Podolsky's higher-order electromagnetism from first principles

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Summary. — Podolsky's higher-order field equations are obtained by generalizing the laws of Podolsky's electrostatics, which follow from Coulomb's generalized law and superposition, to be consistent with special relativity. In addition, it is necessary to take into account the independence of the observed charge of a particle on its speed. It is also shown that the gauge-independent term concerning the Feynman propagator for Podolsky's generalized electrodynamics has a good ultraviolet behaviour at the expense of a negative metric massive ghost which, contrary to what is currently assumed in the literature, is non-tachyonic. A brief discussion on Podolsky's characteristic length is presented as well.

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1. – Introduction

As is well known, any attempt to derive Maxwell equations from Coulomb's law and special relativity ends in failure unless additional assumptions are made [1, 2]. Suppose then we know what these assumptions are (to be more specific, *the principle of superposition* and *the independence of the observed charge on its speed*); suppose also that we are well acquainted with the theory of special relativity. Assume, furthermore, that instead of the usual Coulomb's law the force law for the electrostatic interaction between two point charges Q and Q' is given in Heaviside-Lorentz units with c replaced by unit by

$$\mathbf{F}(\mathbf{r}) = \frac{QQ'}{4\pi}\left[\frac{1-e^{-r/a}}{r^2} - \frac{e^{-r/a}}{ra}\right]\frac{\mathbf{r}}{r}\,, \tag{1}$$

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where a is a real constant with dimension of length. For short, we shall call this force law Coulomb's generalized law, which is nothing but the force law of electrostatics developed by Podolsky and co-authors [3-6]. Incidentally, Podolsky's generalized electrodynamics leads to results free of infinities usually associated with a point source. The question is: can we derive Podolsky's higher-order field equations using the afore-mentioned assumptions simply and solely? Our aim here is to show that the answer to this question is affirmative.

The discussion is divided broadly into two parts. The first develops Podolsky's electrodynamics from first principles. Starting from Coulomb's generalized law and assuming the validity of the principle of superposition, two equations are obtained that make up Podolsky's electrostatics. Taking for granted the invariance of electric charge under Lorentz transformations the previous equations are generalized so that they are consistent with special relativity. As a result, Podolsky's higher-order field equations are obtained (sect. **2**).

In the second part we concentrate our attention on the inverse problem, namely, we show that (1) is indeed the force law for Podolsky's electrostatics. A Lagrangian is found for Podolsky theory and starting from it we calculate the effective nonrelativistic potential for the interaction of two charged bosons of equal mass via a «Podolskian photon» exchange. Equation (1) is then promptly recovered from this potential (sect. **3**).

In sect. **4** a brief discussion concerning Podolsky's characteristic length a is presented. We use natural units throughout.

2. – Podolsky's higher-derivative field equations

From (1) it follows immediately that the electric field at the position $\mathbf{r}$ due to a charge Q at the origin of the radius vector is

$$\mathbf{E}(\mathbf{r}) = \frac{Q}{4\pi}\left[\frac{1-e^{-r/a}}{r^2} - \frac{e^{-r/a}}{ra}\right]\frac{\mathbf{r}}{r}\ .$$

By Coulomb's generalized law and the principle of superposition the electric field due to a static electric charge density $\varrho(\mathbf{r})$ is

$$\mathbf{E}(\mathbf{r}) = \int \mathrm{d}^3 r'\ \frac{\varrho(\mathbf{r}')}{4\pi}\left[\frac{1-e^{-R/a}}{R^2} - \frac{e^{-R/a}}{Ra}\right]\frac{\mathbf{R}}{R}\ , \tag{2}$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$. By taking the curl of (2) we arrive at the familiar result of Maxwell electrostatics,

$$\nabla \times \mathbf{E}(\mathbf{r}) = 0\ , \tag{3}$$

which tells us that Podolsky's electrostatic field is conservative as well. Some more algebra and we have

$$(1 - a^2\nabla^2)\ \nabla\cdot\mathbf{E}(\mathbf{r}) = \varrho(\mathbf{r})\,, \tag{4}$$

whereupon use has been made of the identity

$$\nabla^2_{\mathbf{r}}\ \frac{e^{-R/a}}{R} = \frac{e^{-R/a}}{a^2 R} - 4\pi\delta^3(\mathbf{R})\,.$$

Equations (3) and (4) make up Podolsky's electrostatics.

We are now led to pose the question: had a static field theory, namely, Podolsky's electrostatics, and special relativity been all that was initially known, could Podolsky's equations have been predicted? We will show that this is so, provided that the independence of the observed charge of a particle on its speed is taken into account.

The results of several experiments can be used to support the invariance of electric charge under Lorentz tranformation or, to be more specific, the independence of the observed charge of a particle on its speed. In Bartlett and Ward's experiments [7], only to mention an example, it is assumed that the charge of a slow electron (or proton) varies as $q = e(1 + kv^2)$, where $|k| \ll 1$. A limit of $|k| < 2 \times 10^{-16}$ is then inferred for bound electrons from existing data on the neutrality of various atoms. Therefore we take for granted the invariance of electric charge under Lorentz transformations, which leads us to conclude promptly that ϱ transforms like the zero-component of a four-vector, in this case the electric charge-density current four-vector $j^\mu = (\varrho, \mathbf{j})$. Greek indices range from 0 to 3 and Latin indices from 1 to 3. Moreover, as was shown by Feynman [8] using an argument by Einstein, *if anything is conserved, it must be conserved locally*. Thus, electric charge is conserved locally, which implies the equation of continuity for electric charge

$$\partial_\mu j^\mu = 0 . \tag{5}$$

If we define the quantities $F^{0i} = -F_{0i} = E^i = -E_i$, eqs. (3) and (4) can be rewritten as

$$\varepsilon^{jkl}\, \partial_k F_{0l} = 0 , \tag{6}$$

$$(1 + a^2 \eta^{ij} \partial_i \partial_j)\, \partial_k F^{0k} = j^0 . \tag{7}$$

Here ε^{mnl} is the Levi-Civita density with $\varepsilon^{123} = +1$ and $\eta^{\mu\nu}$ is the contravariant metric tensor which is diagonal and has the components $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = +1$ in our convention. We demand now invariance of eqs. (6) and (7) under Lorentz transformations. It follows then that

$$\varepsilon^{\mu\nu\alpha\beta}\, \partial_\nu F_{\alpha\beta} = 0 , \tag{8}$$

$$(1 + a^2 \Box)\, \partial_\nu F^{\mu\nu} = j^\mu , \tag{9}$$

where $\varepsilon^{\mu\nu\alpha\beta}$ is a completely antisymmetric tensor of rank four with $\varepsilon^{0123} = +1$ and $\Box \equiv \eta^{\mu\nu} \partial_\mu \partial_\nu$. Note that the F^{0i} are not necessarily static anymore and we do not know yet what the other components of $F^{\mu\nu}$ are. Let us then probe and find out whatever symmetries the $F^{\mu\nu}$ may have. From eqs. (4) and (9) we obtain

$$(1 + a^2 \Box)\, \partial_\mu \partial_\nu F^{\mu\nu} = \partial_\mu j^\mu \; . \tag{10}$$

If $F^{\mu\nu}$ is an antisymmetric tensor, then eq. (10) is identically zero. Obviously, $F^{\mu\nu} = -F^{\nu\mu}$ is the simplest solution of (10), which does not mean that it is the only one. On the other hand, the equation of motion for a particle of charge q and mass m in an electrostatic field can be obtained by writing Newton's second law as

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = q\mathbf{E}(\mathbf{r}) .$$

The covariant generalization of this equation is

$$\frac{\mathrm{d}p^\nu}{\mathrm{d}\tau} = qF^{\mu\nu} u_\mu , \tag{11}$$

where τ is the proper time and u^μ the four-velocity. From (11) it follows that

$$F_{\mu\nu} u^\mu u^\nu = 0 \; .$$

Using the previous result, it can be shown that $F^{\mu\nu}$ is antisymmetric [9].

We name therefore the quantities

$$F^{12} = B^3 \, , \qquad F^{13} = -B^2 \, , \; F^{23} = B^1 \; .$$

So, a shrewd physicist who only knew Podolsky's electrostatics and special relativity could predict the existence of the magnetic field **B**, which naturally still lacks physical interpretation. The content of eqs. (8) and (9) can now be seen. For $\mu = 0$, eq. (8) gives

$$\nabla \cdot \mathbf{B} = 0 \, , \tag{12}$$

showing that there are no magnetic monopoles in Podolsky's electrodynamics; while for $\mu = i$ we obtain

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \, , \tag{13}$$

which says that time-varying magnetic fields can be produced by **E** fields with circulation.

The components $\mu = 0$ and $\mu = i$ of eq. (9) give, respectively,

$$(1 + a^2 \Box) \, \nabla \cdot \mathbf{E} = \varrho \, , \tag{14}$$

$$(1 + a^2 \Box) \left[\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} \right] = \mathbf{j} , \tag{15}$$

which for $a = 0$ yield Gauss and Ampère-Maxwell laws in this order. For $\nu = i$, eq. (11) becomes

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \, , \tag{16}$$

which is nothing but Newton's second law with the Lorentz force on the electric charge. For $\nu = 0$, eq. (11) assumes the form

$$\frac{\mathrm{d}U}{\mathrm{d}t} = q \mathbf{v} \cdot \mathbf{E} \, ,$$

where $U = p^0$ is the particle's energy. Hence, our aforementioned clever physicist, who was able to predict the **B** field only from his knowledge of electrostatics and special relativity, can now—by making judicious use of eqs. (16) and (17)—observe, measure and distinguish the **B** field from the **E** field. The new field couples to moving electric charge, does not act on a static charged particle and, unlike the electrostatic field, is capable only of changing the particle's momentum direction.

Equations (12)-(15) make up Podolsky's higher-order field equations.

Of course, in the limit $a = 0$, all the preceding arguments apply equally well to Maxwell's theory.

3. – The effective nonrelativistic potential

We show now that (1) is indeed the force law for Podolsky's electrostatics. To begin with we find a Lagrangian for Podolsky's electrodynamics.

It can be shown that if the Euler form E related to a given set of field equations is an exact local form, then a Lagrangian exists for those field equations [10, 11]. In the case of Podolsky's electrodynamics the corresponding Euler form turns out to be

$$E = [(1 + a^2 \Box)\, \partial_\nu F^{\mu\nu} - j^\mu]\, \delta A_\mu\,, \tag{17}$$

whereupon the relation of $F^{\mu\nu}$ to the vector potential A^μ is $F^{\mu\nu} = A^{\mu,\nu} - A^{\nu,\mu}$. From (17) we obtain

$$\delta E = -\frac{1}{2}\,\delta F^{\mu\nu} \wedge \delta F_{\mu\nu} + \frac{a^2}{2}\,\delta(\partial_\nu F^{\mu\nu}) \wedge \delta(\partial^\alpha F_{\mu\alpha}) = 0\,,$$

saying that E is an exact form. Once assured of the existence of the Lagrangian, an expression for it can be easily found using the algorithm [10, 11]

$$\mathscr{L} = \int_1^0 \mathrm{d}\lambda\, E_\mu(\lambda\phi)\, \phi^\mu\,,$$

where $\phi = (\phi^1, \phi^2, \ldots, \phi^N)$ stands for a set of fields which are supposed to be defined on the space-time.

In the case in hand,

$$\int_0^1 \mathrm{d}\lambda [\lambda(\partial_\nu F^{\mu\nu} + a^2 \Box \partial_\nu F^{\mu\nu})\, A_\mu - j^\mu A_\mu]$$

gives, after integration and a suitable antisymmetrization,

$$\mathscr{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{a^2}{2}\, \partial_\nu F^{\mu\nu} \partial^\alpha F_{\mu\alpha} - j^\mu A_\mu$$

which is the Lagrangian originally proposed by Podolsky.

Let us then compute the effective nonrelativistic potential for the interaction of two spinless charged bosons of equal mass via a "Podolskian photon" exchange, which will allow us to recover eq. (1).

The expression for the nonrelativistic potential is

$$U(\mathbf{r}) = \frac{1}{4m^2}\, \frac{1}{(2\pi)^3} \int \mathrm{d}^3\mathbf{k}\, \mathscr{T}_{\mathrm{NR}}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\,, \tag{18}$$

whereupon $\mathscr{T}_{\mathrm{NR}} = i\, \mathscr{M}_{\mathrm{NR}}$, where $\mathscr{M}_{\mathrm{NR}}$ is the nonrelativistic limit of the Feynman amplitude for the process $S + S \rightarrow S + S$; here S stands for a spinless boson of mass m and charge Q. The corresponding Feynman diagram is shown in fig. 1.

In the Lorentz gauge Podolsky's scalar QED is described by the Lagrangian

$$\mathscr{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{a^2}{2}\, \partial_\nu F^{\mu\nu} \partial^\alpha F_{\mu\alpha} - \frac{1}{2\lambda}(\partial_\mu A^\mu)^2 + (D_\mu \phi)^* D^\mu \phi - m^2 \phi^* \phi\,, \tag{19}$$

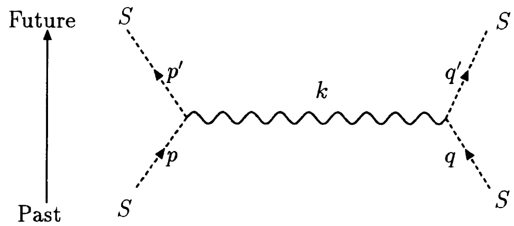

Fig. 1. – Lowest-order contribution to the reaction $S+S \to S+S$, where S stands for a spinless charged boson.

where

$$D_\mu \phi = \partial_\mu \phi + iQA_\mu \phi\ .$$

The free propagator for the field A^μ is fixed by the quadratic part of the above Lagrangian. In momentum space we have

$$D^{\mathrm{F}}_{\mu\nu}(k) = \frac{iM^2}{k^2(k^2 - M^2 + i\varepsilon)}\left[\eta_{\mu\nu} - \frac{1-\lambda(1-k^2/M^2)}{k^2}k_\mu k_\nu\right], \tag{20}$$

with $M^2 \equiv 1/a^2$.

Note that the gauge-independent term of (20) has a good ultraviolet behaviour ($\approx k^{-4}$) at the expense of a negative metric massive ghost which, contrary to what is currently assumed in the literature [12, 14], is a non-tachyonic ghost. Of course, the gauge-dependent term of (20) cancels out in the computation of physical quantities.

The non-quadratic part of the Lagrangian (19) defines the interaction vertices. However, since our calculation will be restricted to trilinear vertices, the interaction Lagrangian reduces to

$$\mathscr{L}_{\mathrm{inter}} = iQA^\mu(\phi\partial_\mu \phi^* - \phi^* \partial_\mu \phi)\,.$$

From the previous expression the Feynman rule for the elementary vertex may readily be deduced. This is shown in fig. 2.

Accordingly, the invariant amplitude for the process shown in fig. 1 is

$$\mathscr{M} = \frac{-iM^2Q^2}{k^2(k^2-M^2)}(2p-k)(2q+k)\,,$$

which in the nonrelativistic limit becomes

$$\mathscr{M}_{\mathrm{NR}} = -4i\,\frac{M^2Q^2m^2}{\mathbf{k}^2(\mathbf{k}^2+M^2)}\ .$$

The effective nonrelativistic potential is then given by

$$U(\mathbf{r}) = \frac{Q^2}{4\pi}\left(\frac{1-e^{-Mr}}{r}\right), \tag{21}$$

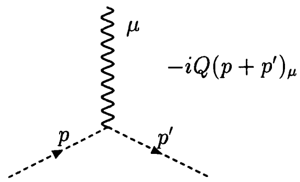

Fig. 2. – Elementary vertex for Podolsky's scalar electrodynamics of a spinless boson with charge Q.

from which it follows immediately the expression for Coulomb's generalized potential

$$V(\mathbf{r}) = \frac{Q}{4\pi}\left(\frac{1-e^{-r/a}}{r}\right). \tag{22}$$

Note that this potential approaches $Q/4\pi a$ as r approaches zero. It is a trivial task to derive (1) from (22). Thus, (1) is indeed the force law for Podolsky's electrostatics.

4. – On Podolsky's characteristic length a

Since Podolsky's equations are a generalization of Maxwell's equations, in the limit of small field strengths, Podolsky's Lagrangian

$$\mathscr{L} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{a^2}{2}[(\nabla\cdot\mathbf{E})^2 - (\nabla\times\mathbf{B} - \dot{\mathbf{E}})^2]$$

has to approach

$$\mathscr{L}^{(0)} = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2),$$

which is the Lagrangian leading to Maxwell's equations. Thereby, in practice Podolsky's characteristic length must be small. Let us then analyse the influence of this parameter on well-known results of Maxwell's theory. To begin with, we compute the flux of the electric field across a spherical surface of radius R with a charge Q at its center. The result is

$$\oint \mathbf{E}\cdot d\mathbf{S} = \begin{cases} 0, & R \ll a, \\ Q, & R \gg a. \end{cases}$$

A comparison of this result with the usual Maxwell one, clearly shows that the electrostatic flux would be modified only at short distances.

Recently, it was shown that at the tree level massive fermions have their helicity flipped on account of their interaction with an electromagnetic field described by Podolsky's generalized electrodynamics; massless fermions, in turn, seem to be unaffected by the electromagnetic field as far as their helicity is concerned [15]. So, at the tree level these results are the same as those of Maxwell's theory [16].

If one computes in the framework of Podolsky's theory the anomalous magnetic moment of the electron, to one-loop order, and takes into account that the experimental results for this quantity agree extremely well with the theoretically predicted values in

QED [17], one finds that Podolsky's characteristic length has an upper bound [18]

$$a_0 \approx 4.72 \times 10^{-16} \text{ cm} .$$

This length characterizes in a phenomenological way a possible modification of QED at short distances. It is interesting to note that a_0 is of the order of magnitude of the Compton wavelength of the neutral vector boson Z, $\lambda(Z) \approx 2.16 \times 10^{-16}$ cm, which mediates the unified weak and electromagnetic interactions [19].

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