UNIVERSIDADE ESTADUAL PAULISTA “Júlio de Mesquita Filho”
FACULDADE DE ENGENHARIA
CAMPUS DE ILHA SOLTEIRA

HAMID KHORASANI

ALGORITMO TABU SEARCH ESPECIALIZADO PARA O PROBLEMA DE PLANEJAMENTO DA EXPANSAO DE SISTEMAS DE TRANSMISSAO

Ilha Solteira

2015

HAMID KHORASANI

STATIC AND DYNAMIC TRANSMISSION NETWORK EXPANSION PLANNING UNDER (N-1) SECURITY CONSTRAINTS

A tese submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering in the Electrical Engineering Faculty of Ilha Solteira UNESP, São Paulo, Brazil. Área do Conhecimento: Automação.

Orientador:

Professor Rubén Augusto Romero Lázaro

Committee in charge:

Professor Rubén Augusto Romero Lázaro

Professor José Roberto Sanches Mantovani

Professora Marina Lavorato de Oliveira

Professor Walmir de Freitas Filho

Professor Carlos Roberto Mendonca Rocha

Ilha Solteira

2015

FICHA CATALOGRÁFICA
Desenvolvido pelo Serviço Técnico de Biblioteca e Documentação

K45a

Khorasani, Hamid.
Algoritmo tabu search especializado para o problema de planejamento da expansao de sistemas de transmissao / Hamid Khorasani. -- Ilha Solteira: [s.n.], 2015
148 f. : il.

Tese (doutorado) - Universidade Estadual Paulista. Faculdade de Engenharia de Ilha Solteira. Área de conhecimento: Automação, 2015

Orientador: Rubén Augusto Romero Lázaro
Inclui bibliografia

1. Planejamento de expansão do sistema de transmissão. 2. Restrições de segurança. 3. Algoritmo híbrido. 4. Algoritmo busca tabu especializado. 5. Algoritmo genético. 6. Algoritmo heurístico construtivo.

unesp UNIVERSIDADE ESTADUAL PAULISTA
CAMPUS DE ILHA SOLTEIRA
FACULDADE DE ENGENHARIA DE ILHA SOLTEIRA

CERTIFICADO DE APROVAÇÃO

TÍTULO: Static and dynamic transmission network expansion planning under (n-1) security constraints

AUTOR: HAMID KHORASANI

ORIENTADOR: Prof. Dr. RUBEN AUGUSTO ROMERO LAZARO

Aprovado como parte das exigências para obtenção do Título de DOUTOR EM ENGENHARIA ELÉTRICA , Área: AUTOMAÇÃO, pela Comissão Examinadora:

Prof. Dr. RUBEN AUGUSTO ROMERO LAZARO
Departamento de Engenharia Elétrica / Faculdade de Engenharia de Ilha Solteira

Prof. Dr. JOSE ROBERTO SANCHES MANTOVANI
Departamento de Engenharia Elétrica / Faculdade de Engenharia de Ilha Solteira

Marina Lavorato de Oliveira
Profa. Dra. MARINA LAVORATO DE OLIVEIRA

Carlos Roberto M. Rocha
Prof. Dr. CARLOS ROBERTO MENDONÇA DA ROCHA
Centro de Engenharias e Ciências Exatas / Universidade Estadual do Oeste do Paraná

Prof. Dr. WALMIR DE FREITAS FILHO
Departamento de Sistemas de Energia Elétrica / Universidade Estadual de Campinas

Data da realização: 20 de fevereiro de 2015.

Acknowledgment

I would like to thank Professor Rubén Romero for his continuous advice and mentorship. His comments and advice have been invaluable and will serve me for a lifetime. This thesis was prepared with a sense of ambition and innovation that would have been impossible to attain without his leadership.

I also thank the rest of my thesis committee: professor José Roberto Sanches Mantovani, professora Marina Lavorato de Oliveira, professor Walmir de Freitas Filho and professor Luis Gustavo Wesz da Silva

I thank all the members of LaPSSE for their support and for helping me learning Portuguese.

I also thank Professor Masoud Rashidinejad, Mahdi Pourakbari, Leili Kheirkhah and Mohsen Rahmani for helping and accompanying me during my doctoral studies. I value their friendship and support as well.

Generous funding was provided by the FAPESP. Without their economic support, this thesis would never have been completed.

Finally, I would like to thank my family, especially my mother and my father for their love, support, patience and understanding.

Thanks to all

Resumo

O sistema de transmissão desempenha um papel muito importante na prevenção de cortes de cargas, blecaute (colapso), etc., fornecendo energia para todos os tipos de consumidores e às vezes em circunstâncias críticas. O problema de planejamento de expansão de sistemas de transmissão (PET) possibilita que a rede transmita a energia gerada para abastecer os centros de carga em todo instante de tempo, a fim de satisfazer a crescente demanda por energia elétrica. No problema PET, além de cumprir com todas as restrições técnicas, deve-se elaborar um plano de expansão econômico. O planejamento da expansão do sistema de transmissão é um tipo de atividade em que as decisões devem são tomadas e planejadas em nível nacional já que implica absorver recursos financeiros significativos. Consequentemente, elaborar um projeto de expansão ideal, com o mínimo custo possível e com a máxima confiabilidade é uma tarefa crucial. Do ponto de vista da estrutura dos sistemas de energia, pode-se afirmar que o problema PET é analisado nos sistemas regulados e desregulados. O principal objetivo do PET em um ambiente regulado é atender a demanda de carga, com mínimo custo, levando em consideração o critério da confiabilidade. Por outro lado, em sistemas de energia reestruturados, a expansão da transmissão destina-se a criar um ambiente competitivo sem qualquer discriminação de acesso à rede de transmissão. Esse mecanismo pode perfeitamente assegurar mercados competitivos. Em geral, o problema PET em ambientes regulados é um problema de programação não-linear inteira mista, o qual envolve algumas dificuldades, tais como um tempo prolongado para executar esse tipo de projetos, assim como a necessidade de uma técnica de otimização não-convexa. Além disso, devido à complicação dos problemas de otimização combinatória, e também por existir muitos mínimos locais neste caso, trata-se de um problema que exige tempos de processamento elevados. Além disso, tendo em vista que os métodos convencionais de programação matemática não funcionam necessariamente bem, foram avaliadas várias técnicas de otimização chamadas de meta-heurísticas para o referido problema. Entretanto, o escopo e a possibilidade de se obter algoritmos ainda melhores ainda são evidentes. Na presente tese, com a finalidade de resolver esses problemas através de algoritmos, foi desenvolvido um algoritmo híbrido para resolver o problema PET considerando restrições de segurança. A metodologia proposta está baseada principalmente na busca tabu e no algoritmo genético. Adicionalmente, o algoritmo híbrido desenvolvido incorporou algumas estratégias especializadas, com o objetivo de diminuir o tamanho da vizinhança e, consequentemente, o

número de problemas de programação linear que devem ser resolvidos de forma iterativa, na busca da solução ótima.
PALAVRA CHAVE: Planejamento de expansão do sistema de transmissão. Restrições de segurança. Algoritmo híbrido. Algoritmo busca tabu especializado. Algoritmo genético. Algoritmo heurístico construtivo.

Abstract

Transmission system plays an undeniable role to avoid load shedding, black out, etc. by supplying the power to all type of consumers under critical circumstances. A transmission network expansion planning (TNEP) enables a network to transmit enough generated power to load centers at a specified times to satisfy the increased electric power demand. In TNEP problem, in addition to satisfying all the technical constraints, an economic plan is demanded. The expansion of transmission network is one of the initiatives, in which the necessary decisions are made and planned at the national level to absorb significant financial resources. Therefore, planning for an optimal expansion project with the least cost and highest reliability is a crucial task. From the viewpoint of the structure of power systems, it can be stated that TNEP is analyzed in both regulated and deregulated environments. The main objective of TNEP in a regulated environment is to meet the load demand at the least cost while the reliability criterion is taken into account. On the other hand, in restructured power systems, the transmission expansion is primarily intended to create a competitive environment without any discrimination to access to the transmission network. This can perfectly ensure competitive markets. In general TNEP problem in regulated environments is a non-linear mixed integer programming problem, which subjects with some difficulties, such as the time-consuming nature of the problem as well as the need for a non-convex optimization technique. In addition, due to the complicacy of the combinatorial optimization problems and also, since there exist many local minima for this problem, it is considered as a time-consuming problem. Moreover, since the conventional mathematical programming methods do not necessarily work very satisfactorily, therefore, various meta-heuristic optimization techniques have been examined for this problem. However, the scope and possibility of having even better algorithms is still indefinite. In this thesis, in order to attempt to solve these problems in algorithms, a hybrid algorithm has been presented in order to solve the TNEP problem considering security constraints. The proposed methodology is principally based on the tabu search and the genetic algorithm. In addition, the mentioned hybrid algorithm has incorporated some improved strategies in order to decrease the number of neighbors and consequently, the number of linear programming problems that should be solved iteratively to find the final solution.

KEYWORDS: Transmission expansion planning. Security constraints. Hybrid algorithm. Special tabu search. Genetic algorithm. Constructive heuristic algorithm.

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

AC	Alternating Current
AI	Artificial Intelligence
AMP	Adaptive Memory Programming
AMPL	A Mathematical Programming Language
BFDEA	Bacteria Foraging-Differential Evaluation Algorithm
CBGA	Genetic Algorithm Proposed by Chu-Beasley
CHA	Constructive Heuristic Algorithm
COA	Chaos Optimal Algorithm
DC	Direct Current
DDO	Discrete Dynamic Optimizing
DM	Disjunctive Model
GA	Genetic Algorithm
GP	Geometric Pro Ngramming
GRASP	Greedy Randomized Adaptive Search Procedure
HLM	Hybrid Linear Model
HS	Harmony Search
IHS	Improved Harmony Search
ILP	Integer Linear Programming
KCL	Kirchhoff Current Law
KVL	Kirchhoff's Voltage Law
LP	Linear Programming
MILP	Mixed Integer Linear Programming Problem
MTEP	Multistage Transmission Expansion Planning
NGA	Niching Genetic Algorithm
NSGA	Nondominated Sorting Genetic Algorithm
NLP	Nonlinear Programming Problem
OPF	Optimal Power Flow
OPM	Operational Planning Model
PDF	Probability Density Functions
PSO	Particle Swarm Optimization
SA	Simulated Annealing
SI	Sensitivity Index
STEP	Static Transmission Expansion Planning
STS	Special Tabu Search
TEP	Transmission Expansion Planning
TNEP	Transmission Network Expansion Planning
TM	Transportation Model
TS	Tabu Search
UC	Unit Commitment
VGS	Villasana, Garver, Solan Algorithm

LIST OF SYMBOLS

The main symbols used in this thesis are listed below for quick reference. Other symbols are defined as needed throughout the text.

Sets

Ω	Set of all right-of-way
Ω_b	Set of indices for all buses
Ω_g	Set of indices for generation buses
Ω_{i}	Set of all buses near at bus *i*
Ω_0	Set of circuits of base case
Y	Set of new circuits

Parameters

c_{ij}	Cost of a circuit that can be added to branch $i{-}j$
n_{ij}^0	Number of circuits in the base case in branch $i{-}j$
d	Vector of demand with elements d_i
d_i	Demand in bus *i*
$\overline{n}_{ij}$	Maximum number of circuits that can be added to branch $i{-}j$
P_{GK}^{max}	Maximum active power generation limits at bus *k*
P_{GK}^{min}	Minimum active power generation limits at bus *k*
Q_{GK}^{max}	Maximum reactive power generation limits at bus *k*
Q_{GK}^{min}	Minimum reactive power generation limits at bus *k*
V_i^{max}	Voltage maximum limits at bus *i*
V_i^{min}	Voltage minimum limits at bus *i*
α	Cost of load shedding
S_{ij}^{max}	Maximum limit of power flows in right-of-way $i{-}j$
g_{ij}	Line conductance of the right-of-way $i - j$
γ_{ij}	Line susceptance of the right-of-way $i - j$
b_{ij}^{sh}	Shunt susceptance of the right-of-way $i - j$
b_i^{sh}	Shunt susceptance at bus *i*
$\bar{f}_{ij}$	Maximum active power flow limit of line $i{-}j$
S	Branch-node incidence matrix
S_0	Transpose incidence branch-node matrix of the base topology
$\bar{g}$	Vector of maximum generation
M	A big parameter

Variables

v	Value of the expansion investment costs for a predefined horizon
n_{ij}	Number of circuits added to branch *i-j*
ρ_i	load shedding percent of the bus *i*
P_{Gi}	Real power generated at bus *i*
Q_{Gi}	Reactive power generated at bus *i*

P_{Di}	Real load powers at bus *i*
Q_{Di}	Reactive load powers at bus *i*
V_i	Voltage magnitude at bus *i*.
θ_i	Phase angle at bus *i*.
r_i	Load shedding at bus *i*
S_{ij}^{from}	The MVA power flows in right-of-way *i*−*j* terminal *i*,
S_{ij}^{to}	The MVA power flows in right-of-way *i*−*j*, terminal *j*
V	Voltage magnitude
θ	Phase angle
n	Number of circuits added vectors
θ_{ij}	Difference of phases angles of the *i* and *j* buses
r	Vector with elements r_i
r_i	Artificial generators added in each load bus
$f_{ij,y}$	Power flow through the candidate circuit y in path $\mathrm{i} - \mathrm{j}$
$w_{ij,y}$	Binary variable related to the candidate circuit y in path $\mathrm{i} - \mathrm{j}$
f	Vector with elements f_{ij}
f_{ij}	Active power flow through line $i - j$
f^0	Vector with elements f_{ij}^0
f_{ij}^0	Active power flow through line $i - j$ of base topology
g	Vector with elements g_i
g_i	Generation at bus *i*

CONTENTS

CHAPTER 1

INTRODUCTION

Transmission network is an important part of the power system aiming to transmit the power from generators to loads. Nowadays the energy demands are increasing, as a result transmission networks must be expanded to satisfy the emerging requirements. Transmission expansion planning (TEP) must be considered in a timely and appropriate manner to address the technical requirements of a network economically. Therefore, the transmission network expansion should be planned in advance. From the viewpoint of the structure of power systems, TEP is analyzed in both regulated and restructured environments. In regulated power systems, the parts of generation, transmission, and distribution are operated via a centralized structure. In other words, for expansion of such systems, the existing of a complete coordination among different parts is ensured. In regulated power systems, the loads are forecasted in advance, and then the generation and transmission systems are expanded in to supply demands at the least cost. Moreover, the reliability of all the aforementioned systems must be addressed that result satisfying the technical constraints of the systems. In such environments, the uncertainties in the optimization problem are modeled deterministically. Due to low efficiency, the regulation in power systems has been gradually abandoned, and the economic structure of power systems, known as the restructured systems, have been transformed from the regulated to privatized sectors. The main objective of deregulation is to increase the efficiency in the electric power generation and consumption via competition. Deregulation has had two major effects on the planning of power systems: firstly, it has greatly increased the uncertainty, and secondly, it has modified some goals of the power system planning. There are many researches in the literature, which have been conducted in deregulated environments. As the main objective of this thesis relies on regulated environments, those studies related with restructured environments are excluded from this thesis.

1.1 POWER SYSTEM PLANNING

The planning of power systems is responsible for supplying electrical energy; in other word, it is responsible for finding the best solution of utilizing the energy. Basically, the consumption of electrical energy, which leads to the expansion of power systems, is a function of the investment rate of industrial production, energy consumption per capita, and the proportion of different consumptions. From the planning horizon`s standpoint planning, three different types of planning have been proposed.

A - Long-term planning

In this horizon, transmission/generation equipment expansion planning and long-term fuel planning are carried out for the period of several years to few decades. It should be mentioned that the intervals considered to perform these studies are generally in the range of 20 to 30 years and in special cases, it is even longer.

B - Operational planning

The operational planning model includes the time frame from several months up to one year. It does not include major investment decisions, except the special cases such as capacitor placement in transmission lines or distribution systems.

Operational planning focuses on number of items such as the purchase of fuel in medium term, the way to transmit it, planning of maintenance operations, loss control strategies, observance of the contracts for power transmission, load management, decentralized production, and pricing procedure.

C - Short-term planning model

The time frame of these studies varies from half an hour to several days, up to a week. Some measures of this model include the unit commitment program, one-hour or half-an-hour production planning, and scheduling of power exchange according to the dynamical constraints in the production and consumption aspects based on the network security. Unit commitment (UC) and optimal power flow (OPF) are two major examples of this category.

It should be noted that TEP problem lies in the area of long-term studies of the system.

1.2 TRANSMISSION EXPANSION PLANNING

The expansion of transmission systems is currently the topic of several studies and this indicates that transmission systems are operating close to its capacity due to insufficient investments in infrastructure. As a result, many researches on different issues related to transmission systems have been carried out, as evident in the special section of the IEEE

Transactions on Power Systems on transmission investment, pricing, and construction. Clearly, there is a need to find better methodologies to manage the expansion and at the same time, take into account various constraints, including varying levels of technical, social, and economic growth constraints.

There are many research studies on transmission expansion planning, which focus on different aspects of the problem. Some propose new planning models, which in addition to their technical aspects, depend on the policy and regulatory issues, such as market-driven expansion models. Other studies offer new mathematical and programming tools for solving this complex problem. In this domain, there are two major groups of studies: (1) exact methods, and (2) approximate methods. The exact methods are based on the classical nonlinear mixed-integer programming, such as the branch-and-bound variations and the Benders decomposition. On the other hand, the approximate methods are heuristic and meta-heuristic methods. The third employed option is to present new mathematical formulations, which are associated with new technical procedures for system expansion. For example, the TEP problem can be represented by a mathematical optimization problem, in which the objective is to define the location and number of the new transmission lines that satisfy the estimated demand at a minimum cost, subject to safe operating conditions. It is worth to state that the research presented here falls within the bounds of second approach to the problem.

The aim of this research is to develop and apply a novel meta-heuristic procedure directly to a DC power flow based model in order to efficiently solve the TEP problem. In this thesis, the security-constrained TEP problem has been investigated in both static and dynamic form. In addition, two cases of the static TEP problem, with and without generation resizing, have also been investigated. The proposed method has achieved solutions with good accuracy, stable convergence characteristics, simple implementation and satisfactory computation time. The analyses have been performed within the mathematical programming environment of AMPL using both CHA and hybrid procedures and a detailed comparison has also been presented.

1.3 OBJECTIVE

The main objectives of this work can be stated as follow:

a. to provide some new models for TEP problem under security constraints

b. to propose a new constructive heuristic algorithm to TEP problem under security constraints
c. to introduce a special tabu search for static and multi-stage TEP problem
d. to propose a hybrid meta-heuristic algorithm based on tabu search and genetic algorithm for solving TEP problem
e. to extend the proposed hybrid algorithm to static and multi-stage TEP problem under security constraints, which is more complex and difficult when compared with the traditional TEP problem

1.4 THESIS OUTLINE

This thesis covers seven main chapters:

Chapter 2 provides an introduction to the TEP problem. It then presents a review of solution methods for static and dynamic TEP and TEP under security constraints.

Chapter 3 demonstrates an overview of formulation of static and dynamic TEP problem and then proposes various models to TEP considering security constraints.

Chapter 4 presents a heuristic algorithm to solve static and multi-stage TEP problem considering security constraints. The aforementioned method has been implemented in order to make use of a part of tabu search algorithm and it is the main objective of thesis. In fact, in tabu search algorithms, the sizes of neighborhoods are extremely large to be evaluated by the algorithm. Therefore, by using this heuristic algorithm, the search space of TS can be reduced.

Chapter 5 provides a review of tabu search and genetic algorithms. In this chapter, a special tabu search for solving TEP is also proposed. In addition, the optimization process and constraint handling techniques of the proposed algorithm are explained.

Chapter 6 exhibits the fundamentals and implementation process of the proposed hybrid algorithm for static and dynamic TEP problem under security constraints.

Chapter 7 provides the results obtained from the application of the hybrid algorithm in five real large-scale power systems. In addition, the results of the proposed method for planning with considering security constrains are compared with the ones in the literature.

Chapter 8 covers the conclusions added to the research activities planned for future.

CHAPTER 2

FUNDAMENTALS OF TRANSMISSION EXPANSION PLANNING PROBLEM

2.1 INTRODUCTION

The main objective of the TEP problem in an electric power system is to define where, how many, and when new transmission lines must be added to the system in order to provide the forecasted power demand and also, to make its operation viable for a pre-defined planning horizon at the minimum cost (VINASCO et al., 2011).

In general, the problem of transmission network expansion planning has a four-step structure as follows:

Initial information:

- Trends of generation and load for the time domain of design studies
- Data of base topology
- Different candidate paths (inc. cost, capacity, and route)
- Types of lines with full details
- Equipment costs
- Generation costs.

Objective function:

The function is used to measure of the acceptability of solutions, usually expressed in terms of costs. This function could include the costs related to investment, operation, and losses.

Constraints:

Including all technical, economic, financial, environmental, social, and other constraints. The constraints are, of course, divided into two equal and unequal constraints.

Criteria:

The plan criteria may vary according to a wide range of parameters including reliability, adequacy, and security.

One of the major problems in the TEP is to identify transmission adequacy under the unavoidable and forced outage of various equipment of the system. Adequacy of transmission network can be investigated using many methods of load distribution. On the other hand, new equipment of the transmission system are added to reduce the non-acceptability of voltage buses and additional loads of lines and transformers according to certain rules.

From a broader perspective, network expansion planning can be classified as static and dynamic, according to the treatment of the study period. The planning is called static, if the planner seeks the optimal circuit additional set for a single year on the planning horizon. In other words, in this case, the planner is not interested in determining when the circuits should be installed, but in finding the final optimal network state for a future single definite situation (static situation). On the other hand, if a period of several years is under consideration, and an optimal expansion strategy is outlined along the whole planning period, the planning is classified as dynamic (i.e. it is a year by year expansion plan). In this case, the coupling among the years makes the problem more complex. In fact, an investment scheduled for a particular year can positively impact on the years afterward and also can contribute to solve the problems elsewhere in the system, given the interconnected nature of transmission networks. The dynamic models are currently in an under-developed status and they have excessive limitations concerning the system’s size and the system modeling complexity. The dynamic planning problem is very complex and large, because it must take into account not only the sizing and placement, but also timing considerations. This results in a large number of variables and restrictions to be considered, and thus, it requires an enormous computational effort to get the optimal solution, especially in real power systems. Few works about dynamic models for real world transmission planning problems can be found in the technical literature (LATTORE et al., 2003).

2.2 SOLUTION METHODS FOR TEP PROBLEM

There are many optimization techniques that have been proposed to solve the TEP problem in regulated power systems. These techniques can be segmented into three major groups: (i) heuristics, the so-called constructive heuristic algorithms in particular, (ii) classical optimization techniques, and (iii) the meta-heuristics.

An overview of these methods is discussed in the follow subsections.

2.2.1 Heuristic Methods

They are the simplest methods to be implemented and understood. The constructive heuristic algorithms (CHAs) are well known in TEP. When the mathematical model is complex for solving, the CHA can be useful. CHA is an iterative process that finds a good quality solution in a step-by-step process. In network transmission planning, a circuit (transmission line or transformer) is chosen by using a sensitivity index and is added to the system in each iteration. The iterative process continues until the sensitivity index indicates that a feasible topology has been found and new circuits are no longer necessary. The difference with the existing CHA algorithms is basically referred to the sensitivity indicator. Usually, the sensitivity index determines the most attractive circuit to update the base topology.

Many of the CHAs in the literature can be divided into two following categories:

i) The algorithms that use electrical system performance to make sensitivity index

ii) The algorithms that use the relaxed version of the mathematical model.

The algorithms similar to least-load-shedding (PEREIRA; PINTO, 1985) and least effort (MONTICELLI et al., 1982) belong to group (i), and Garver (GARVER, 1970), Villasana–Garver–Salon (VGS) (VILLASANA et al., 1985) added to the algorithm proposed in this work belong to group (ii).

In least-load-shedding algorithm (PEREIRA; PINTO, 1985), the sensitivity index tries to identify the circuit that would provide the most significant reduction in load shedding. In this case, CHA solves a linear programing (LP), while the operation constraint is load shedding. It can be stated that sensitivity index is an approximation, due to this fact that the selected circuit may not guarantee the least load shedding. Although the selected circuit may provide a reduction in load shedding, it may not facilitate the process of finding the optimal topology. All these problems may partially arise when the sensitivity index considers the circuits' costs.

In the least-effort algorithm (MONTICELLI et al., 1982), the index tries to identify the circuit that would provide a better power-flow distribution in the system. For this case, CHA solves a linear algebraic system, which can be replaced by an LP whose operation problem would be the possible overloads in the current topology. As occurs with the previous index, it is approximate and therefore, the selected and inserted circuits do not guarantee the best reduction in system overloads. The sensitivity index, as with the previous one, represents a local observation and this drawback can be partially dealt by considering the circuit's cost. A major advantage of using either the least-effort algorithm or the least-load-shedding algorithm is that both employ the DC model directly.

The model that is used in Garver's algorithm is transportation model (TM). The TM is a relaxed version of DC model provided from elimination of the third constraint in DC model. In fact, TM is a mixed-integer linear optimization problem. Garver's algorithm relaxes the integrality of the investment variable and solves TM (i.e. making $n_{ij} \geq 0$ and solving the problem as an LP). The LP solution might not be feasible for TEP problem, and thus, this solution can be deployed as a sensitivity index for CHA. In Garver's algorithm, in each step, an LP with the current topology is solved while the number of new circuits might not be integer, which may facilitate the minimum investment.

On the other hand, VGS algorithm can find a good solution for DC model than the best CHA ever proposed in the literature (VILLASANA et al., 1985). By relaxing the third constraint in DC model, a hybrid model will be produced where an LP solver can be applied to solve such a hybrid model and to identify the most important circuit at each step of the algorithm. It is worth to express that in the hybrid model, the relaxed constraint will only be considered to the circuits of the current topology.

Usually, the generalized CHA finds the optimal and sub-optimal solutions for small and medium-sized systems at a small computational cost. However, for large and complex systems, the algorithms based on a sensitivity index generally find the solutions barely of an acceptable quality, and are frequently too far from the optimal solution. Two types of problem, leading to deviation of the search process from the optimal solution, have been identified by Romero et al. (ROMERO et al., 2003). They relate to the selection criterion.

Some problems may occur when:

- Expensive circuits are selected.
- The selection is based on a low value of sensitivity index.

2.2.2 Mathematical optimization methods

By using a calculation procedure that solves a mathematical formulation of the planning problem, mathematical optimization methods search for the optimal expansion plan. As a result, the TEP is converted into an optimization problem with an objective function subject to a set of constraints in the problem formulation. In order to solve the TEP problem, for instance, linear programming (GARVER., 1970), nonlinear programming (EKWUE, CORY, 1984; YOSSEF; HACKAM, 1989), dynamic programming (DUSONCHET; EL-ABIAD, 1973), branch and bound (HAFFNER et al., 2000; HAFFNER et al., 2001), mixed-integer programming (BAHIENSE et al., 2001; ALGUACIL et al., 2003) and Benders decomposition (BINATO et al., 2001), there have been a number of applications of mathematical methods so far.

2.2.2.1 Linear Programming

In 1970s, a linear programming method was proposed by Garver to solve the TEP problem (GARVER, 1970). The method produced a feasible transmission network with near-minimum circuit miles using as input any existing network plus a load forecast and generation schedule. In addition, it was applied to long-term planning of electrical power systems. In (GARVER, 1970), the two main steps of the method were presented, in which the planning problem is formulated as load flow estimation and new circuit selection could be searched based upon the system overloads. Whereas the result was called "linear flow estimate", the linear programming was applied to solve the minimization problem for the required power movements. A circuit addition was selected based on the location of the largest overland in this flow estimate. These two steps were repeated until no overload remains in the system.

2.2.2.2 Nonlinear Programming

In 1984, Ekwue and Cory (EKWUE; CORY, 1984) proposed and applied an interactive method to optimize the TEP problem. This method was based on a single-stage optimization procedure using sensitivity analysis and the adjoint network approach to transmit power from a new generation station to a loaded AC power system. The non-linear programing technique of gradient projection, which is followed by a round-off procedure, was used for this optimization method.

2.2.2.3 Dynamic Programming

El-Abiad and Dusonchet (DUSONCHET; EL-ABIAD, 1973) proposed the discrete dynamic optimizing (DDO) to solve the transmission planning problem. The primary idea of this method is combining the deterministic search procedure of dynamic programming and then, optimizing a probabilistic search coupled with a heuristic stopping criterion. This method offers the way to deal with two problems, i.e. the size and complexity of the procedures, in order to assess the performance of alternate strategies, through the use of a probabilistic search procedure and dynamic programming. The other advantage of this method is the probability to consider the planner's experience through the neighborhood concept in the solution process.

2.2.2.4 Integer and Mixed-Integer Programming

In 2003, a mixed-linear programming approach for solving the static TEP problem that includes line losses considerations was proposed by Alguacil et al. (ALGUACIL et al., 2003). The proposed mixed-integer formulation offers an accurate optimal solution. Meanwhile, it is flexible enough to build new networks and to reinforce the existing ones. Whereas the results confirm the accuracy and efficiency of this computation approach, the proposed technique was tested by Garver 6-bus system, the IEEE reliability test system, and the Brazilian system.

2.2.2.5 Branch and Bound

A new specialized branch-and-bound algorithm for solving the TEP problem was presented by Haffner et al. (HAFFNER et al., 2001). However, the optimality was the use of a transportation model for representing the transmission network, which was obtained at a cost. Then, the expansion problem became an integer linear programming (ILP), which was solved by the proposed branch-and-bound method. The branch-and-bound algorithm was specialized by employing specific knowledge about the problem for both the selection of candidate problems and for the selection of the next variable to be used for branching in order to control combinatorial explosion. Moreover, in order to reduce the gap between the optimal integer solution (ILP program) and the solution obtained by relaxing the integrality constraints, special constraints were used

2.2.2.6 Benders Decomposition

To solve the real-world power transmission network design, a new Benders decomposition approach was applied by Binato et al. (BINATO et al., 2001). This approach was characterized by using a mixed linear (0-1) disjunctive model. Besides the traditional Benders cuts, it ensures the optimality of the found solution, by using additional constraints and evaluating iteratively. The use of Gomory cuts iteratively evaluated from master sub-problem and the use of Benders cuts evaluated from relaxed versions of slave sub-problem in (BINATO et al., 2001). In order to improve the practical convergence to the optimal solution of Benders approach, Gomory cuts within Benders decomposition were used.

2.2.3 Meta-heuristic methods

In addition to mathematical optimizations and heuristic methods, meta-heuristic methods have become the current alternative to solve the TEP problem. These meta-heuristic techniques are efficient algorithms applied to optimize the transmission planning problem. There have been many applications of meta-heuristic optimization methods to solve TEP problem. For example, tabu search (SILVA et al., 2001), simulated annealing (ROMERO et al., 1996), genetic algorithms (SILVA et al., 2000), artificial neural networks (AL-SABA; EL-AMIN, 2002) particle swarm (JIN et al., 2007) and hybrid artificial intelligent techniques (SONG; IRVING., 2001). The details of these methods are as discussed follow.

2.2.3.1 Tabu Search (TS)

TS is an iterative search procedure, which moves from one solution to another looking for improvements to find the best solution. The basic concepts in TS are movements and memory. A movement is defined as an operation of jumping from one solution to another while the memory is used for different objectives, for instance, for guiding the search to avoid cycles. Using the concept of memory, specific movements are defined as forbidden or taboo. In (SILVA et al., 2001), a tabu search algorithm was proposed for solving static TEP problem are presented. The results from two tested real-world case studies were a robust technique to be applied to this planning problem. In both case studies, the acceptable quality of the results, which were produced from the intensification phase, qualified the applied strategy. In other words, it looks for consistent candidate circuits (in different plans) in order to build a consistent transmission expansion plan. Unlike the classical methods of optimization, this algorithm is able to avoid local optimum solutions and consequently, can have a greater chance to find the global optimum solution.

2.2.3.2 Simulated Annealing (SA)

SA starts the optimization process with a given initial configuration from which, a set of candidate configurations are generated through a cooling process. A candidate configuration will be accepted if its investment cost is smaller than the cost of the current configuration with a certain probability. However, it is important to express that the configurations presenting the costs higher than the cost of the current configuration might be also accepted. This feature is crucial, because it unfavorably allows the simulated annealing process to avoid local optimal solutions. The other essential issue regards about the cooling scheme in simulated annealing, which basically controls how temperature is decreased and is aimed at obtaining the perfect solution. Two other important aspects of simulated annealing are the search space representation and also, the transition mechanism that governs the way the search space is explored.

The SA technique has been successfully applied to several engineering optimization problems, including power system optimization problems. This approach was proposed by Romero (ROMERO et al., 1996) in order to solve the long-term transmission system expansion planning problem. The method was compared with a conventional optimization method, which is based on mathematical decomposition with a zero-one implicit enumeration procedure. In (ROMERO et al., 1996), prior to applying the proposed technique to a large test system for which no optimal solution had been known before, two small test systems were used in order tune the main parameters of the simulated annealing process. Some interesting solutions were achieved with the costs of approximately 7% less than the best solutions for that particular example system, which were obtained by optimization and heuristic methods.

2.2.3.3 Expert Systems

The expert system, which uses the knowledge and interface procedure to solve problems, is a rule-based or a knowledge-based system. Galiana et al. (GALIANA et al., 1992) reviewed the state of expert systems and knowledge engineering in transmission planning. The details of their study include the principal elements of transmission planning, such as the principal activities that constituted transmission planning, general planning methodology, the aim, constraints, prerequisites that must be met by the planner, and a selection to indicate the potential area and to justify the use of expert systems in transmission planning. Furthermore, an expert system approach for multi-year STEP (short-term expansion planning) was presented in (GAJBHIYE et al., 2008). This approach addresses the reactive power management issues in order to ensure the efficiency of transmission system and the adequate quality of voltage supply, which could be measured by network congestion and percentage losses in the system. By using the enhanced fast decoupled load flow (FLDF), an expert system approach for STEP was proposed to address these reactive power issues.

2.2.3.4 Evolutionary Algorithms

An evolutionary algorithm is based on the Darwin's principle of "survival of the fittest". It begins with initializing a population of candidate solutions to a problem and then, new solutions are generated by randomly varying those of initial population. All solutions are evaluated with respect to how well they address the task. At the end, in order to eliminate bad solutions, a selection operation is applied. In (CECILIANO; NIEVA, 1999), an evolutionary programming approach for transmission network planning in electric power systems was presented. The proposed evolutionary programming algorithm was tested in two electric power systems, including Garver 6-bus system and Mexican electric power system.

2.2.3.5 Genetic Algorithms (GA)

GA as a global search approach, based on the mechanics of natural selection and genetics, is different from conventional optimization techniques, since it uses the concept of population genetics to guide the optimization search. GA searches from one population to another, instead of point-to-point search. In order to solve the problem in transmission network expansion planning, Gallego (GALLEGO et al., 1998) presented an extended GA. In their algorithm, there are two main improvements introduced in (GALLEGO et al., 1998). Firstly, the initial population is obtained by using a method based on conventional optimization and secondly, the mutation approach has been inspired from the simulated annealing technique.

Silva et al. (SILVA et al., 2000) also proposed the application of an improved GA for solving the TEP problem. GAs had demonstrated the ability to deal with TEP problem, including non-convex, non-linear, integer-mixed problems. They generate better performance compared with a number of other mathematical methodologies. In order to improve their performance in solving the TNEP problem for three real large-scale transmission systems, some special features have been added to the basic GAs. According to the findings (SILVA et al., 2000), the proposed approach was not only suitable, but a promising technique for solving such problems.

In 2001, a reliable approach for solving the TEP problem using GAs was presented by Gil and Silva (SILVA et al., 2000). The procedure was produced by utilizing unfeasible solutions found by the GA. It was based on the "loss of load limit curve" of the transmission system under study. The modified procedure has made the GA more robust to solve different large-scale transmission expansion problems and this proposed method was proved to be efficient for solving in two real large-scale power systems (GIL; SILVA, 2001). Escober et al. (ESCOBER et al., 2004) proposed an efficient GA to solve a mixed integer nonlinear programming problem, which is a coordinated and multistage transmission planning problem. The mentioned GA, which has a set of specialized genetic operators, utilizes an efficient form for generation of the initial population that finds high-quality suboptimal topologies for large-size and high-complexity transmission systems. According to the achieved results, an efficient GA was efficiently and effectively implemented for multistage planning on medium- and large-size systems.

2.3 SECURITY TEP PROBLEM

It should be stated that the researches on the TEP problem considering security constraints especially for multi-stage planning are scarce. The TEP problem with security constraints is traditionally solved in two phases (SEIFU et al., 1989; MONTICELLI et al., 1982). In the first phase, the TEP problem without considering security constraints is solved. In the second phase, new circuits are added into the system in order to make it operate amply in case of line outages, belonging to a pre-defined list of contingencies. The significant advantage of this strategy is that it finds an expansion plan with a relatively small computational effort. In contrast, the main disadvantage is that the obtained plan is not optimal. Additionally, the obtained plan with security constraints is highly influenced by the expansion plan of the first phase, and this influence can become critical in large and complex systems (SILVA et al., 2005).

In (OLIVEIRA et al., 2004), at first, a mixed integer non-linear programming problem for multistage TEP considering security constraints and considering multiple dispatch scenarios is proposed. Then, a disjunctive formulation is used to linearize the non-linear constraints. A heuristic algorithm is taken to identify the most critical dispatches and contingency scenarios. Their approach has obtained a heuristic method in order to find a solution for the TEP with security based on the solutions of each stage. Therefore, the achieved solution may not be optimum.

A specialized GA was used in (SILVA et al., 2005) to solve the TEP with security constraints. In the mentioned article, a mixed integer programming formulation is employed for modeling the problem. Having analyzed the results, it was revealed that the inclusion of security constraints on TEP increases the investment cost of the expansion plan significantly and therefore, a proper definition should be made in the set of contingencies. The mathematical formulation shown in that article is of this disadvantage that there is no efficient solver on the market to solve it when it is large and also the GA approach does not guarantee to find the optimal solution of the problem.

A static multi objective TEP problem considering security constraints in the electricity market was proposed in (MAGHOULI et al., 2009), in which a GA based on the NSGA II is used to solve the problem followed by a fuzzy decision analysis to find the optimum plan.

In (ZHANG et al., 2012) a disjunctive linear model added to a strategy for solving the MTEP problem with N−1 security constraints were presented. The advantage of the proposed model is that it can be solved by a commercial solver, which guarantees the optimal solution. On the other side, the main disadvantage is that for large scale system, such as North Brazilian system, the solver is unable to converge.

2.4 CONCLUSION

This chapter has covered the basis of TEP problem. It has also provided a literature survey with regard to the application of a variety of solution techniques to the planning problem. Over several past decades, many researchers have focused on TEP problem and set their interest mostly on static/dynamic planning models without considering security constraints. Unfortunately, the security planning models are still in an un-developed status as security planning models are subject to some limitations for their application to real-world transmission systems.

CHAPTER 3

MATHEMATICAL MODEL FOR TRANSMISSOSION EXPANSION PLANNING

3.1 INTRODUCTION

The use of mathematical models in decision making and optimization problems has become an inevitable fact. The ability of mathematical models to analyze, solve, and be understood has resulted that complex problems such as TEP to be also included in this context. Hence, several mathematical models have been considered in this part.

In terms of the number of objectives in modeling, mathematical programming can be of one or more objective functions. As a basis for classification of models, this difference can help to divide them into two main groups of single-objective models and multi-objective models. According to the performed studies, the single-objective models have been used in the majority of cases. The main difference among single-objective models is the way they formulate the constraints.

In a decision making process for complex systems, a variety of criteria are taken into account, which have been usually different with each other in terms of the dimension and the degree of importance. They are often in conflict with each other so that it is not easy to optimize them simultaneously. The need to solve these models (multi-objective models) has caused the creation and development of multiple criteria for decision-making methods. As a result, it allows the designers and planners to simultaneously solve and analyze the problem with details.

Power transmission networks have not been an exception and have been assisted by various multiple-criteria models. In the simplest and most basic form of a multi-criteria problem, if all the objectives are of the same type, dimension, and direction, it will be possible to add them together and convert them into a single-objective problem. Sometimes, in a TEP problem, the criteria of investment cost and ohmic losses both have cost dimensions. Thus, if they are considered equally important, they can be summed up and converted into a single-objective problem.

One of the oldest methods for modeling in decision-making conditions is ideal planning criteria, which has been extensively used to solve multi-objective problems. This method is based on minimization of the sum of absolute values of objective deviations from their ideal amount. For example, if the investment cost, un-supplied load, and space of additional lands are three objectives in the TEP problem, the planning manager will determine a value as ideal or the final desired value for each of these purposes, which can be approximated by GP method. In an ideal planning, it is possible to take into account different weights for different objectives.

In TEP optimization problem, normally the cost of transmission lines is considered as objective function and the basic optimal power flow and security constraints are considered as the limitations of the problem.

Regarding the mathematical modeling to represent the electrical system, the following models for TEP can be employed: (1) AC model, (2) DC model, (3) transportation model, (4) hybrid model, and (5) disjunctive linear model.

3.2 TRANSMISSION EXPANSION PLANNING WITHOUT CONSIDERING SECURITY CONSTRAINTS

In this section, the mathematical models of the static and multistage TEP problem without considering security constraints, using the AC model and the DC model are presented.

3.2.1 Static planning

3.2.1.1 AC model

The full static AC model of the TEP problem can be formulated as below:

$$min\, v = \sum c_{ij} n_{ij} + \alpha \sum_i \rho_i \tag{1a}$$

Subject to:

$$P_i(V,\theta,n) - P_{Gi} + (1-\rho_i)P_{Di} = 0 \tag{2a}$$

$$Q_i(V,\theta,n) - Q_{Gi} + (1-\rho_i)Q_{Di} = 0 \tag{3a}$$

$$P_{GK}^{min} \le P_{GK} \le P_{GK}^{max} \tag{4a}$$

$$Q_{GK}^{min} \le Q_{GK} \le Q_{GK}^{max} \tag{5a}$$

$$V_i^{min} \le V_i \le V_i^{max} \tag{6a}$$

$$(n_{ij} + n_{ij}^0)S_{ij}^{from} \le (n_{ij} + n_{ij}^0)S_{ij}^{max} \tag{7a}$$

$$(n_{ij} + n_{ij}^0)S_{ij}^{to} \le (n_{ij} + n_{ij}^0)S_{ij}^{max} \tag{8a}$$

$$0 \le n_{ij} \le \overline{n}_{ij} \tag{9a}$$

$$S_{ij}^{from} and\ S_{ij}^{to} \ge 0 \tag{10a}$$

$$0 \le \rho_i \le 1 \tag{11a}$$

$$n_{ij}\ integer\ and\ \theta\ unbounded \tag{12a}$$

$$ij \in \Omega, i \in \Omega_b, k \in \Omega_g \tag{13a}$$

Eqs. (2a) and (3a) represent the conventional power flow equations, where

$$P_i(V,\theta,n) = V_i \sum_{j\in\Omega_b} V_j[G_{ij}(n_{ij})cos\theta_{ij} + B_{ij}(n_{ij})sin\theta_{ij}] \tag{14a}$$

$$Q_i(V,\theta,n) = V_i \sum_{j\in\Omega_b} V_j[G_{ij}(n_{ij})sin\theta_{ij} - B_{ij}(n_{ij})cos\theta_{ij}] \tag{15a}$$

The conductance and susceptance matrices are formed as is shown in (16a)-(19a), respectively.

$$G_{ij}(n_{ij}) = -(n_{ij} + n_{ij}^0)g_{ij} \tag{16a}$$

$$G_{ij}(n_{ij}) = \sum_{j\in\Omega_i}(n_{ij} + n_{ij}^0)g_{ij} \tag{17a}$$

$$B_{ij}(n_{ij}) = -(n_{ij} + n_{ij}^0)b_{ij} \tag{18a}$$

$$B_{ij}(n_{ij}) = b_i^{sh} + \sum_{j\in\Omega_i}(n_{ij} + n_{ij}^0)b_{ij} \tag{19a}$$

In this model Eq. (1a) is the objective function of the AC model containing the sum of the investment costs of the newly added transmission lines as well as the penalty load curtailment. The active and reactive power limits of generators are represented by (4a) and (5a), respectively and voltage magnitude ranges are given in (6a). Transfer capacity of MVA power flows is restricted by (7a) and (8a). The capacity of circuits added in right-of-way $i - j$ is restricted by (9a) and the load shedding ranges are given in (11a). The decision variables are the voltage magnitude and angles, the number of circuits added in right of- way $i-j$ and loading factor.

The application of the AC model has several advantages, such as:

I. The possibility of carrying out other types of studies, after solving the AC integrated TEP problem, for example voltage stability, nodal analysis, transient stability analysis, and so on.
II. Incorporating other non-linear operation characteristic devices in the TEP problem, for example, the FACTS controllers.
III. Incorporating the determination of the transmission system's precise real losses in a trivial way and as a sub-product of the optimization process.

The AC model of TEP problem is a mixed integer non-linear programming problem and very difficult to solve, especially for large-scale systems. Therefore, linearization of the non-linear functions becomes important. To solve the TEP problem, network synthesis techniques are generally used. That is, relaxed mathematical models of the TEP problem considering only the active power of the system. In this type of approach, the long-term transmission network planning is solved in the first place and, then, the reactive power planning is solved in a way that these expansion plans are evaluated with considering the operational constraints. In such phase, one must use AC load flow, short circuit, and transient stability analysis programs. The DC model, transportation model, hybrid model and disjunctive model are the simplified models used to solve the TEP problem.

3.2.1.2 DC model

The DC model of TEP problem is the most frequently used model in transmission planning and there are a numerous number of publications discussing this model.

When the power grid is represented by the DC power flow model, the mathematical model of the transmission expansion static planning problem is formulated as follows:

$$min\, v = \sum c_{ij} n_{ij} + \alpha \sum r_i \tag{1b}$$

$S.t.$

$$Sf + g + r = d \tag{2b}$$

$$f_{ij} - \gamma_{ij}\left(n_{ij}^0 + n_{ij}\right)\left(\theta_i - \theta_j\right) = 0 \tag{3b}$$

$$|f_{ij}| \le (n_{ij}^0 + n_{ij})\bar{f}_{ij} \quad (4b)$$

$$0 \le n_{ij} \le \bar{n}_{ij} \quad (5b)$$

$$0 \le g_i \le \bar{g}_i \quad (6b)$$

$$0 \le r_i \le d_i \quad (7b)$$

$$ij \in \mathit{\Omega}, i \in \Omega_b \quad (8b)$$

Equation (1b) is the objective function of the DC model containing the sum of the investment costs of the newly added transmission lines as well as the penalty load curtailment. In Eq. (2b), Kirchhoff's Current Law (KCL) in the equivalent DC network is modeled. Eq. (3b) is an expression of Ohm's law for the equivalent DC network, while Kirchhoff's Voltage Law (KVL) is implicitly taken into consideration. Equations (4b), (5b), (6b), and (7b) are based on the line power flow, generator's capacity, the limitations of line numbers, and load shedding vector, respectively. The TEP problem as formulated above is a mixed integer non-linear problem. It is a difficult combinatorial problem, which may lead to combinatorial explosion on the number of alternatives that need to be searched.

3.2.1.3 Transportation model (TM)

This model is obtained by relaxing the nonlinear constraint Eq. (3b) of the DC model described above. In this case the network is represented by a transportation model, and the resulting expansion problem becomes an integer linear problem (ILP).

The formulation of this model is as follows:

$$min\, v = \sum c_{ij} n_{ij} + \alpha \sum r_i \quad (1c)$$

S.t.

$$Sf + g + r = d \quad (2c)$$

$$|f_{ij}| \le (n_{ij}^0 + n_{ij})\bar{f}_{ij} \quad (3c)$$

$$0 \le n_{ij} \le \bar{n}_{ij} \quad (4c)$$

$$0 \le g_i \le \bar{g}_i \quad (5c)$$

$$0 \le r_i \le d_i \quad (6c)$$

$ij \in \Omega, i \in \Omega_b$ (7c)

This problem is normally easier to solve than the DC model although it maintains the combinatorial characteristic of the original problem. An optimal plan obtained by the transportation model is not necessarily feasible for the DC model, since some part of the constraints have been ignored. Thus, depending on the case, additional circuits are needed in order to satisfy the constraint in Eq. (3b), which apparently implies the higher investment cost.

3.2.1.4 Hybrid linear model

The hybrid model combines the characteristics of the DC model and the transportation model. There are various ways to formulate hybrid models, although the most common is that which preserves the linear features of the transportation model. In this model, it is assumed that the constraint in Eq. (2), KCL, is satisfied for all nodes of the network, whereas the constraint in Eq. (3), which represents Ohm's law (and indirectly, KVL), is satisfied only by the existing circuits (and not necessarily by the added circuits). Therefore, in the hybrid model, the power flows through circuits in the existing transmission lines are represented separately from the flows of candidate transmission lines. The flow of existing transmission lines is represented by variable f_{ij}^0 and for candidate transmission lines, by f_{ij}. Ignoring this constraint for candidate transmission lines results in a linear model.

$min\, v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} + \alpha \sum_{i\in\Omega_b} r_i$ (1d)

S.t.

$Sf + S^0 f^0 + g + r = d$ (2d)

$f_{ij}^0 - \gamma_{ij}(n_{ij}^0 + n_{ij})(\theta_i - \theta_j) = 0 \quad \forall\ (i,j) \in \Omega_0$ (3d)

$|f_{ij}^0| \le (n_{ij}^0 + n_{ij})\bar{f}_{ij} \quad \forall\ (i,j) \in \Omega_0$ (4d)

$|f_{ij}| \le n_{ij}\, \bar{f}_{ij} \quad \forall\ (i,j) \in \Omega$ (5d)

$0 \le n_{ij} \le \bar{n}_{ij} \quad \forall\ (i,j) \in \Omega$ (6d)

$0 \le g_i \le \bar{g}_i \quad \forall\ i \in \Omega_b$ (7d)

$0 \le r_i \le d_i \quad \forall\ i \in \Omega_b$ (8d)

The advantage of the HLM over the DC model is that it is much easier to be solved while the solution may be infeasible for the DC model. In the hybrid linear model, the constraint for the voltage angle difference is usually neglected.

3.2.1.5 Disjunctive model (DM)

The non-linear DC model of TEP can be transformed to a mixed integer linear model. This model is named disjunctive model and was firstly proposed by Bahiense (BAHIENSE et al., 2001). It is always possible to transform a mixed integer non-linear model by bilinear equations to a linear problem with binary variables using a large enough disjunctive coefficient (M). In the DM, a binary variable is considered for each candidate line. This is different from the DC model, where in the DC model, an integer variable was used to represent all the lines in a corridor. The disjunctive model of the TEP problem (DM) is provided in equations (1e)-(11e).

$$min\, v = \sum_{ij \in \Omega} c_{ij} \sum_{y \in Y} w_{ij,y} \tag{1e}$$

s.t.

$$S^0 f^0 + Sf + g = d \tag{2e}$$

$$f_{ij}^0 = \gamma_{ij} n_{ij}^0 (\theta_i - \theta_j) \qquad \forall\, ij \in \Omega_0 \tag{3e}$$

$$|f_{ij}^0| \le n_{ij}^0 \bar{f}_{ij} \;\; \forall\, ij \in \Omega_0 \qquad \forall\, ij \in \Omega_0 \tag{4e}$$

$$|f_{ij,y}/\gamma_{ij} - (\theta_i - \theta_j)| \le M(1 - w_{ij,y}) \qquad \forall\, ij \in \Omega,\; \forall\, y \in Y \tag{5e}$$

$$|f_{ij,y}| \le w_{ij,y} \bar{f}_{ij} \qquad \forall\, ij \in \Omega,\; \forall\, y \in Y \tag{6e}$$

$$f_{ij} = \sum_{y \in Y} f_{ij,y} \qquad \forall\, ij \in \Omega,\; \forall\, y \in Y \tag{7e}$$

$$0 \le g_{\mathrm{i}} \le \bar{g}_{\mathrm{i}} \qquad \forall\, i \in \Omega_b \tag{8e}$$

$$\sum_{y \in Y} w_{ij,y} \le \bar{n}_{ij} \qquad \forall\, ij \in \Omega,\; \forall\, y \in Y \tag{9e}$$

$$w_{ij,y} \le w_{ij,y-1} \qquad \forall\, ij \in \Omega,\; \forall\, y \in Y | y > 1 \tag{10e}$$

$$w_{ij,y} \quad \text{Binary} \qquad \forall\, ij \in \Omega,\; \forall\, y \in Y \tag{11e}$$

In this model Eq. (1e) calculates the investment cost, (2e) represents the power flow balance constraint or Kirchhoff's current law and there is an equation for each bus. Constraint

(3e) represents the Kirchhoff's voltage law for existing circuits, (4e) is a constraint related to the limit of the flows in the existing circuits for the base topology, (5e) signifies Kirchhoff's voltage law for candidate circuits, (6e) denotes the limits of the flows for candidate circuits, (7e) finds the total flow through the candidate circuits in each path i-j, (8e) represents the limits of generation for each bus, (9e) restricts the number of additions in each path $\mathrm{i}-\mathrm{j}$, (10e) requires the sequential addition of circuits in each path $i-j$, i.e., circuit y can be added only if circuit $\mathrm{y}-1$ is already added and (11e) indicates that the decision variable related to the addition of a new circuit must be binary.

The model (1e)-(11e) is a mixed integer (binary) linear programming (MILP) problem. In this model, the variables related to the candidate lines which should be added are represented by binary variables $w_{ij,y}$. The superiority of this model refers to its formulation based on the mixed binary linear programming. Therefore, a significant advantage of the disjunctive model, in comparison with the original DC model, which can be efficiently solved using MILP solvers, is that in each year, it becomes more efficient. Hence, if the solver presents convergence, the solution that has been found for the problem is an optimal solution.

3.2.2 Multi-stage planning

In this subsection, a mathematical representation of the multistage TEP problem is discussed. The purpose of multistage TEP problem is to minimize the present value of investment cost for transmission expansion over the entire planning periods.

In the previous section, the AC model, DC model, hybrid model, TM, and disjunctive model were applied to the static TEP problem. The multistage planning of the AC model is very difficult and there is no report about this problem, but other models can be extended to multistage planning in order to determine the financial investment for the most economical schedule. For multistage planning, we provide the DC model and hybrid model. The transportation model can be easily extracted from DC model. In the multi-stage model, an expansion investment plan should be determined for the referred base year. Considering an annual discount rate of I, the present values of the investment and operation costs for the base year t_0 with a horizon of T years, are as the followings:

$$c(x) = {c_1(x)}\Big/{(1+I)^{t_1-t_0}} + {c_2(x)}\Big/{(1+I)^{t_2-t_0}} + \ldots\ldots + {c_T(x)}\Big/{(1+I)^{t_T-t_0}}$$

$$= \delta_{inv}^{1} c_1(x) + \delta_{inv}^{2} c_2(x) + \cdots + \delta_{inv}^{T} c_T(x) \tag{1f}$$

where

$$\delta_{inv}^{t} = {}^{1}/_{(1+I)^{t_t - t_0}} \tag{2f}$$

3.2.2.1 Dc Model

Using the relation (1f) and (2f), the multi-stage planning for DC model can be presented as fallow:

$$min\, v = \sum_{t=1}^{T} \left[\delta_{inv}^{t} \sum_{(i,j)\in\Omega} c_{ij} n_{ij}^{t} \right] \tag{1g}$$

S.t.

$$S^t F^t + g^t = d^t \tag{2g}$$

$$f_{ij}^{t} - \gamma_{ij}\left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{t}\right)\left(\theta_i{}^{t} - \theta_j{}^{t}\right) = 0 \tag{3g}$$

$$\left|f_{ij}^{t}\right| \le \left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{t}\right)\bar{f}_{ij} \tag{4g}$$

$$0 \le g_i^t \le \bar{g}_i^t \tag{5g}$$

$$n_{ij}^{t} \le \bar{n}_{ij}^{t} \tag{6g}$$

$$\sum_{t=1}^{T} n_{ij}^{t} \le \bar{n}_{ij} \tag{7g}$$

θ_i unbounded, n_{ij} integer, $\forall (i,j) \in \Omega$, $\forall\, t \in T$, $\forall\, i \in \Omega_b$ (8g)

The variables of the multi-stage TEP problem, constraints in (1g)-(8g) are similar to those of static multistage TEP problem in (1b)-(7b). They only difference is the addition of index t, which indicates the specific stage of planning. In this model, Eq. (1g) denotes the investment in transmission lines projected to the base year. Eq. (2g) represents the power flow balance. Eq. (3g) states the expression of Ohm's law for the equivalent DC network. Moreover, Eqs. (4g) and (5g) are related to the limits of power flow of each branch and the limit of generator capacity, respectively. Finally, Eqs. (6g) and (7g) exhibit the limitations of line numbers.

The DC multistage planning problem is a mixed integer nonlinear programming problem and is difficult to be solved especially in medium- and large-scale transmission systems.

3.2.2.2 Hybrid model

The hybrid liner model of static planning presented in (1d)-(8d), can be extended to a more complex multistage TEP problem as well.

$$min\, v = \sum_{t=1}^{T}\left[\delta_{inv}^{t} \sum_{(i,j)\in\Omega} c_{ij} n_{ij}^{t}\right] \tag{1h}$$

S.t.

$$S^{t}F^{t} + S^{0\,t}F^{0\,t} + g^{t} = d^{t} \tag{2h}$$

$$f_{ij}^{0\,t} - \gamma_{ij}\left(n_{ij}^{0}\right)\left(\theta_{i}{}^{t} - \theta_{j}{}^{t}\right) = 0 \qquad \forall\,(i,j) \in \Omega_0\,,\ \forall\, t\ \in T \tag{3h}$$

$$\left|f_{ij}^{0\,t}\right| \le \left(n_{ij}^{0}\right)\bar{f}_{ij} \qquad \forall\,(i,j) \in \Omega_0\,,\ \forall\, t\ \in T \tag{4h}$$

$$\left|f_{ij}^{t}\right| \le \left(\sum_{t=1}^{t} n_{ij}^{t}\right)\bar{f}_{ij} \qquad \forall\,(i,j) \in \Omega\,,\ \forall\, t\ \in T \tag{5h}$$

$$n_{ij}^{t} \le \bar{n}_{ij}^{t} \qquad \forall\,(i,j) \in \Omega\,,\ \forall\, t\ \in T \tag{6h}$$

$$\sum_{t=1}^{T} n_{ij}^{t} \le \bar{n}_{ij} \qquad \forall\,(i,j) \in \Omega\,,\ \forall\, t\ \in T \tag{7h}$$

$$0 \le g_{i}^{t} \le \bar{g}_{i}^{t} \qquad \forall\, i \in \Omega_b,\ \forall\, t\ \in T \tag{8h}$$

$\theta_{i}{}^{t}$ unbounded, n_{ij} integer (9h)

Where the variables of this model are similar to those of hybrid model presented in (1d)-(8d), except the addition of the index *t*, which shows the specific stage of planning involved. In this model, Eq. (1h) denotes the investment in transmission lines projected to the base year. Eq. (2h) represents the power flow balance. Eq. (3h) denotes the expression of Ohm's law for the equivalent DC network for the base topology. Eqs. (4h) and (5h) are related to the limits of power flow of each branch for circuits in the base topology and in candidate circuits, respectively. Eqs. (6h) and (7h) are related to the limits of transmission lines. Eq (8h) presents the generator capacity. Similar to static hybrid model, this model is much easier to solve than DC model, but the solution may stay far from the optimum solution, especially for large scale systems.

3.3 TRANSMISSION EXPANSION PLANNING UNDER SECURITY CONSTRAINTS

The reliable and efficient operation of a power system largely depends on whether the transmission grid has been meticulously planned or not. In the most planning researches, the security constraints are not considered. In other words, in the literature, the optimal pattern of transmission lines is determined without considering the contingencies caused by the outages of transmission line. A rationally planned power system will not only alleviate the real-time operation pressure caused by the reliability issues, but will also contribute positively to the overall efficiency of the power system.

In some practical issues, such as N-1/N-2 contingency, the number of continuous variables and related constraints increases linearly according to the occurrence of lines outages and number of scenarios, respectively. In these cases, solving the problems become very challenging, since it requires an enormous memory size in order to save the nodes of branch and bound tree. Therefore, it is apparent that practical issues make the problem more complicated both in modeling and solving approaches.

In this section, we propose some mathematical models of the static and multi-stage TEP problem under security constraints. In order to consider the security constraints in TEP, it is necessary to provide a list of congested transmission lines. One strategy is to identify the lines with the most frequent outages based on the historical data, or based on the system operator’s experiences. Another strategy is to solve the proposed model without considering security constraints, and to identify the lines based on optimal solution via the active power flows over a percentage of their maximum capacity (usually between 80% and 95%). It should be considered that the main objective is to identify the most overloaded lines in the system. These lines construct the contingency list to consider the N-1 security. In the proposed models, SC represents the list of congested transmission lines.

3.3.1 Static planning

3.3.1.1 DC model

Taking into account the security constraints, the mathematical DC model of the static TEP problem can be presented in the following structure:

$$min\, v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} \tag{1i}$$

S.t.

$S^p F^p + g^p = d$ $\quad\forall p \in SC$ (2i)

$f_{ij}^p - \gamma_{ij}(n_{ij}^0 + n_{ij})(\theta_i{}^p - \theta_j{}^p) = 0$ $\quad\forall\ (i,j) \in \Omega|(i,j) \neq p,\ \forall p \in SC$ (3i)

$f_{ij}^p = max\{0\ , \gamma_{ij}(n_{ij}^0 + n_{ij} - 1)(\theta_i{}^p - \theta_j{}^p)\}$ $\quad\forall\ (i,j) \in \Omega|(i,j) = p,\ \forall p \in SC$ (4i)

$\left|f_{ij}^p\right| \leq (n_{ij}^0 + n_{ij})\bar{f}_{ij}$ $\quad\forall\ (i,j) \in \Omega|(i,j) \neq p,\ \forall p \in SC$ (5i)

$\left|f_{ij}^p\right| \leq max\{0, (n_{ij}^0 + n_{ij} - 1)\bar{f}_{ij}\}$ $\quad\forall\ (i,j) \in \Omega|(i,j) = p,\ \forall p \in SC$ (6i)

$0 \leq g^p \leq \bar{g}$ $\quad\forall\ p \in SC$ (7i)

$0 \leq n_{ij} \leq \bar{n}_{ij}$ $\quad\forall (i,j) \in \Omega$ (8i)

The variables of the security TEP problem constraints in (1i)-(8i) are similar to those of TEP problem without considering security constraints (1b)-(7b), except the addition of the index *p*, which indicates the specific scenario of planning involved.

In the proposed model, Eq. (1i) presents the objective function of the DC model containing the sum of the investment costs of the newly added transmission lines. In Eq. (2i), Kirchhoff's Current Law (KCL) in the equivalent DC network is modeled for all scenarios. Eqs. (3i) and (4i) are related to Kirchhoff's Voltage Law (KVL). In addition, Eqs. (5i) and (6i) present the limits of flow in the lines. Eqs. (7i) and (8i) are based upon the generator capacity and limitations of line numbers respectively.

The model is a mixed integer non-linear programming problem and very difficult to solve, especially for large-scale systems.

3.3.1.2 Hybrid model

Although the hybrid linear model, originally proposed by Villasana, Garver and Solan, does not consider security constraints, it can be extended to a more complex planning, planning with considering security constraints.

$min\, v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij}$ (1j)

S.t.

$S^p F^p + S^{0^p} F^{0^p} + g^p = d$ $\quad\forall p \in SC$ (2j)

$$f_{ij}^{0p} - \gamma_{ij}\left(n_{ij}^0\right)\left(\theta_i{}^p - \theta_j{}^p\right) = 0 \qquad \forall\ (i,j) \in \Omega_0\ |(i,j) \neq p,\ \forall p \in SC \tag{3j}$$

$$f_{ij}^{0p} - \gamma_{ij}\left(n_{ij}^0 - 1\right)\left(\theta_i{}^p - \theta_j{}^p\right) = 0 \qquad \forall\ (i,j) \in \Omega_0 | \forall\ (i,j) = p,\ \forall p \in SC \tag{4j}$$

$$\left|f_{ij}^{0\,p}\right| \leq \left(n_{ij}^0\right)\bar{f}_{ij} \qquad \forall\ (i,j) \in \Omega_0\ |(i,j) \neq p,\ \forall p \in SC \tag{5j}$$

$$\left|f_{ij}^{0\,p}\right| \leq \left(n_{ij}^0 - 1\right)\bar{f}_{ij} \qquad \forall\ (i,j) \in \Omega_0 | \forall\ (i,j) = p,\ \forall p \in SC \tag{6j}$$

$$\left|f_{ij}^{p}\right| \leq n_{ij}\,\bar{f}_{ij} \qquad \forall\ (i,j) \in \Omega |\ (i,j) \neq p\ , \forall p \in SC \tag{7j}$$

$$\left|f_{ij}^{p}\right| \leq max\{0, \left(n_{ij} - 1\right)\bar{f}_{ij}\,\} \qquad \forall\ (i,j) \in \Omega | ((i,j) \notin \Omega_0 \text{ and } (i,j) = p)\ , \forall p \in SC \tag{8j}$$

$$0 \leq g^p \leq \bar{g} \qquad \forall\ p \in SC \tag{9j}$$

$$0 \leq n_{ij} \leq \bar{n}_{ij} \qquad \forall (i,j) \in \Omega \tag{10j}$$

$$\theta_i{}^p \ \textit{unbounded and}\ n_{ij}\ \textit{integer} \tag{11j}$$

For static hybrid linear model under security constraints, the variables are similar to those of static hybrid model without considering security constraints (1d)-(7d) except the addition of the index p, which indicates the specific scenario of planning involved.

In the proposed model, Eq. (1j) presents the objective function. In Eq. (2j), KCL is modeled for all scenarios. Eqs. (3j) and (4j) are related to KVL for the circuits of the base topology. Eqs. (5j) and (6j) show the limit of flow in lines for the base topology. Eqs. (7j) and (8j) are related to the limits of flow in candidate lines. Eqs. (9j) and (10j) are based on the generator capacity and limitations of line numbers, respectively.

The advantage of the proposed hybrid model over the DC model presented in (1i)-(8i) refers to this fact that it is much easier to be solved while the solution may be infeasible for the DC model.

3.3.2 Multistage planning

The dynamic planning problem is very complex and very large, because it must take into account not only the sizing and placement, but also timing considerations. This results in a large number of variables and restrictions to be considered, and requires an enormous computational effort to get the optimal solution, especially in real power systems. The problem will become more complicated when the security constraints are taken into account while in some large-scale systems, it is a dream to find the optimal solution. Very few works about dynamic models considering security constraints for real world transmission planning problems can be found in the technical literature.

In this section, we propose two mathematical model for MTEP considering security constraints, the DC model and hybrid model. We will use these models in the next chapters.

3.3.2.1 DC model

When the power grid is represented by the DC power flow model, the mathematical model for the multistage TEP problem under security constrains problem can be formulated as follows:

$$min\, v = \sum_{t=1}^{T}\left[\delta_{inv}^{t} \sum_{(i,j)\in\Omega} c_{ij} n_{ij}^{t}\right] \tag{1l}$$

S.t.

$$S^{t,p}F^{t,p} + g^{t,p} = d^{t}$$

$$\forall\, p \in SC,\ \forall\, t \in T \tag{2l}$$

$$f_{ij}^{t,p} - \gamma_{ij}\left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{t}\right)\left({\theta_i}^{t,p} - {\theta_j}^{t,p}\right) = 0$$

$$\forall\, (i,j) \in \Omega | (i,j) \neq p, \forall\, p \in SC, \forall\, t \in T \tag{3l}$$

$$f_{ij}^{t,p} = max\left\{0\,, \gamma_{ij}\left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{t} - 1\right)\left({\theta_i}^{t,p} - {\theta_j}^{t,p}\right)\right\}$$

$$\forall\, (i,j) \in \Omega | (i,j) \neq p,\ \forall\, p \in SC, \forall\, t \in T \tag{4l}$$

$$\left|f_{ij}^{t,p}\right| \leq \left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{t}\right)\bar{f}_{ij}$$

$$\forall\, (i,j) \in \Omega | (i,j) \neq p,\ \forall\, p \in SC, \forall\, t \in T \tag{5l}$$

$$\left|f_{ij}^{t,p}\right| \leq max\left\{0\,, \left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{t} - 1\right)\bar{f}_{ij}\right\}$$

$$\forall\, (i,j) \in \Omega | (i,j) \neq p,\ \forall\, p \in SC, \forall\, t \in T \tag{6l}$$

$$n_{ij}^{t} \leq \bar{n}_{ij}^{t}$$

$$\forall (i,j) \in \Omega\,, \forall\, t \in T \tag{7l}$$

$$\sum_{t=1}^{T} n_{ij}^{t} \leq \bar{n}_{ij}$$

$\forall (i,j) \in \Omega\ , \forall\ t \in T$ (8l)

$0 \leq g^{t,p} \leq \bar{g}^{t}$

$\forall\ p \in SC, \forall\ t \in T$ (9l)

${\theta_i}^{t,p}$ *unbounded and* n_{ij} *integer* (10l)

In this model, the variables are similar to those of DC model of TEP presented in (1b)-(6b), except the addition of the index t and index p, where index t and index p indicate the specific stage and specific scenario of planning, respectively.

Where (1l) denotes the investment in transmission lines projected to the base year; (2l) represents the power flow balance constraint; (3l) and (4l) show the expression of Ohm's law for the equivalent DC network; (5l) and (6l) are the limits of power flow of each branch and transmission line. (7l) and (8l) present the limitations of line numbers and (9l) is related to generator capacity.

The proposed model is non-linear, mixed integer, very large, and very complex to solve.

3.3.2.2 Hybrid model

The hybrid linear model for MTEP can be extended to the MTEP considering security constraints. Like the other hybrid models presented in this work, assuming the constraint related to KCL, is satisfied for all nodes of the network, whereas the constraints related to KVL are satisfied only by the existing circuits, and consequently, the hybrid model of multi-stage TEP considering security constraints can be obtained.

Thus, having taken the security constraints into account, the mathematical formulation for the TEP problem in multi-stage is of the following format:

$min\, v = \sum_{t=1}^{T} \left[\delta_{inv}^{t} \sum_{(i,j)\in\Omega} c_{ij} n_{ij}^{t}\right]$ (1k)

S.t.

$S^{t,p} F^{t,p} + {S^0}^{t,p} {F^0}^{t,p} + g^{t,p} = d^{t}$

$\forall p \in SC, \forall t \in T$ (2k)

${f_{ij}^{0}}^{t,p} - \gamma_{ij}(n_{ij}^{0})({\theta_i}^{t,p} - {\theta_j}^{t,p}) = 0$

$$\forall\ (i,j) \in \Omega_0\ |(i,j) \neq p, \forall\ p \in SC\ , \forall\ t \in T \tag{3k}$$

$$f_{ij}^{0\,t,p} - \gamma_{ij}\left(n_{ij}^{0} - 1\right)\left(\theta_i^{\ t,p} - \theta_j^{\ t,p}\right) = 0$$

$$\forall\ (i,j) \in \Omega_0 |(i,j) = p,\ \ \forall p \in SC\ , \forall\ t \in T \tag{4k}$$

$$\left|f_{ij}^{0\,t,p}\right| \leq \left(n_{ij}^{0}\right)\bar{f}_{ij}$$

$$\forall\ (i,j) \in \Omega_0\ |(i,j) \neq p, \forall\ p \in SC\ \ , \forall\ t \in T \tag{5k}$$

$$\left|f_{ij}^{0\,t,p}\right| \leq \left(n_{ij}^{0} - 1\right)\bar{f}_{ij}$$

$$\forall\ (i,j) \in \Omega_0 |(i,j) = p,\ \ \forall p \in SC\ , \forall\ t \in T \tag{6k}$$

$$\left|f_{ij}^{t,p}\right| \leq \left(\sum\nolimits_{t=1}^{t} n_{ij}^{t}\right)\bar{f}_{ij}$$

$$\forall (i,j) \in \Omega\ , \forall\ t \in T \tag{7k}$$

$$\left|f_{ij}^{t,p}\right| \leq max\{0\ , \left(\sum\nolimits_{t=1}^{t} n_{ij}^{t} - 1\right)\bar{f}_{ij}\ \}$$

$$\forall\ (i,j) \in \Omega |(i,j) \notin \Omega_0, (i,j) = p, \forall p \in SC\ \ , \forall\ t \in T \tag{8k}$$

$$n_{ij}^{t} \leq \bar{n}_{ij}^{t}$$

$$\forall (i,j) \in \Omega, \forall t \in T \tag{9k}$$

$$\sum\nolimits_{t=1}^{T} n_{ij}^{t} \leq \bar{n}_{ij}$$

$$\forall (i,j) \in \Omega, \forall t \in T \tag{10k}$$

$$0 \leq g^{t,p} \leq \bar{g}^{t}$$

$$p \in SC, \forall\ t \in T \tag{11k}$$

$$\theta_i^{\ t,p}\ unbounded\, and\ n_{ij}\ integer \tag{12k}$$

Where (1k) denotes the investment in transmission lines projected to the base year; (2k) represents the power flow balance constraint; (3k) and (4k) reflect the expression of Ohm's law for circuits of the base topology; (5k) and (6k) are the limits of power flow of each branch of the base topology; (7k) and (8k) refer to the limits of power flow of each candidate circuit; (9k) and (10k) present the limitations of line numbers and (11k) is related to the generator capacity.

In this model, the variables of the MTEP with considering security constraints are similar to those of hybrid model of TEP presented in (1d)-(7d), except the addition of the index t and index p, in which index t is related to the specific stage of planning involved and index p indicates the specific scenario of planning.

3.4 CONCLUSION

This chapter covered several mathematical models for static and multi-stage TEP problem with/without considering security constraints. Since the DC model is the best and widely accepted model, in the current research, in order to solve TEP problem, we use this model in a hybrid algorithm based on the TS and GA. In addition, the proposed hybrid algorithm also employs the relaxed models (such as TM and hybrid model) as a part of the specialization strategy in order to generate a high-quality initial population in the GA process and to decrease the number of neighbors in the TS process.

CHAPTER 4

CONSTRUCTIVE HEURISTIC ALGORITHM TO STATIC AND MULTISTAGE TRANSMISSION EXPANSION PLANNING UNDER N-1 SECURITY CONSTRAINTS

4.1 INTRODUCTION

The TEP problem in modern power systems is a large-scale, mixed-integer, non-linear, and non-convex problem. The problem becomes even more complicated when the security constraints are taken into account as in some large-scale systems, finding the optimal solution is inaccessible. The recent methodologies for solving the TEP problem can be divided into three groups: (1) classic optimization algorithms (2) Heuristic algorithms (3) meta-heuristics. From the mathematical point of view, the TEP is a mixed integer programming and non-deterministic polynomial yield to the complexity of its algorithm. In general, the mathematics-based methods, which are basically applied to medium- or large-scale power systems, are time consuming and if the additional constraints (e.g. N-1 security) are taken into account, the computational effort will be increased substantially. On the other hand, for solving the problem by using heuristic methods, although they can normally obtain a solution with less computational effort, they might be trapped in local solution. However, according to the nature of the problem and by using the modified search procedure, it is probable to achieve an acceptable result. Meta-heuristic methods, e.g. genetic algorithm, simulated annealing, etc., are mostly similar to heuristic methods. The highlighted point in meta-heuristic methods in the search process is about an embedded mechanism to escape from the local optima. The advantage of meta-heuristic methods is that they are normally capable of providing a more feasible solution with less computational effort. However, the drawback is still obvious: the solutions given by meta-heuristic methods typically do not include a mathematical indicator (e.g., duality gap), and hence, they provide few clues regarding the quality of the feasible solution. These methods are usually more time consuming than the heuristic ones.

In this chapter, a CHA to solve security constraints static transmission system expansion planning problem is proposed. The solution technique has been also extended to multi-stage planning. A CHA can find a solution with good quality in an iterative process. In fact, in each step, a circuit is chosen using a sensitivity index and is subsequently added to the system. The stopping criterion would be to obtain a feasible solution, where there would be no need to add more circuits to the system. The robustness and fastness are the main aspect of a CHA. The proposed CHA algorithm employs a hybrid model, considering the (N-1) security criterion, which is the most frequently used criterion in recent researches on transmission network planning. The (N-1) security criterion implies that the system should be expanded in such a way that, if the system gets a line outage, it could still operate accurately.

4.2 APPLICATION OF CHA TO TEP

In this section, some fundamental components and main characteristics of CHA are presented. In fact, CHA is capable of finding a good quality solution in an iterative process. It is worth to state that the fastness and robustness are considered as the main characteristics of CHA. In order to obtain a high quality solution in each iteration, a circuit is added to the network, where the aforementioned circuit is selected based on a sensitivity index. The iterative process continues until the sensitivity index indicates that a feasible topology has been found and new circuits are no longer necessary. The difference between existing algorithms is typically based on the sensitivity indicator. Usually the sensitivity index determines the most attractive circuit to update the base topology.

General CHA process is explained through number of steps as provided in the following:

Step 1: Assume a base topology as the current topology.

Step 2: Choose a mathematical model for TEP.

Step 3: Solve the LP/NLP to determine the parameters used in the sensitivity index, which considers operational conditions. If the LP or NLP solution indicates that the system is adequately operated in new additions, it means that a feasible solution is acquired. Then, skip to step 5.

Step 4: Use a sensitivity index to indicate the most attractive circuit. Update the current topology by adding the selected circuit, and return back to step 3.

Step 5: Arrange all the added circuits in a descending order of costs. Simulate the removal of the first circuit and verify the feasibility with an LP (or NLP). If the system is feasible in the simulation, remove the circuit; otherwise, maintain it. Repeat the process for each circuit in the list until all are analyzed. The circuits, which have not been removed, represent the solution of CHA.

The mentioned sensitivity index, in step 4 can be defined by Eq. (1m).

$$IS = \ max\{IS_{ij} = n_{ij}\bar{f}_{ij} \ ; \ n_{ij} \neq 0\} \tag{1m}$$

Where: n_{ij} is the solution of LP after relaxing integrality of n_{ij}.

In step 3, HLM as LP can be solved to identify the most important circuit at each step of algorithm. It is worth to mention that in the hybrid model, the relaxed constraint will only consider the circuits of the current topology.

4.3 CHA FOR EXTENDED HYBRID MODEL

Unlike the ordinary CHAs that solve only a simple model without considering security constraints, in the proposed CHA, an extended hybrid model considering security constraints is used. The sensitivity indicator is defined based on the optimal solution given by the proposed hybrid linear model presented in (1j)_(11j). It can be observed that if integrality constraints of investment variables are relaxed, i.e., $n_{ij} \geq 0$, the proposed model will become a linear programming (LP) problem and LP optimal solution will provide optimal solution of the relaxed problem. Furthermore, LP solution can be used to identify the best circuit to be included in the system. The sensitivity indicator is the power flow in the circuit with $n_{ij} \neq 0$ for the LP. The circuit to be added is identified by (1m). The topology is updated at each CHA's step. The current topology is formed by circuits of base topology and from circuits added during the iterative process.

Another feature presented in this algorithm is that every circuit added in the process must comply with both KCL and KVL. It means that there is a compatibility between the current solution and the DC model solution. It should be noted that the major drawback of this method is that at each CHA step, a very large LP must be solved, which becomes even greater for large scale power systems. The proposed CHA employed in this work is provided as follows:

Step 1: Assume the base topology as the current topology.

Step 2: Solve the LP (Model (1j)_(11j)) to determine the parameters used in the sensitivity index Eq. (1m). If the LP solution indicates that the system is adequately operated in new additions, it means that a new solution has been obtained, thus, skip to step 4.

Step 3: Use sensitivity index Eq. (1m) to identify the most attractive circuit. Update the current topology with the selected circuit. Then, return back to step 2.

Step 4: Sort the added circuits in a descending order of costs. By using an LP, it verifies whether the removal circuit keeps the system in the satisfactory operational conditions or not. If so, remove the circuit; otherwise, maintain it. Continue the circuit removal until all circuits are examined. All the added circuits that were not removed represent the CHA's solution.

It can be expressed that although this CHA uses a hybrid linear model to identify the best circuit to add in an iterative process, it complies with both Kirchhoff's laws after adding a new circuit. As a result, the final solution is also a feasible solution of the DC model.

The second step of the algorithm solves an LP, which is slightly different from model (1j-11j). During the solution process, step 2 of CHA algorithm for static planning of the LP assumes the following form:

$$min\, v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} \tag{1n}$$

S.t.

$$S^p F^p + S^{0^p} F^{0^p} + S^{1^p} F^{1^p} + g^p = d$$

$$\forall p \in SC \tag{2n}$$

$$f_{ij}^{0^p} - \gamma_{ij}\left(n_{ij}^0 + n_{ij}^1\right)\left(\theta_i{}^p - \theta_j{}^p\right) = 0$$

$$\forall\ (i,j) \in \Omega_0|\ (i,j) \neq p,\ \forall p \in SC \tag{3n}$$

$$f_{ij}^{0^p} - \gamma_{ij}\left(n_{ij}^0 + n_{ij}^1 - 1\right)\left(\theta_i{}^p - \theta_j{}^p\right) = 0$$

$$\forall\ (i,j) \in \Omega_0|\forall\ (i,j) = p,\ \forall p \in SC \tag{4n}$$

$$\left|f_{ij}^0\right| \leq \left(n_{ij}^0 + n_{ij}^1\right)\bar{f}_{ij}$$

$$\forall\ (i,j) \in \Omega_0|\ (i,j) \neq p,\ \forall p \in SC \tag{5n}$$

$$\left|f_{ij}^0\right| \leq \left(n_{ij}^0 + n_{ij}^1 - 1\right)\bar{f}_{ij}$$

$\forall\ (i,j) \in \Omega_0 | \forall\ (i,j) = p,\ \forall p \in SC$ (6n)

$|f_{ij}| \leq n_{ij}\, \bar{f}_{ij} \quad \forall (i,j) \in \Omega$ (7n)

$0 \leq n_{ij} \leq \bar{n}_{ij} - n_{ij}^1 \quad \forall (i,j) \in \Omega$ (8n)

$0 \leq g^p \leq \bar{g} \qquad p \in SC$ (9n)

θ_i *unbounded*

Where n_{ij}^1 represents the inserted circuits during the process.

At the end of CHA, the solution may have useless additions. An addition is useless if it can be removed from the transmission expansion plan and the network continues feasible (without overloads). The procedure to remove these additions is simple and direct: try to remove, one at a time, all candidate circuits previously added in reverse order of their investment costs, checking for network feasibility using the LP presented in (1p)-(9p). Those circuits that if removed do not cause any overload are definitely removed from the expansion plan.

$min\, v = \sum_{p \in SC} r^p$ (1p)

S.t.

$S^p F^p + g^p + r^p = d \qquad \forall p \in SC$ (2p)

$f_{ij}^p = \gamma_{ij}(n_{ij}^0 + n_{ij})(\theta_i{}^p - \theta_j{}^p) \qquad \forall\ (i,j) \in \Omega |\ (i,j) \neq p,\ \forall p \in SC$ (3p)

$f_{ij}^p = max\{0, \gamma_{ij}(n_{ij}^0 + n_{ij} - 1)(\theta_i{}^p - \theta_j{}^p)\}$

$\forall\ (i,j) \in \Omega | \forall\ (i,j) = p,\ \forall p \in SC$ (4p)

$|f_{ij}^p| \leq (n_{ij}^0 + n_{ij})\bar{f}_{ij} \qquad \forall\ (i,j) \in \Omega |\ (i,j) \neq p,\ \forall p \in SC$ (5p)

$|f_{ij}^p| \leq max\{0, (n_{ij}^0 + n_{ij} - 1)\bar{f}_{ij}\} \qquad \forall\ (i,j) \in \Omega | \forall\ (i,j) = p,\ \forall p \in SC$ (6p)

$0 \leq g^p \leq \bar{g} \qquad \forall\ p \in SC$ (8p)

$0 \leq r^p \leq d \qquad \forall\ p \in SC$ (9p)

4.4 FIND FEASIBLE SOLUTIONS FOR DC MODEL BY SOLVING HYBRID MODEL

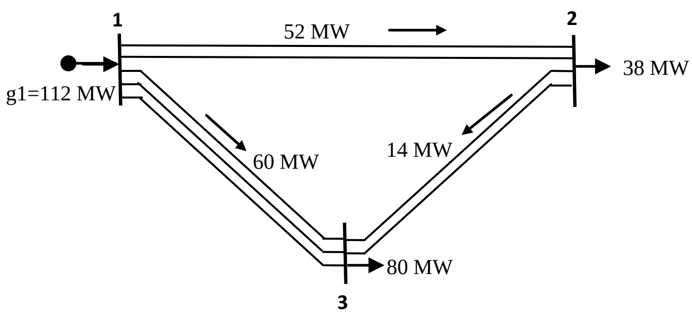
This section by using an example describes how hybrid linear model can produce feasible solutions for DC model similar to non-linear model. Fig. 1 exhibits a system with 3 buses. System data are the following:

g_1 = 150 MW, d2 = 38 MW, d3 = 80 MW, γ_{1-2} = 1 p.u., γ_{1-3} = γ_{2-3} = 0.5 p.u., c_{1-2} = 3 m.u., c_{1-3} = c_{2-3} = 2 m.u., $\bar{f}_{1-2}$ = 30 MW, $\bar{f}_{1-3}$ = $\bar{f}_{2-3}$ = 20 MW, n_{1-2} = 2, n_{1-3} = 3, n_{2-3} = 2 and $\overline{n}_{1-2}$ = $\overline{n}_{1-2}$ = $\overline{n}_{1-2}$ = 4.

Without considering security constraints, when the optimal dispatch of this problem is solved, the system presents a load shedding of 6 MW.

Moreover, the amounts of load shedding can be larger when security constraints are considered. In this case, when line 1-2 get outage, the load shedding increases to 28 MW. In addition, when line 1-3 get outage, the load shedding increases to 26 MW and when line 2-3 get outage, the load shedding increases to 11.6 M

Figure 1- Three-bus system before expansion

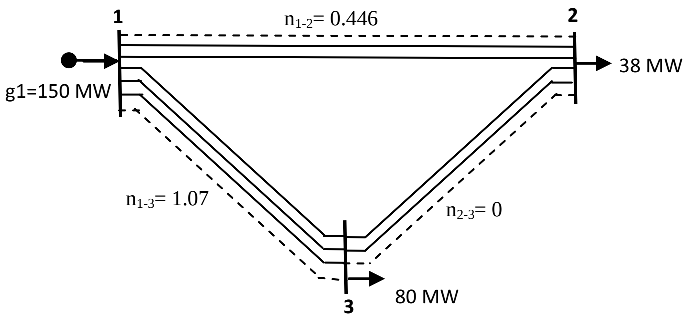


Source: The author

Therefore, according to the classical approach, additional lines are required to satisfy the demand under N-1 security constraints.

Fig 2. presents a solution of the hybrid model, in which n_{ij} is assumed to be the real positive variable. According to this solution, transmission lines 1-2 and 1-3 are introduced by CHA in order to add to the base topology. The value of the n_{ij} corresponding to the line 1-3 is more than line 1-2. Thus, a new circuit is added in corridor 1-3.

Figure 2 - Solution of the hybrid model at first iteration

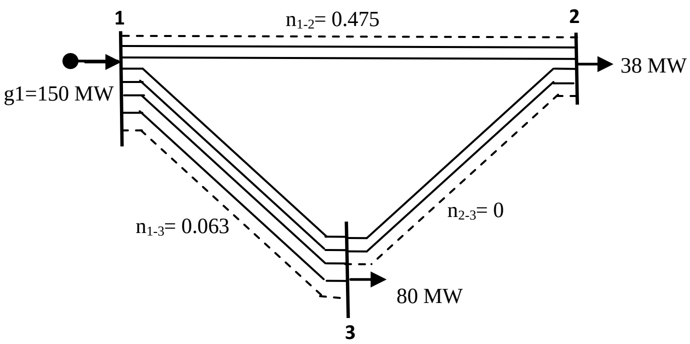


Source: The author

Accordingly, load shedding of the network is calculated in two states. In the first state, security constraints are not considered, while in the second state, they are taken into account. In the first state, load shedding of the system decreases to zero. Similarly, in the second state, outage of lines 1-2 and 1-3 respectively apply 14 and 6 MW load shedding to the system. Moreover, considering the outage of line 2-3, the demand loads of the network are satisfied. As a result, it can be realized that the network needs to install new transmission lines.

Secondly, the hybrid model is applied to the network shown in Fig 3. According to the solution shown in Fig 3., lines 1-2 and 1-3 are introduced by the algorithm in order to add to the network. But this time, line 1-2 is attractive.

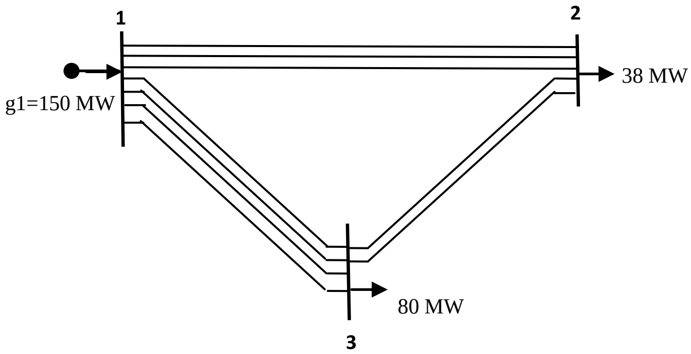
Figure 3 - Solution of the hybrid model at second iteration



Source: The author

Therefore, a new line is added in corridor 1-2 as shown in fig 4. By adding this line, the network is able to supply demand loads while the security constraints are satisfied. Indeed, by considering the obtained topology, any outage of transmission line cannot apply the load shedding to the network.

Figure 4 - Final solution for three-bus system



Source: The author

If the network shown in Fig 4. is calculated using the hybrid model, no new line will be introduced to add to the system in the third effort. Therefore, a solution is found for the DC model. This example demonstrates that how hybrid model as a linear model finds a solution with good quality for the DC model as nonlinear model through a step-by-step procedure. Additionally, it is worth to note that the DC model with considering security constraints may be solved relatively more difficult than the DC model without considering security constraints.

4.5 HEURISTIC ALGORITHM FOR THE MULTI-STAGE PLANNING

The constructive heuristic algorithm for the static planning under security constraints can be extended to multi-stage planning using the same approach for the transportation model as presented in (ROMERO et al., 2003). In multi-stage planning, the planning problem for different stages must be solved in an integrated way. In this section, an extension of the previous CHA to multistage planning is shown. The critical point of a CHA for multi-stage planning is about the choice of sensitivity indicator. The proposed CHA for multi-stage planning using HLM presents the following format:

Step 1. Consider the base topology as the current topology and use the hybrid linear model. Make the current stage k = 1.

Step 2. Solve the LP's corresponding problem for the current topology. If $n_{ij}^{k} = 0 \; \forall \; (i,j) \in \Omega$ then the addition phase at stage k is over. If the local search procedure must be implemented, skip to step 4.

Step 3. Use a sensitivity indicator to find the most attractive circuit at stage k. Update the current topology with the chosen circuit and return back to step 2.

Step 4. Update k = k+1 and return back to step 2.

Step 5. Execute step 4 of the CHA algorithm to remove unnecessary circuits. Thus, the circuits, which are not removed, will present the solution for multi-stage planning.

In step 2, the algorithm solves an LP for the current topology. The LP is slightly different from the problem model presented in (1k)-(11k) because of the addition, which needs to be taken into account independently. If the modified algorithm for the hybrid model is utilized in step 2 of the algorithm, the LP will become:

$$min\, v = \sum_{t=1}^{T} \left[\delta_{inv}^{t} \sum_{(i,j)\in\Omega} c_{ij} n_{ij}^{t} \right] \tag{1q}$$

S.t.

$$S^{t,p} F^{t,p} + S^{0\,t,p} F^{0\,t,p} + S^{1\,t,p} F^{1\,t,p} + g^{t,p} = d^{t}$$

$$\forall p \in SC\,, \forall t \in T \tag{2q}$$

$$f_{ij}^{0\,t,p} - \gamma_{ij} \left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{1t} \right) \left(\theta_{i}^{\,t,p} - \theta_{j}^{\,t,p} \right) = 0$$

$$\forall \; (i,j) \in \Omega_0 \; |(i,j) \neq p, \;\; \forall p \in SC \tag{3q}$$

$$f_{ij}^{0\,t,p} - \gamma_{ij} \left(n_{ij}^{0} + \left(\sum_{t=1}^{t} n_{ij}^{1t} \right) - 1 \right) \left(\theta_{i}^{\,t,p} - \theta_{j}^{\,t,p} \right) = 0$$

$$\forall \; (i,j) \in \Omega_0 |(i,j) = p, \;\; \forall p \in SC \tag{4q}$$

$$\left| f_{ij}^{0\,t,p} \right| \leq \left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{1t} \right) \bar{f}_{ij}$$

$$\forall \; (i,j) \in \Omega_0 |(i,j) \neq p, \;\; \forall p \in SC \tag{5q}$$

$$\left|f_{ij}^{0\,t,p}\right| \le \left(n_{ij}^{0} + (\sum_{t=1}^{t} n_{ij}^{1t}) - 1\right)\bar{f}_{ij}$$

$$\forall\ (i,j) \in \Omega_0 | \forall\ (i,j) = p,\ \forall p \in SC \tag{6q}$$

$$\left|f_{ij}^{t,p}\right| \le \left(\sum_{t=1}^{t} n_{ij}^{t}\right)\bar{f}_{ij} \quad \forall (i,j) \in \Omega$$

$$0 \le n_{ij}^{t} \le \bar{n}_{ij}^{t} - n_{ij}^{1t}\ \ \forall (i,j) \in \Omega \tag{7q}$$

$$\sum_{t=1}^{T} n_{ij}^{t} \le \bar{n}_{ij} - (\sum_{t=1}^{T} n_{ij}^{1t}) \tag{8q}$$

$$0 \le g^{t,p} \le \bar{g}^{t} \quad\ \ p \in SC \tag{9q}$$

Where n_{ij}^{1t} shows the inserted circuits during the process and represents the known values to the LP. n_{ij}^{t} displays the circuits given by the LP solution. The proposed sensitivity indicator identifies the most attractive circuit in path (i, j) ($n_{ij} \ne 0$) at stage k as the circuit that carries the largest amount of power flow considering all circuits added by the LP subroutine for all stages $t \ge k$. Consequently, for each path, with $n_{ij}^{k} \ne 0, f_{ij} = f_{ij}^{k} + f_{ij}^{k+1} + \cdots + f_{ij}^{T}$ can be found, where f_{ij}^{k+1} is the power flow in path (i, j) at stage k+1. It is worthwhile to mention that all of these values become available through the LP solution.

Considering a given proposal for the transmission lines, which means n_{ij}^{t} is given, we have a linear model of the DC model explained in (1r)_(8r). We use this model in step 5 of CHA in order to remove unnecessary lines.

$$min\, v = \sum_{t=1}^{T} \sum_{p \in SC} r^{t,p} \tag{1r}$$

S.t.

$$S^{t,p} F^{t,p} + g^{t,p} + r^{t,p} = d^{t} \qquad \forall p \in SC \tag{2r}$$

$$f_{ij}^{t,p} - \gamma_{ij}\left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{t}\right)\left({\theta_i}^{t,p} - {\theta_j}^{t,p}\right) = 0$$

$$\forall\ (i,j) \in \Omega | (i,j) \ne p,\ \ p \in SC \tag{3r}$$

$$f_{ij}^{t,p} = max\{0, \gamma_{ij}\left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{t} - 1\right)\left({\theta_i}^{t,p} - {\theta_j}^{t,p}\right)\}$$

$$\forall\ (i,j) \in \Omega | (i,j) = p,\ \ p \in SC \tag{4r}$$

$$\left|f_{ij}^{t,p}\right| \le \left(n_{ij}^{0} + \sum_{t=1}^{t} n_{ij}^{t}\right)\bar{f}_{ij}$$

$$\forall\ (i,j) \in \Omega | (i,j) \neq p,\ \ p \in SC \tag{5r}$$

$$\left|f_{ij}^{t,p}\right| \leq max\{0, \left(n_{ij}^{0} + \sum\nolimits_{t=1}^{t} n_{ij}^{t} - 1\right) \bar{f}_{ij}\}$$

$$\forall\ (i,j) \in \Omega | (i,j) = p,\ \ p \in SC \tag{6r}$$

$$0 \leq g^{t,p} \leq \bar{g}^{t} \quad\ p \in SC, \forall t \in T \tag{7r}$$

$$0 \leq r^{t,p} \leq r^{t} \quad\ p \in SC, \forall t \in T \tag{8r}$$

4.6 TEST WITH PROPOSED ALGORITHM

The algorithm has been implemented using an AMPL structure by CPLEX solver to solve the LP problem at each step of the constructive algorithm. Five systems, which are well known in specialized literature, were tested and partial results are presented in detail.

4.6.1 Results - static planning

The proposed algorithm, which was developed to solve the static TEP problem under security constraints, was tested by using three electrical power systems. The first system is the system originally proposed by Garver (GARVER., 1970), the second one is the IEEE 24 bus system and third one is South Brazilian system.

4.6.1.1 Garver 6-bus test system

The Garver system has 6 buses and 15 candidate branches. The total demand is 760 MW and the maximum possible number of added lines per branch equals five. The electrical system data for this system was extracted from (ROMERO et al., 2003). There are two options of planning that can be made with this system; with and without generation rescheduling. Having carried out the simulations with the algorithm, the following configurations were obtained:

4.6.1.1.1 Plan with generation rescheduling

The obtained solution of the expansion planning problem without security constraints by the proposed algorithm has an investment cost equal to v= US$130,000 with the addition of the following lines to the base topology: $n_{2\text{-}3}=1$, $n_{2\text{-}6}=1$, $n_{3\text{-}5}=1$ and $n_{4\text{-}6}=2$.

The algorithm converges after solving 10 LPs without removing circuits in step 4.

The planning with security constraints to this system can be found using the proposed CHA resulting in an investment of v=US$ 190,000 considering the following added lines: $n_{2-6}=2$, $n_{3-5}=2$, and $n_{4-6}=3$.

Table 1 presents the solution process through iterations. vlp is the investment given by LP and v is the partial investment resulted from additions provided by the algorithm.

Table 1- Garver's system with rescheduling

Iter	*Addition by LP*	*Sensitivity index*	*Vlp*	*V*
1	$n_{2-3} = 0.266667$ $n_{2-6} = 1.7$ $n_{3-5} = 1.93333$ $n_{4-6} = 0.8$	$IS_{2-3} = 26.6667$ $IS_{2-6} = 170$ $\mathbf{IS_{3-5} = 193.333}$ $IS_{4-6} = 80$	*119*	*0*
2	$n_{2-3} = 0.0967742$ $n_{2-6} = 1.50484$ $n_{3-5} = 1.12258$ $n_{4-6} = 0.995161$	$IS_{2-3} = 9.67742$ $\mathbf{IS_{2-6} = 150.484}$ $IS_{3-5} = 112.258$ $IS_{4-6} = 99.5161$	*99.3871*	*20*
3	$n_{2-6} = 1.25909$ $n_{3-5} = 0.818182$ $n_{4-6} = 1.24091$	$\mathbf{IS_{2-6} = 125.909}$ $IS_{3-5} = 81.8182$ $IS_{4-6} = 124.091$	*91.3636*	*50*
4	$n_{2-6} = 0.259091$ $n_{3-5} = 0.818182$ $n_{4-6} = 1.24091$	$IS_{2-6} = 25.9091$ $IS_{3-5} = 81.8182$ $\mathbf{IS_{4-6} = 124.091}$	*61.3636*	*80*
5	$n_{2-3} = 0.0272266$ $n_{2-6} = 0.0891094$ $n_{3-5} = 0.852976$ $n_{4-6} = 0.410891$	$IS_{2-3} = 2.72266$ $IS_{2-6} = 8.91094$ $\mathbf{IS_{3-5} = 85.2976}$ $IS_{4-6} = 41.0891$	*32.6041*	*110*
6	$n_{2-3} = 0.0526316$ $n_{2-6} = 0.0789474$ $n_{4-6} = 0.421053$	$IS_{2-3} = 5.26316$ $IS_{2-6} = 7.89474$ $\mathbf{IS_{4-6} = 42.1053}$	*16.0526*	*130*
7	$n_{4-6} = 0.0869894$	$\mathbf{IS_{4-6} = 8.69894}$	*2.60968*	*160*
8			*0*	*190*

Source: The author

The sequence of circuits' additions performed by the algorithm is shown in Table 1, which is provided in the following :

n_{3-5}, n_{2-6}, n_{2-6}, n_{4-6} , n_{3-5}, n_{4-6} and n_{4-6}. The algorithm converges after solving 11 LPs and without removing any circuit in step 4.

The optimal solution for the case without contingency needs 5 new transmission lines while in other case with contingency, 7 transmission lines are needed. The investment cost of the planning with contingency is 31.57% greater than planning without contingency with respect to the investment cost of planning with contingency.

It shuld be noted that there are 4 common lines, ($n_{2-6}=1$, $n_{3-5}=1$, $n_{4-6}=2$), in both planning case.

4.6.1.1.2 Plan without generation rescheduling

When it is impossible to reprogram the generation, the optimal solution of the planning problem without security constraints has an investment v= US$ 200,000 and the following lines are added:
$n_{2-6}=4$, $n_{3-5}=1$ and $n_{4-6}=2$.

The CHA converges after solving 11 LPs without removing any circuit in step 4. Using the proposed algorithm, a planning test is realized, with security constraints and without considering generation rescheduling. In this situation, an investment v=US$300,000 results, and the final optimal topology has the addition of the following lines:
$n_{2-6}=5$, $n_{3-5}=2$, $n_{4-6}=3$ and $n_{2-3}=1$.

The proposed CHA converges after solving 16 LPs and without removing any circuit in step 4. The sequence of adding line is as follows:
n_{2-6}, n_{2-6}, n_{2-6}, n_{4-6}, n_{2-6}, n_{3-5}, n_{4-6}, n_{3-5}, n_{4-6}, n_{2-6} and n_{2-3}.

Table 2 shows the results of each iteration for this case.

When the security constraints are not taken into account, 7 new transmission lines are needed; but when security constraints are taken into consideration, 11 transmission lines are required. The difference between the investment cost in the cases with and without contingency is 33% with respect to the investment cost of planning with contingency. All of the lines of the first plan exist in the second plan.

Table 2- Garver's system without rescheduling

Iter	*Addition by LP*	*Sensitivity index*	*Vlp*	*V*
1	$n_{1-5} = 0.15$ $n_{2-6} = 3.05$ $n_{3-5} = 1.25$ $n_{4-6} = 2.4$	$IS_{1-5} = 15$ **$IS_{2-6} = 305$** $IS_{3-5} = 125$ $IS_{4-6} = 24$	*191.5*	*0*
2	$n_{1-5} = 0.15$ $n_{2-6} = 2.25$ $n_{3-5} = 1.25$ $n_{4-6} = 2.2$	$IS_{1-5} = 15$ **$IS_{2-6} = 225$** $IS_{3-5} = 125$ $IS_{4-6} = 220$	*191.5*	*30*
3	$n_{1-5} = 0.15$ $n_{2-6} = 2.25$ $n_{3-5} = 1.25$ $n_{4-6} = 2.2$	$IS_{1-5} = 15$ **$IS_{2-6} = 225$** $IS_{3-5} = 125$ $IS_{4-6} = 220$	*161.5*	*60*
4	$n_{1-5} = 0.15$ $n_{2-6} = 1.25$ $n_{3-5} =1.25$ $n_{4-6} = 2.2$	$IS_{1-5} = 15$ $IS_{2-6} = 125$ $IS_{3-5} =125$ **$IS_{4-6} = 220$**	*131.5*	*90*
5	$n_{1-5} =0.15$ $n_{2-6} = 1.4875$ $n_{3-5} =1.25$ $n_{4-6} = 0.9625$	$IS_{1-5} =15$ **$IS_{2-6} = 148.75$** $IS_{3-5} =125$ $IS_{4-6} = 96.25$	*101.5*	*120*
6	$n_{1-5} =0.15$ $n_{2-6} = 0.4875$ $n_{3-5} =1.25$ $n_{4-6} = 0.9625$	$IS_{1-5} =15$ $IS_{2-6} = 48.75$ **$IS_{3-5} =125$** $IS_{4-6} = 96.25$	*71.5*	*150*
7	$n_{2-6} =0.375$ $n_{3-5} = 0.8$ $n_{3-6} =0.15$ $n_{4-6} = 0.925$	$IS_{2-6} =37.5$ $IS_{3-5} = 80$ $IS_{3-6} =15$ **$IS_{4-6} = 92.5$**	*62.2*	*170*
8	$n_{2-3} =0.15$ $n_{2-6} = 0.207893$ $n_{3-5} = 0.762002$ $n_{4-6} = 0.739186$	$IS_{2-3} =15$ $IS_{2-6} = 20.7893$ **$IS_{3-5} = 76.2002$** $IS_{4-6} = 73.9186$	*46.6524*	*200*
9	$n_{1-5} = 0.120784$ $n_{2-3} = 0.0292157$ $n_{2-6} = 0.239304$ $n_{4-6} = 0.736161$	$IS_{1-5} = 12.0784$ $IS_{2-3} = 2.92157$ $IS_{2-6} = 23.9304$ **$IS_{4-6} = 73.6161$**	*32.264*	*220*
10	$n_{1-5} = 0.0963311$ $n_{2-3} = 0.0536689$ $n_{2-6} = 0.239003$	$IS_{1-5} = 9.63311$ $IS_{2-3} = 5.36689$ **$IS_{2-6} = 23.9003$**	*10.1701*	*250*
11	$n_{2-3} =0.15$	**$IS_{2-3} =15$**	*3*	*280*
12			*0*	*300*

Source: The author

4.6.1.2 IEEE 24-bus system

This system consists of 24 buses, 41 candidate circuits and 8550MW of total demand. Maximum possible number of added lines per branch equals 3. For this system only planning without generation resizing is considered. The electrical data and generation/load data have been taken for plans' G1–G4 of (ROMERO et al., 2005).

4.6.1.2.1 Plan without security constraints

Considering generation plan G1, the CHA founds the expansion plan of v = US$ 438,000,000 with the following topology:

$n_{5-1}=1$, $n_{3-24}=1$, $n_{6-10}=1$, $n_{7-8}=2$, $n_{14-16}=1$, $n_{15-21}=1$, $n_{15-24}=1$, $n_{16-17}=2$, $n_{16-19}=1$, $n_{17-18}=1$.

Taking into account the plan G2, the expansion plan of v = US$ 494,000,000 is derived from the proposed CHA founds considering the following topology:

$n_{5-1}=1$, $n_{3-24}=1$, $n_{6-10}=1$, $n_{7-8}=1$, $n_{10-11}=1$, $n_{14-16}=2$, $n_{15-21}=1$, $n_{15-24}=1$, $n_{16-17}=2$ and $n_{17-18}=1$.

For generation plan G3, the CHA founds the expansion plan of v=US$ 218,000,000 with the following topology:

$n_{6-10}=1$, $n_{7-8}=2$, $n_{10-12}=1$, $n_{14-16}=1$, $n_{16-17}=1$ and $n_{20-23}=1$.

And finally considering generation plan G4, the expansion plan of v = US$ 376,000,000 using the proposed CHA with the following topology is in hand:

$n_{3-24}=1$, $n_{6-10}=1$, $n_{7-8}=2$, $n_{10-12}=1$, $n_{12-13}=1$, $n_{14-16}=1$, $n_{15-24}=1$ and $n_{17-18}=1$.

4.6.1.2.2 Plan with security constraints

The investment of the expansion planning considering security constraints for plan G1 using the proposed CHA is v = US$ 949,000,000 where the added lines are as follows:

$n_{1-5}=1$, $n_{2-8}=1$, $n_{3-24}=2$, $n_{4-9}=1$, $n_{5-10}=1$, $n_{6-7}=2$, $n_{6-10}=1$, $n_{7-8}=2$, $n_{10-11}=1$, $n_{11-14}=1$, $n_{14-16}=2$, $n_{15-16}=1$, $n_{15-24}=2$, $n_{16-17}=3$,$n_{16-19}=2$ and $n_{17-18}=2$.

The CHA converges after solving 42 LPs and without removing any circuit in step 4.

Considering generation plan G2, the CHA founds the expansion plan of v = US$ 964,000,000 with the following topology:

$n_{1-5}=1$, $n_{3-24}=2$, $n_{4-9}=1$, $n_{6-10}=2$, $n_{7-8}=1$, $n_{9-11}=1$, $n_{10-11}=2$, $n_{11-14}=1$, $n_{14-16}=3$, $n_{15-21}=1$,$n_{15-24}=2$, $n_{16-17}=3$, $n_{17-18}=2$ and $n_{2-8}=1$.

For this plan the CHA converges after solving 124 LPs by removing circuits of n_{6-7}, n_{6-7}, n_{2-8}, n_{6-7} and n_{15-16} in step 4 respectively.

Considering generation plan G3, the investment of expansion plan founded by CHA is v = US$ 722,000,000 with the following topology:

$n_{1-5}=1$, $n_{2-8}=1$, $n_{3-24}=1$, $n_{4-9}=1$, $n_{6-7}=2$, $n_{6-10}=1$, $n_{7-8}=2$, $n_{10-12}=1$, $n_{12-13}=1$, $n_{14-16}=2$, $n_{15-16}=1$, $n_{15-24}=1$, $n_{16-17}=2$, $n_{17-18}=1$ and $n_{20-23}=1$.

For this plan the CHA converges after solving 35 LPs and without removing any circuit in step 4.

Finally for plan G4, the CHA found the expansion plan of v = US$ 818,000,000 with the following topology:

$n_{1\text{-}5}=1$, $n_{2\text{-}8}=1$, $n_{3\text{-}24}=2$, $n_{4\text{-}9}=1$, $n_{6\text{-}7}=2$, $n_{6\text{-}10}=2$, $n_{7\text{-}8}=1$, $n_{9\text{-}11}=1$, $n_{10\text{-}12}=2$, $n_{12\text{-}13}=1$, $n_{14\text{-}16}=2$, $n_{15\text{-}24}=1$, $n_{16\text{-}17}=2$ and $n_{17\text{-}18}=1$.

For this plan the CHA converges after solving 35 LPs and without removing any circuit in step 4.

4.6.1.3 Brazilian 46-bus system

The third test system is the Brazilian 46-bus system. The system comprises 46 buses, 79 circuits, 6880 MW of total demand. The electrical system data, which consist of transmission line, load and generation data including generation resizing range in MW, are available in (ROMERO et al., 1996). This system represents a good test to the proposed approach because it is a real-world transmission system. The addition of parallel transmission lines to existing lines is again allowed in this case with a limit of 3lines for each branch.

4.6.1.3.1 Plan with generation rescheduling

The planning without security constraints for this system using the CHA, resulted in an investment of v= US$ 95,795,000, and the following lines are added.

$n_{19\text{-}21}=1$, $n_{20\text{-}23}=1$, $n_{20\text{-}21}=2$, $n_{42\text{-}43}=1$, $n_{05\text{-}06}=2$, $n_{46\text{-}06}=1$

The algorithm converges after 15 LPs without circuits removal in step 4.

When security constraints are considered, the optimal solution is v= US$ 252,460,000 and the final optimal topology has the addition of the following lines:

$n_{02\text{-}05}=1$, $n_{05\text{–}06}=3$, $n_{12\text{–}14}=1$, $n_{19\text{–}21}=1$, $n_{20\text{–}21}=3$, $n_{20\text{–}23}=1$, $n_{21\text{–}25}=1$, $n_{24\text{–}25}=1$, $n_{31\text{–}32}=1$, $n_{31\text{–}41}=1$, $n_{40\text{–}41}=1$, $n_{40\text{–}45}=1$, $n_{41\text{–}43}=1$ $n_{42\text{–}43}=2$, $n_{46\text{–}06}=2$.

The proposed CHA reached this solution after the solution of 36 LPs without removing any circuit in step 4. The sequence of additions is the following:

$n_{19\text{–}21}$, $n_{20\text{–}21}$ $n_{20\text{–}21}$, $n_{46\text{–}06}$, $n_{05\text{–}06}$, $n_{31\text{–}41}$, $n_{40\text{–}41}$, $n_{24\text{–}25}$, $n_{21\text{–}25}$ $n_{42\text{–}43}$, $n_{41\text{–}43}$, $n_{12\text{–}14}$ $n_{42\text{–}43}$, $n_{46\text{–}06}$, $n_{05\text{–}06}$, $n_{05\text{–}06}$ $n_{02\text{-}05}$, $n_{31\text{–}32}$ $n_{20\text{–}23}$, $n_{40\text{–}45}$, $n_{20\text{–}21}$.

The optimal plan of the case without security constraint needs 8 new transmission lines in 6 distinct corridors. When considering the security constraints, the optimal plan needs 21 new transmission lines in 15 different corridors.

The difference between the investment cost in cases with and without contingency is 62%. Note that, all of the lines in the plan without considering security constraints are in plan with considering security constraints.

4.6.1.3.2 Plan without generation rescheduling

The best found solution of the expansion planning problem without security constraints when it is not possible to reprogram the generation, is equal to v =US$166,041,000. The addition lines to the base topology are as fallow:
$n_{20\text{-}21}=2$, $n_{42\text{-}43}=1$, $n_{46\text{-}06}=1$, $n_{25\text{-}32}=1$,$n_{31\text{-}32}=1$,$n_{28\text{-}31}=1$, $n_{31\text{-}41}=1$, $n_{24\text{-}25}=2$, $n_{40\text{-}41}=1$, $n_{05\text{-}06}=2$.

For this plan after solving 24 LPs, the algorithm converges and there is no removal in step 4.

The necessary investment to solve the planning problem with security constraints for this system is v =US$ 375,941,000 with the addition of the following lines to the base topology:
$n_{02\text{-}05}=1$, $n_{12\text{–}14}=1$, $n_{17\text{-}19}=1$, $n_{19\text{–}21}=1$, $n_{14\text{–}22}=1$, $n_{22\text{-}26}=1$, $n_{20\text{–}21}=2$, $n_{42\text{–}43}=3$, $n_{46\text{-}06}=2$, $n_{21\text{-}25}=1$, $n_{25\text{-}32}=1$, $n_{31\text{-}32}=1$, $n_{28\text{-}31}=2$, $n_{31\text{-}41}=2$, $n_{41\text{-}43}=1$, $n_{40\text{-}45}=1$, $n_{24\text{-}25}=3$, $n_{40\text{-}41}=2$ and $n_{05\text{-}06}=3$.

In this Plan convergence is reached after solving 50 LPs and without removing any circuit in step 4. The sequence of addition is the following:
$n_{20\text{–}21}$, $n_{19\text{–}21}$, $n_{24\text{-}25}$, $n_{46\text{-}06}$, $n_{05\text{-}06}$, $n_{21\text{-}25}$, $n_{31\text{-}41}$, $n_{31\text{-}32}$, $n_{40\text{-}41}$, $n_{42\text{–}43}$ $n_{41\text{-}43}$, $n_{42\text{–}43}$, $n_{28\text{-}31}$ $n_{28\text{-}31}$, $n_{17\text{-}19}$, $n_{20\text{–}21}$, $n_{25\text{-}32}$, $n_{24\text{-}25}$, $n_{05\text{-}06}$, $n_{46\text{-}06}$ $n_{05\text{-}06}$, $n_{12\text{–}14}$, $n_{24\text{-}25}$, $n_{31\text{-}41}$, $n_{02\text{-}05}$, $n_{42\text{–}43}$ $n_{14\text{–}22}$, $n_{40\text{-}45}$, $n_{22\text{-}26}$ and $n_{40\text{-}41}$.

In the plan without security constraint 13 new transmission lines are needed to add to the network; on the other hand when security constraints are taken into account the optimum plan needs 30 new transmission lines.

The difference between the investment cost in cases with and without contingency is 55.85%. Note that, all of the lines in plan without considering security constraints exist in plan with considering security constraints.

4.6.2 Test and results - multi-stage planning

The proposed CHA was tested to solve the multistage TEP problem too. The algorithm is applied in the Brazilian North-Northeastern System and Colombian 93-bus system. The algorithm has implemented using an AMPL structure using CPLEX solver to solve the LP problem at each step of the constructive algorithm.

4.6.2.1 Brazilian North-Northeastern system

The system has 87 buses, 183 circuits and 29748MWof load for the entire planning horizon. The system data is in (ROMERO et al., 2002) and the available data allows planning without generation rescheduling and a multi-stage planning with two stages. The system is very complex, the optimal solution is unknown and there is no any solution for planning considering security constraints. The year 1998 was considered as the base year and plans P1 and P2 were considered in 2002 and 2008, respectively. Consequently, the needed circuits for 2002 were considered as the cost of 1998 (original values) and the needed circuits in 2008 were considered as were built in 2002 and their costs were updated for the base year. In the tests, the factor of discount was considered 10% (I = 10%). Therefore, the costs for a transmission line added in P2 were multiplied by 0.683

In multi-stage planning, the best topology without security constraints was found using the CHA as in the following:

Stage P1: 1998-2002: $v1 = 1,650,770,000 US\$$

$n_{02\text{-}87}=2$, $n_{03\text{-}83}=1$, $n_{03\text{-}87}=2$, $n_{04\text{-}68}=1$, $n_{04\text{-}81}=1$, $n_{05\text{-}56}=1$, $n_{05\text{-}58}=2$, $n_{05\text{-}68}=2$, $n_{06\text{-}67}=1$, $n_{10\text{-}11}=1$ $n_{13\text{-}14}=1$, $n_{13\text{-}15}=3$,$n_{14\text{-}59}=1$, $n_{15\text{-}16}=2$, $n_{15\text{-}46}=1$, $n_{16\text{-}44}=4$, $n_{16\text{-}61}=1$, $n_{18\text{-}50}=6$, $n_{18\text{-}74}=3$, $n_{21\text{-}57}=1$ $n_{22\text{-}58}=1$, $n_{24\text{-}43}=1$, $n_{25\text{-}55}=2$, $n_{26\text{-}54}=1$, $n_{30\text{-}31}=1$, $n_{30\text{-}63}=2$, $n_{36\text{-}46}=2$, $n_{40\text{-}45}=1$, $n_{41\text{-}64}=2$, $n_{43\text{-}55}=2$ $n_{43\text{-}58}=2$, $n_{48\text{-}49}=1$, $n_{49\text{-}50}=2$, $n_{52\text{-}59}=1$, $n_{53\text{-}54}=1$, $n_{54\text{-}63}=1$, $n_{56\text{-}57}=1$, $n_{61\text{-}64}=1$, $n_{61\text{-}58}=2$, $n_{67\text{-}68}=1$ $n_{68\text{-}69}=1$, $n_{68\text{-}83}=1$, $n_{72\text{-}73}=1$, $n_{72\text{-}83}=1$, $n_{73\text{-}74}=1$.

Stage P2: 2002-2008: $v2 = 1,142,450,000 US\$$

$n_{01\text{-}02}=1$, $n_{04\text{-}05}=1$, $n_{04\text{-}06}=1$, $n_{04\text{-}81}=2$, $n_{05\text{-}58}=1$, $n_{12\text{-}13}=1$,$n_{12\text{-}15}=1$, $n_{13\text{-}15}=1$$n_{14\text{-}45}=1$, $n_{15\text{-}16}=2$, $n_{16\text{-}44}=2$, $n_{16\text{-}61}=1$, $n_{18\text{-}50}=5$, $n_{18\text{-}74}=3$, $n_{21\text{-}57}=1$, $n_{25\text{-}55}=2$, $n_{30\text{-}31}=1$, $n_{35\text{-}51}=1$,$n_{36\text{-}39}=1$, $n_{36\text{-}46}=1$, $n_{40\text{-}45}=1$, $n_{43\text{-}55}=2$, $n_{43\text{-}58}=1$, $n_{49\text{-}50}=2$, $n_{61\text{-}85}=1$, $n_{68\text{-}83}=1$, $n_{73\text{-}74}=1$,$n_{73\text{-}75}=1$, $n_{75\text{-}81}=1$.

In the multi-stage planning the algorithm converges after solving 210 LPs and removing 2 circuits from first stage and 10 circuits from second stage in step 5. The total investment cost is v=2,431,060,000 US$.

The planning with security constraints to this system can be found using the methodology that is proposed in this work, resulting in an investment of v = US$ 3,980,109,067 and the addition of the following lines:

Stage P1: 1998-2002: *v1 = US$ 3,113,212,000*

$n_{01\text{-}02}=1$, $n_{02\text{-}60}=2$, $n_{02\text{-}87}=1$, $n_{04\text{-}60}=2$, $n_{04\text{-}68}=1$, $n_{04\text{-}69}=1$,$n_{04\text{-}81}=2$, $n_{05\text{-}56}=2$, $n_{05\text{-}58}=3$, $n_{05\text{-}60}=3$, $n_{08\text{-}73}=1$, $n_{10\text{-}11}=1$, $n_{12\text{-}13}=1$, $n_{13\text{-}14}=1$, $n_{13\text{-}15}=4$, $n_{14\text{-}45}=1$, $n_{14\text{-}59}=1$, $n_{15\text{-}16}=3$, $n_{15\text{-}46}=1$, $n_{16\text{-}44}=4$, $n_{16\text{-}61}=2$, $n_{18\text{-}50}=7$, $n_{18\text{-}74}=5$, $n_{21\text{-}57}=3$, $n_{22\text{-}58}=2$, $n_{24\text{-}43}=1$, $n_{25\text{-}55}=3$, $n_{27\text{-}53}=1$, $n_{30\text{-}63}=2$, $n_{35\text{-}47}=1$, $n_{36\text{-}39}=1$, $n_{36\text{-}46}=3$, $n_{40\text{-}45}=2$, $n_{41\text{-}64}=3$, $n_{43\text{-}55}=2$, $n_{43\text{-}58}=2$, $n_{49\text{-}50}=4$, $n_{52\text{-}59}=2$, $n_{53\text{-}54}=1$, $n_{54\text{-}55}=1$, $n_{54\text{-}63}=2$, $n_{56\text{-}57}=2$, $n_{61\text{-}64}=1$, $n_{61\text{-}58}=3$, $n_{63\text{-}64}=1$, $n_{67\text{-}69}=1$, $n_{67\text{-}71}=3$, $n_{68\text{-}83}=1$, $n_{69\text{-}87}=1$, $n_{71\text{-}72}=1$, $n_{71\text{-}83}=1$, $n_{72\text{-}73}=2$, $n_{72\text{-}83}=1$, $n_{73\text{-}74}=3$, $n_{73\text{-}75}=1$, $n_{75\text{-}81}=1$, $n_{81\text{-}83}=1$.

Stage P2: 2002-2008: *v2* = US$ 1,269,249,000

$n_{01\text{-}02}=1$, $n_{04\text{-}81}=3$, $n_{06\text{-}75}=1$, $n_{11\text{-}12}=1$, $n_{12\text{-}13}=2$, $n_{13\text{-}14}=1$, $n_{13\text{-}15}=2$, $n_{15\text{-}16}=1$, $n_{15\text{-}45}=1$, $n_{15\text{-}46}=1$, $n_{16\text{-}44}=3$, $n_{16\text{-}45}=1$, $n_{17\text{-}18}=1$, $n_{17\text{-}59}=1$, $n_{18\text{-}50}=5$, $n_{18\text{-}74}=2$, $n_{22\text{-}58}=1$, $n_{24\text{-}43}=1$, $n_{25\text{-}55}=2$, $n_{30\text{-}31}=2$, $n_{30\text{-}63}=1$, $n_{35\text{-}51}=1$, $n_{36\text{-}39}=1$, $n_{36\text{-}46}=1$, $n_{39\text{-}42}=1$, $n_{40\text{-}45}=1$, $n_{41\text{-}64}=1$, $n_{43\text{-}55}=2$, $n_{43\text{-}58}=2$, $n_{47\text{-}48}=1$, $n_{48\text{-}49}=1$, $n_{49\text{-}50}=1$, $n_{58\text{-}78}=3$, $n_{61\text{-}85}=1$, $n_{68\text{-}69}=1$, $n_{68\text{-}83}=2$, $n_{71\text{-}72}=1$, $n_{72\text{-}73}=1$, $n_{78\text{-}80}=1$, $n_{80\text{-}83}=1$ $n_{81\text{-}83}=1$

The proposed algorithm converges after solving 287 LPs and removing 10 circuits from second stage in step 5. The topologies found by CHA have good quality and more important, they are feasible to the DC model and can be used as useful tools in system planning.

The optimal plan for the case without contingency requires 112 new lines, while by considering security constraints the plan needs 170 new lines. The difference between the investment costs is 38.92% with respect to the investment cost of planning with contingency.

4.6.2.2 The Colombian 93-bus system

The second transmission network is selected to test the CHA that is the Colombian system. The system consists of 93 buses, 155 possible right-of-ways and 14559 MW of total demand for the entire planning horizon. The required electrical system data, which consist of transmission line, generation and load data including the load growth along the study horizon, are available in (ESCOBAR, 2002). The addition of parallel transmission lines to existing lines is allowed in this case with a limit of 4 lines in each branch. Three planning stages P1, P2 and P3 are considered in this case. The P1 stage is the first stage that is the period from

2002 until 2005 and 2002 is the base year for this stage. The P2 stage is the period from 2005 until 2009 and 2005 is the base year for the second stage. The P3 stage is the period from 2009 until 2012 and 2009 is the base year for the third stage. Furthermore, the total transmission expansion investment plan is obtained with reference to the base year 2002 and an annual interest rate value I = 10 %. Hence, the total investment cost can again be calculated by using equation (1q).

The best topology without security constraints in the multi-stage planning, found using the CHA is the following:

Stage P1: 2002-2005: *v1 =363,173,000US$*

$n_{52\text{-}88}=1$, $n_{56\text{-}81}=1$, $n_{57\text{-}84}=1$, $n_{55\text{-}84}=1$, $n_{56\text{-}57}=1$, $n_{45\text{-}81}=1$, $n_{55\text{-}82}=1$, $n_{82\text{-}85}=1$.

Stage P2: 2005-2009: *v2 =291,672,000US$*

$n_{57\text{-}84}=1$, $n_{55\text{-}84}=1$, $n_{56\text{-}57}=1$, $n_{55\text{-}62}=1$, $n_{27\text{-}29}=1$, $n_{62\text{-}73}=1$, $n_{64\text{-}74}=1$, $n_{19\text{-}82}=1$.

Stage P3: 2009-2012: *v3 =70,912,000US$*

$n_{73\text{-}74}=1$, $n_{29\text{-}64}=1$, $n_{68\text{-}86}=1$.

In this case the algorithm converges after solving 53 LPs and removing 4 circuits from first stage and 2 circuits from second stage in step 5. The total investment cost is v=610,173,000 US$.

When the security constraints are taken into account, CHA can find the optimal solution after solving 124 LPs. In this case the following circuits are added to base topology:

Stage P1: 2002-2005: *v1 =930,538,000US$*

$n_{52\text{-}88}=1$, $n_{43\text{-}88}=1$, $n_{2\text{-}83}=1$, $n_{9\text{-}83}=1$, $n_{56\text{-}81}=2$, $n_{55\text{-}57}=2$, $n_{56\text{-}57}=2$, $n_{55\text{-}62}=2$, $n_{9\text{-}69}=3$, $n_{60\text{-}69}=1$

$n_{27\text{-}29}=1$, $n_{19\text{-}66}=1$, $n_{8\text{-}71}=1$, $n_{8\text{-}9}=1$, $n_{62\text{-}73}=1$, $n_{45\text{-}81}=2$, $n_{54\text{-}56}=1$, $n_{19\text{-}82}=2$, $n_{83\text{-}85}=1$, $n_{82\text{-}85}=2$.

Stage P2: 2005-2009: *v2 =225,157,000US$*

$n_{15\text{-}18}=1$, $n_{30\text{-}64}=1$, $n_{30\text{-}72}=1$, $n_{56\text{-}57}=1$, $n_{3\text{-}71}=1$, $n_{31\text{-}72}=1$, $n_{27\text{-}64}=1$, $n_{48\text{-}63}=1$, $n_{62\text{-}73}=1$, $n_{64\text{-}74}=1$, $n_{72\text{-}73}=1$, $n_{7\text{-}90}=1$.

Stage P3: 2009-2012: *v3 =380,630,000US$*

$n_{52\text{-}88}=1$, $n_{13\text{-}14}=1$, $n_{45\text{-}50}=1$, $n_{55\text{-}84}=2$, $n_{59\text{-}67}=1$, $n_{55\text{-}62}=1$, $n_{18\text{-}21}=1$, $n_{27\text{-}44}=1$, $n_{27\text{-}29}=1$, $n_{73\text{-}74}=2$, $n_{29\text{-}64}=2$, $n_{64\text{-}74}=1$, $n_{19\text{-}82}=1$, $n_{68\text{-}86}=2$.

The CHA converges after adding 59 circuits to base topology, 29 circuits in first stage, 12 circuits in second stage and 18 circuits in three stages. The total investment cost for the Colombian system consider security constraints is equal to 1,276,620,000 US$ and in step 5 of CHA, 5 circuits are removed, 4 circuits from first stage and 1 from three stage.

4.7 CONCLUSION

This chapter presented one version of constructive heuristic algorithm (CHA) for the hybrid linear model (HLM) considering security constraints. The proposed algorithm was extended to multi-stage planning, where excellent results were demonstrated. The tests verified the efficient performance of the algorithm. The proposed version exhibited itself as a fast and robust algorithm for solving TEP problem by considering security constraints. An important issue about the algorithm is that the topology found by an LP is of high quality and is also feasible to the DC model. Finally, it is important to observe that besides its conventional application in the transmission network synthesis, the CHA is suited specifically for generating good quality initial topologies for evaluative algorithms or elite topologies in a meta-heuristic Tabu Search. Furthermore, it can be stated that the other application of the CHA is to generate excellent bounds for branch-and-bound algorithms.

CHAPTER 5

FUNDAMENTALS OF TABU SEARCH AND GENETIC ALGORITHM

5.1 INTRODUCTION

TS is an iterative search procedure, in which the algorithm moves from one solution to another in order to improve the solution and find the most optimal one. The movements and memory are the basic concepts of TS. The movement is an operation to jump from a solution to another, while the memory is used for different objectives. For instance, the memory can guide the search to avoid cycles. Using the concept of memory helps the search to make the specific movements forbidden or tabu.

GA is grounded on the natural selection strategy. It selects the best solution out of the present iterations and applies it as the base generation of the next iteration, which helps to obtain better solutions. GA carries out a multi-point probabilistic search and employs some genetic operations such as crossover, mutation, reproduction, etc. to diversify the solution candidates. It should be expressed that the tuning-up of the genetic operators' parameters in large-scale systems is one of their main difficulties.

In this thesis, a novel hybrid algorithm is proposed to be applied directly to DC power flow. The algorithm is established based on a model of TEP problem with security constraints. In addition, genetic algorithms alongside a Tabu search algorithm are employed to compare their achieved results with the proposed hybrid method, in the planning without consideration of security constraints. These two optimization techniques are introduced and discussed in this chapter.

5.2 TABU SEARCH

The significant practical difficulties of optimization problems have led to the development of several robust optimization techniques. Many of such newly developed methodologies have their roots in the existing natural and physical phenomena. For instance, Tabu search is based on the concepts that are utilized the fields of artificial intelligence and operations research.

Tabu search is rapidly becoming prevalent and being extensively applied to solve hard combinatorial problems that frequently occur in practical settings. In recent years, many successful applications of tabu search have been witnessed, including several variations of the scheduling problem, character recognition using neural networks, path assignment in telecommunication networks, and so forth.

Moreover, tabu search can be viewed as a powerful extension of the standard bill-climbing method, which can be defined according to the neighborhood structure and the move evaluation function. It is based on the procedures that cross the boundaries of local optimality and explore new regions in the quest for the local optimum. The basic ideas behind tabu search were originated in the 1970s and its initial form was proposed by Glover in (GLOVER, 1986).

The limitations in classical methods further extended the application of tabu search and its variants in several diverse fields. Tabu search along with genetic algorithms and simulated annealing have been widely recognized by the Committee On the Next Decade of Optimization Research (CONDOR) as an extremely promising technique for solving difficult combinatorial optimization problems.

The meta-heuristic refers to a principal strategy that works on the top of other heuristic procedures in an attempt to guide the lower level heuristic procedures beyond local optimality. In the essence, tabu search is a meta-heuristic, which uses memory-based strategies to impose certain restrictions on the search process and thereby, allow the exploration of difficult regions (GLOVER et al., 1993) Adaptive memory and responsive exploration are the key features that make tabu search intelligent. Tabu search contrasts with other memory models that are mainly reliable on random sampling of neighborhood of the promising solutions and population-based approaches.

The emphasis on the responsive exploration in tabu search, whether in a deterministic or probabilistic implementation, can be obtained from this supposition that a bad strategic choice can yield more information than a good random choice. TS is also superior compared to other techniques like the branch-and-bound technique that offers a rigid form of memory.

5.2.1 Memory and tabu search

Memory in tabu search can be both explicit and attributive. The explicit memory records complete solutions that can be assessed as promising. This information can be used to explore the neighborhood of these elite solutions at the later stage. This form is memory-intensive, which requires very intelligent data structures. Attributive memory, on the other hand, is primarily aimed at directing the search. This type of memory structure records the information about the attributes that change while moving from one solution to another. It can be considerably helpful to avoid cycles and to guide the search to un-explored regions in the search space.

5.2.2 Short-term memory

The privilege of tabu search is embedded in it short-term memory process. Both long-term and short-term memories are intended to modification of the structure of neighborhood of the current solution. The short-term memory is capable of modifying the neighborhood into a subset of it, while the long-term memory can add new solutions to those originally found in the given neighborhood. This dynamically changing neighborhood structure is one of the key futures behind tabu search. Short-term memory facilitates the aggressive exploration of a given neighborhood resulting in the selection of the best available move.

5.2.3 Tabu list and tabu tenure

A tabu list can be perceived as a FIFO (First-In-First-Out) list based on the certain attributes of k preceding moves (the parameter 'k' is usually associated with the number of completed iterations). The list of such attributes is recorded in the same sequence, as which their corresponding solution is generated. A move, which should be considered as forbidden (or as tabu) is added to the tabu list and it continues to be maintained in the list for a specified number of iterations. The period that a move remains tabu is usually referred to the tabu tenure of that move. Depending on the problem's nature, tabu tenure may remain constant throughout the search or alternatively, may dynamically vary as the search progresses.

The intuitive observations reported that the length of the tabu list should grow with the size of the problem in hand. In fact, there is no general rule that could notify us the size of tabu list for a given problem. The size can be determined experimentally. An operator should observe for the occurrence of a cycle with low dues of tabu size and then, observe the maximum size, which can be reached without compromising the quality of obtained solution. Therefore, the ideal tabu size is to be positioned between these bounds.

5.2.4 Candidate list strategy

The idea of examining the entire neighborhoods of a solution is desirable as it can yield high quality solutions. However, in a practical setting, this may turn out to be extremely time-consuming in terms of CPU time and in several cases, it may be unattainable due to the inherent limitations in the amount of the accessible computing power. In such situations, we are compelled to analyze a subset of moves present in the solution's neighborhood.

Moreover, in the literature, several strategies have been proposed, which provide broad guidelines for selecting an elite subset of promising moves out of the huge set of available moves. This elite set of moves known as the candidate list.

5.2.5 Aspiration criteria

A precise evaluation of the tabu restrictions reveals this fact that TS might be sometimes very restrictive by forbidding the moves leading to attractive un-visited solutions. Thus, there might be many situations where we want the guidance procedure to override the tabu status of a move and accept it accordingly. This can be functioned by employing the aspiration level conditions. The aspiration level component will permit the guidance procedure to override the tabu status if a particular move is sufficiently attractive to be performed instead. Furthermore, the aspiration criteria ensure that the search does not get into the cycles.

In several commercial applications of tabu search, utilization of short-term memory can provide superior solutions. However, long-term memory can be advantageous in obtaining good solutions for hard problems. It can be indicated that long-term memory is primarily used to intensify or diversify the search.

5.2.6 Long-term memory

The scope of memory used to implement tabu conditions operates on a very short-term horizon and is primarily intended to prevent cycling. Memory in tabu search can also be applied to learn more about the properties of good solutions. As they are being visited during a search, the notion of learning forms the motivation in such a way to introduce the intensification and diversification schemes for the search. In some literatures, this approach is referred as adaptive memory programming (AMP). As mentioned earlier, the purpose of both long-term and short-term memories is to create the modified neighborhoods of the current solution. The modified neighborhood in the case of short-term memory is always a subset of the neighborhood of the current solution. Long-term memory can comprise those moves that cannot be simply found in the immediate neighborhood of the current solution. This is the key difference between long-term and short-term memories.

5.2.7 Intensification and diversification

The purpose of intensification strategies is to reinforce the move combinations and solution features, which were historically found good. The goal is to give preferentiality to the moves that have similar properties to those that were identified as good by the ranking mechanism.

Intensification alone is insufficient to yield the best solutions as it primarily tries to intensify the search only in a particular region. In order to offset this weakness, the complementary notion of diversification is introduced that directs the search over the un-explored regions in the search space. Diversifications are particularly useful for escaping from local optimality. A finer look at the short-term memory strategies can unveil this fact that they embody the fundamentals of intensification and diversification.

On the other hand, several other techniques like strategic oscillation and target analysis have been lately positioned in the literature and are used in association with long-term memory to constructively drive the search towards the global optimum.

Recently, some experiments have been implemented that demonstrated a similar ability to tabu search to produce superior solutions, which can match or even surpass the best known solutions of certain hard problems.

5.3 SPECIAL TABU SEARCH

Due to the large memory requirement, TS cannot store all the information about the solutions, and thus can store only part of this information, which is stored in taboo lists. If there is a possibility for tabu search to fully store all the data, the need to define taboo list and hence aspiration criteria disappears.

One of the main problems of TS is to set the parameters related to taboo list and aspiration criteria, which is removed by eliminating taboo list, and therefore, TS converges with higher speed and accuracy. Accordingly, this thesis gives specific TS to solve the TEP, which lacks taboo list and hence Aspiration Criteria.

The idea behind the proposed TS algorithm to solve the TEP problem is based on the following experiences and observations:

I. In comparison to other optimization problems such as travelling salesman problem, TEP problem can converge in less iteration.

II. In comparison to other optimization problems, the objective function is calculated in the TEP problem by very expensive method, which means that LP must be solved to compute the objective function; while in the travelling salesman problem, the objective function can be calculated with a regular multiplication.

5.4 GENETIC ALGORITHMS

GA is another evolutionary search strategy, which is used for optimization of complex problems, especially when the objective function is not smooth, or there are multiple local optimal functions and there exist a large number of parameters.

GA was firstly introduced in the book “Adaptation in Natural and Artificial Systems” in 1975 and afterwards, it was principally developed in the USA by J. H. Holland (HOLLAND, 1975). Genetic algorithms have shown their proven ability in solving various engineering optimization problems, especially electrical power system problems such as economic dispatch (SONG et al., 1997), unit commitment (KAZARLIS et al., 1996; MAIFELD; SHEBLE, 1996), generator/hydrothermal scheduling (ORERO; ILVING, 1996; ORERO; ILVING, 1998), optimal power flow (BAKIRTZIS et al., 2002), voltage/reactive power control (YOKOYAMA et al., 1993), capacitor placement (SUNDHARARAJAN; PAHWA, 1994; DELFANTI, et al., 2000), generation expansion planning (FUKUYAMA., CHIANG., 1996), transmission expansion planning (GALLEGO et al., 1998; SILVA et al., 2000). It provides a mechanism to explore possible solutions in a wider solution space by applying crossover, mutation, and selection operators to individual solutions.

GA mimics the way the life and intelligence can evolve in a natural environment. Like the evolution process in a natural environment, evolution in GA takes place as the result of natural selection and reproduction. GA follows the Darwinian principle of natural selection, where only the fittest individuals can evolve into the next generation, while the unfit individuals cease to exist. This principle commonly known as the survival of the fittest.

The generic algorithm involves the representation mechanism, the initialization of the individuals' population (candidate solutions), credit assignment/evaluation criteria, selection mechanism, reproduction mechanism, and replacement mechanism.

5.4.1 The representation mechanism

In genetic algorithm, individuals or chromosomes are mostly represented as a string. The strings can be encoded in binary, decimal or independent bits formats.

5.4.2 The reproduction mechanism

This mechanism involves the production of additional individuals within the existing population of chromosomes. This can be simply accomplished by duplicating the current individuals within the population. However, since the essential characteristic of GA is increasing the average fitness value of the population, the choice of population for reproduction will be based on the fitness value associated with the individual. Therefore, in order to increase the average fitness value of the population over a period of time, individuals with better fitness will benefit from a better chance of being selected. It should be also expressed that in some of the problems, individuals with the best fitness may not be selected as the best individual.

5.4.3 The selection operator

The selection operator selects the individuals from the population of chromosomes in accordance with their fitness value. The individuals with better fitness values will benefit from the preference to be chosen. The selected individuals by the selection operator will be directed to the process of genetic operations (crossover and mutation) to produce fitter offspring.

There are various ways through which individuals are selected in genetic algorithm. Some of the most widespread selection methods include the elitist, fitness-proportionate, roulette, and tournament selection methods. In the elitist selection method, the fitter individuals in the population are guaranteed to be selected. In fitness-proportionate selection, the fitter individuals are more likely to be selected, but it is not guaranteed. Fitness-proportionate selection is also known as roulette-wheel selection. The tournament selection method selects fitter individuals from subgroups and only one individual from each single subgroup can be selected.

5.4.4 The genetic operators

Genetic algorithm involves two main genetic operators: crossover and mutation. The following section provides the explanation for each of these operators.

5.4.4.1 The crossover operator

In the crossover operation, two individuals are firstly chosen by using the selection operator. Then, the crossover site is randomly chosen along with the bit strings and also, the value of the chosen string is exchanged at the chosen crossover point. Finally, the two new offspring generated by this process are added to the population and they will take the place of unfit individuals.

5.4.4.2 The mutation operator

The mutation operation alters the value of one or more bits within the bit string with a very small probability value. The main purpose is to maintain the diversity within the population and inhibit the premature convergence of the search algorithm. Mutation allows the search algorithm to make a random walk among the population.

5.4.5 Replacement

In this step, the offspring replaces the other individuals in the population depending on the fitness criterion. In most cases, the worst individuals are replaced. However, it should be mentioned that in some other cases, the best, similar, or immediate parents are replaced incorrectly. Moreover, replacement may be also performed randomly in some problems.

5.5 SPECIAL GENETIC ALGORITHM

In this section, the genetic algorithm proposed by Chu-Beasley (known as CBGA) is presented, which was initially designed to solve the generalized assignment problem (CHU; BEASLEY, 1997). In the two next chapters the CBGA is applied with special TS to resolve the TEP problem under security constraints. The CBGA has some special characteristics as provided in the below:

I. CBGA uses a fitness function to identify the value of the objective function, and an unfitness function to quantify the unfeasibility of the tested solution.
II. It is different from the GA proposed by Holland, because in each iteration, it substitutes only one individual in the population.

III. It performs an efficient strategy of local improvement for each tested individual.

The CBGA presents a superior performance compared to the traditional GA. It is generally employed for planning with deterministic load, according to the tests carried out by the research group. The CBGA does not discard the unfeasible individuals. In the beginning of the calculations, unfeasible individuals are applied and inserted into the population until they converge to a point, where all the individuals are feasible. At this convergence process point, only better quality individuals are inserted into the population, or rather feasible individuals. The convergence process for the optimum or sub-optimum solution makes the population converged in a uniform way. This is the chief characteristic of the CBGA. For a small population, a small number of topologies should be chosen in the selection process. This can help maintaining the genetic diversity in the population, and thus avoiding any premature GA convergence. In this way, the genetic diversity can be raised by increasing the size of population or by incrementing the requirements for diversity. For example, it can be done by substituting an element of the population for a descendant, which is different from the entire elements of the population in a specified number of positions for the codification vector.

The Chu-Beasley genetic algorithm performs in the following structure:

i. Specify the control parameters.
ii. Create the initial population.
iii. Transform the initial population into the current population.
iv. Choose two parent topologies using the tournament selection.
v. Recombine these two topologies and preserve only one offspring.
vi. Implement the mutation in the preserved offspring.
vii. Eliminate the unfeasibility of the offspring, if it is unfeasible.
viii. Improve the objective function of the created offspring.
ix. Substitute the offspring in the population. It should be noticed that the created offspring can be incorporated into the current population according to the following conditions:
    a. If the offspring is unfeasible and there are unfeasible topologies in the current population, the created offspring substitutes the topology that has more unfeasibility, as long as this unfeasibility is greater than the unfeasibility of the created offspring. If this does not happen, the offspring will be discarded.

b. If the offspring is feasible, then the topology with more unfeasibility must be substituted.
c. If all members of the population are feasible, then the created offspring will substitute the feasible topology with the worst quality as long as the created offspring has a better quality. If this does not happen, the offspring will be discarded.
d. If the offspring is already in the current population, it must be discarded to avoid equal individuals being stored.

x. Stop the process, if the stop criterion is satisfied. Otherwise, update the iteration counter and return back to step iv.

5.6 CONCLUSIONS

This chapter presents two artificial intelligence (AI) techniques, which are tabu search and genetic algorithm. They were employed to solve TEP problem under security constraints in the next chapters. These methods have much more potential and efficiency to apply a wide variety of practical engineering problems, particularly the electrical power system problems.

CHAPTER 6

FUNDAMENTALS AND APPLICATION OF PROPOSED HYBRID ALGORITHM TO SECURITY TRANSMISSION EXPANSION PLANNING

6.1 INTRODUCTION

TEP is a nonlinear mixed-integer programming problem that encounters some difficulties, such as its time-consuming nature and the need for a non-convex optimization technique. Owing to the complicated nature of the combinatorial optimization problem and the existence of many local minima for such a problem, TEP is considered a time-consuming problem. As conventional mathematical programming does not necessarily work well for this problem, various heuristic optimization techniques such as CHAs have been employed for it. Usually, the CHAs find optimal and sub-optimal solutions for small- and medium-sized systems; however, for large and complex systems, CHAs generally find solutions of barely acceptable quality, which are frequently too far from the optimal solution. Algorithms that are more successful in relation to the quality of the solution attained for complex problems such as TEP are hybrid algorithms or meta-heuristics, which have the ability to find high-quality solutions to complex problems.

Among the metaheuristic algorithms, which have been applied to solve the TEP problem, it can be witnessed that GA has been used more frequently than others. In particular, a specific type of GA, which was firstly introduced by Chu-Beasley (CBGA), has been repeatedly employed to solve the TEP problem and it has demonstrated an acceptable performance. Moreover, the improvement phase is the main indicator for the superiority of the CBGA over the traditional GA. In almost all cases, where the CBGA has been applied for solving the TEP, the CHA has been used in the improvement phase of the CBGA. In general, CHAs are the algorithms, which solve the TEP during an iterative process. The main advantage of CHA is the rapid convergence, and wherever a high speed is needed, this algorithm can be the best choice. The CHA algorithm is able to find acceptable solutions for the networks with small-scale dimensions. However, with the increase of the system dimensions, the imperfections of this algorithm will be exposed, in a way that for large-scale systems, the obtained solution is far from the optimum solution. That is why the metaheuristic algorithms, which are aided by the CHA algorithm in a part of their optimization process, despite their successfulness in quick finding of optimal solutions for small- and medium-sized systems, do not exhibit a significant success for large-scale systems. The root of this problem has to be explored in the CHA algorithm's structure. Normally, the CHA algorithms do not observe KVL and many of the problems arise from this point. For example, this algorithm, without considering the KVL, introduces a line to be added to the network, but when it adds to the network, it has no option but to observe the KVL. Observing this law can make the situation even worse than when the line has not been added up. Another problem of the CHA algorithm refers to its intense convergence to local optimum points, so the metaheuristic algorithms that are assisted by the CHA algorithm, have to try intensively in order to get out of these points.

TS algorithm is another metaheuristic algorithm, which has been utilized to solve the TEP problem. Unlike the CBGA, this algorithm has demonstrated a high ability in solving the TEP for large-scale systems (SILVA et al., 2001). Due to the large memory requirement, TS cannot store all the information about the solutions, and hence, it stores only a part of these information on tabu lists. TS uses these information to avoid the creation of cycles in the problem-solving process. Adjusting the parameters related to the tabu lists has been one of the inseparable problems of the TS. It is worth mentioning that TS is fundamentally a slow algorithm, but if proper parameters are not selected, the slowness of the algorithm becomes more sensible.

This chapter suggests a modified CBGA algorithm for solving the TEP considering security constraints. The proposed algorithm applies STS algorithm in the improvement phase. The employed STS algorithm has been provisioned in such a way that it stores all the information related to the solutions. As a result, the need for defining tabu lists and parameter adjustment will be eliminated. Such an algorithm has a faster convergence speed in comparison with the traditional TS algorithms and can be an appropriate choice for the improvement phase of the CBGA. Furthermore, the proposed algorithm uses several strategies to reduce the search space.

The proposed algorithm, in order to consider the security constraints, applies the (N-1) criterion, which is the most prevalent criterion. The (N-1) criterion states that the network should be expanded in such a way that if a line gets outage, the network can still provide power without any difficulty.

In this chapter, the proposed hybrid algorithm is directly applied to a DC-based power flow model in order to solve static and multistage TEP problems under security constraints. The flowchart of this algorithm is shown in Fig. 5.

6.2 COMPARISON BETWEEN GA AND TS

In this section, we discuss and compare the main mechanisms of each of the two combinatorial algorithms (GA and TS). Specifically, we discuss the use of memory and the intensification and diversification mechanisms.

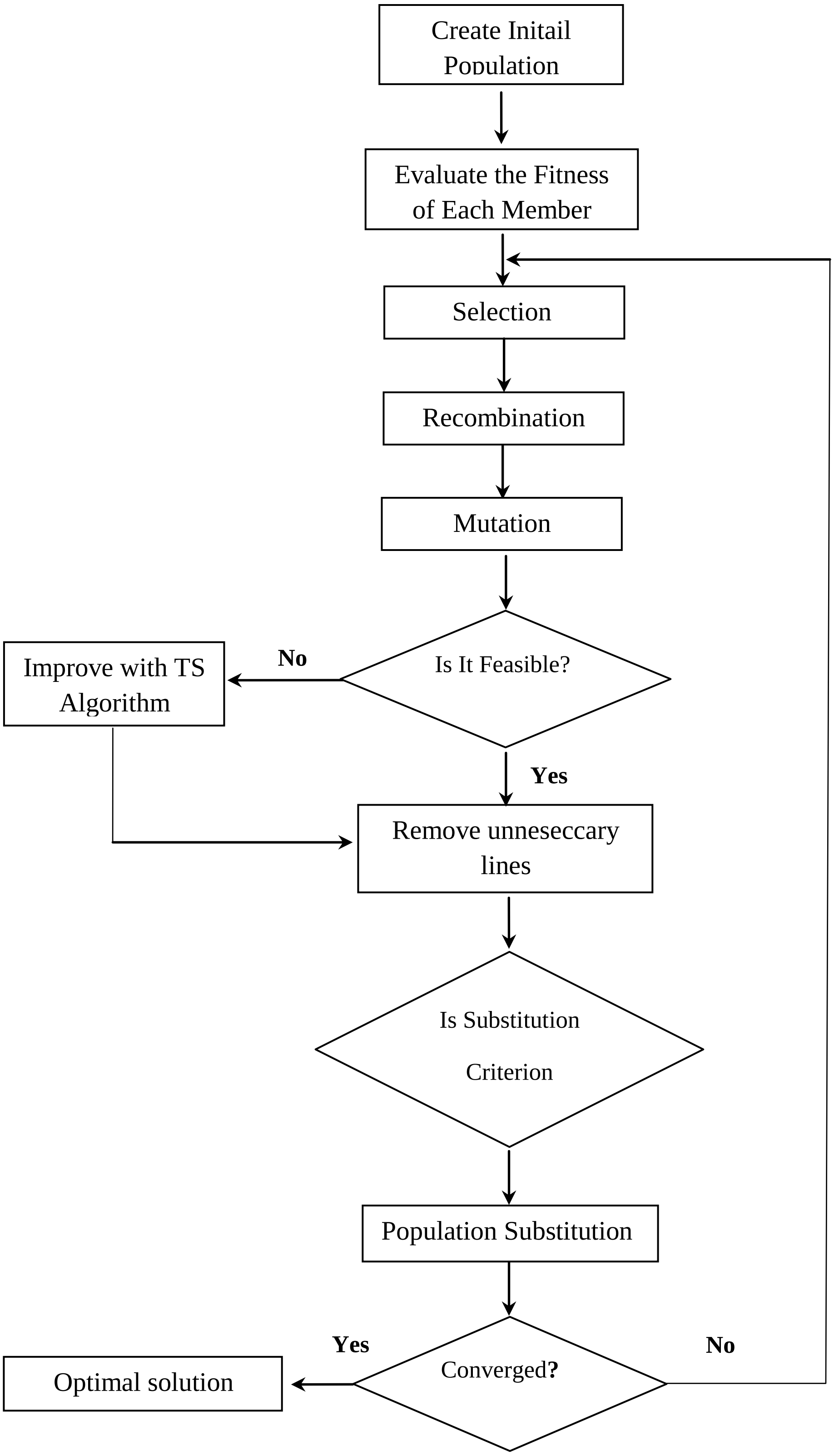
6.2.1 Short term and long term memories

GAs make implicit use of memory by keeping track of attractive building blocks. Furthermore, the fact that GA works with populations (a sequence of generations) can be related to the use of implicit memory. Explicit memory is present when a subset of a population (the current generation) is not subject to crossover and is thus retained as is for the next generation in order to preserve valuable genetic information.

In the TS approach, memory structures are used to direct the search process. Four different dimensions are considered in memory structures: recency, frequency, quality and influence. Recency-based memory is one of the most important features of TS: it is a type of short-term memory that keeps track of solution attributes that have changed during the most recent moves made by the algorithm.

The information contained in this memory allows for labelling of selected attributes of the recently visited solutions as tabu-active; this feature helps avoid revisiting the solution that was already visited in recent past.

Figure 5- Flowchart of hybrid algorithm



Source: The author

Tabu lists are used for managing recency-based memory. The frequency dimension is used in connection with long-term memory mechanisms; for instance, in the TEP problem, one could keep track of the frequencies with which line/transformer additions appeared in the configurations visited by the algorithm. This information could be used later on in diversifying the search by changing the choice rules in such a way that configurations containing yet unused features could be visited by the algorithm. The quality dimension refers to the costs associated with different solutions searched by the algorithm. For instance, certain groups of lines/transformers greatly improve candidate solutions when added together to the current configuration; these building blocks can be used as part of the path re-linking strategy, which allows the creation of a new configuration from high-quality building blocks (such building blocks are present in elite configurations stored in the long-term memory). Finally, the influence dimension takes account of the impact that a line/transformer addition may have on the loss of load: the removal of these lines/transformers will then remain tabu-active as long as possible. In summary, the TS algorithm makes extensive use of memory structures to direct the search to attractive regions and to avoid revisiting solutions that have already been considered.

6.2.2 Intensification

In GA, intensification is carried out in a non-systematic way. The mechanism of mutation allows relatively small changes to be made in candidate configurations; therefore, it is a means of searching in a neighbourhood, i.e. a mechanism for implementing intensification.

TS algorithms explore intensification in a more systematic way. Configurations found during the search are stored and their neighbourhoods are then explored more thoroughly; the local optimal solutions closest to these are found in the intensification phase.

6.2.3 Diversification

The crossover mechanism is primarily responsible for the introduction of diversification in GAs. During crossover, two high-performance configurations exchange parts of their elements (chromosomes), there by generating new configurations that are usually located in unvisited regions of the search space. Diversification is guaranteed by the random selection of configurations that are used for crossover and by the random choice of the crossover point (or points), where the strings representing the configurations are broken.

In TS algorithms, diversification is performed in a more planned way than it is in GAs. As mentioned earlier, the objective of diversification is to move the search to unvisited regions of the search space. In the TEP problem, this is achieved by temporarily changing the rules for finding new configurations: for instance, certain lines are removed from a configuration and they become tabu-active, i.e. their re-introduction is prohibited for a certain time. The removal of key lines or transformers usually changes the architecture of a solution in such a way that the algorithm is forced to visit unexplored regions as desired.

6.3 FUNDAMENTALS OF HYBRID ALGORITHM

The mathematical models presented in chapter 2 can be solved using many optimization techniques, such as heuristic algorithms, classical optimization techniques like branch and bound algorithms, Benders decomposition and meta-heuristics. In this work, we use a hybrid algorithm that belongs to the meta-heuristic group. The main advantage in the use of meta-heuristics in TEP problem is that for a specific codification proposal in which the investment variable n_{ij} is known, the TEP problem with (or without) considering security constraints is changed in to linear one. As a result, the planning problem with security can be solved using the same meta-heuristic algorithm used in planning without security. This is one of the main characteristics of meta-heuristics. Obviously, the computational effort to find good solutions to the planning problem with security is nl higher when compared to the computational effort needed to solve the planning problem without security. The proposed algorithm such as GA analyzes, in an intelligent way, a very reduced number of proposed solutions with a strategy that allows finding high quality solutions. This contrasts with CHA where a set of transitions is also carried out, but a unique solution is found that is a local optimum, and in the case of large and complex problems, generally of poor quality. Also, this strategy contrasts with the classic techniques of optimization where the optimal or suboptimal solution is generally obtained only after the algorithm converges.

6.4 APPLICATION OF HYBRID ALGORITHM TO TEP PROBLEM

In the proposed hybrid algorithm, some alterations were made so that it could solve the security TEP problem in a more efficient way. In this section, these alterations are analysed in detail.

6.4.1 Codification

The codification proposal, i.e. how a candidate solution is represented, is the most important aspect in the structure of GA. Codification can facilitate or complicate the implementation of the mechanisms of GA.

There are three alternative codification approaches as follow:

a. binary codification
b. codification by independent bits
c. decimal codification

In this work, we apply the decimal codification approach to the number of circuits added (a solution proposal). Although binary representation is normally favoured in conventional GA applications, there is at least one good reason for not using it in the TEP problem: mutation and crossover can generate offspring that are too different from their parent configurations, which sometimes induces chaotic behaviour in the GA.

An individual is a solution proposal for the planning problem, or better, it is the topology made up of all the lines added to the system corresponding to an investment proposal.

In the TEP problem, the individual of the hybrid algorithm is represented by a vector size *ni*, i.e. the number of right-of-way. Each member of this vector corresponds to a right-of-way of the system that is being analyzed, and where new lines can be constructed. Each member can vary its value from 0 to the maximum number of lines that can be added to the respective branch. Thus, in the codification shown in Fig. 6, branch $1-2$ has one new line, branch $1-5$ has two new lines, etc. The method proposed in this work does not require the characteristics of the lines between two buses to be equal; it can work with various types of circuits between the two buses. In this case, the only change occurs in the form of the linear problem to be solved. For the TEP problem, the number of individuals in the population depends on the dimensions of the system.

Figure 6- Static Codification

1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6		4-6	5-6
1	0	0	2	0	0	0	1	3	....	0	3

Source: The author

In multistage planning, a solution proposal (that is, an individual) is represented by a matrix of size *(*$t \times ni$*)*. In this way, for example, one configuration of a system that has ni paths to add circuits and t planning stages, should be represented by a vector of size $t \times ni$. This vector should be divided into t sectors and each sector must contain the number of circuits added in each planning stage. For example, in the codification shown in Fig. 7, the branch $2 - 4$ has two lines added in the first stage, and one line added in the n^{th} stage.

Figure 7- Dynamic Codification

	1-2	1-3	1-4	1-5	1-6	2-3	2-4	2-5	2-6		4-6	5-6
Stage 1	0	0	0	1	0	0	2	1	3	....	0	0
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Stage n	1	0	0	0	0	0	1	1	3	....	0	2

Source: The author

6.4.2 Initial population

The generation of the initial population is very important in order to achieve a globally optimal solution. The members of the initial population should have high quality and also be different from each other. In fact, the initial members, that are indifferent from each other, lead to the convergence of the problem to local minimums. There are many methods to produce the initial population. The simplest one is the random method, in which the initial population is arbitrarily chosen in those lines and then, added to the system. Random selection of the initial population makes the process difficult in later stages, because the process will involve a lot of calculations and it might fail to achieve an overall optimal solution in many situations. Especially in the case of large-size systems, the random initial population causes the process fail in acquiring the optimal solution. Therefore, better methods can be employed in order to generate the initial solutions that in one hand, could reduce the computational volume, and on the other hand, could largely guarantee an optimal solution.

CHA and GRASP algorithms are the methods that have been widely used in numerous papers, but their solutions require a lot of time, since each solution is obtained by solving a large number of LPs. In this method, after each LP, a line that has the greatest attractiveness for the system is added to the network. They continue to be added until the network reaches the optimal performance, which means that all equal and unequal equations are satisfied, and no variable gets out of the range. In addition, the attractiveness of a transmission line can be defined with different indices. For instance, an index can be the reduction of overload by the line. In this case, the line with the highest decrease in overload will be added to the system.

In this work, a method is proposed by which, high-quality solutions can be produced that are firstly different from each other, and secondly, are generated by solving the problem once for all. It has been proven that the transportation model is able to produce high-quality solutions for the TEP problem. To produce different solutions, some noise are added to the cost of lines, and then, the transportation model is used to generate solutions.

Although n_{ij} is a discrete variable, it is considered as a continuous variable in the transportation model and as a result, the model becomes a linear one. In order to generate each initial solution, this model is solved every time by adding a different disturbance. After solving the problem, for each path, a real number is obtained for n_{ij}. In addition, since the number of lines must be an integer number, the numbers will be rounded to integers. Moreover, it should be stated that the number of the initial solutions depends on the problem size. In fact, larger problems need the initial solutions more than the smaller ones in order to able to expand the search space.

6.4.3 Objective function

In meta-heuristic algorithms, the objective function represents the quality of a configuration (the value associated with an individual of a population). In this case, the objective function has two components: investment costs and penalties associated with the loss of load. Investment costs are functions of the decision variables (integer variables representing the addition of new transmission equipment); penalties are functions of the continuous operation variables (power flows).

For the proposed codification, the objective function is obtained by solving an LP problem that solves the planning stages or by security planning in a single plan in a simultaneous and coordinated manner.

The topologies found using the proposed codification method can be feasible or unfeasible. If LP shows a loss of load in some stages or scenarios, the topology is unfeasible from the viewpoint of operation. In the proposed algorithm, the unfeasible topologies are penalized in their objective functions by selecting a value α that is sufficiently large. In this way, in the optimization process, all topologies are considered as feasible and the optimization process eliminates those topologies that have a loss of load.

6.4.4 Selection

Once the population is defined, the hybrid algorithm applies the selection, recombination and mutation operators. Several proposals have been made on how to implement selection, but most of the selection methods work in a similar way in a planning problem. Therefore, tournament selection, which is a fast and efficient method, is used in this work. This method executes 2 or 3 games for a population size n_p. In each game, the algorithm chooses k topologies randomly and the winning topology (with the right to generate a descendant) is that whose objective function is the best. This method is efficient when a small value for k is chosen, this method is implemented in the following steps. k individuals are randomly chosen from the present population and the individual that has better fitness will be parent number 1. After this, the procedure is repeated and parent number 2 is subsequently determined. Parents 1 and 2 must be different from each other. Then, the parents go on to the recombination phase.

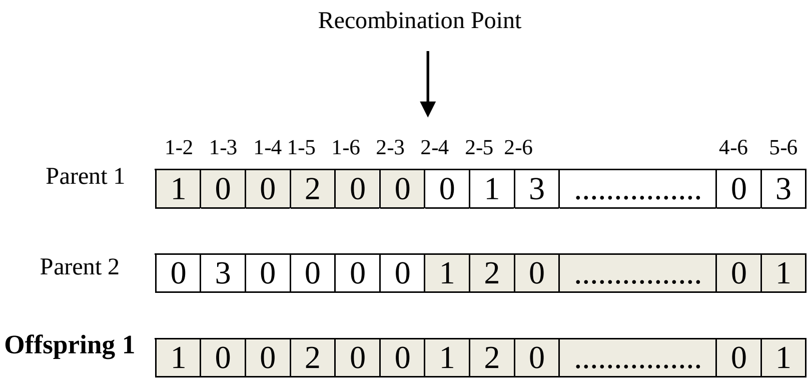
The use of a larger population is usually more efficient for solving large problems. In this case, this means great topological diversity. However, this enlargement of population should be followed by an increase in the number of individuals in the tournament selection procedure.

6.4.5 Recombination

In the recombination phase, the two selected members are combined together to create only one child. In this solution, single-point recombination is used, in which a recombination point is randomly chosen and two offspring are created. Each offspring has a piece of each parent's topology, separated by the recombination point.

In conventional GA, the two offspring can be part of the population in the next generation. However, in the CBGA, only one offspring can do so. Thus, with the same probability, only one offspring is chosen, whereas the other one is eliminated. Figure 8 shows an example of recombination wherein the second offspring is eliminated.

Figure 8- Single Point Recombination (Static Codification)



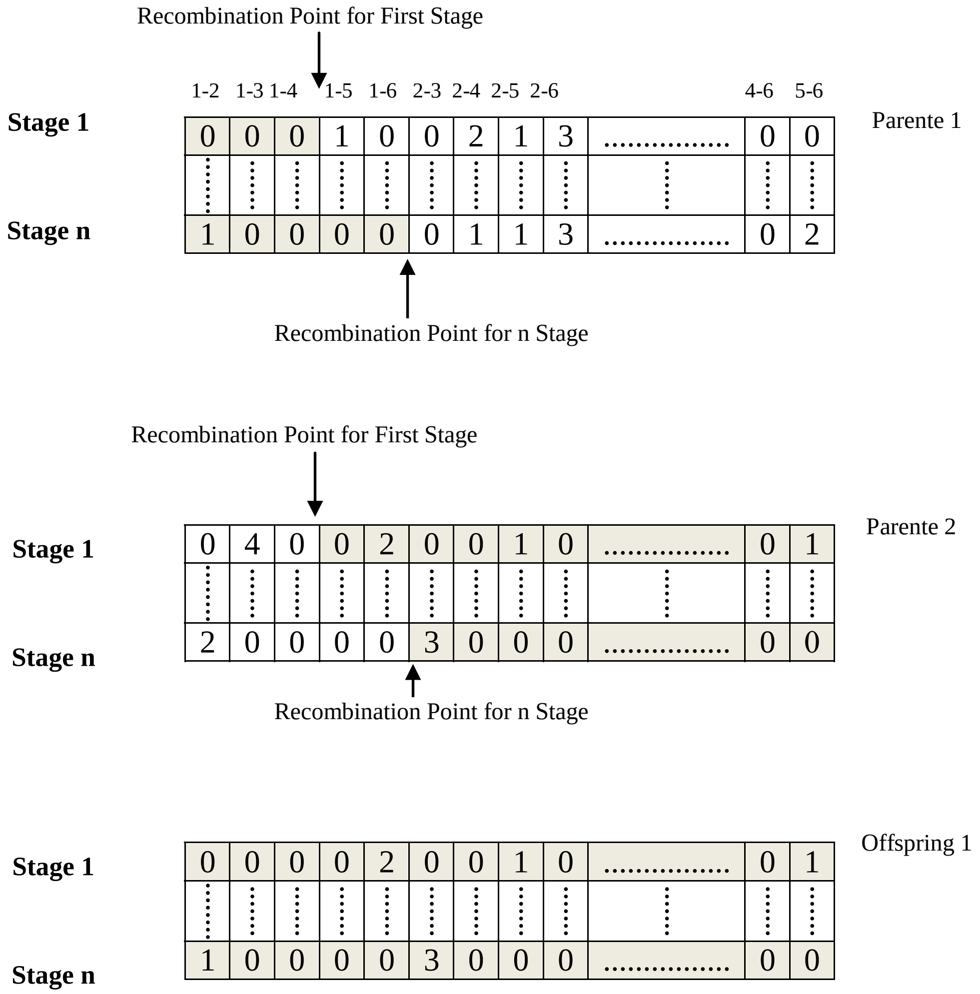
Source: The author

Figure 9 shows a recombination between multistage codifications when the second offspring is eliminated. The recombination must be performed between stages with the same order, but the recombination point can be chosen aleatorily for different stages.

6.4.6 Mutation

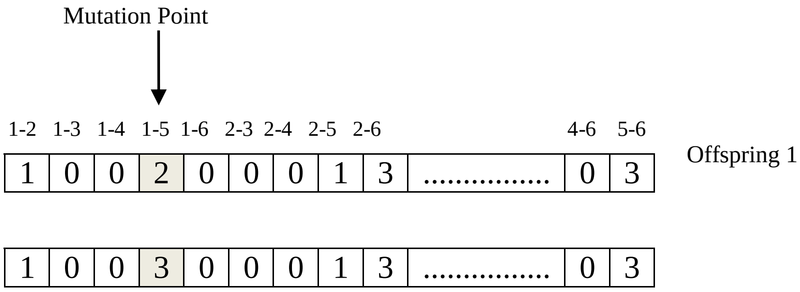
The mutation operator is easily implemented when decimal codification is used. For this reason, this codification method was used in the problem of multistage and coordinated planning in a similar way to that of static planning. In this way, once a mutation point, or path for addition or subtraction (remove) of new circuits, is chosen, the decision to increase or decrease the number of circuits in one unit should be taken randomly. Obviously, if the number of circuits in the selected branch is zero, then the decision is to add circuits, and if the number of circuits in a branch is the maximum permitted, then the decision is to eliminate circuits. Another strategy is to add two or more circuits; this represents a larger mutation. Figures 10 and 11 show examples of mutation to the static and multistage codifications, respectively.

Figure 9- Single Point Recombination (Multistage) Codification)



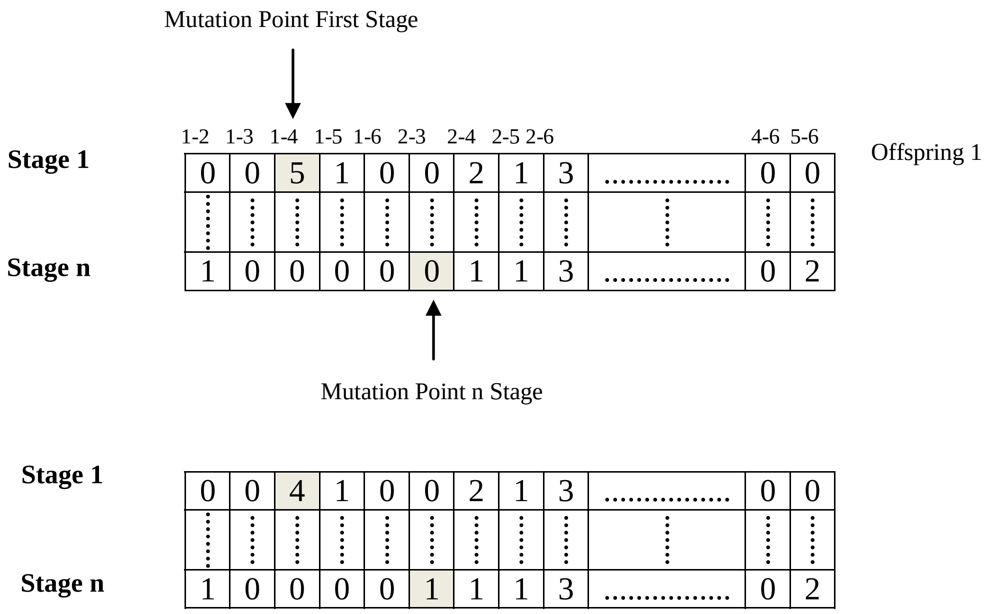
Source: The author

Figure 10- Mutation (Static Codification)



Source: The author

Figure 11- Mutation (Multistage Codification



Source: The author

6.4.7 Improvement phase

The improvement stage is the most important superiority index of the proposed hybrid algorithm in solving the TEP problem, in comparison with other meta-heuristic algorithms. If the created offspring is unfeasible, i.e., if it has load shedding in the base case, or in some contingency, or in some stages, the unfeasibility should be improved by using a suitable algorithm. Furthermore, the TS is used for adding lines to the individuals in order to eliminate the unfeasibility. Therefore, in this step, at each iteration, the best circuit (transmission line or transformer) is chosen and added to the system. In addition, the iterative process of—adding the best candidate circuit to the base topology— will be stopped, as soon as a high quality feasible solution is obtained.

Since TS is greedier than other meta-heuristics such as simulated annealing and GAs, the definition of a neighbourhood is more critical for this method; one of the main strategies of TS is to move to the best configuration in a given neighbourhood.

In TEP, the size of neighbourhoods is much larger than the size expected by an evaluation of neighbourhoods by the algorithm. However, working with full neighbourhoods is practical only for relatively small sized problems, since for larger systems, the exploration of a neighbourhood utilizes large numbers of LP routine calls. Thus, only the most attractive part of a neighbourhood is actually explored. Having the means to efficiently finding such attractive parts is a key issue in the TS methodology. This is particularly important in the TEP problem. In TEP, a neighbourhood is then considered as a reduced list of circuit additions or swaps. One of the main difficulties faced in using TS pertains to the need for reducing the size of a solution neighbourhood without compromising on quality, since TS tries to find the best configuration in the neighbourhood before making a move during the intensification phase. When dealing with practical problems, dramatic reductions in neighbourhood sizes are normally required, and this should be done adaptively.

On the other hand, a number of algorithms and heuristics have been used in the past to generate approximate solutions to the TEP problem, such as the Garver algorithm (GARVER, 1970) and the least-effort algorithm (MONTICELLI et al., 1982). These procedures are based on special features of the problem and may provide satisfactory performance under appropriate conditions.

There are three basic issues in the application of combinatorial algorithms, as listed below, for which an approximate algorithm can be of some help:

I. generation of initial configurations,
II. representation of specific knowledge about the problem and
III. use of special strategies from approximate algorithms to guide and simplify the search process.

Approximate algorithms can be extremely useful at this stage (reductions in neighbourhood sizes). For instance, when the current configuration is infeasible, i.e. it has some loss of load owing to a lack of transmission capacity, approximate methods such as those presented in (PEREIRA; PINTO, 1985; MONTICELLI et al., 1982; GARVER, 1970; VILLASANA et al., 1985) can, in addition to reducing the size of the neighbourhood significantly (to approximately one-tenth of its actual size), provide lists of attractive line/transformer additions that can be used to adaptively guide the search in a neighbourhood.

In this work, the list is obtained using the best alternatives found with three different methods:

I. Approximate algorithm such as the Graver algorithm based on a transportation model, VGS method based on a hybrid model, least-effort criterion based on DC power flow, and a minimum-loss-of-load criterion based on a specialised LP algorithm.

II. The indexes corresponding to the Lagrange multiplier of a DC model.

The first index is related to the susceptance of candidate circuits and is expressed as

$\delta_{ij} = (\lambda_i - \lambda_j)(\theta_i - \theta_j)$

where λ_i is a dual variable related to equation (2v) in the DC model expressed in equations (1v)–(7v).

$$min\, v = \sum_i r_i \tag{1v}$$

$S.t.$

$$Sf + g + r = d \tag{2v}$$

$$f_{ij} - \gamma_{ij}\left(n_{ij}^0 + n_{ij}\right)\left(\theta_i - \theta_j\right) = 0 \tag{3v}$$

$$\left|f_{ij}\right| \le \left(n_{ij}^0 + n_{ij}\right)\bar{f}_{ij} \tag{4v}$$

$$0 \le g \le \bar{g} \tag{6v}$$

$$0 \le r \le d \tag{7v}$$

It should be noted that in this model, n_{ij} is assumed to be given.

The second index is based on the maximum flow limit of lines. This index is obtained using dual variables related to equation (4v), at the optimum solution of the LP problem.

III. Circuits or sets of circuits adjacent to buses with significant unserved load, generation (buses with a large loss of load)

IV. A set of randomly chosen additions/swaps.

The list of neighbors found by these methods is a compromise between greedy algorithms (which are good for local searches) and random search (which are less prone to be trapped into local optimal points). For situations of so far unconnected load/generation sites, the list may contain elements with multiple simultaneous additions (for example a connected set of circuits and transformers or paths).

At each stage of the STS, the most important line of the list is identified through solving a LP and is then added to the network. Afterwards, the network will be updated by adding the line, and the steps will be repeated. The process will continue until the network does not require any new line. The process of finding a feasible solution by the STS is as follows:

Step 1: Solve an LP in order to obtain load shedding of the network. If load shedding is equal to zero, indicate that the network does not need any new line, stop the process, and go to step 5.

Step 2: Create the list of neighborhoods by the three explained methods.

Step 3: Add the first line of the list to the network, solve an LP in order to obtain load shedding, and save the load shedding. Then, remove this line, and add another line. This process continues until all lines of the list are tested.

Step 4: Select the most attractive line of the list (the line that results in minimum load shedding), add it to the network, and go to step 1.

Step 5: Check the obtained solution. If this solution is different from the other members of the population, stop; otherwise, remove all the added lines in this stage, and go back to step 1.

6.4.8 Remove unnecessary lines

At the end of the pervious step, the solution may have some useless additions. For elucidation, an addition is called useless, if it can be removed from the transmission expansion plan and at the same time, the network can still maintain its feasibility (without overloads). Removal of unnecessary lines is performed in this way that the added lines to the network are arranged in descending order of price. The first line is removed from the network, and an LP is solved for it. If this line is removed from the network without causing any problem (i.e., the level of load shedding from the LP solution is zero), then, the line will be removed from the network; otherwise, the line will be kept, and the next line will be examined. This process continues until all the added lines are tested.

6.4.9 Population substitution

In GA, at each iteration, the created population almost completely replaces the previous population, regardless of their compatibility. But in the proposed algorithm, the replacement has been defined as a different process, where there is only one offspring at each iteration, and under certain conditions, it can replace one of the population members. For replacement, the conditions of the new offspring should be better than the worst population member in order to replace. If all members of the population are feasible solutions to the problem, then, the worst population member is the member that has the worst adaptation, i.e. the maximum cost. However, if there is (are) infeasible solution(s) for the problem among the members, the worst member will be selected from the infeasible solution(s). In this case, the worst member is the member that has the highest amount of incompatibility, i.e. the highest load-shedding.

Now, the process of replacement can be expressed in this way that if the new offspring is a feasible solution to the problem, and if there is an infeasible solution among the population members, it will replace the member who has the highest load-shedding. In addition, in case there is no infeasible solution among the population members, it will replace if the investment cost is less than the worst member. If the new child is an infeasible solution to the problem, it can only replace the member, who has a worse situation in terms of incompatibility. In other words, it must have more load shedding.

6.4.10 Stop conditions

The proposed algorithm will stop, if one of the following two conditions is met:

- The maximum number of LPs is achieved.
- The best member of the population has not been improved in a certain number of iterations, which depends on the problem dimensions.

6.5 CONCLUSION

In this chapter, we presented an integrated algorithm of two non-convex optimization approaches (TS and CBGA). The proposed algorithm combines the most interesting features of both these approaches. The main objective of using this algorithm is to improve the convergence speed as well as the quality of the final solution of the TEP problem. TS is implemented in the improvement stage of the CBGA to generate some percentage of new members in the new population. In the TEP problem, the neighbourhood of a solution is the set of all solutions that could be obtained by adding or removing any candidate circuit; however, working with full neighbourhoods is practical only for relatively small sized problems. Thus, we proposed an efficient method to decrease the number of neighbourhoods.

CHAPTER 7

SIMULATION AND RESULTS

7.1 INTRODUCTION

The proposed hybrid algorithm has been implemented in AMPL and tested on five electrical transmission networks, the Garver 6-bus system, the IEEE 24-bus system, the Brazilian 46-bus system, the Brazilian 87-bus system and the Colombian 93-bus system. The results of proposed method for planning without considering security constrains are compared with the results of CBGA and special tabu search that have also been implemented in this research. In addition, the results of proposed method for planning with considering security constrains are compared with the ones in the literature.

7.2 STATIC PLANNING

7.2.1 Garver

The first test system adopted in this research is the well-known Garver system. The Garver system has 6 buses, 15 candidate branches, a total demand of 760 MW and a maximum possible number of added lines per branch equal to 5. In this research, static TEP problem is analyzed in both cases, with and without power generation resizing.

7.2.1.1 Planning without generation resizing

The optimal solution of the expansion planning problem without generation rescheduling and without security constraints is equal to 200 million dollars and the following lines are added to the base topology:

$n_{2-6}=4$, $n_{3-5}=1$, *and* $n_{4-6}=2$.

The hybrid proposed algorithm found this solution after 151 LPs executions, on average, with 10 individuals in the initial population and parent selection by tournament with k = 2. The planning with security constraints to this system can be found using the methodology that is proposed in this thesis, resulting in an investment of 298 million dollars and the addition of the following lines:

$n_{2-6}=4$, $n_{3-5}=2$, $n_{3-6}=1$, *and* $n_{4-6}=3$.

The proposed hybrid algorithm found the optimal solution after 50 LPs executions, on average, using an initial population with 10 individuals, and selection of parents by tournament with k = 2.

7.2.1.2 Planning with generation resizing

When it is possible to reprogram the generation, the optimal solution of the planning problem without security constraints has an investment 110 million dollars with the addition of the following lines to the base topology:

$n_{3\text{–}5}=1$ and $n_{4\text{–}6}=3$.

The proposed hybrid algorithm found the previous solution after solving 230 LPs executions, on average, with 10 individuals in the population and parent selection by tournament with k = 2.

Using the proposed algorithm, a planning test was realized, with security constraints and the possibility of generation rescheduling. In this situation, an investment 180 million dollars results, and the final optimal topology has the addition of the following lines:

$n_{2\text{–}3}=1$, $n_{2\text{–}6}=1$, $n_{3\text{–}5}=2$, *and* $n_{4\text{–}6}=3$.

The hybrid algorithm found this solution after 80 LPs executions, on average, with 10 individuals in the initial population. The parents were selected by tournament with k = 2.

The number of LPs solved to obtain the optimal solution for proposed method and GA in (SILVA et al., 2005), is shown in Table 3.

Table 3 - Computational effort for the Garver 6-bus system

Plan	Plan with repro	Plan without repro
GA	2233	3286
Hybrid algorithm	80	50

Source: The author

It can be noticed that for the Garver system, the proposed algorithm shows better performance than the GA tested in the security TEP problem because it executes fewer LPs to find the optimal solution.

7.2.2 IEEE 24-bus system

This system consists of 24 buses, 41 candidate circuits and 8550MW of total demand. Maximum possible number of added lines per branch equals 3. In this system for all the planning the following parameters were used:

1. Initial population with 20 individuals.
2. Selection of parents by tournament with k = 2.

7.2.2.1 Planning with generation resizing

For this system the necessary investment to solve the planning problem with generation resizing and without security constraints is 152 million dollars, with the addition of the following lines to the base topology:

$n_{06–10}=1$; $n_{07–08}=2$; $n_{10–12}=1$;$n_{14–16}=1$.

The algorithm found this solution after solving 260 LPs executions, on average.

The expansion planning with security constraints for this system, using the methodology proposed in this work, resulted in an investment value 441 million dollars and the added lines are:

$n_{01–05}=1$; $n_{03–24}=1$; $n_{04–09}=1$; $n_{06–10}=2$; $n_{07–08}=2$; $n_{10–11}=1$; $n_{11–13}=1$; $n_{14–16}=1$; $n_{15–24}=1$; $n_{16–17}=1$.

This solution is reported also in (CORREA et al., 2013). The algorithm found the optimal solution after solving 1593 LPs executions, on average.

7.2.2.2 Planning without generation resizing

There is four generation plans for this system when it is not possible to reprogram the generation. The electrical data and generation/load data have been taken for plans' G1–G4 of (ROMERO et al., 2005).

a) *Planning without considering security constraints*

Considering generation plan G1, the algorithm founds the expansion plan of v = 390 million dollars with the following topology:

$n_{5-1}=1$, $n_{3-24}=1$, $n_{6-10}=1$, $n_{7-8}=2$, $n_{14-16}=1$, $n_{15-21}=1$, $n_{15-24}=1$, $n_{16-17}=2$, $n_{16-19}=1$, $n_{17-18}=1$.

Taking into account the plan G2, the expansion plan of v = 392 million dollars is derived from the proposed algorithm founds considering the following topology:

$n_{5-1}=1$, $n_{3-24}=1$, $n_{6-10}=1$, $n_{7-8}=2$, $n_{10-12}=1$, $n_{14-16}=1$, $n_{15-24}=1$, $n_{16-17}=2$, $n_{17-18}=2$.

For generation plan G3, the hybrid algorithm founds the expansion plan of v = 218 million dollars with the following topology:

$n_{6-10}=1$, $n_{7-8}=2$, $n_{10-12}=1$, $n_{14-16}=1$, $n_{16-17}=1$, $n_{20-23}=1$.

And finally considering generation plan G4, the expansion plan of v = 342 million dollars using the hybrid algorithm with the following topology is in hand:

$n_{3\text{-}24}=1$, $n_{6\text{-}10}=1$, $n_{7\text{-}8}=2$, $n_{9\text{-}11}=2$, $n_{10\text{-}12}=1$, $n_{14\text{-}16}=2$, $n_{16\text{-}17}=1$.

The proposed hybrid algorithm found the optimal solution for plans G1, G2, G3 and G4 after solving 250, 190, 180 and 300 LPs respectively.

b) Planning with Considering Security Constraints

The investment of the expansion planning considering security constraints for plan G1 using the proposed algorithm is v = 787 million dollars where the algorithm converges after solving 1579 LPs and the added lines are as follows:

$n_{1\text{-}5}=1$, $n_{3\text{-}24}=2$, $n_{4\text{-}9}=1$, $n_{6\text{-}10}=2$, $n_{7\text{-}8}=2$, $n_{9\text{-}11}=1$, $n_{10\text{-}11}=1$, $n_{11\text{-}14}=1$, $n_{14\text{-}16}=2$, $n_{15\text{-}16}=1$,$n_{15\text{-}24}=1$, $n_{16\text{-}17}=3$, $n_{16\text{-}19}=2$, $n_{17\text{-}18}=2$.

Considering generation plan G2, the hybrid algorithm founds the expansion plan of v = 854 million dollars with the following topology:

$n_{1\text{-}5}=1$, $n_{3\text{-}9}=1$, $n_{3\text{-}24}=2$, $n_{4\text{-}9}=1$, $n_{6\text{-}10}=2$, $n_{7\text{-}8}=1$, $n_{10\text{-}11}=1$, $n_{10\text{-}12}=1$, $n_{11\text{-}14}=1$, $n_{14\text{-}16}=2$, $n_{15\text{-}21}=1$, $n_{15\text{-}24}=2$, $n_{16\text{-}17}=3$, $n_{17\text{-}18}=2$.

For this plan the algorithm found the solution after solving 1348 LPs executions, on average.

Considering generation plan G3, the investment of expansion plan founded by the algorithm is v = 589 million dollars with the following topology:

$n_{1\text{-}5}=1$, $n_{3\text{-}24}=1$, $n_{4\text{-}9}=1$, $n_{6\text{-}10}=2$, $n_{7\text{-}8}=2$, $n_{9\text{-}12}=1$, $n_{10\text{-}11}=1$, $n_{14\text{-}16}=2$, $n_{15\text{-}16}=1$, $n_{15\text{-}24}=1$,$n_{16\text{-}17}=2$, $n_{17\text{-}18}=1$, $n_{20\text{-}23}=1$.

The algorithm found the optimal solution after solving 7730 LPs executions, on average.

Finally for plan G4, the algorithm found the expansion plan of v = 644 million dollars after solving 4304 LPs, with the following topology:

$n_{3\text{-}24}=2$, $n_{4\text{-}9}=1$, $n_{5\text{-}10}=1$, $n_{6\text{-}10}=2$, $n_{7\text{-}8}=2$, $n_{10\text{-}11}=1$, $n_{10\text{-}12}=1$, $n_{11\text{-}14}=1$, $n_{14\text{-}16}=2$, $n_{15\text{-}24}=1$, $n_{16\text{-}17}=2$, $n_{17\text{-}18}=1$.

Tables 4 and 5 contain a comparison between the results of the proposed method and the methods in the literature for the IEEE 24-bus system. With considering cost in Table 4, it shows that the proposed method can get very better results than the other presented methods in (ORFANOS et al., 2012; VERMA et al., 2010).

Table 4- Comparison of cost for the IEEE 24-bus system

Plan	Plan G1	Plan G2	Plan G3	Plan G4
HS	1033	1034	829	913
GA	978	977	903	899
BFDEA	975	974	898	882
IHS	964	942	837	882
CHA	949	964	722	818
Hybrid Algorithm	787	854	589	644

Source: The author

The other superiority of the method is manifested when the number of fitness function or LP is taken into consideration. As it can be seen the proposed method has a perfect performance than other methods in terms of LP numbers.

Table 5- Computational effort for the IEEE 24-bus system

Plan	Plan G1	Plan G2	Plan G3	Plan G4
GA	1945090	313167	2753166	2690833
BFDEA	1157900	737300	553500	2753166
IHS	118280	20450	58400	220500
CHA	42	124	35	35
Hybrid Algorithm	1579	1348	7730	4304

Source: The author

7.2.3 Brazilian 46-Bus system

The third test system is the Brazilian 46-bus system. The system comprises 46 buses, 79 circuits, 6880 MW of total demand. This system represents a good test to the proposed approach because it is a real-world transmission system. The addition of parallel transmission lines to existing lines is again allowed in this case with a limit of 3 lines for each branch. In this system for all the planning the following parameters were used:

1. Initial population with 30 individuals.
2. Selection of parents by tournament with k = 3.

7.2.3.1 Planning with generation resizing

The necessary investments to solve the planning problem without security constraints for the south Brazilian system is 72.87 million dollars and the following lines are added:

$n_{02–05}=1$, $n_{05–06}=2$, $n_{13–20}=1$, $n_{20–21}=2$, $n_{20–23}=1$, $n_{42–43}=1$, $n_{46–06}=1$.

The algorithm found the solution after solving 3600 LPs executions, on average.

The planning with security constraints for this system using the methodology proposed in this thesis resulted in an investment of 213 million dollars where the added lines are as follows:

$n_{02–05}=1$, $n_{05–06}=3$, $n_{12–14}=1$, $n_{19–21}=1$, $n_{20–21}=3$, $n_{20–23}=2$, $n_{31–32}=1$, $n_{32–43}=1$, $n_{42–43}=2$, $n_{42–44}=1$, $n_{44–45}=1$, $n_{46–06}=2$.

It should be noted that the hybrid algorithm found the optimal solution after solving 6134 LPs while the GA proposed in (SILVA et al., 2005), find this solution after solving 303950 LPs.

7.2.3.2 Planning without generation resizing

In this case the optimal solution of the planning problem without security constraints needs an investment of 154.420 million dollars and the follow lines are added:

$n_{20-21}=1$, $n_{42-43}=2$, $n_{46-6}=1$, $n_{19-25}=1$, $n_{31-32}=1$, $n_{28-30}=1$, $n_{26-29}=3$, $n_{24-25}=2$,$n_{29-30}=2$, $n_{5-6}=2$.

The hybrid algorithm found this solution after solving 5100 LPs executions, on average. In this example that considers the planning problem with security constraints, the value of the investment is 356 million dollars and the final topology has the addition of the following lines:

$n_{2-5}=1$, $n_{12-14}=1$, $n_{19-21}=1$, $n_{17-19}=1$, $n_{14-22}=1$, $n_{32-43}=1$, $n_{20-21}=2$, $n_{42-43}=3$, $n_{46-6}=2$ $n_{19-25}=1$, $n_{21-25}=1$, $n_{31-32}=2$, $n_{28-31}=2$, $n_{31-41}=1$, $n_{40-45}=1$, $n_{24-25}=3$, $n_{40-41}=1$, $n_{5-6}=3$.

The algorithm found this solution after 2539 LPs executions, on average.

Table 6 shows a comparison between the results of the proposed method and the methods in the literature (QUA et al., 2010; FAN, SHENG, 2009; SARRAFAN, 2014; VERMA et al., 2010). As it can be seen for this case the proposed algorithm provides a better (low cost) solution while the less number of fitness function evaluations is needed.

Table 6- Comparison of cost and number of LPs for 46-bus system without repro

Plan	Investment cost (million)	Number of LPs
COA	535	
NGA	474	
PSO	432	
GA	361	298000
BFDEA	432	267000
CHA	376	50
Hybrid Algorithm	356	2539

Source: The author

7.2.4 Colombian 93-bus system

This system has 93 buses and 155 circuits, and a maximum of three circuits can be added to each corridor.

Solution to TEP without security constraints results in an investment cost of 562.417 million dollars with the addition of following 19 lines:
$n_{43\text{-}88}=2$, $n_{15\text{-}18}=1$, $n_{30\text{-}65}=1$, $n_{30\text{-}72}=1$, $n_{55\text{-}57}=1$, $n_{55\text{-}84}=1$, $n_{56\text{-}57}=1$, $n_{55\text{-}62}=1$, $n_{27\text{-}64}=1$, $n_{27\text{-}29}=1$, $n_{50\text{-}54}=1$, $n_{62\text{-}73}=1$, $n_{54\text{-}56}=1$, $n_{72\text{-}73}=1$, $n_{19\text{-}82}=2$, $n_{82\text{-}85}=1$, $n_{68\text{-}86}=1$

The hybrid algorithm found this solution after 47100 LPs executions, on average. The population had 50 individuals and the parents were selected by tournament with k = 4.

The solution to TEP with security constraints results in an investment cost of 1271 million dollars with the addition of following 47 lines:
$n_{43\text{-}88}=3$, $n_{57\text{-}81}=3$, $n_{13\text{-}14}=1$, $n_{2\text{-}83}=1$, $n_{9\text{-}83}=1$, $n_{15\text{-}18}=1$, $n_{45\text{-}50}=1$, $n_{55\text{-}57}=1$, $n_{57\text{-}84}=1$, $n_{55\text{-}84}=2$ $n_{1\text{-}59}=1$, $n_{59\text{-}67}=1$, $n_{3\text{-}71}=1$, $n_{55\text{-}62}=2$, $n_{18\text{-}58}=1$, $n_{18\text{-}21}=1$, $n_{27\text{-}44}=1$, $n_{27\text{-}29}=2$, $n_{73\text{-}74}=2$, $n_{29\text{-}64}=3$ $n_{48\text{-}63}=1$, $n_{67\text{-}68}=1$, $n_{62\text{-}73}=2$, $n_{45\text{-}81}=2$, $n_{64\text{-}74}=2$, $n_{19\text{-}82}=3$, $n_{83\text{-}85}=1$, $n_{82\text{-}85}=2$, $n_{68\text{-}86}=2$, $n_{7\text{-}90}=1$.

The hybrid algorithm found this solution after 21255 LPs executions, on average. The population had 50 individuals and the parents were selected by tournament with k=4.

7.2.5 Brazilian 87-bus system plan P2 (2008)

This system is a reduced version of the Brazilian north northeastern network: the reduced model has 87 buses, 183 right-of-ways for the addition of new circuits, and a total demand of 20316 MW for plan P1 and 29748 MW for plan P2. There is no limit for the number of circuit additions in each right-of-way.

The necessary investment to solve the planning problem without considering security constraints is 2571.870 million dollars, with the addition of the following lines to the base topology:

$n_{01-02}=1$, $n_{02-87}=1$, $n_{04-05}=4$, $n_{04-68}=1$, $n_{04-81}=3$, $n_{05-56}=1$, $n_{05-58}=4$, $n_{12-15}=1$ $n_{13-15}=4$, $n_{14-45}=1$ $n_{15-16}=4$, $n_{15-46}=1$, $n_{16-44}=6$, $n_{16-61}=2$, $n_{18-50}=11$, $n_{18-74}=6$, $n_{21-57}=2$, $n_{22-58}=2$, $n_{24-43}=1$, $n_{25-55}=4$ $n_{27-53}=1$, $n_{30-31}=1$, $n_{30-63}=2$, $n_{35-51}=2$, $n_{36-39}=1$, $n_{36-46}=3$, $n_{40-45}=2$, $n_{41-64}=2$, $n_{43-55}=3$, $n_{43-58}=3$ $n_{48-49}=1$, $n_{49-50}=4$ $n_{52-59}=1$, $n_{54-55}=1$, $n_{54-63}=1$, $n_{56-57}=1$, $n_{61-64}=1$, $n_{61-85}=3$, $n_{67-69}=2$, $n_{67-71}=3$ $n_{69-87}=1$, $n_{71-72}=1$, $n_{72-73}=1$, $n_{73-74}=2$, $n_{73-75}=1$, $n_{75-81}=1$.

The algorithm found this solution after solving 221000 LPs executions, on average.

The solution to TEP with security constraints results in an investment cost of 3963.300 million dollars with the addition of following lines.

$n_{01-02}=2$, $n_{02-60}=2$, $n_{04-05}=1$, $n_{04-60}=2$, $n_{04-68}=1$, $n_{04-81}=4$, $n_{05-56}=2$, $n_{05-58}=4$, $n_{05-60}=4$, $n_{07-08}=1$, $n_{08-73}=1$, $n_{10-11}=1$, $n_{11-15}=1$, $n_{12-13}=2$, $n_{13-14}=2$, $n_{13-15}=5$, $n_{13-45}=1$, $n_{14-45}=1$, $n_{14-59}=2$, $n_{15-16}=5$, $n_{15-45}=1$, $n_{15-46}=2$, $n_{16-44}=7$, $n_{16-61}=2$, $n_{17-18}=1$, $n_{17-59}=1$, $n_{18-50}=12$, $n_{18-74}=7$, $n_{21-57}=3$, $n_{22-58}=3$, $n_{24-43}=2$, $n_{25-55}=5$, $n_{26-54}=2$, $n_{27-53}=1$, $n_{30-31}=2$, $n_{30-63}=3$, $n_{35-47}=1$, $n_{36-39}=2$, $n_{36-46}=4$, $n_{40-45}=3$, $n_{41-64}=4$, $n_{42-44}=1$, $n_{43-55}=4$, $n_{43-58}=4$, $n_{48-49}=5$, $n_{48-50}=4$, $n_{49-50}=2$, $n_{52-59}=2$, $n_{53-54}=1$, $n_{54-63}=2$, $n_{56-57}=2$, $n_{60-87}=1$, $n_{61-64}=1$, $n_{61-85}=4$, $n_{63-64}=1$,$n_{67-69}=3$, $n_{67-71}=3$, $n_{68-69}=1$, $n_{68-83}= 1$, $n_{69-87}=1$, $n_{71-72}=2$, $n_{71-83}=1$, $n_{72-73}=3$, $n_{72-83}=1$, $n_{73-74}=3$, $n_{81-83}=2$.

The algorithm found the optimal solution after solving 48000 LPs executions, on average. In this case for plan without security constraints and plan with security constraints, the population had 50 individuals and the parents were selected by tournament with k=4.

7.3 MULTI-STAGE PLANNING

In the previous section to estimate the effectiveness of the proposed hybrid algorithm, some comparison was made between the results of the proposed method and the methods in the literature. It can be perceived that the hybrid algorithm presented one better efficiency than the other meta-heuristics tested in the security static transmission expansion planning problem because provides a better (low cost) solution for all the cases while the less number of fitness function evaluations is needed. As a consequence of the successful results obtained from solving static security TEP problem, the hybrid algorithm is then re-implemented to solve multi-stage TEP problem under security constraints, which is classed as a mixed integer nonlinear optimization problem.

Multi-stage TEP problem is more complex and difficult to be solved than the static one as not only the optimal number of new transmission lines and their locations but also the most appropriate times to carry out the investment must be considered.

The hybrid algorithm was tested to solve the multi-stage TEP problem under security constraints too. The algorithm was tested in the Colombian system and Brazilian 87-Bus System.

7.3.1 Colombian 93-bus system

For this system the available data allows a three-stage planning, namely P1, P2 and P3. The P1 stage is between the years 2002 and 2005, the P2 stage is between the years 2005 and 2009, and the P3 stage is between the years 2009 and 2012. The annual discount rate I is equal to 10%. Obviously, the circuits added to P1 appear in the objective function with their nominal costs and those added to P2 and P3 are multiplied by 0.729 and 0.478, respectively. In this system for planning without security and planning with security constraints the following parameters were used:

1. Initial population with individuals between 80 and 100.
2. Selection of parents by tournament with k = 5.

The optimum solution of this system without contingency in transmission lines has an investment of 491 million dollars and installation of 18 transmission lines, as follows:

Plan P1: 2002-2005: v1=338,744,000 US$

$n_{57\text{-}81}=2,\ n_{55\text{-}57}=1,\ n_{55\text{-}62}=1,\ n_{45\text{-}81}=1,\ n_{82\text{-}85}=1$

Plan P2: 2005-2009: v2=104,750,000 US$

$n_{27\text{-}29}=1,\ n_{62\text{-}73}=1,\ n_{72\text{-}73}=1,\ n_{19\text{-}82}=1$

Plan P3: 2009-2012: v3=158,798,000 US$

$n_{43\text{-}88}=1,\ n_{15\text{-}18}=1,\ n_{30\text{-}65}=1,\ n_{30\text{-}72}=1,\ n_{55\text{-}84}=1,\ n_{29\text{-}64}=1,\ n_{19\text{-}82}=1,\ n_{68\text{-}86}=1$

The hybrid algorithm solves about 127000 LPs to find this solution. It should be noted that the solution presents negligible-loss of load (1.3 MW).

When security constraints are taken into account, the optimal solution has an investment of 1131 million dollars and following lines are added to base topology:

Stage P1: 2002-2005: $v1 = 729{,}867{,}000$ *US$*

$n_{57\text{-}81}=3$, $n_{2\text{-}83}=1$, $n_{9\text{-}83}=1$, $n_{57\text{-}84}=2$, $n_{55\text{-}84}=2$, $n_{55\text{-}62}=2$, $n_{27\text{-}29}=1$, $n_{8\text{-}71}=1$, $n_{8\text{-}9}=1$, $n_{45\text{-}81}=2$, $n_{19\text{-}82}=2$, $n_{62\text{-}82}=1$, $n_{83\text{-}85}=1$, $n_{82\text{-}85}=2$.

Stage P2: 2005-2009: $v2 = 409{,}684{,}000$ *US$*

$n_{13\text{-}14}=1$, $n_{27\text{-}80}=1$, $n_{57\text{-}84}=1$, $n_{55\text{-}84}=1$, $n_{3\text{-}71}=1$, $n_{27\text{-}29}=1$, $n_{73\text{-}74}=2$, $n_{29\text{-}64}=2$, $n_{48\text{-}63}=1$, $n_{62\text{-}73}=2$, $n_{45\text{-}81}=1$, $n_{67\text{-}74}=2$, $n_{19\text{-}82}=1$, $n_{7\text{-}90}=1$,

Stage P3: 2009-2012: $v3 = 214{,}399{,}000$*US$*

$n_{52\text{-}88}=2$, $n_{15\text{-}18}=1$, $n_{59\text{-}67}=1$, $n_{55\text{-}62}=1$, $n_{18\text{-}58}=1$, $n_{18\text{-}21}=1$, $n_{27\text{-}44}=1$, $n_{29\text{-}64}=1$, $n_{68\text{-}86}=2$.

For security planning the algorithm found the optimal solution after solving 28000 LPs executions.

7.3.2 North-Northeast Brazilian system

The North-Northeast Brazilian System is used as another case study. This system consists of 87 buses and 183 circuits. This system represents a benchmark in the transmission planning problem, due to its high complexity and the unknown global optimal solution. There are two levels of demand, one considered for 2002 (P1) with level of 20316 MW and the other for 2008 (P2) with level of 29748 MW. In this test the factor of discount used is considered to be 10% (I = 10%). Therefore, the costs for a transmission line added in P2 are multiplied by 0.683.

In the multi-stage planning, the best topology without security constraints, found, with the actual value of investment projected to the base year 2002 equal to US$ 2,200,062,000 and loss of load 0 MW, for the two operation stages, is the following.

Stage P1: 1998-2002: $v1 = 1{,}428{,}062{,}000$*US$*

$n_{02\text{-}04}=1$, $n_{02\text{-}60}=1$, $n_{04\text{-}05}=1$, $n_{05\text{-}38}=1$, $n_{05\text{-}58}=2$, $n_{05\text{-}60}=1$, $n_{12\text{-}15}=1$, $n_{13\text{-}15}=2$, $n_{14\text{-}45}=1$, $n_{15\text{-}16}=2$, $n_{16\text{-}44}=3$, $n_{16\text{-}61}=1$, $n_{18\text{-}50}=6$, $n_{18\text{-}74}=3$, $n_{20\text{-}21}=2$, $n_{20\text{-}38}=1$, $n_{22\text{-}58}=1$, $n_{24\text{-}43}=1$, $n_{25\text{-}55}=2$, $n_{26\text{-}29}=2$, $n_{26\text{-}54}=1$, $n_{27\text{-}53}=1$, $n_{29\text{-}30}=1$, $n_{35\text{-}51}=1$, $n_{36\text{-}39}=1$, $n_{36\text{-}46}=2$, $n_{40\text{-}45}=2$, $n_{41\text{-}64}=2$, $n_{43\text{-}55}=1$, $n_{43\text{-}58}=1$, $n_{49\text{-}50}=3$, $n_{54\text{-}58}=1$, $n_{61\text{-}64}=1$, $n_{61\text{-}58}=2$, $n_{67\text{-}68}=1$, $n_{67\text{-}69}=1$, $n_{67\text{-}71}=3$, $n_{71\text{-}72}=1$, $n_{72\text{-}73}=1$, $n_{73\text{-}74}=1$.

Stage P2: 2002-2008: $v2 = 1{,}177{,}685{,}000$*US$*

$n_{01\text{-}02}=1$, $n_{04\text{-}05}=2$, $n_{04\text{-}81}=3$, $n_{05\text{-}58}=2$, $n_{13\text{-}15}=2$, $n_{15\text{-}16}=2$, $n_{15\text{-}46}=1$, $n_{16\text{-}44}=3$,$n_{16\text{-}61}=1$, $n_{18\text{-}50}=5$, $n_{18\text{-}74}=3$, $n_{20\text{-}21}=1$, $n_{20\text{-}38}=1$, $n_{20\text{-}66}=1$, $n_{22\text{-}58}=1$, $n_{25\text{-}55}=1$, $n_{26\text{-}29}=1$, $n_{26\text{-}54}=1$, $n_{29\text{-}30}=2$, $n_{30\text{-}31}=1$,

$n_{35\text{-}51}=1$, $n_{36\text{-}46}=1$, $n_{42\text{-}85}=1$, $n_{43\text{-}55}=1$,$n_{43\text{-}58}=1$, $n_{49\text{-}50}=2$, $n_{52\text{-}59}=1$, $n_{61\text{-}85}=1$,$n_{65\text{-}66}=1$, $n_{65\text{-}87}=1$, $n_{73\text{-}74}=1$, $n_{73\text{-}75}=1$, $n_{75\text{-}81}=1$.

The hybrid algorithm found the this solution after solving about 400,000 LPs where the population has between 80 and 100 individuals, and the selection of parents was made by tournament with k = 5.

The planning considering security constraints to this system can be found using the methodology that is proposed in this work, resulting in an investment of v = US$ 3,700,370,000 and the addition of the following lines:

Stage P1: 1998-2002: *v1 =2,736,980,000 US$*

$n_{01\text{-}02}=1$, $n_{02\text{-}60}=3$, $n_{04\text{-}68}=1$, $n_{04\text{-}81}=1$, $n_{05\text{-}56}=2$, $n_{05\text{-}58}=3$, $n_{05\text{-}60}=3$, $n_{08\text{-}73}=1$, $n_{10\text{-}11}=1$, $n_{11\text{-}53}=1$, $n_{12\text{-}13}=1$, $n_{13\text{-}14}=1$, $n_{13\text{-}15}=3$, $n_{14\text{-}45}=1$, $n_{14\text{-}59}=1$, $n_{15\text{-}16}=3$, $n_{15\text{-}45}=1$, $n_{16\text{-}44}=4$, $n_{16\text{-}61}=1$, $n_{17\text{-}18}=1$, $n_{17\text{-}59}=1$, $n_{18\text{-}50}=7$, $n_{18\text{-}74}=3$, $n_{18\text{-}74}=4$, $n_{21\text{-}57}=3$, $n_{22\text{-}58}=1$, $n_{24\text{-}43}=2$, $n_{25\text{-}55}=3$, $n_{26\text{-}29}=1$, $n_{27\text{-}53}=1$, $n_{29\text{-}30}=1$, $n_{30\text{-}63}=1$, $n_{35\text{-}47}=1$, $n_{36\text{-}46}=1$, $n_{39\text{-}86}=2$, $n_{40\text{-}45}=2$, $n_{41\text{-}64}=3$, $n_{43\text{-}55}=2$, $n_{43\text{-}58}=3$, $n_{47\text{-}48}=1$ $n_{48\text{-}49}=1$, $n_{49\text{-}50}=3$, $n_{52\text{-}59}=2$, $n_{53\text{-}54}=1$, $n_{53\text{-}86}=1$, $n_{54\text{-}55}=1$, $n_{54\text{-}63}=1$, $n_{56\text{-}57}=2$, $n_{61\text{-}64}=1$, $n_{61\text{-}58}=3$, $n_{61\text{-}86}=1$, $n_{63\text{-}64}=1$, $n_{67\text{-}69}=1$, $n_{67\text{-}71}=3$, $n_{68\text{-}69}=1$, $n_{68\text{-}83}=2$, $n_{71\text{-}72}=1$, $n_{71\text{-}83}=1$, $n_{72\text{-}73}=1$, $n_{72\text{-}83}=1$, $n_{73\text{-}74}=2$, $n_{81\text{-}83}=1$.

Stage P2: 2002-2008: *v2 = 1,483,830,000 US$*

$n_{01\text{-}02}=1$, $n_{04\text{-}05}=2$, $n_{04\text{-}68}=1$, $n_{04\text{-}81}=3$, $n_{05\text{-}58}=1$, $n_{06\text{-}75}=1$, $n_{12\text{-}15}=1$, $n_{13\text{-}15}=2$, $n_{15\text{-}16}=2$, $n_{15\text{-}46}=1$, $n_{16\text{-}44}=3$, $n_{16\text{-}61}=1$, $n_{18\text{-}50}=5$, $n_{18\text{-}74} =3$, $n_{25\text{-}58}=2$, $n_{25\text{-}55}=2$, $n_{30\text{-}31}=2$, $n_{30\text{-}63}=2$, $n_{35\text{-}51}=1$, $n_{36\text{-}46}=1$, $n_{40\text{-}45}=1$, $n_{41\text{-}64}=1$, $n_{43\text{-}55}=1$, $n_{43\text{-}58}=1$, $n_{49\text{-}50}=2$, $n_{54\text{-}63}=1$, $n_{61\text{-}85}=1$, $n_{71\text{-}72}=1$, $n_{72\text{-}73}=1$, $n_{73\text{-}74}=1$, $n_{73\text{-}75}=1$, $n_{75\text{-}81}=1$, $n_{81\text{-}83}=2$.

The proposed hybrid algorithm obtains the solution after solving about 35000 LPs. Its population has between 80 and 100 individuals, and the selection of parents was made by tournament with k = 5.

At final a comparison of results is presented for the Brazilian 46-bus system, the Brazilian 87-bus system and the Colombian 93-bus system with the one obtained with CBGA and special TS to confirm the potential of the proposed approach. The CBGA and STS were implemented in this thesis to compare the results. The results are shown for static planning in Table 7 and for multi-stage planning in Table 8. It should be noted that this results are only for the case of TEP without security constraints.

Table 7- Comparison of cost (million dollars) by three algorithms, for static planning, without security constraints

System	CBGA	STS	CBGA-STS
South Brazilian 46-bus System with generation Rescheduling	72.870	72.870	72.870
South Brazilian 46-bus System without generation Rescheduling	161.37	154.420	154.420
Colombian 93-bus System	616.44	567.785	562
North Brazilian 87-bus System plan 2008	2662.560	2674.450	2571.780

Source: The author

Table 8- Comparison of cost by three algorithms, for multi-stage planning, without security constraints

System	CBGA	STS	CBGA-STS
Colombian 93-bus System	503.8	508.035	491
North Brazilian 87-bus System	2281110	2367540	2200210

Source: The author

7.4 CONCLUSION

In this chapter the proposed hybrid algorithm in chapter 5 was used to solve the new mathematical model of the static/dynamic expansion planning problem with security constraints.

The results obtained using small, medium and large size known systems show the excellent performance of the proposed methodology. The proposed algorithm presented one better efficiency than the other meta-heuristics to solve the static planning problem, solving less LP problems to find the optimal solutions for small size system and finding better solution for medium and large scale system.

CHAPTER 8

CONCLUSIONS AND FUTURE

8.1 CONCLUSIONS

In this work at first, two mathematical models for static and dynamic transmission expansion problem under security constraints have been presented then two algorithms such as constructive heuristic algorithm and hybrid algorithm have been presented to solve the proposed models.

The proposed constructive heuristic algorithm firstly has been applied to solve static planning problems under security constraints. As the results for the static model showed an improvement in both cost and execution time comparing the works in the literature, then it has been extended to solve the multi-stage security planning problem, where in this work it is the first time that this complex and large-scale problem is solved. The principal advantage of this CHA is that it works with the security constrained TEP model.

Finally a combinatorial approach consisting of the TS and GA methods has been proposed for security constrained transmission expansion planning as an efficient method for achieving the optimal solutions. In the proposed algorithm some improved strategies to decrease the number of neighbors is employed and consequently it decreases the number of linear programming problems needed to be solved. At first the proposed algorithm has been applied to one-stage and multi-stage TEP problem without considering security and afterwards it has been extended to a more complex planning, where the security constrains are considered. Results of the medium and large-scale systems show a significant performance enhancement comparing with previous plausible techniques.

8.2 FUTURE

The fact that the proposed hybrid algorithm requires to solve less LP problems in order to find the optimal solutions shows that the proposed algorithm presents a better performance and higher efficiency than the other meta-heuristic techniques to solve TEP problems. It can be emphasized that unlike the other algorithms, the proposed algorithm can be applied to the more complex TEP problems as follows.

- transmission expansion planning considering FACT devices.
- transmission expansion planning considering uncertainty in demand.
- transmission expansion planning considering uncertainty in generations.
- transmission expansion planning using AC model
- transmission expansion planning considering N-1 security in generators
- multiple scenario transmission expansion planning

REFERENCES

AL-SABA, T.; EL-AMIN, I. The application of artificial intelligent tools to the transmission expansion problem. **Electric Power Systems Research,** Lausanne, v. 62, n. 1, p.117-126, 2002.

ALGUACIL, N.; MOTTO, A. L.; CONEJO, A. J. Transmission expansion planning: a mixed-integer LP approach. **IEEE Transactions on Power Systems,** Piscataway, v. 18, n. 3, p. 1070–1077, Aug. 2003.

BAHIENSE, L.; OLIVEIRA, G. C.; PEREIRA, M.; GRANVILLE, S. A mixed integer disjunctive model for transmission network expansion. **IEEE Transactions on Power Systems,** Piscataway, v. 16, n. 3, p. 560–565, Aug. 2001.

BAKIRTZIS, A. G.; BISKAS, P. N.; ZOUMAS, C. E.; PETRIDIS, V. Optimal power flow by enhanced genetic algorithm. **IEEE Transactions on Power Systems,** Piscataway, v. 17, n. 2, p. 229-236, May 2002.

BINATO, S.; PEREIRA, M.V.F.; GRANVILLE, S. A new benders decomposition approach to solve power transmission network design problems. **IEEE Transactions on Power Systems,** Piscataway, v. 16, n. 2, p. 235-240, 2001.

CECILIANO, J. L.; NIEVA, R. Transmission network planning using evolutionary programming. In: CONGRESS ON EVOLUTION COMPUTATION- CEC99, 3., 1999, Washington. **Proceedings...** Piscataway: IEEE, 1999. v. 3, p. 1796-1803.

CHU, P. C.; BEASLEY, J. E. A genetic algorithm for the generalized assignment problem. **Computers and Operations Research,** Oxford, v. 24, n. 1, p. 17-23, 1997.

CORREA, A. A.; BOLANOS, R.; SANCHEZ, A.; GARCES, A.; MOLINA, A. Multi-objective Transmission Planning with Security Constraints. In: EUROCON CONFERENCE - EUROCON, 10., 2013, Zagreb. **Proceedings...** Piscataway: IEEE, 2013. v. 1, p. 1215-1221.

DELFANTI, M.; GRANELLI, G. P.; MARANNINO, P.; MONTAGNA, M. Optimal capacitor placement using deterministic and genetic algorithms. **IEEE Transactions on Power Systems,** Piscataway, v. 15, n. 3, p. 1041-1046, Aug. 2000.

DUSONCHET, Y.P.; EL-ABIAD, A.H. Transmission planning using discrete dynamic optimization. **IEEE Transactions on Power Systems,** Piscataway, v. PAS-92, n. 4, p. 1358–1371, Jul. 1973.

EKWUW, A. O.; CORY, B. J. Transmission system expansion planning by interactive methods. **IEEE Transactions on Power Systems**, Piscataway, v. PAS-103, n. 7, p.1583-1591, jul. 1984.

ESCOBAR, A. H.; GALLEGO, R. A.; ROMERO, R. Multistage and coordinated planning of the expansion of transmission systems. **IEEE Transactions on Power Systems,** Piscataway, v. 19, n. 2, p. 735-744, May 2004.

FAN, H.; CHENG, H. Transmission network expansion planning with security constraints based on bi-level linear programming. **European Transactions on Electrical Power,** West Sussex, v. 19, n. 2, p. 388–399, 2009.

FUKUYAMA, Y.; CHIANG, H. D. A parallel genetic algorithm for generation expansion planning. **IEEE Transactions on Power Systems,** Piscataway, v. 11, n. 2, p. 955-961, May 1996.

GAJBHIYE, R. K.; NAIK, D.; DAMBHARE, S.; SOMAN, S. A. An expert system approach for multi-year short-term transmission system expansion planning: an Indian experience. **IEEE Transactions on Power Systems,** Piscataway, v. 23, n. 1, p. 226-237, Feb. 2008.

GALLEGO, R. A.; MONTICELLI, A.; ROMERO, R. Transmission system expansion planning by an extended genetic algorithm. **IEE Proceedings - Generation, Transmission and Distribution,** Stevenage, v. 145, n. 3, p. 329-335, May 1998.

GARVER, L. L. Transmission network estimation using linear programming. **IEEE Transactions on Power Systems,** Piscataway, v. 89, n. 7, p. 1688–1697, 1970.

GALIANA, F. D.; MCGILLIS, D. T.; MARIN, M. A expert systems in transmission planning. **Proc. the IEEE,** Piscataway, v. 80, n. 5, p. 712-726, May 1992.

GIL, A. H; SILVA, E. L. A reliable approach for solving the transmission network expansion planning problem using genetic algorithms. **Electric Power Systems Research,** Lausanne, v. 58, p.45-51, 2001.

GLOVER, F. Future paths for integer programming and links to artificial intelligence. **Computers and Operations Research,** Oxford, v. 13, n. 1, p. 533–549, 1986.

GLOVER, F.; TAILLARD, E.; WERRA, D. A user's guide to tabu search. **Annals of Operations Research**, New York, v. 41, n. 1, p. 3-28, 1993.

HAFFNER. S.; MONTICELLI. A.; GARCIA. J.; MANTOVANI, M.; ROMERO, R. Branch and bound algorithm for transmission system expansion planning using transportation model. **IEE Proceedings - Generation, Transmission and Distribution**, Stevenage, v. 147, n. 3, p. 149-156, May 2000.

HAFFNER. S.; MONTICELLI. A.; GARCIA. J.; ROMERO, R. Specialized branch and bound algorithm for transmission network expansion planning. **IEE Proceedings - Generation, Transmission and Distribution**, Stevenage, v. 148, n. 5, p. 482-488, Sep. 2001.

JIN, Y. X.; CHENG, H. Z. J. Y.; ZHANG, L. New discrete method for particle swarm optimization and its application in transmission network expansion planning. **Electric Power Systems Research**, Lausanne, v. 77, n. 1, p.227-233, 2007.

KAZARLIS, S. A.; BAKIRTZIS, A. G.; PETRIDIS, A. A genetic algorithm solution to the unit commitment problem. **IEEE Transactions on Power Systems,** Piscataway, v. 11, n. 1, p. 83-92, Feb. 1996

MAGHOULI, P.; HOSSEINI, S. H.; BUYGI, M. O.; SHAHIDEHPOUR, M. A. Multi-objective framework for transmission expansion planning in deregulated environments. **IEEE Transactions on Power Systems**, Piscataway, v. 24, n. 2, p. 1051 –1061, 2009.

MAIFELD, T. T.; SHEBLE, G. B. Genetic-based unit commitment algorithm. **IEEE Transactions on Power Systems,** Piscataway, v. 11, n. 3, p. 1359-1370, Aug. 1996.

MONTICELLI, A.; SANTOS JUNIOR, A.; PEREIRA, M. V. F. Interactive transmission network planning using a least-effort criterion. **IEEE Transactions on Power Systems,** Piscataway, v. 101, n. 10, p. 3919–3925, 1982.

OLIVEIRA, G. C. S; PEREIRA, M. V. F.; THOM, L. M. Multi-stage transmission expansion planning considering multiple dispatches and contingency criterion. In: CONGRESSO BRASILEIRO DE AUTOMATICA- CBA, 15., 2004, Gramado. **Anais…** Gramado: Universidade Federal do Rio Grande do Sul, 2004. p. 1-6.

ORERO, S. O.; IRVING, M.R. A genetic algorithm for generator scheduling in power system. **Int. Journal of Electrical Power and Energy Systems**, London, v. 18, n. 1, p. 19-26, 1996.

ORERO, S. O.; IRVING, M. R. A genetic algorithm modeling framework and solution technique for short term optimal hydrothermal scheduling. **IEEE Transactions on Power Systems,** Piscataway, v. 13, n. 2, p. 501-518, May 1998.

ORFANOS, G. A.; ANASTOPOULOU, A. G.; GEORGILAKIS, P. S.; HATZIARGYRIOU, N. D. Static transmission expansion planning under an improved harmony search algorithm. In: CONFERENCE ON POWER GENERATION, TRANSMISSION, DISTRIBUTION AND ENERGY CONVERSION- MEDPOWER, 8., 2012, Cagliari. **Proceedings...** Piscataway: IEEE, 2012. v. 1, p. 1–6.

ORFANOS, G. A.; GEORGILAKIS, P. S.; HATZIARGYRIOU, N. D. A recursive harmony search algorithm for security constrained transmission expansion planning. In: POWERTECH CONFERENCE - POWERTECH, 11., 2013, Grenoble. **Proceedings...** Piscataway: IEEE, 2013. v. 1, p. 1–6.

LATTORE, G.; CRUZ, R.D.; AREIZA, J.M; VILLAGES, A. Classification of publications and models on transmission expansion planning. **IEEE Transactions on Power Systems,** Piscataway, v. 18, n. 2, p. 938-945, May 2003.

PEREIRA, M. V. F; PINTO, L. M. V. G. Application of sensitivity analysis of load supplying capability to interactive transmission expansion planning. **IEEE Transactions on Power Systems,** Piscataway, v. 104, n. 2, p. 381–389, 1985.

QUA, G.; CHENGA, H. G.; YAOB, L.; MAC, Z.; ZHUC, Z. Transmission surplus capacity based power transmission expansion planning. **Electric Power Systems Research,** Lausanne, v. 80, n. 1, p. 19–27, 2010.

ROMERO, R.; GALLEGO, R. A.; MONTICELLI, A. Transmission system expansion planning by simulated annealing. **IEEE Transactions on Power Systems,** Piscataway, v. 11, n. 1, p. 364–369, Feb. 1996.

ROMERO, R.; MONTICELLI, A.; GARCIA, A.; HAFFNER, S. Test systems and mathematical models for transmission network expansion planning**. IEE Proceedings - Generation, Transmission and Distribution,** Stevenage, v. 149, n. 1, p. 29-36, 2002.

ROMERO, R.; RIDER, M. J.; SILVA, J. A meta-heuristic to solve the transmission expansion planning. **IEEE Transactions on Power Systems,** Piscataway, v. 22, n. 4, p. 2289-2291, 2007

ROMERO, R.; ROCHA, C.; MANTOVANI, J. R. S.; SANCHEZ, I. G. Constructive heuristic algorithm for the DC model in network transmission expansion planning. **IEE Proceedings - Generation, Transmission and Distribution,** Stevenage, v. 152, n. 2, p. 277-282, 2005.

ROMERO, R.; ROCHA, C.; MANTOVANI, M.; MANTOVANI, J. R. S. Analysis of heuristic algorithms for the transportation model in static and multistage planning in network, expansion systems. **IEE Proceedings - Generation, Transmission and Distribution**, Stevenage, v. 150, n. 5, p. 521–526, 2003.

SARRAFAN, E. Application of particle swarm optimization for transmission network expansion planning with security constraints. **Research Journal of Applied Sciences, Engineering and Technology,** London, v. 7, n. 13, p. 2605-2611, 2014.

SILVA, E. L.; GIL, A. H; AREIZA, J. M. Transmission network expansion planning under an improved genetic algorithm. **IEEE Transactions on Power Systems,** Piscataway, v. 15, n. 3, p. 1168-1175, Aug. 2000.

SILVA, E. L.; ORTIZ, J. M. A.; OLIVEIRA, G. C.; BINATO, S. Transmission network expansion planning under a taboo search approach. **IEEE Transactions on Power Systems**, Piscataway, v. 16, n. 1, p. 62-68, Feb. 2001.

SILVA, I. J.; RIDER, M. J.; ROMERO, R.; GARCIA, A. V.; MURARI, C. A. Transmission network expansion planning with security constraints. **IEE Proceedings - Generation, Transmission and Distribution,** Stevenage, v. 152, n. 6, p. 828–836, 2005.

SONG, Y. H.; CHOU, C. S. V. Advanced engineered-conditioning genetic approach to power economic dispatch. **IEE Proceedings - Generation, Transmission & Distribution,** Stevenage, v. 144, n. 3, p. 285-292, May 1997.

SONG, Y. H.; IRVING M. R. Optimization techniques for electrical power systems: part 2 heuristic optimization methods. **Power Engineering Journal,** Stevenage, v. 15, n. 3, p. 151-160, Jun. 2001.

SUNDHARARAJAN, S.; PAHWA, A. Optimal selection of capacitors for radial distribution systems using a genetic algorithm. **IEEE Transactions on Power Systems,** Piscataway, v. 9, n. 3, p. 1499-1507, Aug. 1994.

VERMA, A.; PANIGRAHI, B. K.; BIJWE, P. R. Harmony search algorithm for transmission network expansion planning. **IEE Proceedings - Generation, Transmission & Distribution,** Stevenage, v. 4, n. 6, p. 663–673, 2010.

VILLASANA, R.; GARVER, L. L.; SALON, S. J. Transmission network planning using linear programming. **IEEE Transactions on Power Systems,** Piscataway, v. 104, n. 2, p. 349–356, 1985.

VINASCO, G.; RIDER, M.J.; ROMERO, R. A strategy to solve the multistage transmission expansion planning problem. **IEEE Transactions on Power Systems,** Piscataway, v. 26, n. 4, p. 2574-2576, 2011.

YOKOYAMA, R.; NIIMURA, T.; NAKANISHI, Y. A coordinated control of voltage and reactive power by heuristic modeling and approximate reasoning. **IEEE Transactions on Power Systems,** Piscataway, v. 8, n. 2, p. 636-645, May 1993.

YOUSSEF, H. K.; HACKAM, R. New transmission planning model. **IEEE Transactions on Power Systems**, Piscataway, v. 4, n. 1, p. 9–18, Feb. 1989.

ZHANG, H.; VITTAL, V.; HEYDT, G. T.; QUINTERO, J. A Mixed-integer linear programming approach for multi-stage security-constrained transmission expansion planning. **IEEE Transactions on Power Systems,** Piscataway, v. 27, n. 2, p. 1125-1133, 2012.

SYSTEMS DATA FOR THE TEP PROBLEM

In this appendix, the data for Garver system, the Southern Brazilian System, the Colombian System, and the North-Northeast Brazilian systems are provided. First, the base topology for each system presented, then, when available, the bus data for each stage or for planning with and without generation rescheduling is provided. Finally, transmission lines data are given.

A. I. SYSTEMS DATA FORMAT

The data provided in following sections are useful when the DC model of power systems is used. There are three types of data for these systems:

System Stages

1 Stage Number

2 Discount factor to find the net present value for transmission investment (%).

This data is available for systems with multistage planning in which the discount factor needed to find the net present value for the transmission investment is provided.

System Buse

1 Bus Number

2 Bus Type: 0 -> Load, 1 -> Generator, 2 -> Slack

3 Load (MW)

4 Maximum generation limit (MW)

Transmission Lines

1 Line Number

2 "From" Bus

3 "To" Bus

4 Line reactance (p.u)

5 Number of existence lines

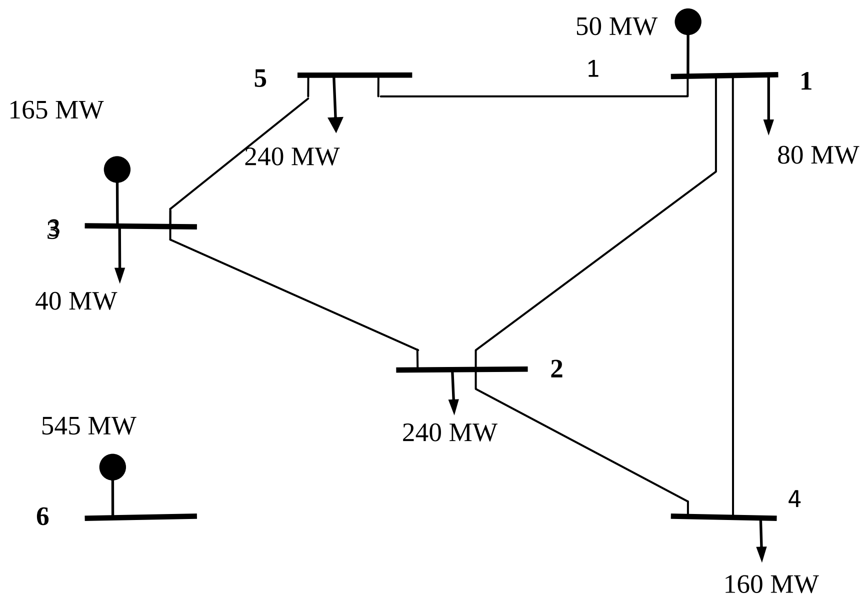
6 Maximum power flow limit in the line (MW)

7 Investment cost of the line (US$)

8 Maximum number of transmission lines in corridor

A. II. GARVER SYSTEM DATA

Figure 12- Garver system base line



Source: The Author

Table 9- Garver system bus data for TEP

Bus Number	Type	Load (MW)	Without Generation Rescheduling (MW)	With Generation Rescheduling (MW)
1	2	80	50	150
2	0	240	0	0
3	1	40	165	365
4	0	160	0	0
5	0	240	0	0
6	1	0	545	600

Source**:** Unesp (2014).

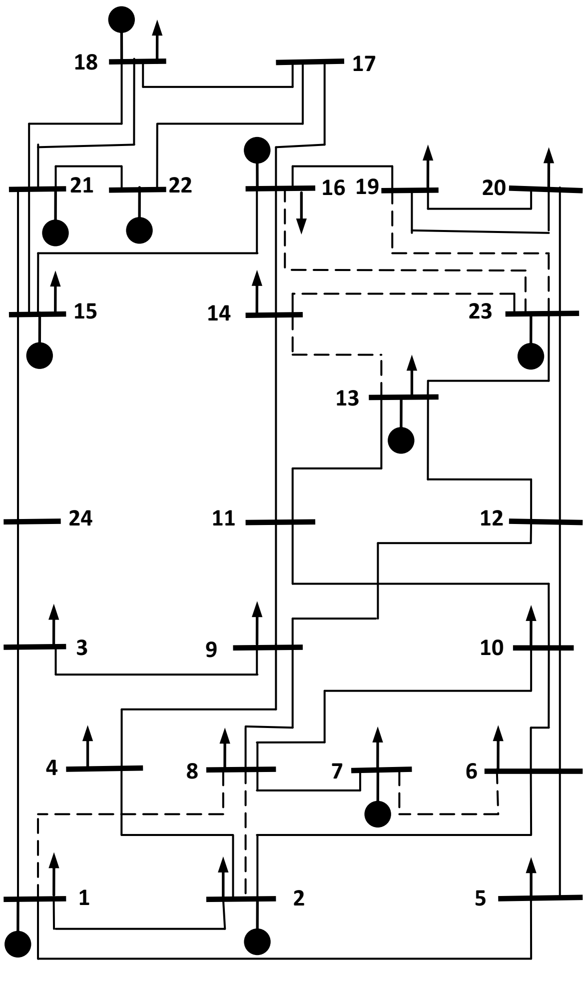
Table 10- Garver system transmission lines data

Line No.	Line From-To	Reactance (p.u)	Existing Lines	Flow Capacity (MW)	COST (US$)	Max. Lines
1	1-2	0.4	1	100	40	5
2	1-3	0.38	0	100	38	5
3	1-4	0.6	1	80	60	5
4	1-5	0.2	1	100	20	5
5	1-6	0.68	0	70	68	5
6	2-3	0.2	1	100	20	5
7	2-4	0.4	1	100	40	5
8	2-5	0.31	0	100	31	5
9	2-6	0.3	0	100	30	5
10	3-4	0.59	0	82	59	5
11	3-5	0.2	1	100	20	5
12	3-6	0.48	0	100	48	5
13	4-5	0.63	0	75	63	5
14	4-6	0.3	0	100	30	5
15	5-6	0.61	0	78	61	5

Source**:** Unesp (2014).

A. III.IEEE 24-BUS SYSTEM DATA

Figure 13- IEEE 24-bus system base topology



Source: The Author.

Table 11- IEEE 24-bus System Bus Data

Bus number	Type	Load (MW)	Maximum generation limit(MW)	Plan G1 (MW)	Plan G2 (MW)	Plan G3 (MW)	Plan G4 (MW)
1	0	324	576	576	465	576	520
2	0	291	576	576	576	576	520
3	0	540	0	0	0	0	0
4	0	222	0	0	0	0	0
5	0	213	0	0	0	0	0
6	0	408	0	0	0	0	0
7	0	375	900	900	722	900	812
8	0	513	0	0	0	0	0
9	0	525	0	0	0	0	0
10	0	585	0	0	0	0	0
11	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0
13	0	795	1773	1773	1424	1457	1599
14	0	582	0	0	0	0	0
15	0	951	645	645	645	325	581
16	0	300	465	465	465	282	419
17	0	0	0	0	0	0	0
18	0	0	1200	1200	1200	603	718
19	0	543	0	0	0	0	0
20	0	384	0	0	0	0	0
21	0	0	1200	1200	1200	951	1077
22	0	0	900	900	900	900	900
23	2	0	1980	315	953	1980	1404
24	0	0	0	0	0	0	0

Source: Unesp (2014).

Table 12- IEEE 24-bus transmission lines data

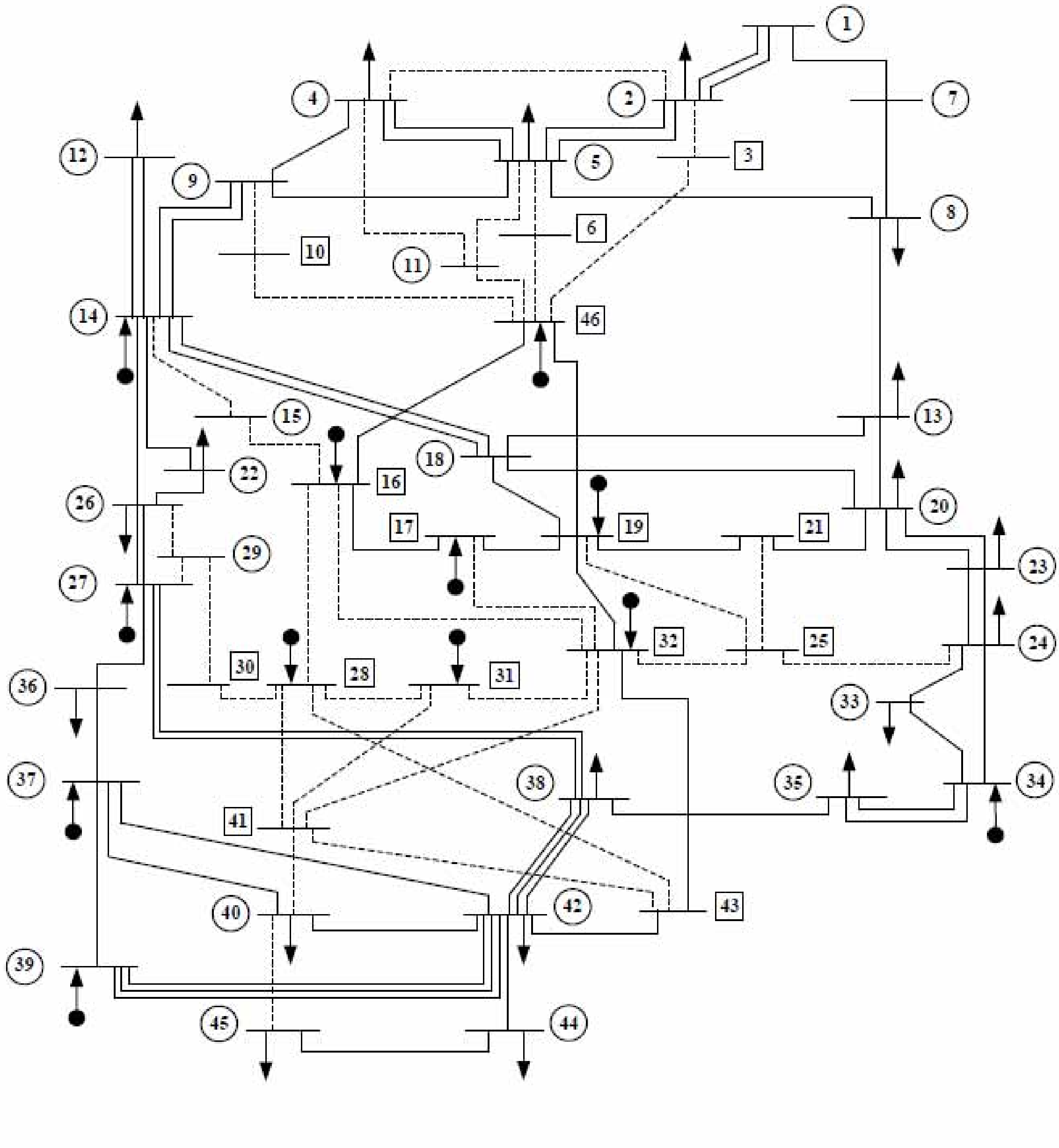
Line No.	Line From-To	Reactance (p.u)	Existing Lines	Flow Capacity (MW)	COST (US$)	Max. Lines
1	1- 2	0.0139	1	175	3.00	3
2	1- 3	0.2112	1	175	55.00	3
3	1- 5	0.0845	1	175	22.00	3
4	2- 4	0.1267	1	175	33.00	3
5	2- 6	0.1920	1	175	50.00	3
6	3- 9	0.1190	1	175	31.00	3
7	3-24	0.0839	1	400	50.00	3
8	4- 9	0.1037	1	175	27.00	3
9	5-10	0.0883	1	175	23.00	3
10	6-10	0.0605	1	175	16.00	3
11	7- 8	0.0614	1	175	16.00	3
12	8- 9	0.1651	1	175	43.00	3
13	8-10	0.1651	1	175	43.00	3
14	9-11	0.0839	1	400	50.00	3
15	9-12	0.0839	1	400	50.00	3
16	10-11	0.0839	1	400	50.00	3
17	10-12	0.0839	1	400	50.00	3
18	11-13	0.0476	1	500	66.00	3
19	11-14	0.0418	1	500	58.00	3
20	12-13	0.0476	1	500	66.00	3
21	12-23	0.0966	1	500	134.00	3
22	13-23	0.0865	1	500	120.00	3
23	14-16	0.0389	1	500	54.00	3
24	15-16	0.0173	1	500	24.00	3
25	15-21	0.0490	2	500	68.00	3
26	15-24	0.0519	1	500	72.00	3
27	16-17	0.0259	1	500	36.00	3
28	16-19	0.0231	1	500	32.00	3
29	17-18	0.0144	1	500	20.00	3
30	17-22	0.1053	1	500	146.00	3
31	18-21	0.0259	2	500	36.00	3
32	19-20	0.0396	2	500	55.00	3
33	20-23	0.0216	2	500	30.00	3
34	21-22	0.0678	1	500	94.00	3
35	1- 8	0.1344	0	500	35.00	3
36	2- 8	0.1267	0	500	33.00	3

37	6- 7	0.1920	0	500	50.00	3
38	13-14	0.0447	0	500	62.00	3
39	14-23	0.0620	0	500	86.00	3
40	16-23	0.0822	0	500	114.00	3
41	19-23	0.0606	0	500	84.00	3

Source: Unesp (2014).

A. IV. SOUTHERN BRAZILIAN SYSTEM DATA

Figure 14- Southern Brazilian system



Source**:** The author

Table 13- Southern Brazilian bus system data

Bus Number	Type	Load (MW)	Without Generation Rescheduling (MW)	With Generation rescheduling (MW)
1	0	0	0	0
2	0	443.1	0	0
3	0	0	0	0
4	0	300.7	0	0
5	0	238	0	0
6	0	0	0	0
7	0	0	0	0
8	0	72.2	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	511.9	0	0
13	0	185.8	0	0
14	2	0	1257	944
15	0	0	0	0
16	1	0	2000	1366
17	1	0	1050	1000
18	0	0	0	0
19	1	0	1670	773
20	0	1091.2	0	0
21	0	0	0	0
22	0	81.9	0	0
23	0	458.1	0	0
24	0	478.2	0	0
25	0	0	0	0
26	0	231.9	0	0
27	1	0	220	54
28	1	0	800	730
29	0	0	0	0
30	0	0	0	0
31	1	0	700	310
32	1	0	500	450
33	0	229.1	0	0
34	1	0	748	221
35	0	216	0	0

36	0	90.1	0	0
37	1	0	300	212
38	0	216	0	0
39	1	0	600	221
40	0	262.1	0	0
41	0	0	0	0
42	0	1607.9	0	0
43	0	0	0	0
44	0	79.1	0	0
45	0	86.7	0	0
46	1	0	700	599

Source**:** Unesp (2014).

Table 14- Southern Brazilian transmission lines data

Line No.	Line From-To	Reactance (p.u)	Existing Lines	Flow Capacity (MW)	COST (US$×$10^3$)	Max. Lines
1	1-7	0.0616	1	270	4349	3
2	1-2	0.1065	2	270	7076	3
3	4-9	0.0924	1	270	6217	3
4	5-9	0.1173	1	270	7732	3
5	5-8	0.1132	1	270	7480	3
6	7-8	0.1023	1	270	6823	3
7	4-5	0.0566	2	270	4046	3
8	2-5	0.0324	2	270	2581	3
9	8-13	0.1348	1	240	8793	3
10	9-14	0.1756	2	220	11267	3
11	12-14	0.074	2	270	5106	3
12	14-18	0.1514	2	240	9803	3
13	13-18	0.1805	1	220	11570	3
14	13-20	0.1073	1	270	7126	3
15	18-20	0.1997	1	200	12732	3
16	19-21	0.0278	1	1500	32632	3
17	16-17	0.0078	1	2000	10505	3
18	17-19	0.0061	1	2000	8715	3
19	14-26	0.1614	1	220	10409	3
20	14-22	0.084	1	270	5712	3
21	22-26	0.079	1	270	5409	3
22	20-23	0.0932	2	270	6268	3
23	23-24	0.0774	2	270	5308	3
24	26-27	0.0832	2	270	5662	3
25	24-34	0.1647	1	220	10611	3
26	24-33	0.1448	1	240	9399	3
27	33-34	0.1265	1	270	8288	3
28	27-36	0.0915	1	270	6167	3
29	27-38	0.208	2	200	13237	3
30	36-37	0.1057	1	270	7025	3
31	34-35	0.0491	2	270	3591	3
32	35-38	0.198	1	200	12631	3
33	37-39	0.0283	1	270	2329	3
34	37-40	0.1281	1	270	8389	3
35	37-42	0.2105	1	200	13388	3
36	39-42	0.203	3	200	12934	3

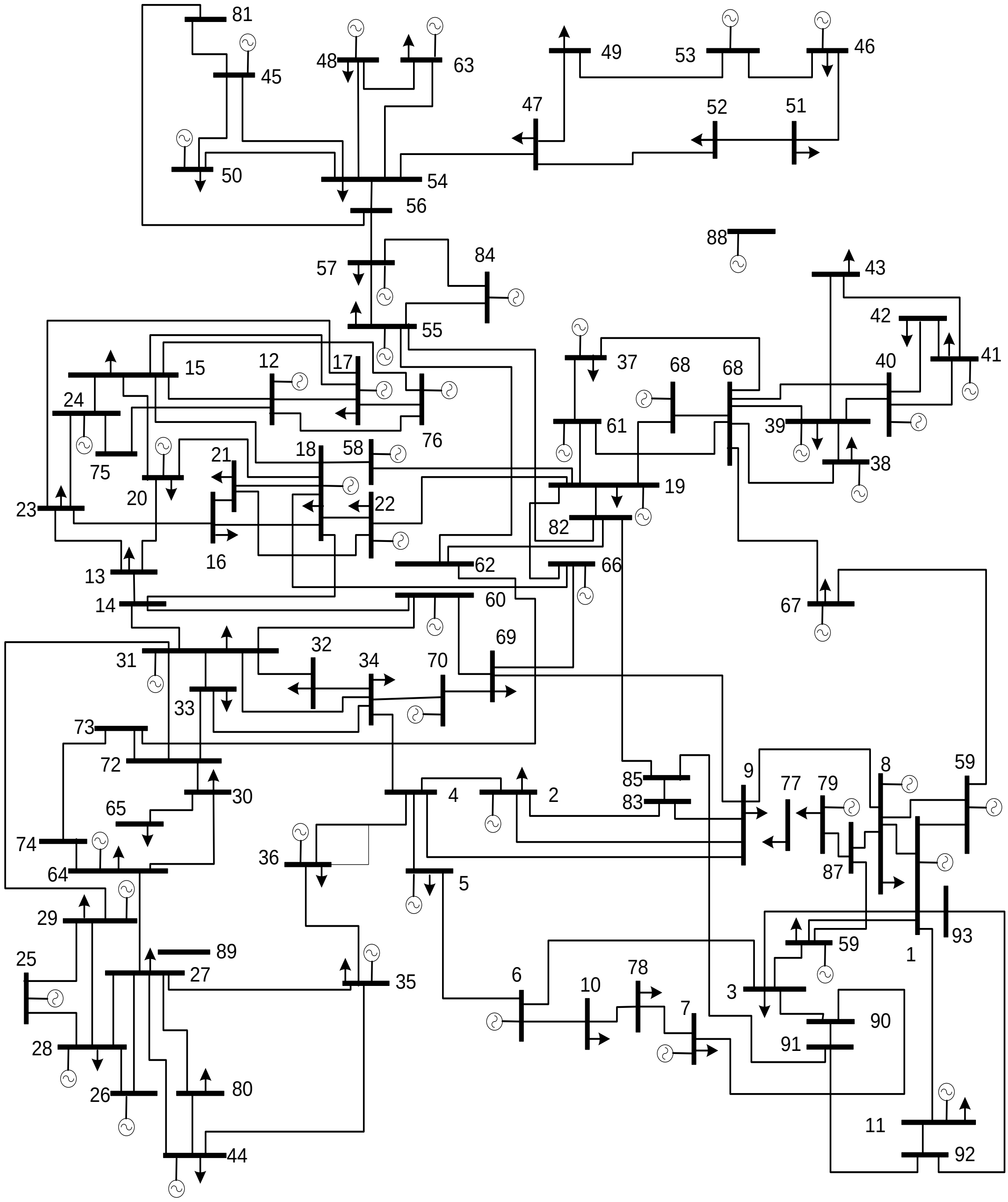
37	40-42	0.0932	1	270	6268	3
38	38-42	0.0907	3	270	6116	3
39	32-43	0.0309	1	1400	35957	3
40	42-44	0.1206	1	270	7934	3
41	44-45	0.1864	1	200	11924	3
42	19-32	0.0195	1	1800	23423	3
43	46-19	0.0222	1	1800	26365	3
44	46-16	0.0203	1	1800	24319	3
45	18-19	0.0125	1	600	8178	3
46	20-21	0.0125	1	600	8178	3
47	42-43	0.0125	1	600	8178	3
48	2-4	0.0882	0	270	5965	3
49	14-15	0.0374	0	270	2884	3
50	46-10	0.0081	0	2000	10889	3
51	4-11	0.2246	0	240	14247	3
52	5-11	0.0915	0	270	6167	3
53	46-6	0.0128	0	2000	16005	3
54	46-3	0.0203	0	1800	24319	3
55	16-28	0.0222	0	1800	26365	3
56	16-32	0.0311	0	1400	36213	3
57	17-32	0.0232	0	1700	27516	3
58	19-25	0.0325	0	1400	37748	3
59	21-25	0.0174	0	2000	21121	3
60	25-32	0.0319	0	1400	37109	3
61	31-32	0.0046	0	2000	7052	3
62	28-31	0.0053	0	2000	7819	3
63	28-30	0.0058	0	2000	8331	3
64	27-29	0.0998	0	270	6672	3
65	26-29	0.0541	0	270	3894	3
66	28-41	0.0339	0	1300	39283	3
67	28-43	0.0406	0	1200	46701	3
68	31-41	0.0278	0	1500	32632	3
69	32-41	0.0309	0	1400	35957	3
70	41-43	0.0139	0	2000	17284	3
71	40-45	0.2205	0	180	13994	3
72	15-16	0.0125	0	600	8178	3
73	46-11	0.0125	0	600	8178	3
74	24-25	0.0125	0	600	8178	3
75	29-30	0.0125	0	600	8178	3
76	40-41	0.0125	0	600	8178	3

77	2-3	0.0125	0	600	8178	3
78	5-6	0.0125	0	600	8178	3
79	9-10	0.0125	0	600	8178	3

Source: Unesp (2012).

A. V. COLOMBIAN SYSTEM DATA

Figure 15- Colombian 93 bus system initial topology



Source: The author

Colombian system planning stages:

Stage	Discount factor
1	1
2	0.729
3	0.478

Table 15- Colombian system bus data for three planning stages

Bus Number	Type	Load (2005) (MW)	Generation (2005) (MW)	Load (2009) (MW)	Generation (2009) (MW)	Load (2012) (MW)	Generation (2012) (MW)
1	1	0	241	0	241	0	241
2	0	352.9	0	406.53	165	486.66	165
3	0	393	0	490.5	0	587.08	0
4	0	0	0	0	0	0	0
5	1	235	40	293.56	40	351.42	40
6	1	0	34	0	34	0	34
7	0	300	0	374.26	0	448.03	136
8	1	339	100	423	230	505.87	230
9	0	348	0	434.12	0	519.69	0
10	0	60	0	74.21	0	88.84	0
11	1	147	80	183.9	108	220.15	108
12	1	0	47	0	47	0	47
13	0	174	0	217.26	0	260.08	0
14	0	0	0	0	0	0	0
15	0	377	0	470.17	0	562.84	0
16	0	236	0	294	0	351.9	0
17	1	136	35	169.57	35	203	35
18	1	36.2	480	45.2	540	54.1	539
19	1	19.6	900	24.46	1340	29.28	1340
20	0	202.4	0	252.5	0	302.27	45
21	0	186	0	231.7	0	277.44	0
22	1	53	200	66.13	200	79.17	200
23	0	203	0	252.5	0	302.27	0
24	1	0	120	0	150	0	150
25	1	0	86	0	86	0	86

26	1	0	70	0	70	0	70
27	0	266	0	331.4	0	396.71	0
28	0	326	0	406.3	0	486.39	14
29	1	339	618	422.6	617	505.96	618
30	0	137	0	166.7	0	199.55	0
31	1	234	189	327.3	189	391.88	189
32	0	126	0	157.3	0	188.33	0
33	0	165	0	206.53	0	247.24	0
34	0	77.5	0	96.7	0	115.81	0
35	1	172	200	214.6	200	256.86	200
36	0	112	0	140	0	167.29	44
37	1	118	138	147.3	138	176.3	138
38	0	86	0	108.4	15	129.72	15
39	0	180	0	224	0	268.19	15
40	1	0	305	0	305	0	305
41	1	54.8	70	68.4	100	81.85	100
42	0	102	0	127.3	0	152.39	0
43	0	35.4	0	44.2	0	52.9	0
44	1	257	23	321.3	23	384.64	23
45	1	0	950	0	1208	0	1208
46	1	121	150	151.7	150	181.62	150
47	0	41.15	0	51.5	0	61.6	0
48	1	600	775	750	885	896.26	885
49	0	130	0	162	0	193.27	0
50	1	424	240	528	240	632.75	240
51	0	128	0	159	0	190.45	0
52	0	38	0	46.5	0	55.6	0
53	1	0	280	0	320	0	320
54	0	76	0	95.3	0	114.19	0
55	1	223	39	279	40	333.59	40
56	0	0	0	0	0	0	0
57	0	226	0	281	130	336.94	130
58	1	0	190	0	190	0	190
59	1	0	160	0	160	0	160
60	2	0	1191	0	1216	0	1216
61	1	0	155	0	155	0	155
62	0	0	0	0	0	0	0
63	1	35	900	44	1090	52.77	1090
64	0	88	0	110.55	0	132.35	280
65	0	132	0	165	0	197.58	0

66	1	0	200	0	300	0	300
67	1	266	474	332.45	474	397.98	474
68	0	0	0	0	0	0	0
69	0	71.4	0	89	0	106.61	0
70	1	0	30	0	180	0	180
71	0	315	0	393	211	471.21	424
72	0	0	0	0	0	0	0
73	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0
76	1	0	40	0	40	0	40
77	0	55	0	70	0	82.85	0
78	0	36.65	0	45.1	0	54.07	0
79	0	98	0	123	0	146.87	300
80	0	60	0	72	0	88.34	0
81	0	0	0	0	0	0	0
82	0	0	0	0	0	0	0
83	0	0	0	0	0	0	0
84	0	0	0	0	0	0	500
85	0	0	0	0	0	0	0
86	0	0	0	0	300	0	850
87	0	0	0	0	0	0	0
88	0	0	0	0	0	0	300
89	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0
91	0	0	0	0	0	0	0
92	0	0	0	0	0	0	0
93	0	0	0	0	0	0	0

Source: Unesp (2012).

Table 16- Colombian System transmission lines data

Line No.	Line From-To	Reactance (p.u)	Existing Lines	Flow Capacity (MW)	COST (US$×$10^6$)	Max. Lines
1	52-88	0.098	0	300	34.19	5
2	43-88	0.1816	0	250	39.56	5
3	57-81	0.0219	0	550	58.89	5
4	73-82	0.0374	0	550	97.96	5
5	27-89	0.0267	0	450	13.27	5
6	74-89	0.0034	0	550	14.57	5
7	73-89	0.0246	0	550	66.65	5
8	79-83	0.0457	0	350	15.4	5
9	8-67	0.224	0	250	29.2	5
10	39-86	0.0545	0	350	9.88	5
11	25-28	0.0565	1	320	9.77	5
12	25-29	0.057	1	320	9.88	5
13	13-14	0.0009	2	350	3.9	5
14	13-20	0.0178	1	350	5.74	5
15	13-23	0.0277	1	350	7.01	5
16	14-31	0.1307	2	250	18.62	5
17	14-18	0.1494	2	250	20.23	5
18	14-60	0.1067	2	300	15.98	5
19	2-4	0.0271	2	350	6.66	5
20	2-9	0.0122	1	350	5.28	5
21	2-83	0.02	1	570	5.97	5
22	9-83	0.02	1	400	5.97	5
23	15-18	0.0365	1	450	7.93	5
24	15-17	0.0483	1	320	9.42	5
25	15-20	0.0513	1	320	9.65	5
26	15-76	0.0414	1	320	9.88	5
27	15-24	0.0145	1	350	5.28	5
28	37-61	0.0139	1	350	4.94	5
29	19-61	0.1105	2	250	16.09	5
30	61-68	0.0789	1	250	12.41	5
31	37-68	0.0544	1	320	9.65	5
32	40-68	0.132	1	320	18.16	5
33	12-75	0.0641	1	320	11.49	5
34	24-75	0.0161	1	350	5.51	5
35	35-36	0.2074	1	250	27.36	5
36	27-35	0.1498	1	250	22.07	5

37	35-44	0.1358	2	250	20.35	5
38	38-68	0.0389	1	350	7.93	5
39	38-39	0.03	1	350	6.32	5
40	27-80	0.0242	1	350	7.01	5
41	44-80	0.1014	1	250	17.59	5
42	56-81	0.0114	1	550	32.86	5
43	45-54	0.0946	1	320	13.56	5
44	45-50	0.007	2	350	4.36	5
45	10-78	0.0102	1	350	4.94	5
46	7-78	0.0043	1	350	4.13	5
47	30-64	0.1533	1	250	20.58	5
48	30-65	0.091	1	250	13.68	5
49	30-72	0.0173	2	350	5.51	5
50	55-57	0.0174	1	600	46.81	5
51	57-84	0.0087	1	600	26.66	5
52	55-84	0.0087	1	600	26.66	5
53	56-57	0.024	2	600	62.62	5
54	9-77	0.019	1	350	5.86	5
55	77-79	0.0097	1	350	5.17	5
56	1-59	0.0232	2	350	6.2	5
57	59-67	0.118	2	250	16.67	5
58	8-59	0.1056	2	250	15.4	5
59	1-3	0.104	1	250	15.86	5
60	3-71	0.0136	1	450	5.17	5
61	3-6	0.0497	1	350	9.42	5
62	55-62	0.0281	1	550	70.99	5
63	47-52	0.0644	1	350	10.57	5
64	51-52	0.0859	1	250	12.87	5
65	29-31	0.1042	2	250	32.98	5
66	41-42	0.0094	1	350	4.71	5
67	40-42	0.0153	1	350	5.17	5
68	46-53	0.1041	2	250	14.6	5
69	46-51	0.1141	1	250	16.32	5
70	69-70	0.0228	2	350	6.2	5
71	66-69	0.1217	2	250	17.13	5
72	9-69	0.1098	2	350	15.75	5
73	60-69	0.0906	2	350	13.68	5
74	31-32	0.0259	1	350	6.55	5
75	32-34	0.054	1	350	9.77	5
76	16-18	0.0625	1	350	10.92	5

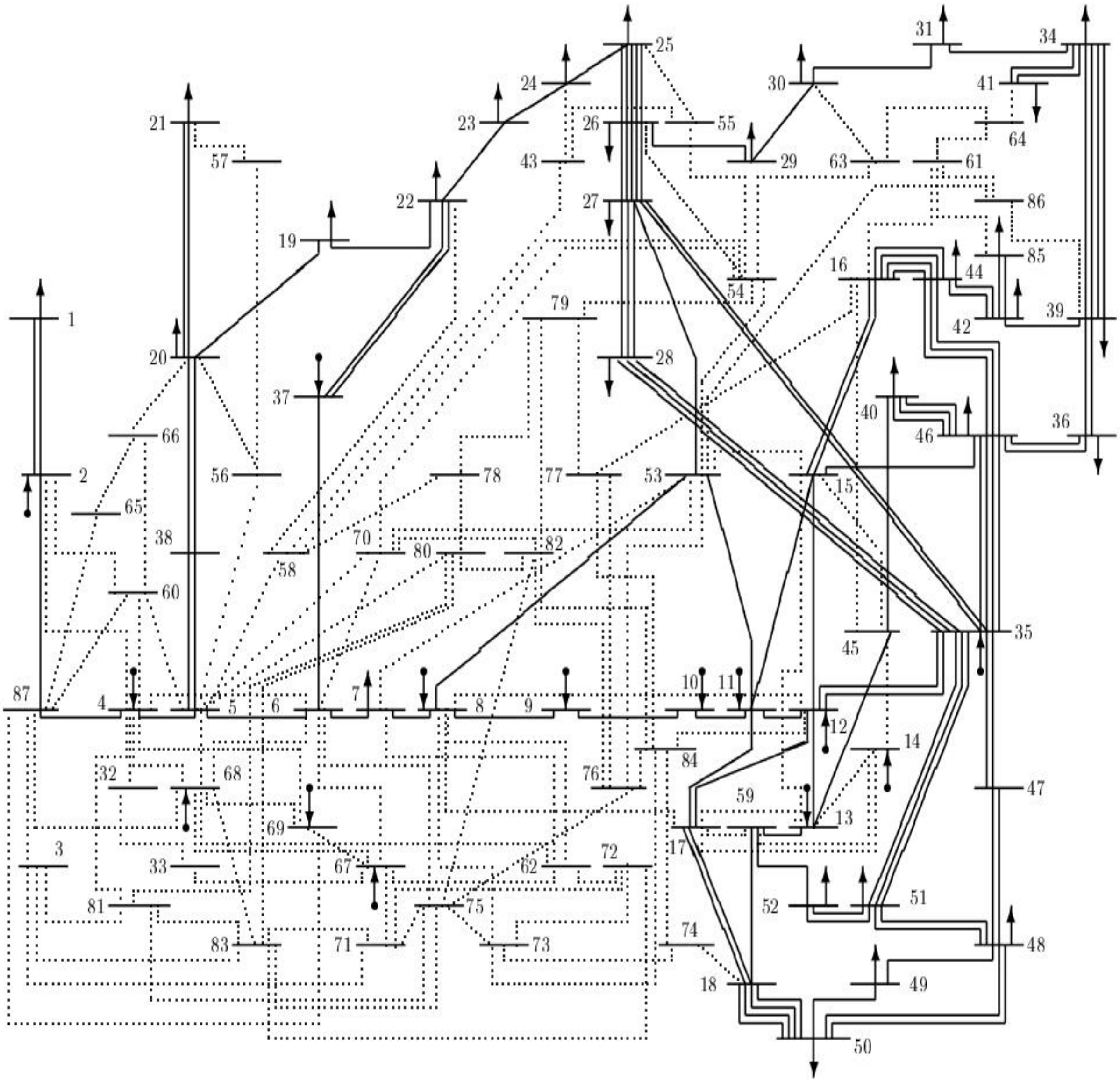
77	16-23	0.0238	1	350	6.89	5
78	16-21	0.0282	1	350	6.89	5
79	31-34	0.0792	1	250	12.41	5
80	31-33	0.0248	2	350	6.43	5
81	31-60	0.1944	2	250	25.98	5
82	31-72	0.0244	2	350	6.32	5
83	47-54	0.1003	2	250	14.25	5
84	47-49	0.0942	2	250	13.56	5
85	18-58	0.0212	2	350	5.74	5
86	18-20	0.0504	1	350	9.54	5
87	18-66	0.0664	2	350	11.38	5
88	18-21	0.0348	1	350	7.47	5
89	18-22	0.0209	1	350	6.43	5
90	19-22	0.0691	1	350	11.72	5
91	4-5	0.0049	3	350	4.25	5
92	5-6	0.0074	2	350	4.48	5
93	17-23	0.0913	1	250	12.99	5
94	17-76	0.002	1	350	3.9	5
95	12-17	0.0086	1	350	4.71	5
96	1-71	0.0841	2	250	14.37	5
97	1-8	0.081	1	250	13.22	5
98	1-11	0.0799	1	250	12.53	5
99	4-36	0.085	2	250	13.56	5
100	19-58	0.0826	1	320	11.72	5
101	27-64	0.028	1	350	6.78	5
102	27-28	0.0238	1	350	6.2	5
103	27-44	0.0893	1	250	16.32	5
104	26-27	0.0657	1	350	10.92	5
105	27-29	0.0166	1	350	5.05	5
106	19-66	0.0516	1	350	9.31	5
107	73-74	0.0214	1	600	58.28	5
108	64-65	0.0741	1	350	11.84	5
109	29-64	0.0063	1	350	4.36	5
110	4-34	0.1016	2	270	14.94	5
111	34-70	0.0415	2	350	8.27	5
112	33-34	0.1139	1	320	16.32	5
113	8-71	0.0075	1	400	4.48	5
114	54-63	0.0495	3	320	9.08	5
115	48-63	0.0238	1	350	6.32	5
116	67-68	0.166	2	250	22.07	5

117	39-68	0.0145	1	350	5.28	5
118	8-9	0.0168	1	350	5.97	5
119	79-87	0.0071	1	350	4.48	5
120	8-87	0.0132	1	350	5.17	5
121	39-43	0.1163	1	250	16.55	5
122	41-43	0.1142	1	250	16.32	5
123	23-24	0.0255	1	350	6.32	5
124	21-22	0.0549	1	350	9.88	5
125	26-28	0.0512	1	350	9.31	5
126	28-29	0.0281	1	350	6.78	5
127	6-10	0.0337	1	350	7.58	5
128	33-72	0.0228	1	350	6.2	5
129	39-40	0.102	2	250	16.21	5
130	12-76	0.0081	1	350	4.71	5
131	48-54	0.0396	3	350	8.04	5
132	50-54	0.0876	2	250	12.87	5
133	62-73	0.0272	1	750	73.16	5
134	49-53	0.1008	2	250	14.25	5
135	40-41	0.0186	1	350	5.74	5
136	45-81	0.0267	1	450	13.27	5
137	64-74	0.0267	1	500	13.27	5
138	54-56	0.0267	3	450	13.27	5
139	60-62	0.0257	3	450	13.27	5
140	72-73	0.0267	2	500	13.27	5
141	19-82	0.0267	1	450	13.27	5
142	55-82	0.029	1	550	77.5	5
143	62-82	0.0101	1	600	31	5
144	83-85	0.0267	2	450	13.27	5
145	82-85	0.0341	1	700	89.9	5
146	19-86	0.1513	1	300	20.92	5
147	68-86	0.0404	1	350	8.27	5
148	7-90	0.005	2	350	4.25	5
149	3-90	0.0074	1	350	4.59	5
150	90-91	0.0267	1	550	13.27	5
151	85-91	0.0139	1	600	40.3	5
152	11-92	0.0267	1	450	13.27	5
153	1-93	0.0267	1	450	13.27	5
154	92-93	0.0097	1	600	30.07	5
155	91-92	0.0088	1	600	27.59	5

Source: Unesp (2012).

A. VI.BRAZILIAN NORTH-NORTHEAST SYSTEM DATA

Figure 16- Brazilian North-Northeast system



Source: Escobar (2002).

Brazilian North-Northeast system planning stages:

Stage	Discount factor
1	1
2	0.656

Table 17- Brazilian North-Northeast system bus data

Bus Number	Type	Load (2002) (MW)	Generation (2002) (MW)	Load (2008) (MW)	Generation (2008) (MW)
1	0	1857	0	2747	0
2	1	0	4048	0	4550
3	0	0	0	0	0
4	1	0	517	0	6422
5	0	0	0	0	0
6	0	0	0	0	0
7	0	31	0	31	0
8	1	0	403	0	82
9	1	0	465	0	465
10	1	0	538	0	538
11	1	0	2200	0	2260
12	1	0	2257	0	4312
13	2	0	4510	0	5900
14	1	0	542	0	542
15	0	0	0	0	0
16	0	0	0	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	86	0	125	0
20	0	125	0	181	0
21	0	722	0	1044	0
22	0	291	0	446	0
23	0	58	0	84	0
24	0	159	0	230	0
25	0	1502	0	2273	0
26	0	47	0	68	0
27	0	378	0	546	0
28	0	189	0	273	0
29	0	47	0	68	0
30	0	189	0	273	0
31	0	110	0	225	0
32	0	0	0	0	0
33	0	0	0	0	0
34	0	28	0	107	0

35	1	0	1635	0	1531
36	0	225	0	325	0
37	1	0	169	0	114
38	0	0	0	0	0
39	0	186	0	269	0
40	0	1201	0	1738	0
41	0	520	0	752	0
42	0	341	0	494	0
43	0	0	0	0	0
44	0	4022	0	5819	0
45	0	0	0	0	0
46	0	205	0	297	0
47	0	0	0	0	0
48	0	347	0	432	0
49	0	777	0	1124	0
50	0	5189	0	7628	0
51	0	290	0	420	0
52	0	707	0	1024	0
53	0	0	0	0	0
54	0	0	0	0	0
55	0	0	0	0	0
56	0	0	0	0	0
57	0	0	0	0	0
58	0	0	0	0	0
59	0	0	0	0	0
60	0	0	0	0	0
61	0	0	0	0	0
62	0	0	0	0	0
63	0	0	0	0	0
64	0	0	0	0	0
65	0	0	0	0	0
66	0	0	0	0	0
67	1	0	1242	0	1242
68	1	0	888	0	888
69	1	0	902	0	902
70	0	0	0	0	0
71	0	0	0	0	0
72	0	0	0	0	0
73	0	0	0	0	0
74	0	0	0	0	0

75	0	0	0	0	0
76	0	0	0	0	0
77	0	0	0	0	0
78	0	0	0	0	0
79	0	0	0	0	0
80	0	0	0	0	0
81	0	0	0	0	0
82	0	0	0	0	0
83	0	0	0	0	0
84	0	0	0	0	0
85	0	487	0	705	0
86	0	0	0	0	0
87	0	0	0	0	0

Source: Unesp (2014).

Table 18 - Brazilian North-Northeastern transmission lines data

Line No.	Line From-To			Reactance (p.u)	Existing Lines	Flow Capacity (MW)	COST (US$×$10^3$)	Max. Lines
1	1	-	2	0.0374	2	1000	44056	16
2	2	-	4	0.0406	0	1000	48880	16
3	2	-	60	0.0435	0	1000	52230	16
4	2	-	87	0.0259	1	1000	31192	16
5	3	-	71	0.0078	0	3200	92253	16
6	3	-	81	0.0049	0	3200	60153	16
7	3	-	83	0.0043	0	3200	53253	16
8	3	-	87	0.0058	0	1200	21232	16
9	4	-	5	0.0435	1	1000	52230	16
10	4	-	6	0.0487	0	1000	58260	16
11	4	-	32	0.0233	0	300	7510	16
12	4	-	60	0.0215	0	1000	26770	16
13	4	-	68	0.007	0	1000	10020	16
14	4	-	69	0.0162	0	1000	20740	16
15	4	-	81	0.0058	0	1200	21232	16
16	4	-	87	0.0218	1	1000	26502	16
17	5	-	6	0.0241	1	1000	29852	16
18	5	-	38	0.0117	2	600	8926	16
19	5	-	56	0.0235	0	1000	29182	16
20	5	-	58	0.022	0	1000	27440	16

21	5	-	60	0.0261	0	1000	32130	16
22	5	-	68	0.0406	0	1000	48880	16
23	5	-	70	0.0464	0	1000	55580	16
24	5	-	80	0.0058	0	1200	21232	16
25	6	-	7	0.0288	1	1000	35212	16
26	6	-	37	0.0233	1	300	7510	16
27	6	-	67	0.0464	0	1000	55580	16
28	6	-	68	0.0476	0	1000	56920	16
29	6	-	70	0.0371	0	1000	44860	16
30	6	-	75	0.0058	0	1200	21232	16
31	7	-	8	0.0234	1	1000	29048	16
32	7	-	53	0.0452	0	1000	54240	16
33	7	-	62	0.0255	0	1000	31460	16
34	8	-	9	0.0186	1	1000	23420	16
35	8	-	12	0.0394	0	1000	47540	16
36	8	-	17	0.0447	0	1000	53570	16
37	8	-	53	0.0365	1	1200	44190	16
38	8	-	62	0.0429	0	1000	51560	16
39	8	-	73	0.0058	0	1200	21232	16
40	9	-	10	0.0046	1	1000	7340	16
41	10	-	11	0.0133	1	1000	17390	16
42	11	-	12	0.0041	1	1200	6670	16
43	11	-	15	0.0297	1	1200	36284	16
44	11	-	17	0.0286	1	1200	35078	16
45	11	-	53	0.0254	1	1000	31326	16
46	12	-	13	0.0046	1	1200	7340	16
47	12	-	15	0.0256	1	1200	31594	16
48	12	-	17	0.0246	1	1200	30388	16
49	12	-	35	0.0117	2	600	8926	16
50	12	-	84	0.0058	0	1200	21232	16
51	13	-	14	0.0075	0	1200	10690	16
52	13	-	15	0.0215	0	1200	26770	16
53	13	-	17	0.0232	0	1200	28780	16
54	13	-	45	0.029	1	1200	35480	16
55	13	-	59	0.0232	1	1200	28780	16
56	14	-	17	0.0232	0	1200	28780	16
57	14	-	45	0.0232	0	1200	28780	16
58	14	-	59	0.0157	0	1200	20070	16
59	15	-	16	0.0197	2	1200	24760	16
60	15	-	45	0.0103	0	1200	13906	16

61	15	-	46	0.0117	1	600	8926	16
62	15	-	53	0.0423	0	1000	50890	16
63	16	-	44	0.0117	4	600	8926	16
64	16	-	45	0.022	0	1200	27440	16
65	16	-	61	0.0128	0	1000	16720	16
66	16	-	77	0.0058	0	1200	21232	16
67	17	-	18	0.017	2	1200	21678	16
68	17	-	59	0.017	0	1200	21678	16
69	18	-	50	0.0117	4	600	8926	16
70	18	-	59	0.0331	1	1200	40170	16
71	18	-	74	0.0058	0	1200	21232	16
72	19	-	20	0.0934	1	170	5885	16
73	19	-	22	0.1877	1	170	11165	16
74	20	-	21	0.0715	1	300	6960	16
75	20	-	21	0.1032	1	170	6435	16
76	20	-	38	0.1382	2	300	12840	16
77	20	-	56	0.0117	0	600	8926	16
78	20	-	66	0.2064	0	170	12210	16
79	21	-	57	0.0117	0	600	8926	16
80	22	-	23	0.1514	1	170	9130	16
81	22	-	37	0.2015	2	170	11935	16
82	22	-	58	0.0233	0	300	7510	16
83	23	-	24	0.1651	1	170	9900	16
84	24	-	25	0.2153	1	170	12705	16
85	24	-	43	0.0233	0	300	7510	16
86	25	-	26	0.1073	2	300	29636	16
87	25	-	26	0.1691	3	170	10120	16
88	25	-	55	0.0117	0	600	8926	16
89	26	-	27	0.1404	2	300	25500	16
90	26	-	27	0.2212	3	170	12760	16
91	26	-	29	0.1081	1	170	6710	16
92	26	-	54	0.0117	0	600	8926	16
93	27	-	28	0.0826	3	170	5335	16
94	27	-	35	0.1367	2	300	25000	16
95	27	-	53	0.0117	1	600	8926	16
96	28	-	35	0.1671	3	170	9900	16
97	29	-	30	0.0688	1	170	4510	16
98	30	-	31	0.0639	1	170	4235	16
99	30	-	63	0.0233	0	300	7510	16
100	31	-	34	0.1406	1	170	8525	16

101	32	-	33	0.1966	0	170	11660	16
102	33	-	67	0.0233	0	300	7510	16
103	34	-	39	0.116	2	170	7150	16
104	34	-	39	0.2968	2	80	6335	16
105	34	-	41	0.0993	2	170	6215	16
106	35	-	46	0.2172	4	170	12705	16
107	35	-	47	0.1327	2	170	8085	16
108	35	-	51	0.1602	3	170	9625	16
109	36	-	39	0.1189	2	170	7315	16
110	36	-	46	0.0639	2	170	4235	16
111	39	-	42	0.0973	1	170	6105	16
112	39	-	86	0.0233	0	300	7510	16
113	40	-	45	0.0117	1	600	8926	16
114	40	-	46	0.0875	3	170	5500	16
115	41	-	64	0.0233	0	300	7510	16
116	42	-	44	0.0698	2	170	4565	16
117	42	-	85	0.0501	2	170	3465	16
118	43	-	55	0.0254	0	1000	31326	16
119	43	-	58	0.0313	0	1000	38160	16
120	44	-	46	0.1671	3	170	10010	16
121	47	-	48	0.1966	2	170	11660	16
122	48	-	49	0.0757	1	170	4895	16
123	48	-	50	0.0256	2	170	2090	16
124	48	-	51	0.2163	2	170	12760	16
125	49	-	50	0.0835	1	170	5335	16
126	51	-	52	0.056	2	170	3795	16
127	52	-	59	0.0117	1	600	8926	16
128	53	-	54	0.027	0	1000	32120	16
129	53	-	70	0.0371	0	1000	44860	16
130	53	-	76	0.0058	0	1200	21232	16
131	53	-	86	0.0389	0	1000	46870	16
132	54	-	55	0.0206	0	1000	25028	16
133	54	-	58	0.051	0	1000	60940	16
134	54	-	63	0.0203	0	1000	25430	16
135	54	-	70	0.036	0	1000	43520	16
136	54	-	79	0.0058	0	1200	21232	16
137	56	-	57	0.0122	0	1000	16050	16
138	58	-	78	0.0058	0	1200	21232	16
139	60	-	66	0.0233	0	300	7510	16
140	60	-	87	0.0377	0	1000	45530	16

141	61	-	64	0.0186	0	1000	23420	16
142	61	-	85	0.0233	0	300	7510	16
143	61	-	86	0.0139	0	1000	18060	16
144	62	-	67	0.0464	0	1000	55580	16
145	62	-	68	0.0557	0	1000	66300	16
146	62	-	72	0.0058	0	1200	21232	16
147	63	-	64	0.029	0	1000	35480	16
148	65	-	66	0.3146	0	170	18260	16
149	65	-	87	0.0233	0	300	7510	16
150	67	-	68	0.029	0	1000	35480	16
151	67	-	69	0.0209	0	1000	26100	16
152	67	-	71	0.0058	0	1200	21232	16
153	68	-	69	0.0139	0	1000	18060	16
154	68	-	83	0.0058	0	1200	21232	16
155	68	-	87	0.0186	0	1000	23240	16
156	69	-	87	0.0139	0	1000	18060	16
157	70	-	82	0.0058	0	1200	21232	16
158	71	-	72	0.0108	0	3200	125253	16
159	71	-	75	0.0108	0	3200	125253	16
160	71	-	83	0.0067	0	3200	80253	16
161	72	-	73	0.01	0	3200	116253	16
162	72	-	83	0.013	0	3200	149253	16
163	73	-	74	0.013	0	3200	149253	16
164	73	-	75	0.013	0	3200	149253	16
165	73	-	84	0.0092	0	3200	107253	16
166	74	-	84	0.0108	0	3200	125253	16
167	75	-	76	0.0162	0	3200	185253	16
168	75	-	81	0.0113	0	3200	131253	16
169	75	-	82	0.0086	0	3200	101253	16
170	75	-	83	0.0111	0	3200	128253	16
171	76	-	77	0.013	0	3200	149253	16
172	76	-	82	0.0086	0	3200	101253	16
173	76	-	84	0.0059	0	3200	70953	16
174	77	-	79	0.0151	0	3200	173253	16
175	77	-	84	0.0115	0	3200	132753	16
176	78	-	79	0.0119	0	3200	137253	16
177	78	-	80	0.0051	0	3200	62253	16
178	79	-	82	0.0084	0	3200	98253	16
179	80	-	81	0.0101	0	3200	117753	16
180	80	-	82	0.0108	0	3200	125253	16

181	80	-	83	0.0094	0	3200	110253	16
182	81	-	83	0.0016	0	3200	23253	16
183	82	-	84	0.0135	0	3200	155253	16

Source: Unesp (2014).