, 1976).
[3] A. Zee, Phys. Rev. Lett. **42**, 417 (1979); A. J. Accioly, Lett. Nuovo Cimento **44**, 48 (1985); A. J. Accioly and B. M. Pimentel, Can. J. Phys. **68**, 1183 (1990).
[4] J. D. Bekenstein, Ann. Phys. (N.Y.) **82**, 535 (1974); Y. Deng and P. D. Mannheim, Astrophys. J. **324**, 1 (1988).
[5] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Spaces* (Cambridge University Press, Cambridge, England, 1982).
[6] L. Amendola, Phys. Lett. B **301**, 175 (1993), and references therein.
[7] F. G. Peet, Can. J. Phys. **48**, 923 (1970).
[8] S. F. Novaes and D. Spehler, Phys. Rev. D **44**, 3990 (1991).
[9] R. Aldrovandi, S. F. Novaes, and D. Spehler, Gen. Relativ. Gravit. **26**, 409 (1994).
[10] F. A. Berends and R. Gastmans, Nucl. Phys. **B88**, 99 (1975).
[11] T. Futamase and K. Maeda, Phys. Rev. D **39**, 399 (1989); H. J. Schmidt, Phys. Lett. B **214**, 519 (1988); A. J. Accioly, U. F. Wichoski, S. F. Kwok, and N. L. Pereira da Silva, Class. Quantum Grav. **10**, L215 (1993).
[12] J. E. Chase, Commun. Math. Phys. **19**, 276 (1970); A. G. Agnese and M. La Camera, Lett. Nuovo Cimento **35**, 365 (1982); Phys. Rev. D **31**, 1280 (1985).
[13] J. D. Bekenstein, Ann. Phys. (N.Y.) **91**, 75 (1975).
[14] K. A. Bronnikov and Yu. N. Kireyev, Phys. Lett. **67A**, 95 (1978).