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Simplified $D = 11$ pure spinor b ghost

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Abstract: A b-ghost was constructed for the $D = 11$ non-minimal pure spinor superparticle by requiring that $\{Q, b\} = T$ where $Q = \Lambda^\alpha D_\alpha + R^\alpha \bar{W}_\alpha$ is the usual non-minimal pure spinor BRST operator. As was done for the $D = 10$ b-ghost, we will show that the $D = 11$ b-ghost can be simplified by introducing an SO(10, 1) fermionic vector $\bar{\Sigma}^i$ constructed out of the fermionic spinor D_α and pure spinor variables. This simplified version will be shown to satisfy $\{Q, b\} = T$ and $\{b, b\}$ = BRST - trivial.

Keywords: BRST Quantization, M-Theory, Supergravity Models

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Contents

1 Introduction

The $D = 11$ pure spinor superparticle is a useful tool to describe $D = 11$ linearized supergravity in a manifestly covariant way [1]. This formalism describes physical states as elements of the cohomology of a BRST operator defined by $Q_{\text{min}} = \Lambda^\alpha D_\alpha$, where Λ^α is a $D = 11$ pure spinor[1] satisfying the constraint $\Lambda\Gamma^a\Lambda = 0$, a is an SO(10, 1) vector index, and D_α are the first-class constraints of the $D = 11$ Brink-Schwarz-like superparticle [3]. The spectrum found by using this formalism coincides with that obtained via the BV quantization of $D = 11$ linearized supergravity and includes the graviton, gravitino, and 3-form at ghost-number 3, as well as their ghosts and antifields at other ghost number [1, 4],

[1] In this paper, a d=11 pure spinor Λ^α will be defined to satisfy $\Lambda\Gamma^a\Lambda = 0$. A d=11 pure spinor is sometimes [2] defined to satisfy both $\Lambda\Gamma^a\Lambda = 0$ and $\Lambda\Gamma^{ab}\Lambda = 0$.

each one of them satisfying certain equations of motion and gauge invariances as dictated by the BV prescription.

Motivated by the non-minimal version of the pure spinor superstring [5], Cederwall formulated the $D = 11$ non-minimal pure spinor superparticle by introducing a new set of variables $\bar{\Lambda}_\alpha$, R_β and their respective momenta $\bar{W}^\alpha$, S^β, where $\bar{\Lambda}_\alpha$ is a $D = 11$ bosonic spinor and R_β is a $D = 11$ fermionic spinor satisfying the constraints $\bar{\Lambda}\Gamma^a\bar{\Lambda} = 0$ and $\bar{\Lambda}\Gamma^a R = 0$ [6, 7]. In order for the new variables to not affect the physical spectrum, the BRST operator should be modified to $Q = \Lambda^\alpha D_\alpha + R_\alpha \bar{W}^\alpha$, as in the quartet argument of [8]. In the non-minimal pure spinor formalism of superstring, one can formulate a consistent prescription to compute scattering amplitudes by constructing a non-fundamental b ghost satisfying $\{Q, b\} = T$. Therefore, it is important to know if a similar b ghost can be constructed in the $D = 11$ superparticle case.

The $D = 11$ b-ghost was first constructed in [9] in terms of quantities which are not manifestly invariant under the gauge symmetries of w_α generated by $\Lambda\Gamma^a\Lambda = 0$. This b-ghost was later shown in [10] to be Q-equivalent to one written in terms of the gauge-invariant quantities N_{ab} and J, and we will focus on this manifestly gauge-invariant version of the b-ghost.

The complicated form of the b-ghost in [10] makes it difficult to treat, so for instance its nilpotency property $\{b, b\}$ has not yet been analyzed. A similar complication exists in $D = 10$ dimensions, however, it was shown in [11] that the $D = 10$ b-ghost could be simplified by defining new fermionic vector variables. In this paper, a similar simplification involving fermionic vector variables will be found for the $D = 11$ b-ghost which will simplify the computations of $\{Q, b\} = T$ and $\{b, b\}$.

The paper is organized as follows: in section 2 we review the $D = 10$ non-minimal pure spinor superparticle, constructing the corresponding pure spinor b-ghost and its simplification. In section 3 we review the $D = 11$ pure spinor superparticle, constructing the manifestly gauge-invariant b-ghost and explaining how to translate the simplification of the $D = 10$ b-ghost to the $D = 11$ b-ghost by defining the SO(10, 1) composite fermionic vector $\bar{\Sigma}^j$. Finally we construct the simplified $D = 11$ b-ghost and show that it satisfies the relations $\{Q, b\} = T$ and $\{b, b\} =$ BRST-trivial. Some comments are given at the end of the paper concerning the relation between the b-ghost found in [10] and this simplified b-ghost.

2 $D = 10$ non-minimal pure spinor superparticle

The $D = 10$ (*minimal*) pure spinor superparticle action is given by [12]:

$$S = \int d\tau \left(\dot{X}^m P_m + \dot{\theta}^\mu p_\mu - \frac{1}{2} P^m P_m + \dot{\lambda}^\mu w_\mu \right) \tag{2.1}$$

where m, μ are SO(9, 1) vector/spinor indices, θ^μ is an SO(9, 1) Majorana-Weyl spinor, p_μ is its corresponding conjugate momentum and P^m is the momentum. The variable λ^μ is a $D = 10$ pure spinor satisfying the constraint $\lambda\gamma^m\lambda = 0$ where m is an SO(9, 1) vector index, and w_μ is its corresponding conjugate momentum. Because of the pure spinor constraint this SO(9, 1) antichiral spinor is defined up to the gauge transformation $\delta w_\mu = (\gamma^m\lambda)_\mu f_m$, where f_m is an arbitrary vector. The SO(9,1) gamma matrices denoted by γ^m satisfy the Clifford algebra $(\gamma^m)_{\mu\nu}(\gamma^n)^{\nu\rho}+(\gamma^n)_{\mu\nu}(\gamma^m)^{\nu\rho}=2\eta^{mn}\delta^\rho_\mu$. The physical states are defined as elements of the cohomology of the BRST operator $Q=\lambda^\mu d_\mu$, where $d_\mu=p_\mu-P_m(\gamma^m\theta)_\mu$ are the first-class constraints of the $D=10$ Brink-Schwarz superparticle [13]. The spectrum turns out to describe the BV version of $D=10$ (abelian) Super Yang-Mills [12, 14, 15].

In the *non-minimal* version of the pure spinor superparticle [5, 16], one introduces a new pure anti-Weyl spinor $\bar\lambda_\mu$, and a fermionic field r_μ satisfying the constraint $\bar\lambda\gamma^m r=0$, together with their respective conjugate momenta $\bar w^\mu$, s^μ. In order to not affect the cohomology corresponding to $Q_{\min}$, the *non-minimal* BRST operator is defined as $Q_{non-min}=\lambda^\mu d_\mu+\bar w^\mu r_\mu$. Thus the $D=10$ non-minimal pure spinor superparticle is described by the action:

$$S=\int d\tau\left(\dot X^m P_m+\dot\theta^\mu p_\mu-\frac{1}{2}P^m P_m+\dot\lambda^\mu w_\mu+\bar w^\mu\dot{\bar\lambda}_\mu+\dot r_\mu s^\mu\right) \tag{2.2}$$

and the BRST operator $Q=\lambda^\mu d_\mu+\bar w^\mu r_\mu$. By construction, the physical spectrum also describes BV $D=10$ (abelian) Super Yang-Mills.

2.1 $D=10$ b-ghost

As discussed in [16, 17] a consistent scattering amplitude prescription can be defined using a composite b-ghost satisfying $\{Q,b\}=T$, where Q is the non-minimal BRST operator and $T=-\frac{1}{2}P^aP_a$ is the stress-energy tensor. This superparticle b-ghost is obtained by dropping the worldsheet non-zero modes in the superstring b ghost and is

$$\begin{aligned}b=&\frac{1}{2}\frac{(\bar\lambda\gamma_m d)}{\bar\lambda\lambda}P^m-\frac{1}{192}\frac{(\bar\lambda\gamma^{mnp}r)[(d\gamma_{mnp}d)+24N_{mn}P_p]}{(\bar\lambda\lambda)^2}+\frac{1}{16}\frac{(r\gamma_{mnp}r)(\bar\lambda\gamma^m d)N^{np}}{(\bar\lambda\lambda)^3}\\&-\frac{1}{128}\frac{(r\gamma_{mnp}r)(\bar\lambda\gamma^{pqr}r)N^{mn}N_{qr}}{(\bar\lambda\lambda)^4}\end{aligned} \tag{2.3}$$

where $N_{mn}=\frac{1}{2}\lambda\gamma_{mn}w$.

The complicated nature of this expression makes it difficult to prove nilpotence [18], however it was shown in [11] that the b-ghost can be simplified by introducing an SO(9,1) composite fermionic vector $\bar\Gamma^m$ satisfying the constraint $(\gamma_m\bar\lambda)^\mu\bar\Gamma^m=0$. In the expression (2.3), the terms involving d_μ always appear in the combination

$$\bar\Gamma^m=\frac{1}{2}\frac{(\bar\lambda\gamma^m d)}{(\bar\lambda\lambda)}-\frac{1}{8}\frac{(\bar\lambda\gamma^{mnp}r)N_{np}}{(\bar\lambda\lambda)^2}, \tag{2.4}$$

and using this $\bar\Gamma^m$, the b-ghost can be written in the simpler form:

$$b=P^m\bar\Gamma_m-\frac{1}{4}\frac{(\lambda\gamma^{mn}r)}{(\bar\lambda\lambda)}\bar\Gamma_m\bar\Gamma_n \tag{2.5}$$

This simplified $D=10$ b-ghost was shown to satisfy the property $\{Q,b\}=T$ in [19], and the nilpotence property $\{b,b\}=0$ easily follows from $\{\bar\Gamma_m,\bar\Gamma_n\}=0$ and $[\bar\Gamma_m,\bar\lambda\lambda]=0$.

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3 $D = 11$ non-minimal pure spinor superparticle

The $D = 11$ non-minimal pure spinor superparticle action is given by [1]

$$S = \int d\tau \left(\dot{X}^a P_a + \dot{\Theta}^\alpha P_\alpha - \frac{1}{2} P^a P_a + \dot{\Lambda}^\alpha W_\alpha + \dot{\bar{\Lambda}}_\alpha \bar{W}^\alpha + \dot{R}_\alpha S^\alpha \right) \tag{3.1}$$

We use letters of the beginning of the Greek alphabet $(\alpha, \beta, \ldots)$ to denote SO(10, 1) spinor indices and henceforth we will use Latin letters $(a, b, \ldots, l, m, \ldots)$ to denote SO(10, 1) vector indices, unless otherwise stated. In (3.1) Θ^α is an SO(10, 1) Majorana spinor and P_α is its corresponding conjugate momentum, and P_a is the momentum for X^a. The variables Λ^α, $\bar{\Lambda}_\alpha$ are $D = 11$ pure spinors and W_α, $\bar{W}^\alpha$ are their respective conjugate momenta, R_α is an SO(10, 1) fermionic spinor satisfying $\bar{\Lambda}\Gamma^a R = 0$ and S^α is its corresponding conjugate momentum. The SO(10, 1) gamma matrices denoted by Γ^a satisfy the Clifford algebra $(\Gamma^a)_{\alpha\beta}(\Gamma^b)^{\beta\sigma} + (\Gamma^b)_{\alpha\beta}(\Gamma^a)^{\beta\sigma} = 2\eta^{ab}\delta^\sigma_\alpha$. In $D = 11$ dimensions there exist an antisymmetric spinor metric $C_{\alpha\beta}$ (and its inverse $(C^{-1})^{\alpha\beta}$) which allows us to lower (and raise) spinor indices (e.g. $(\Gamma^a)^{\alpha\beta} = C^{\alpha\sigma}C^{\beta\delta}(\Gamma^a)_{\sigma\delta}$, $(\Gamma^a)^\alpha{}_\beta = C^{\alpha\sigma}(\Gamma^a)_{\sigma\beta}$, etc).

The physical states described by this theory are defined as elements of the cohomology of the BRST operator $Q = \Lambda^\alpha D_\alpha + R_\alpha \bar{W}^\alpha$ where $D_\alpha = P_\alpha - P_a(\Gamma^a\Theta)_\alpha$ and describe $D = 11$ linearized supergravity.

3.1 $D = 11$ b-ghost and its simplification

As in the $D = 10$ case, a composite $D = 11$ b-ghost can be constructed satisfying the properties $\{Q, b\} = T$ where $T = -P^a P_a$, and was found in [9, 10, 20] to be:

$$\begin{aligned} b = {} & \frac{1}{2}\eta^{-1}(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})(\Lambda\Gamma^{ab}\Gamma^i D)P_i + \eta^{-2}L^{(1)}_{ab,cd}\Big[(\Lambda\Gamma^a D)(\Lambda\Gamma^{bcd}D) + 2(\Lambda\Gamma^{abc}{}_{ij}\Lambda)N^{di}P^j \\ & + \frac{2}{3}(\eta^b{}_p\eta^d{}_q - \eta^{bd}\eta_{pq})(\Lambda\Gamma^{apcij}\Lambda)N_{ij}P^q\Big] - \frac{1}{3}\eta^{-3}L^{(2)}_{ab,cd,ef}\Big\{(\Lambda\Gamma^{abcij}\Lambda)(\Lambda\Gamma^{def}D)N_{ij} \\ & - 12\Big[(\Lambda\Gamma^{abcei}\Lambda)\eta^{fj} - \frac{2}{3}\eta^{f[a}(\Lambda\Gamma^{bce]ij}\Lambda\Big](\Lambda\Gamma^d D)N_{ij}\Big\} \\ & + \frac{4}{3}\eta^{-4}L^{(3)}_{ab,cd,ef,gh}(\Lambda\Gamma^{abcij}\Lambda)\Big[(\Lambda\Gamma^{defgk}\Lambda)\eta^{hl} - \frac{2}{3}\eta^{h[d}(\Lambda\Gamma^{efg]kl}\Lambda)\Big]\{N_{ij}, N_{kl}\} \end{aligned} \tag{3.2}$$

where

$$\eta = (\Lambda\Gamma^{ab}\Lambda)(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda}) \tag{3.3}$$

$$L^{(n)}_{a_0b_0,a_1b_1,\ldots,a_nb_n} = (\bar{\Lambda}\Gamma_{[\![a_0b_0}\bar{\Lambda})(\bar{\Lambda}\Gamma_{a_1b_1}R)\ldots(\bar{\Lambda}\Gamma_{a_nb_n]\!]}R) \tag{3.4}$$

and $[\![]\!]$ means antisymmetrization between each pair of indices. The $D = 11$ ghost current is defined by $N_{ij} = \Lambda\Gamma_{ij}W$.

To simplify this complicated expression for the D=11 b-ghost, we shall mimic the procedure explained above for the D=10 b-ghost and look for a similar object to $\bar{\Gamma}^m$. A hint comes from looking at the quantity multiplying the momentum P^i in the expression for the D=11 b-ghost:

$$
\begin{aligned}
b = P^i\Big[\frac{1}{2}\eta^{-1}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\Lambda\Gamma^{ab}\Gamma_i D) + \eta^{-2}L^{(1)}_{ab,cd}\Big[2(\Lambda\Gamma^{abc}{}_{ki}\Lambda)N^{dk} \\
+\frac{2}{3}(\eta^b{}_p\eta^d{}_i - \eta^{bd}\eta_{pi})(\Lambda\Gamma^{apcqj}\Lambda)N_{qj}\Big]\Big] + \dots
\end{aligned} \tag{3.5}
$$

Therefore our candidate to play the analog role to $\bar\Gamma^m$ is:

$$
\bar\Sigma^i = \bar\Sigma^i_0 + \frac{2}{\eta^2}L^{(1)}_{ab,cd}(\Lambda\Gamma^{abcki}\Lambda)N^d_k + \frac{2}{3\eta^2}L^{(1)}_{ab,c}{}^i(\Lambda\Gamma^{abcqj}\Lambda)N_{qj} - \frac{2}{3\eta^2}L^{(1)}_{ad,c}{}^d(\Lambda\Gamma^{aicqj}\Lambda)N_{qj} \tag{3.6}
$$

where $\bar\Sigma^i_0 = \frac{1}{2}\eta^{-1}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\Lambda\Gamma^{ab}\Gamma^i D)$ is the only term containing D_α's. Using the identities (A.1), (A.2) in appendix A, one finds that $\bar\Sigma^j$ satisfies the constraint:

$$
(\bar\Lambda\Gamma_{ab}\bar\Lambda)\bar\Sigma^a = 0. \tag{3.7}
$$

Furthermore, it will be shown in appendix B that the D_α's appearing in $\bar\Sigma^i_0$ are the same as those appearing in the b-ghost. Therefore a plausible assumption for the simplification of the b-ghost would be $b = P^i\bar\Sigma_i + O(\bar\Sigma^2)$. As will now be shown, the simplified form of the b-ghost satisfying $\{Q, b\} = T$ is indeed

$$
b = P^i\bar\Sigma_i - \frac{2}{\eta}(\bar\Lambda\Gamma_{ac}R)(\Lambda\Gamma^{aj}\Lambda)\bar\Sigma_j\bar\Sigma^c - \frac{1}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)\bar\Sigma_j\bar\Sigma_k \tag{3.8}
$$

3.2 Computation of $\{Q, \bar\Sigma^j\}$

To show that the b-ghost of (3.8) satisfies $\{Q, b\} = T$, it will be convenient to first compute $\{Q, \bar\Sigma^i\}$ where, using the identities (A.10), (A.13),

$$
\begin{aligned}
\bar\Sigma^i = \bar\Sigma^i_0 + \frac{2}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{abcki}\Lambda)N^d{}_k + \frac{2}{3\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^{abcqj}\Lambda)N_{qj} \\
- \frac{2}{3\eta^2}(\bar\Lambda\Gamma_{ac}\bar\Lambda)(\bar\Lambda R)(\Lambda\Gamma^{aicqj}\Lambda)N_{qj}
\end{aligned} \tag{3.9}
$$

Using equation (3.9) and the identities (A.20), (A.21), (A.22), (A.23):

$$
\begin{aligned}
\{Q,\bar\Sigma^i\} = & -P^i - \frac{2}{\eta}[(\bar\Lambda\Gamma^{mb}\bar\Lambda)(\Lambda\Gamma_b{}^i\Lambda) - (\bar\Lambda\Gamma^{ib}\bar\Lambda)(\Lambda\Gamma_b{}^m\Lambda)]P_m + \frac{2}{\eta}(\bar\Lambda\Gamma_{mn}R)(\Lambda\Gamma^{mn}\Lambda)\bar\Sigma^i_0 \\
& + \frac{4}{\eta}(\bar\Lambda\Gamma^{mn}R)(\Lambda\Gamma_{mn}\Lambda)(\bar\Sigma^i - \bar\Sigma^i_0) - \frac{1}{\eta}(\bar\Lambda\Gamma_{ab}R)(\Lambda\Gamma^{ab}\Gamma^i D) \\
& - \frac{2}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{abcki}\Lambda)(\Lambda\Gamma^d_k D) - \frac{2}{3\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^{abcdk}\Lambda)(\Lambda\Gamma_{dk}D) \\
& - \frac{2}{3\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda R)(\Lambda\Gamma^{iabdk}\Lambda)(\Lambda\Gamma_{dk}D) - \frac{4}{\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{abcki}\Lambda)N^d_k \\
& - \frac{4}{3\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^{abcdk}\Lambda)N_{dk} - \frac{4}{3\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda R)(\Lambda\Gamma^{iabdk}\Lambda)N_{dk} \\
& - \frac{2}{3\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(RR)(\Lambda\Gamma^{iabdk}\Lambda)N_{dk}
\end{aligned} \tag{3.10}
$$

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As shown in appendix D, this expression in invariant under the same gauge transformations under which $\bar{\Sigma}_0^i$ is invariant:

$$\delta D_\alpha = (\Gamma^{ij}\Lambda')_\alpha f_{ij} \tag{3.11}$$

where $(\Lambda')^\alpha = \frac{1}{2\eta}(\bar{\Lambda}\Gamma_{mn}\bar{\Lambda})(\Lambda\Gamma^{mn})^\alpha$ is a pure spinor, and f_{ij} is an antisymmetric gauge parameter. Therefore we can write all D_α's in this object in terms of $\bar{\Sigma}_0^i$, and the result is (see appendix D):

$$\begin{aligned}\{Q,\bar{\Sigma}^i\} = &-P^i - \frac{2}{\eta}[(\bar{\Lambda}\Gamma^{mb}\bar{\Lambda})(\Lambda\Gamma_b{}^i\Lambda) - (\bar{\Lambda}\Gamma^{ib}\bar{\Lambda})(\Lambda\Gamma_b{}^m\Lambda)]P_m + \frac{4}{\eta}(\bar{\Lambda}\Gamma_{mn}R)(\Lambda\Gamma^{mn}\Lambda)(\bar{\Sigma}^i - \bar{\Sigma}_0^i)\\ &-\frac{2}{\eta}(\bar{\Lambda}\Gamma^{ci}R)(\Lambda\Gamma_{ck}\Lambda)\bar{\Sigma}_0^k + \frac{4}{\eta}(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)\bar{\Sigma}_0^d + \frac{2}{\eta}(\bar{\Lambda}R)(\Lambda\Gamma^{ik}\Lambda)\bar{\Sigma}_{0k}\\ &-\frac{2}{\eta^2}(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{cd}\Lambda)(\bar{\Lambda}\Gamma^{in}\bar{\Lambda})(\Lambda\Gamma^{nk}\Lambda)\bar{\Sigma}_{0k} - \frac{4}{\eta^2}(\bar{\Lambda}\Gamma_{ab}R)(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{abcki}\Lambda)N^d_k\\ &-\frac{4}{3\eta^2}(\bar{\Lambda}\Gamma_{ab}R)(\bar{\Lambda}\Gamma_c{}^iR)(\Lambda\Gamma^{abcdk}\Lambda)N_{dk} - \frac{4}{3\eta^2}(\bar{\Lambda}\Gamma_{ab}R)(\bar{\Lambda}R)(\Lambda\Gamma^{iabdk}\Lambda)N_{dk}\\ &-\frac{2}{3\eta^2}(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})(RR)(\Lambda\Gamma^{iabdk}\Lambda)N_{dk}\end{aligned} \tag{3.12}$$

After plugging (3.9) into (3.12), all of the terms explicitly depending on N_{ab} are cancelled and we get (see appendix E):

$$\begin{aligned}\{Q,\bar{\Sigma}^i\} = &-P^i - \frac{2}{\eta}[(\bar{\Lambda}\Gamma^{mb}\bar{\Lambda})(\Lambda\Gamma_b{}^i\Lambda) - (\bar{\Lambda}\Gamma^{ib}\bar{\Lambda})(\Lambda\Gamma_b{}^m\Lambda)]P_m - \frac{2}{\eta}(\bar{\Lambda}\Gamma^{ci}R)(\Lambda\Gamma_{ck}\Lambda)\bar{\Sigma}^k\\ &+\frac{4}{\eta}(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)\bar{\Sigma}^d + \frac{2}{\eta}(\bar{\Lambda}R)(\Lambda\Gamma^{ik}\Lambda)\bar{\Sigma}_k - \frac{2}{\eta^2}(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{cd}\Lambda)(\bar{\Lambda}\Gamma^{in}\bar{\Lambda})(\Lambda\Gamma^{nk}\Lambda)\bar{\Sigma}_k\end{aligned} \tag{3.13}$$

3.3 $\{Q, b\} = T$

Using (3.13) it is now straightforward to compute $\{Q, b\}$:

$$\begin{aligned}\{Q,b\} = &P^i\{Q,\bar{\Sigma}_i\} - \frac{4}{\eta^2}(\Lambda\Gamma^{mn}\Lambda)(\bar{\Lambda}\Gamma_{mn}R)(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^k\bar{\Sigma}_j\\ &+\frac{2}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)(\{Q,\bar{\Sigma}^k\})\bar{\Sigma}_j - \frac{2}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^k(\{Q,\bar{\Sigma}_j\})\\ &-\frac{2}{\eta^2}(\Lambda\Gamma^{mn}\Lambda)(\bar{\Lambda}\Gamma_{mn}R)(\bar{\Lambda}R)(\Lambda\Gamma^{jk}\Lambda)\bar{\Sigma}_j\bar{\Sigma}_k + \frac{1}{\eta}(RR)(\Lambda\Gamma^{jk}\Lambda)\bar{\Sigma}_j\bar{\Sigma}_k\\ &+\frac{1}{\eta}(\bar{\Lambda}R)(\Lambda\Gamma^{jk}\Lambda)(\{Q,\bar{\Sigma}_j\})\bar{\Sigma}_k - \frac{1}{\eta}(\bar{\Lambda}R)(\Lambda\Gamma^{jk}\Lambda)\bar{\Sigma}_j(\{Q,\bar{\Sigma}_k\})\end{aligned} \tag{3.14}$$

To make the computations transparent, each term in (3.14) involving $\{Q, \bar{\Sigma}_i\}$ will be simplified separately:

$$\begin{aligned}M_1 &= P^i\{Q,\bar{\Sigma}_i\}\\ &= P_i\Big\{ - P^i - \frac{2}{\eta}\Big[(\Lambda\Gamma^{ib}\Lambda)(\bar{\Lambda}\Gamma_{bm}\bar{\Lambda}) - (\Lambda\Gamma^{mb}\Lambda)(\bar{\Lambda}\Gamma^{bi}\bar{\Lambda})\Big]P_m\\ &\quad -\frac{2}{\eta}(\bar{\Lambda}\Gamma^{ci}R)(\Lambda\Gamma_{ck}\Lambda)\bar{\Sigma}^k + \frac{4}{\eta}(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)\bar{\Sigma}^d\\ &\quad +\frac{2}{\eta}(\bar{\Lambda}R)(\Lambda\Gamma^{ik}\Lambda)\bar{\Sigma}_k - \frac{2}{\eta^2}(\bar{\Lambda}\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)(\bar{\Lambda}\Gamma^{in}\bar{\Lambda})(\Lambda\Gamma_{nk}\Lambda)\bar{\Sigma}^k\Big\}\end{aligned}$$

$$
\begin{aligned}
&= -P^2 - \frac{2}{\eta}(\bar{\Lambda}\Gamma^{ci}R)(\Lambda\Gamma_{ck}\Lambda)P_i\bar{\Sigma}^k + \frac{4}{\eta}(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)P_i\bar{\Sigma}^d \\
&\quad + \frac{2}{\eta}(\bar{\Lambda}R)(\Lambda\Gamma^{ik}\Lambda)P_i\bar{\Sigma}_k - \frac{2}{\eta^2}(\bar{\Lambda}\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)(\bar{\Lambda}\Gamma^{in}\bar{\Lambda})(\Lambda\Gamma_{nk}\Lambda)P_i\bar{\Sigma}^k \qquad (3.15)
\end{aligned}
$$

$$
\begin{aligned}
M_2 &= \frac{2}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)(\{Q,\bar{\Sigma}^k\})\bar{\Sigma}_j \\
&= \frac{2}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\Big[-P^k + \frac{2}{\eta}(\Lambda\Gamma_{mb}\Lambda)(\bar{\Lambda}\Gamma^{bk}\bar{\Lambda})P^m - \frac{2}{\eta}(\bar{\Lambda}\Gamma^{ck}R)(\Lambda\Gamma_{cp}\Lambda)\bar{\Sigma}^p \\
&\quad - \frac{2}{\eta^2}(\bar{\Lambda}\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)(\bar{\Lambda}\Gamma^{kn}\bar{\Lambda})(\Lambda\Gamma_{np}\Lambda)\bar{\Sigma}^p\Big]\bar{\Sigma}_j \\
&= -\frac{2}{\eta}\bar{\Lambda}\Gamma^{aj}R(\Lambda\Gamma_{ak}\Lambda)P^k\bar{\Sigma}_j + \frac{4}{\eta^2}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)(\Lambda\Gamma_{mb}\Lambda)(\bar{\Lambda}\Gamma^{bk}\bar{\Lambda})P^m\bar{\Sigma}_j \\
&\quad - \frac{4}{\eta^2}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)(\bar{\Lambda}\Gamma^{ck}R)(\Lambda\Gamma_{cp}\Lambda)\bar{\Sigma}^p\bar{\Sigma}_j \\
&\quad - \frac{4}{\eta^3}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)(\bar{\Lambda}\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)(\bar{\Lambda}\Gamma^{kn}\bar{\Lambda})(\Lambda\Gamma_{np}\Lambda)\bar{\Sigma}^p\bar{\Sigma}_j
\end{aligned}
$$

Using (A.2), we get

$$
\begin{aligned}
M_2 &= -\frac{2}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)P^k\bar{\Sigma}_j + \frac{2}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ma}\Lambda)P^m\bar{\Sigma}_j \\
&\quad - \frac{2}{\eta^2}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ap}\Lambda)(\bar{\Lambda}\Gamma^{ck}R)(\Lambda\Gamma_{ck}\Lambda)\bar{\Sigma}^p\bar{\Sigma}_j \\
&\quad - \frac{2}{\eta^2}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{pa}\Lambda)(\bar{\Lambda}\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)\bar{\Sigma}^p\bar{\Sigma}_j \\
&= -\frac{4}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)P^k\bar{\Sigma}_j \qquad (3.16)
\end{aligned}
$$

$$
\begin{aligned}
M_3 &= -\frac{2}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^k(\{Q,\bar{\Sigma}_j\}) \\
&= -\frac{2}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^k\Big\{-P_j - \frac{2}{\eta}\Big[(\Lambda\Gamma_{jb}\Lambda)(\bar{\Lambda}\Gamma^{bm}\bar{\Lambda}) - (\Lambda\Gamma^{mb}\Lambda)(\bar{\Lambda}\Gamma_{bj}\bar{\Lambda})\Big]P_m \\
&\quad - \frac{2}{\eta}(\bar{\Lambda}\Gamma_{cj}R))(\Lambda\Gamma^{cp}\Lambda)\bar{\Sigma}_p + \frac{4}{\eta}(\bar{\Lambda}\Gamma^{cd}R)(\Lambda\Gamma_{cj}\Lambda)\bar{\Sigma}_d + \frac{2}{\eta}(\bar{\Lambda}R)(\Lambda\Gamma_{jp}\Lambda)\bar{\Sigma}^p \\
&\quad - \frac{2}{\eta^2}(\bar{\Lambda}\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)(\bar{\Lambda}\Gamma_{jn}\bar{\Lambda})(\Lambda\Gamma^{np}\Lambda)\bar{\Sigma}_p\Big\} \\
&= \frac{2}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^kP_j + \frac{4}{\eta^2}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^k(\Lambda\Gamma_{jb}\Lambda)(\bar{\Lambda}\Gamma^{bm}\bar{\Lambda})P_m \\
&\quad - \frac{4}{\eta^2}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^k(\Lambda\Gamma^{mb}\Lambda)(\bar{\Lambda}\Gamma_{bj}\bar{\Lambda})P_m + \frac{4}{\eta^2}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^k(\bar{\Lambda}\Gamma^{cj}R)(\Lambda\Gamma_{cp}\Lambda)\bar{\Sigma}^p \\
&\quad - \frac{8}{\eta^2}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^k(\bar{\Lambda}\Gamma^{cd}R)(\Lambda\Gamma_{cj}\Lambda)\bar{\Sigma}_d - \frac{4}{\eta^2}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}R)\bar{\Sigma}^k(\bar{\Lambda}R)(\Lambda\Gamma_{jp}\Lambda)\bar{\Sigma}^p \\
&\quad + \frac{4}{\eta^3}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^k(\bar{\Lambda}\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)(\bar{\Lambda}\Gamma_{jn}\bar{\Lambda})(\Lambda\Gamma^{np}\Lambda)\bar{\Sigma}_p
\end{aligned}
$$

Using (A.2), (A.5), (A.19):

$$
\begin{aligned}
M_3 &= \frac{2}{\eta}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar{\Sigma}^kP_j + \frac{2}{\eta^2}(\bar{\Lambda}\Gamma^{aj}R)(\Lambda\Gamma_{aj}\Lambda)(\Lambda\Gamma^{kb}\Lambda)(\bar{\Lambda}\Gamma_{bm}\bar{\Lambda})\bar{\Sigma}_kP^m \\
&\quad + \frac{2}{\eta}(\bar{\Lambda}R)(\Lambda\Gamma^{mk}\Lambda)P_m\bar{\Sigma}_k - \frac{2}{\eta^2}(\bar{\Lambda}\Gamma_{ac}R)(\Lambda\Gamma^{ac}\Lambda)(\bar{\Lambda}R)(\Lambda\Gamma^{kp}\Lambda)\bar{\Sigma}_k\bar{\Sigma}_p
\end{aligned}
$$

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$$
\begin{aligned}
&-\frac{1}{\eta}(RR)(\Lambda\Gamma^{kp}\Lambda)\bar\Sigma_k\bar\Sigma_p+\frac{4}{\eta^2}(\bar\Lambda\Gamma^{aj}R)(\Lambda\Gamma_{aj}\Lambda)(\bar\Lambda\Gamma^{cd}R)(\Lambda\Gamma_{ck}\Lambda)\bar\Sigma^k\bar\Sigma_d\\
&+\frac{2}{\eta^2}(\bar\Lambda\Gamma^{aj}R)(\Lambda\Gamma_{aj}\Lambda)(\bar\Lambda R)(\Lambda\Gamma^{kp}\Lambda)\bar\Sigma_k\bar\Sigma_p-\frac{2}{\eta^2}(\bar\Lambda R)(\Lambda\Gamma^{kp}\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{cd}\Lambda)\bar\Sigma_k\bar\Sigma_p\\
=&\frac{2}{\eta}(\bar\Lambda\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar\Sigma^k P_j+\frac{2}{\eta^2}(\bar\Lambda\Gamma^{aj}R)(\Lambda\Gamma_{aj}\Lambda)(\Lambda\Gamma^{kb}\Lambda)(\bar\Lambda\Gamma_{bm}\bar\Lambda)\bar\Sigma_k P^m\\
&+\frac{2}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{mk}\Lambda)P_m\bar\Sigma_k-\frac{1}{\eta}(RR)(\Lambda\Gamma^{kp}\Lambda)\bar\Sigma_k\bar\Sigma_p+\frac{4}{\eta^2}(\bar\Lambda\Gamma^{aj}R)(\Lambda\Gamma_{aj}\Lambda)(\bar\Lambda\Gamma^{cd}R)(\Lambda\Gamma_{ck}\Lambda)\bar\Sigma^k\bar\Sigma_d\\
&-\frac{2}{\eta^2}(\bar\Lambda R)(\Lambda\Gamma^{kp}\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{cd}\Lambda)\bar\Sigma_k\bar\Sigma_p
\end{aligned}\tag{3.17}
$$

$$
\begin{aligned}
M_4=&\frac{1}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)(\{Q,\bar\Sigma_j\})\bar\Sigma_k\\
=&-\frac{1}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)P_j\bar\Sigma_k+\frac{2}{\eta^2}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)(\Lambda\Gamma^{mb}\Lambda)(\bar\Lambda\Gamma_{bj}\bar\Lambda)P_m\bar\Sigma_k\\
&-\frac{2}{\eta^2}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)(\bar\Lambda\Gamma_{cj}R)(\Lambda\Gamma^{cp}\Lambda)\bar\Sigma_p\bar\Sigma_k-\frac{2}{\eta^3}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{cd}\Lambda)(\bar\Lambda\Gamma_{jn}\bar\Lambda)(\Lambda\Gamma^{np}\Lambda)\bar\Sigma_p\bar\Sigma_k\\
=&-\frac{1}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)P_j\bar\Sigma_k+\frac{1}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{km}\Lambda)P_m\bar\Sigma_k\\
&-\frac{1}{\eta^2}(\bar\Lambda R)(\Lambda\Gamma^{pk}\Lambda)(\bar\Lambda\Gamma^{cj}R)(\Lambda\Gamma_{cj}\Lambda)\bar\Sigma_p\bar\Sigma_k-\frac{1}{\eta^2}(\bar\Lambda R)(\bar\Lambda\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)(\Lambda\Gamma^{kp}\Lambda)\bar\Sigma_p\bar\Sigma_k\\
=&-\frac{2}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)P_j\bar\Sigma_k
\end{aligned}\tag{3.18}
$$

$$
\begin{aligned}
M_5=&-\frac{1}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)\bar\Sigma_j(\{Q,\bar\Sigma_k\})\\
=&\frac{1}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)\bar\Sigma_j P_k-\frac{2}{\eta^2}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)\bar\Sigma_j(\Lambda\Gamma^{mb}\Lambda)(\bar\Lambda\Gamma_{bk}\bar\Lambda)P_m\\
&+\frac{2}{\eta^2}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)\bar\Sigma_j(\bar\Lambda\Gamma_{ck}R)(\Lambda\Gamma^{cp}\Lambda)\bar\Sigma_p+\frac{2}{\eta^3}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)\bar\Sigma_j(\bar\Lambda\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)(\bar\Lambda\Gamma_{kn}\bar\Lambda)(\Lambda\Gamma^{np}\Lambda)\bar\Sigma_p\\
=&\frac{1}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{kj}\Lambda)P_j\bar\Sigma_k-\frac{1}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{mj}\Lambda)P_m\bar\Sigma_j\\
&+\frac{1}{\eta^2}(\bar\Lambda R)(\Lambda\Gamma^{jp}\Lambda)\bar\Sigma_j(\bar\Lambda\Gamma^{ck}R)(\Lambda\Gamma_{ck}\Lambda)\bar\Sigma_p+\frac{1}{\eta^2}(\bar\Lambda R)(\Lambda\Gamma^{pj}\Lambda)\bar\Sigma_j(\bar\Lambda\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)\bar\Sigma_p\\
=&-\frac{2}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)P_j\bar\Sigma_k
\end{aligned}\tag{3.19}
$$

Putting together all the terms in (3.14):

$$
\begin{aligned}
\{Q,b\}=&\sum_{i=1}^{5}M_i-\frac{4}{\eta^2}(\Lambda\Gamma^{mn}\Lambda)(\bar\Lambda\Gamma_{mn}R)(\bar\Lambda\Gamma^{aj}R)(\Lambda\Gamma_{ak}\Lambda)\bar\Sigma^k\bar\Sigma_j\\
&-\frac{2}{\eta^2}(\Lambda\Gamma^{mn}\Lambda)(\bar\Lambda\Gamma_{mn}R)(\bar\Lambda R)(\Lambda\Gamma^{jk}\Lambda)\bar\Sigma_j\bar\Sigma_k+\frac{1}{\eta}(RR)(\Lambda\Gamma^{jk}\Lambda)\bar\Sigma_j\bar\Sigma_k\\
=&-P^2
\end{aligned}\tag{3.20}
$$

Recalling that $T=-P^2$ is the stress-energy tensor, we have checked that $\{Q,b\}=T$.

3.4 $\{b,b\}$ = BRST-trivial

In the D=10 case, the identity $\{\bar\Gamma^m,\bar\Gamma^n\}=0$ was crucial for showing that $\{b,b\}=0$. However, in the D=11 case, it is shown in appendix F that $\{\bar\Sigma^j,\bar\Sigma^k\}$ is non-zero and is proportional to R_α. This implies that

$$
\{b,b\}=R^\alpha G_\alpha(\Lambda,\bar\Lambda,R,W,D)\tag{3.21}
$$

for some $G_\alpha(\Lambda,\bar\Lambda,R,W,D)$.

Note that $[Q, \{b, b\}] = 0$ since $[b, T] = 0$ where $T = -P_a P^a$. Since $Q = \Lambda^\alpha D_\alpha + R^\alpha \bar{W}_\alpha$, the quartet argument implies that the cohomology of Q is independent of R_α, which allows us to conclude that $\{b, b\}$ = BRST-trivial. It would be interesting to investigate if this BRST-triviality of $\{b, b\}$ is enough for the scattering amplitude prescription using the b-ghost to be consistent.

4 Remarks

We have succeeded in finding a considerably simpler form in (3.8) for the D=11 b-ghost than that of equation (3.2) which was presented in [10]. Although this simplified version is not strictly nilpotent, it satisfies the relation $\{b, b\}$ = BRST-trivial which may be good enough for consistency.

It is natural to ask if the simplified $D = 11$ b-ghost (3.8) is the same as the b-ghost presented in (3.2). These two expressions are compared in appendix G and we find that they coincide up to normal-ordering terms coming from the position of N_{mn} in each expression. Note that the product of N_{mn}'s appears as an anticommutator in (3.2) whereas it appears as an simple ordinary product in (3.8). However, because we have ignored normal-ordering questions in our analysis, we will not attempt to address this issue.

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A $D = 11$ pure spinor identities

We list some pure spinor identities in eleven dimensions:

$$(\bar{\Lambda}\Gamma^{ab}\bar{\Lambda})(\Gamma_b\bar{\Lambda})_\alpha = 0 \tag{A.1}$$

$$(\bar{\Lambda}\Gamma^{[ab}\bar{\Lambda})(\bar{\Lambda}\Gamma^{c]d}\bar{\Lambda}) = 0 \tag{A.2}$$

$$(\bar{\Lambda}\Gamma^{[ab}\bar{\Lambda})(\bar{\Lambda}\Gamma^{cd]}\bar{\Lambda}) = 0 \tag{A.3}$$

$$(\bar{\Lambda}\Gamma^{[ab}\bar{\Lambda})(\bar{\Lambda}\Gamma^{cd]}R) = 0 \tag{A.4}$$

$$(\bar{\Lambda}\Gamma_{ij}R)(\bar{\Lambda}\Gamma_k{}^jR) = (\bar{\Lambda}\Gamma_{ik}R)(\bar{\Lambda}R) + \frac{1}{2}(\bar{\Lambda}\Gamma_{ik}\bar{\Lambda})(RR) \tag{A.5}$$

$$(\bar{\Lambda}\Gamma_{ab}R)(\bar{\Lambda}\Gamma_{cd}R)f^{ac}g^{bd} = 0 \tag{A.6}$$

$$(\Lambda\Gamma_{sk}\Lambda)(\Lambda\Gamma^{abcdk}\Lambda) = 0 \tag{A.7}$$

$$(\Gamma_i\Lambda)_\alpha(\Lambda\Gamma^{abcdi}\Lambda) = 6(\Gamma^{[ab}\Lambda)_\alpha(\Lambda\Gamma^{cd]}\Lambda) \tag{A.8}$$

$$(\Gamma_{ij}\Lambda)_\alpha(\Lambda\Gamma^{abcij}\Lambda) = -18(\Gamma^{[a}\Lambda)_\alpha(\Lambda\Gamma^{bc]}\Lambda) \tag{A.9}$$

where f^{ac}, g^{bd} are antisymmetric in (a, c), (b, d) respectively. In addition, using (A.4) it can be shown that

$$L^{(1)}_{ab,cd}f^{abc} = (\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})(\bar{\Lambda}\Gamma_{cd}R)f^{abc} \tag{A.10}$$

$$L^{(1)}_{ab,cd}f^{abc} = -(\bar\Lambda\Gamma_{cd}\bar\Lambda)(\bar\Lambda\Gamma_{ab}R)f^{abc} \tag{A.11}$$

$$L^{(2)}_{ab,cd,ef}f^{abce} = (\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\bar\Lambda\Gamma_{ef}R)f^{abce} \tag{A.12}$$

where f^{abc}, f^{abce} are antisymmetric in all of their indices.
Other useful identities:

$$L^{(1)\ \ d}_{ad,c} = (\bar\Lambda\Gamma_{ac}\bar\Lambda)(\bar\Lambda R) \tag{A.13}$$

$$L^{(2)\ \ \ a}_{ab,cd,e} = \frac{1}{3}[2(\bar\Lambda\Gamma_{eb}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\bar\Lambda R) - (\bar\Lambda\Gamma_{cd}\bar\Lambda)(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda\Gamma_e{}^aR)] \tag{A.14}$$

$$L^{(2)\ \ \ c}_{ab,cd,e} = \frac{1}{3}[(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\bar\Lambda\Gamma_e{}^cR) - 2(\bar\Lambda\Gamma_{ed}\bar\Lambda)(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda R)] \tag{A.15}$$

$$(\bar\Lambda\Gamma_{ab}\bar\Lambda)\bar\Sigma^b = 0 \tag{A.16}$$

Some useful commutation relations

$$[\bar\Sigma^i,\eta]=0 \tag{A.17}$$

$$[\bar\Sigma^j,(\Lambda\Gamma^{mn}\Lambda)]=\frac{2}{\eta^2}(\bar\Lambda\Gamma_{ef}\bar\Lambda)(\bar\Lambda\Gamma_{gh}R)[(\Lambda\Gamma^{efgmj}\Lambda)(\Lambda\Gamma^{hn}\Lambda)-(\Lambda\Gamma^{efgnj}\Lambda)(\Lambda\Gamma^{hm}\Lambda)] \tag{A.18}$$

$$\{\bar\Sigma^j,(\bar\Lambda\Gamma_{mn}R)(\Lambda\Gamma^{mn}\Lambda)\}=0 \tag{A.19}$$

$$[Q,\eta]=-2(\Lambda\Gamma^{mn}\Lambda)(\bar\Lambda\Gamma_{mn}R) \tag{A.20}$$

$$[Q,(\bar\Lambda\Gamma^{ab}\bar\Lambda)]=-2(\bar\Lambda\Gamma^{ab}R) \tag{A.21}$$

$$[Q,N^{hi}]=(\Lambda\Gamma^{hi}D) \tag{A.22}$$

$$[Q,D_\beta]=-2(\Gamma^m\Lambda)_\beta P_m \tag{A.23}$$

$$\begin{aligned}[N^{hi},\eta]&=-2(\bar\Lambda\Gamma_{ab}\bar\Lambda)[-2\eta^{ai}(\Lambda\Gamma^{bh}\Lambda)+2\eta^{ah}(\Lambda\Gamma^{bi}\Lambda)]\\&=4(\bar\Lambda\Gamma^i{}_b\bar\Lambda)(\Lambda\Gamma^{bh}\Lambda)-4(\bar\Lambda\Gamma^h{}_b\bar\Lambda)(\Lambda\Gamma^{bi}\Lambda)\end{aligned} \tag{A.24}$$

$$\begin{aligned}[N^{hi},(\Lambda\Gamma^{lmnpq}\Lambda)]&=-2\eta^{iq}(\Lambda\Gamma^{chlmn}\Lambda)+2\eta^{in}(\Lambda\Gamma^{chlmq}\Lambda)-2\eta^{im}(\Lambda\Gamma^{chlnq}\Lambda)\\&\quad+2\eta^{il}(\Lambda\Gamma^{chmnq}\Lambda)+2\eta^{hq}(\Lambda\Gamma^{cilmn}\Lambda)-2\eta^{hn}(\Lambda\Gamma^{cilmq}\Lambda)\\&\quad+2\eta^{hm}(\Lambda\Gamma^{cilnq}\Lambda)-2\eta^{hl}(\Lambda\Gamma^{cimnq}\Lambda)+2\eta^{ci}(\Lambda\Gamma^{hlmnq}\Lambda)\\&\quad-2\eta^{ch}(\Lambda\Gamma^{ilmnq}\Lambda)\end{aligned} \tag{A.25}$$

$$-4\delta^{ac}_{mn}N^{bp}+4\delta^{ab}_{np}N^{cm}-4\delta^{ab}_{mp}N^{cn}+4\delta^{ab}_{mn}N^{cp}-2\Lambda\Gamma^{abcmnp}W \tag{A.26}$$

B The b-ghost and $\bar\Sigma^j$ have the same D_α's

We should figure out which are the D_α's appearing in the expressions for $\bar\Sigma^i$ and the b-ghost. For this, we will decompose the eleven dimensional Lorentz group in the following way: $\mathrm{SO}(10,1) \to \mathrm{SO}(3,1) \times \mathrm{SO}(7)$. In addition, we conveniently choose the special direction for $\bar\Lambda_\alpha$ to be $\bar\Lambda^{++0} \neq 0$. So

$$\bar\Lambda\Gamma^a R = 0 \to R^{+-0} = R^{-+0} = R^{--j} = 0 \quad , \quad \text{where } j = 1,\dots,7 \tag{B.1}$$

On the other hand, from the pure spinor constraint $\Lambda\Gamma^a\Lambda = 0$ we have:

$$\Lambda^{-+0} = -\frac{\Lambda^{-+j}\Lambda^{--j}}{\Lambda^{--0} \tag{B.2}}$$

$$\Lambda^{+-0} = -\frac{\Lambda^{+-j}\Lambda^{--j}}{\Lambda^{--0}} \tag{B.3}$$

$$\Lambda^{++j} = \frac{1}{\Lambda^{--0}}[\Lambda^{--j}\Lambda^{++0} - \Lambda^{-+j}\Lambda^{+-0} + \Lambda^{+-j}\Lambda^{-+0}] \tag{B.4}$$

where $j = 1, \ldots, 7$ and we have assumed that $\Lambda^{--0} \neq 0$. This allows us to expand the quadratic term in D_α in the b-ghost in terms of these components:

$$\begin{aligned}
b_1 \propto \; & \frac{(\bar\Lambda^{++0}\bar\Lambda^{++0})(\bar\Lambda^{++0}R^{--0})}{(\bar\Lambda^{++0}\bar\Lambda^{++0})^2(\Lambda^{--0}\Lambda^{--0}+\Lambda^{--k}\Lambda^{-k})^2}\{[\Lambda^{--0}D^{+-0}+\Lambda^{--k}D^{+-k}+\Lambda^{+-0}D^{--0} \\
& +\Lambda^{+-k}D^{--k}] \times [\Lambda^{--0}D^{-+0}+\Lambda^{--k}D^{-+k}-\Lambda^{-+0}D^{--0}-\Lambda^{-+k}D^{--k}]\} \\
& +\frac{(\bar\Lambda^{++0}\bar\Lambda^{++0})(\bar\Lambda^{++0}R^{--0})}{(\bar\Lambda^{++0}\bar\Lambda^{++0})^2(\Lambda^{--0}\Lambda^{--0}+\Lambda^{--k}\Lambda^{--k})^2}\{[\Lambda^{--0}D^{-+0}+\Lambda^{--k}D^{-+k}+\Lambda^{-+0}D^{--0} \\
& +\Lambda^{-+k}D^{--k}] \times [\Lambda^{--0}D^{+-0}+\Lambda^{--k}D^{+-k}-\Lambda^{+-0}D^{--0}-\Lambda^{+-k}D^{--k}]\} \\
& +\frac{(\bar\Lambda^{++0}\bar\Lambda^{++0})(\bar\Lambda^{++0}R^{+-j})}{(\bar\Lambda^{++0}\bar\Lambda^{++0})^2(\Lambda^{--0}\Lambda^{--0}+\Lambda^{--k}\Lambda^{-k})^2}\{[\Lambda^{--0}D^{-+0}+\Lambda^{--k}D^{-+k} \\
& +\Lambda^{-+0}D^{--0}+\Lambda^{-+k}D^{--k}] \times [\Lambda^{--0}D^{--j}-\Lambda^{--j}D^{--0}]\} \\
& -\frac{(\bar\Lambda^{++0}\bar\Lambda^{++0})(\bar\Lambda^{++0}R^{-+j})}{(\bar\Lambda^{++0}\bar\Lambda^{++0})^2(\Lambda^{--0}\Lambda^{--0}+\Lambda^{--k}\Lambda^{--k})^2}\{[\Lambda^{--0}D^{+-0}+\Lambda^{--k}D^{+-k} \\
& +\Lambda^{+-0}D^{--0}+\Lambda^{+-k}D^{--k}] \times [\Lambda^{--0}D^{--j}-\Lambda^{--j}D^{--0}]\}
\end{aligned} \tag{B.5}$$

Now, we write $\bar\Sigma_0^i$ in the convenient form:

$$\bar\Sigma_0^i = \frac{1}{2\eta}[2(\bar\Lambda\Gamma^{ai}\bar\Lambda)(\Lambda\Gamma_a D) + (\bar\Lambda\Gamma_{ab}\bar\Lambda)(\Lambda\Gamma^{abi}D)] \tag{B.6}$$

After using the particular direction chosen above, $\bar\Sigma_0^i$ presents the following SO(3,1)×SO(7) components:

$$\bar\Sigma_0^{1+2i} = 0 \tag{B.7}$$

$$\bar\Sigma_0^{3+4i} = 0 \tag{B.8}$$

$$\bar\Sigma_0^{1-2i} \propto \frac{\bar\Lambda^{++0}\bar\Lambda^{++0}}{(\bar\Lambda^{++0}\bar\Lambda^{++0})(\Lambda^{--0}\Lambda^{--0}+\Lambda^{--k}\Lambda^{--k})}(\Lambda^{--0}D^{+-0}+\Lambda^{--k}D^{+-k}) \tag{B.9}$$

$$\bar\Sigma_0^{3-4i} \propto \frac{\bar\Lambda^{++0}\bar\Lambda^{++0}}{(\bar\Lambda^{++0}\bar\Lambda^{++0})(\Lambda^{--0}\Lambda^{--0}+\Lambda^{--k}\Lambda^{--k})}(\Lambda^{--0}D^{-+0}+\Lambda^{--k}D^{-+k}) \tag{B.10}$$

$$\bar\Sigma_0^{j} \propto \frac{\bar\Lambda^{++0}\bar\Lambda^{++0}}{(\bar\Lambda^{++0}\bar\Lambda^{++0})(\Lambda^{--0}\Lambda^{--0}+\Lambda^{--k}\Lambda^{--k})}(\Lambda^{--0}D^{--j}-\Lambda^{--j}D^{--0}) \tag{B.11}$$

where $k, j = 1, \ldots, 7$. Therefore, after using the pure spinor constraint, we see that the expression for b_1 contains the same combinations of D_α's as those contained in the expression for $\bar\Sigma_0^i$ ((B.9), (B.10), (B.11)).

C D_α in terms of $\bar{\Sigma}_0^j$

Let us define the quantity:

$$H_\alpha = (\Lambda\Gamma_i)_\alpha \bar{\Sigma}_0^i = \frac{1}{2\eta}(\Gamma_i\Lambda)_\alpha(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})(\Lambda\Gamma^{ab}\Gamma^i D) \tag{C.1}$$

Now we will assume that there exist a matrix $(M^{-1})_\alpha{}^\beta$ such that:

$$D_\alpha = (M^{-1})_\alpha{}^\beta H_\beta \tag{C.2}$$

and let us check that the following ansatz for $(M^{-1})_\alpha{}^\beta$:

$$(M^{-1})_\alpha{}^\beta = 2\delta_\alpha^\beta + \frac{2}{\eta}(\Lambda\Gamma_m)_\alpha(\bar{\Lambda}\Gamma^{mn}\bar{\Lambda})(\Lambda\Gamma_n)^\beta \tag{C.3}$$

is right. This can be seen easily as follows

$$\begin{aligned}
H_\alpha &= \frac{1}{2\eta}(\Gamma_i\Lambda)_\alpha(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})(\Lambda\Gamma^{ab}\Gamma^i M^{-1}H) \\
&= \frac{1}{\eta}(\Gamma_i\Lambda)_\alpha(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})[(\Lambda\Gamma^{ab}\Gamma^i\Gamma^j\Lambda)\bar{\Sigma}_{0\,j} + \frac{1}{\eta}(\Lambda\Gamma^{ab}\Gamma^i\Gamma^m\Lambda)(\bar{\Lambda}\Gamma_{mn}\bar{\Lambda})(\Lambda\Gamma^{nj}\Lambda)\bar{\Sigma}_{0\,j}] \\
&= (\Gamma^j\Lambda)_\alpha\bar{\Sigma}_{0\,j} - \frac{2}{\eta}(\Gamma_i\Lambda)_\alpha(\bar{\Lambda}\Gamma^i{}_b\bar{\Lambda})(\Lambda\Gamma^{bj}\Lambda)\bar{\Sigma}_{0\,j} + \frac{1}{\eta^2}[\eta(\Gamma_i\Lambda)_\alpha(\bar{\Lambda}\Gamma^i{}_n\bar{\Lambda})(\Lambda\Gamma^{nj}\Lambda)\bar{\Sigma}_{0\,j} \\
&\quad -2(\Gamma_i\Lambda)_\alpha(\bar{\Lambda}\Gamma^i{}_b\bar{\Lambda})(\Lambda\Gamma^b{}_m\Lambda)(\bar{\Lambda}\Gamma^m{}_n\bar{\Lambda})(\Lambda\Gamma^{nj}\Lambda)\bar{\Sigma}_{0\,j}] \\
&= (\Gamma^j\Lambda)_\alpha\bar{\Sigma}_{0\,j}
\end{aligned}$$

where the identity (A.3) was used. Therefore we have the relation:

$$D_\alpha = 2(\Lambda\Gamma^c)_\alpha\bar{\Sigma}_{0\,c} + \frac{2}{\eta}(\Lambda\Gamma^m)_\alpha(\bar{\Lambda}\Gamma_{mn}\bar{\Lambda})(\Lambda\Gamma^{nj}\Lambda)\bar{\Sigma}_{0\,j} \tag{C.4}$$

Furthermore, from the constraint $(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})\bar{\Sigma}_0^b = 0$, one immediately concludes that

$$\bar{\Sigma}_0^k = \frac{1}{\eta}(\bar{\Lambda}\Gamma_{ij}\bar{\Lambda})(\Lambda\Gamma^{ijk}H) \tag{C.5}$$

which is the inverse relation between $\bar{\Sigma}^k$ and H_α.

D The D_α's in $\{Q, \bar{\Sigma}^i\}$ are gauge invariant

We will show that the D_α's appearing in (3.10) are invariant under the gauge transformations (3.11). Therefore they are the same D_α's as those contained in the definition of $\bar{\Sigma}^i$. In this appendix and the next ones we have made use of the GAMMA package [21] because of the heavy manipulation of gamma matrix identities which computations demanded. Let us call I^i to the terms containing D_α's explicitly in (3.10). The identities (A.8), (A.9) allow

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us simplify this object:

$$
\begin{aligned}
I^i &= -\frac{1}{\eta}(\bar\Lambda\Gamma_{ab}R)(\Lambda\Gamma^{ab}\Gamma^i D) - \frac{2}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{abcki}\Lambda)(\Lambda\Gamma^d_{\ k}D) \\
&\quad - \frac{2}{3\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_c{}^i R)(\Lambda\Gamma^{abcdk}\Lambda)(\Lambda\Gamma_{dk}D) - \frac{2}{3\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda R)(\Lambda\Gamma^{iabdk}\Lambda)(\Lambda\Gamma_{dk}D) \\
&= -\frac{1}{\eta}(\bar\Lambda\Gamma_{ab}R)(\Lambda\Gamma^{abi}D) - \frac{2}{\eta}(\bar\Lambda\Gamma^{ai}R)(\Lambda\Gamma_a D) - \frac{2}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{abcik}\Lambda)(\Lambda\Gamma_k^{\ d}D) \\
&\quad + \frac{8}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_c{}^i R)(\Lambda\Gamma^{bc}\Lambda)(\Lambda\Gamma^a D) + \frac{4}{\eta}(\bar\Lambda\Gamma^{ci}R)(\Lambda\Gamma_c D) \\
&\quad + \frac{8}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda R)(\Lambda\Gamma^{bi}\Lambda)(\Lambda\Gamma^a D) + \frac{4}{\eta}(\bar\Lambda R)(\Lambda\Gamma^i D)
\end{aligned}
\tag{D.1}
$$

The third term of this expression requires more careful manipulations:

$$
\begin{aligned}
I^{*i} &= -\frac{2}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{abcik}\Lambda)(\Lambda\Gamma_k{}^d D) \\
&= -\frac{4}{\eta^2}(\bar\Lambda\Gamma_{ac}\bar\Lambda)(\bar\Lambda R)(\Lambda\Gamma^{ci}\Lambda)(\Lambda\Gamma^a D) - \frac{2}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)(\Lambda\Gamma^{abd}D) \\
&\quad + \frac{4}{\eta^2}(\bar\Lambda\Gamma_{bc}\bar\Lambda)(\bar\Lambda R)(\Lambda\Gamma^{bi}\Lambda)(\Lambda\Gamma^c D) + \frac{4}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{bi}\Lambda)(\Lambda\Gamma^{acd}D) \\
&\quad - \frac{4}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_c{}^i R)(\Lambda\Gamma^{bc}\Lambda)(\Lambda\Gamma^a D) - \frac{4}{\eta}(\bar\Lambda R)(\Lambda\Gamma^i D) - \frac{4}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{bc}\Lambda)(\Lambda\Gamma^{aid}D) \\
&\quad - \frac{2}{\eta}(\bar\Lambda\Gamma^{ci}R)(\Lambda\Gamma_c D) - \frac{2}{\eta}(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{cid}D)
\end{aligned}
\tag{D.2}
$$

Furthermore, the identity (A.4) allows us to cast this result as

$$
\begin{aligned}
I^{*i} &= -\frac{8}{\eta^2}(\bar\Lambda\Gamma_{ac}\bar\Lambda)(\bar\Lambda R)(\Lambda\Gamma^{ci}\Lambda)(\Lambda\Gamma^a D) - \frac{2}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)(\Lambda\Gamma^{abd}D) \\
&\quad + \frac{4}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{bi}\Lambda)(\Lambda\Gamma^{acd}D) - \frac{4}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_c{}^i R)(\Lambda\Gamma^{bc}\Lambda)(\Lambda\Gamma^a D) \\
&\quad - \frac{4}{\eta}(\bar\Lambda R)(\Lambda\Gamma^i D) + \frac{1}{\eta}(\bar\Lambda\Gamma_{ad}R)(\Lambda\Gamma^{aid}D) + \frac{1}{\eta^2}(\bar\Lambda\Gamma_{ad}\bar\Lambda)(\bar\Lambda\Gamma_{bc}R)(\Lambda\Gamma^{bc}\Lambda)(\Lambda\Gamma^{aid}\Lambda) \\
&\quad - \frac{2}{\eta}(\bar\Lambda\Gamma^{ci}R)(\Lambda\Gamma_c D) - \frac{2}{\eta}(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{cid}D)
\end{aligned}
\tag{D.3}
$$

Plugging this result into (D.1), we find

$$
\begin{aligned}
I^i &= \frac{4}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_c{}^i R)(\Lambda\Gamma^{bc}\Lambda)(\Lambda\Gamma^a D) + \frac{2}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)(\Lambda\Gamma^{abd}D) \\
&\quad + \frac{1}{\eta^2}(\bar\Lambda\Gamma_{ad}\bar\Lambda)(\bar\Lambda\Gamma_{bc}R)(\Lambda\Gamma^{bc}\Lambda)(\Lambda\Gamma^{aid}D)
\end{aligned}
\tag{D.4}
$$

After applying the transformation (3.11) and using the identities (A.2), (A.3), (A.4) one can show that this expression is invariant as mentioned above.

Therefore we can replace the inverse relation (C.4) in (3.10). After doing this for each term in (D.4), we get:

$$
I_1^i = -\frac{2}{\eta}(\bar\Lambda\Gamma^{ci}R)(\Lambda\Gamma_{ck}\Lambda)\bar\Sigma_0^k \tag{D.5}
$$

$$
I_2^i = \frac{4}{\eta}(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)\bar\Sigma_0^d + \frac{2}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{ik}\Lambda)\bar\Sigma_{0\,k} \tag{D.6}
$$

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$$I_3^i = -\frac{2}{\eta}(\bar\Lambda\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)\bar\Sigma_0^i - \frac{2}{\eta^2}(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{cd}\Lambda)(\bar\Lambda\Gamma^{in}\bar\Lambda)(\Lambda\Gamma^{nk}\Lambda)\bar\Sigma_{0\,k} \tag{D.7}$$

Replacing these expressions in (D.4) and putting all together in (3.10) we obtain

$$\begin{aligned}\{Q,\bar\Sigma^i\} =& -P^i - \frac{2}{\eta}[(\bar\Lambda\Gamma^{mb}\bar\Lambda)(\Lambda\Gamma_b{}^i\Lambda) - (\bar\Lambda\Gamma^{ib}\bar\Lambda)(\Lambda\Gamma_b{}^m\Lambda)]P_m + \frac{4}{\eta}(\bar\Lambda\Gamma_{mn}R)(\Lambda\Gamma^{mn}\Lambda)\bar\Sigma^i\\ &-\frac{2}{\eta}(\bar\Lambda\Gamma_{mn}R)(\Lambda\Gamma^{mn}\Lambda)\bar\Sigma_0^i - \frac{2}{\eta}(\bar\Lambda\Gamma^{ci}R)(\Lambda\Gamma_{ck}\Lambda)\bar\Sigma_0^k + \frac{4}{\eta}(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)\bar\Sigma_0^d\\ &+\frac{2}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{ik}\Lambda)\bar\Sigma_{0k} - \frac{2}{\eta}(\bar\Lambda\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)\bar\Sigma_0^i - \frac{2}{\eta^2}(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{cd}\Lambda)(\bar\Lambda\Gamma^{in}\bar\Lambda)(\Lambda\Gamma^{nk}\Lambda)\bar\Sigma_{0k}\\ &-\frac{4}{\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{abcki}\Lambda)N^d_{\ k} - \frac{4}{3\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^{abcdk}\Lambda)N_{dk}\\ &-\frac{4}{3\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda R)(\Lambda\Gamma^{iabdk}\Lambda)N_{dk} - \frac{2}{3\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(RR)(\Lambda\Gamma^{iabdk}\Lambda)N_{dk}\end{aligned} \tag{D.8}$$

E Cancellation of all of the N_{ab} contributions in the equation (3.12)

We will show this cancellation in two steps. First we will simplify the expression depending explicitly on $\bar\Sigma_0^i$ and then simplify the expression depending explicitly on N_{ab}. Finally we will see that these two expressions identically cancel out. We start with the following equation

$$\begin{aligned}J^i =& \frac{4}{\eta}(\bar\Lambda\Gamma^{mn}R)(\Lambda\Gamma_{mn}\Lambda)(\bar\Sigma^i - \bar\Sigma_0^i) - \frac{2}{\eta}(\bar\Lambda\Gamma^{ci}R)(\Lambda\Gamma_{ck}\Lambda)\bar\Sigma_0^k + \frac{4}{\eta}(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)\bar\Sigma_0^d\\ &+\frac{2}{\eta}(\bar\Lambda R)(\Lambda\Gamma^{ik}\Lambda)\bar\Sigma_{0\,k} - \frac{2}{\eta^2}(\bar\Lambda\Gamma^{cd}R)(\Lambda\Gamma_{cd}\Lambda)(\bar\Lambda\Gamma^{in}\bar\Lambda)(\Lambda\Gamma_{nk}\Lambda)\bar\Sigma_0^k\end{aligned} \tag{E.1}$$

One can show that the term proportional to $\bar\Lambda R$ can be cast as

$$\begin{aligned}J_1^i =& \frac{8}{\eta^3}(\Lambda\Gamma^{bi}\Lambda)(\bar\Lambda R)(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{ck}R)(\Lambda\Gamma^{ck}\Lambda)(\Lambda\Gamma^a W)\\ &-\frac{24}{\eta^2}(\Lambda\Gamma^{ik}\Lambda)(\bar\Lambda R)(\bar\Lambda\Gamma_{ck}R)(\Lambda\Gamma^c W)\\ &-\frac{16}{\eta^2}(\bar\Lambda\Gamma^{mn}R)(\Lambda\Gamma_{mn}\Lambda)(\bar\Lambda R)(\Lambda\Gamma^i W)\end{aligned} \tag{E.2}$$

The use of the identity (A.9) allows us to write the term proportional to $(\bar\Lambda\Gamma^{ci}R)$ in the form

$$\begin{aligned}J_2^i =& -\frac{16}{\eta^3}(\bar\Lambda\Gamma^{mn}R)(\Lambda\Gamma_{mn}\Lambda)(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^{bc}\Lambda)(\Lambda\Gamma^a W) - \frac{12}{\eta^2}(\bar\Lambda\Gamma^{mn}R)(\Lambda\Gamma_{mn}\Lambda)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^c W)\\ &-\frac{8}{\eta^2}(\bar\Lambda\Gamma^{ci}R)(\Lambda\Gamma_{ck}\Lambda)(\bar\Lambda\Gamma_f{}^kR)(\Lambda\Gamma^f W)\end{aligned} \tag{E.3}$$

Finally, with a little algebra and the use of the identities (A.4), (A.8) one gets the following result

$$\begin{aligned}J^i =& -\frac{4}{\eta^2}(\bar\Lambda\Gamma^{mn}R)(\Lambda\Gamma_{mn}\Lambda)(\bar\Lambda\Gamma_{bd}R)(\Lambda\Gamma^{bdi}W) + \frac{4}{\eta^2}(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)(\bar\Lambda\Gamma_{ef}R)(\Lambda\Gamma^{efd}W)\\ &-\frac{4}{\eta^2}(\bar\Lambda\Gamma^{mn}R)(\Lambda\Gamma_{mn}\Lambda)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^c W) - \frac{8}{\eta^2}(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^{ck}\Lambda)(\bar\Lambda\Gamma_{fk}R)(\Lambda\Gamma^f W)\end{aligned} \tag{E.4}$$

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Now we will simplify the expressions containing N_{mn} explicitly:

$$S^i = -\frac{4}{\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{abcki}\Lambda)N^d{}_k - \frac{4}{3\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^{abcdk}\Lambda)N_{dk}$$
$$-\frac{4}{3\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda R)(\Lambda\Gamma^{iabdk}\Lambda)N_{dk} - \frac{2}{3\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(RR)(\Lambda\Gamma^{iabdk}\Lambda)N_{dk} \quad \text{(E.5)}$$

The first term in (E.5) can be written as follows

$$S_1^i = -\frac{8}{\eta^2}(\bar\Lambda\Gamma_{ac}R)(\bar\Lambda R)(\Lambda\Gamma^{ci}\Lambda)(\Lambda\Gamma^aW) - \frac{4}{\eta^2}(\bar\Lambda\Gamma_{ac}\bar\Lambda)(RR)(\Lambda\Gamma^{ci}\Lambda)(\Lambda\Gamma^aW)$$
$$-\frac{4}{\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)(\Lambda\Gamma^{abd}W) - \frac{8}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^{ca}\Lambda)(\Lambda\Gamma^bW)$$
$$+\frac{8}{\eta^2}(\bar\Lambda\Gamma_{ca}R)(\bar\Lambda R)(\Lambda\Gamma^{ca}\Lambda)(\Lambda\Gamma^iW) - \frac{4}{\eta}(RR)(\Lambda\Gamma^iW)$$
$$-\frac{4}{\eta^2}(\bar\Lambda\Gamma^{ab}R)(\Lambda\Gamma_{ab}\Lambda)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^cW) + \frac{4}{\eta^2}(\bar\Lambda\Gamma^{ab}R)(\Lambda\Gamma_{ab}\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{cdi}W)$$
$$-\frac{8}{\eta^2}(\bar\Lambda\Gamma_{ac}R)(\bar\Lambda R)(\Lambda\Gamma^{ia}\Lambda)(\Lambda\Gamma^cW) - \frac{4}{\eta^2}(\bar\Lambda\Gamma_{ac}\bar\Lambda)(RR)(\Lambda\Gamma^{ia}\Lambda)(\Lambda\Gamma^cW)$$
$$+\frac{8}{\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{ia}\Lambda)(\Lambda\Gamma^{bcd}W) \quad \text{(E.6)}$$

The last three terms in (E.5) can be put into the form:

$$S_2^i = \frac{16}{\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^{bc}\Lambda)(\Lambda\Gamma^aW) + \frac{8}{\eta^2}(\bar\Lambda\Gamma_{ab}R)(\Lambda\Gamma^{ab}\Lambda)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^cW)$$
$$+\frac{8}{\eta^2}(\bar\Lambda\Gamma_{ab}R)(\Lambda\Gamma^{ab}\Lambda)(\bar\Lambda R)(\Lambda\Gamma^iW) + \frac{16}{\eta^2}(\bar\Lambda\Gamma_{ab}R)(\bar\Lambda R)(\Lambda\Gamma^{bi}\Lambda)(\Lambda\Gamma^aW)$$
$$+\frac{4}{\eta}(RR)(\Lambda\Gamma^iW) + \frac{8}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(RR)(\Lambda\Gamma^{bi}\Lambda)(\Lambda\Gamma^aW) \quad \text{(E.7)}$$

When summing $S_1^1 + S_2^i$ we obtain

$$S^i = \frac{4}{\eta^2}(\bar\Lambda\Gamma^{mn}R)(\Lambda\Gamma_{mn}\Lambda)(\bar\Lambda\Gamma_{bd}R)(\Lambda\Gamma^{bdi}W) - \frac{4}{\eta^2}(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{ci}\Lambda)(\bar\Lambda\Gamma_{ef}R)(\Lambda\Gamma^{efd}W)$$
$$+\frac{4}{\eta^2}(\bar\Lambda\Gamma^{mn}R)(\Lambda\Gamma_{mn}\Lambda)(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^cW) + \frac{8}{\eta^2}(\bar\Lambda\Gamma_c{}^iR)(\Lambda\Gamma^{ck}\Lambda)(\bar\Lambda\Gamma_{fk}R)(\Lambda\Gamma^fW) \quad \text{(E.8)}$$

Thus we have a full cancellation $J^i + S^i = 0$.

F Calculation of $\{\bar\Sigma^i, \bar\Sigma^j\}$

The object $\bar\Sigma^i$ has a part depending on D_α and other part depending on N_{mn}, as it can be seen in (3.9). The part depending on N_{mn} will be called $\bar\Sigma_1^i$ and as before we use $\bar\Sigma_0^i$ to denote the part depending on D_α. Therefore

$$\bar\Sigma^i = \bar\Sigma_0^i + \bar\Sigma_1^i \quad \text{(F.1)}$$

It is easy to see that $\{\bar{\Sigma}^i_0, \bar{\Sigma}^j_0\} = 0$. To compute the anticommutator $\{\bar{\Sigma}^i_0, \bar{\Sigma}^j_1\}$ we write $\bar{\Sigma}^j_1$ in the more convenient way:

$$\begin{aligned}\bar{\Sigma}^j_1 &= -\frac{2}{\eta^2}(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{cj}\Lambda)(\Lambda\Gamma^{abd}W) - \frac{1}{\eta}(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{cdj}W)\\ &+\frac{2}{\eta^2}(\bar{\Lambda}\Gamma^{bj}\bar{\Lambda})(\bar{\Lambda}\Gamma^{ca}R)(\Lambda\Gamma_{ca}\Lambda)(\Lambda\Gamma_bW) - \frac{4}{\eta^2}(\bar{\Lambda}\Gamma^{cj}\bar{\Lambda})(\bar{\Lambda}\Gamma^{ba}R)(\Lambda\Gamma_{ca}\Lambda)(\Lambda\Gamma_bW)\\ &+\frac{1}{\eta^2}(\bar{\Lambda}\Gamma_{bd}\bar{\Lambda})(\bar{\Lambda}\Gamma_{ac}R)(\Lambda\Gamma^{ac}\Lambda)(\Lambda\Gamma^{bdj}W)\end{aligned} \tag{F.2}$$

The result is

$$\begin{aligned}\{\bar{\Sigma}^i_0, \bar{\Sigma}^j_1\} &= -\frac{4}{\eta^2}(\bar{\Lambda}\Gamma_{cj}R)(\bar{\Lambda}\Gamma^{in}\bar{\Lambda})(\Lambda\Gamma^{cn}D) + \frac{2}{\eta^2}(\bar{\Lambda}\Gamma^{ij}R)(\bar{\Lambda}\Gamma_{mn}\bar{\Lambda})(\Lambda\Gamma^{mn}D)\\ &+\frac{2}{\eta^2}(\bar{\Lambda}\Gamma_c{}^jR)(\bar{\Lambda}\Gamma_{mn}\bar{\Lambda})(\Lambda\Gamma^{cimn}D) - \frac{4}{\eta^2}(\bar{\Lambda}R)(\bar{\Lambda}\Gamma^i{}_n\bar{\Lambda})(\Lambda\Gamma^{jn}D)\\ &+\frac{2}{\eta^2}\eta^{ij}(\bar{\Lambda}R)(\bar{\Lambda}\Gamma_{mn}\bar{\Lambda})(\Lambda\Gamma^{mn}D) - \frac{2}{\eta}(\bar{\Lambda}R)(\bar{\Lambda}\Gamma_{mn}\bar{\Lambda})(\Lambda\Gamma^{ijmn}D)\end{aligned} \tag{F.3}$$

Analogously, one can show that $\{\bar{\Sigma}^i_1, \bar{\Sigma}^j_1\}$ depends linearly and quadratically on R_α. This allows us to find the R_α-dependence of $\{b, b\}$ which turns out to be of the form:

$$\{b, b\} = R^\alpha f^{(1)}_\alpha + \ldots + R^\alpha R^\beta R^\delta R^\sigma R^\rho R^\lambda f^{(6)}_{\alpha\beta\delta\sigma\rho\lambda} \tag{F.4}$$

where $f^{(i)}_{\alpha_1\ldots\alpha_i}$ for $i = 1, \ldots, 6$ are functions of pure spinor variables $\Lambda^\alpha, \bar{\Lambda}_\alpha, W_\alpha$ and the fermionic constraints D_α.

This can be used to check that $\{b, b\} = Q\Omega$ where Ω is an arbitrary function of pure spinor variables and the constraints D_α. To see this let us expand Ω in terms of R^α:

$$\Omega = \Omega^{(0)} + R^\alpha\Omega^{(1)}_\alpha + R^{\alpha\beta}\Omega^{(2)}_{\alpha\beta} + \ldots + R^{\alpha_1\ldots\alpha_{23}}\Omega^{(23)}_{\alpha_1\ldots\alpha_{23}} \tag{F.5}$$

Thus the action of the BRST operator $Q = Q_0 + R^\alpha\bar{W}_\alpha$ on Ω gives us

$$Q\Omega = Q_0\Omega^{(0)} + R^\alpha\left(\frac{\partial}{\partial\bar{\Lambda}^\alpha}\Omega^{(0)} + Q_0\Omega^{(1)}_\alpha\right) + R^\alpha R^\beta\left(\frac{\partial}{\partial\bar{\Lambda}^\alpha}\Omega^{(1)}_\beta + Q_0\Omega^{(2)}_{\alpha\beta}\right) + \ldots \tag{F.6}$$

The comparison of this result with the equation (F.4) determines the functions $\Omega^{(k)}$ for $k = 1, \ldots, 23$:

$$0 = Q_0\Omega^{(0)} \tag{F.7}$$

$$f^{(1)}_\alpha = \frac{\partial}{\partial\bar{\Lambda}^\alpha}\Omega^{(0)} + Q_0\Omega^{(1)}_\alpha \tag{F.8}$$

$$f^{(2)}_{\alpha\beta} = \frac{\partial}{\partial\bar{\Lambda}^\alpha}\Omega^{(1)}_\beta + Q_0\Omega^{(2)}_{\alpha\beta} \tag{F.9}$$

$$\vdots$$

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Therefore if we make the following definitions:

$$\Omega^{(0)} = \bar{\Lambda}^{\alpha} f^{(1)}_{\alpha} \tag{F.10}$$

$$\Omega^{(1)}_{\beta} = \bar{\Lambda}^{\alpha} f^{(2)}_{\alpha\beta} \tag{F.11}$$

$$\Omega^{(2)}_{\beta\delta} = \bar{\Lambda}^{\alpha} f^{(3)}_{\alpha\beta\delta} \tag{F.12}$$

$$\vdots$$

$$\Omega^{(5)}_{\beta\delta\sigma\rho\lambda} = \bar{\Lambda}^{\alpha} f^{(6)}_{\alpha\beta\delta\sigma\rho\lambda} \tag{F.13}$$

$$\Omega^{(6)}_{\beta\delta\sigma\rho\lambda\gamma} = 0 \tag{F.14}$$

$$\vdots$$

$$\Omega^{(23)} = 0 \tag{F.15}$$

the equations above are automatically solved.

G Expanding the simplified $D = 11$ b-ghost

In this appendix we will show explicitly the terms contained in $O(\bar{\Sigma}^2)$ in the expression for the simplified $D = 11$ b-ghost. We will work with the expression

$$b_{\text{simpl}} = P^i\bar{\Sigma}_i - \frac{4}{\eta^2}(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{aj}\Lambda)\bar{\Sigma}_j\left[(\Lambda\Gamma^{bd}\Lambda)\bar{\Sigma}^c + \frac{1}{\eta}(\Lambda\Gamma^{bd}\Lambda)(\bar{\Lambda}\Gamma^{cs}\bar{\Lambda})(\Lambda\Gamma_{sk}\Lambda)\bar{\Sigma}^k\right] \tag{G.1}$$

Therefore,

$$\begin{aligned} b_{\text{simpl}} = {} & P^i\bar{\Sigma}_i - \frac{4}{\eta^2}(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{bd}\Lambda)(\Lambda\Gamma^{a}{}_{j}\Lambda)\Big[\bar{\Sigma}^j_0 + \frac{2}{\eta^2}(\bar{\Lambda}\Gamma_{ef}\bar{\Lambda})(\bar{\Lambda}\Gamma_{gh}R)(\Lambda\Gamma^{efgij}\Lambda)N^h{}_i \\ & + \frac{2}{3\eta^2}(\bar{\Lambda}\Gamma_{ef}\bar{\Lambda})(\bar{\Lambda}\Gamma_g{}^jR)(\Lambda\Gamma^{efghi}\Lambda)N_{hi} - \frac{2}{3\eta^2}(\bar{\Lambda}\Gamma_{eg}\bar{\Lambda})(\bar{\Lambda}R)(\Lambda\Gamma^{ejghi}\Lambda)N_{hi}\Big]\times \\ & \times\Big\{\bar{\Sigma}^c_0 + \frac{2}{\eta^2}(\bar{\Lambda}\Gamma_{lm}\bar{\Lambda})(\bar{\Lambda}\Gamma_{np}R)(\Lambda\Gamma^{lmnqc}\Lambda)N^p{}_q + \frac{2}{3\eta^2}(\bar{\Lambda}\Gamma_{lm}\bar{\Lambda})(\bar{\Lambda}\Gamma_n{}^cR)(\Lambda\Gamma^{lmnpq}\Lambda)N_{pq} \\ & - \frac{2}{3\eta^2}(\bar{\Lambda}\Gamma_{ln}\bar{\Lambda})(\bar{\Lambda}R)(\Lambda\Gamma^{lcnpq}\Lambda)N_{pq} + \frac{1}{\eta}(\bar{\Lambda}\Gamma^c{}_s\bar{\Lambda})(\Lambda\Gamma^s{}_k\Lambda)\Big[\bar{\Sigma}^k_0 + \frac{2}{\eta^2}(\bar{\Lambda}\Gamma_{rt}\bar{\Lambda})(\bar{\Lambda}\Gamma_{uw}R)(\Lambda\Gamma^{rtuyk}\Lambda)N^w{}_y \\ & + \frac{2}{3\eta^2}(\bar{\Lambda}\Gamma_{rt}\bar{\Lambda})(\bar{\Lambda}\Gamma_u{}^kR)(\Lambda\Gamma^{rtuwy}\Lambda)N_{wy} - \frac{2}{3\eta^2}(\bar{\Lambda}\Gamma_{ru}\bar{\Lambda})(\bar{\Lambda}R)(\Lambda\Gamma^{rkuwy}\Lambda)N_{wy}\Big]\Big\} \end{aligned} \tag{G.2}$$

The contributions proportional to D^2 are:

$$\begin{aligned} b^{(2)}_{simp} &= -\frac{4}{\eta^2}(\bar{\Lambda}\Gamma_{ab}\bar{\Lambda})(\bar{\Lambda}\Gamma_{cd}R)(\Lambda\Gamma^{aj}\Lambda)\bar{\Sigma}_{0\,j}\left[(\Lambda\Gamma^{bd}\Lambda)\bar{\Sigma}^c_0 + \frac{1}{\eta}(\Lambda\Gamma^{bd}\Lambda)(\bar{\Lambda}\Gamma^{cs}\bar{\Lambda})(\Lambda\Gamma_{sk}\Lambda)\bar{\Sigma}^k_0\right] \\ &= \frac{1}{\eta^2}L^{(1)}_{mc,rs}(\Lambda\Gamma^m D)(\Lambda\Gamma^{crs}D) \end{aligned}$$

where the identities (A.3), (A.4) were used.

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The terms proportional to η^{-3}, η^{-4} can be found by using the identities (A.2), (A.4) and (A.7). The result is

$$\begin{aligned}
b^{(3)}_{simp} &= \frac{4}{\eta^3}(\bar\Lambda\Gamma_{lm}\bar\Lambda)(\bar\Lambda\Gamma_{cx}R)(\bar\Lambda\Gamma_n{}^pR)(\Lambda\Gamma^{lmcnq}\Lambda)(\Lambda\Gamma^xD)N_{qp}\\
&\quad-\frac{2}{\eta^3}(\bar\Lambda\Gamma_{lm}\bar\Lambda)(\bar\Lambda\Gamma_{nx}R)(\bar\Lambda R)(\Lambda\Gamma^{lmnpq}\Lambda)(\Lambda\Gamma^xD)N_{pq}\\
&\quad-\frac{1}{3\eta^3}(\bar\Lambda\Gamma_{ef}\bar\Lambda)(\bar\Lambda\Gamma_{gc}R)(\bar\Lambda\Gamma_{mn}R)(\Lambda\Gamma^{efghi}\Lambda)N_{hi}(\Lambda\Gamma^{cmn}D) \qquad \text{(G.3)}\\
b^{(4)}_{simp} &= \frac{4}{3\eta^4}(\bar\Lambda\Gamma_{ef}\bar\Lambda)(\bar\Lambda\Gamma_{gc}R)(\bar\Lambda\Gamma_{lm}R)(\bar\Lambda\Gamma_{np}R)(\Lambda\Gamma^{efghi}\Lambda)N_{hi}(\Lambda\Gamma^{clmnq}\Lambda)N_{qp}\\
&\quad-\frac{2}{3\eta^4}(\bar\Lambda\Gamma_{ef}\bar\Lambda)(\bar\Lambda\Gamma_{gm}R)(\bar\Lambda\Gamma_{ln}R)(\bar\Lambda R)(\Lambda\Gamma^{efghi}\Lambda)N_{hi}(\Lambda\Gamma^{lnmpq}\Lambda)N_{pq} \qquad \text{(G.4)}
\end{aligned}$$

In order to compare the two ways for the b-ghost, we should move all of the N_{ab}'s at the end of the expressions showed above. After doing this one concludes that $b^{(3)}_{\text{simpl}}$ changes by the factor $-\frac{1}{3\eta^3}(\bar\Lambda\Gamma_{ef}\bar\Lambda)(\bar\Lambda\Gamma_{gc}R)(\bar\Lambda\Gamma_{mn}R)(\Lambda\Gamma^{efghi}\Lambda)(\Lambda\Gamma^{chimn}D)$, and $b^{(4)}_{\text{simpl}}$ does not receive any contribution. Therefore, the simplified b-ghost has the following form:

$$\begin{aligned}
b_{\text{simpl}} &= P^i\Bigg[\frac{1}{2}\eta^{-1}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\Lambda\Gamma^{ab}\Gamma_iD)+\eta^{-2}L^{(1)}_{ab,cd}\Bigg[2(\Lambda\Gamma^{abc}{}_{ki}\Lambda)N^{dk}\\
&\quad+\frac{2}{3}(\eta^b{}_p\eta^d{}_i-\eta^{bd}\eta_{pi})(\Lambda\Gamma^{apcqj}\Lambda)N_{qj}\Bigg]\Bigg]+\frac{1}{\eta^2}L^{(1)}_{mc,rs}(\Lambda\Gamma^mD)(\Lambda\Gamma^{crs}D)+\\
&\quad+\frac{4}{\eta^3}(\bar\Lambda\Gamma_{lm}\bar\Lambda)(\bar\Lambda\Gamma_{cx}R)(\bar\Lambda\Gamma_n{}^pR)(\Lambda\Gamma^{lmcnq}\Lambda)(\Lambda\Gamma^xD)N_{qp}\\
&\quad-\frac{2}{\eta^3}(\bar\Lambda\Gamma_{lm}\bar\Lambda)(\bar\Lambda\Gamma_{nx}R)(\bar\Lambda R)(\Lambda\Gamma^{lmnpq}\Lambda)(\Lambda\Gamma^xD)N_{pq}\\
&\quad-\frac{1}{3\eta^3}(\bar\Lambda\Gamma_{ef}\bar\Lambda)(\bar\Lambda\Gamma_{gc}R)(\bar\Lambda\Gamma_{mn}R)(\Lambda\Gamma^{efghi}\Lambda)(\Lambda\Gamma^{cmn}D)N_{hi}\\
&\quad+\frac{2}{3\eta^4}(\bar\Lambda\Gamma_{ef}\bar\Lambda)(\bar\Lambda\Gamma_{gc}R)(\bar\Lambda\Gamma_{lm}R)(\bar\Lambda\Gamma_{np}R)(\Lambda\Gamma^{efghi}\Lambda)(\Lambda\Gamma^{clmnq}\Lambda)\{N_{hi},N_{qp}\}\\
&\quad-\frac{1}{3\eta^4}(\bar\Lambda\Gamma_{ef}\bar\Lambda)(\bar\Lambda\Gamma_{gm}R)(\bar\Lambda\Gamma_{ln}R)(\bar\Lambda R)(\Lambda\Gamma^{efghi}\Lambda)(\Lambda\Gamma^{lnmpq}\Lambda)\{N_{hi},N_{pq}\}\\
&\quad-\frac{1}{3\eta^3}(\bar\Lambda\Gamma_{ef}\bar\Lambda)(\bar\Lambda\Gamma_{gc}R)(\bar\Lambda\Gamma_{mn}R)(\Lambda\Gamma^{efghi}\Lambda)(\Lambda\Gamma^{chimn}D) \qquad \text{(G.5)}
\end{aligned}$$

where we have written the anticommutator instead of the ordinary product of N_{ab}'s after using the relation $2N_{hi}N_{pq} = [N_{hi},N_{pq}] + \{N_{hi},N_{pq}\}$ and the fact that the commutator contribution vanishes because of the identity (A.7). We can compare this result to the expansion of the b-ghost in (3.2)

$$\begin{aligned}
b &= \frac{1}{2}\eta^{-1}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\Lambda\Gamma^{ab}\Gamma^iD)P_i+\eta^{-2}L^{(1)}_{ab,cd}\Bigg[(\Lambda\Gamma^aD)(\Lambda\Gamma^{bcd}D)+2(\Lambda\Gamma^{abc}{}_{ij}\Lambda)N^{di}P^j\\
&\quad\times\frac{2}{3}(\eta^b{}_p\eta^d{}_q-\eta^{bd}\eta_{pq})(\Lambda\Gamma^{apcij}\Lambda)N_{ij}P^q\Bigg]-\frac{1}{3}\eta^{-3}L^{(2)}_{ab,cd,ef}\Big\{(\Lambda\Gamma^{abcij}\Lambda)(\Lambda\Gamma^{def}D)N_{ij}
\end{aligned}$$

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$$- 12[(\Lambda\Gamma^{abcei}\Lambda)\eta^{fj} - \frac{2}{3}\eta^{f[a}(\Lambda\Gamma^{bce]ij}\Lambda](\Lambda\Gamma^{d}D)N_{ij}\Big\}$$

$$+ \frac{4}{3}\eta^{-4}L^{(3)}_{ab,cd,ef,gh}(\Lambda\Gamma^{abcij}\Lambda)[(\Lambda\Gamma^{defgk}\Lambda)\eta^{hl} - \frac{2}{3}\eta^{h[d}(\Lambda\Gamma^{efg]kl}\Lambda)]\{N_{ij}, N_{kl}\} \qquad \text{(G.6)}$$

The quadratic term in D_α is easy to obtain using the identity (A.10)

$$b^{(2)} = \frac{1}{\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\Lambda\Gamma^{a}D)(\Lambda\Gamma^{bcd}D) \qquad \text{(G.7)}$$

With a little algebra the terms proportional to η^{-3}, η^{-4} can be found. The result is

$$\begin{aligned} b^{(3)} &= \frac{4}{\eta^3}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\bar\Lambda\Gamma_{e}{}^{j}R)(\Lambda\Gamma^{abcei}\Lambda)(\Lambda\Gamma^{d}D)N_{ij} \\ &\quad -\frac{2}{\eta^3}(\bar\Lambda\Gamma_{bc}\bar\Lambda)(\bar\Lambda\Gamma_{ed}R)(\bar\Lambda R)(\Lambda\Gamma^{bceij}\Lambda)(\Lambda\Gamma^{d}D)N_{ij} \\ &\quad -\frac{1}{3\eta^2}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\bar\Lambda\Gamma_{ef}R)(\Lambda\Gamma^{abcpq}\Lambda)(\Lambda\Gamma^{def}D)N_{pq} \qquad \text{(G.8)} \\ b^{(4)} &= \frac{4}{3\eta^4}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cd}R)(\bar\Lambda\Gamma_{ef}R)(\bar\Lambda\Gamma_{gh}R)(\Lambda\Gamma^{abcij}\Lambda)(\Lambda\Gamma^{defgk}\Lambda)\eta^{hl}\{N_{ij}, N_{kl}\} \\ &\quad -\frac{2}{3\eta^4}(\bar\Lambda\Gamma_{ab}\bar\Lambda)(\bar\Lambda\Gamma_{cg}R)(\bar\Lambda\Gamma_{ef}\bar\Lambda)(\bar\Lambda R)(\Lambda\Gamma^{abcij}\Lambda)(\Lambda\Gamma^{efgkl}\Lambda)\{N_{ij}, N_{kl}\} \qquad \text{(G.9)} \end{aligned}$$

which should be compared with the analog expressions corresponding to b_{simpl}, equations (G.3), (G.4).

The differences between this equation (G.6) and (G.5) is the non-zero extra term proportional to $(\Lambda\Gamma^{chimn}D)$. This might be related to normal-ordering ambiguities.

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