

Supersymmetric 3-3-1 model with right-handed neutrinos

J. C. Montero* and V. Pleitez†

Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona, 14501405-900 São Paulo, SP, Brazil

M. C. Rodriguez‡

*Fundação Universidade Federal do Rio Grande/FURG, Departamento de Física,
Avenida Itália, km 8, Campus Carreiros, 96201-900, Rio Grande, RS, Brazil*

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We consider the supersymmetric extension of the 3-3-1 model with right-handed neutrinos. We study the mass spectra in the scalar and pseudoscalar sectors, and for a given set of the input parameters, we find that the lightest scalar in the model has a mass of 130 GeV and the lightest pseudoscalar has mass of 5 GeV. However, this pseudoscalar decouples from the Z^0 at high energy scales since it is almost a singlet under $SU(2)_L \otimes U(1)_Y$.

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I. INTRODUCTION

Models with $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ gauge symmetry (called 3-3-1 models for short) are interesting possibilities for the physics at the TeV scale [1–3]. At low energies they coincide with the standard model and some of them give at least partial explanation to some fundamental questions that are accommodated but not explained by the standard model. For instance, i) in order to cancel the triangle anomalies, together with asymptotic freedom in QCD, the model predicts that the number of generations must be three and only three; (ii) the model of Ref. [1] predicts that $(g'/g)^2 = \sin^2\theta_W/(1 - 4\sin^2\theta_W)$, thus there is a Landau pole at the energy scale μ at which $\sin^2\theta_W(\mu) = 1/4$. According to recent calculations $\mu \sim 4$ TeV [4,5]; (iii) the quantization of the electric charge [6] and the vectorial character of the electromagnetic interactions [7] do not depend on the nature of the neutrinos, i.e., if they are Dirac or Majorana particles; (iv) as a consequence of item ii) above, the model possesses perturbative $\mathcal{N} = 1$ supersymmetry naturally at the μ scale [8,9]; (v) the Peccei-Quinn [10] symmetry occurs also naturally in these models [11]; (vi) since one generation of quarks is treated differently from the others this may be lead to a natural explanation for the large mass of the top quarks [12]. Moreover, if right-handed neutrinos are considered transforming nontrivially [3], 3-3-1 models [1,2] can be embedded in a model with 3-4-1 gauge symmetry in which leptons transform as $(\nu_l, l, \nu_l^c, l^c)_L \sim (1, 4, 0)$ under each gauge factor [13]. The $SU(3)_L$ symmetry is possibly the largest symmetry involving the known leptons (and $SU(4)_L$ if right-handed neutrinos do really exist). This make 3-3-1 or 3-4-1 models interesting by their own.

Models with $SU(3)$ (or $SU(4)$) electroweak symmetry may have doubly charged vector bosons. These sort of bileptons may be detected by measuring several left-right asymmetries in Møller scattering [14] or, in future lepton-lepton accelerators. It is interesting that one of these parity nonconserving asymmetry was observed in Møller scattering for the first time only recently, and its value is, at present, in agreement with the standard model prediction [15]. However, in the future these asymmetries in e^-e^- colliders [14] may also be used for searching doubly charged particles and a heavy neutral Z'^0 vector bosons, which is also a prediction of these models, may be discovered in $e^-\mu^-$ colliders [16]. Singly and doubly charged vector bileptons may also be produced in $e^-\gamma$ [17] or $\gamma\gamma$ [18] or hadron [19] colliders. New heavy quarks are also part of the electroweak quark multiplets in the minimal model representation. They are singlets under the standard model $SU(2)_L \otimes U(1)_Y$ group symmetry and in some versions of the 3-3-1 models the electric charges of these heavy quarks are different from the usual ones, so that it can be used to distinguish such models from their viable competitors. In fact, the $p\bar{p}$ production of these exotic quarks at the energies of the Tevatron have been studied in Ref. [20] where a lower bound of 250 GeV on their masses was found. This sort of models are also predictive with respect to neutrino masses [21]: they can implement the large mixing angle MSW solution to the solar neutrino issue [22], and also the almost bimaximal mixing matrix in the lepton sector [23].

Gauge models based on the 3-3-1 gauge symmetry can have several representation contents [1,3] depending on the embedding of the charge operator in the $SU(3)_L$ generators,

$$\frac{Q}{e} = \frac{1}{2}(\lambda_3 - \vartheta\lambda_8) + NI, \quad (1)$$

where the ϑ parameter defines two different representation contents, N denotes the $U(1)_N$ charge and λ_3, λ_8 are

*Electronic address: montero@ift.unesp.br

†Electronic address: vicente@ift.unesp.br

‡Electronic address: mcrodriguez@fisica.furg.br

the diagonal generators of $SU(3)$. The supersymmetric version of the model of Ref. [1], with $\vartheta = \sqrt{3}$, has already been considered in Refs. [8,9]. In this work we build the supersymmetric model based on the representation content of the 3-3-1 model of Refs. [3] which corresponds to the case when $\vartheta = 1/\sqrt{3}$.

The outline of this paper is as follows. In Sec. II we review the nonsupersymmetric 3-3-1 model with right-handed neutrinos, introduce the respective superpartners and the superfields. The Lagrangian of this 3-3-1 supersymmetric model, including the soft term, is considered in Sec. III; the scalar potential and the scalar mass spectra are given in Sec. IV. Finally, the last section is devoted to our conclusions.

II. THE MODEL

In this section (Sec. II A) we review the nonsupersymmetric 3-3-1 model with right-handed neutrinos of Refs. [3] and add the superpartners (Sec. II B). The superfields are introduced in Sec. II C.

A. The representation content

Let us first summarize the nonsupersymmetric model with the charge operator defined by Eq. (1) with $\vartheta = 1/\sqrt{3}$ [3], i.e.,

$$\frac{Q}{e} = \frac{1}{2} \left(\lambda_3 - \frac{1}{\sqrt{3}} \lambda_8 \right) + N\mathbf{I}, \quad (2)$$

which implies leptons transforming under the 3-3-1 factors as

$$L_{aL} = \begin{pmatrix} \nu_a \\ l_a \\ \nu_a^c \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}, -1/3), \quad (3)$$

with $a = e, \mu, \tau$ and $\nu_a^c = C\bar{\nu}_a^T$, plus the singlets

$$l_{aL}^c \sim (\mathbf{1}, \mathbf{1}, 1). \quad (4)$$

In the quark sector we have the first two families transforming as antitriplets of $SU(3)_L$

$$Q_{\alpha L} = \begin{pmatrix} d_\alpha \\ u_\alpha \\ d_\alpha^{\prime c} \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}^*, 0), \quad \alpha = 1, 2; \quad (5)$$

with the respective singlets

$$u_{\alpha L}^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3), d_{\alpha L}^c, d_{\alpha L}^{\prime c} \sim (\mathbf{3}^*, \mathbf{1}, 1/3). \quad (6)$$

The third family transforms as triplet under $SU(3)_L$ (in such a way that, the triangle anomaly cancels out only among the three families, taken also into account also the color charges):

$$Q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ u^{\prime c} \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, 1/3), \quad (7)$$

and their respective singlets

$$u_{3L}^c, u_L^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3), d_{3L}^c \sim (\mathbf{3}^*, \mathbf{1}, 1/3). \quad (8)$$

In the scalar sector only two triplets $\eta \sim (\mathbf{1}, \mathbf{3}, -1/3)$ and $\rho \sim (\mathbf{1}, \mathbf{3}, 2/3)$ are necessary to break appropriately the gauge symmetry and also to give the correct mass to all the fermions in the model. However, to eliminate flavor changing neutral currents we add an extra scalar triplet χ transforming like η :

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta^- \\ \eta_2^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_1^0 \\ \chi^- \\ \chi_2^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -1/3), \quad (9)$$

$$\rho = \begin{pmatrix} \rho_1^+ \\ \rho^0 \\ \rho_2^+ \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, 2/3),$$

and we will denote the vacuum expectation values which are different from zero as $v = \langle \eta_1^0 \rangle / \sqrt{2}$, $w = \langle \chi_2^0 \rangle / \sqrt{2}$, and $u = \langle \rho^0 \rangle / \sqrt{2}$.

B. Supersymmetric partners

Here we will follow the usual notation writing for a given fermion f , the respective sfermions by $\tilde{f}$ i.e., $\tilde{l}$ and $\tilde{q}$ denote sleptons and squarks, respectively. Then, we have the following additional representations

$$\tilde{Q}_{\alpha L} = \begin{pmatrix} \tilde{d}_\alpha \\ \tilde{u}_\alpha \\ \tilde{d}_\alpha^{\prime c} \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}^*, 0),$$

$$\tilde{Q}_{3L} = \begin{pmatrix} \tilde{u}_3 \\ \tilde{d}_3 \\ \tilde{u}^{\prime c} \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{3}, 1/3), \quad (10)$$

$$\tilde{L}_{aL} = \begin{pmatrix} \tilde{\nu}_a \\ \tilde{l}_a \\ \tilde{\nu}_a^c \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{3}, -1/3),$$

$$\tilde{l}_{aL}^c \sim (\mathbf{1}, \mathbf{1}, 1), \quad \tilde{u}_{iL}^c, \tilde{u}_L^c \sim (\mathbf{3}^*, \mathbf{1}, -2/3), \quad (11)$$

$$\tilde{d}_{iL}^c, \tilde{d}_L^c \sim (\mathbf{3}^*, \mathbf{1}, 1/3),$$

with $\alpha = 1, 2$; $a = e, \mu, \tau$; and $i = 1, 2, 3$.

The supersymmetric partners of the scalar Higgs fields, the Higgsinos, are

$$\tilde{\eta} = \begin{pmatrix} \tilde{\eta}_1^0 \\ \tilde{\eta}^- \\ \tilde{\eta}_2^0 \end{pmatrix}, \quad \tilde{\chi} = \begin{pmatrix} \tilde{\chi}_1^0 \\ \tilde{\chi}^- \\ \tilde{\chi}_2^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, -1/3), \quad (12)$$

$$\tilde{\rho} = \begin{pmatrix} \tilde{\rho}_1^+ \\ \tilde{\rho}^0 \\ \tilde{\rho}_2^+ \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}, 2/3),$$

and the respective extra Higgsinos, needed to cancel the chiral anomaly of the Higgsinos in Eq. (12), are

$$\begin{aligned}\tilde{\eta}' &= \begin{pmatrix} \tilde{\eta}_1'^0 \\ \tilde{\eta}'^+ \\ \tilde{\eta}_2'^0 \end{pmatrix}, & \tilde{\chi}' &= \begin{pmatrix} \tilde{\chi}_1'^0 \\ \tilde{\chi}'^+ \\ \tilde{\chi}_2'^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}^*, 1/3), \\ \tilde{\rho}' &= \begin{pmatrix} \tilde{\rho}_1'^- \\ \tilde{\rho}'^0 \\ \tilde{\rho}_2'^- \end{pmatrix} \sim (\mathbf{1}, \mathbf{3}^*, -2/3),\end{aligned}\quad (13)$$

and the corresponding scalar partners denoted by η', χ', ρ' , with the same charge assignment as in Eq. (13), and with the following VEVs: $v' = \langle \eta_1'^0 \rangle / \sqrt{2}$, $w' = \langle \chi_2'^0 \rangle / \sqrt{2}$, and $u' = \langle \rho'^0 \rangle / \sqrt{2}$. This completes the representation content of this supersymmetric model.

Concerning the gauge bosons and their superpartners, if we denote the gluons by g^b the respective superparticles, the gluinos, are denoted by λ^b , with $b = 1, \dots, 8$; in the electroweak sector we have V^b , the gauge boson of $SU(3)_L$, and their gauginos partners λ_A^b ; finally we have the gauge boson of $U(1)_N$, denoted by V' , and its supersymmetric partner λ_B . This is the total number of fields in the minimal supersymmetric extension of the 3-3-1 model of Refs. [3].

C. Superfields

The superfields formalism is useful in writing the Lagrangian which is manifestly invariant under the supersymmetric transformations [24] with fermions and scalars put in chiral superfields while the gauge bosons in vector superfields. As usual the superfield of a field ϕ will be denoted by $\hat{\phi}$ [25]. The chiral superfield of a multiplet ϕ is denoted by

$$\begin{aligned}\hat{\phi} &\equiv \hat{\phi}(x, \theta, \bar{\theta}) \\ &= \tilde{\phi}(x) + i\theta\sigma^m\bar{\theta}\partial_m\tilde{\phi}(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square\tilde{\phi}(x) \\ &\quad + \sqrt{2}\theta\phi(x) + \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\bar{\theta}\partial_m\phi(x) + \theta\theta F_\phi(x),\end{aligned}\quad (14)$$

while the vector superfield is given by

$$\begin{aligned}\hat{V}(x, \theta, \bar{\theta}) &= -\theta\sigma^m\bar{\theta}V_m(x) + i\theta\theta\bar{\theta}\bar{\theta}\bar{V}(x) - i\bar{\theta}\bar{\theta}\theta\theta\tilde{V}(x) \\ &\quad + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x).\end{aligned}\quad (15)$$

The fields F and D are auxiliary fields which are needed to close the supersymmetric algebra and eventually will be eliminated using their motion equations.

For fermion superfields we use the notation

$$\hat{L}_{aL}, \hat{l}_{aL}^c, \hat{Q}_{\alpha L}, \hat{Q}_{3L}, \hat{u}_{iL}^c, \hat{d}_{iL}^c, \hat{u}_{iL}'^c, \hat{d}_{\alpha L}'^c. \quad (16)$$

For scalar superfields we write: $\hat{\eta}, \hat{\chi}, \hat{\rho}$ and similar expressions for $\hat{\eta}', \hat{\chi}', \hat{\rho}'$ changing (field) by (field)'.

The vector superfield for the gauge bosons of each factor $SU(3)_C$, $SU(3)_L$ and $U(1)_N$ are denoted by

$\hat{V}_C, \hat{V}_C^b, \hat{V}, \hat{V}^b$; and $\hat{V}'$, respectively, where we have defined $\hat{V}_C = T^b \hat{V}_C^b$, $\hat{V} = T^b \hat{V}^b$; $\hat{V}_C = \bar{T}^b \hat{V}_C^b$, $\hat{V} = \bar{T}^b \hat{V}^b$; $T^b = \lambda^b/2$, $\bar{T}^b = -\lambda^{*b}/2$ are the generators of triplet and antitriplets representations, respectively, and λ^b are the Gell-Mann matrices.

III. THE LAGRANGIAN

The Lagrangian of the model has the following form

$$\mathcal{L}_{331S} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}, \quad (17)$$

where $\mathcal{L}_{\text{SUSY}}$ is the supersymmetric part and $\mathcal{L}_{\text{soft}}$ the soft terms breaking explicitly the supersymmetry.

A. The supersymmetric Lagrangian

The supersymmetric part of the Lagrangian is decomposed in the lepton, quark, gauge, and the scalar sectors as follows:

$$\mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{Lepton}} + \mathcal{L}_{\text{Quark}} + \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Scalar}}, \quad (18)$$

where

$$\mathcal{L}_{\text{Leptons}} = \int d^4\theta [\hat{L}_{aL} e^{2g\hat{V} - \frac{g'}{3}\hat{V}'} \hat{L}_{aL} + \hat{l}_{aL}^c e^{g'\hat{V}'} \hat{l}_{aL}^c], \quad (19)$$

in the lepton sector, we have omitted the sum over the three lepton family for simplicity, and

$$\begin{aligned}\mathcal{L}_{\text{Quarks}} &= \int d^4\theta [\hat{Q}_{\alpha L} e^{[2(g_s\hat{V}_C + g\hat{V}) + \frac{g'}{3}\hat{V}']} \hat{Q}_{\alpha L} \\ &\quad + \hat{Q}_{3L} e^{[2(g_s\hat{V}_C + g\hat{V}) + \frac{g'}{3}\hat{V}']} \hat{Q}_{3L} \\ &\quad + \hat{u}_{iL}^c e^{[2g_s\hat{V}_C - \frac{2g'}{3}\hat{V}']} \hat{u}_{iL}^c + \hat{d}_{iL}^c e^{[2g_s\hat{V}_C + \frac{g'}{3}\hat{V}']} \hat{d}_{iL}^c \\ &\quad + \hat{u}_{iL}'^c e^{[2g_s\hat{V}_C - \frac{2g'}{3}\hat{V}']} \hat{u}_{iL}'^c + \hat{d}_{\alpha L}'^c e^{[2g_s\hat{V}_C + \frac{g'}{3}\hat{V}']} \hat{d}_{\alpha L}'^c],\end{aligned}\quad (20)$$

in the quark sector, and we have denoted g_s, g, g' the gauge coupling constants for the $SU(3)_C, SU(3)_L, U(1)_N$ factors, respectively. In the gauge sector we have

$$\begin{aligned}\mathcal{L}_{\text{Gauge}} &= \frac{1}{4} \int d^2\theta [\mathcal{W}_C \mathcal{W}_C + \mathcal{W} \mathcal{W} + \mathcal{W}' \mathcal{W}'] \\ &\quad + \frac{1}{4} \int d^2\bar{\theta} [\bar{\mathcal{W}}_C \bar{\mathcal{W}}_C + \bar{\mathcal{W}} \bar{\mathcal{W}} + \bar{\mathcal{W}}' \bar{\mathcal{W}}'],\end{aligned}\quad (21)$$

where $\mathcal{W}_C, \mathcal{W}$ e $\mathcal{W}'$ are fields that can be written as follows

$$\begin{aligned}\mathcal{W}_{\zeta C} &= -\frac{1}{8\mathcal{G}_S} \bar{\mathcal{D}} \bar{\mathcal{D}} \mathcal{E}^{-2\mathcal{G}_S} \hat{V}_C \mathcal{D}_\zeta \mathcal{E}^{2\mathcal{G}_S} \hat{V}_C, \\ \mathcal{W}_\zeta &= -\frac{1}{8\mathcal{G}} \bar{\mathcal{D}} \bar{\mathcal{D}} \mathcal{E}^{-2\mathcal{G}} \hat{V} \mathcal{D}_\zeta \mathcal{E}^{2\mathcal{G}} \hat{V}, \\ \mathcal{W}'_\zeta &= -\frac{1}{4} \bar{\mathcal{D}} \bar{\mathcal{D}} \mathcal{D}_\zeta \hat{V}', \quad \zeta = 1, 2.\end{aligned}\quad (22)$$

Finally, in the scalar sector we have

$$\begin{aligned} \mathcal{L}_{\text{Scalar}} = & \int d^4\theta [\hat{\eta} e^{[2g\hat{V} - \frac{g'}{3}\hat{V}']} \hat{\eta} + \hat{\chi} e^{[2g\hat{V} - \frac{g'}{3}\hat{V}']} \hat{\chi} \\ & + \hat{\rho} e^{[2g\hat{V} + \frac{2g'}{3}\hat{V}']} \hat{\rho} + \hat{\eta}' e^{[2g\hat{V} + \frac{g'}{3}\hat{V}']} \hat{\eta}' \\ & + \hat{\chi}' e^{[2g\hat{V} + \frac{g'}{3}\hat{V}']} \hat{\chi}' + \hat{\rho}' e^{[2g\hat{V} - \frac{2g'}{3}\hat{V}']} \hat{\rho}'] \\ & + \int d^2\theta W + \int d^2\bar{\theta} \bar{W}, \end{aligned} \quad (23)$$

where W is the superpotential.

B. Superpotential

The superpotential of the model is decomposed as follows

$$W = \frac{W_2}{2} + \frac{W_3}{3}, \quad \bar{W} = \frac{\bar{W}_2}{2} + \frac{\bar{W}_3}{3}, \quad (24)$$

$W_2(\bar{W}_2)$ having two chiral superfields and $W_3(\bar{W}_3)$, three superfields. Explicitly we have that the term with two superfields is given by:

$$\begin{aligned} W_2 = & \sum_{a=e}^{\tau} [\mu_{0a} \hat{L} \hat{\eta}' + \mu_{1a} \hat{L} \hat{\chi}'] + \mu_{\eta} \hat{\eta} \hat{\eta}' + \mu_{\chi} \hat{\chi} \hat{\chi}' \\ & + \mu_{\rho} \hat{\rho} \hat{\rho}', \end{aligned} \quad (25)$$

where $\hat{L} \hat{\eta}' \equiv \hat{L}_i \hat{\eta}'_i$. The term with three superfields is written explicitly as

$$\begin{aligned} W_3 = & \sum_{a=e}^{\tau} \sum_{b=e}^{\tau} \lambda_{1ab} \hat{L}_{aL} \hat{\rho}' \hat{L}_{bL}^c + \sum_{a=e}^{\tau} [\lambda_{2a} \epsilon \hat{L}_{aL} \hat{\chi} \hat{\rho} + \lambda_{3a} \epsilon \hat{L}_{aL} \hat{\eta} \hat{\rho}] + \sum_{a=e}^{\tau} \sum_{b=e}^{\tau} \lambda_{4ab} \epsilon \hat{L}_{aL} \hat{L}_{bL} \hat{\rho} + \sum_{i=1}^3 \kappa_{1i} \hat{Q}_{3L} \hat{\eta}' \hat{u}_{iL}^c \\ & + \kappa'_1 \hat{Q}_{3L} \hat{\eta}' \hat{u}_L^c + \sum_{i=1}^3 \kappa_{2i} \hat{Q}_{3L} \hat{\chi}' \hat{u}_{iL}^c + \kappa'_2 \hat{Q}_{3L} \hat{\chi}' \hat{u}_L^c + \sum_{i=1}^3 \kappa_{3i} \hat{Q}_{3L} \hat{\rho}' \hat{d}_{iL}^c + \sum_{\alpha=1}^2 \kappa'_{3\alpha} \hat{Q}_{3L} \hat{\rho}' \hat{d}_{\alpha L}^c + \sum_{\alpha=1}^2 \sum_{i=1}^3 \kappa_{3\alpha i} \hat{Q}_{\alpha L} \hat{\eta} \hat{d}_{iL}^c \\ & + \sum_{\alpha=1}^2 \sum_{\beta=1}^2 \kappa'_{3\alpha\beta} \hat{Q}_{\alpha L} \hat{\eta} \hat{d}_{\beta L}^c + \sum_{\alpha=1}^2 \sum_{i=1}^3 \kappa_{4\alpha i} \hat{Q}_{\alpha L} \hat{\rho} \hat{u}_{iL}^c + \sum_{\alpha=1}^2 \kappa'_{4\alpha} \hat{Q}_{\alpha L} \hat{\rho} \hat{u}_L^c + \sum_{\alpha=1}^2 \sum_{i=1}^3 \kappa_{5\alpha i} \hat{Q}_{\alpha L} \hat{\chi} \hat{d}_{iL}^c \\ & + \sum_{\alpha=1}^2 \sum_{\beta=1}^2 \kappa'_{5\alpha\beta} \hat{Q}_{\alpha L} \hat{\chi} \hat{d}_{\beta L}^c + \sum_{\alpha=1}^2 \sum_{\beta=1}^2 \sum_{\gamma=1}^2 \epsilon f_{1\alpha\beta\gamma} \hat{Q}_{\alpha L} \hat{Q}_{\beta L} \hat{Q}_{\gamma L} + f_2 \epsilon \hat{\rho} \hat{\chi} \hat{\eta} + f'_2 \epsilon \hat{\rho}' \hat{\chi}' \hat{\eta}' + \sum_{i=1}^3 \sum_{\beta=1}^2 \sum_{j=1}^3 \xi_{1i\beta j} \hat{d}_{iL}^c \hat{d}_{\beta L}^c \hat{u}_{jL}^c \\ & + \sum_{i=1}^3 \sum_{\beta=1}^2 \xi_{2i\beta} \hat{d}_{iL}^c \hat{d}_{\beta L}^c \hat{u}_L^c + \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \xi_{3ijk} \hat{d}_{iL}^c \hat{d}_{jL}^c \hat{u}_{kL}^c + \sum_{i=1}^3 \sum_{j=1}^3 \xi_{4ij} \hat{d}_{iL}^c \hat{d}_{jL}^c \hat{u}_L^c + \sum_{\alpha=1}^2 \sum_{\beta=1}^2 \sum_{i=1}^3 \xi_{5\alpha\beta i} \hat{d}_{\alpha L}^c \hat{d}_{\beta L}^c \hat{u}_{iL}^c \\ & + \sum_{\alpha=1}^2 \sum_{\beta=1}^2 \xi_{6\alpha\beta} \hat{d}_{\alpha L}^c \hat{d}_{\beta L}^c \hat{u}_L^c + \sum_{a=e}^{\tau} \sum_{\alpha=1}^2 \sum_{i=1}^3 \xi_{7a\alpha j} \hat{L}_{aL} \hat{Q}_{\alpha L} \hat{d}_{jL}^c + \sum_{a=e}^{\tau} \sum_{\alpha=1}^2 \sum_{\beta=1}^2 \xi_{8a\alpha\beta} \hat{L}_{aL} \hat{Q}_{\alpha L} \hat{d}_{\beta L}^c. \end{aligned} \quad (26)$$

Finally, as usual it is necessary to introduce the so-called soft terms that break the supersymmetry explicitly.

C. The soft terms

The soft terms can be written as

$$\mathcal{L}_{\text{soft}} = \mathcal{L}_{\text{GMT}} + \mathcal{L}_{\text{Scalar}}^{\text{soft}} + \mathcal{L}_{\text{SMT}}, \quad (27)$$

where

$$\mathcal{L}_{\text{GMT}} = -\frac{1}{2} \left[m_{\lambda_C} \sum_{b=1}^8 (\lambda_C^b \lambda_C^b) + m_{\lambda_A} \sum_{b=1}^8 (\lambda_A^b \lambda_A^b) + m' \lambda_B \lambda_B + \text{H.c.} \right], \quad (28)$$

give mass to the boson superpartners and

$$\begin{aligned} \mathcal{L}_{\text{Scalar}}^{\text{soft}} = & -m_{\eta}^2 \eta^{\dagger} \eta - m_{\rho}^2 \rho^{\dagger} \rho - m_{\chi}^2 \chi^{\dagger} \chi - m_{\eta'}^2 \eta'^{\dagger} \eta' - m_{\rho'}^2 \rho'^{\dagger} \rho' - m_{\chi'}^2 \chi'^{\dagger} \chi' + [k_1 \epsilon_{ijk} \rho_i \chi_j \eta_k + k'_1 \epsilon_{ijk} \rho'_i \chi'_j \eta'_k \\ & + \text{H.c.}], \end{aligned} \quad (29)$$

in order to give mass to the scalars (omitting the sum upon repeated indices, $i, j, k = 1, 2, 3$). Finally, we have to add

$$\begin{aligned}
-\mathcal{L}_{SMT} = & m_{\tilde{L}_a}^2 \tilde{L}_a^\dagger \tilde{L}_a + m_{\tilde{L}_{aL}}^2 \tilde{L}_{aL}^\dagger \tilde{L}_{aL} + m_{\tilde{Q}_3}^2 \tilde{Q}_3^\dagger \tilde{Q}_3 + m_{\tilde{Q}_{aL}}^2 \tilde{Q}_{aL}^\dagger \tilde{Q}_{aL} + m_{\tilde{u}_i}^2 \tilde{u}_i^\dagger \tilde{u}_i + m_{\tilde{d}_i}^2 \tilde{d}_i^\dagger \tilde{d}_i + m_{\tilde{u}'_L}^2 \tilde{u}'_L{}^\dagger \tilde{u}'_L \\
& + m_{\tilde{d}'_L}^2 \tilde{d}'_L{}^\dagger \tilde{d}'_L + [M_a^2 \tilde{L}_a \eta'^\dagger + M_a'^2 \tilde{L}_a \chi'^\dagger + \varepsilon_{1a} \tilde{L}_a \rho'^\dagger \tilde{L}_a + \varepsilon_{2a} \epsilon \tilde{L}_a \chi \rho + \varepsilon_{3a} \epsilon \tilde{L}_a \eta \rho + \varepsilon_{4ab} \epsilon \tilde{L}_a \tilde{L}_b \rho \\
& + \varrho_{1i} \tilde{Q}_3 \eta' \tilde{u}'_i + \varrho'_1 \tilde{Q}_3 \eta' \tilde{u}'_i + \varrho_{2i} \tilde{Q}_3 \chi' \tilde{u}'_i + \varrho'_2 \tilde{Q}_3 \chi' \tilde{u}'_i + \varrho_{3\alpha i} \tilde{Q}_\alpha \eta \tilde{d}'_i + \varrho'_{3\alpha\beta} \tilde{Q}_\alpha \eta \tilde{d}'_\beta \\
& + \varrho_{4\alpha i} \tilde{Q}_\alpha \rho \tilde{u}'_i + \varrho'_{4\alpha} \tilde{Q}_\alpha \rho \tilde{u}'_i + \varrho_{5i} \tilde{Q}_3 \rho' \tilde{d}'_i + \varrho'_{5\alpha} \tilde{Q}_3 \rho' \tilde{d}'_\alpha + \varrho_{6\alpha i} \tilde{Q}_\alpha \chi \tilde{d}'_i + \varrho'_{6\alpha\beta} \tilde{Q}_\alpha \chi \tilde{d}'_\beta \\
& + \varrho_{7\alpha\beta\gamma} \tilde{Q}_\alpha \tilde{Q}_\beta \tilde{Q}_\gamma + v_{1i\beta j} \tilde{d}'_i \tilde{d}'_\beta \tilde{u}'_j + v_{2i\beta} \tilde{d}'_i \tilde{d}'_\beta \tilde{u}'_L + v_{3ijk} \tilde{d}'_i \tilde{d}'_j \tilde{u}'_k + v_{4ij} \tilde{d}'_i \tilde{d}'_j \tilde{u}'_L + v_{5\alpha\beta i} \tilde{d}'_\alpha \tilde{d}'_\beta \tilde{u}'_i \\
& + v_{6\alpha\beta} \tilde{d}'_\alpha \tilde{d}'_\beta \tilde{u}'_L + v_{7a\alpha j} \tilde{L}_a \tilde{Q}_\alpha \tilde{d}'_j + v_{8a\alpha\beta} \tilde{L}_a \tilde{Q}_\alpha \tilde{d}'_\beta + \text{H.c.}], \quad (30)
\end{aligned}$$

in order to give appropriate masses to the sfermions.

IV. THE SCALAR POTENTIAL

In the present model the scalar potential is written as

$$V_{331} = V_F + V_D + V_{\text{soft}}, \quad (31)$$

where

$$\begin{aligned}
V_F = -\mathcal{L}_F = & \sum_m F_m^\dagger F_m \\
= & \sum_{ijk} \left[\left| \frac{\mu_\eta}{2} \eta_i' + \frac{f_2}{3} \epsilon_{ijk} \rho_j \chi_k \right|^2 + \left| \frac{\mu_\chi}{2} \chi_i' + \frac{f_2}{3} \epsilon_{ijk} \eta_j \rho_k \right|^2 + \left| \frac{\mu_\rho}{2} \rho_i' + \frac{f_2}{3} \epsilon_{ijk} \chi_j \eta_k \right|^2 + \left| \frac{\mu_\eta}{2} \eta_i \right. \right. \\
& \left. \left. + \frac{f_2'}{3} \epsilon_{ijk} \rho_j' \chi_k' \right|^2 + \left| \frac{\mu_\chi}{2} \chi_i + \frac{f_2'}{3} \epsilon_{ijk} \eta_j' \rho_k' \right|^2 + \left| \frac{\mu_\rho}{2} \rho_i + \frac{f_2'}{3} \epsilon_{ijk} \chi_j' \eta_k' \right|^2 \right], \quad (32)
\end{aligned}$$

and

$$\begin{aligned}
V_D = -\mathcal{L}_D = & \frac{1}{2} (D^a D^a + DD) \\
= & \frac{g'^2}{18} (-\eta^\dagger \eta + \eta'^\dagger \eta' - \chi^\dagger \chi + \chi'^\dagger \chi' + 2\rho^\dagger \rho - 2\rho'^\dagger \rho')^2 + \frac{g^2}{8} (\eta_i^\dagger \lambda_{ij}^b \eta_j - \eta_i'^\dagger \lambda_{ij}^{*b} \eta_j' + \chi_i^\dagger \lambda_{ij}^b \chi_j - \chi_i'^\dagger \lambda_{ij}^{*b} \chi_j' \\
& + \rho_i^\dagger \lambda_{ij}^b \rho_j + \rho_i'^\dagger \lambda_{ij}^{*b} \rho_j'), \quad (33)
\end{aligned}$$

finally,

$$\begin{aligned}
V_{\text{soft}} = -\mathcal{L}_{\text{soft}} \\
= & m_\eta^2 \eta^\dagger \eta + m_\rho^2 \rho^\dagger \rho + m_\chi^2 \chi^\dagger \chi + m_{\eta'}^2 \eta'^\dagger \eta' + m_{\rho'}^2 \rho'^\dagger \rho' + m_{\chi'}^2 \chi'^\dagger \chi' - \epsilon_{ijk} (k_1 \rho_i \chi_j \eta_k + k'_1 \rho'_i \chi'_j \eta'_k + \text{H.c.}), \quad (34)
\end{aligned}$$

where we have used the scalar multiplets in Eqs. (9) and those related to Higgsinos in Eq. (13).

With Eqs. (32)–(34) we can work out the mass spectra of the scalar and pseudoscalar fields by making the usual shift $X^0 \rightarrow \frac{1}{\sqrt{2}}(v_X + H_X + iF_X)$. The analysis is similar to that of Ref. [8] and we will not write the constraints equation.

By using as input the following values for the parameters: $\sin^2 \theta_W = 0.2314$, $g = 0.6532$, $g' = 1.1466$; $f_2 = 2$, $f_2' = 10^{-3}$; $k_1 = k'_1 = 10$ GeV; $\mu_\eta = \mu_\rho = \mu_\chi = -10^3$ GeV; $m_\eta = 15$ GeV, $m_\rho = 10$ GeV, $m_{\rho'} = 244.99$ GeV; $m_{\chi_2} = m_{\chi'_2} = 10^3$ GeV and $m_{\rho'} = 13$ GeV, we obtain the masses

$$\begin{aligned}
m_1 \approx 1702, \quad m_2 \approx 1449, \quad m_3 \approx 387, \\
m_4 \approx 380, \quad m_5 \approx 361, \quad m_6 \approx 130, \quad (35)
\end{aligned}$$

for the scalar sector (all masses are in GeV). Note that the lightest neutral scalar is heavier than the lower limit of the Higgs scalar of the standard model, i.e., $m_H \gtrsim 114$ GeV [26]. For the pseudoscalar sector we obtain

$$\begin{aligned}
M_1 \approx 1702, \quad M_2 \approx 1449, \quad M_3 \approx 363, \\
M_4 \approx 5, \quad M_5 = 0, \quad M_6 = 0, \quad (36)
\end{aligned}$$

only the two massless pseudoscalars are exact values, i.e., there are two would be Goldstone bosons as it should be. Notice that there is a light pseudoscalar. Although the values of the masses above are only an exercise, and it is possible that other values of the input parameters would give another set of masses, we would like to point out the following. In the basis $(F_{\eta_1}, F_\rho, F_{\chi_2}, F_{\eta'_1}, F_{\rho'}, F_{\chi'_2})$ the lightest pseudoscalar is given by $(0.0099, 0.0012, 0.7070, 0.0071, 0.0170, 0.7070)$, hence we see that it is

mainly F_{χ_2} and $F_{\chi'_2}$. So, we need to investigate the couplings of these pseudoscalars with the Z^0 .

In the (W_3, W_8, B) basis we have the mass square of the real vector bosons given by:

$$\frac{g^2}{4} \begin{pmatrix} V^2 + U^2 & \frac{1}{\sqrt{3}}(V^2 - U^2) & -\frac{2t}{3}(V^2 + 2U^2) \\ \frac{1}{3}(V^2 + U^2 + 4W^2) & -\frac{2t}{3\sqrt{3}}(V^2 - 2U^2 - 2W^2) & \frac{4t^2}{9}(V^2 + 4U^2 + W^2) \end{pmatrix}, \quad (37)$$

where we have defined $V^2 = v^2 + v'^2$, $U^2 = u^2 + u'^2$ and $W^2 = w^2 + w'^2$, and

$$t^2 = \left(\frac{g'}{g}\right)^2 = \frac{\sin^2 \theta_W}{1 - \frac{4}{3} \sin^2 \theta_W}. \quad (38)$$

The eigenstates of Eq. (36) are

$$A = \frac{\sqrt{3}}{4t^2 + 3} \left(tW_3 - \frac{t}{\sqrt{3}} W_8 + B \right), \quad (39)$$

for the photon, and

$$Z^0 \approx \frac{3t}{4t^2 + 15t^2 + 9} \left[-\left(\frac{t^2 + 3}{3t}\right) W_3 - \frac{t}{\sqrt{3}} W_8 + B \right], \quad (40)$$

and

$$Z^{0'} \approx \frac{t}{t^2 + 3} \left(\frac{\sqrt{3}}{t} W_8 + B \right), \quad (41)$$

for the Z^0 and $Z^{0'}$, where we have neglected the mixing among Z^0 and $Z^{0'}$, so that $M_Z^2/M_W^2 \approx (3 + 4t^2)/(3 + t^2) = 1/\cos^2 \theta_W$. With this at hand, we were able to verify that the couplings of $F_{\chi_2}, F_{\chi'_2}$ are given by the usual vertex of the Higgs scalar in the standard model times a factor proportional to $(W/v_W)(U/W)^4$ or $(W/v_W) \times (V/W)^4$, where $v_W \approx 246$ GeV, and these couplings go to zero as $W \rightarrow \infty$. This behavior is expected since both $\chi_2^0, \chi_2^{0'}$ are singlets of $SU(2)_L \otimes U(1)_Y$ and do not couple to the Z^0 in this limit.

For completeness we show that the lightest scalar, in the basis $(H_{\eta_1}, H_\rho, H_{\chi_2}, H_{\eta'_1}, H_\rho, H_{\chi'_2})$, is written as $(-0.0581, -0.9775, 0.0610, -0.0394, -0.0592, 0.1800)$, that is, it is mainly H_ρ which transforms as doublet under $SU(2)_L \otimes U(1)_Y$.

V. CONCLUSIONS

We have put forward a supersymmetric version of the 3-3-1 model of Refs. [3] which includes right-handed neutrinos transforming nontrivially under $SU(3)_L$. This sort of models is an interesting possibility since neutrinos are massive particles and right-handed neutrinos should be added eventually to any extension of the standard model (See Ref. [26]).

Concerning the scalar and pseudoscalar mass spectra we have found two different situations: for the scalar sector we were able to find all the Higgs scalars with masses above the Z^0 mass and above the lower limit of the standard model Higgs boson obtained by LEP: $m_H \gtrsim 114$ GeV [26]; for the pseudoscalars, for the same set of the input parameters, we have found a considerably light one ($M_4 = 5$ GeV) which in principle can bring some problems. However, a carefully analysis have shown that the mass eigenstate corresponding to M_4 is mainly formed by the symmetry eigenstates F_{χ_2} and $F_{\chi'_2}$, and studying the couplings of these pseudoscalars with the Z^0 we have noted that they vanish in the limit where w and w' go to infinity, i.e., both pseudoscalars decouple from Z^0 in this limit since they are singlets under $SU(3)_L \otimes U(1)_Y$.

In the other supersymmetric 3-3-1 model [8], the proton decay modes are $p \rightarrow K^+ \bar{\nu}_\mu$ and $p \rightarrow K^+ e^\mp \mu^\pm \bar{\nu}_\tau$ but in the present one, only the mode $p \rightarrow \pi^+ \bar{\nu}_\mu$ is possible. It means that the constraints coming from SuperKamiokande data on $p \rightarrow K^+ \bar{\nu}_e$, which implies that $\tau_p > 10^{33}$ years in this decay mode [27], are avoided. The proton decay in the context of the present model has been considered recently in Ref. [28].

However, there are higher-dimension (nonrenormalizable) operators, that arise from new physics at some scale Λ . For instance, there are dimension-5 operators that violate baryon or lepton number, that are allowed by the gauge invariance, and that contribute to the proton decay unless they are sufficiently suppressed. In the context of the Minimal Supersymmetric Standard Model, we have [29]

$$\frac{\kappa_{ijkl}^1}{\Lambda} \hat{Q}_i \hat{Q}_j \hat{Q}_k \hat{L}_l + \frac{\kappa_{ijka}^2}{\Lambda} \hat{u}_i^c \hat{u}_j^c \hat{d}_k^c \hat{t}_a^c. \quad (42)$$

These terms contribute to the proton decay with diagrams at 1-loop level involving gauginos and gluinos, known as dressing diagrams. These are the dangerous terms that induce the decay mode $p \rightarrow K^+ \bar{\nu}_e$. This channel is enough to almost exclude the minimal supersymmetric SU(5) model [30] since the SuperKamiokande data [27].

In the present model dimension-5 operators that violate lepton and baryon number, that are allowed by the 3-3-1 symmetry, may arise in unification theories [31], and are given by

$$\begin{aligned}
& \frac{\kappa_{a\alpha\beta}^1}{\Lambda} (\hat{L}_a \hat{Q}_\alpha) (\hat{Q}_3 \hat{Q}_\beta) + \frac{\kappa_{aijk}^2}{\Lambda} \hat{f}_a^c \hat{u}_i^c \hat{u}_j^c \hat{d}_k^c + \frac{\kappa_{abi}^3}{\Lambda} (\hat{L}_a \hat{L}_b) (\hat{Q}_3 \hat{d}_i^c) + \frac{\kappa_{aij}^4}{\Lambda} (\hat{Q}_\alpha \hat{Q}_3) \hat{u}_i^c \hat{d}_j^c + \frac{\kappa_{ab}^5}{\Lambda} (\hat{L}_a \hat{\eta}') (\hat{L}_b \hat{\eta}') + \frac{\kappa_{ab}^6}{\Lambda} (\hat{L}_a \hat{\chi}') (\hat{L}_b \hat{\chi}') \\
& + \frac{\kappa_{\alpha\beta}^7}{\Lambda} (\hat{Q}_3 \hat{Q}_\alpha) (\hat{Q}_\beta \hat{\eta}) + \frac{\kappa_{\alpha\beta}^8}{\Lambda} (\hat{Q}_3 \hat{Q}_\alpha) (\hat{Q}_\beta \hat{\chi}), \tag{43}
\end{aligned}$$

while in the SUSY 3-3-1 model of Ref. [8] these operators are

$$\frac{\kappa_{a\alpha\beta}^1}{\Lambda} (\hat{L}_a \hat{Q}_\alpha) (\hat{Q}_3 \hat{Q}_\beta) + \frac{\kappa_{\alpha\beta}^2}{\Lambda} (\hat{Q}_\alpha \hat{J}^c) (\hat{Q}_3 \hat{J}_\beta^c) + \frac{\kappa_{abi}^3}{\Lambda} (\hat{L}_a \hat{L}_b) (\hat{Q}_3 \hat{u}_i^c) + \frac{\kappa_{\alpha\beta}^4}{\Lambda} (\hat{\eta} \hat{Q}_\alpha) (\hat{Q}_3 \hat{Q}_\beta) + \frac{\kappa_{ab}^5}{\Lambda} (\hat{L}_a \hat{\eta}') (\hat{L}_b \hat{\eta}'). \tag{44}$$

The suppression of the effective operators in Eqs. (42) and (43) in the context of SUSY 3-3-1 models will be considered elsewhere.

Finally, we would like to say that the 3-3-1 model with nonsupersymmetric right-handed neutrinos furnishes a good candidate for self-interacting dark matter since there are two Higgs bosons, one scalar, and one pseudoscalar, which have the properties of the candidates for dark matter like, stability, neutrality, and that they must not overpopulate the universe [32] and, in particular, for a self-interacting dark matter candidates, they have large scattering cross-section and negligible annihilation or dissipation. It means that the candidate for self-interacting dark matter has not to be introduced *ad hoc* as in other models [33]. This feature remains valid in the supersymmetric version that we have developed in this work, but it deserves a more careful study.

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